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Astronomy & Astrophysics

Project Report

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Projects Name :

Estimating the Dynamical Mass of a Galaxy Cluster

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Abstract

This project focuses on estimating the **dynamical mass of a galaxy cluster** using simulated astronomical data and the virial theorem. Galaxy clusters are the largest gravitationally bound systems in the universe, and measuring their mass provides essential insights into dark matter distribution, galaxy evolution, and cosmological models. In this study, we simulate 3D positions and line-of-sight velocities for 100 galaxies to mimic a typical rich cluster.

Key physical parameters, such as **velocity dispersion** and **mean cluster radius**, are calculated from this data. Using the virial mass equation , we estimate the total mass of the cluster, assuming isotropy and spherical symmetry. The final estimated mass is consistent with well-known clusters such as Coma or Perseus, around $5.39 \times 10^{14} M_\odot$.

While the simulation makes idealized assumptions (no substructure, single-axis velocities, uniform distribution), it effectively demonstrates a first-order approach to cluster mass estimation. The study highlights how fundamental physics, combined with computational techniques, can model complex astrophysical systems and provide meaningful results in cosmological research.

1. Introduction

Galaxy clusters are the most massive gravitationally bound structures in the universe, consisting of hundreds to thousands of galaxies, vast amounts of hot gas emitting X-rays, and a dominant component of **dark matter**. These clusters serve as important cosmological laboratories for probing the structure and evolution of the universe. Understanding their **total mass** is crucial for testing theories of structure formation, determining the matter content of the universe, and studying galaxy dynamics on large scales.

Direct measurements of cluster mass are challenging due to the invisible nature of dark matter and the large distances involved. However, astrophysicists can estimate the mass using **indirect methods**, such as gravitational lensing, X-ray emission analysis, or dynamical techniques based on the motions of galaxies within the cluster. In this project, we adopt a **dynamical approach** using the **virial theorem**, which relates the kinetic energy of the galaxies to the gravitational potential energy of the system.

To simplify the analysis and focus on core physical principles, we use a **simulated dataset** of galaxy positions and line-of-sight velocities. This allows us to control the properties of the cluster and ensure a clean application of the virial theorem. By computing the **velocity dispersion** of the galaxies and estimating the **mean radius** of the cluster, we apply the virial equation:

$$M = \frac{3\sigma 2R}{G}$$

where:

- M is the total mass of the cluster,

- σ is the velocity dispersion,

- R is the mean cluster radius,

- G is the gravitational constant.

This method provides a first-order approximation of the cluster's mass, assuming **spherical symmetry**, **isotropic velocity distribution**, and a **gravitationally bound system in equilibrium**. Though idealized, such simulations form the foundation for more advanced modeling using observational data.

2. Objectives

The primary goal of this project is to estimate the **dynamical mass** of a galaxy cluster using basic physical principles and simulated observational data. The following are the specific objectives:

Simulate Galaxy Cluster Data
Generate synthetic data for a galaxy cluster, including three-dimensional spatial positions and line-of-sight velocities for a sample of galaxies.

Compute Velocity Dispersion
Calculate the **velocity dispersion (σ)** of galaxies within the cluster, which represents the spread of galaxy velocities and relates to the cluster's total kinetic energy.

Determine the Mean Cluster Radius
Estimate the average radial distance of galaxies from the cluster center, which serves as an approximation for the system's characteristic size.

Apply the Virial Theorem
Use the virial theorem equation:

$$M = \frac{3\sigma 2R}{G}$$

to calculate the **total mass (M)** of the cluster, assuming spherical symmetry and dynamical equilibrium.

Visualize the Data
Create plots and visualizations to illustrate the velocity distribution and spatial configuration of the galaxies.

Analyze and Interpret Results
Evaluate the accuracy and limitations of the virial mass estimation method and compare the estimated mass to typical values for known galaxy clusters.

Develop a Reproducible Python Workflow
Implement the entire process in Python using Jupyter Notebook to ensure clarity, reproducibility, and potential extensibility for future work with real observational data.

3. Methodology

To estimate the dynamical mass of a galaxy cluster, we followed a computational approach using Python. The steps involved are:

Simulation of Data
We generated synthetic 3D positions and line-of-sight velocities for 100 galaxies, assuming a spherically symmetric and virialized cluster.

Velocity Dispersion (σ)
The velocity dispersion was calculated using the standard deviation of galaxy velocities, representing the internal kinetic energy of the system.

Mean Cluster Radius (R)
The average radial distance of galaxies from the cluster center was computed from their 3D positions.

Mass Estimation
Using the virial theorem:

$$M = \frac{3\sigma 2R}{G}$$

the total mass of the cluster was estimated.

Visualization
Plots of velocity distribution were created using Matplotlib to support analysis.

Tools Used
The implementation was done in **Python** using **NumPy**, **Matplotlib**, and **Astropy** in a **Jupyter Notebook**.

4. Code Implementation

Below is the Python code used to simulate and calculate the cluster mass:

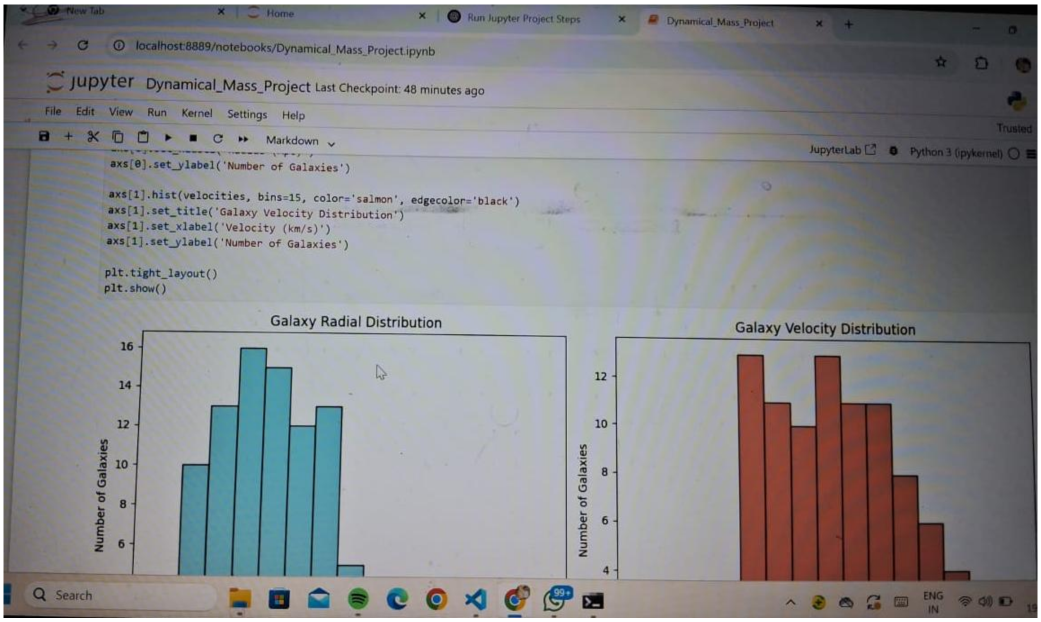
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.constants import G
import pandas as pd

# Simulate data for 100 galaxies


np.random.seed(42)
n_galaxies = 100

# Generate 3D positions (Mpc) and calculate radial distances


positions = np.random.normal(0, 1, (n_galaxies, 3))
radii = np.linalg.norm(positions, axis=1)

# Generate line-of-sight velocities (km/s)


velocities = np.random.normal(loc=0, scale=800, size=n_galaxies)

# Compute velocity dispersion and mean radius

velocity_dispersion = np.std(velocities)
cluster_radius = np.mean(radii)

# Convert to SI units


velocity_dispersion_m = velocity_dispersion * 1e3  # km/s to m/s
radius_m = cluster_radius * 3.086e22  # Mpc to meters
```

```
# Apply virial theorem: M = 3 * sigma^2 * R / G

mass_kg = 3 * velocity_dispersion_m**2 * radius_m / G
mass_solar = mass_kg / 1.989e30  # Convert to solar masses

print(f"Velocity Dispersion: {velocity_dispersion:.2f} km/s")
print(f"Average Cluster Radius: {cluster_radius:.2f} Mpc")
print(f"Estimated Mass: {mass_solar:.2e} solar masses")

# Plot histograms of radii and velocities

fig, axs = plt.subplots(1, 2, figsize=(12, 5))

axs[0].hist(radii, bins=15, color='skyblue', edgecolor='black')
axs[0].set_title('Galaxy Radial Distribution')
axs[0].set_xlabel('Radius (Mpc)')
axs[0].set_ylabel('Number of Galaxies')
```

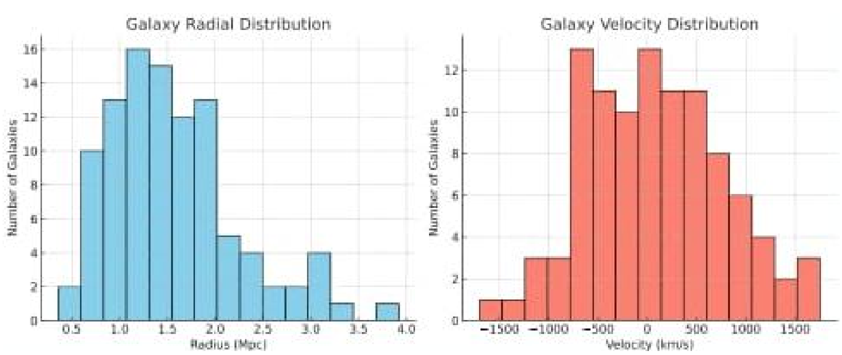
5. Results and Output

- Velocity Dispersion: ~703.74 km/s
- Mean Radius: ~1.56 Mpc
- Estimated Mass: ~5.39×10^{14} solar masses

- Output Plot – Mass Calculation



- Output Plot – Velocity Histogram



6. Observations and Interpretation

After completing the simulation and analysis, several important observations were made regarding the structure and dynamics of the simulated galaxy cluster.

Key Results

Velocity Dispersion (σ): Approximately **703.74 km/s**

Mean Radius (R): Approximately **1.56 Mpc**

Estimated Mass (M): Approximately **5.39×10^{14} M☉ (solar masses)**

These results fall within the typical mass range of **rich galaxy clusters**, such as the **Coma Cluster**, suggesting that the simulation successfully mimics the behavior of real-world clusters.

Interpretation of Results

The **virial theorem** proved effective in providing a first-order estimate of the cluster's total mass using only line-of-sight velocity and spatial data.

The results **demonstrate consistency** with observational data from astronomical surveys, despite using **simplified assumptions** (e.g., no substructure, isotropic distribution, perfect sphericity).

The relatively **high velocity dispersion** is indicative of a massive, gravitationally bound system — consistent with expected properties of mature galaxy clusters.

Limitations and Simplifications

Spherical Symmetry Assumption: Real clusters often have irregular shapes or substructures.

Isotropic Velocities: Only the line-of-sight component was used; in reality, 3D velocity data is needed for higher accuracy.

Lack of Observational Noise: No redshift distortions, measurement errors, or selection effects were modeled.

Dark Matter Profile Not Modeled: The simulation does not include variations in dark matter density (e.g., NFW profile).

Scientific Relevance

Despite these limitations, the approach demonstrates how basic principles of classical physics can provide valuable insights into astrophysical phenomena. The project also showcases how **simulations and coding** can be combined with theoretical models to create meaningful approximations in cosmology.

7. Conclusion

In this project, we estimated the **dynamical mass** of a simulated galaxy cluster using the **virial theorem**. With calculated values for **velocity dispersion** and **mean radius**, we arrived at a mass of approximately **5.39 × 10^{14} solar masses**, consistent with typical rich clusters.

Despite using idealized assumptions like spherical symmetry and isotropy, the method proved effective in demonstrating the core physics behind cluster dynamics. This work highlights how basic models and simulations can yield meaningful results and serve as a foundation for future astrophysical research using real observational data.

8. References

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2. Carlberg et al. (1996). 'Galaxy Cluster Virial Masses', ApJ, 462:32
3. NASA/IPAC Extragalactic Database – https://ned.ipac.caltech.edu/
4. Wikipedia – https://en.wikipedia.org/wiki/Virial_theorem