Bank Liquidity and the Cost of Debt

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Abstract

Since the crisis, tougher bank liquidity regulation has been imposed which aims to ensure banks can survive a severe funding stress. Critics of this regulation suggest that it raises the cost of maturity transformation and reduces productive lending. In this paper we build a bank run model with a unique equilibrium where solvent banks can fail due to illiquidity. We endogenise banks' funding costs as a function of their liquid asset holdings and show how they are negatively related to liquidity, therefore offsetting some of the costs from higher liquidity requirements. We find evidence for this relationship using post-crisis data for US banks, implying that liquidity requirements may be less costly than previously thought.

JEL codes: G21, G28

Keywords: bank runs; global games; liquidity

1 Introduction

During the global financial crisis many banks which had adequate capital failed or experienced financial distress as they were not liquid. Previously bank regulation had focused on capital but the crisis showed the importance of liquidity and the potential for solvent banks to experience runs. The introduction of global liquidity standards - the Net Stable Funding Ratio (NSFR) and the Liquidity Coverage Ratio (LCR) - in the Basel III accord are aimed at ensuring banks could survive runs. The objective of the LCR is to enhance the short-term resilience of banks, and the NSFR requires banks to maintain a stable funding profile in relation to the composition of their assets. Both of these regulations have been criticised for forcing banks to hold more liquid assets which reduces their ability to perform maturity and liquidity transformation; which reduces lending in productive assets, and likely imposes costs, because yields on liquid assets are generally below the yield on illiquid assets.

If liquidity requirements have their desired effect in making bank runs less likely or increasing a bank's ability to survive a run, then this implies that higher liquidity requirements should make bank failure less likely. If bank liabilities are not guaranteed and investors respond to the risks that banks are taking - including liquidity risk - then reducing the probability of failure should *ceteris paribus* reduce banks' cost of funding. And in turn this fall in the cost of funding reduces the private cost to banks of complying with the liquidity regulation.

The literature on liquidity regulation is relatively spare compared to that of capital regulation Allen and Gale (2017). A few papers such as Roger and Vlcek (2011) and Boissay and Collard (2016) in the literature examine the cost of liquidity requirements, usually - like us - in the context of banks having to forego investment in the more profitable long-term asset. They examine the cost of liquidity regulation together with capital regulation. The cost of liquidity regulation is mitigated as liquid assets are low or zero risk weighted; meaning as liquid assets increase, risk-weighted assets decrease, and so required capital - which is assumed to be expensive - is lower. However, to our knowledge no paper has formally modeled lower funding costs as a result of increased resilience to runs.

In this paper we use a simple framework of a bank's 'liquidity creation' with a bank which invests in an illiquid project but also has debt investors which can draw on demand. We endogenise the probability of a successful bank run as a function of the liquid assets a bank has, in order to capture some of the effects of liquidity regulation. We then use this model to examine the effect of higher liquid holdings on both a bank's probability of a run and of it being solvent. Given these probabilities we use the investors' incentive to invest to endogenously derive the cost of demandable debt funding, the bank's choice of liquid assets and bank profitability. When we introduce a social cost of bank failure the endogenous probability of a bank run means we can then use this framework to consider the level of optimal liquidity regulation at a microprudential level.

We build on the seminal work by Rochet and Vives (2004), to show how holding more liquid assets can reduce the probability of a bank run. We endogenise the probability of a run by using Global Games as an equilibrium selection technique. While most global games models focus on the investors' decision to run, our focus is on the initial period. In this way we can determine the return the investor demands which depends on the bank's liquid asset holdings and the bank's choice of investment in the risky long-term asset. We show that as a solvent bank holds more cash then the probability of a run decreases (up to the point when it becomes run-proof). The bank can therefore pay less for its funding and that this can offset some of the costs of liquidity regulation. However, this only holds above a certain equity threshold. If the bank is poorly capitalised then the lower return on liquid assets means that the bank's solvency decreases as liquid assets increase and increasing liquid assets actually makes a bank more likely to experience a run. We empirically test our model's prediction that banks with more liquidity should have lower funding costs using post-crisis data for large US banks. We find a negative association between asset liquidity and credit-default swap (CDS) spreads, indicating that investors may now be more conscious of liquidity risk and are pricing it into firms' funding costs. In turn this implies that the cost of liquidity requirements could be lower than previously thought.

The rest of this paper proceeds as follows. Section 2 reviews the relevant literature. Section 3 provides our theoretical model while section 4 provides empirical support. Finally section 5 concludes and offers some policy considerations.

2 Literature review

There is a long literature in banking such as Diamond and Dybvig (1983) and Bryant (1980) which shows that banks are susceptible to runs due to coordination failures amongst depositors. In many of these models there are multiple equilibria as depositors coordinate following a sunspot, and the probability of investors seeing this sunspot is exogenous. Diamond and Kashyap (2016) modify this framework to study the effect of liquidity regulation by allowing for partial runs in which some depositors do not run following the sunspot. In the model depositors are uncertain as to whether the bank's liquidity holdings are sufficient to allow it to survive a run. The bank then faces a tradeoff between being more robust to a run against the foregone profits form investing in a risky asset. They find that the additional liquidity needed to survive a run will turn out to be excessive whenever a run is avoided.

This is not the approach that we take in our model as we endogenise the probability of a run based on both the liquid asset holdings a bank has and also the fundamentals of the risky asset, rather than liquidity needs by investors. This approach, whereby runs are related to fundamentals is used by Goldstein and Pauzner (2005), Rochet and Vives (2004), Jacklin and Bhattacharya (1988) amongst others. In the seminal work of Morris and Shin (2000) they use Global Games techniques to solve for the unique equilibrium in a bank run game, which is the method that we employ. Our model builds on Rochet and Vives (2004) who build a model of bank runs where a solvent bank can fail due to a coordination failure and creditor run, and they use global games to solve for a unique equilibrium. In their model banks have to resort to the repo market whereby it can raise funds only at a discount. We deviate from this by employing a Lender of Last Resort (as they do in their final section) who also receives a signal about the outcome of the project but still demands a haircut. This avoids the slightly awkward problem of the repo market perfectly revealing information about the bank and thus invalidating the global games equilibrium. In Rochet and Vives (2004) they explore a LOLR and show that the LOLR can only be fully effective when the interest rate it charges is arbitrarily close to zero. We go beyond their model by solving for the bank's optimal ex ante liquidity choice, as in Ahnert (2016), and endogenising the bank's funding costs. We are unaware of any papers that have taken the latter step.

The second strand of the literature is the cost for banks of holding liquid assets and therefore the cost of liquidity regulation. This is a perennial feature of banking models which have a storage technology and a longer-term higher return asset (e.g. Diamond and Dybvig (1983), Bhattacharya and Gale (1987), Freixas and Rochet (2008)) In these models often banks hold the storage or liquid asset to allow agents to insure against their own idiosyncratic liquidity shocks. Liquid asset holdings have a dual purpose - they protect against bank runs but also provide for consumption for agents with liquidity shocks. In our model liquid assets do not provide this second purpose - while it is a potential extension it would complicate the model unnecessarily.

Given the renewed post-crisis interest in liquidity requirements a few papers have tried to examine the macroeconomic impact of liquidity requirements and whether there are mitigating effects. Boissay and Collard (2016) examine the problem of a social planner which sets capital and liquidity requirements. Liquidity requirements are costly as they reduce investment in the risky asset, but some of the cost if offset as increasing liquidity holdings de facto reduces risky assets, making capital requirements less binding. Roger and Vlcek (2011) build a DSGE model with a banking sector and introduce liquidity requirements as a requirement to hold a share of their assets in liquid government securities. Liquid assets have a cost as they have a lower yield meaning that bank revenues decline as they increase holdings of liquid assets. However, as holdings of liquid assets increase, banks risk-weighted assets decline, allowing them to reduce capital.

A few papers have attempted to empirically assess the effect of liquidity regulation on individual banks. Banerjee and Mio (2015) examine the effect of Individual Liquidity Guidance applied by the UK's Financial stability Authority and find that banks which were subject to the regulation increased their share of high-quality liquid assets and reduced intra-financial lending. However, banks did not reduce lending to the non-financial sector. Bruno, Onali, and Schaeck (2016) examine market reactions to announcements about liquidity regulation, which were made as the Basel framework was negotiated and find that liquidity regulation announcements are associated with negative abnormal returns. But these are mainly driven by announcements which also tighten capital regulation. The authors interpret this as suggesting that markets do not consider liquidity regulation to be binding.

A number of papers use a similar approach to our econometric specification when examining the cost of higher capital requirements. Miles, Yang, and Marcheggiano (2013), Yang and Tsatsaronis (2012) and Hanson, Kashyap, and Stein (2011) all use a CAPM framework to examine the cost of higher capital requirements on the cost of bank equity. All of them find that there is a Modigliani-Miller 'offset': as a bank's capital increases the volatility of its equity falls and that this decreases the cost of holding extra equity, which in turn implies higher optimal capital requirements. A further stand examines the effect of bank *solvency* on the cost of funding for banks. While there was a lot of pre financial crisis work which found no market discipline for banks more recent work such as **?** and Dent and Panagiotopoulos (2017) both find evidence of a relationship between bank funding costs and various bank fundamentals in the post-crisis period. Our econometric framework is similar but we instead focus on bank liquidity, for which there is very little empirical literature thus far.

3 The model

Consider a three-period economy with time periods t={0,1,2} in which there are two types of agent. The first is a bank whose size is normalised to 1. The liability side of the bank's balance sheet is fixed with uninsured short-term debt (D) and equity (E).[1] The bank optimises over its assets consisting of cash (C) and loans (L). The return on cash is 1 and the return on loans R is random with density f(R) and distribution F(R), which is common knowledge.

The second type of agent is a continuum of investors of size 1 that each provide D units of funding to the bank in period 0. The investors have preferences $u()$ and outside option utility of $U > 1$. The bank offers investors a contract in period 0 that follows Table 1. If the investors withdraw in period 1, they receive 1 with certainty. If they wait until period 2, they receive r_D (which is endogenous) if the bank succeeds but 0 if it fails.

Table 1: Payoffs for investors

Action	Bank fails	Bank Survives
Withdraw in period 1	1	1
Don't withdraw	0	r_D

The bank's profit is any surplus left after repaying investors at time 2. In period 0 the bank chooses cash and r_D to maximise expected profit, subject to the investors' participation constraint of $u(invest) \geq U$. Endogenising r_D is the main difference in our model compared to previous papers.

[1]In this model we abstract from the reason why the bank must issue uninsured short-term debt rather than long-term debt. There are numerous reasons in the literature from the disciplining role of short-term deb, to the need for liquidity and payment services.

In period 1 a fraction of investors $w \in [0, 1]$ investors decide to withdraw based on a private signal over the risky asset's return $x_i = R + e_i$, where e_i is independently and identically distributed $N(0, \sigma^2)$. The purpose of early withdrawals is to allow investors (who get better information in period 1) to close a bad project down early, because project failures in period 2 are very costly. The bank can pay withdrawing investors using cash or via interest-free secured borrowing from the central bank. We assume that there are no other available forms of funding.[2]

The central bank is essentially passive: they run a committed facility and lend at a haircut θ, where $\theta \in (0, 1)$. The bank can therefore borrow up to $\theta R(1 - c)$ and will receive more central bank funding if its assets are high quality. For simplicity we assume the central bank knows R with a high degree of precision ($x_C B \to R$), due to its supervision of banks, but cannot reveal it to the market. The central bank only lends to the bank if R is high enough such that the bank will be solvent - this, together with the near=perfect information of the central bank, eliminates the issue of the bank choosing to sell 'lemons'. The central bank lends with a haircut of θ despite knowing the value of the asset with near-certainty. In practice central banks charge steep haircuts in their liquidity facilities to protect themselves against possible further falls in the value of collateral. While there is no moral hazard in the basic model the central bank's choice of θ could also arise from a need to induce banks to hold their own liquid assets rather than relying on central bank facilities. Another justification could come from the return R only being realised if the bank itself is the owner of the project: if the bank failed the assets would either have to be sold or the central bank would have to manage them with worse technology.

We solve the model first by solving for the investor's strategy in the middle period, given the signal that they receive and the bank's choice of cash. Given this strategy we learn the failure frequency of the bank, and can then solve backwards to the interest rate needed for investors to participate. Finally given this interest rate we find the bank's optimal cash holdings. As with Rochet and Vives (2004) the bank is a corporation which acts in the interest of its equity holders, not the investors.

3.1 Critical thresholds

The bank will fail in period 1 if withdrawals exceed available funds i.e.

$$wD > \theta R(1 - c) + c. \tag{1}$$

Withdrawing investors receive 1 and other investors receive 0. If the bank survives then it repays the central bank in period 2 along with r_D to the remaining investors. The firm will fail from insolvency in period 2 if its remaining deposit liabilites exceeds its assets i.e.

[2] We could, as Rochet and Vives (2004) add a repo market, but we wanted to reflect the increased committment to public liquidity support in the post-crisis era.

$$R < \frac{(1-w)r_d D + wD - c}{1-c} = R_s \tag{2}$$

Note that the total value of its assets is invariant to period 1 withdrawals, because the central bank does not charge interest. Therefore runs will not harm the bank's solvency position.[3] In fact solvency can improve if some depositors withdraw early, because they receive 1 in period 1 but would have received r_D if they had waited for period 2. To isolate the impact of cash choice on solvency risk, assume that no depositors withdraw i.e. $w = 0$. Evaluating the partial derivative:

$$\frac{\partial R_s}{\partial c} = \frac{Dr_D - 1}{(1-c)^2} \tag{3}$$

Holding cash can both reduce or increase the bank's solvency risk, depending on D and r_D. The bank's negative interest margin from holding cash to pay depositors is $Dr_D - 1$. Equation 3 shows that if $Dr_D - 1 > 0$ then holding more cash will make the bank less likely to be solvent (higher R_s), because they do not have enough equity to absorb the negative margin from holding cash. In the limit $c \to 1$ the bank is always insolvent, because their assets yield 1 with certainty which is not enough to pay depositors.

However if $Dr_D - 1 < 0$ then holding more cash makes the bank more likely to be solvent (lower R_s), because they have enough equity to absorb the negative margin from holding cash. In the limit $c \to 1$ the bank is always solvent, because their assets yield 1 with certainty which is enough to pay depositors.

An implication is, given that $r_D > 1$, if the bank has no equity then holding cash will always raise their solvency risk. We explore this dynamic further in section 3.4

3.2 Investors' withdrawing decisions

As stated before, the bank fails from illiquidity at $t = 1$ if:

$$wD > \theta R(1-c) + c$$

Therefore, investors will decide to wait until the end of the contract if:

$$\Delta u \equiv u(wait, w, x) - u(run, w, x) \geq 0 \tag{4}$$

where $u(a, w, x$ is the investor's payoff when the investor takes action $a \in \{wait, run\}$, w is the proportion of other investors that run and x is the signal they receive. Therefore

[3]This was for simplicity but will hold so long as $r_{cb} \leq r_D$, because repaying the CB will be cheaper than repaying depositors.

we have that:

$$\Delta u = \begin{cases} r_D - 1 \text{ if } wD < \theta R(1-c) + c \text{ and } R(1-c) + c \geq r_D(1-w)D + wD \\ -1 \text{ otherwise} \end{cases} \tag{5}$$

The first condition in equation (5) is that the bank is not illiquid in period 1. The second is that it is solvent in the final period. We will show that a solvent bank can fail due to illiquidity but an insolvent bank will never survive period 1, so only the first condition is relevant.

3.3 Period 1 equilibrium run decision

We use techniques from the Global Games literature to solve for the period 1 equilibrium investor strategy. We will show that there is a unique equilibrium asset return, R^*, under which the bank fails and a unique equilibrium failure frequency, $F(R^*)$. The equilibrium is fully determined by:

- Exogenous parameters - $F(R), U, \theta$ and D.
- The bank's period 0 choices of c and r_D, which are taken as given in period 1.

This differs from the classic Diamond and Dybvig (1983) setup, where there are multiple equilibria and we cannot determine ex ante which will occur.

Proposition 1: There exists a threshold strategy equilibrium where all investors stay if they receive a signal above some value x, and run if they receive a signal below x:

$$s_i(x_i) = \begin{cases} \text{stay if } x_i \geq x \\ \text{run if } x_i < x \end{cases} \tag{6}$$

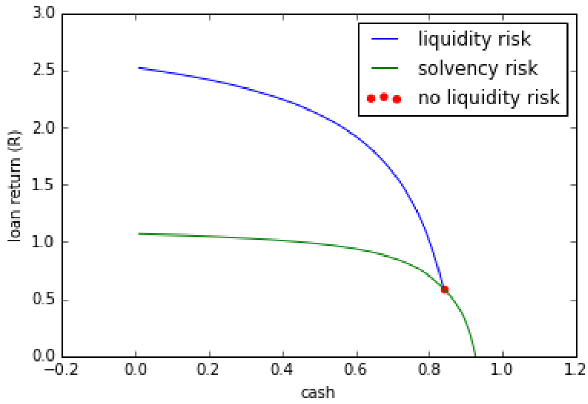
At this equilibrium:

$$R^* = \frac{1}{\theta(1-c)} \left(\frac{D}{r_D} - c \right) \tag{7}$$

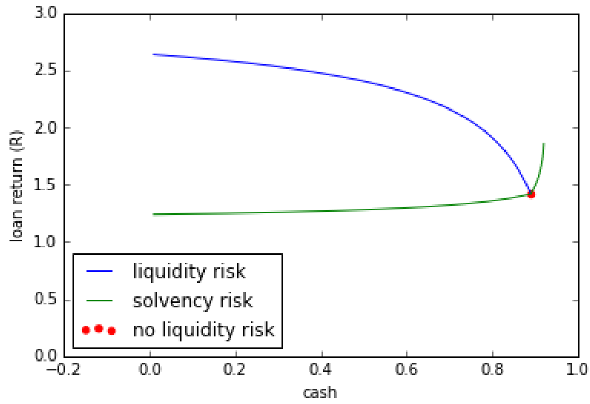
Proof: See Annex A.1. An interesting property of the threshold equilibrium is that it exists for low values of σ, which is counter-intuitive. Suppose σ is just above zero. I could receive some x_i just below R^*, but well above the insolvency threshold. I also know that other investors will have received a signal well above the insolvency threshold, given that σ is near zero, but in equilibrium we will all run anyway. We unpack the intuition further in Annex A.2.

Proposition 2: The threshold strategy equilibrium given by R^* is the only strategy surviving iterated deletion of dominated strategies. It is therefore the unique equilibrium for the investor decision in period 1. Proof: See Annex A.2. The uniqueness property is crucial for analysing comparative statics - we can see that the equilibrium run threshold is:

Figure 1: Failure point vs. cash choice



(a) High-equity firm



(b) Low-equity firm

- Falling in c - banks with more cash can survive more withdrawals at any given R.
- Falling in θ - higher θ means the bank can raise more liquidity from the central bank at any given R.
- Falling in r_D - greater payoff from rolling over means investors are less incentivised to run after a given signal.
- Increasing in D - banks that have more flighty funding are more vulnerable to runs.

The first two illustrate the substitutability between cash and funding from the central bank. The risky asset provides two functions: it yields profit if the project succeeds but it can also be used to raise cash from the central bank.

Proposition 3: If the firm holds sufficient cash $c \geq \hat{c}$ then there will be no liquidity risk in the model. Only insolvent firms will fail in period 1. Proof: see Annex A.3. The bank could choose to eliminate its liquidity risk but will never do so in equilibrium. We discuss this further in section 4.5.

Figure 1 shows how the failure point depends on cash choice. The return at which the bank is insolvent is always below the return at which the bank is potentially illiquid i.e. a solvent firm sometimes fails due to illiquidity and an insolvent firm will always be illiquid. The left diagram shows that, for a firm with high equity, failure always becomes less likely as cash increases. Raising cash reduces both liquidity risk and solvency risk, so raising cash improves its chance of survival even after the point where there is no liquidity risk . However for a poorly capitalised firm, shown in the right diagram, cash makes it more likely to fail after liquidity risk is eliminated, as discussed in section 3.1.

Knowing that the unique period 1 equilibrium is failure for R below R^*, we can now solve backwards for the period 0 equilibrium. We do this by first solving for the r_D

that investors demand for a given amount of cash held by the firm - the participation constraint. Then the firm optimises its cash choice to maximise expected profits, subject to the participation constraint.

3.3.1 Period 0 equilibrium - parameter restrictions

From Proposition 3, the firm can choose in period 0 to eliminate its liquidity risk by holding $c = \hat{c}$, or it can hold $c \in [0, \hat{c})$ and suffer some runs while solvent. Below we derive their optimal choice of $\{c, r_D\}$.

For tractability we assume that R is distributed uniformly on $[0, \bar{R}]$ and investors are risk neutral. This yields some parameter restrictions:

1. E(R) $= \frac{1}{2}\bar{R} > U$.
2. $\theta \bar{R} \geq D$

The first restriction is from the bank's need to make positive expected profits - it is sufficient to assume that the expected loan return exceeds the expected payout to investors. The second restriction is from the Global Games framework. There needs to be a state of the world for which it is strictly dominant not to run, because the bank has access to sufficient liquidity to survive a run from all investors.

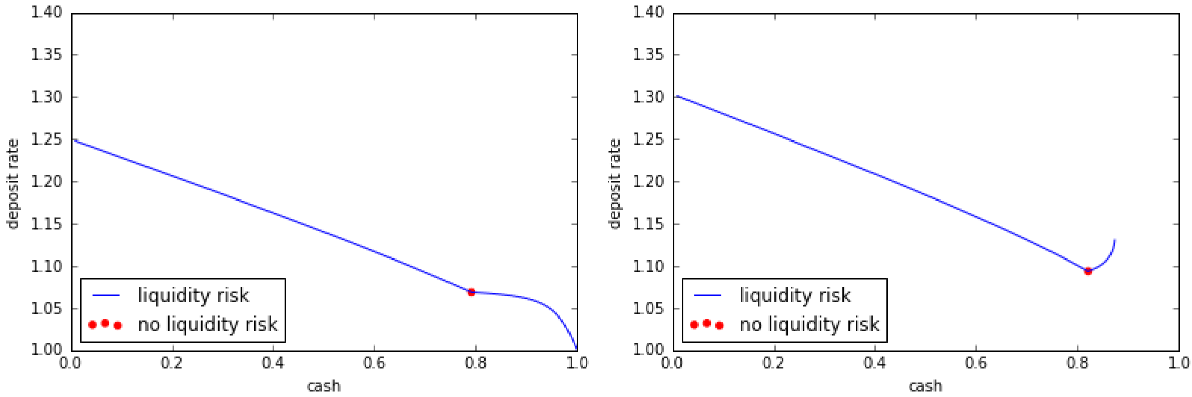
3.4 Period 0 equilibrium - deposit rate and cash choice

3.4.1 Deposit rate

Now that we have pinned down the parameter restrictions, shown that there is a unique equilibrium run decision which depends on c (cash and the interest rate r_D we are now able to solve for the interest rate paid on bank debt. Investors have a safe outside option U where is $U > 1$ otherwise the investor will never invest in the bank but $U < E(R)$ to reflect that the bank's project has some value. In our model, as in Rochet and Vives (2004) the bank is a profit maximising bank. It does not exist to act as liquidity insurance for depositors and therefore the depositors' participation constraint strictly binds and we are able to solve for the deposit rate.

Equation 7 pins down the highest value of R for which the firm will fail, R^*, for a given cash choice and deposit rate. Depositors will run for signals below R^* and receive 1.

Figure 2: Deposit rate vs. cash choice



(a) High-equity firm

(b) Low-equity firm

They will stay for signals above R^* and receive r_D.[4]

The participation constraint is therefore:

$$P(R < R^*) * 1 + P(R \geq R^*) * r_D = U \tag{8}$$

$$\frac{1}{\bar{R}}(\frac{1}{\theta(1-c)}(\frac{D}{r_D} - c)) + r_D(1 - \frac{1}{\bar{R}}(\frac{1}{\theta(1-c)}(\frac{D}{r_D} - c))) - U = 0 \tag{9}$$

$$r_D^* = \frac{D + c + U\theta\bar{R}(1-c) + \sqrt{(D + c + U\theta(1-c))^2 - 4D(\theta\bar{R}(1-c) + c)}}{2(\theta\bar{R}(1-c) + c)} \tag{10}$$

Equation 10 follows from applying the quadratic formula to 9. The deposit rate is falling in the firm's cash choice, up until the point that the firm is run-proof. Whether it falls after that will depend on how much equity the firm has. Figure 2 shows the relationship graphically for both a firm with high equity and a firm with low equity.

For a firm with high enough equity the deposit rate falls for all levels of cash. For a firm with low equity the deposit rate falls until the point at which it could survive all 'liquidity' based runs . It then increases after this point because the low return on holding cash means that the bank effectively becomes less solvent when it holds more cash, even though it becomes more liquid.

This is a key result of the paper *that when a bank holds more cash, the probability of a*

[4] We note that there must be some reason for investors to have a debt contract which involves demandable debt in the first period. While it could be the case that there is some small demand for first period liquidity this does not meaningfully add to the model but further complicates it. In this model it is efficient to liquidate the bank in the first period if signals about R are low enough and this too can justify the existence of demandable debt

run can decrease and the interest rate it pays to depositors also decreases.

3.4.2 Cash choice

The bank then maximises profits by choosing cash, subject to the participation constraint (equation 10).

$$c^* = argmax_c(\frac{1}{\bar{R}} \int_{R^*}^{\bar{R}} R(1-c) + c - Dr_D^* dR) \tag{11}$$

We know that $c^* \in [0, \hat{c}]$ where $\hat{c}$ is the cash choice that eliminates liquidity risk. If there is an interior solution i.e. then it will satisfy the following first-order condition:

$$-(\bar{R} - R^*)(\frac{1}{2}(\bar{R} + R^*) - 1) - \frac{dR^*}{dc}(R^*(1-c) + c^* - Dr_D^*) - \frac{Dr_D{}^*}{dc}(\bar{R} - R^*) = 0 \tag{12}$$

The first term is the cost of insuring against runs, given by the expected return on foregone loans. The second term is the positive effect from less frequent runs. The final term is the *funding cost offset* from investors knowing the bank is less risky. The optimal cash choice will trade off these 3 effects.

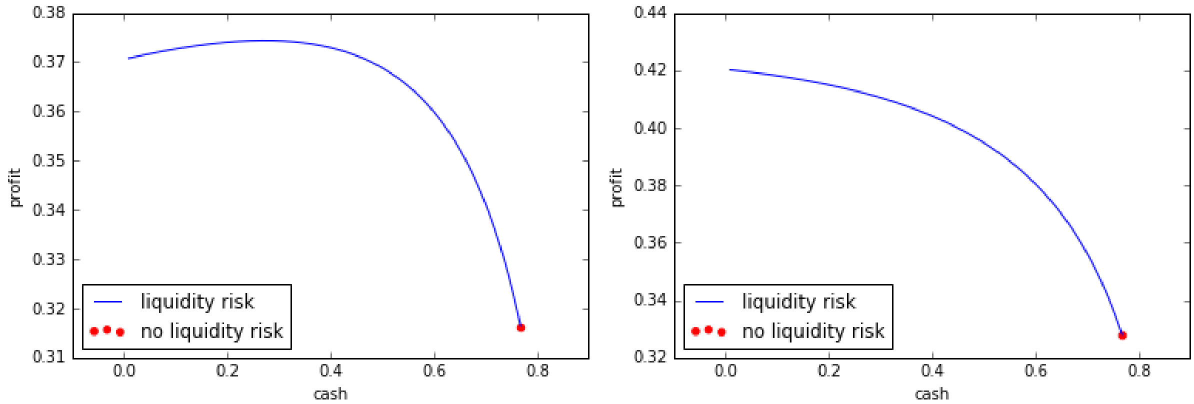
Both interior and exterior ($c = 0$) solutions are numerically possible. Figure 4 shows an example of each. Cash choice will generally be higher when:

- R is low, because the opportunity cost of holding cash is low.
- U is low, because this will relax the participation constraint and reduce r_D. Furthermore if r_D is low then deposits become more "flighty" because the relative pay-off from running is higher.
- D is low, so that the firm has more "skin in the game" and is able to absorb the loss of $r_D - 1$ from holding cash.

Proposition 4: If the bank has no equity then it will never choose to hold $c > 0$, regardless of the other parameters. Proof: See Appendix B. This result is driven by two dynamics:

- The bank has no "skin in the game" and therefore little incentive to insure against illiquidity.
- Holding cash would make the bank less solvent because they have no equity to absorb the margin of $-(r_D - 1)$ from holding cash.

Figure 3: Profits vs. cash choice



(a) Cash > 0, low E(R) (b) Cash = 0, high E(R)

3.5 The funding cost offset and the cost of liquidity regulation

If the firm chooses $c = \hat{c}$ there will only be solvency risk, and investors will only run if the firm is insolvent. Let R_s denote the loan return for which the firm is just solvent.

$$(1 - \hat{c})R_s + c^* = Dr_D \tag{13}$$

$$R_s = R^* = \frac{Dr_D - \hat{c}}{1 - \hat{c}} \tag{14}$$

This outcome is never supported in equilibrium. The bank has no incentive to prevent runs for states s.t. $R \leq R_s$, because their equity will be wiped out in period 2. They will be indifferent over survival and failure, because their payoff will be zero in either case. In equilibrium banks will therefore always choose some liquidity risk.

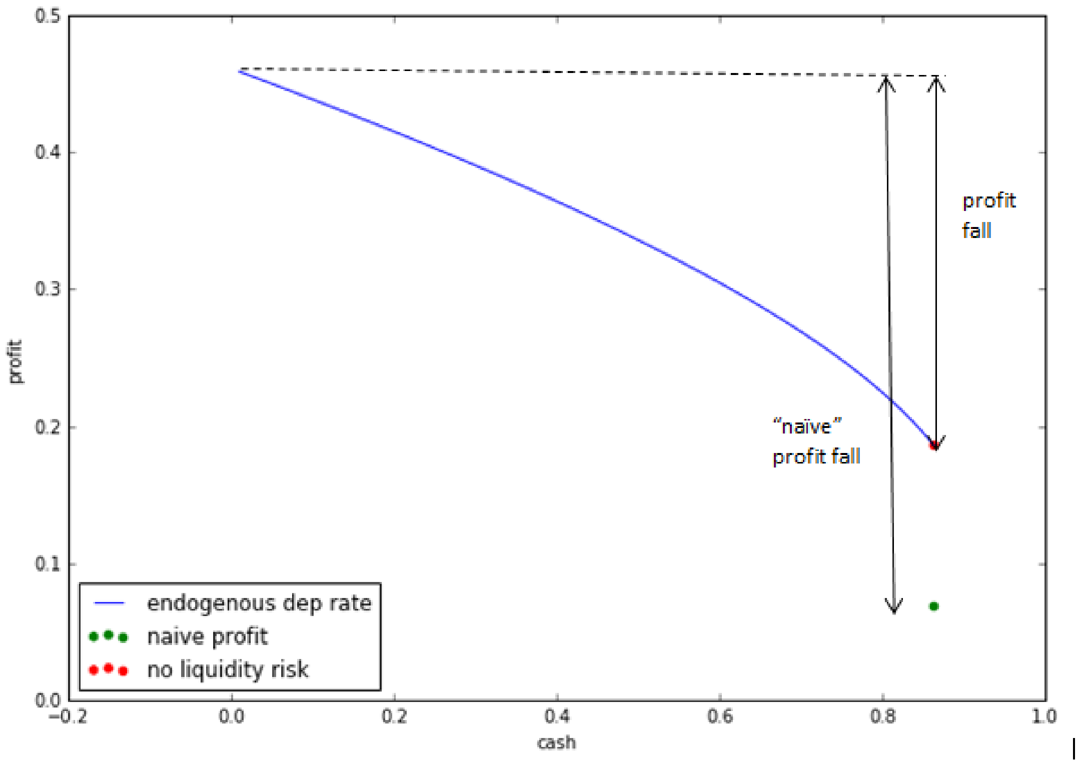
However, bank runs may be socially costly. There are many ways we could justify this, for example if the risky project represented a relationship banking style arrangement which would be lost if the bank failed. Or socially costly runs could be motivated by externalities from fire sales such as in Ahnert (2016) and Acharya, Shin, and Yorulmazer (2011), but it is not the focus of the paper. Instead we consider the thought experiment that under some circumstances a social planner would want the bank to hold $c = \hat{c}$ such that they only fail when insolvent.

By definition this will reduce the bank's profits. However this reduction will be somewhat offset by the falling deposit rate. Ignoring this offset would lead us to overestimate the negative impact of liquidity requirements on firm profitability. Let profits under the assumption of an exogenous deposit rate be "naive profits". We define the offset below and show it graphically in Figure 4.

$$\text{offset} = \text{profit}_{|c=c^*} - \text{"naive" profit}_{|c=\hat{c}} \tag{15}$$

The offset will be larger when the opportunity cost of holding cash is low, and when the deposit rate is more elastic with respect to cash.

Figure 4: Funding cost offset



4 Empirics

4.1 Empirical specification

We test our model's prediction that funding costs are negatively related to banks' liquidity positions. Our empirical specification is:

$$\text{cost of funding}_{it} = \beta_1 \text{LAR}_{it} + \beta_2 \text{STD}_{it} + \beta_3 \text{LEV}_{it} + VIX_t + UST_t + \alpha_i + \epsilon_{it} \tag{16}$$

- cost of funding$_{it}$ is the 5 year senior credit default swap (CDS) spread for firm i in period t. Although CDS spreads are not actually a funding cost, they are a good proxy(Beau, Hill, Hussain, and Nixon (2014)). We use them instead of direct measures of funding cost, such as secondary market bond yields, because the CDS market is very deep and liquid, whereas any given funding instrument may have periods of non-trading.

- LAR is the ratio of liquid assets to total assets. β_1 is our main coefficient of interest, because it measures the association between a firm's asset-side liquidity position and cost of funding. Our model predicts a negative relationship.

- STD is the ratio of short term deposits to total assets, which is a measure of funding risk. β_2 is therefore a secondary coefficient of interest - our model predicts a positive coefficient. Funding fragility is also an important control because a firm may have more liquid assets because of an increased reliance on short term funding.

- LEV is the ratio of equity to total assets i.e. their leverage ratio. Augustin, Subrahmanyam, Tang, and Wang (2014) shows these are correlated to firms' CDS spreads. Leverage could also be correlated to liquidity as a more prudent firm may have higher capital and higher liquidity.

- VIX_t is a control for S&P 500 volatility in period t, which has been found to be a significant driver of CDS spreads in previous studies (Fama and French (1989)). The VIX also serves as proxy for investor sentiment.

- UST_t is a control for the average yield on 5 year US treasuries in period t, which we use as a proxy for the risk-free rate. Risk free rates should be negatively related to CDS yields for two reasons. Firstly Longstaff and Schwartz (1995) finds that higher risk-free rates are generally associated with better macroeconomic conditions. Second Annaert, Ceuster, Roy, and Vespro (2009) argues that higher risk-free rates reduce default probabilities.

- α_i are firm-level fixed effects. These control for time invariant firm-level unobservables, such as business model, which may affect CDS spreads.

We run our specification in logs because we expect there to be diminishing returns from holding more liquidity. The coefficients therefore estimate the percentage change in CDS spreads associated with a marginal percentage change in each variable. Also logs are invariant to whether total assets are on the numerator or denominator, whereas for levels this would matter. For example if our independent variable were $\frac{\text{liquid assets}}{\text{total assets}}$, the specification would be linear in liquid assets and non-linear in total assets. But if our independent variable were $\frac{\text{total assets}}{\text{liquid assets}}$ then it would be non-linear in liquid assets and linear in total assets.

4.2 Data

We use the Federal Reserve's Financial Reports (form FRY9-C) to obtain data for the balance sheet variables:

- Liquid assets are the sum of cash, withdrawable reserves and US treasury securities. Our liquid asset measure is quite narrow - it excludes demand deposits at other banks and non-US government securities.
- Leverage is the ratio of Core Equity Tier 1 capital to total assets. Again this is a relatively narrow definition as it excludes other forms of loss-absorbing capacity.
- Short term debt is time deposits with remaining maturity of less than a year. This covers both retail and wholesale deposits.

The FRY9-C Reports are publically available so it is plausible that investors may use them directly when making decisions about banks. They date back to 1986 at quarterly frequency for bank holding companies. However Goldman Sachs and Morgan Stanley were purely investment firms, not banks, until Q4 2008. Therefore their first Call Report submission is Q4 2008 and the time period for our sample is Q4 2008 - Q1 2017. We have a balanced panel of 198 firm-quarter observations.

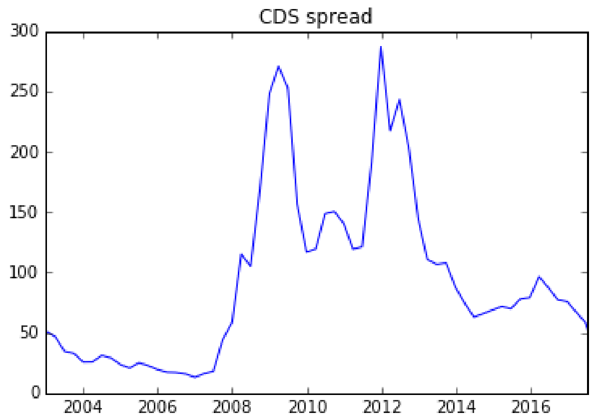
Bloomberg is the source of our data CDS data. These are available daily for 6 of the largest US firms: Bank of America, Citigroup, Goldman Sachs, JPMorgan Chase, Morgan Stanley and Wells Fargo. Bloomberg is also the source of the VIX and US treasury yield data.

The balance sheet variables are reported for quarter-end dates and we aggregate the daily CDS data to match the period following that reporting date. For example: the call report for Q1 2009 would refer to the firm's balance sheet on 31st March 2009, which we would match to the average of CDS spreads from 1st April to 30th June 2009. Therefore our balance sheet variables are "lagged" by a quarter. We think this helps deal with reverse causality issues e.g. higher CDS spreads may trigger a run, causing the firm's liquidity to fall.

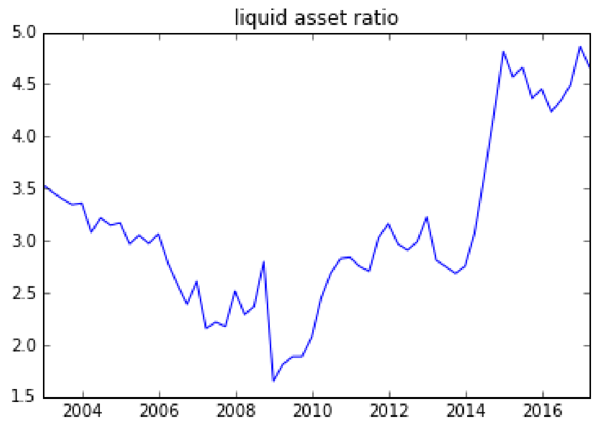
Figure 5 shows how our variables have evolved over time. Annex B provides a fuller breakdown of descriptive statistics and how these variables evolve over time by firm.

CDS spreads spiked during the financial crisis, when investors suddenly became aware of risks that had built up in the banking system. There was another spike in 2012 during the Eurozone crisis, as banks were very exposed to distressed Eurozone sovereign debt. Since then spreads have been more stable, although higher than the pre-crisis period. This could reflect permanently higher awareness of financial risks, or a better resolution regime reducing the likelihood of future bailouts.

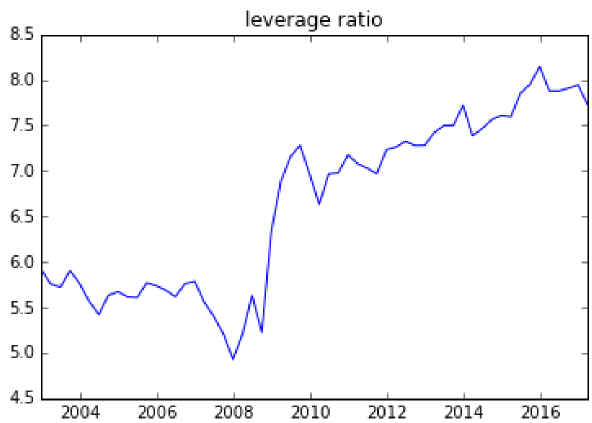
Figure 5: Variables over time



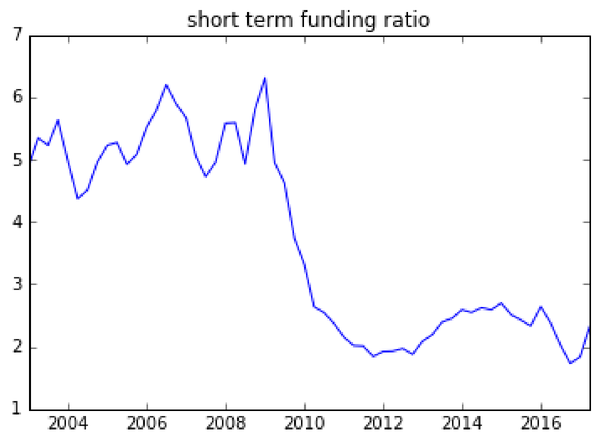
(a) CDS spreads



(b) Liquid asset ratio

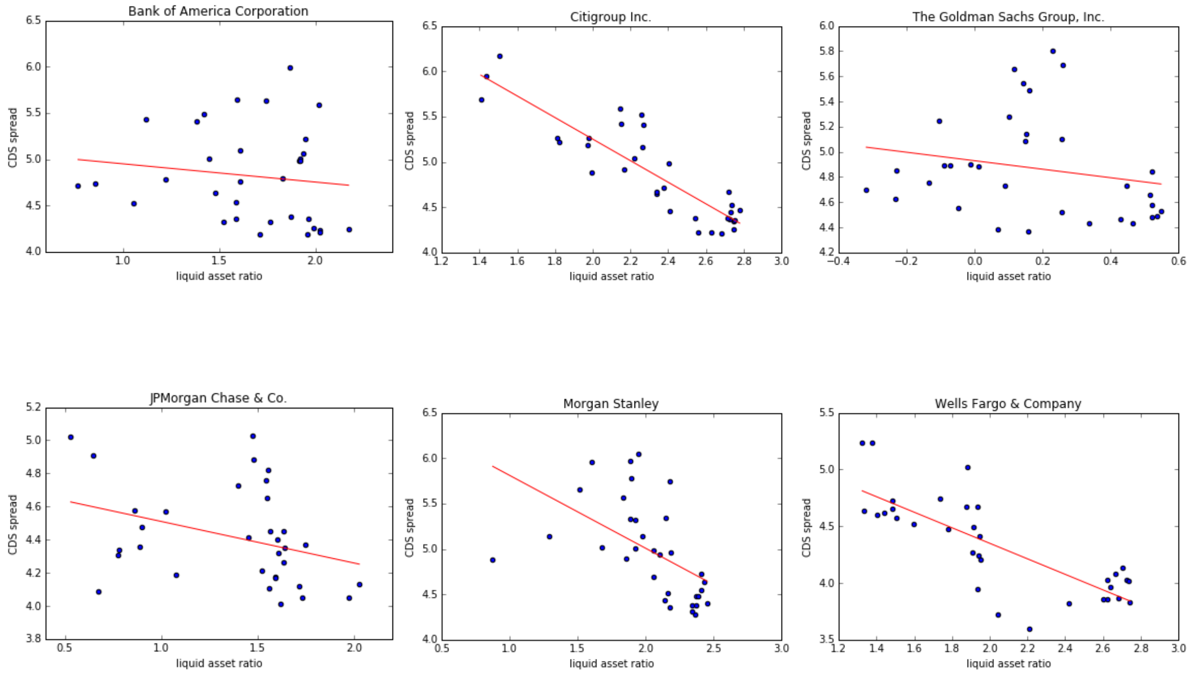


(c) Leverage



(d) Short term debt ratio

Figure 6: Within-firm correlations (variables in logs)



Liquidity positions were very poor pre-crisis. There was high reliance on short-term funding and banks held few liquid assets. There was a general belief that financial markets had become so efficient that a solvent firm could always find liquidity. Therefore we would not expect a funding cost offset pre-crisis, because there was little belief in liquidity risk. Banks built up their liquid assets after the crisis and reduced short-term funding. They have continued to build liquidity, likely in response to more stringent regulation [5].

Capital positions were also poor pre-crisis and firms took significant losses during the crisis. However firms were forced to re-capitalise quickly and have continued to build capital since then.

Figure 6 shows the within-firm correlations between liquid assets and CDS spreads. These are generally negative, although the strength of the relationship varies.

4.3 Initial results

Table 2 presents the regression results as we build up the specification. Column 1 includes only the liquid asset ratio and firm fixed effects. There is a significant negative association between liquidity and CDS spreads. Column 2 adds controls for the firm's leverage and reliance on short term funding. The estimated size of the association between liquidity

[5] The US implemented the LCR at the end of 2014.

Table 2: Regression results

VARIABLES	(1) FE only	(2) FE + BS Variables	(3) FE + BS Variables + Controls
liq asset ratio	-0.465**	-0.389***	-0.243***
	(-3.086)	(-4.251)	(-4.276)
leverage ratio		-1.813***	-1.115***
		(-4.947)	(-6.007)
ST debt ratio		0.0398	0.0130
		(0.915)	(0.609)
Constant	5.178***	8.704***	6.921***
	(34.47)	(11.80)	(14.15)
Observations	198	198	198
R-squared	0.181	0.301	0.706
Number of firmid	6	6	6
Firm FE	YES	YES	YES
Controls	NO	NO	YES

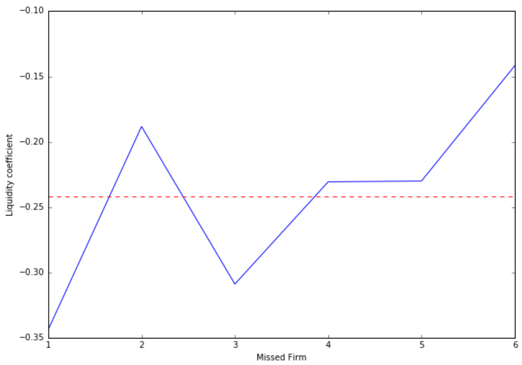
Robust t-statistics in parentheses
*** p<0.01, ** p<0.05, * p<0.1

and funding costs falls, but it gains significance. Column 3 adds firm-invariant controls for stock market volatility and the risk-free rate. The magnitude of the liquidity coefficient falls but it remains highly significant.

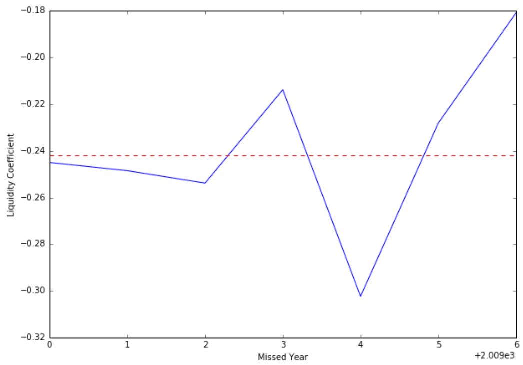
The coefficient in column 3 implies that a 1% rise in a firm's liquid asset ratio is associated with a -0.24% decline in their CDS spreads. Note that this is *not* a percentage point change i.e. a firm that raised its liquid asset ratio from 10% to 11% would have raised their ratio by 10%, but only 1 percentage points. We also find evidence of a negative association between leverage and CDS spreads, consistent with previous research on leverage and funding costs.

We do not find evidence that funding fragility (the variable STD) is associated with funding costs in any of our specifications. This may be due to collinearity with leverage - debt funding is negatively correlated with equity funding. Alteratively, investors are perhaps less informed on funding risks than liquidity risks. The funding data is not very granular as the only maturity buckets we have are greater than / less than a year. In practice runs can be much faster than this, so the funding data may not be a good proxy for funding risk.

Figure 7: Robustness checks



(a) Dropping each firm out



(b) Dropping each year out

4.4 Robustness to outliers

We perform two basic outlier robustness checks of our specification. The first is to drop out each firm individually and re-estimate without that firm. The second test is similar; we re-estimate without each of the years in our sample. If the association between liquidity and funding costs is relatively stable then we can conclude that our results are not being driven by any given firm or year.

Figure 7 shows the results of these checks. The liquid asset ratio coefficient remains fairly stable in both cases. Dropping out each year yields a range of coefficients from -0.18 to -0.30. Therefore we can conclude that our results are not being driven by a single firm or year.

4.5 Robustness to specification changes

We also test the robustness of our results to changes in specification. If changes to the specification were to dramatically change our results, the underlying relationship may not be robust.

Table 3 presents the results. The liquidity coefficient varies but is significant to at least the 10% level in all specifications. Column 1 broadens the liquid asset measure to include other government securities, such as local governments, and interest-bearing demand deposits at other banks. The significance of the relationship falls but the point estimate is fairly similar to our baseline result of -0.24. Columns 2-4 deepen the lag of the independent variables by 1, 2 and 3 periods respectively. The coefficient is slightly smaller but still significant at the 5% level. Finally column 5 re-estimates with a linear specification,

Table 3: Robustness - different specifications

VARIABLES	(1) Broader Liquidity	(2) Lag 1	(3) Lag 2	(4) Lag 3	(5) Linear spec
liq asset ratio	-0.300*	-0.205**	-0.234**	-0.176**	-7.587*
	(-2.124)	(-3.988)	(-3.311)	(-4.029)	(-2.151)
leverage ratio	-0.928**	-1.034***	-0.983**	-1.395***	-19.06**
	(-3.182)	(-5.228)	(-3.793)	(-7.347)	(-3.773)
ST debt ratio	0.000187	0.00337	0.00910	0.00539	-0.897
	(0.00709)	(0.171)	(0.416)	(0.203)	(-0.235)
R-squared	0.706	0.698	0.717	0.740	0.614
Firm FE	YES	YES	YES	YES	YES
CONTROLS	YES	YES	YES	YES	YES

Robust t-statistics in parentheses
*** p<0.01, ** p<0.05, * p<0.1

rather than logs, therefore we cannot compare coefficient size. Reduced significance and R^2 suggest this is a poorer fit, but the association is still significant.

5 Conclusion

In this paper, we have built a model of bank runs with a unique equilibrium where solvent banks may fail due to illiquidity. We go beyond the existing literature by endogenising the firm's funding costs to take account of this risk, and solve for the bank's optimal choice of liquidity. While forcing the bank to hold more liquidity may impact the bank's profits, we have shown that it can reduce their reduce their funding costs, because they are less likely to fail, thereby offsetting some of the fall in profits.

However our model is very simple and the numerical results are unlikely to be useful. For example we assumed, for tractability, that asset risk is uniformly distributed. Further work on enriching the model would generate better numerical predictions that could be used to analyse optimal liquidity requirements.

We test our model's prediction that banks with stronger liquidity positions have lower funding costs. Using post-crisis data for US banks, we find evidence of such an association. This result is robust to removing any individual firm or year from the sample. Our baseline estimate suggests doubling a bank's liquid asset ratio would be associated with a 24.4% decline in their CDS spreads. However we find no evidence for a relationship between funding fragility and CDS spreads.

We view our empirical work as a "first step", rather than drawing strong causal conclusions from it. Our sample size is small with 198 observations (and only 6 in the cross-section). The main issue is finding consistent publicly available measures of both funding costs and liquidity risk. We think there will be better data in the near future with the introduction of mandatory LCR disclosures in Europe.

Our results show that liquidity requirements may be less costly than previously thought. This has a clear policy implication: any analysis of optimal liquidity requirements should account for the beneficial effect on funding costs. We believe this could have a similar role to the "Modigliani-Miller" offset for bank capital requirements.

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A Proofs

We will first show that a threshold strategy equilibrium exists, where all investors stay if their signal exceeds a certain value but run if the signal drops below that value. We then show that this threshold strategy equilibrium is the unique equilibrium surviving iterated deletion of dominated strategies. Finally we show that banks can hold enough cash to become "run-proof", such that they will only fail due to insolvency.

A.1 Proof of existence of a threshold strategy equilibrium

Proposition 1: There exists a threshold strategy equilibrium where all investors stay if they receive a signal above some value x, and run if they receive a signal below x:

We begin by assuming that each investor is using a threshold strategy around some signal x.

$$s_i(x_i) = \begin{cases} \text{stay if } x_i \geq x \\ \text{run if } x_i < x \end{cases} \tag{17}$$

The key question for investors in the intermediate period: given that my signal x_i is the threshold signal, what is the probability that enough investors run for the bank to become illiquid? Remember that if $w > \frac{\theta R(1-c)+c}{D} = z$ investors run then the bank will become illiquid. Investors want to know $Pr(w > z|x_i)$ i.e. the bank fails.

Given that x is the threshold signal, the bank will fail if fewer than z investors get a signal of at least x. We want to know the maximum value of R that would cause less than z investors to get a signal of at least x. Given that we have a continuum of investors and $x_i = R + e_i$, the fraction of investors with a signal higher than x will be $1 - \Phi(\frac{x-R}{\sigma})$.

$$1 - \Phi(\frac{x-R}{\sigma}) \leq z \tag{18}$$

$$R \leq x - \sigma\Phi^{-1}(1-z) = R^* \tag{19}$$

Therefore the bank will fail due to illiquidity if $R < R^*$, given that depositors withdraw when they receive a signal lower than x. To find the probability that the bank will fail, given I have received the threshold signal x, we simply need to find the probability that R is below R^*.

$$P(R < R^*|x_i = x) = \Phi(\frac{R^* - x}{\sigma}) \tag{20}$$

$$= 1 - \Phi(\frac{x - R^*}{\sigma}) \tag{21}$$

$$= 1 - \Phi(\frac{x - (x - \sigma\Phi^{-1}(1-z)}{\sigma}) = z \tag{22}$$

So if I receive the threshold signal then I believe the bank fails with probability z, where z is the critical number of people needed to run i.e. $Pr(w \geq z) = z$. This is the $U[0,1]$ distribution.

Now that we know beliefs over the distribution of investors that will run is $U[0,1]$, we can determine the threshold equilibrium loan return R^* that will cause the bank to fail. At equilibrium, investors must be indifferent staying (LHS) and running (RHS).

$$\int_{W=0}^{\frac{\theta R^*(1-c)+c}{D}} r_D dl + \int_{W=\frac{\theta R^*(1-c)+c}{D}}^{1} 0 dl \tag{23}$$

$$= \int_{W=0}^{1} 1 dl R^* = \frac{1}{\theta(1-c)}(\frac{D}{r_D} - c) \tag{24}$$

So a threshold equilibrium is that depositors will only stay if they receive $x_i \geq \frac{1}{\theta(1-c)}(\frac{1}{r_D} - c)$. We know this is a stable equilibrium because the expected staying payoff (LHS) is strictly increasing in R^*, so investors receiving a signal above R^* would strictly prefer to stay and those receiving a signal below R^* would strictly prefer to run.

A.2 Proof of Uniqueness

Proposition 2: The threshold strategy equilibrium given by R^* is the only strategy surviving iterated deletion of dominated strategies. It is therefore the unique equilibrium for the investor decision in period 1.

The key to understanding the threshold equilibrium is iterated deletion of dominated strategies. Let's continue with the assumption that $\sigma \to 0$, such that investors disregard their prior in forming an expectation over R. Let Pr(bank fails) = P. Investors will run if $P > (1-P)r_D$ i.e. $P > 1 - \frac{1}{r_D} = \gamma$.

Let $\underline{R}_0$ be the lowest return at which the firm is solvent i.e. $\underline{R}_0(1-c)+c = Dr_D$. The probability that the firm is insolvent, given signal x, is $Pr(R < \underline{R}_0|x) = \Phi(\frac{\underline{R}_0 - x}{\sigma})$. Therefore it is strictly dominant to run if investors observe any signal x such that $\Phi(\frac{\underline{R}_0-x}{\sigma}) > \gamma$. Denote $\underline{x}_0$ the highest signal for which it is strictly dominant to run s.t. $\underline{x}_0 = \underline{R}_0 - \sigma\Phi^{-}1(\gamma)$. We can delete all strategies that rollover at $x < \underline{x}_0$.

Let $\underline{R}_1 > \underline{R}_0$ be the highest return for which a firm will fail from illiquidity given that investors with a signal below $\underline{x}_0$ run, because the proportion of investors that run will exceed available liquidity i.e. $\Phi(\frac{\underline{x}_0-\underline{R}_1}{\sigma}) > \theta\underline{R}_1(1-c)+c$. The probability that the firm fails due to illiquidity because $R \leq \underline{R}_1$, given signal x, is at least $\Phi(\frac{\underline{R}_1-x}{\sigma})$. Given the previous round of deletion, it is now strictly dominant to run if investors recevie signal x such that $\Phi(\frac{\underline{R}_1-x}{\sigma}) > \gamma$. Therefore investors will run below any signal $\underline{x}_1 = \underline{R}_0 - \sigma\Phi^{-}1(\gamma) > \underline{x}_0$ and we can delete all strategies that rollover with signal $x < \underline{x}_1$.

We can iterate this deletion until we reach some pair $\underline{x}_k = \underline{R}_k - \sigma\Phi^{-}1(\gamma)$ s.t. $\nexists \underline{R}_{k+1} > \underline{R}_k$ where $\Phi(\frac{\underline{x}_k-\underline{R}_{k+1}}{\sigma}) > \theta\underline{R}_{k+1}(1-c)+c$. In English, the firm will hold enough liquidity to survive a run at any return $R_{k+1} > \underline{R}_k$, where the proportion of runners is given by the threshold signal $\underline{x}_k$ from the previous round of deletion. We have therefore deleted all strategies that involve rolling over with signals $x < \underline{x}_k$.

Now denote $\bar{R}_0$ as the lowest loan return that the bank is immune to runs i.e. $\theta\bar{R}_0(1-c)+c = 1$. The probability of this, given signal x, is $Pr(R > \bar{R}_0|X) = 1 - \Phi(\frac{\bar{R}_0-x}{\sigma})$. Therefore $Pr(fail) \leq \Phi(\frac{\bar{R}_0-x}{\sigma})$. Denote $\bar{x}_0$ as the lowest signal for which it is strictly dominant to roll over because $\Phi(\frac{\bar{R}_0-\bar{x}_0}{\sigma}) = \gamma \therefore \bar{x}_0 = \bar{R}_0 - \sigma\Phi^{-}1(\gamma)$. We can delete all strategies that run with signals $x \geq \bar{x}_0$, because any investor expect the bank to survive often enough even if all other investors run.

Let $\bar{R}_1 < \bar{R}_0$ be the smallest return for which a firm cannot fail due to illiquidity given that investors with a signal above $\underline{x}_0$ roll over i.e. $\Phi(\frac{\bar{x}_0-\bar{R}_1}{\sigma}) < \theta\bar{R}_1(1-c)+c$. The probability that a firm cannot fail due to illiquidity because $R \geq \bar{R}_1$, given signal x, is at least $1 - \Phi(\frac{\bar{R}_1-x}{\sigma})$. Denote $\bar{x}_1$ as the lowest signal for which it is dominant to roll over because $\Phi(\frac{\bar{R}_1-\bar{x}_1}{\sigma}) = \gamma \therefore \bar{x}_1 = \bar{R}_1 - \sigma\Phi^{-}1(\gamma) < \bar{x}_0$. There we can delete all strategies that run with signals $x \geq \bar{x}_1$.

We can iterate this deletion until we reach some pair $\bar{x}_k = \bar{R}_k - \sigma\Phi^{-}1(\gamma)$ s.t. $\nexists \bar{R}_{k+1} < \bar{R}_k$ where $\Phi(\frac{\bar{x}_k-\bar{R}_{k+1}}{\sigma}) < \theta\bar{R}_{k+1}(1-c)+c$. In other words, we eventually we reach some $\bar{R}_{k+1}$ s.t. it is no longer strictly dominant to roll over, where the proportion of investors definitely rolling over is given by the threshold signal $\bar{x}_k$ from the previous round of deletion. We have therefore deleted all strategies that involve running with signals $x > \bar{x}_k$.

Given continuity of the distributions and payoff functions, the limits of these two sequences will converge i.e. $\underline{x}_k = \bar{x}_k = x^*$ and $\underline{R}_k = \bar{R}_k = R^*$. Therefore there will be a unique equilibrium where investors roll over if they observe signals above x^*, and run if they receive signals below. Moreover $\lim_{\sigma\to 0} x^* = R^*$. Our model satisfies the general conditions laid out in Morris and Shin (2000) for existence of a unique equilibrium.

A.3 Proof of existence of cash choice that eliminates liquidity risk

Proposition 3: If a firm holds sufficient liquidity $\hat{c}$ s.t. $\theta\underline{R}_0(1-\hat{c})+\hat{c} \geq D - \frac{1}{r_D} = \gamma$, there will be no liquidity risk as investors will only run if they observe $x < \underline{x}_0$.

We prove this by contradiction. Recall that the point at which it is strictly dominant to run, due to solvency concerns, is $\underline{x}_0 = \underline{R}_0 - \sigma\Phi^-1(\gamma)$. Suppose $c \geq \hat{c}$ and $R^* > \underline{R}_0$ i.e. there is some liquidity risk because solvent banks can fail. There must exist at least 1 possible value R_1 s.t.$\underline{R}_0 < \underline{R}_1 < R^*$ where it is still strictly dominant to run. At signal $\underline{R}^1$, the firm has liquidity of $\theta\underline{R}^1(1-\hat{c})+\hat{c} > \gamma$ therefore it can no longer be strictly dominant to run, so $\nexists\ R^* > R_0$. There is no liquidity risk if $c \geq \hat{c}$ - only insolvent banks will fail.

A.4 Proof of unique solution for deposit rate

We show that we can rule out the lower root of the quadratic solution for the deposit rate, given by **??**:

$$r_D^* = \frac{D + c + U\theta\bar{R}(1-c) \pm \sqrt{(D + c + U\theta(1-c))^2 - 4D(\theta\bar{R}(1-c)+c)}}{2(\theta\bar{R}(1-c)+c)} \tag{25}$$

We know that $r_D^* \geq U$ is a necessary condition to satisfy the participation constraint. The lower root is bounded above by:

$$B = \frac{1 + c + U\theta\bar{R}(1-c)}{2(\theta\bar{R}(1-c)+c)} \tag{26}$$

The derivative wrt. c:

$$\frac{\delta B}{\delta c} = \frac{1}{4(\theta\bar{R}(1-c)+c)**2}(1-U\theta\bar{R})(\theta\bar{R}(1-c)+c) - 2(1+c+U\theta\bar{R}(1-c))(1-\theta\bar{R}) = \frac{1}{4(\theta\bar{R}(1-c)}$$

This could be positive or negative, depending on the values of the exogenous parameters. However whether it is positive or negative is independent of c. Therefore if $\frac{\delta B}{\delta c}_{c=0} > (<)0$ then $\frac{\delta B}{\delta c}_{c\in[0,1]} > (<)0$.

It is therefore sufficient to show that that $B_{c\in\{0,1\}} < U$, because if this condition holds at the extremes then it will hold at all intermediate points.

$$B_{c=0} = \frac{1}{2\theta\bar{R}}(1 + U\theta\bar{R}) = \frac{1}{2\theta\bar{R}} + \frac{U}{2} < UB_{c=1} = \frac{2}{2} < U \tag{28}$$

Therefore the lower root of r_D^* is always less than U, so we can rule it out.

A.5 Proof that interior solutions exist only if the bank has equity

We show that the bank will never choose $c > 0$ in equilibrium unless they have some equity. It's sufficient to show that $\frac{\delta\pi}{\delta c}|(c = 0, D = 1) < 0$.

$$\frac{\delta\pi}{\delta c}_{|c=0} = -(\bar{R} - R^*)(\frac{1}{2}(\bar{R} + R^*) - 1) - \frac{dR^*}{dc}(R^* - Dr_D^**) - \frac{Dr_D{}^{**}}{dc}(\bar{R} - R^*) \tag{29}$$

We evaluate each of these terms individually at $c = 0$. The firm is most likely to hold cash if it can raise less liquidity from its loans e.g. $\theta\bar{R} = D$. Evaluating $r_D|c = 0$ and its derivative:

$$r_{D|c=0,\theta\bar{R}=D} = \frac{1}{2}(1 + U + \sqrt{(U+3)(U-1)}) \tag{30}$$

$$\frac{\delta r_D}{\delta c}_{|c=0,\theta\bar{R}=D} = \frac{1}{2D}[1 - UD - 2(D(U+2) - 1)\sqrt{\frac{U-1}{U+3}} - (1 - D)(1 + U + \sqrt{(U+3)(U-1)})] \tag{31}$$

Evaluate the failure point R^* and its derivative:

$$R^*_{|c=0,\theta\bar{R}=D} = \frac{\bar{R}}{r_D} \tag{32}$$

$$\frac{\delta R^*}{\delta c}_{|c=0,\theta\bar{R}=D} = -\bar{R}[\frac{1}{2}(1 + U + \sqrt{(U+3)(U-1)}) - D] \tag{33}$$

We can make one final simplification by evaluating at $lim_{U\to 1}$, because lower reserve utilities reduce the loss from holding cash.

$$\frac{\delta\pi}{\delta c}_{|c=0,\theta\bar{R}=D,U\to 1} = \bar{R}(1 - D)(\bar{R} - D) > 0 \text{ iff } D < 1 \tag{34}$$

If the bank has no equity ($D = 1$), it will never hold any cash because the profit function is downward sloping at $c = 0$, even with the parameter choices that most incentivise holding cash. The intuition is that without equity, the bank has no "skin in the game" and therefore little incentive to insure against runs.

However if the bank has equity then there will be parameters for which this derivative is positive, therefore the bank may hold some cash.

B Descriptive Statistics

Table 4: Summary statistics

VARIABLES	(1) N	(2) mean	(3) sd	(4) min	(5) max
cds	210	125.2	80.00	36.49	477.9
5y UST yield	210	1.532	0.485	0.662	2.453
VIX	210	18.88	7.084	10.92	44.84
capital ratio	198	7.451	0.870	5.965	10.04
liq asset ratio	198	3.407	2.229	0.682	8.099
ST debt ratio	198	2.469	1.274	0.0149	7.076

Figure 8: CDS spread by firm

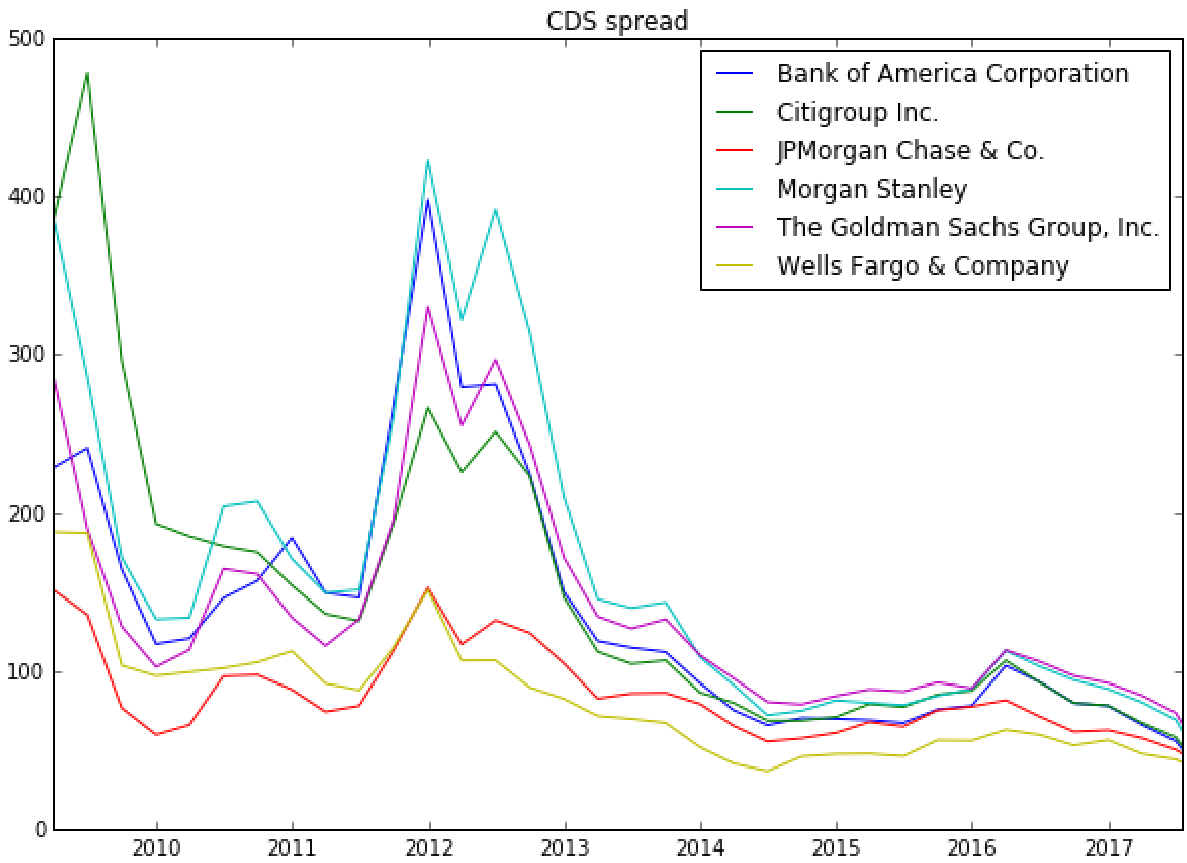

Figure 9: Liquid asset ratio by firm

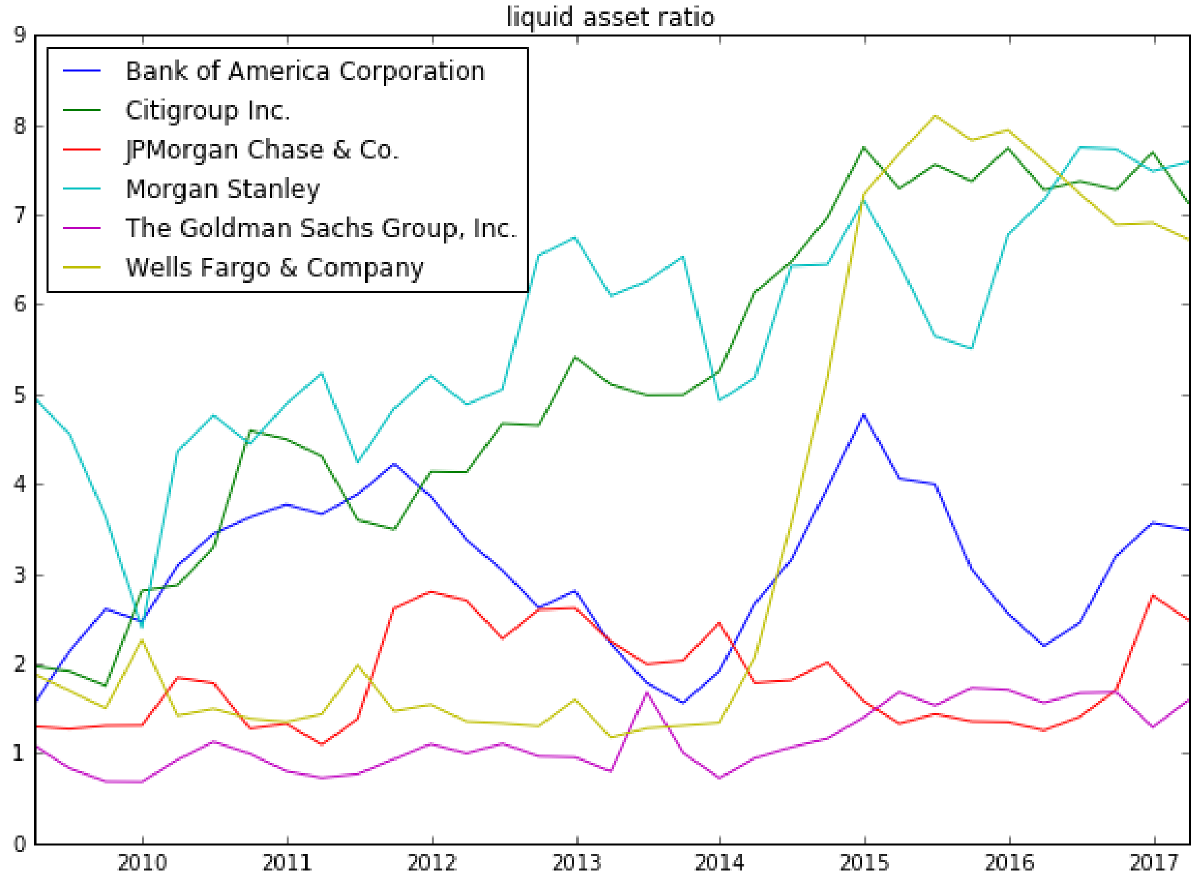

Figure 10: Leverage ratio by firm

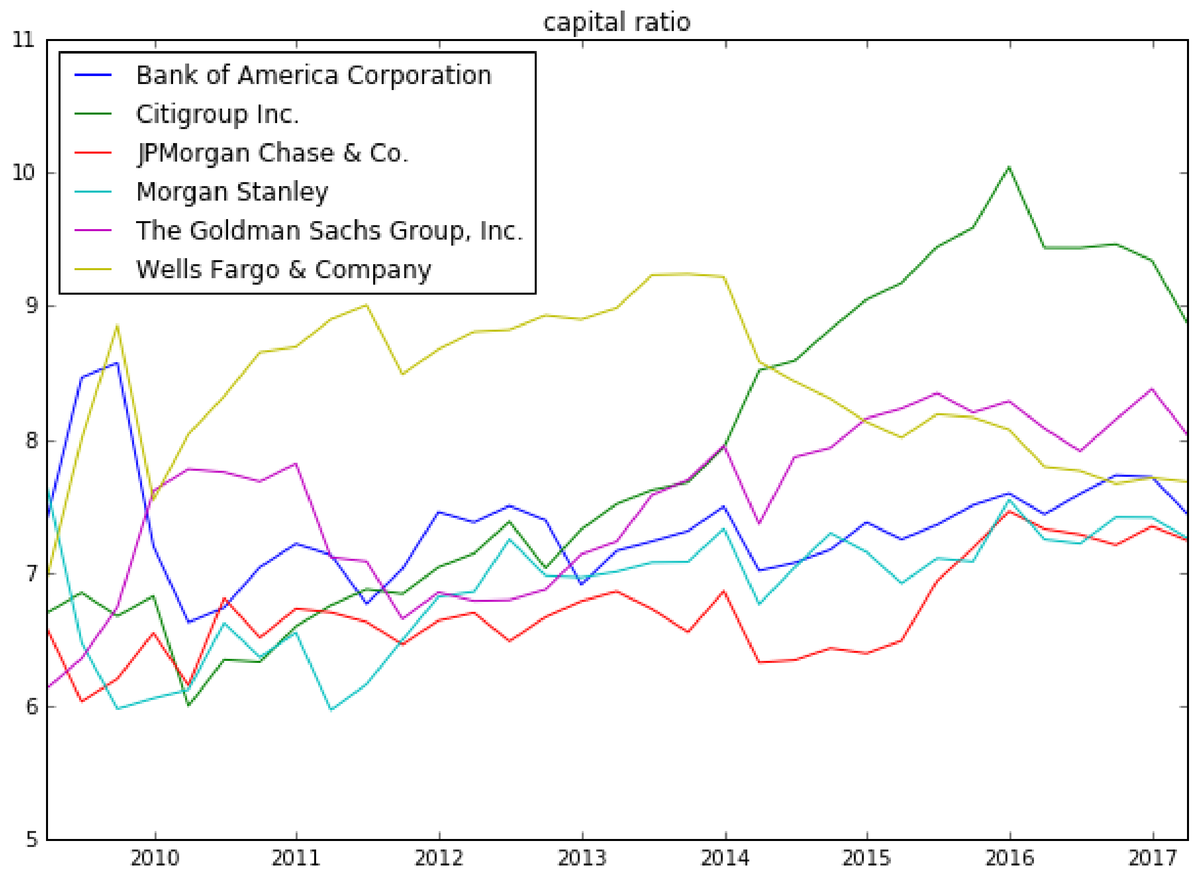

Figure 11: Short term funding ratio by firm