Proofs Portfolio

MAT 3100W: Intro to Proofs

Sam Ly

October 6, 2025

1 Introduction

(Leave this blank for now. Here's an outline of course topics for your reference.)

2 Proof techniques

(Here we give examples of some proof techniques.)

2.1 Proof by Induction

As an example of Proof by Induction, we will prove the following.

Proposition 1. *Let F_n be the n-th Fibonacci number, where $F_0 = F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$. Prove that $F_n \leq 1.9^n$ for all $n \geq 1$.*

Proof. We begin by verifying the relation for small n to create our base cases:

$$n = 1, F_1 = 1 \leq 1.9^1 = 1.9$$

$$n = 2, F_2 = 2 \leq 1.9^2 = 3.61.$$

We then form our inductive hypothesis by assumimg $F_n \leq 1.9^n$ for $1 \leq n \leq k$.

$$F_{k+1} \leq 1.9^{k+1}$$

$$F_k + F_{k-1} \leq 1.9^{k+1}.$$

Using our inductive hypothesis, we see that $F_k \leq 1.9^k$ and $F_{k-1} \leq 1.9^{k-1}$.
So, $F_k + F_{k-1} \leq 1.9^k + 1.9^{k-1}$.
By refactoring $1.9^k + 1.9^{k-1}$, we get:

$$1.9^k + 1.9^{k-1} = 1.9(1.9^{k-1}) + 1.9^{k-1} = 2.9(1.9^{k-1}).$$

Also, 1.9^{k+1} can be rewritten as $1.9^2(1.9^{k-1}) = 3.61(1.9^{k-1})$.
Finally, we see

$$F_{k+1} = F_k + F_{k-1} \leq 2.9(1.9^{k-1}) \leq 3.61(1.9^{k-1}) = 1.9^{k+1}$$

$$F_{k+1} \leq 1.9^{k+1}.$$

Therefore, $F_n \leq 1.9^n$ for all $n \geq 1$. □

Appendix

(The first section, "Course objectives and student learning outcomes" is just here for your reference.)

A Course objectives and student learning outcomes

1. Students will learn to identify the logical structure of mathematical statements and apply appropriate strategies to prove those statements.
2. Students learn methods of proof including direct and indirect proofs (contrapositive, contradiction) and induction.
3. Students learn the basic structures of mathematics, including sets, functions, equivalence relations, and the basics of counting formulas.
4. Students will be able to prove multiply quantified statements.
5. Students will be exposed to well-known proofs, like the irrationality of $\sqrt{2}$ and the uncountability of the reals.

A.1 Expanded course description

- Propositional logic, truth tables, DeMorgan's Laws
- Sets, set operations, Venn diagrams, indexed collections of sets
- Conventions of writing proofs
- Proofs
  - Direct proofs
  - Contrapositive proofs
  - Proof by cases
  - Proof by contradiction
  - Existence and Uniqueness proofs
  - Proof by Induction
- Quantifiers
  - Proving universally and existentially quantified statements
  - Disproving universally and existentially quantified statements
  - Proving and disproving multiply quantified statements
- Number systems and basic mathematical concepts
  - The natural numbers and the integers, divisibility, and modular arithmetic
  - Counting: combinations and permutations, factorials
  - Rational numbers, the irrationality of $\sqrt{2}$
  - Real numbers, absolute value, and inequalities
- Relations and functions
  - Relations, equivalence relations
  - Functions
  - Injections, surjections, bijections

- Cardinality
  - Countable and uncountable sets
  - Countability of the rational numbers, $\mathbb{Q}$
  - Uncountability of the real numbers, $\mathbb{R}$