UNIT-2

Angle Modulation

Note Title 27-04-2021

Angle modulation:

Angle of the carrier wave is varied according to the amplitude of message signal. In this method of modulation the amplitude of the carrier wave is maintained constant.

Angle modulation provides better discrimination against noise and interference than amplitude modulation.

There are two forms of angle modulation.

1. Frequency Modulation
2. Phase modulation

Basic definitions:

→ Let $\theta_i(t)$ = Angle of a modulated sinusoidal carrier.

→ Angle modulated wave is expressed as $s(t) = A_c \cos \theta_i(t)$

→ A_c = Amplitude of carrier wave.

A complete oscillation occurs whenever $\theta_i(t)$ changes by 2π radians.

$$\theta_i(t) = 2\pi f_i t$$

$\theta_i = \omega_i t$

$\theta_i = 2\pi f_i t$

$$f_i = \frac{1}{2\pi t}\{\theta_i(t)\}$$

If $\theta_i(t)$ increases monotonically with time, the average frequency over an interval from t to $t+\Delta t$ is given by

$$\rightarrow f_i(t) = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi(\cancel{t}+\Delta t) - \cancel{t}} = \frac{\overbrace{\theta_i(t+\Delta t) - \theta_i(t)}^{d\theta}}{2\pi \underbrace{(\Delta t)}_{dt}}$$

$$f_i(t) = \frac{1}{2\pi}\left[\frac{d\theta}{dt}\right] \longrightarrow (1)$$

↳ instantaneous frequency

$$d\theta = 2\pi f_i(t)\,dt$$

Integrate on both sides

$$\int d\theta = \int 2\pi f_i(t)\,dt$$

$$\theta = 2\pi \int f_i(t)\,dt$$

So, equation of Angle modulated wave

$S(t) = A_c \cos \theta_i(t) = A_c \cos(2\pi \int f_i(t)\,dt)$

$$S(t) = A_c \cos\left(2\pi \int f_i(t)\,dt\right) \longrightarrow (1)$$

Frequency modulation:

↳ It is a form of angle modulation in which the instantaneous frequency $f_i(t)$ is varied linearly with the baseband signal $m(t)$ as shown by

$$f_i(t) = f_c + k_f m(t)$$

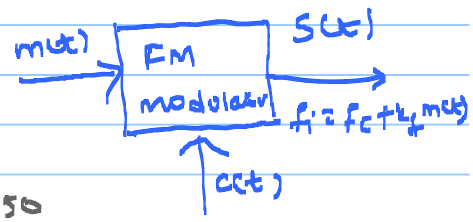
where k_f = frequency sensitivity = Hz/volt

$k_f = 5\text{kHz/volt}$

$m(t) = 10 \quad f_i = f_c + 50$

$m(t) = 0 \quad f_i = f_c$

$m(t) = -10 \quad f_i = f_c - 50$



If $m(t) = 0$, the o/p of FM modulator is carrier wave whose frequency is f_c.

If $m(t)$ is maximum f_i will be maximum, If $m(t)$ is minimum f_i will be minimum.

consider $S(t) = A_c \cos\left[2\pi \int f_i(t)\,dt\right]$

$\rightarrow f_i(t) = f_c + k_f m(t)$

$$S(t) = A_c \cos\left[2\pi \int (f_c + k_f m(t))\,dt\right]$$

$$= A_c \cos\left[2\pi\left(\int f_c\,dt + k_f \int m(t)\,dt\right)\right]$$

$$S(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\,dt\right]$$

↳ Time domain equation of frequency modulation wave.

Single tone frequency modulation:

Consider time domain equation of frequency modulation:

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\,dt\right]$$

$m(t) = A_m \cos 2\pi f_m t$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int A_m \cos 2\pi f_m t\,dt\right]$$

$$= A_c \cos\left[2\pi f_c t + \cancel{2\pi} k_f A_m \frac{\sin 2\pi f_m t}{\cancel{2\pi} f_m}\right]$$

$$= A_c \cos\left[2\pi f_c t + \underbrace{\frac{k_f A_m}{f_m}}_{\beta} \sin 2\pi f_m t\right]$$

$$=$$

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right]}$$

where $\beta = \frac{k_f A_m}{f_m}$ = modulation index

$\Delta f = k_f A_m$ = frequency deviation

$$\beta = \frac{\Delta f}{f_m}$$

Depending on the value of modulation index β, frequency modulation can be classified into

(i) Narrow Band FM ($\beta < 1$) (β is small)

(ii) Wide band FM ($\beta > 1$) (β is large)

Narrow Band Frequency modulation: ($\beta < 1$)

$\hookrightarrow \beta$ is small

Consider time domain equation of single tone frequency modulation:

$$S(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

$$S(t) = A_c[\cos 2\pi f_c t \cos(\beta \sin 2\pi f_m t) - \sin 2\pi f_c t \sin(\beta \sin 2\pi f_m t)]$$

Consider $\theta = \beta \sin 2\pi f_m t$

$$S(t) = A_c[\cos 2\pi f_c t \cos\theta - \sin 2\pi f_c t \sin\theta]$$

$\beta < 1$, $\theta < 1$, as $\theta = \beta \sin 2\pi f_m t$

so, $\cos\theta \approx 1$, $\sin\theta = \theta$

$$S(t) = A_c[\cos 2\pi f_c t - \sin 2\pi f_c t(\theta)]$$

$$S(t) = A_c[\cos 2\pi f_c t - \theta \sin 2\pi f_c t]$$

$$S(t) = A_c[\cos 2\pi f_c t - \beta \sin 2\pi f_m t \sin 2\pi f_c t]$$

$$= A_c\left[\cos 2\pi f_c t - \frac{\beta}{2}(2 \sin 2\pi f_c t \sin 2\pi f_m t)\right]$$

$[\cos(A-B) - \cos(A+B)] = 2\sin A \sin B$

$$= A_c\left[\cos 2\pi f_c t - \frac{\beta}{2}\{\cos 2\pi(f_c - f_m)t - \cos 2\pi(f_c + f_m)t\}\right]$$

$$S(t) = A_c\left[\cos 2\pi f_c t - \frac{\beta}{2}\cos 2\pi(f_c - f_m)t + \frac{\beta}{2}\cos 2\pi(f_c + f_m)t\right]$$

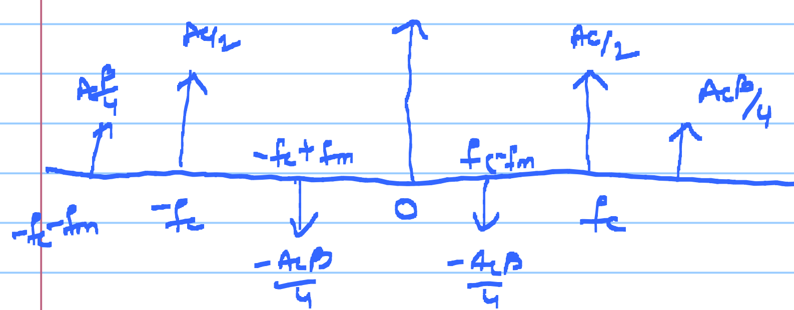
Apply Fourier transform:

$$S(t) = A_c\left[\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2} - \frac{\beta}{2}\left(\frac{e^{j2\pi(f_c - f_m)t} + e^{-j2\pi(f_c - f_m)t}}{2}\right) + \frac{\beta}{2}\left(\frac{e^{j2\pi(f_c + f_m)t} + e^{-j2\pi(f_c + f_m)t}}{2}\right)\right]$$

$$s(t) = \frac{A_c}{2}\left[e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right] - \frac{A_c\beta}{4}\left[e^{j2\pi(f_c-f_m)t} + e^{-j2\pi(f_c-f_m)t}\right]$$
$$+ \frac{A_c\beta}{4}\left[e^{j2\pi(f_c+f_m)t} + e^{-j2\pi(f_c+f_m)t}\right]$$

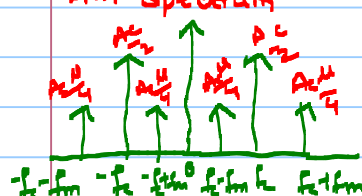
$$S(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] - \frac{A_c\beta}{4}\left[\delta(f-(f_c-f_m)) + \delta(f+(f_c-f_m))\right]$$
$$+ \frac{A_c\beta}{4}\left[\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m))\right]$$

Spectrum of NBFM:

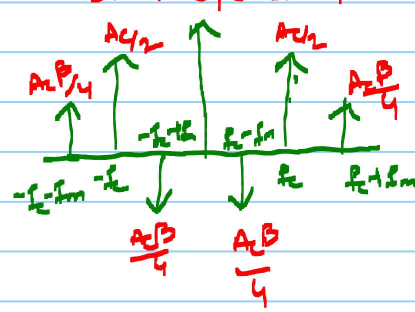


Spectral Analysis:

AM Spectrum



NBFM Spectrum



→ **AM equation:**

$$S(f) = \underbrace{\frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]} + \underbrace{\frac{A_c\mu}{4}\left[\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m))\right]}$$
$$+ \underbrace{\frac{A_c\mu}{4}\left[\delta(f-(f_c-f_m)) + \delta(f+(f_c-f_m))\right]}_{LSB}$$

→ **FM equation:**

$$S(f) = \underbrace{\frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]} + \underbrace{\frac{A_c\beta}{4}\left(\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m))\right)}$$
$$- \underbrace{\frac{A_c\beta}{4}\left(\delta(f-(f_c-f_m)) - \delta(f+(f_c-f_m))\right)}_{LSB}$$

Note:

The basic difference between the NBFm and Am wave is that sign of lower side band in the NBFm is reversed.

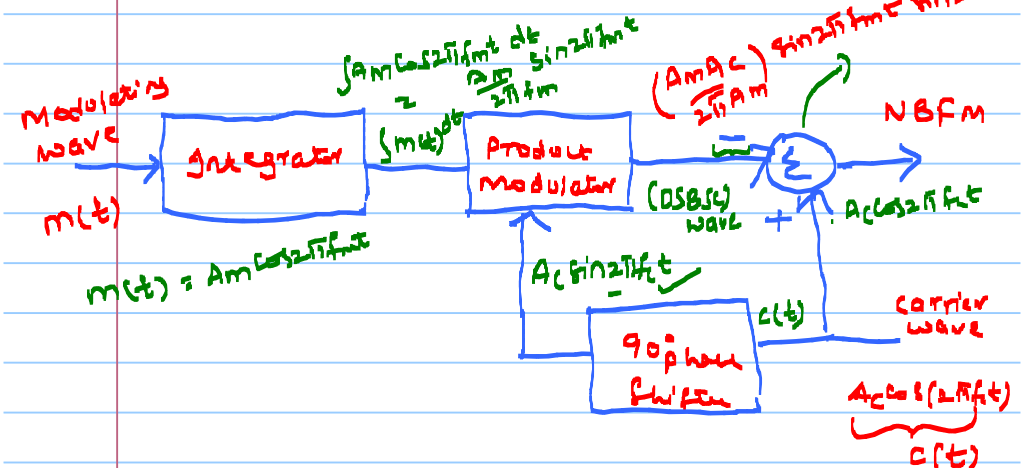
Bandwidth of NBFM:

$$BW = f_H - f_L = f_c + f_m - (f_c - f_m)$$
$$= \cancel{f_c} + f_m - \cancel{f_c} + f_m$$
$$= 2f_m$$

Block diagram for generating NBFM:

Time domain equation of NBFM

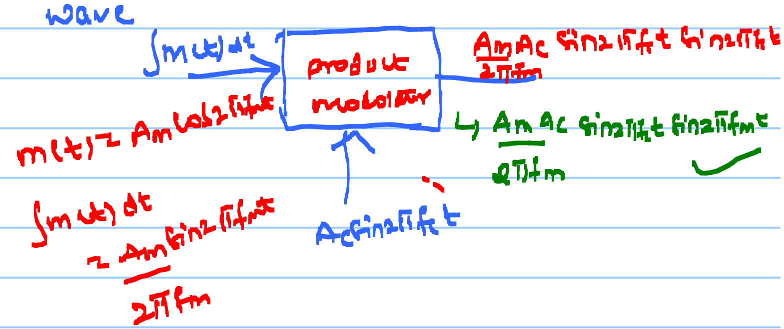
→ $s(t) = A_c \cos 2\pi f_c t - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$



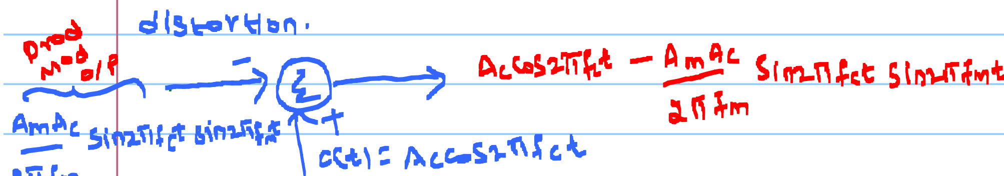
Block diagram for generating NBFM:

→ This modulator involves splitting the carrier wave $A_c \cos(2\pi f_c t)$ into two paths. One path is direct and other contains a 90° phase shifting network.

→ Product modulator generates a DSBSC wave



→ Combination of these two signals produces a narrow band FM wave with some distortion.



$$S(t) = A_c\left[\cos 2\pi f_c t - \frac{A_m}{2\pi f_m} \sin 2\pi f_c t \sin 2\pi f_m t\right] \rightarrow (1)$$

Compare eq (1) time domain eq. of NBFM

→ Time domain eq. of NBFM, $\beta = \frac{k_f A_m}{f_m}$

(*) $S(t) = A_c[\cos 2\pi f_c t - \beta \sin 2\pi f_c t \sin 2\pi f_m t]$

Consider eq (1)

→ $S(t) = A_c\left[\cos 2\pi f_c t - \left(\frac{1}{2\pi}\right)^{k_f} \frac{A_m}{f_m} \sin 2\pi f_c t \sin 2\pi f_m t\right]$

where $k_f = 1/2\pi$

→ $S(t) = A_c\left[\cos 2\pi f_c t - \frac{k_f A_m}{f_m} \sin 2\pi f_c t \sin 2\pi f_m t\right]$

So, finally

$$S(t) = A_c(\cos 2\pi f_c t - \beta \sin 2\pi f_c t \sin 2\pi f_m t]$$

Phase Modulation:

→ It is a form of angle modulation in which the angle $\theta_i(t)$ is varied linearly with the baseband signal $m(t)$.

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

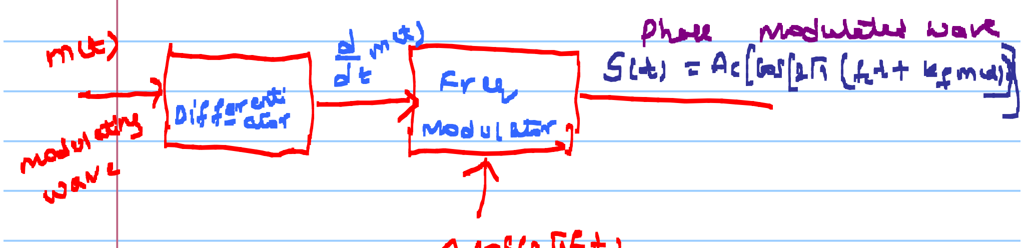
k_p = phase sensitivity

Time domain equation of phase-modulated wave is given by

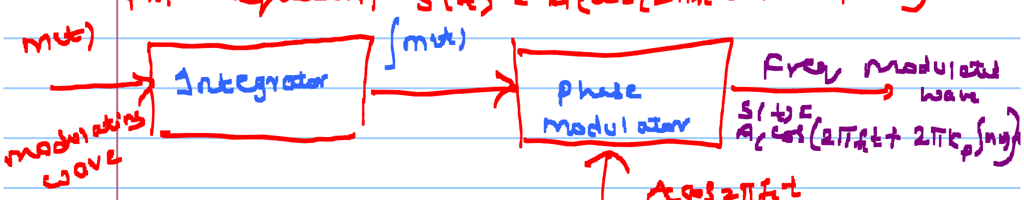
$$S(t) = A_c \cos \theta_i(t)$$

$$S(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

FM equation: $S(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int m(t)dt]$.



PM equation; $S(t) = A_c \cos[2\pi f_c t + 2\pi k_p m(t)]$



Wide Band FM: $(\beta > 1)$

Note Title 12-05-2021

↳ β = modulation index

Consider time domain equation of FM wave for single tone modulation.

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t) \rightarrow (1)$$

According to exponential Fourier series

$$\rightarrow g_p(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$\rightarrow C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p(t)\, e^{-jn\omega_0 t}$$

$\omega_0 = \frac{2\pi}{T_0} = 2\pi f_m$

$f_m = \frac{1}{T_0}$

T_0 : Time period of periodic signal

$$\cos\theta = Re(e^{j\theta})$$

Consider $s(t)$

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$$

$$s(t) = A_c\, Re\left[e^{j(2\pi f_c t) + j\beta \sin 2\pi f_m t}\right]$$

$$= A_c\, Re\left[e^{j2\pi f_c t}\, \underbrace{e^{j\beta \sin 2\pi f_m t}}_{g_p(t)}\right]$$

$$= A_c\, Re\left(e^{j2\pi f_c t}\, g_p(t)\right) \rightarrow (2)$$

$$g_p(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn2\pi f_m t}$$

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p(t)\, e^{-jn(2\pi f_m)t}$$

$$g_p(t) = e^{j\beta \sin 2\pi f_m t}, \quad f_m = \frac{1}{T_0}$$

$$c_n = f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} e^{j\beta \sin 2\pi f_m t}\, e^{-jn(2\pi f_m t)}\, dt$$

$$= f_m \int_{-1/2f_m}^{1/2f_m} e^{j\beta \sin 2\pi f_m t - jn(2\pi f_m t)}\, dt$$

Consider $x = \beta$,

$\theta = 2\pi f_m t$

$d\theta = 2\pi f_m dt$

$dt = \frac{1}{2\pi f_m} d\theta$

Limits: at $t = \frac{-1}{2f_m}$

$\rightarrow \theta = 2\pi f_m\left(\frac{-1}{2f_m}\right)$

$\theta = -\pi$

$\rightarrow t = \frac{1}{2f_m}$

$\theta = 2\pi f_m\left(\frac{1}{2f_m}\right) = \pi$

$$c_n = f_m \int_{-\pi}^{\pi} e^{jx\sin\theta - jn\theta} \left(\frac{1}{2\pi f_m}\right) d\theta$$

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{jx\sin\theta - jn\theta} \, d\theta$$

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(x\sin\theta - n\theta)} \, d\theta$$

↳ same as Bessel function:

$$J_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(x\sin\theta - n\theta)} \, d\theta$$

↳ $$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin(2\pi f_m t) - n 2\pi f_m t)} \, d\theta$$

↳ $\boxed{c_n = J_n(\beta)}$

Consider eq (2)

$$s(t) = A_c \, Re\left[e^{j2\pi f_c t} g_p(t)\right]$$

$$g_p(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn2\pi f_m t}$$

$$c_n = J_n(\beta)$$

$$s(t) = A_c \, Re\left[e^{j2\pi f_c t} \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn2\pi f_m t}\right]$$

$$= A_c \, Re\left(\sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi(f_c + n f_m)t}\right)$$

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi(f_c + n f_m)t.$$

↳ time domain eq. of WBFM.

Spectrum:

$$S(f) = \frac{A_c}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f-(f_c+nf_m)) + \delta(f+(f_c+nf_m))\right]$$

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f-(f_c+nf_m)) + \delta(f+(f_c+nf_m)) \right]$$

$n=0$ $\quad S(f) = \frac{A_c}{2} \left(\delta(f-f_c) + \delta(f+f_c) \right)$

$n=1$

$$S(f) = \frac{A_c}{2} \sum_{n=-1}^{1} J_n(\beta) \left[\delta(f-(f_c+nf_m)) + \delta(f+(f_c+nf_m)) \right]$$

$$= \frac{A_c}{2} \Big[J_{-1}(\beta) \left(\delta(f-(f_c-f_m)) + \delta(f+(f_c-f_m)) \right)$$
$$+ J_0(\beta) \left[\delta(f-f_c) + \delta(f+f_c) \right]$$
$$+ J_1(\beta) \left(\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m)) \right)$$

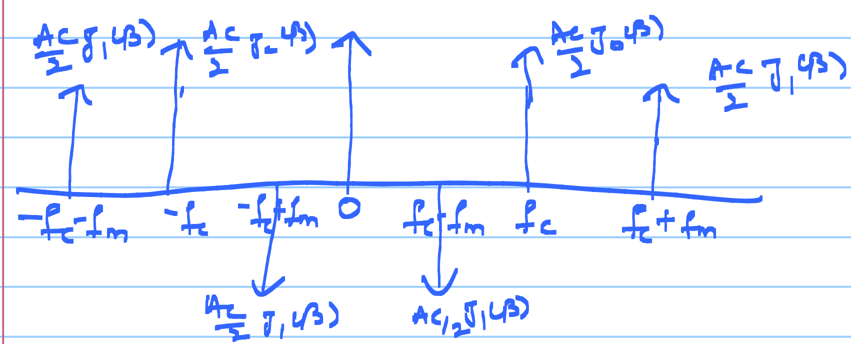
$$J_{-1}(x) = -J_1(x)$$

$$J_{-1}(\beta) = -J_1(\beta)$$

1

$$S(f) = \frac{A_c}{2} \Big\{ -J_1(\beta) \left[\delta(f-(f_c-f_m)) + \delta(f+(f_c-f_m)) \right]$$
$$+ J_0(\beta) \left[\delta(f-(f_c)) + \delta(f+f_c) \right]$$
$$+ J_1(\beta) \left[\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m)) \right]$$

$\underbrace{n=1}$



Analysis of Spectrum:

The spectrum consists of carrier and infinite number of upper and lower side band frequency components

Theoritical Bandwidth = ∞

Transmission Bandwidth using Carson's Rule:

→ The theoritical Bandwidth of the FM signal is ∞ but the bandwidth of a signal should be as low as possible. To reduce the BW insignificant frequency are eliminated.

According to carson's Rule $(\beta+1)$ upper and lower sidebands will have significant magnitudes and contains 99% of the total power.

So $\boxed{BW = 2(\beta+1)f_m}$, $\beta = \frac{\Delta f}{f_m}$

where f_m = Highest frequency for message signal

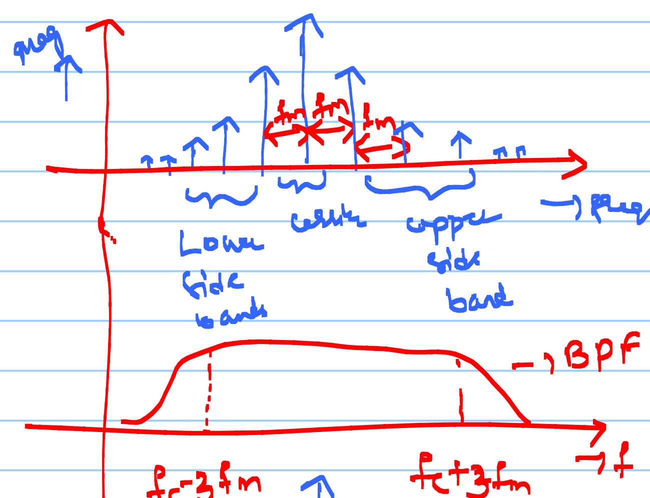
Δf : frequency deviation

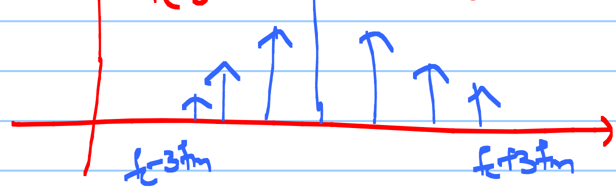
$$BW = 2\left(\frac{\Delta f}{f_m}+1\right)f_m$$

$$\boxed{BW = 2(\Delta f + f_m)}$$

When $\beta = 2$, total 7 comp we get

7 comp → 3 Lower sidebands
→ 1 carrier comp.
→ 3 upper Side band





$BW = f_c+3f_m-(f_c-3f_m) = \cancel{f_c}+3f_m-\cancel{f_c}+3f_m$
$= 6f_m$

Power calculations:

For WBFM : $\beta > 1$

Time domain equation of WBFM signal

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi (f_c + n f_m) t$$

$$\begin{aligned}
s(t) = \; & \ldots\ldots A_c J_2(\beta) \cos 2\pi (f_c - 2f_m) t \\
& - A_c J_1(\beta) \cos 2\pi (f_c - f_m) t \\
& + A_c J_0(\beta) \cos 2\pi f_c t \\
& + A_c J_1(\beta) \cos 2\pi (f_c + f_m) t \\
& + A_c J_2(\beta) \cos 2\pi (f_c + 2 f_m) t \\
& + \\
& \quad \vdots \\
& + A_c J_n(\beta) \cos 2\pi (f_c + n f_m) t
\end{aligned}$$

Power $P = \dfrac{V_{rms}^2}{R}$

$$P = \left(\frac{A_c J_0(\beta)}{\sqrt{2}}\right)^2 \Big/ R = \frac{A_c^2 J_0^2(\beta)}{2R}$$

$$P_{f_c + f_m} = \frac{A_c^2 J_1(\beta)}{2R}$$

$$P_{f_c - f_m} = \frac{A_c^2}{2R} J_1^2(\beta)$$

$$P_{f_c + 2f_m} = \frac{A_c^2}{2R} J_2^2(\beta)$$

$$P'_{f_c - 2f_m} = \frac{A_c^2}{2R} J_2^2(\beta)$$

$$\vdots$$

$$P_{f_c + n f_m} = \frac{A_c^2}{2R} J_n^2(\beta)$$

↳ Add above all sideband freq. components

total power $P_t = \dfrac{A_c^2}{2R} \displaystyle\sum_{n=-\infty}^{\infty} J_n^2(\beta)$

Total power $P_t = \dfrac{A_c^2}{2R}$ $\left(\text{where } \displaystyle\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1\right)$

$$P_t = \frac{A_c^2}{2R} \quad \text{(Carrier power)}$$

Power calculation for NBFM:

Time domain equation of NBFM

$$s(t) = A_c\left(\cos(2\pi f_c t) - \frac{\beta}{2}\cos 2\pi (f_c - f_m)t + \frac{\beta}{2}\cos 2\pi(f_c+f_m)t\right)$$

$$P_t = P_c + P_{LSB} + P_{USB}$$

$$P_c = \frac{A_c^2}{2R}, \quad P_{LSB} = \frac{A_c^2\beta^2}{8R}, \quad P_{USB} = \frac{A_c^2\beta^2}{8R}$$

$$P_{total} = \frac{A_c^2}{2R} + \frac{A_c^2\beta^2}{8} + \frac{A_c^2\beta^2}{8}$$

$$= \frac{A_c^2}{2R} + \frac{A_c^2\beta^2}{4}$$

$$= \frac{A_c^2}{2R}\left[1 + \frac{\beta^2}{2}\right]$$

$$P_{total} = P_c\left[1+\beta^2/2\right]$$

↳ Same as AM power.

where β = Modulation index

→ Bandwidth of NBFM:

$BW = 2f_m$

Bandwidth of WBFM $= 2(\beta+1)f_m$

→ Total power WBFM $= P_c\left(1+\frac{\beta^2}{2}\right)$

Total power WBFM $= \frac{A_c^2}{2R} = P_c$

→ WBFM $(\beta > 1)$

NBFM $(\beta < 1)$

Comparison between AM and FM:

	Amplitude modulation	(WBFM) Frequency Modulation:
Def:	Process of varying the amplitude of carrier w.r.t the amplitude of msg. signal.	Process of varying the frequency of carrier w.r.t the amplitude of msg. signal.
Power:	$P_t = P_c\left(1+\frac{\mu^2}{2}\right)$ Power varies with modulation index	$P_t = \frac{A_c^2}{2R} = P_c$ Power is independent of Modulation index.
Bandwidth:	$BW = 2f_m$ BW is independent of Modulation index.	$BW = 2(\beta+1)f_m$ BW varies with Modulation index.
Receiver complexity:	AM Receiver is simple.	FM Receiver is more complex. (Amplitude limiter, De emphasis ckt is used).
Effect of noise:	The effect of noise is more. (Sny is in the form of voltage which effects easily)	Effect of noise is less. (Sny is the form of freq. and we will use amplitude limiter to remove noise)
→ practically:	→ (550K − 1650 KHz) carrier freq range → BW = 10KHz, → IF = 455KHz	→ practically: → (88MHz − 108MHz) carrier freq. range BW = 200KHz IF = 10.7MHz

Generation of FM wave:

Two basic methods of generating frequency Modulated waves namely direct method and Indirect method.

In the direct method of producing frequency Modulation the carrier frequency is directly varied in accordance with amplitude of message signal.

In the Indirect method of producing frequency Modulation, the modulating wave is first used to produce a narrow band FM wave and frequency multipliers are used to ~~increase~~ increase the frequency deviation to the desired level.

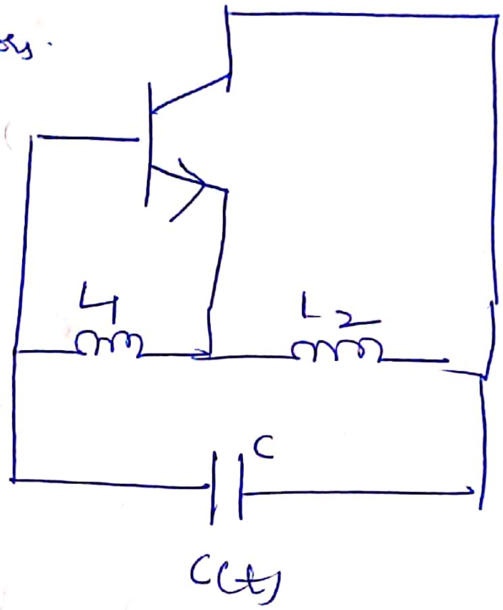
Direct Frequency Modulation:

→ In a direct FM system, the instantaneous frequency of the carrier ~~frequency~~ wave is varied directly in accordance with the baseband signal. by means of a device known as a Voltage Controlled oscillator.

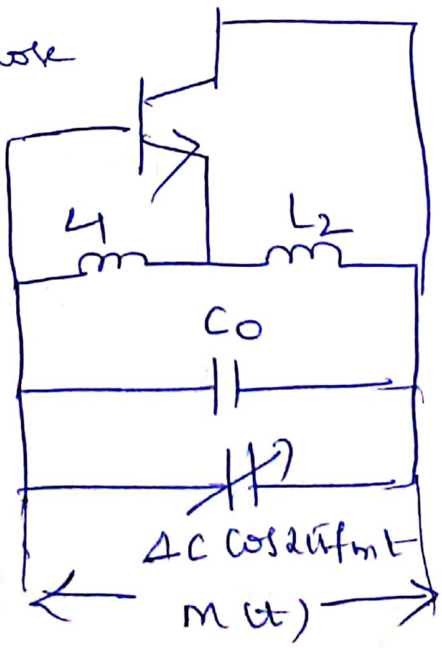
→ This device should be a relatively high quality factor frequency determining network.

→ An example of VCO is Hartly oscillator.

Circuit diagram of Hartly oscillator:



→ L_1, L_2 are Inductors. and the capacitive component of this device $C(t)$ consists of a fixed capacitor shunted ~~~~ by a voltage variable capacitor commonly called as a varicap or varactor.



→ Varicap is one whose capacitance depends upon the voltage applied across its electrodes.

→ The variable voltage capacitor may be obtained by using a p-n junction diode that is biased in the reverse direction. (varactor diode).

→ The larger the reverse voltage applied the smaller the transition capacitance.

$$C = \frac{\varepsilon A}{W}, \quad C = \frac{Q}{V}$$

W = width of the depletion region.

C = Capacitance.

V = Applied voltage.

$$C \propto \frac{1}{W}, \quad C \propto \frac{1}{V}, \quad C \propto \frac{1}{f}.$$

→ V↑ W↑ C↓ frequency increases.

→ V↓ W↓ C↑ frequency decreases.

→ The frequency of oscillation of the Hartley oscillator is given by

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C(t)}}$$

C(t) = Total capacitance of the fixed capacitor and the variable-voltage capacitor.

$$\boxed{C(t) = \underbrace{C_0}_{\text{fixed Capacitor}} + \underbrace{\Delta C\,(\cos(2\pi f_m t))}_{\text{Variable voltage Capacitor}}}$$

→ In the absence of modulation $m(t) = 0$,

$c(t) = C_0 \quad (\Delta C \cos 2\pi f_m t = 0)$

→ $f_i(t)$ becomes $= \dfrac{1}{2\pi\sqrt{(L_1+L_2)C_0}} = f_0$ = unmodulated frequency of oscillatio

→ f_0 = unmodulated freq. of oscillation. $= \dfrac{1}{2\pi\sqrt{(L_1+L_2)C_0}}$

→ Consider $f_i(t) = \dfrac{1}{2\pi\sqrt{(L_1+L_2)c(t)}}$

Substitute $C(t) = C_0 + \Delta C \cos 2\pi f_m t$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_0+\Delta C\cos 2\pi f_m t)}}$$

$$= \frac{1}{2\pi\sqrt{(L_1+L_2)C_0\left[1+\frac{\Delta C}{C_0}\cos 2\pi f_m t\right]}}$$

$$= \underbrace{\frac{1}{2\pi\sqrt{(L_1+L_2)C_0}}}_{f_0}\left(\frac{1}{\sqrt{\left(1+\frac{\Delta C}{C_0}\cos 2\pi f_m t\right)}}\right)$$

$$= f_0\left[\frac{1}{\left(1+\frac{\Delta C}{C_0}\cos 2\pi f_m t\right)^{1/2}}\right] = f_0\left(1+\frac{\Delta C}{C_0}\cos 2\pi f_m t\right)^{-1/2}$$

$$f_i(t) = f_0\left[1+\frac{\Delta C}{C_0}\cos 2\pi f_m t\right]^{-1/2}.$$

By applying Binomial expansion formula,

$$f_i(t) = f_o\left(1 - \frac{\Delta C}{2C_o}\cos 2\pi f_m t + \cdots\right)$$

$\Delta C << C_o$, So, $\frac{\Delta C}{C_o} << 1$, all other higher order terms are neglected.

$$f_i(t) \simeq f_o\left(1 - \frac{\Delta C}{2C_o}\cos 2\pi f_m t\right)$$

Replace $\frac{-\Delta C}{2C_o} = \frac{\Delta f}{f_o}$

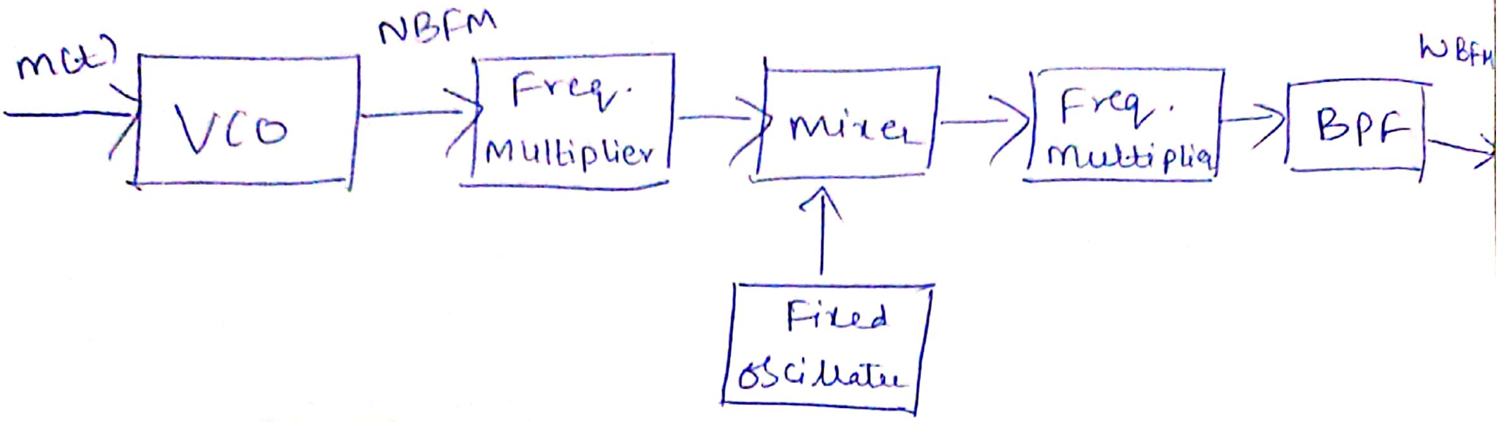
$$f_i(t) = f_o\left[1 + \frac{\Delta f}{f_o}\cos 2\pi f_m t\right)$$

$$\boxed{f_i(t) = f_o + \Delta f \cos 2\pi f_m t}$$

where $f_o = \frac{1}{2\pi\sqrt{(L_1+L_2)C_o}}$

↳ the Instantaneous frequency of the oscillator which is being frequency modulated by varying the Capacitance.

⟶ In order to generate wide band FM wave with ~~the~~ required frequency deviation and modulation index β. following block diagram is used.



NBFM: Narrow Band FM signal.

WBFM: Wide band FM signal.

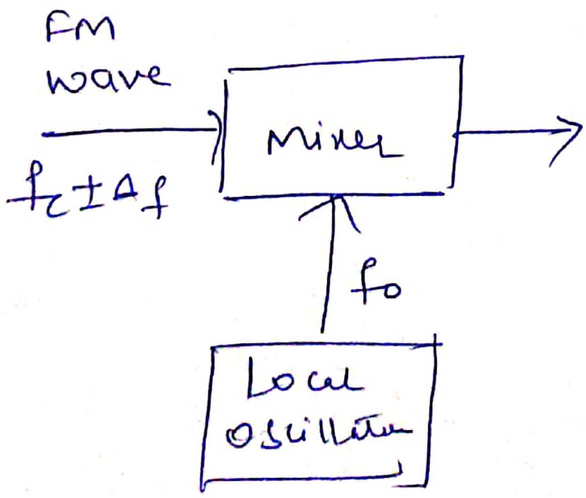
VCO : Voltage controlled oscillator.

Block diagram of WBFM by using VCO

Mixer:

Mixing of FM wave with a local oscillator will produce sum and difference frequency components at the o/p of the frequency mixer.

Mixer:



$f_c \pm \Delta f + f_o$

↳ sum comp.

$f_c \pm \Delta f - f_o$

↳ difference comp

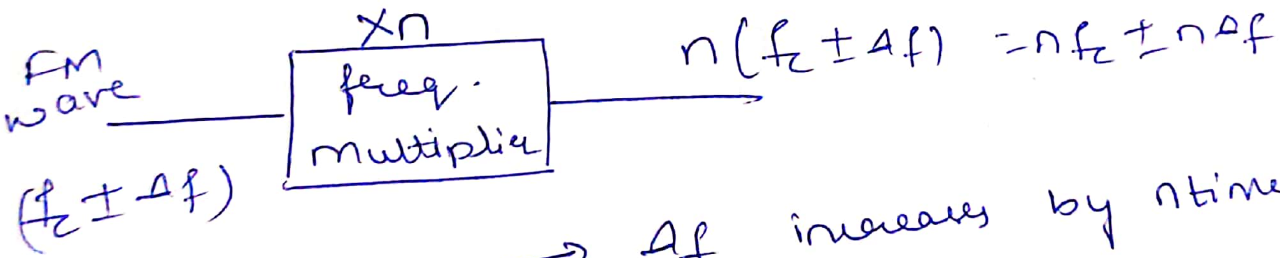
↳ o/p of mixer has $\boxed{\begin{array}{l} f_c + f_o \pm \Delta f \\ f_c - f_o \pm \Delta f \end{array}}$

↳ Carrier frequency f_c will increase or decrease by f_o.

↳ But Δf, frequency deviation remains unchanged, and modulation index β remains unchanged as $\boxed{\beta = \frac{\Delta f}{f_m}}$.

Frequency Multiplier:

→ If FM wave with carrier frequency f_c and deviation Δf are applied to a frequency multiplier, the carrier frequency f_c and deviation Δf both are multiplied equally.



→ Δf increases by n times.

→ as $\boxed{\beta = \frac{n\Delta f}{f_m}}$, β increases by n times

→ $\boxed{f_c = nf_c}$, carrier freq. increases by n times.

→ The modulation index β of the FM wave will also get multiplied by n in the process of frequency multiplication

Draw back of Direct FM method:

1, Difficult to maintain the stability of carrier frequency.

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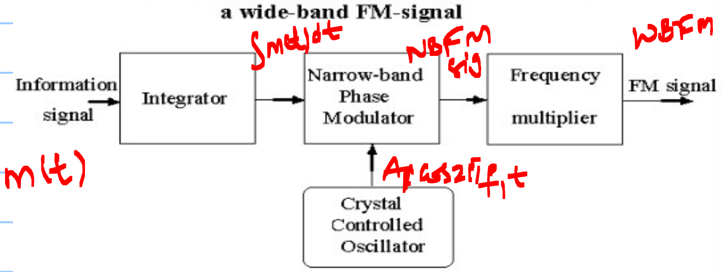
Indirect method:

→ Block diagram:

Block diagram of the indirect method of generating a wide-band FM-signal



↳ It was proposed by Armstrong and called as Armstrong method.

↳ The block diagram of Indirect method of generating a wideband FM signal Consists of
1. Integrator
2. Narrow band phase Modulator
3. crystal controlled oscillator
4. Frequency Multipliers.

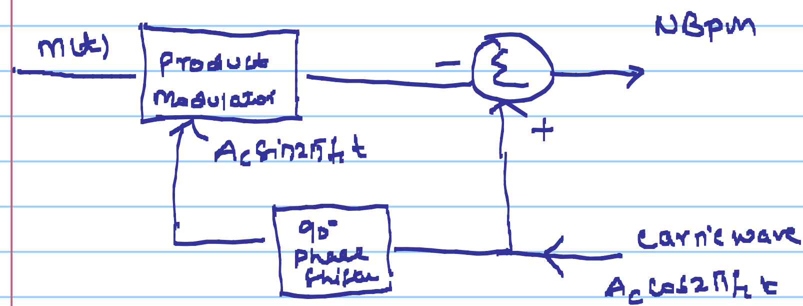
Working operation:

→ The baseband signal m(t) is first integrated and then used to phase modulate a crystal controlled oscillator.

→ In order to minimize the distortion inherent in phase modulator, the modulation index β is kept small. there by resulting in a narrow band FM wave.

→ The signal is next multiplied in frequency by means of a frequency multiplier to produce the desired wide band FM wave.

Block diagram of NBPM:



→ Let $s_1(t)$ denote the output of of the phase modulator.

$$s_1(t) = A_1 \cos\left(2\pi f_1 t + 2\pi k_p \int_0^t m(t)\,dt\right)$$

f_1 = freq. of crystal controlled oscillator.

k_p = constant

→ If $m(t) = A_m \cos 2\pi f_m t$

$$s(t) = A_1 \cos\left[2\pi f_1 t + 2\pi k_f \int_0^t A_m \cos 2\pi f_m t\right]$$

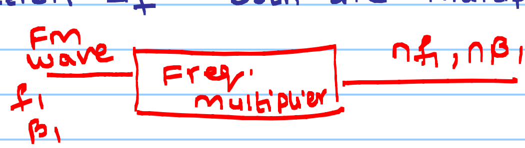
$$= A_1 \cos\left[2\pi f_1 t + \cancel{2\pi} k_f \left(\frac{A_m \sin 2\pi f_m t}{\cancel{2\pi} f_m}\right)\right]$$

$$= A_1 \cos\left[2\pi f_1 t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t\right]$$

$$s(t) = A_1 \cos\left[2\pi f_1 t + \beta_1 \sin 2\pi f_m t\right]$$

↳ The phase modulator O/p is next multiplied n times in frequency by the frequency multiplier producing the desired WBFM wave

↳ If FM wave with carrier freq f_c and deviation Δf are applied to a freq multiplier, the carrier freq f_c and deviation Δf both are multiplied equally.



$$s(t) = A_c \cos\left[2\pi n f_1 t + 2n\pi k_f \int_0^t m(t)\,dt\right]$$

→ $m(t) = A_m \cos 2\pi f_m t$

$$s(t) = A_c \cos\left[2\pi n f_1 t + \cancel{2n\pi} k_f \left(\frac{A_m \sin 2\pi f_m t}{\cancel{2\pi} f_m}\right)\right.$$

$$s(t) = A_c \cos\left[2\pi n f_1 t + \frac{n k_f A_m}{f_m} \sin 2\pi f_m t\right]$$

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right]$$

where $f_c = n f_1$, $\beta = \frac{n k_f A_m}{f_m} = \frac{n \Delta f}{f_m}$

$\beta = n \beta_1$

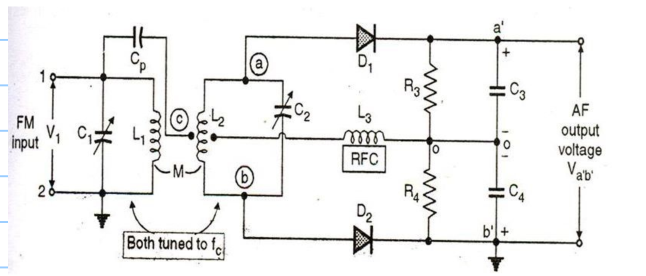
Demodulation of FM wave:

- Foster Seeley discriminator
- Phase-Locked loop (PLL)

Foster seeley discriminator:

- Also known as phase discrimination method

Circuit diagram:



- It consists of primary tank circuit (C_1 and L_1) and secondary tank circuit (C_2 and L_2).
- Both tank circuits are tuned to carrier frequency f_c.
- C_p: coupling capacitor, blocks dc and couples primary to centre tapping.
- RFC (Radio frequency choke) L_3 is the dc return path for diode D_1 and D_2
- Resistors R_3 and R_4 are load resistors by passed by capacitors C_3 and C_4.
- Connection of D_1, R_3 and C_3 form a envelope detector 1.
- Connection of D_2, R_4 and C_4 form a envelope detector 2

Working operation:

- The primary and secondary tuned circuits are tuned to the same carrier frequency, the voltage applied to the two diodes D_1 and D_2 are not constant
- They may vary depending on the frequency of the input signal.
- This is due to the change in phase shift between the primary and secondary windings depending on the input frequency.

(i) At $f_{in} = f_c$

f_{in} = frequency of i/p signal (FM mod signal)

f_c = freq. of carrier signal

→ At $f_{in} = f_c$ the phase shift between primary winding voltage V_1 and secondary winding voltage V_2 is 90°.

→ The individual o/p voltages of two diodes will be equal and opposite.

$$|V_{01}| = |V_{02}|$$

$$V_0 = |V_{01}| - |V_{02}| = 0$$

where

V_0 = Resultant o/p voltage

V_{01} = o/p voltage of Envelope detector 1

V_{02} = o/p voltage of Envelope detector 2

(ii) For $f_{in} > f_c$.

↳ The phase shift between primary winding voltage V_1 and secondary winding voltage V_2 is less than 90°

↳ so, the o/p of Diode D_1 is higher than D_2, the resultant o/p voltage is positive.

$$V_{01} > V_{02}, \quad V_0 = |V_{01}| - |V_{02}| = \text{positive.}$$

(iii) $f_{in} < f_c$

↳ The phase shift between the primary winding voltage V_1 and secondary winding voltage V_2 is greater than 90°

↳ The o/p voltage of Diode 1 is less than o/p voltage of Diode 2. making the resultant o/p negative.

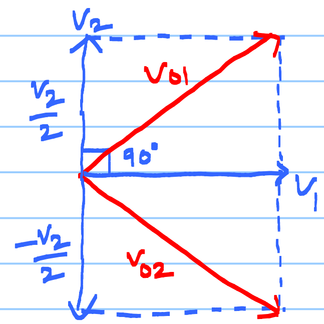
$$V_{01} < V_{02}, \quad V_0 = |V_{01}| - |V_{02}| = \text{Negative}$$

Note:

As o/p is dependent on the primary - secondary phase relationship, this circuit is called Phase-Discriminator.

Phasor diagram.

(i) at $f_{in} = f_c$.

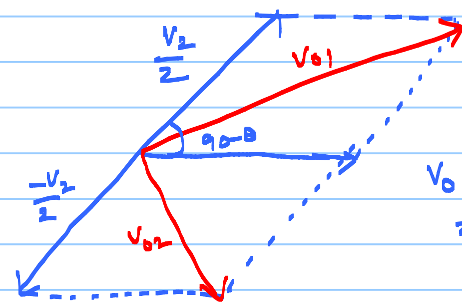


$|V_{01}| = |V_{02}|$

$V_0 = 0$

Phase shift between primary and secondary winding is 90°

(ii) $f_{in} > f_c$



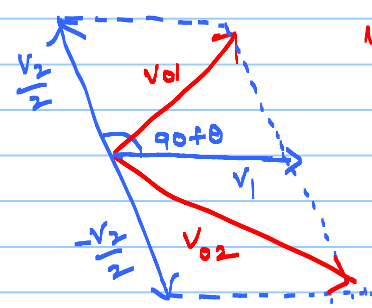
$V_{01} > V_{02}$

$V_0 = |V_{01}| - |V_{02}|$

= Positive

Phase shift between primary and second-ary winding voltages < 90°.

(iii) $f_{in} < f_c$



$V_{01} < V_{02}$

$V_0 = |V_{01}| - |V_{02}|$

V_0 = Negative.

Phase shift between primary and secondary winding voltages greater than 90°

Advantage:

↳ Better linearity

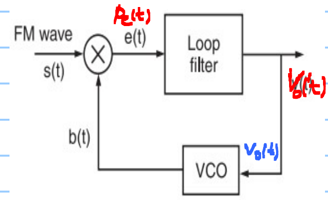
Disadvantage:

↳ High cost

Phase Locked Loop (PLL):

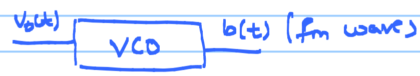
PLL is a negative feedback system which consists of three major components.
(1) a multiplier
(2) a loop filter
(3) a voltage controlled oscillator.

Circuit diagram:



$s(t)$ = FM wave
$b(t)$ = o/p of VCO
$V_0(t)$ = control voltage.
$\phi_e(t)$ = phase error
$e(t)$ = error signal.

→ VCO is a sine wave generator whose frequency is determined by a voltage applied to it from an external source.



→ If the signal feedback is not equal to the i/p signal, the error signal e(t) will change the value of feedback signal until it is equal to the i/p signal.

→ The error signal e(t) is utilized to adjust the VCO in such a way the instantaneous phase angle of b(t) close to the angle of incoming signal s(t).

→ At this point, s(t) and b(t) are synchronized and PLL is locked to the incoming signal. (Lock mode)

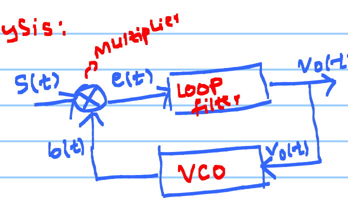
Working operation:

→ Initially adjust the VCO so that when control voltage is zero Two conditions are satisfied:

→ (1) The frequency of the VCO is set at the unmodulated carrier frequency f_c.

→ (2) The VCO output has a 90° phase shift with respect to the unmodulated carrier wave.

Mathematical Analysis:



Equation of i/p signal applied to PLL is an FM wave, defined by

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\,dt\right]$$

Consider $\phi_1(t) = 2\pi k_f \int m(t)\,dt$

So, $s(t) = A_c \cos\left[2\pi f_c t + \phi_1(t)\right] \rightarrow (1)$

A_c = Amplitude of carrier

$m(t)$ = message signal, or modulating signal

k_f = frequency sensitivity

output of Voltage controlled oscillator is given by

$$b(t) = A_v \sin\left(2\pi f_c t + 2\pi K_v \int v_o(t)\,dt\right)$$

where A_v = Amplitude

$v_o(t)$ = control voltage $v_o(t)$ applied to VCO

consider, $\phi_2(t) = 2\pi K_v \int v_o(t)\,dt$

so, $b(t) = A_v \sin(2\pi f_c t + \phi_2(t)) \longrightarrow (2)$

→ The incoming wave $s(t)$ and o/p of VCO $b(t)$ applied to multiplier and o/p of multiplier is given as

$$e(t) = (s(t) * b(t)) = A_c \cos(2\pi f_c t + \phi_1(t)) \left[A_v \sin(2\pi f_c t + \phi_2(t))\right]$$

$$= A_c A_v (\sin(2\pi f_c t + \phi_2(t)) \cos(2\pi f_c t + \phi_1(t)))$$

$$= \frac{A_c A_v}{2}\left[2\sin(2\pi f_c t + \phi_2(t)) \cos(2\pi f_c t + \phi_1(t))\right]$$

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$$= \frac{A_c A_v}{2}\left[\sin(2\pi f_c t + \phi_2(t) + 2\pi f_c t + \phi_1(t)) + \sin(\cancel{2\pi f_c t} + \phi_2(t) - \cancel{2\pi f_c t} - \phi_1(t))\right]$$

× high freq comp

$$= \frac{A_c A_v}{2}\left[\sin(4\pi f_c t + \phi_1(t) + \phi_2(t)) + \sin(\phi_1(t) - \phi_2(t))\right]$$

* $\boxed{\phi_1(t) > \phi_2(t)}$

→ High frequency component is eliminated.

$$e(t) = \sin[\phi_1(t) - \phi_2(t)]$$

$$e(t) = \sin(\phi_e(t))$$

where $\phi_e(t) = \phi_1(t) - \phi_2(t)$ = phase error

Under capture mode, $\phi_e(t)$ is very small

$$e(t) = \phi_e(t),$$

where $\phi_e(t)$ is very small value.

$\sin\phi_e(t) \approx \phi_e(t)$

Consider $e(t) = \phi_1(t) - \phi_2(t)$

$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$

Substitute $\phi_2(t) = 2\pi k_v \int v_o(t)\,dt$

$$\phi_e(t) = \phi_1(t) - 2\pi k_v \int v_o(t)\,dt$$

Differentiate w.r.t 't' on both sides

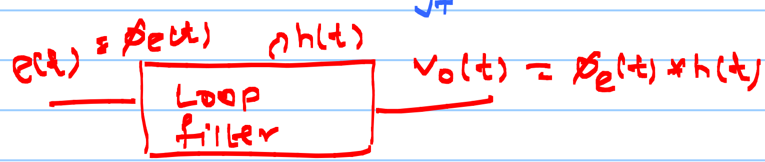
$$\frac{d}{dt}\phi_e(t) = \frac{d}{dt}\phi_1(t) - 2\pi k_v \frac{d}{dt}\int v_o(t)\,dt$$

Apply Fourier transform:

$$j2\pi f\, \phi_e(f) = j2\pi f\, \phi_1(f) - 2\pi k_v\, V_o(f)$$

$$\cancel{j2\pi f}\left(\phi_e(f)\right) = \cancel{j2\pi f}\left(\phi_1(f) - \frac{k_v}{jf} V_o(f)\right)$$

$$\phi_e(f) = \phi_1(f) - \frac{k_v}{jf} V_o(f) \longrightarrow (3)$$



$h(t)$ = Impulse response of loop filter

$$V_o(t) = \phi_e(t) * h(t)$$

Apply Fourier transform

$$V_o(f) = \phi_e(f)\, H(f)$$

where $H(f)$ = Transfer function of Loop filter

Substitute $V_o(f) = \phi_e(f) H(f)$ in eq (3)

$$\phi_e(f) = \phi_1(f) - \frac{k_v}{jf} V_o(f)$$

$$\phi_e(f) = \phi_1(f) - \frac{k_v}{jf}(\phi_e(f))H(f)$$

$$\phi_e(f) + \frac{k_v}{jf}\phi_e(f)H(f) = \phi_1(f)$$

$$\phi_e(f)\left[1 + \frac{k_v}{jf}H(f)\right) = \phi_1(f)$$

$$\phi_e(f) = \frac{\phi_1(f)}{\left(1 + \frac{k_v}{jf}H(f)\right)}$$

As we know

$$V_o(f) = \phi_e(f)\,H(f)$$

$$V_o(f) = \frac{\phi_1(f)}{1 + \frac{k_v}{jf}H(f)}(H(f))$$

$\longrightarrow$ $H(f) \gg 1$, $\quad 1 + \frac{k_v}{jf}H(f) \approx \frac{k_v}{jf}H(f)$

$$V_o(f) = \frac{\phi_1(f)\,\cancel{H(f)}}{\frac{k_v}{jf}\cancel{H(f)}} = \frac{jf}{k_v}\phi_1(f)$$

$$V_o(f) = \frac{jf}{k_v}\phi_1(f)$$

$$V_o(f) = \frac{j2\pi f}{2\pi k_v}\phi_1(f)$$

$$V_o(f) = \frac{1}{2\pi k_v}\left(j2\pi f\,\phi_1(f)\right)$$

Apply Inverse Fourier transform:

$$V_o(t) = \frac{1}{2\pi k_v}\left(\frac{d}{dt}\phi_1(t)\right)$$

where $\phi_1(t) = 2\pi k_f \int m(t)\,dt$

$$V_o(t) = \frac{1}{2\pi k_v}\frac{d}{dt}\left(2\pi k_f \int m(t)\,dt\right)$$

$$= \frac{1}{\cancel{2\pi} k_v}\cancel{2\pi} k_f\left(\frac{d}{dt}\int m(t)\,dt\right)$$

(*)

$$\boxed{v_o(t) = \frac{k_f}{k_v}m(t)}$$

Problems:

1Q A 100 MHz carrier is frequency modulated by a sinusodial signal of amplitude 20V and frequency 100KHz. The frequency sensitivity of the modulator is 25 KHz/volt determine frequency deviation Δf, Modulation index β, and Bandwidth.

Frequency of carrier $f_c = 100\,MHz = 100 \times 10^6\,Hz$

A_m = Amplitude of msg signal = 20V

f_m = frequency of msg signal = 100KHz $= 100 \times 10^3\,Hz$

K_f = Frequency sensitivity = 25 KHz/volt $= 25 \times 10^3\,Hz/volt$.

→ Frequency deviation $\Delta f = K_f A_m$

$= 25 \times 10^3 \times 20V$

$\boxed{\Delta f = 500 \times 10^3\,Hz = 500KHz}$

→ Modulation index $\beta = \frac{\Delta f}{f_m} = \frac{500KHz}{100KHz}$

$\beta = 5KHz$

→ $\beta > 1$, WBFM

$BW = 2(\beta+1)f_m = (2(5+1)100)\,KHz$

$= 1200\,KHz = 1.2\,MHz$

$\boxed{BW = 1.2\,MHz}$

2(Q) The time domain equation of the FM signal is $s(t) = 10\cos[2\pi 10^6 t + 8\sin 4\pi 10^3 t]$ Determine β, Δf, BW and power.

$S(t) = 10\cos[2\pi 10^6 t + 8\sin 4\pi 10^3 t] \rightarrow (1)$

Time domain equation of FM:

$S(t) = A_c\cos[2\pi f_c t + \beta\sin 2\pi f_m t] \rightarrow (2)$

Compare eq (1) & (2)

$A_c = 10$ $f_c = 10^6\,Hz$ $\beta = 8$, $f_m = 2\times 10^3\,Hz$

→ frequency deviation $\Delta f = \beta f_m$ $\left(\beta = \frac{\Delta f}{f_m}\right)$

$= 8 \times 2 \times 10^3$

$= 16 \times 10^3 = 16\,KHz$

→ Bandwidth $= 2(\beta+1)f_m = 2(8+1)2\times 10^3$

$\beta > 1$ (WBFM)

$= 36 \times 10^3$

$BW = 36\,KHz$

→ Power of WBFM signal $= \frac{A_c^2}{2R}$

$A_c = 10V$

$R = 1\Omega$, $P = \frac{A_c^2}{2} = \frac{100}{2} = 50W$

(3Q) A carrier signal is frequency modulated by a sinusoidal signal and frequency deviation is 50KHz, Determine β, BW if $f_m = 500$KHz.

Δf = frequency deviation = 50KHz

$f_m = 500$KHz

$$\beta = \frac{\Delta f}{f_m} = \frac{\cancel{50K}^{1}}{\cancel{500K}_{10}} = 0.1 \quad (\beta < 1, \text{NBFM})$$

$$BW = 2f_m = 2 \times 500\text{KHz} = 1000\text{KHz}$$

$$= 1000 \times 10^3 \text{Hz}$$

$$= 10^6 \text{Hz} = 1\text{MHz}$$

Noise:

Note Title 06-05-2021

→ Noise is a unwanted signal which interfers with the original message signal and corrupts the parameters of the message signal.

→ Noise is a random and unpredictable.

Types of Noise:

The classification of Noise is done depending on the type of the source.

There are two main ways in which noise is produced.

1. External source
2. Internal source

→ External Noise.

Noise produced by External source is called External noise.

→ Examples:

Atmospheric noise, Extra terrestrial noise, and cosmic noise, Industrial Noise.

→ Internal Noise:

The noise produced by internal sources is called internal noise i.e, noise produced by receiver components while functioning.

Types of Internal Noise:

→ Shot noise
→ Partition Noise
→ Thermal Noise
→ flicker noise or low frequency noise
→ transient noise or high frequency noise.

Shot Noise:

→ It is produced in amplifying devices rather than in all.

→ It is produced because of random variation of electrons and holes.

→ It has uniform power spectral density like thermal noise.

→ The exact formula for shot noise can be calculated for diodes:

$$I_n^2 = 2(I + 2I_o)qB$$

Where

I_n = Shot noise current

I = direct current

I_o = Reverse saturation current

q = charge

B = Bandwidth.

→ For amplifying devices, shot noise is directly proportional to o/p current.

Partition Noise:

→ Partition noise is generated when current gets divided into two or more parts.

→ partition noise is higher in transistor compared to diode

Low frequency noise or flicker noise:

→ It will appear at frequency below few khz.

→ It is generated because of fluctuations in current density.

→ This will change conductivity of material which produces fluctuations in voltage and current.

→ The mean square value of flicker noise is directly proportional to the square of direct current flowing through the device.

Transient noise or high frequency Noise:

→ If the time taken by electron from emitter to collector becomes comparable to period of signal then the transit time effect take place.

→ This effect is observed at high freq.

→ Due to this some carries may diffuse back to emitter and this gives rise to input Impedance.

→ So minor change at input generates random fluctuations at output.

→ once the noise appears it goes on increasing with frequency at a rate 6dB/octave.

Thermal Noise:

↳ It arises due to random motion of free charge particles.

↳ mainly electrons in conducting media.

↳ The intensity of random motion is proportional to thermal (heat energy) supplied.

↳ The net motion of the electrons gives rise to an electric current to flow through the resistor causing the noise.

Noise power $P = kTB$

Noise voltage $V = \sqrt{4kTBR}$

Noise current $I = \sqrt{4kTBG}$

where k = Boltzmann's constant

T = Temperature

B = Band width

R = Resistance

G = Conductance.

Modelling of Noise and AWGN

→ **Thermal Noise or White noise or AWGN Noise:**

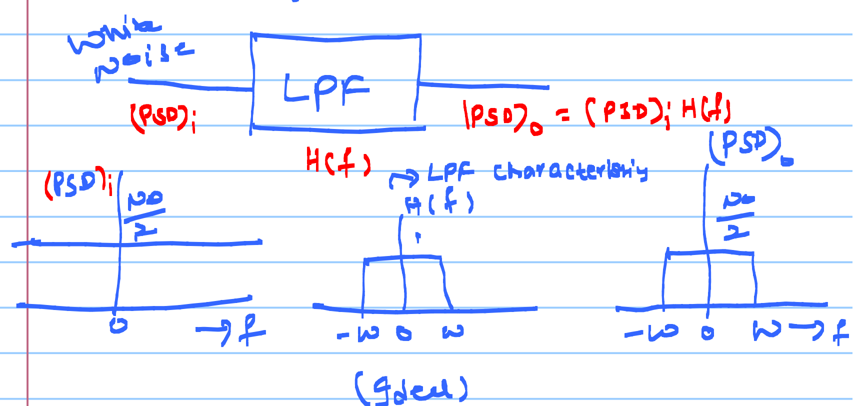
AWGN: Additive White Gaussian Noise.

$\underbrace{\text{Power Spectral Density}}$ of AWGN Noise:

↳ Distribution of Power at all frequencies.

↳ Thermal noise effects at all frequencies and Noise power is uniformly distributed at all frequencies.

→ Transmission of white noise or thermal noise through a Low pass filter:



$$\text{Noise power} = \int_{-W}^{W} \frac{N_0}{2}\, df = \left\{\frac{N_0}{2}(f)\right\}_{-W}^{W}$$

$$= \frac{N_0}{2}(W+W) = \frac{N_0}{\cancel{2}}(\cancel{2}W)$$

$$\text{Noise power} = N_0 W \text{ (watts)}$$

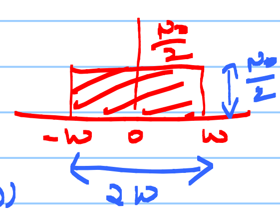
(OR)

Noise power = Area under (-W to W)

$$\text{Area} = L \times b$$

$$= \cancel{2}W \times \frac{N_0}{\cancel{2}}$$

$$= N_0 W \text{ (watts)}$$

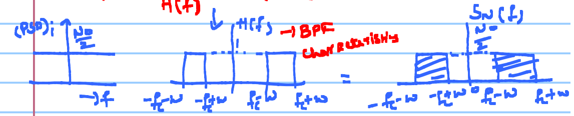


$L = 2W$

$b = \frac{N_0}{2}$

Transmission of Noise through BPF (Band pass filter):

$(PSD)_o = (PSD)_i H(f)$



$$\text{Noise Power} = \int_{-f_c-W}^{-f_c+W} \frac{N_0}{2}\,df + \int_{f_c-W}^{f_c+W} \frac{N_0}{2}\,df$$

$$= \left[\frac{N_0}{2} f\right]_{-f_c-W}^{-f_c+W} + \left[\frac{N_0}{2} f\right]_{f_c-W}^{f_c+W}$$

$$= \frac{N_0}{2}\left(\cancel{-f_c}+W+\cancel{f_c}+W\right) + \frac{N_0}{2}\left[f_c+W-f_c+W\right]$$

$$= \frac{N_0}{2}\left(2W+2W\right)$$

Noise power $= 2N_0W$

(OR)

Noise power $= \frac{N_0}{2} \times 2W + \frac{N_0}{2} \times 2W = 2N_0W$

If the white noise passed through a BPF the resultant noise is called Narrow band noise.

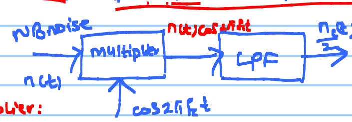
The narrow band noise is represented as

$$n(t) = n_c(t)\cos 2\pi f_c t + n_s(t)\sin 2\pi f_c t$$

Where $n_c(t)$ = Inphase component

$n_s(t)$ = Quadrature phase component.

Power spectral density of Inphase component:



O/p of multiplier:

$n(t)(\cos 2\pi f_c t)$

$$= \left(n_c(t)\cos 2\pi f_c t + n_s(t)\sin 2\pi f_c t\right)\cos 2\pi f_c t$$

$$= n_c(t)\cos^2 2\pi f_c t + n_s(t)\sin 2\pi f_c t\cos 2\pi f_c t$$

$$= n_c(t)\left[\frac{1+\cos 4\pi f_c t}{2}\right] + \frac{n_s(t)}{2}\left(\sin 4\pi f_c t\right)$$

O/p of LPF

$n(t)\cos 2\pi f_c t = n_c(t)/2$

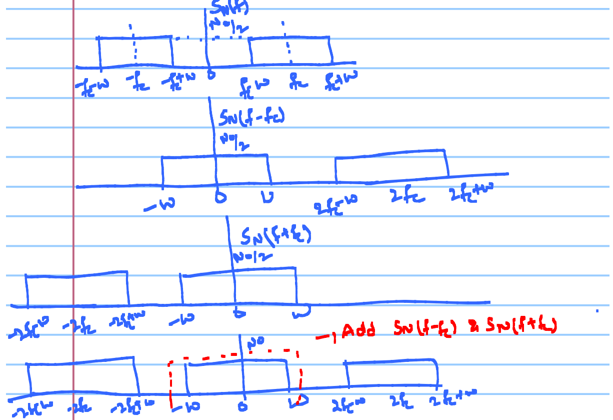
Apply fourier transform:

$$\frac{N(f-f_c)+N(f+f_c)}{2} = \frac{N_c(f)}{2}$$

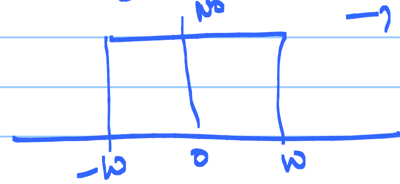
$$N_c(f) = N(f-f_c) + N(f+f_c)$$

$$S_{N_c}(f) = S_N(f-f_c) + S_N(f+f_c)$$

$S_N(f)$ = Power spectral density of Narrow band noise $n(t)$



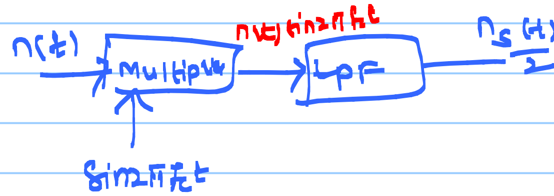
O/p of LPF : $S_{N_c}(f) = S_N(f-f_c) + S_N(f+f_c)$



→ PSD of $n_c(t)$

$$S_{N_c}(f) = \begin{cases} S_N(f-f_c) + S_N(f+f_c) & -W \le f \le W \\ 0 & \text{elsewhere.} \end{cases}$$

Quadrature phase component $n_s(t)$:



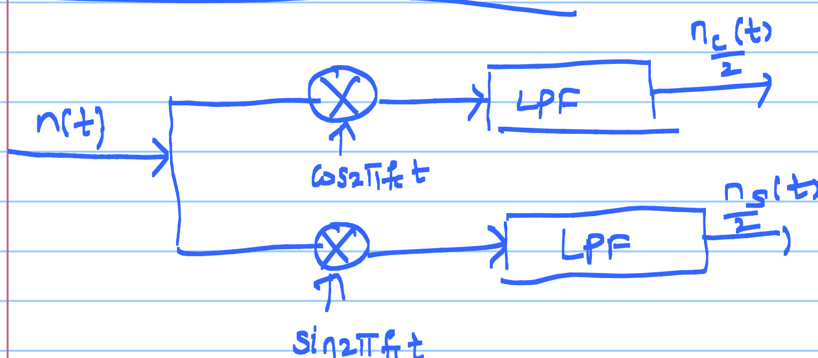
O/p of multiplier:

$$n(t)\sin 2\pi f_c t$$
$$= \left(n_c(t)\cos 2\pi f_c t + n_s(t)\sin 2\pi f_c t\right)\sin 2\pi f_c t$$
$$= n_c(t)\cos 2\pi f_c t \sin 2\pi f_c t + n_s(t)\sin^2 2\pi f_c t$$
$$= \frac{n_c(t)}{2}\left(2\sin 2\pi f_c t \cos 2\pi f_c t\right) + n_s(t)\left(\frac{1-\cos 4\pi f_c t}{2}\right)$$

O/p of LPF

$$= \frac{n_s(t)}{2}$$

$$\boxed{n(t)\sin 2\pi f_c t = \frac{n_s(t)}{2}}$$



Where $n(t)$ = Narrow Band noise

$n_c(t)$ = Inphase comp $n_s(t)$ = quadrature phase comp.

Properties of Narrow Band Noise:

(1) If the narrow band noise $n(t)$ has zero mean, then inphase comp $n_c(t)$ and quadrature component $n_s(t)$ has zero mean.

(2) If the narrow band noise $n(t)$ is Gaussian, then its inphase comp $n_c(t)$ and quadrature comp $n_s(t)$ are jointly Gaussian.

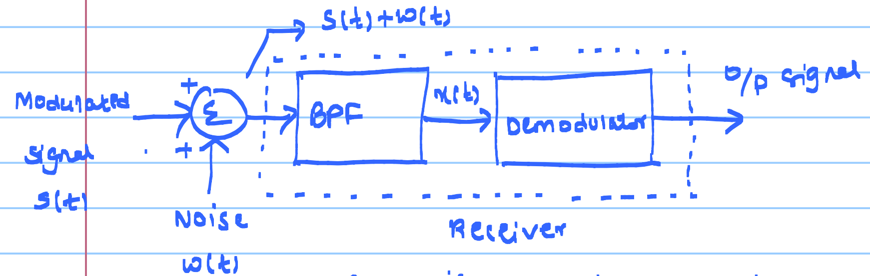
(3) If $n_c(t)$ and $n_s(t)$ have same power spectral density,

then $S_{N_c}(f) = S_{N_s}(f) = S_N(f-f_c) + S(f+f_c)$ $\quad -W \le f \le W$

$= 0$ elsewhere.

(4) If a narrow band noise $n(t)$ has zero mean, the $n_c(t)$ and $n_s(t)$ has same variance.

Receiver Model:



A noisy receiver model

$x(t) = s(t) + n(t)$

where $n(t)$ = Narrow Band noise

$n(t) = n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t$

$n_c(t)$ = Inphase comp

$n_s(t)$ = quadrature comp

Output signal to noise ratio: $(SNR)_o$

$$(SNR)_o = \frac{\text{Average power of the demodulated msg signal}}{\text{Average power of noise measured at the o/p of the demod}}$$

Factors affecting $(SNR)_o$

→ Type of Modulation at the transmitter

→ Type of demodulation at the Receiver

→ **Channel signal to Noise Ratio $(SNR)_c$ or input signal to Noise ratio:**

↳ Denoted as $(SNR)_c$

↳ $$(SNR)_c = \frac{\text{Average signal power at receiver input}}{\text{Average noise power at receiver input}}$$

→ **Figure of merit:**

↳

$$\text{Figure of merit} = \frac{SNR_o}{SNR_c}$$

Note:

The figure of merit must be as high as possible, because higher value of figure of merit better noise performance of the receiver.

Comparison of Noise performance in AM, DSBSC, SSB, and FM

Figure of merit:

Figure of Merit $= \dfrac{(SNR)_o}{(SNR)_c}$

$$\text{FoM} = \frac{\text{Signal to Noise Ratio at the o/p of receiver}}{\text{channel signal to Noise Ratio}}$$

Figure of Merit for AM Receiver:

$$\text{FoM} = \frac{\mu^2}{\mu^2+2}$$ where (μ = Modulation index)

Figure of merit for DSBSC Receiver:

$$\text{FoM} = 1$$

Figure of merit for SSB Receiver

$$\text{FoM} = 1$$

Figure of merit for FM Receiver

$$\text{FoM} = \frac{3}{2}\beta^2$$

	AM	DSBSC	SSB	FM
FoM → formula	$\frac{\mu^2}{\mu^2+2}$	1	1	$\frac{3}{2}\beta^2$
FoM →	Less	moderate	moderate	high for WBFM
Noise performance of a Receiver →	Less	Moderate	Moderate	Good.