Problem Formulation

We consider a team of autonomous agents, such as unmanned aerial vehicles (UAVs), indexed by $i \in \mathcal{U}$, and a set of tasks indexed by $j \in \mathcal{T}$. Each UAV can be assigned a subset of tasks, subject to individual task capacity and global exclusivity constraints. The objective is to assign UAVs to tasks in a manner that maximizes total utility, discounted by the estimated time until task execution.

Let $x_{ij} \in \{0, 1\}$ be a binary decision variable indicating whether UAV i is assigned to task j. Each assignment yields a utility u_{ij}, and incurs an estimated delay t_{ij}, representing the time UAV i takes to reach and begin task j, accounting for prior tasks in its sequence. The utility is discounted using an exponential decay factor $e^{-\lambda t_{ij}}$, where $\lambda > 0$ is a tunable parameter reflecting temporal urgency.

The optimization problem is formulated as:

$$\max_{\{x_{ij}\}} \quad \sum_{i \in \mathcal{U}} \sum_{j \in \mathcal{T}} u_{ij} \cdot e^{-\lambda t_{ij}} \cdot x_{ij} \tag{1}$$

$$\text{subject to} \quad \sum_{j \in \mathcal{T}} x_{ij} \leq C_i, \quad \forall i \in \mathcal{U} \tag{2}$$

$$\sum_{i \in \mathcal{U}} x_{ij} \leq 1, \quad \forall j \in \mathcal{T} \tag{3}$$

$$x_{ij} \in \{0, 1\}, \quad \forall i \in \mathcal{U},\ \forall j \in \mathcal{T} \tag{4}$$

Constraint (2) ensures that each UAV is assigned no more than C_i tasks, its maximum task capacity. Constraint (3) ensures that each task is assigned to at most one UAV. Constraint (4) enforces the binary nature of the task assignments.

Variable Definitions

Variable	Description
$\mathcal{U}$	Set of UAVs (agents), indexed by i
$\mathcal{T}$	Set of tasks, indexed by j
$x_{ij} \in \{0, 1\}$	Assignment decision variable: 1 if UAV i is assigned to task j, 0 otherwise
$u_{ij} \in \mathbb{R}_{\geq 0}$	Base utility of UAV i completing task j
$t_{ij} \in \mathbb{R}_{\geq 0}$	Estimated time for UAV i to begin and execute task j
$\lambda > 0$	Time decay coefficient controlling urgency weighting
$C_i \in \mathbb{Z}_{>0}$	Maximum number of tasks UAV i can be assigned
$B_i \subseteq \mathcal{T}$	Ordered bundle of tasks assigned to UAV i (used in CBBA)
$b_{ij} \in \mathbb{R}$	Bid value computed by UAV i for task j during CBBA

Stochastic Task-Arrival Model (Poisson Process)

Arrival process. Let $\{N(\tau),\ \tau \geq 0\}$ be a homogeneous Poisson process with intensity[1] $\alpha > 0$. Then

$$\Pr\big[N(\tau) = n\big] = e^{-\alpha\tau}\frac{(\alpha\tau)^n}{n!}, \quad n \in \mathbb{N},\ \tau \in [0, T_{\text{end}}],$$

and the inter-arrival times $E_k \triangleq A_k - A_{k-1} \overset{\text{iid}}{\sim} \text{Exp}(\alpha)$, where A_k is the k-th task-arrival epoch ($A_0 = 0$).

Task set at time τ. Each newly revealed task is indexed consecutively $j = N(\tau)$; the set of tasks known at mission time τ is

$$\mathcal{T}(\tau) \ = \ \big\{1, \dots, N(\tau)\big\}, \quad \mathcal{T}(0) = \varnothing.$$

Availability constraint. For tasks not yet arrived we impose

$$x_{ij}(\tau) = 0, \qquad \forall i \in \mathcal{U},\ \forall j > N(\tau). \tag{5}$$

Rolling-horizon re-optimisation (centralised view). At each review epoch $\tau_k = k\Delta\tau$ ($k = 1, 2, \dots$) or immediately after an arrival ($\tau = A_{N(\tau)}$), solve

$$\max_{\boldsymbol{x}(\tau)} \sum_{i\in\mathcal{U}} \sum_{j\leq N(\tau)} u_{ij}\, e^{-\lambda t_{ij}(\tau)} x_{ij}(\tau),$$

subject to capacity, exclusivity, binary constraints (2)–(4) *and* the availability constraint (5).[2] This produces a *recourse* assignment that adapts to the realised Poisson arrivals.

Decentralised (CBBA) implementation. Every time a new task arrives ($N(\tau) \leftarrow N(\tau) + 1$),

1. The discovering UAV broadcasts the task descriptor to its neighbours.
2. All UAVs append the task to their local task lists and compute bids $b_{i,N(\tau)} = u_{i,N(\tau)}\, e^{-\lambda \hat{t}_{i,N(\tau)}}$, where $\hat{t}_{i,N(\tau)}$ is the predicted start time *given the current bundle order.*
3. Standard CBBA consensus rounds resume, restricted to the updated set $\mathcal{T}(\tau)$; feasibility is enforced by (5).

Remark. Because arrivals are Poisson, the expected number of tasks revealed by time τ is $\mathbb{E}[N(\tau)] = \alpha\tau$, allowing tractable mission-length sizing and Monte-Carlo studies of algorithmic performance under stochastic workload.

[1] For time-varying intensities $\alpha(\tau)$, replace $\alpha\tau$ by $\int_0^\tau \alpha(s)\, ds$ throughout.
[2] Completed or in-progress tasks are fixed as in (A2) of the static model.