Exercise 1: `f(x) = sin(x)`

Naive:

```
f'(x) ~ -0.9190600938410731, h = 1.0E-4
f'(x) ~ -0.9190600949854355, h = 5.0E-5
f'(x) ~ -0.9190600952724282, h = 2.5E-5
f'(x) ~ -0.9190600953434824, h = 1.25E-5
f'(x) ~ -0.9190600953967731, h = 6.25E-6
```

Richardson's:

```
D(0, 0) = -0.9190600938410731
D(1, 0) = -0.9190600949854355
D(1, 1) = -0.9190600953668896
D(2, 0) = -0.9190600952724282
D(2, 1) = -0.9190600953680924
D(2, 2) = -0.9190600953681726
D(3, 0) = -0.9190600953434824
D(3, 1) = -0.9190600953671672
D(3, 2) = -0.9190600953671054
D(3, 3) = -0.9190600953670884
D(4, 0) = -0.9190600953967731
D(4, 1) = -0.9190600954145367
D(4, 2) = -0.9190600954176946
D(4, 3) = -0.9190600954184976
D(4, 4) = -0.9190600954186993
D(5, 0) = -0.9190600954056549
D(5, 1) = -0.9190600954086156
D(5, 2) = -0.9190600954082208
D(5, 3) = -0.9190600954080704
D(5, 4) = -0.9190600954080296
D(5, 5) = -0.9190600954080191
actual: -0.9190600953708519
Best: D(2, 2) = -0.9190600953681726
```

Exercise 1: `f(x) = 1 + ln(x)`

Naive:

```
f'(x) ~ 0.28195223736648956, h = 1.0E-4
f'(x) ~ 0.28195223730875796, h = 5.0E-5
f'(x) ~ 0.2819522372998762, h = 2.5E-5
f'(x) ~ 0.2819522372909944, h = 1.25E-5
f'(x) ~ 0.28195223730875796, h = 6.25E-6
```

Richardson's:

```
D(0, 0) = 0.28195223736648956
D(1, 0) = 0.28195223730875796
D(1, 1) = 0.2819522372895141
D(2, 0) = 0.2819522372998762
D(2, 1) = 0.2819522372969156
D(2, 2) = 0.28195223729740904
D(3, 0) = 0.2819522372909944
D(3, 1) = 0.2819522372880338
D(3, 2) = 0.2819522372874417
D(3, 3) = 0.28195223728728347
D(4, 0) = 0.28195223730875796
D(4, 1) = 0.2819522373146 7917
D(4, 2) = 0.2819522373164555
D(4, 3) = 0.28195223731691604
D(4, 4) = 0.28195223731703223
D(5, 0) = 0.2819522372732308
D(5, 1) = 0.28195223726138846
D(5, 2) = 0.28195223725783575
D(5, 3) = 0.28195223725690527
D(5, 4) = 0.28195223725666996
D(5, 5) = 0.28195223725661095
actual: 0.2819522372910029
Best: D(3, 0) = 0.2819522372909944
```

Exercise 1: `f(x) = x^2 - 3x + 5`

Naive:

```
f'(x) ~ 4.09340000000924, h = 1.0E-4
f'(x) ~ 4.093399999991476, h = 5.0E-5
f'(x) ~ 4.0933999999737125, h = 2.5E-5
f'(x) ~ 4.0933999999737125, h = 1.25E-5
f'(x) ~ 4.093400000044767, h = 6.25E-6
```

Richardson's:

```
D(0, 0) = 4.09340000000924
D(1, 0) = 4.093399999991476
D(1, 1) = 4.093399999985555
D(2, 0) = 4.0933999999737125
D(2, 1) = 4.093399999967791
D(2, 2) = 4.093399999966607
D(3, 0) = 4.0933999999737125
D(3, 1) = 4.0933999999737125
D(3, 2) = 4.093399999974107
D(3, 3) = 4.093399999974226
D(4, 0) = 4.093400000044767
D(4, 1) = 4.093400000068452
D(4, 2) = 4.093400000074768
D(4, 3) = 4.093400000763655
D(4, 4) = 4.093400000076766
D(5, 0) = 4.093400000044767
D(5, 1) = 4.093400000044767
D(5, 2) = 4.093400000043188
D(5, 3) = 4.093400000042687
D(5, 4) = 4.093400000042554
D(5, 5) = 4.093400000042521
actual: 4.0934
Best: D(1, 0) = 4.093399999991476
```

Exercise 2: `f(x) = sin(x)`

Trapezoid

Actual: 0.2566401204049135

No. of Trapezoids: 5
A0 = 0.7261274462744367
A1 = 0.5957396010798638
A2 = 0.10398410791785301
A3 = -0.45084674977616795
A4 = -0.7322000188306907
A = 0.2428043866652949
%err = 5.391103198435731

No. of Trapezoids: 10
A0 = 0.36538414295927135
A1 = 0.39185947217333106
A2 = 0.35646880693955571
A3 = 0.26479955512821096
A4 = 0.13132427597626645
A5 = -0.022884218793392627
A6 = -0.17347979859390988
A7 = -0.2966867326712613
A8 = -0.3730533552094104
A9 = -0.3905230556593206
A = 0.25320909224934224
%err = 1.3369024882617657

No. of Trapezoids: 20
A0 = 0.17735100707751228
A1 = 0.19174888159556866
A2 = 0.19850233330299655
A3 = 0.19734212339197005
A4 = 0.18831450577038772
A5 = 0.17177938306452722
A6 = 0.14839595843707412
A7 = 0.11909645523726152
A8 = 0.0850489521977369
A9 = 0.04761081582157715
A10 = 0.008274586463228624
A11 = -0.031391524545441266
A12 = -0.06980615453216854
A13 = -0.10543783342375733
A14 = -0.1368660386250649
A15 = -0.16283782677215175
A16 = -0.1823177846303105
A17 = -0.1945293077522981
A18 = -0.1989855612469305
A19 = -0.19550888834989785
A = 0.25578408248181983
%err = 0.33355576740810294

Romberg:

```
R(0, 0) = -0.2349065797104839
R(1, 0) = 0.16478672626449248
R(1, 1) = 0.2980178282561513
R(2, 0) = 0.23488829464999975
R(2, 1) = 0.2582554841118355
R(2, 2) = 0.25560466116888114
R(3, 0) = 0.2512710400016468
R(3, 1) = 0.2567319551188625
R(3, 2) = 0.25663038651933096
R(3, 3) = 0.2566466678741
R(4, 0) = 0.25530205867416406
R(4, 1) = 0.25664573156500314
R(4, 2) = 0.2566399833280792
R(4, 3) = 0.2566401356583768
R(4, 4) = 0.25664011004184456
R(5, 0) = 0.256305866526472
R(5, 1) = 0.25664046914390803
R(5, 2) = 0.25664011831583505
R(5, 3) = 0.2566401204584978
R(5, 4) = 0.25664012039889045
R(5, 5) = 0.2566401204090146
Best: R(5, 5) = 0.2566401204090146
```

Exercise 2: `f(x) = f(x) = 1 + ln(x)`

Trapezoid

```
Actual: 8.047189562170502

No. of Trapezoids: 5
A0 = 1.0351146659608477
A1 = 1.4173192439718223
A2 = 1.6717147506598211
A3 = 1.8635439827645754
A4 = 2.0178089750893693
A = 8.005501618446436
%err = 0.5180435157143415

No. of Trapezoids: 10
A0 = 0.46729444732424263
A1 = 0.5848517803046664
A2 = 0.6752488050532779
A3 = 0.7487937610783413
A4 = 0.8108247467391092
A5 = 0.864477544058045
A6 = 0.9117552996708911
A7 = 0.9540171184043325
A8 = 0.9922281657568743
A9 = 1.02709884318583
A = 8.036590511575609
%err = 0.13171120815543397
```

```
No. of Trapezoids: 20
A0 = 0.2182321556793955
A1 = 0.2518793793415168
A2 = 0.28064758658669486
A3 = 0.30577902941478546
A4 = 0.3280933845462064
A5 = 0.3481604540924216
A6 = 0.36639260977181703
A7 = 0.3830980182381336
A8 = 0.3985130862208595
A9 = 0.41282317058492685
A10 = 0.42617630984737914
A11 = 0.4386926241427797
A12 = 0.45047092770841807
A13 = 0.4615934912194406
A14 = 0.4721295427852232
A15 = 0.4821378886409214
A16 = 0.4916689066213539
A17 = 0.5007660844419265
A18 = 0.5094672221408896
A19 = 0.5178053830347946
A = 8.0445272550598
%err = 0.03308368828705456
```

Romberg:

```
R(0, 0) = 7.218875824868201
R(1, 0) = 7.80666248977032
R(1, 1) = 8.002591378071026
R(2, 0) = 7.982772786564996
R(2, 1) = 8.041476218829889
R(2, 2) = 8.044068541547146
R(3, 0) = 8.03068449590948
R(3, 1) = 8.046655065690974
R(3, 2) = 8.04700032214838
R(3, 3) = 8.047046858348399
R(4, 0) = 8.04303347406756
R(4, 1) = 8.047149800120254
R(4, 2) = 8.04718278241554
R(4, 3) = 8.047185678610257
R(4, 4) = 8.04718622300344
R(5, 0) = 8.046148565255043
R(5, 1) = 8.047186928984205
R(5, 2) = 8.047189404241802
R(5, 3) = 8.047189509350156
R(5, 4) = 8.047189524372666
R(5, 5) = 8.04718952759981
Best: R(5, 5) = 8.04718952759981
```

Exercise 2: f(x) = x^2 - 3x + 5

Trapezoid

```
Actual: 25.333333333333332          No. of Trapezoids: 20
                                    A0 = 0.584
No. of Trapezoids: 5                A1 = 0.5600000000000002
A0 = 2.336                          A2 = 0.552
A1 = 2.72                           A3 = 0.5599999999999999
A2 = 4.128                          A4 = 0.584
A3 = 6.56                           A5 = 0.624
A4 = 10.016                         A6 = 0.68
A = 25.759999999999998             A7 = 0.7520000000000001
%err = 1.6842105263157863          A8 = 0.8400000000000003
                                    A9 = 0.9440000000000004
No. of Trapezoids: 10               A10 = 1.0640000000000005
A0 = 1.1520000000000001            A11 = 1.2000000000000006
A1 = 1.12                           A12 = 1.3520000000000008
A2 = 1.216                          A13 = 1.520000000000001
A3 = 1.4399999999999997            A14 = 1.7040000000000006
A4 = 1.7919999999999998            A15 = 1.9040000000000008
A5 = 2.272                          A16 = 2.120000000000001
A6 = 2.879999999999999             A17 = 2.3520000000000016
A7 = 3.6159999999999983            A18 = 2.6000000000000023
A8 = 4.47999999999999              A19 = 2.8640000000000025
A9 = 5.472                          A = 25.360000000000014
A = 25.439999999999998             %err = 0.10526315789479536
%err = 0.4210526315789431
```

Romberg:

```
R(0, 0) = 36.0
R(1, 0) = 28.0
R(1, 1) = 25.333333333333332
R(2, 0) = 26.0
R(2, 1) = 25.333333333333332
R(2, 2) = 25.33333333333332
R(3, 0) = 25.5
R(3, 1) = 25.333333333333332
R(3, 2) = 25.333333333333332
R(3, 3) = 25.333333333333332
R(4, 0) = 25.375
R(4, 1) = 25.333333333333332
R(4, 2) = 25.333333333333332
R(4, 3) = 25.333333333333332
R(4, 4) = 25.333333333333332
R(5, 0) = 25.34375
R(5, 1) = 25.333333333333332
R(5, 2) = 25.333333333333332
R(5, 3) = 25.333333333333332
R(5, 4) = 25.333333333333332
R(5, 5) = 25.333333333333332
Best: R(1, 1) = 25.333333333333332
```