

Title

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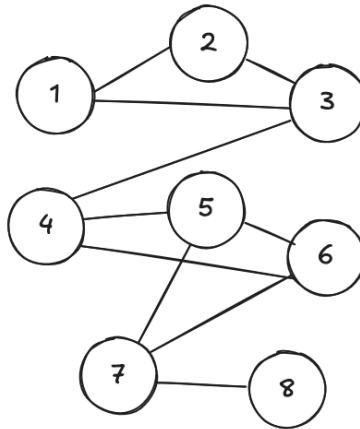
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## Part A: By Hand (15 points)

**Edges:**  $(1, 2), (1, 3), (2, 3), (3, 4), (4, 5), (4, 6), (5, 6), (5, 7), (6, 7), (7, 8)$

**Nodes:**  $\{1, 2, 3, 4, 5, 6, 7, 8\}$

**No. Edges:** 10



### 1. Density (3 points)

- Write the formula for density.

$$D = \frac{2E}{N(N-1)}$$

- Count nodes and edges.  
8 nodes, 10 edges.
- Compute the density of this graph.

$$D = \frac{2(10)}{8(7)} = 0.3555$$

### 2. Local Clustering Coefficient for Node 3 (3 points)

- Write the formula.

$$C_i = \frac{2E_i}{k_i(k_i - 1)}$$

- Identify Node 3's neighbors.  
Node 3's neighbors are node 1, node 2, and node 4.

- Count edges among them.  
There is one edge between node 3's neighbors: (1, 2).
- Compute  $C_3$ .

$$C_3 = \frac{2(1)}{3(2)} = \frac{1}{3}$$

### 3. Global Clustering Coefficient (3 points)

- Compute the local clustering coefficient for each node with degree  $\geq 2$ .

Node $i$	$k_i$	$E_i$	$C_i$
1	2	1	1
2	2	1	1
3	3	1	$\frac{1}{3}$
4	3	1	$\frac{1}{3}$
5	3	2	$\frac{2}{3}$
6	3	2	$\frac{2}{3}$
7	2	1	1
8	-	-	-

- Average them.

$$C = \frac{1}{7} \sum_{i=1}^7 C_i = \frac{1 + 1 + \frac{1}{3} + \frac{1}{3} + \frac{2}{3} + \frac{2}{3} + 1}{7} = \frac{5}{7}$$

### 4. Average Path Length (4 points)

- List all unique pairs of nodes.

**Starting at 1** (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8)

**Starting at 2** (2, 3), (2, 4), (2, 5), (2, 6), (2, 7), (2, 8)

**Starting at 3** (3, 4), (3, 5), (3, 6), (3, 7), (3, 8)

**Starting at 4** (4, 5), (4, 6), (4, 7), (4, 8)

**Starting at 5** (5, 6), (5, 7), (5, 8)

**Starting at 6** (6, 7), (6, 8)

**Starting at 7** (7, 8)

- Find the shortest distance  $d(i, j)$  for each pair.

	1	2	3	4	5	6	7	8
1		1	1	2	3	4	4	5
2			1	2	3	4	4	5
3				1	2	2	3	4
4					1	1	2	3
5						1	1	2
6							1	2
7								1
8								

- Compute the average.