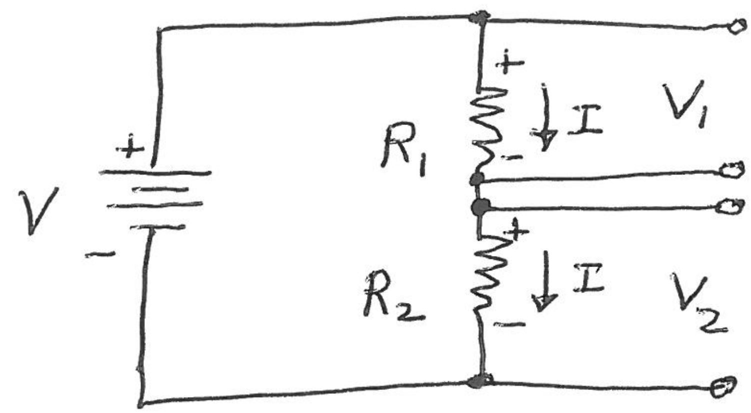
Physics 264 – Lecture 2

Voltage Divider



$$I = \frac{V}{R_1 + R_2}$$

$$V_1 = I R_1 = \frac{V}{R_1 + R_2} R_1 = \left(\frac{R_1}{R_1 + R_2}\right) V$$

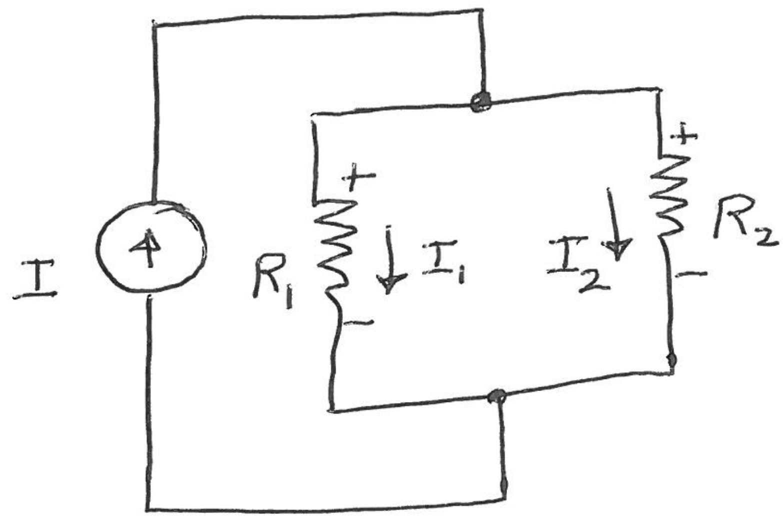
$$V_2 = I R_2 = \frac{V}{R_1 + R_2} R_2 = \left(\frac{R_2}{R_1 + R_2}\right) V$$

Example: $V = 10\,V$, $R_1 = 2\,\Omega$, $R_2 = 3\,\Omega$

$$V_1 = \left(\frac{2}{2+3}\right) 10 = \left(\frac{2}{5}\right) 10 = 4\,V$$

$$V_2 = \left(\frac{3}{2+3}\right) 10 = \left(\frac{3}{5}\right) 10 = 6\,V$$

Current Divider



$$R = \frac{R_1 R_2}{R_1 + R_2}$$

$$V = IR$$

$$I_1 = \frac{V}{R_1} = \frac{I \cancel{R_1} R_2}{(R_1 + R_2)\cancel{R_1}} = \left(\frac{R_2}{R_1 + R_2}\right) I$$

$$I_2 = \frac{V}{R_2} = \frac{I R_1 \cancel{R_2}}{(R_1 + R_2)\cancel{R_2}} = \left(\frac{R_1}{R_1 + R_2}\right) I$$

Example: $I = 10A$, $R_1 = 2\Omega$, $R_2 = 3\Omega$

$$I_1 = \left(\frac{3}{2+3}\right)10 = \left(\frac{3}{5}\right)10 = 6A$$

$$I_2 = \left(\frac{2}{2+3}\right)10 = \left(\frac{2}{5}\right)10 = 4A$$

Node – A point in an electrical circuit where three or more wires connect.

Branch – The series connected elements between two nodes.

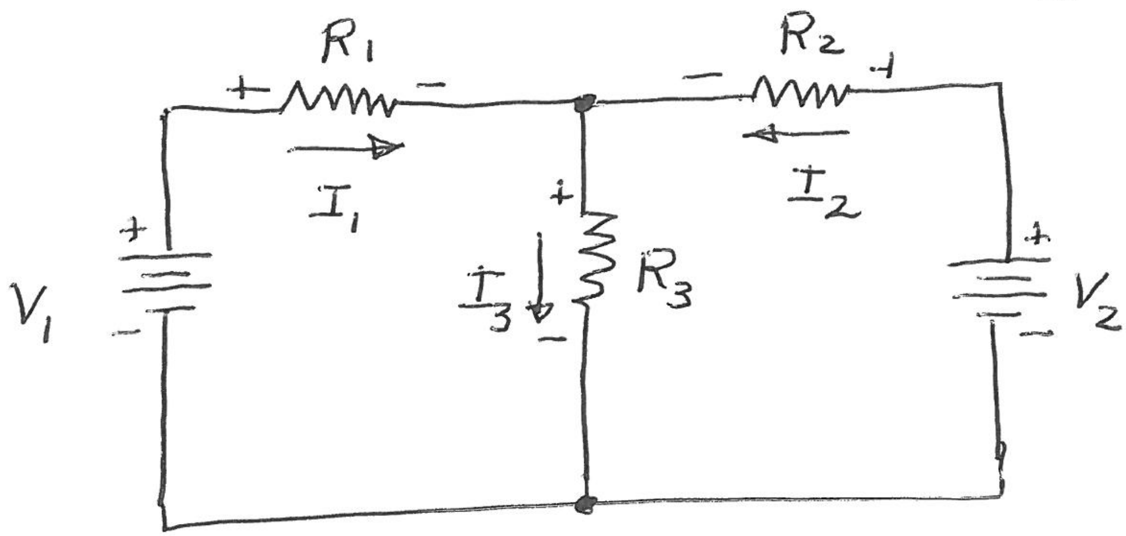
Kirchhoff's Rules

1. The algebraic sum of the currents at any node in an electrical circuit is equal to zero.
2. The algebraic sum of the potential differences around any closed loop in an electrical circuit is equal to zero.

N – nodes B – branches

N−1 independent node equations

B−(N−1) = B−N+1 independendant loop equations



$$I_1 + I_2 - I_3 = 0$$

$$V_1 - I_1 R_1 - I_3 R_3 = 0$$

$$V_2 - I_2 R_2 - I_3 R_3 = 0$$

$$1\,I_1 + 1\,I_2 - 1\,I_3 = 0$$

$$R_1 I_1 + 0\,I_2 + R_3 I_3 = V_1$$

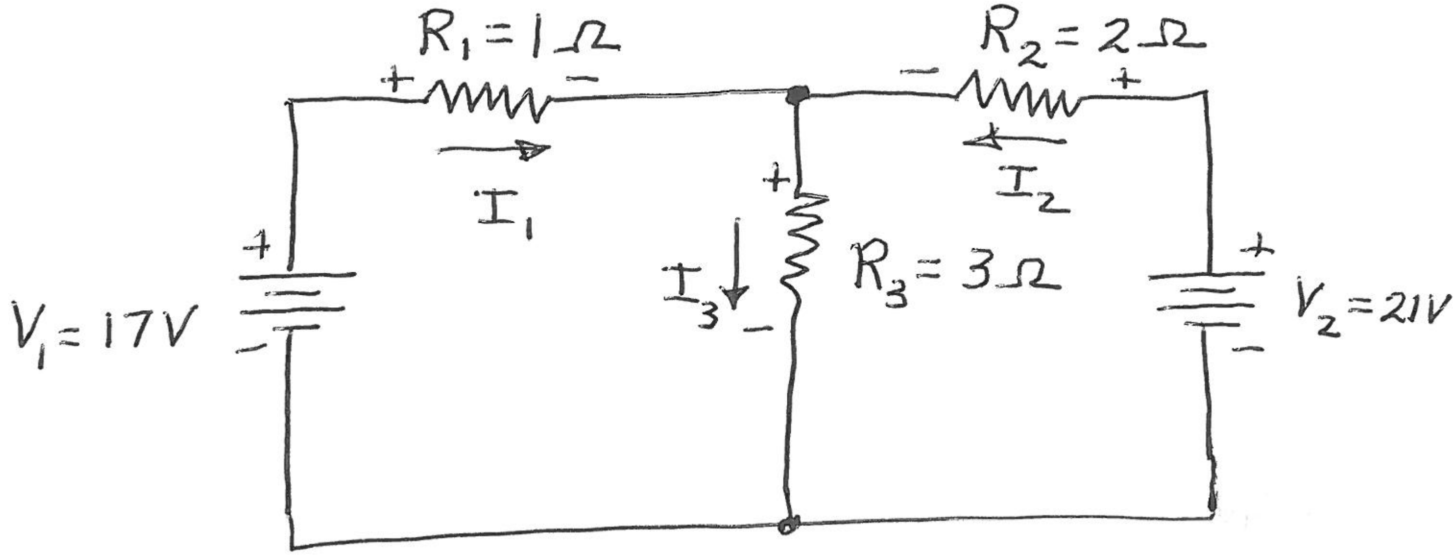
$$0\,I_1 + R_2 I_2 + R_3 I_3 = V_2$$

$$\begin{pmatrix} 1 & 1 & -1 \\ R_1 & 0 & R_3 \\ 0 & R_2 & R_3 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ V_1 \\ V_2 \end{pmatrix}$$

$$R I = V$$

$$I = R^{-1} V$$

$$I = R \backslash V \leftarrow \text{Matlab}$$



$$I_1 + I_2 - I_3 = 0$$

$$17 - 1 I_1 - 3 I_3 = 0$$

$$21 - 2 I_2 - 3 I_3 = 0$$

$$I_1 = 17 - 3 I_3$$

$$I_2 = \frac{21}{2} - \frac{3}{2} I_3$$

$$\left(17 - 3I_3\right) + \left(\frac{21}{2} - \frac{3}{2}I_3\right) - I_3 = 0$$

$$\left(17 + \frac{21}{2}\right) = \left(3 + \frac{3}{2} + 1\right) I_3$$

$$\left(\frac{6+3+2}{\cancel{2}}\right) I_3 = \frac{34+21}{\cancel{2}}$$

$$11 I_3 = 55$$

$$\boxed{I_3 = 5A}$$

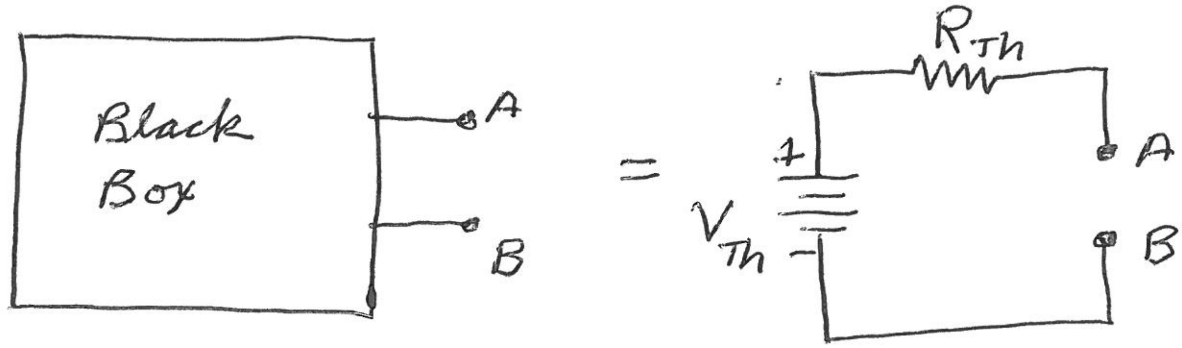
$$I_1 = 17 - 3I_3$$

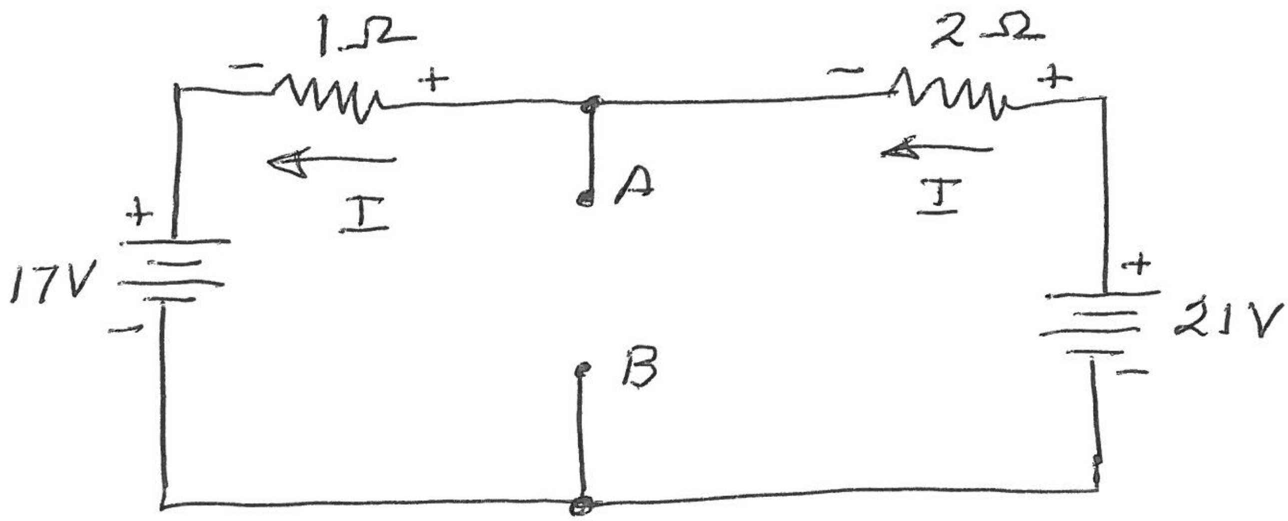
$$I_1 = 17 - (3)(5) = \boxed{2A}$$

$$I_2 = I_3 - I_1$$

$$I_2 = 5 - 2 = \boxed{3A}$$

Thevenin's Theorem





$$21 - 2I - 1I - 17 = 0$$

$$3I = 4$$

$$I = \frac{4}{3}\ A$$

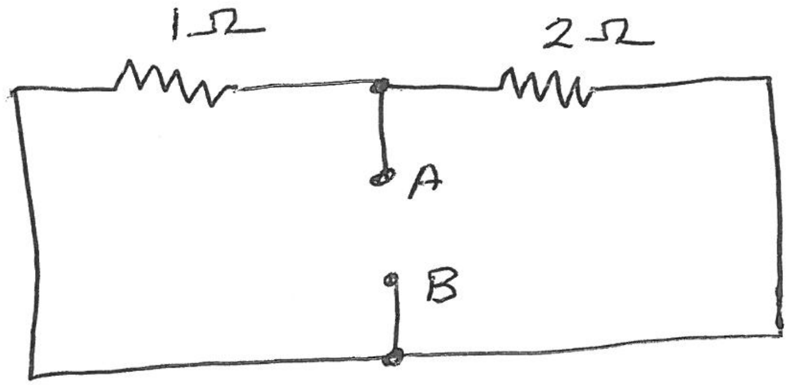
$$V_{Th} = 21 - 2I = 21 - (2)\left(\frac{4}{3}\right)$$

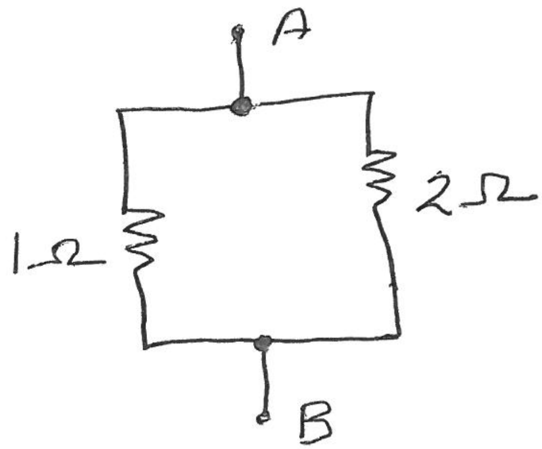
$$V_{Th} = 21 - \frac{8}{3} = \frac{63-8}{3} = \boxed{\frac{55}{3}\ V}$$

Check:

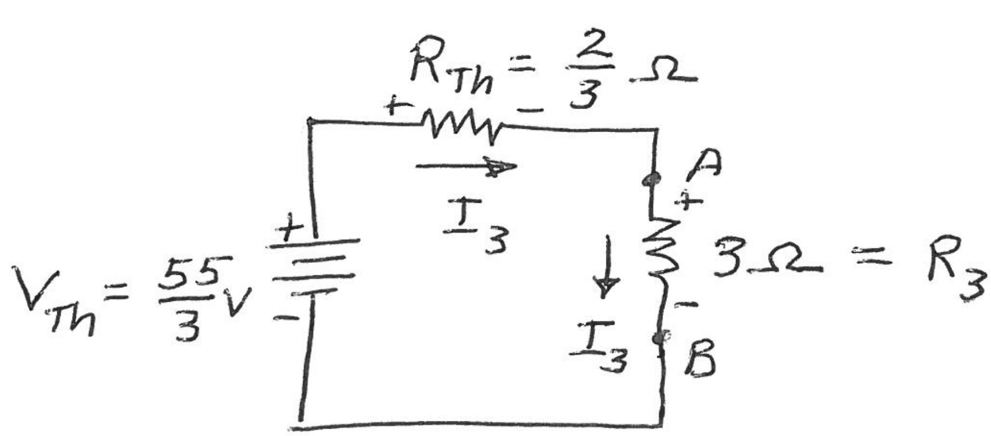
$$V_{Th} = 17 + 1I = 17 + \frac{4}{3}$$

$$V_{Th} = \frac{51+4}{3} = \boxed{\frac{55}{3}\,V}$$





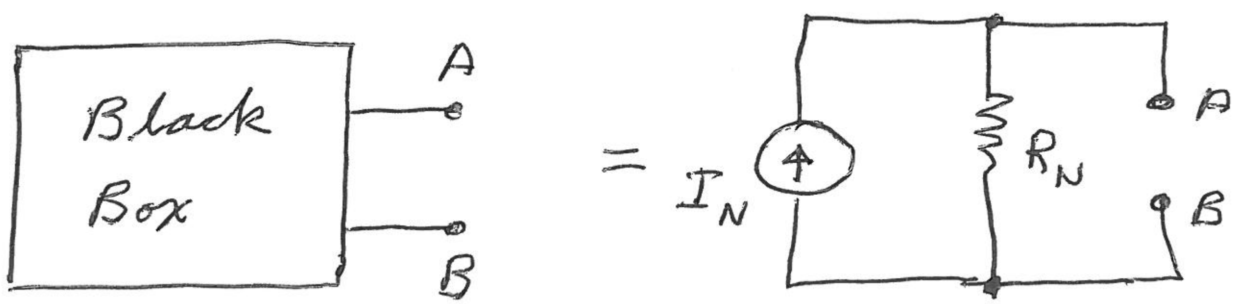
$$R_{Th} = \frac{(1)(2)}{1+2} = \frac{2}{3}\,\Omega$$



$$I_3 = \frac{V_{Th}}{R_{Th} + R_3}$$

$$I_3 = \frac{\frac{55}{3}}{\frac{2}{3} + 3} = \frac{\frac{55}{\cancel{3}}}{\frac{11}{\cancel{3}}} = \frac{55}{11} = \boxed{5A}$$

<u>Norton's Theorem</u>



$$R_N = R_{Th} \qquad I_N = \frac{V_{Th}}{R_{Th}}$$

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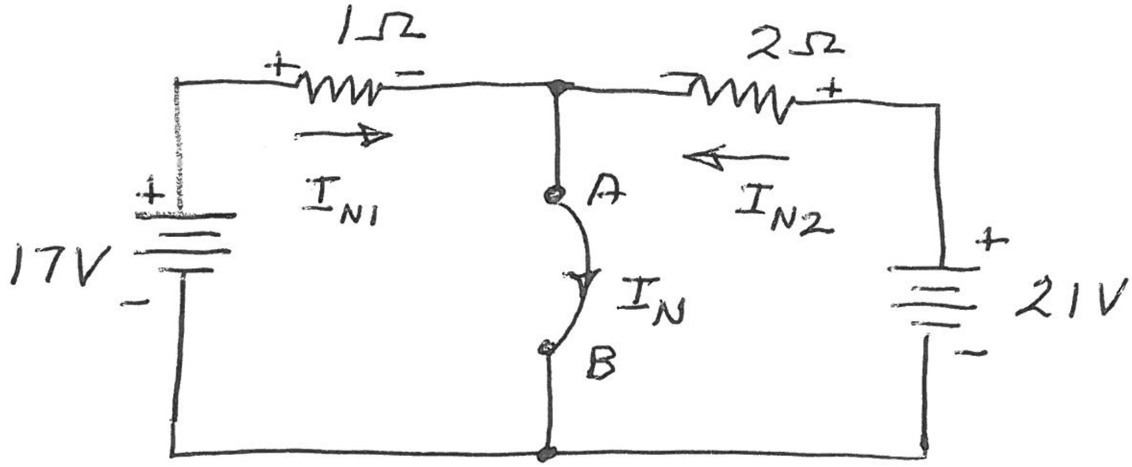


$$17 - 1\,I_{N1} = 0$$

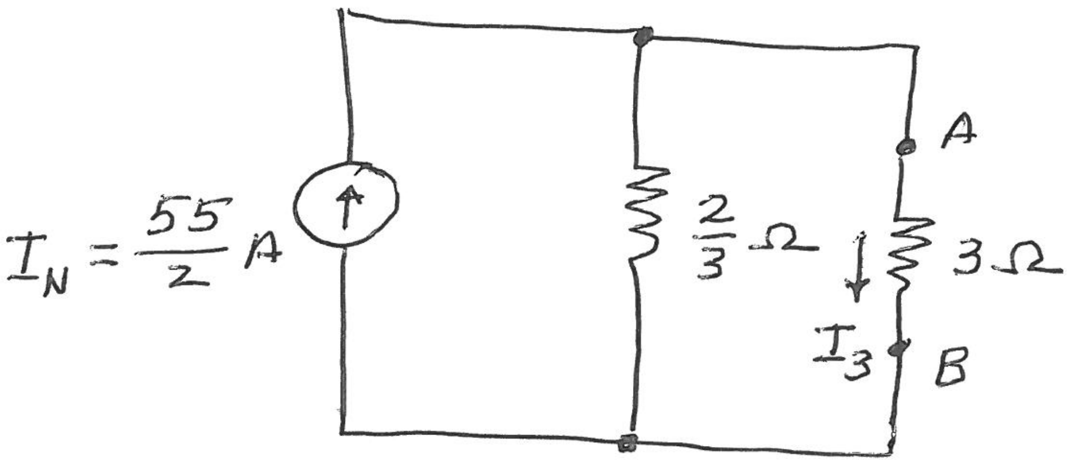
$$I_{N1} = 17A$$

$$21 - 2\,I_{N2} = 0$$

$$I_{N2} = \frac{21}{2}A$$

$$I_N = 17A + \frac{21}{2}A = \frac{34+21}{2}$$

$$\boxed{I_N = \frac{55}{2}A}$$

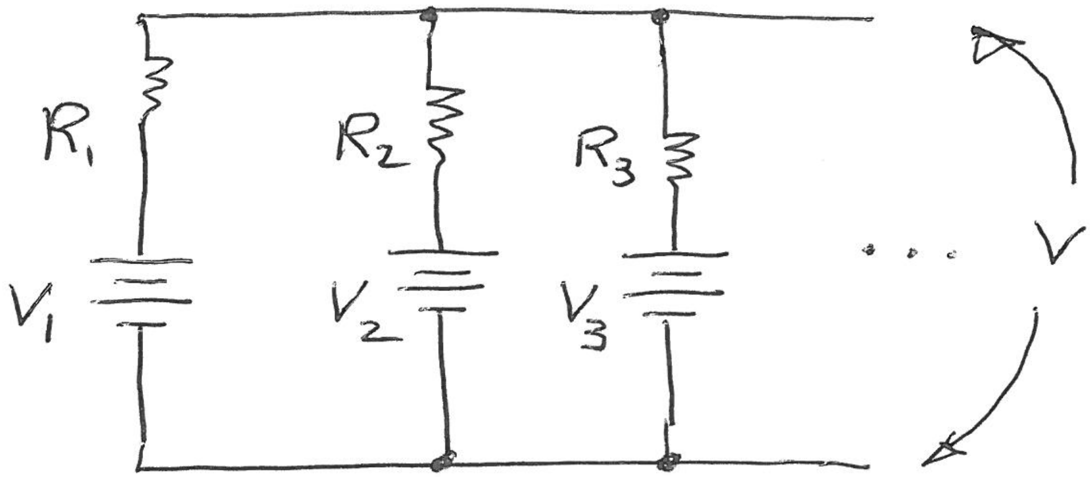


$$I_3 = \left(\frac{\frac{2}{3}}{\frac{2}{3}+3}\right)\left(\frac{55}{2}\right)$$

$$I_3 = \left(\frac{\frac{2}{\cancel{3}}}{\frac{11}{\cancel{3}}}\right)\left(\frac{55}{2}\right)$$

$$I_3 = \left(\frac{\cancel{2}}{11}\right)\left(\frac{55}{\cancel{2}}\right) = \boxed{5\,A}$$

Millman's Theorem



$$V = \frac{\sum \frac{V_i}{R_i}}{\sum \frac{1}{R_i}}$$

$$V = \frac{\frac{17}{1} + \frac{0}{3} + \frac{21}{2}}{\frac{1}{1} + \frac{1}{3} + \frac{1}{2}}$$

$$V = \frac{\frac{34+21}{2}}{\frac{6+2+3}{6}}$$

$$V = \left(\frac{\overset{5}{\cancel{55}}}{\cancel{2}}\right)\left(\frac{\overset{3}{\cancel{6}}}{\cancel{11}}\right)$$

$$\boxed{V = 15\,V}$$

$$I_3 = \frac{V}{R_3} = \frac{15}{3} = \boxed{5\,A}$$