Uppsala University

Modelling Complex Systems

1MA256

Lab5 report

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1 Task 1: Boids model

For this lab we are implementing boids model which simulates the flocking behaviour of birds, and related group motion. We start with choosing three radii $r_1 < r_2 < r_3$ which reflect the different zones for each boid. Boids is an example of emergent behavior; that is, the complexity of Boids arises from the interaction of individual agents (the boids, in this case) adhering to a set of simple rules. The rules applied in the simplest Boids world are as follows:

- **separation:** A particle gets repelled away from the common center of mass of particles in zone 1. That is it's new velocity is aligned along the vector from the particle to the center of mass, and away from the center of mass. This gives the angle θ^1
- **alignment:** A particle aligns its velocity according to the polarization measure in zone 2. This produces angle θ^2. This can be realised by implementing the Vicsek polarization model in this zone.
- **cohesion:** A particle gets attracted to the center of mass of all particles in zone 3. That is it's new velocity is aligned along the vector from the particle to the center of mass, and towards the center of mass. This gives angle θ^3.

For the second item alignment, the Vicsek polarization resulted in a drift to the right, so another adaptation to simulate alignment is to compute the average velocity $v^2(t)$ of all particles in zone 2, and include it instead so this was done.

For each boid the influence from the other boids were determined based on the positions relative to the different zones, after this the velocity for a time step Δt could be calculated, let e^k be the corresponding unit vectors corresponding to θ^k, k=1,2,3.

$$v_j(t + \Delta t) = \rho_1 e^1(t) + \rho_2 \underbrace{e^2(t)}_{v^2(t)} + \rho_3 e^3(t) + \rho_4 v_j(t), \quad \rho_1 + \rho_2 + \rho_3 + \rho_4 = 1 \tag{1}$$

where $v_j(t)$ is the velocity of this particle at time t (inertia). Now, the vector $v_j(t + \Delta t)$ specifies the new direction of motion for the bird. Its angle $\theta_j(t + \Delta t)$ is the new angle.

Additionally to the velocity a random walk (=Brownian motion) is added to the motion of the boids. Since a random walk is a stationary stochastic process, it means that when a particle is at position $r_j(t)$, we can think of that position as a starting point for the next random walk which will be happening on time scale Δt. Thus we can randomize the update of the location as

$$r_j(t + \Delta t) = r_j(t) + \alpha v_j(t)\Delta t + \beta X(\Delta t), \quad \alpha + \beta = 1. \tag{2}$$

where $X(\Delta t)$ is a random walk. One pitfall for the brownian motion is that very far from 0 the Gaussian curve has a small, but finite area under the curve. This would mean that the particle is allowed to perform huge jumps with a small probability. To make the model more physical, you can impose the maximum value, r_max of $|X(\delta t)|$, and modify the distribution function.

This code is simulated for different parameters ρ_i and α, β.

1.1 Implementation

I am not sure the way i handled the boundary conditions was the correct way that we were supposed to do but for my code if a boid goes "over" an edge it reappears in the opposite side. In retrospect looking at the simulation on wikipedia im not sure if this way was the right way to handle it or if I was to make the boids avoid the edges and change direction accordingly. Anyways this worked as well, although it created some complications in deciding which boids were close to each other taking into acount the floating boundary and calculating the new directions.

The use of the angles was not really necessary as well since for each rule in 1 we could just return the vector straight away. To help aid in the visualization circles were created around the boids for each zone as well as arrows pointing in the direction of the boids.

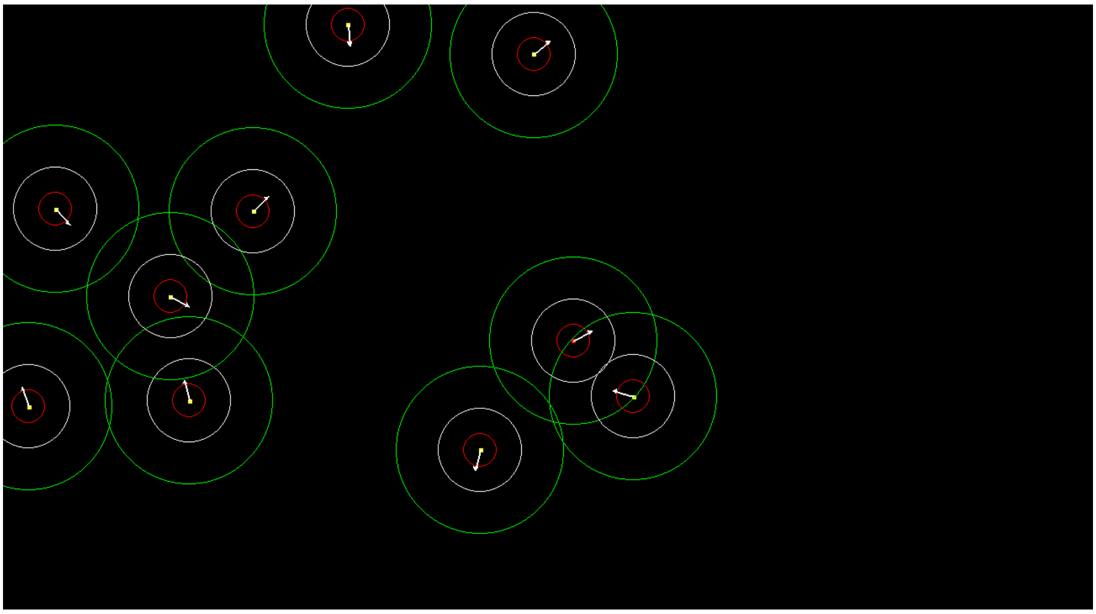


Figure 1: Initial random configuration for the simulation

Figure 1 shows how the boids are initialized with random positions and directions. The red inner circle is zone 1 (repelling), the white circle around is zone 2 (alignment) and the green circle around is zone 3 (attraction). The white arrow is the direction of velocity for each boid.

2 Experiments

The first experiments we do is to check if the boids move in a way we want them to for the three rules. For this we start with the repel, we set $\rho_1 = 1, \rho_2 = \rho_3 = \rho_4 = 0$ and $\alpha = 1, \beta = 0$ and then simulate. We only use the red circle for zone 1 around the boids since that is the only one that is necessary for now. The results can be found here `https://youtu.be/R6GgR3NKIFs`.

For the align we do the same thing but for $\rho_2 = 1$ and the rest 0 and with the white circle for zone 2 the video for that is here `https://youtu.be/9TiqTagQ42E`. And lastly video for attract is her `https://youtu.be/q0cWoMQtRPc` with the green circle for zone 3.

The videos shows that the rules work as they should. If a boid comes inside of zone 1 it changes direction away from it, if they are in zone 2 they align their direction and if they are inside zone 3 they get attracted to each other. Isolating the effects show very clearly how they all work, it is interesting to see that with only align they eventually end up together with $R2$ distance from each other and all heading in the same direction. For only attract they all eventually end up in one spot and for only repel they try to be separated the whole time.

We can now test for the effects from the random walk, in the previous videos $\alpha = 1$ and $\beta = 0$ but now we will see how the Brownian motion impacts the boids. For the first test we set $\rho_1 = \rho_2 = \rho_3 = 0.3, \rho_4 = 0.1$ so that it has equal "force" of atraction, repulsion and alignment and smaller force to stay on the same path. We set $\alpha = 0.8$ and $\beta = 0.2$. The video for that is here `https://youtu.be/bpffSqhUZH8`. The same is simulated for a higher beta ($\alpha = 0.2, \beta = 0.8$) and the video for that is here `https://youtu.be/W9SEuSo3tDk`

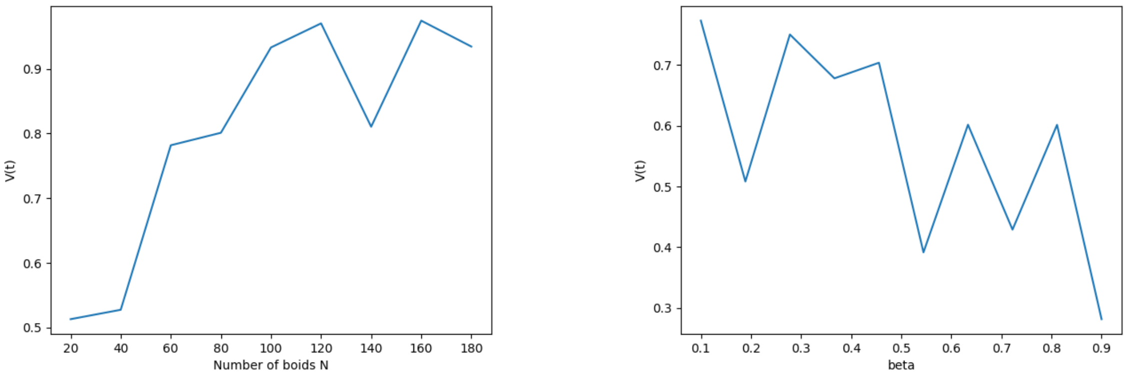
What the videos tell us is that for a low beta the boids act less random, and the "forces" from the velocity calculation directs the boids to where they will go. The video with the higher β is more random as it should but since the forces from the velocity still exists the boids will still do the same thing as the case with low β only slower and a bit more jumpier.

The case with high beta is a bit weird (at least for me) since the boids does not really travel (or fly like the bird like objects they are supposed to be) but rather they jump or teleport some distance. One solution that could make the appearance more streamlined would possibly be to incorporate the randomness in the velocity direction maybe. This way the boids would fly but still sometimes more randomly.

3 Task 2

In task 2 we are going to model a phase transition in the Vicsek model. For that, we turn down all parameters, but β and ρ_2. We then do two measurements we change the average density of the flock, and β. We measure V(t), where it is calculated as

$$V(t) = \frac{|\sum v_j(t)|}{N} \tag{3}$$



(a) Density dependence

(b) β dependence

Figure 2: Density and β dependence for V(t)

In figure 2 we can see in (a) how V(t) depends on the amount of boids N (or density if you will) and in (b) we see the dependence of β. For the density dependence β is constant and for the β dependence N is constant. Generally we see that V(t) increases as the density increases and V(t) decreases when β increases. However this result was very hard to simulate, there are several problems (at least for my code). To start with for the boids at t=0 the boids placement and velocity directions are random and thus in an ideal world V(0) = 0. However randomness does not really work this way and especially for low N the impact if for example two boids have velocities in the same direction V(t) > 0, through experiments V(0) ranged anywhere from 0.005 to 0.3. This of course impacted the results significantly. Another thing was the radius R2, a small R2 meant that the boids did not align at all or that they formed smaller flocks which could have totally different directions and essentially changed the simulation to an n boid simulation where n«N. The R2 together with the choice of N made a big difference in the β dependence simulation, here again if we had to few N the randomness of the initialization could impact to much and even for small β the boids would not "find" each other. A to big N made the boids all have the same direction pretty much straight away and ruined the findings. The last aspect that was difficult to decide was the time, since V(t) is average for the time it affected the results. For $t \to \infty$ and 100% contribution of align the boids would eventually all have the same direction. I did not know how long i was supposed to take the average of so the graph represents a time of 30 seconds for each data-point.

The figures are expected based on the theory for the Vicsek model that V(t) (v_a) decreases when the noice increases and V(t) increases when the density increases. The code for this assignment is linked in studium for this assignment.