Title

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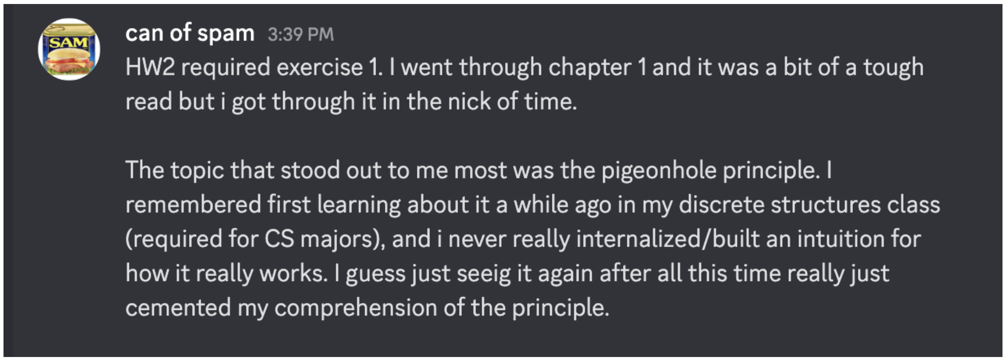
[3 pts] Required Exercise 1.

Done.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HW2 required exercise 1. I went through chapter 1 and it was a bit of a tough read but i got through it in the nick of time.

The topic that stood out to me most was the pigeonhole principle. I remembered first learning about it a while ago in my discrete structures class (required for CS majors), and i never really internalized/built an intuition for how it really works. I guess just seeig it again after all this time really just cemented my comprehension of the principle.

[3 pts] Required Exercise 2.

Figure 1: Do I look like I know what a JPEG is?

Try the following settings for [width=...], and discuss what they do:

1. `[width=2cm]`

   This makes the figure very small, like a thumbnail. The sizing is abosulte w.r.t the literal page size.

2. `[width=5cm]`

   A bit bigger than the previous, but is still absolute sizing.

3. `[width=0.5\textwidth]`

   This size fills a good amount of the page without being overly intrusive. I usually like using this.

4. `[width=\linewidth]`

   Seems to do the same as ”textwidth”. Internet says it has to do with pages with multiple columns.

[4 pts] Required Exercise 3.

[2 + 2 + 3 + 3 + 2] Choice Exercise 4.

1. Prove that given 5¢ coins and 6¢ coins, you can make change for any amount of money that is 20¢ or more.

   Proof. We begin by defining ”being able to make change from 5¢ and 6¢ coins” as:

   $$n = 5a + 6b$$

   Now, we see that we can make change of n¢ if we can write n as $5a + 6b$.

   For the base cases $20 \leq n \leq 24$, we have:

   - $20 = 5(4) + 6(0)$
   - $21 = 5(3) + 6(1)$
   - $22 = 5(2) + 6(2)$
   - $23 = 5(1) + 6(3)$
   - $24 = 5(0) + 6(4)$

   Now we make the inductive hypothesis that we *can* make change of any value $n = k$ such that $k \geq 20$.

   If this is true, we can also make change for $n = k + 1$, because $k + 1 = k - 4 + 5$.

   Therefore, we can make change for any amount of money that is 20¢ or more using only 5¢ and 6¢ coins. □

2. Prove that $9^n + 5^n - 2$ is divisible by 4 for all integers $n \geq 1$.

   Proof. An integer m is divisible by 4 if it can be written in the form $m = 4c$ for some integer c.

   For $n = 1$, we have $9^1 + 5^1 - 2 = 12 = 4(3)$.

   For $n = 2$, we have $9^2 + 5^2 - 2 = 104 = 4(26)$.

   We can make the inductive hypothesis that $9^n + 5^n - 2 = 4m$ holds true for $1 \leq n \leq k$.

   Thus,

   $$9^k = 4m - 5^k + 2$$

and

$$5^k = 4m - 9^k + 2.$$

Then, we see if the relation holds for $n = k + 1$.

$$9^{k+1} + 5^{k+1} - 2$$

$$9(9^k) + 5(5^k) - 2$$

$$9(4m - 5^k + 2) + 5(4m - 9^k + 2) - 2$$

$$36m - 9(5^k) + 18 + 20m - 5(9^k) + 10 - 2$$

$$56m - 9(5^k) + 5(9^k) + 26$$

$$56m - 45(\frac{5^k}{5} + \frac{9^k}{9}) + 26$$

$$56m - 45(5^{k-1} + 9^{k-1} - 2) - 2(45) + 26$$

$$56m - 45(5^{k-1} + 9^{k-1} - 2) - 64$$

Now, we can use our inductive hypothesis to say $5^{k-1} + 9^{k-1} - 2 = 4m_1$.

Thus we have

$$9^{k+1} + 5^{k+1} - 2 = 56m - 45(4m_1) - 64 = 4(14m - 45m_1 - 16).$$

Therefore, $9^n + 5^n - 2$ is divisible by 4 for all $n \geq 1$. □

3. Let F_n be the n-th Fibonacci number, where $F_0 = F_1 = 1$ and $F_n = F_{n-1}+F_{n-2}$. Prove that $F_n \leq 1.9^n$ for all $n \geq 1$.

Proof. We begin by verifying the relation for small n.

$$n = 0, F_0 = 1 \leq 1.9^0 = 1$$

$$n = 1, F_1 = 1 \leq 1.9^1 = 1.9$$

$$n = 2, F_2 = 2 \leq 1.9^2 = 3.61.$$

Assume $F_n \leq 1.9^n$ for $0 \leq n \leq k$.

$$F_{k+1} \leq 1.9^{k+1}$$

$$F_k + F_{k-1} \leq 1.9^{k+1}.$$

Using our inductive hypothesis, we see that

$$F_k \leq 1.9^k \text{ and } F_{k-1} \leq 1.9^{k-1}.$$

So,

$$F_k + F_{k-1} \leq 1.9^k + 1.9^{k-1}$$

By refactoring $1.9^k + 1.9^{k-1}$, we get:

$$1.9^k + 1.9^{k-1} = 1.9(1.9^{k-1}) + 1.9^{k-1} = 2.9(1.9^{k-1})$$

Also, 1.9^{k+1} can be rewritten as $1.9^2(1.9^{k-1}) = 3.61(1.9^{k-1})$.

Finally, we see

$$F_{k+1} = F_k + F_{k-1} \leq 2.9(1.9^{k-1}) \leq 3.61(1.9^{k-1}) = 1.9^{k+1}$$

$$F_{k+1} \leq 1.9^{k+1}$$

Therefore, $F_n \leq 1.9^n$ for all $n \geq 1$. □