Speed Running Calculus

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November 8, 2023

Contents

Author's Notes

This actually isn't the first copy of my discrete math notes…I had an earlier copy that got corrupted, so now I'm just going to constantly upload to GitHub because I'm afraid of them getting wiped again 😭

These notes are based off of the textbook *Discrete Mathematics and Its Applications* by *Kenneth H. Rosen*, as well as from the lectures of *Serdar Erbatur* from the *Fall 2023* semester.

If you have any complaints, or suggestions regarding these notes, please email me at mdn220004@utdallas.edu

1 Propositional Logic

Definition **Proposition**

Declarative statements that are either `true` or `false`

Table 1: Examples of propositions

Statement	Proposition?
$1 + 1 = 2$	✔️
What class are ya takin?	⛔
I am happy	✔️

Propositons can be combined to form another proposition with the use of *logical operators*

Note

Common labels for propositions in discrete mathematics include letters like $P, Q, R, S...$

Table 2: Logical Operators in Discrete Mathematics

Symbol	Meaning	Expression
$\neg$	negation / not	$\neg P$
$\wedge$	conjunction / and	$P \wedge Q$
$\vee$	disjunction / or	$P \vee Q$
$\oplus$	exclusive disjunction/ xor	$P \oplus Q$
$\implies$	implication / conditional	$P \implies Q$
$\iff$	biconditional	$P \iff Q$

Note **Logical Operator Precedence**

Order matters when it comes to evaluating logical operators:

1. negaiton
2. conjunction
3. disjunctions
4. conditionals

Where negation is evaluated first, and conditionals last.

Theorem More on Conditionals

Conditionals can also be expressed in three other ways:

Contrapositive: $\neg Q \implies \neg P$

Converse: $Q \implies P$

Inverse: $\neg P \implies \neg Q$

Example

Conditional: If it is raining, then I am not going to town

Contrapositive: If I go to town, then it is not raining

Converse: If I do not go to town, then it is raining

Inverse: If it is not raining, then I am going to town

Definition **Truth Table**

They are used as a way of seeing all possible values of a proposition

Table 3: Truth Table of $P \implies Q$

P	Q	$P \implies Q$
F	F	T
F	T	T
T	F	F
T	T	T

1.1 Propositional Equivalencies

Definition **Equivalencies**

A proposition is a...

- **Tautology** if it is `true` in *every case*
- **Contradiction / Fallacy** if it is `false` in *every case*
- **Contingency** if *neither* is the case

Table 4: Example of Tautology and Contradiction

P	$\neg P$	$P \vee \neg P$	$P \wedge \neg P$
T	F	T	F
F	T	T	F

Definition **Logical Equivalency ($\equiv$)**

Compound propositions P and Q are *logically equivalent* if $P \iff Q$ is a *tautology*. This is expressed as

$$P \equiv Q$$

Below is a table of useful logical equivalencies

Table 5: Logical Equivalencies

Name	**Equivalence**
Identity	$P \land \mathrm{T} \equiv P$ $P \lor \mathrm{F} \equiv P$
Idempotent	$P \land P \equiv P$ $P \lor P \equiv P$
Domination	$P \lor \mathrm{T} \equiv \mathrm{T}$ $P \land \mathrm{F} \equiv \mathrm{F}$
Negation	$P \lor \neg P \equiv \mathrm{T}$ $P \land \neg P \equiv \mathrm{F}$
Double Negation	$\neg(\neg P) \equiv P$
Commutative	$P \land Q \equiv Q \land P$ $P \lor Q \equiv Q \lor P$
Associative	$(P \land Q) \land R \equiv P \land (Q \land R)$ $(P \lor Q) \lor R \equiv P \lor (Q \lor R)$
Distributive	$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$ $P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)$
De Morgan's	$\neg(P \land Q) \equiv \neg P \lor \neg Q$ $\neg(P \lor Q) \equiv \neg P \land \neg Q$
Absorption	$P \land (P \lor Q) \equiv P$ $P \lor (P \land Q) \equiv P$

The use of showing the equivalencies between two compound propositions is called a *conditional-disjunction equivalence*

Example **Conditional-Disjunction Equivalence**

$$P \implies Q \equiv \neg P \lor Q$$

This can be proven with the use of a truth table 🥰

Continued on Next Page...

Table 6: Conditional-Disjunction Equivalence Proof

P	Q	$\neg P$	$\neg P \vee Q$	$P \implies Q$
F	F	T	T	T
F	T	T	T	T
T	F	F	F	F
T	T	F	T	T

1.2 Predicates and Quantifiers

1.3 Nested Predicates

1.4 Inference Rules and Proofs

2 Set Theory

2.1 Cardinality

2.2 Set Operations

2.3 Functions

2.4 Floor and nCeiling Functions

3 Algorithms