Last Minute Notes – Engineering Mathematics

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GATE CSE is a national-level engineering entrance exam in India specifically for Computer Science and Engineering. It's conducted by top Indian institutions like IISc Bangalore and various IITs. In GATE CSE, engineering mathematics is a significant portion of the exam, typically constituting 15% of the total marks. Key topics in engineering mathematics tested in the exam include:

- Linear Algebra
- Probability and Statistics
- Calculus

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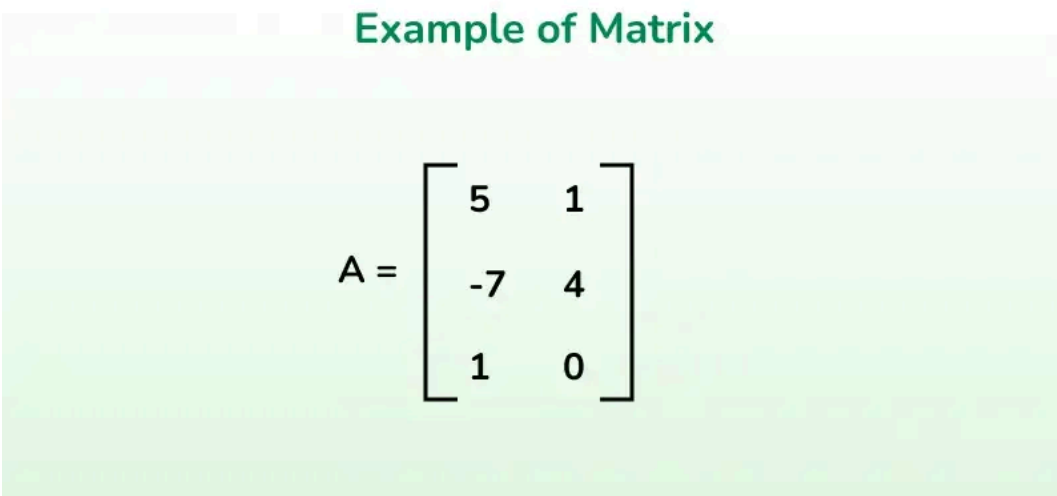
- [Application of Integrals]

Linear Algebra

Matrices

- A matrix is a rectangular array of numbers arranged in rows and columns.
- **Order**: Denoted as m × n, where m is the number of rows and n is the number of columns.



This Matrix [M] has 3 rows and 2 columns i.e., order of 3 × 2. Each element of matrix [M] can be referred to by its row and column number. For example, $a_{32} = 0$.

Types of Matrices

Some common types of matrices include:

- **Square Matrix**: m = n.
- **Diagonal Matrix**: All non-diagonal elements are zero.
- **Identity Matrix (I)**: A diagonal matrix with all diagonal elements as 1.
- **Zero Matrix**: All elements are zero.
- **Symmetric Matrix**: $A = A^T$ (Transpose equals the original matrix).
- **Skew-Symmetric Matrix**: $A = -A^T$.
- **Orthogonal Matrix**: $A^T A = I$, where A^T is the transpose.
- **Singular Matrix**: Determinant is 0 (det(A) = 0).
- **Non-Singular Matrix**: Determinant is non-zero (det(A) ≠ 0).
- **Idempotent Matrix:** A matrix is said to be idempotent if $A^2 = A$
- **Involutory Matrix:** A matrix is said to be Involutory if $A^2 = I$.
- **Nilpotent Matrix:** A square matrix of order n is said to be nilpotent if $A^k = 0$, $k \le n$.

Operations on Matrices

Some common operations on matrices includes:

- Matrix Addition
- Matrix Multiplication
- Transpose of a Matrix
- Inverse of Matrix

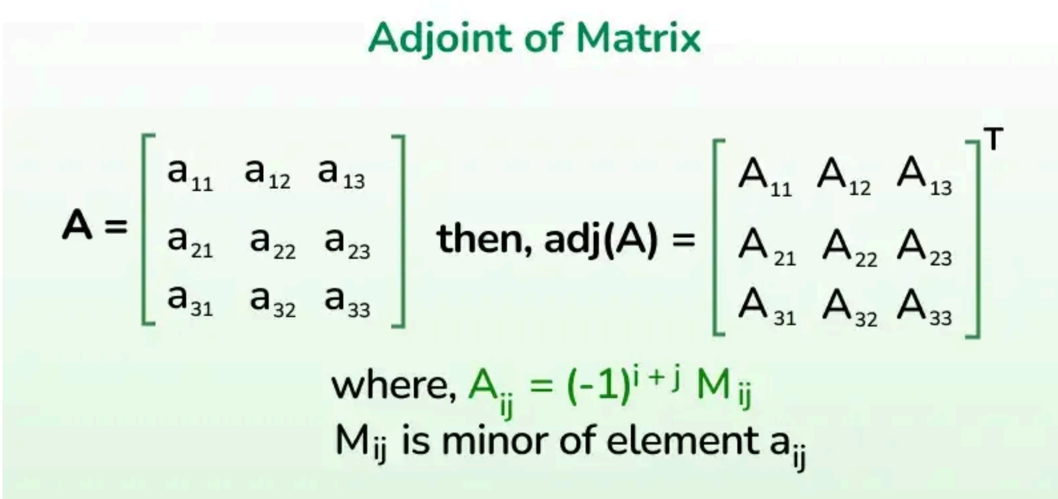
Transpose of a Matrix: The transpose $[M]^T$ of an m x n matrix [M] is the n × m matrix obtained by interchanging the rows and columns of [M]. if A= $[a_{ij}]$ m × n , then $A^T = [b_{ij}]$ n × m where $b_{ij} = a_{ji}$

Properties of transpose of a Matrix:

Some common properties of transpose of matrix include:

- $(A^T)^T = A$
- $(kA)^T = k(A^T)$
- $(A \pm B)^T = A^T \pm B^T$
- $(AB)^T = B^T \cdot A^T$
- $(A^{-1})^T = (A^T)^{-1}$

Adjoint of a Matrix: The adjoint of a square matrix is the transpose of its cofactor matrix.



Read More about Minor and Cofactors.

Properties of Adjoint

Some important properties of adjoint include:

- $A(\text{Adj } A) = (\text{Adj } A)\, A = |A|\, I_n$
- $\text{Adj}(AB) = (\text{Adj } B) \cdot (\text{Adj } A)$
- $|\text{Adj } A| = |A|^{n-1}$
- $\text{adj}(\text{adj}(A)) = |A|^{n-2} \cdot A$
- $\text{adj}(A^T) = (\text{adj}(A))^T$
- $\text{Adj}(kA) = k^{n-1}\, \text{Adj}(A)$

Inverse of a Matrix

For any square matrix A,

$$A^{-1} = \frac{Adj A}{|A|}$$

Here |A| should not be equal to zero, means matrix A should be non-singular.

Properties of Inverse

- $A^{-1} \cdot A = I$
- $(A^{-1})^{-1} = A$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $(kA)^{-1} = (1/k) \cdot A^{-1}$

> ***Note:*** *Only a non-singular square matrix can have an inverse.*

Conjugate of a Matrix: If A is a matrix with elements a_{ij}, then the conjugate matrix A is obtained by taking the complex conjugate of each element a_{ij}.

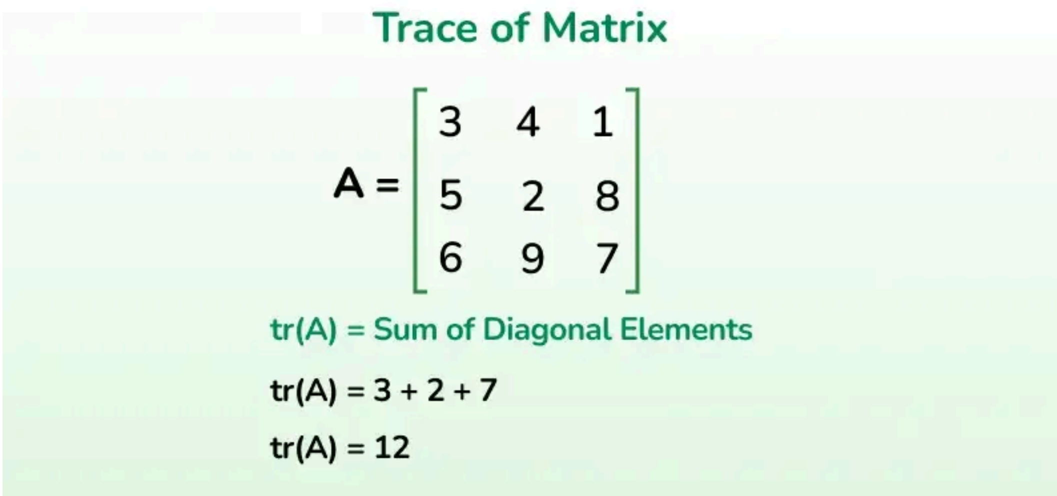
Mathematically conjugate of m × n matrix is given by: $A = \begin{matrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{matrix}$

Here, a_{ij} represents the complex conjugate of a_{ij}.

Trace of a Matrix: Let $A = [a_{ij}]_{n \times n}$ is a square matrix of order n, then the sum of diagonal elements is called the trace of a matrix which is denoted by tr(A).

> $$tr(A) = a_{11} + a_{22} + a_{33} + \ldots\ldots\ldots + a_{nn}$$

Remember trace of a matrix is also equal to the sum of eigen value of the matrix. For example:



Properties of Trace of Matrix:

Let A and B be any two square matrices of order n, then

1. tr(kA) = k tr(A) where k is a scalar.
2. tr(A+B) = tr(A)+tr(B)
3. tr(A-B) = tr(A)-tr(B)
4. tr(AB) = tr(BA)

Determinant of Matrices

Determinant represents the scaling factor of the linear transformation associated with the matrix. For example, in a 2 × 2 matrix, the determinant represents the area scaling factor.

Properties of Determinant

- det(A^T) = det(A)
- det(AB) = det(A) × det(B)
- det(kA) = k^n × det(A)
- det(A) = 0 implies A is singular.
- |adj(adj(A))| = $|A|^{(n-1)^2}$
- If we interchange the row or column in determinant its sign changes from positive to negative.

Rank of a Matrix: Rank of matrix is the number of non-zero rows in the row reduced form or the maximum number of independent rows or the maximum number of independent columns. Rank is denoted as rank(A) or ρ(A). if A is a non-singular matrix of order n, then rank of A = n i.e. ρ(A) = n.

Let A be any m × n matrix and it has square sub-matrices of different orders. A matrix is said to be of rank r, if it satisfies the following properties:

- It has at least one square sub-matrices of order r who has non-zero determinant.
- All the determinants of square sub-matrices of order (r+1) or higher than r are zero.

Properties of Rank of a Matrix

Some important properties of rank of matrix are:

- If A is a null matrix then ρ(A) = 0 i.e. Rank of null matrix is zero.
- If I_n is the n × n unit matrix then ρ(A) = n.
- Rank of a matrix A m × n , ρ(A) ≤ min(m,n). Thus ρ(A) ≤m and ρ(A) ≤ n.
- $\rho(A_{n \times n}) = n$ if |A| ≠ 0
- If ρ(A) = m and ρ(B)=n then ρ(AB) ≤ min(m,n).
- If A and B are square matrices of order n then ρ(AB) ≥ ρ(A) + ρ(B) – n.
- If $A_{m\times1}$ is a non zero column matrix and $B_{1\times n}$ is a non zero row matrix then ρ(AB) = 1.
- The rank of a skew symmetric matrix cannot be equal to one.

Solution of a System of Linear Equations

Linear equations can have three kind of possible solutions:

- No Solution
- Unique Solution
- Infinite Solution

System of homogeneous linear equations AX = 0.

1. X = 0. is always a solution; means all the unknowns has same value as zero. (This is also called trivial solution)
2. If ρ(A) = number of unknowns, unique solution.
3. If ρ(A) < number of unknowns, infinite number of solutions.

System of non-homogeneous linear equations AX = B.

1. If ρ[A:B] ≠ ρ(A), No solution.

2. If P[A:B] = ρ(A) = the number of unknown variables, unique solution.
3. If ρ[A:B] = ρ(A) ≠ number of unknown, infinite number of solutions.

Here ρ[A:B] is rank of gauss elimination representation of AX = B.

There are two states of the Linear equation system:

- **Consistent State:** A System of equations having one or more solutions is called a consistent system of equations.
- **Inconsistent State:** A System of equations having no solutions is called inconsistent system of equations.

Linear dependence and Linear independence of Vector:

- **Linear Dependence:** A set of vectors $X_1, X_2, \ldots, X_r$ is said to be linearly dependent if there exist r scalars $k_1, k_2, \ldots, k_r$ such that: $k_1 X_1 + k_2 X_2 + \ldots + k_r X_r = 0$.
- **Linear Independence**: A set of vectors $X_1, X_2, \ldots, X_r$ is said to be linearly independent if for all r scalars $k_1, k_2, \ldots, k_r$ such that $k_1 X_1 + k_2 X_2 + \ldots + k_r X_r = 0$, then $k_1 = k_2 = \ldots = k_r = 0$.

How to determine linear dependency and independency?

Let $X_1, X_2 \ldots X_r$ be the given vectors. Construct a matrix with the given vectors as its rows.

1. If the rank of the matrix of the given vectors is less than the number of vectors, then the vectors are linearly dependent.
2. If the rank of the matrix of the given vectors is equal to the number of vectors, then the vectors are linearly independent.

Eigen Value and Eigen Vector

Eigen vector of a matrix A is a vector represented by a matrix X such that when X is multiplied with matrix A, then the direction of the resultant matrix remains the same as vector X.

Mathematically, above statement can be represented as:

$$AX = \lambda X$$

Where A is any arbitrary matrix, λ are eigen values and X is an eigen vector corresponding to each eigen value.

Here, we can see that AX is parallel to X. So, X is an eigen vector.

How to Find Eigenvalues and Eigen Vectors of Given Matrices?

We know that,

$AX = \lambda X$
$\Rightarrow AX - \lambda X = 0$
$\Rightarrow (A - \lambda I)\, X = 0 \ldots (1)$

Above condition will be true only if $(A - \lambda I)$ is singular. That means,

$|A - \lambda I| = 0 \ldots (2)$

This is known as characteristic equation of the matrix and the roots of the characteristic equation are the eigen values of the matrix A.

Properties of Eigen Values

Some important properties of eigen values are:

- Eigen values of real symmetric and hermitian matrices are real.
- Eigen values of real skew symmetric and skew hermitian matrices are either pure imaginary or zero.
- Eigen values of unitary and orthogonal matrices are of unit modulus $|\lambda| = 1$.
- If $\lambda_1 = \lambda_2 = \ldots = \lambda_n$ are the eigen values of A, then $k\lambda_1, k\lambda_2 \ldots k\lambda_n$ are eigen values of kA.
- If $\lambda_{1,}\ \lambda_2 \ldots \lambda_n$ are the eigen values of A, then $1/\lambda_1, 1/\lambda_2 \ldots 1/\lambda_n$ are eigen values of A^{-1}.
- If $\lambda_{1,}\ \lambda_2 \ldots \lambda_n$ are the eigen values of A, then $\lambda_1^k, \lambda_2^k \ldots \lambda_n^k$ are eigen values of A^k.
- Eigen values of A = Eigen Values of A^T (Transpose).
- Sum of Eigen Values = Trace of A (Sum of diagonal elements of A).
- Product of Eigen Values = $|A|$.
- Maximum number of distinct eigen values of A = Size of A.
- If A and B are two matrices of same order then, Eigen values of AB = Eigen values of BA.

LU Decomposition

LU Decomposition (also known as LU Factorization) is a method used to solve a system of linear equations. It decomposes a given square matrix **A** into the product of two matrices **L** (Lower Triangular Matrix) and **U** (Upper Triangular Matrix), such that:

$A = L \cdot U$

Where:

- **L** is a lower triangular matrix with ones on the diagonal.
- **U** is an upper triangular matrix.

Read More about Triangular Matrix.

Probability and Statistics

Probability

Probability refers to the extent of occurrence of events. When an event occurs like throwing a ball, picking a card from deck, etc ., then the must be some probability associated with that event.

Basic Terminologies:

- **Sample Space (S)**: The set of all possible outcomes of an experiment.
- **Event (E)**: Any subset of the sample space.
- **Probability of an Event (P(E))**: A measure of the likelihood of an event occurring, where 0≤P(E)≤10 \leq P(E) \leq 10≤P(E)≤1.
  - P(S)=1P(S) = 1P(S)=1 (for the sample space).
  - P(∅)=0P(\emptyset) = 0P(∅)=0 (for the empty set).

Important Rules:

- **Addition Rule**:
  - For two mutually exclusive events **A** and **B**: **P(A∪B) = P(A) + P(B)**.
  - For two non-mutually exclusive events A and B: P(A∪B) = P(A) + P(B) – P(A∩B).
- **Multiplication Rule**:
  - For independent events A and B: P(A∩B) = P(A) · P(B).
  - For conditional probability: P(A|B) = P(A∩B)/P(B), provided P(B) > 0.

Events in Probaility

In probability, **events** are subsets of a sample space and represent the outcomes or collections of outcomes of a random experiment.

Types of Events

Some common types of events in probability are:

- **Simple Event**: Contains a single outcome.
- **Compound Event:** Contains multiple outcomes.
- **Sure Event:** The entire sample space and it always occurs.
- **Impossible Event**: An empty set, ∅ as it never occurs.
- **Mutually Exclusive Events**: Events that cannot occur simultaneously.
- **Exhaustive Events**: A set of events is exhaustive if their union covers the entire sample space.
- **Independent Events**: Two events are independent if the occurrence of one does not affect the probability of the other.
- **Dependent Events**: Events where the occurrence of one affects the probability of the other.

Theorems: General – Let A, B, C are the events associated with a random experiment, then

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cup B) = P(A) + P(B)$ if A and B are mutually exclusive
- $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
- $P(A \cap B') = P(A) - P(A \cap B)$
- $P(A' \cap B) = P(B) - P(A \cap B)$

Extension of Multiplication Theorem: Let $A_1, A_2, \dots, A_n$ are n events associated with a random experiment, then $P(A_1 \cap A_2 \cap A_3 \dots \cap A_n) = P(A_1)P(A_2/A_1)P(A_3/A_2 \cap A_1) \dots P(A_n/A_1 \cap A_2 \cap A_3 \cap \dots \cap A_{n-1})$

Total Law of Probability: Let S be the sample space associated with a random experiment and $E_1, E_2, \dots, E_n$ be n mutually exclusive and exhaustive events associated with the random experiment . If A is any event which occurs with E_1 or E_2 or . . . or E_n, then

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \dots + P(E_n)P(A/E_n)$$

Conditional Probability: Conditional probability P(A | B) indicates the probability of event ‘A’ happening given that event B happened.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Product Rule: Derived from above definition of conditional probability by multiplying both sides with P(B)

$$P(A \cap B) = P(B) \times P(A/B)$$

Bayes’s Formula: If $A_1, A_2, \ldots, A_n$ are mutually exclusive and exhaustive events:

$$P(A_k|B) = \frac{P(B|A_k)P(A_k)}{\sum_{i=1}^{n} P(B|A_i)P(A_i)}$$

Random Variables

A random variable is basically a function which maps from the set of sample space to set of real numbers.

- **Discrete Random Variable**: Takes finite or countably infinite values.
  - **Example:** Number of heads in n coin tosses.
- **Continuous Random Variable**: Takes values in an interval.
  - **Example:** Time taken to complete a task.

Probability Distribution

A **Probability Distribution** describes how probabilities are assigned to outcomes or ranges of outcomes for a random variable.

Probability Mass Function (PMF)

- Used for **discrete random variables**.
  - P(X = x) = f(x), where f(x) satisfies:
    - $0 \le f(x) \le 1$,
    - $\sum_x f(x) = 1$

Probability Density Function (PDF)

- Used for **continuous random variables**.
    - $f(x) \geq 0, \quad \int_{-\infty}^{\infty} f(x)\,dx = 1$
        - Probability for a range $P(a \leq X \leq b) = \int_a^b f(x)\,dx$.

Cumulative Distribution Function (CDF)

- For both discrete and continuous random variables.
    - F(x) = P(X ≤ x)
        - For discrete X: $F(x) = \sum_{x_i \leq x} P(X = x_i)$.
        - For continuous X: $F(x) = \int_{-\infty}^{x} f(t)\,dt$

Expected Value (Mean):

- Discrete: E[X] = $\sum x_i P(X = x_i)$.
- Continuous: E[X] = $\int_{-\infty}^{\infty} x f(x)\,dx$.

- **Variance**: Var(X) = E[X^2] – $(E[X])^2$
- **Standard Deviation**: $\sigma =$ Var(X).

Important Distributions

Binomial Distribution

- For n independent trials with success probability p:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, \ldots, n$$

Mean: E[X] = np, Variance: Var(X) = np(1 – p).

Poisson Distribution:

- For rare events with rate λ:

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k = 0, 1, 2, \ldots$$

Mean and Variance: E[X] = Var(X) = λ.

Uniform Distribution:

- Continuous distribution over [a, b]:

$f(x) = 1/(b-a),\ a \le x \le b$

Mean: $E[X] = (a + b)/2$, Variance: $Var(X) = (b - a)^2/12$.

Normal Distribution:

- Bell-shaped curve with parameters μ (mean) and σ^2 (Varience):

$$f(x) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

68% of values lie within $\mu \pm \sigma$, 95% within $\mu \pm 2\sigma$.

Exponential Distribution: For a positive real number λ the probability density function of a Exponentially distributed Random variable is given by:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & if x \in R_X \\ 0 & if x \notin R_X \end{cases}$$

Where R_x is exponential random variables.

Mean = $1/\lambda$ and Variance = $1/\lambda^2$

Statistics

Descriptive Statistics

- **Measures of Central Tendency**:
  - **Mean**: Average of the dataset.
  - **Median**: Middle value when the data is sorted.
  - **Mode**: Most frequently occurring value in the dataset.
- **Measures of Spread**:

- **Range**: Difference between the maximum and minimum values.
- **Variance and Standard Deviation**: Measures of how much the data deviates from the mean.

Inferential Statistics

- **Sampling Distribution**: A probability distribution of a statistic (like the sample mean) obtained through repeated sampling.
- **Central Limit Theorem (CLT)**: States that the sampling distribution of the sample mean approaches a normal distribution as the sample size increases, regardless of the population's distribution.

Calculus

Existence of Limit: The **limits of a function** f(x) at x = a exists only when its left hand limit and right hand limit exist and are equal i.e. $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

Also Read **Formal Definition of Limit**.

Some Common Limits:

- $\lim_{x \to 0} \frac{\sin x}{x} = 1$
- $\lim_{x \to 0} \cos x = 1$
- $\lim_{x \to 0} \frac{\tan x}{x} = 1$
- $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$
- $\lim_{x \to 0} \frac{\sin x^\circ}{x} = \frac{\pi}{180}$
- $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$
- $\lim_{x \to \infty} \left(1 + \frac{k}{x}\right)^{mx} = e^{mk}$
- $\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$
- $\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a$
- $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$
- $\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$
- $\lim_{x \to \infty} x^{\frac{1}{x}} = 1$

Read More about **Properties of Limits**.

L'Hospital Rule:

If the given limit $\lim_{x \to a} \frac{f(x)}{g(x)}$ is of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ i.e. both f(x) and g(x) are either 0 or ∞, then the limit can be solved by **L'Hospital Rule**.

If the limit is of the form described above, then the L'Hospital Rule says that:

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Where f'(x) and g'(x) obtained by differentiating f(x) and g(x). If after differentiating, the form still exists, then the rule can be applied continuously until the form is changed.

Continuity:

A function is said to be continuous over a range if it's graph is a single unbroken curve. Formally, a real valued function f(x) is said to be continuous at a point x = x_0 in the domain if: $\lim_{x\to x_\circ} f(x)$ exists and is equal to f(x_0).

If a function f(x) is continuous at x = x_0 then:

$$\lim_{x\to x_\circ^+} f(x)=\lim_{x\to x_\circ^-} f(x)=\lim_{x\to x_\circ} f(x)$$

Functions that are not continuous are said to be discontinuous.

Also Read about Continuity at a Point.

Differentiability:

The derivative of a real valued function f(x) wrt x is the function f'(x) and is defined as:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

A function is said to be **differentiable** if the derivative of the function exists at all points of its domain. For checking the differentiability of a function at point x = c, $\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$ must exist.

Note: If a function is differentiable at a point, then it is also continuous at that point, but if a function is continuous at a point does not imply that the function is also differentiable at that point. For example, f(x) = |x| is continuous at x = 0 but it is not differentiable at that point.

Lagrange's Mean Value Theorem:

Suppose $f : [a, b] \rightarrow R$ be a function satisfying three conditions:

- f(x) is continuous in the closed interval a ≤ x ≤ b
- f(x) is differentiable in the open interval a < x < b

Then according to Lagrange's Theorem, there exists **at least one** point 'c' in the open interval (a, b) such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Rolle's Mean Value Theorem:

Suppose f(x) be a function satisfying three conditions:

- f(x) is continuous in the closed interval a ≤ x ≤ b
- f(x) is differentiable in the open interval a < x < b
- f(a) = f(b)

Then according to Rolle's Theorem, there exists **at least one** point 'c' in the open interval (a, b) such that:

f '(c) = 0

Differentiation Formulas

Some of the most common formula used to find derivative are tabulated below:

d/dx(c)	0
d/dx{c.f(x)}	c.f'(x)
d/dx(x)	1
d/dx(x^n)	nx^{n-1}
d/dx{f(g(x))}	f'(g(x)).g'(x)
d/dx(a^x)	a^x.ln(a)

d/dx{ln(x)} {Note: ln(x) = $\log_e(x)$}	1/x, x>0
d/dx($\log_a x$)	1/xln(a)
d/dx(e^x)	e^x
d/dx{sin(x)}	cos(x)
d/dx{cos(x)}	-sin(x)
d/dx{tan(x)}	$\sec^2 x$
d/dx{sec(x)}	sec(x).tan(x)
d/dx{cosec(x)}	-cosec(x).cot(x)
d/dx{cot(x)}	$-\text{cosec}^2(x)$
d/dx{$\sin^{-1}(x)$}	$1/\sqrt{(1 - x^2)}$
d/dx{$\cos^{-1}(x)$}	$-1/\sqrt{(1 - x^2)}$
d/dx{$\tan^{-1}(x)$}	$1/(1+x^2)$

Maxima and Minima

- **Critical Points**: Points where the derivative f'(x) = 0 or f'(x) is undefined.
- **Local Maximum**: f(x) has a local maximum at x = c if f(c) ≥ f(x) for all x in a small neighborhood around c.
- **Local Minimum**: f(x) has a local minimum at x = c if f(c) ≤ f(x) for all x in a small neighborhood around c.
- **Global Maximum/Minimum**: The highest/lowest value of f(x) over its entire domain.

Read More about Maxima and Minima.

First Derivative Test:

- If f'(x) changes:

- From **positive to negative** at x = c: Local Maximum at c.
- From **negative to positive** at x = c: Local Minimum at c.
- If no change: x = c is a point of inflection.

Second Derivative Test:

- Compute the second derivative $f''(x)$:
  - If $f''(c) > 0$: Local Minimum at c.
  - If $f''(c) < 0$: Local Maximum at c.
  - If $f''(c) = 0$: Test is inconclusive (use the first derivative test or higher-order derivatives).

Concavity:

- f(x) is:
  - **Concave Up** if $f''(x) > 0$.
  - **Concave Down** if $f''(x) < 0$.
- Point of inflection: f(x) changes concavity (where $f''(x) = 0$.

Maxima and Minima in Multivariable Functions:

For f(x, y):

1. **Critical Points**: Solve $\partial f/\partial x = 0$ and $\partial f/\partial y = 0$.
2. **Second Partial Derivatives**: Compute:
   - $f_{xx} = \partial^2 f/\partial x^2$,
   - $f_{yy} = \partial^2 f/\partial y^2$,
   - $fxy = \partial^2 f/\partial x \partial y$.

3. **Hessian Determinant**: $H = f_{xx}f_{yy} - (f_{xy})^2$.
   - If $H > 0$ and $f_{xx} > 0$: Local Minimum.
   - If $H > 0$ and $f_{xx} < 0$: Local Maximum.
   - If $H < 0$: Saddle Point.
   - If $H = 0$: Test is inconclusive.

Integrals

Integrals can be classified as:

- **Definite Integral**

- Indefinite Integral
- Improper Integrals

Indefinite Integrals:

Let f(x) be a function. Then the family of all its antiderivatives is called the indefinite integral of a function f(x) and it is denoted by ∫f(x)dx.

- The symbol ∫f(x)dx is read as the indefinite integral of f(x) with respect to x. Thus ∫f(x)dx= ∅(x) + C. Thus, the process of finding the indefinite integral of a function is called integration of the function.

Fundamental Integration Formulas:

Some common integration formulas include:

1. $\int x^n dx = (x^{n+1}/(n+1))+C$
2. $\int (1/x)dx = (\log_e|x|)+C$
3. $\int e^x dx = (e^x)+C$
4. $\int a^x dx = ((a^x)/(\log_e a))+C$
5. ∫sin(x)dx = -cos(x)+C
6. ∫cos(x)dx = sin(x)+C
7. $\int \sec^2(x)dx = \tan(x)+C$
8. $\int \operatorname{cosec}^2(x)dx = -\cot(x)+C$
9. ∫sec(x)tan(x)dx = sec(x)+C
10. ∫cosec(x)cot(x)dx = -cosec(x)+C
11. ∫cot(x)dx = log|sin(x)|+C
12. ∫tan(x)dx = log|sec(x)|+C
13. ∫sec(x)dx = log|sec(x)+tan(x)|+C
14. ∫cosec(x)dx = log|cosec(x)-cot(x)|+C

Definite Integrals:

Definite integrals are the extension after indefinite integrals, definite integrals have limits [a, b]. It gives the area of a curve bounded between given limits.

> $\int_a^b F(x)dx$*, it denotes the area of curve F(x) bounded between a and b, where a is the lower limit and b is the upper limit.*

Note: If f is a continuous function defined on the closed interval [a, b] and F be an anti derivative of f. Then $\int_a^b f(x)dx = [F(x)]_a^b = F(a) – F(b)$.

Here, the function f needs to be well defined and continuous in [a, b].

1. $\int_a^b f(x)dx = \int_a^b f(t)dt$
2. $\int_a^b f(x)dx = -\int_b^a f(x)dx$
3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
4. $\int_a^b f(x) = \int_a^b f(a+b-x)dx$
5. $\int_0^b f(x) = \int_0^b f(b-x)dx$
6. $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$
7. $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$, if f(x) is even function i.e f(x) = f(-x)
8. $\int_{-a}^a f(x)dx = 0$, if f(x) is odd function

Newton-Leibniz Rule

For a definite integral $F(x) = \int_{a(x)}^{b(x)} f(t)\,dt$:

$$\frac{d}{dx}\left[\int_{a(x)}^{b(x)} f(t)\,dt\right] = f(b(x)) \cdot b'(x) – f(a(x)) \cdot a'(x)$$

Application of Integrals

Some common application of integrals are:

- Area Under a Curve
  - Area Between Curves
  - Area Between Polar Curves
- Arc Length of Curves
- Volume of Solids of Revolution
- Surface Area of Revolution

Area Under a Curve

The area enclosed between a curve y = f(x), the x-axis, and the limits x = a and x = b is:

$$\text{Area} = \int_a^b f(x)\,dx$$

- If f(x) < 0, take the absolute value of the integral.

Between Two Curves

The area between two curves y = f(x) and y = g(x) from x = a to x = b is:

$$Area = \int_a^b \; f(x) - g(x) \; dx.$$

Length of a Curve

The length of a curve y = f(x) from x = a to x = b is:

$$Length = \int_a^b \quad 1 + \left(\frac{dy}{dx}\right)^2 dx$$

For parametric equations x = x(t), y = y(t), the arc length is:

$$Length = \int_{t_1}^{t_2} \quad \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 dt$$

Volume of Solids of Revolution

- **Disk Method**: When a curve y = f(x) is revolved about the x-axis: Volume = $\pi \int_a^b [f(x)]^2 \; dx$
- **Shell Method**: When a curve y = f(x) is revolved about the y-axis: Volume = $2\pi \int_a^b x \cdot f(x) \, dx.$
- For parametric equations x = x(t), y = y(t):
    - Revolved about x-axis: Volume = $\pi \int_{t_1}^{t_2} [y(t)]^2 \frac{dx}{dt} \, dt$

Surface Area of Solids of Revolution

1. **Revolution about the x-axis**:Surface Area = $2\pi \int_a^b f(x) \quad 1 + \left(\frac{dy}{dx}\right)^2 dx$
2. **Revolution about the y-axis**: Surface Area = $2\pi \int_a^b x \quad 1 + \left(\frac{dy}{dx}\right)^2 dx$

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