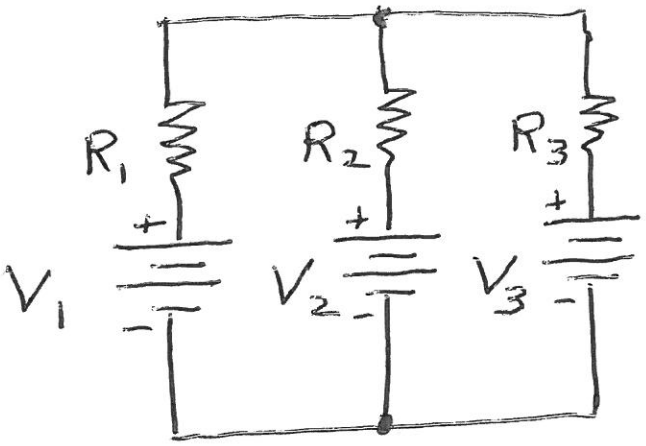
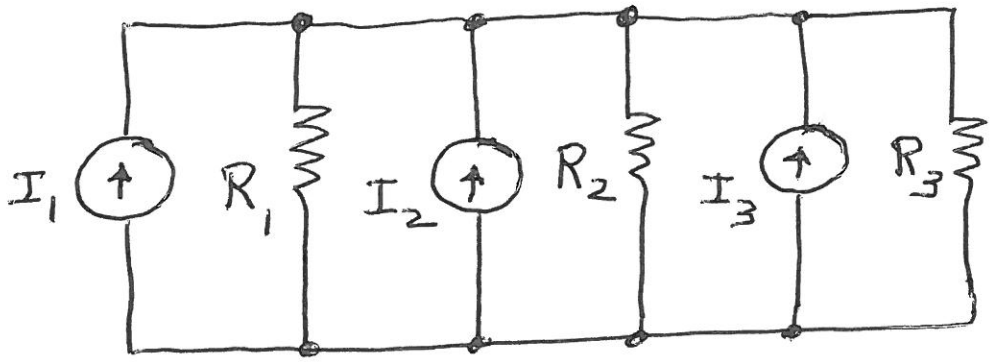
Physics 264 - Lecture 3

Millman's Theorem

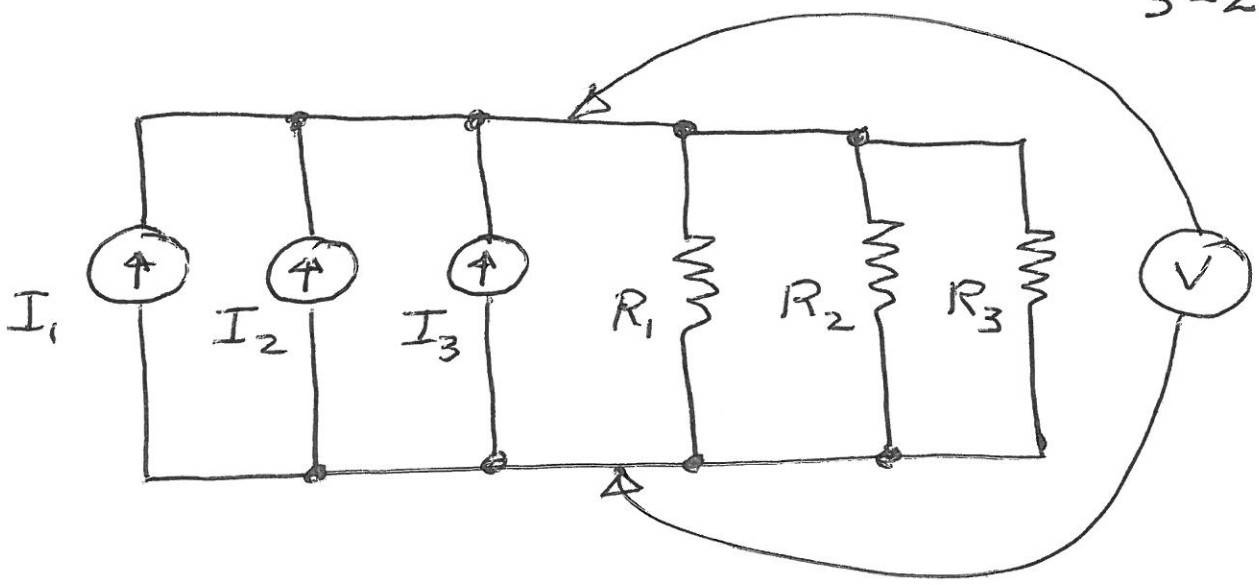


Transform to Norton's Circuit



$$I_1 = \frac{V_1}{R_1} \qquad I_2 = \frac{V_2}{R_2} \qquad I_3 = \frac{V_3}{R_3}$$

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

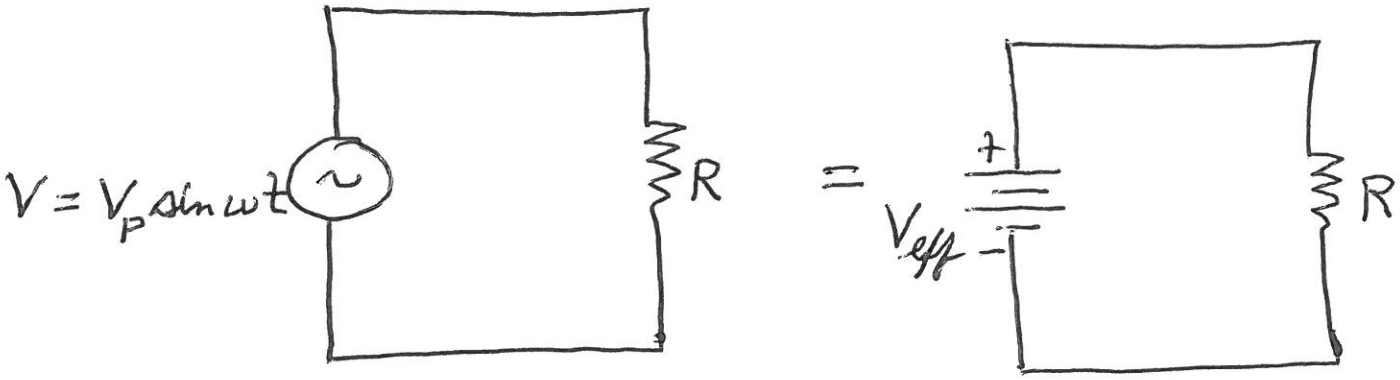


$$V = (I_1 + I_2 + I_3) R$$

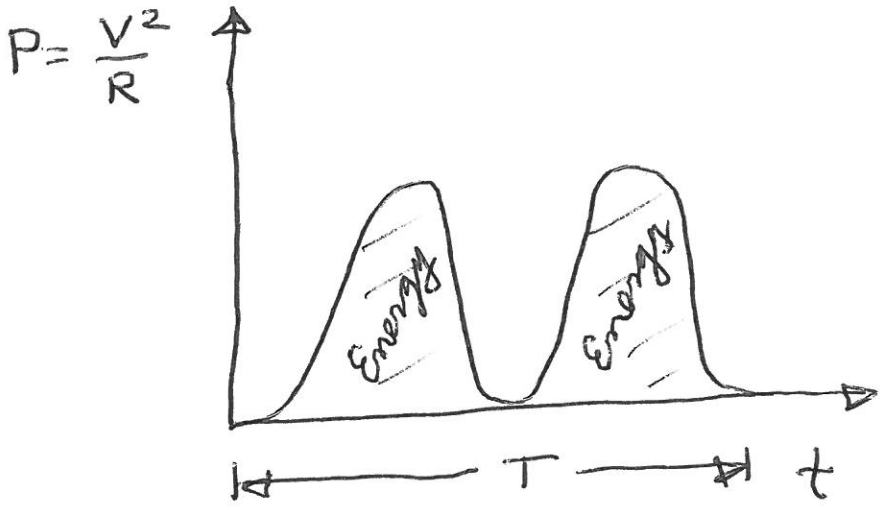
$$V = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

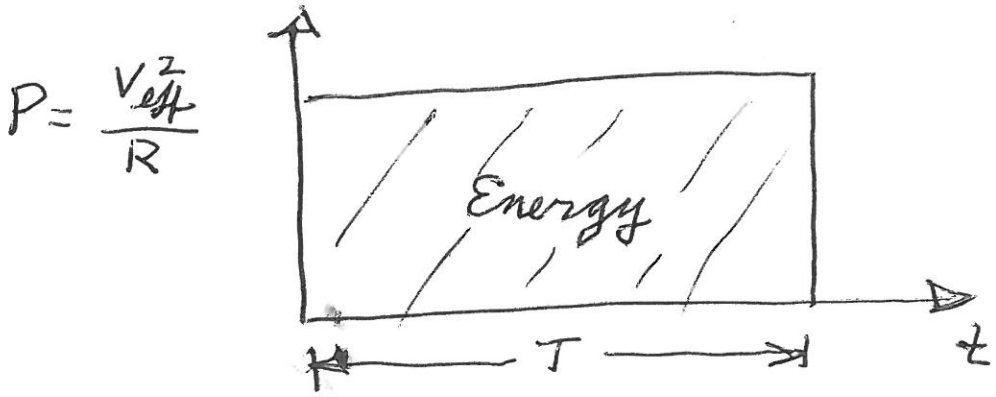
AC Circuit

$$V = IR$$



$$P = \text{Power} = IV = I^2R = \frac{V^2}{R}$$



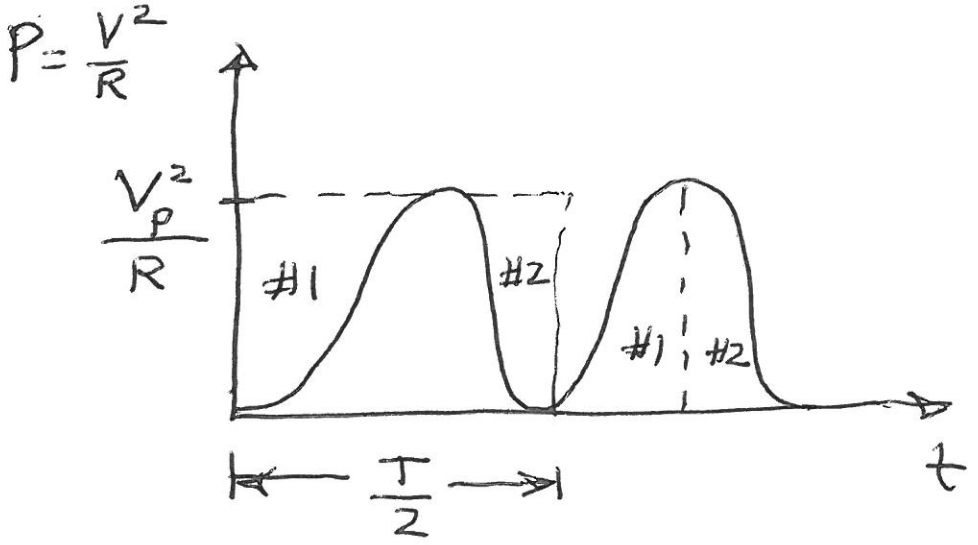


$$\text{Energy} = \int_0^T \frac{V^2}{R}\,dt = \frac{V_{eff}^2}{R}\,T$$

$$\text{Energy} = \frac{\overline{V^2}}{R}\,T = \frac{V_{eff}^2}{R}\,T$$

$$V_{eff} = \sqrt{\overline{V^2}} = V_{rms}$$

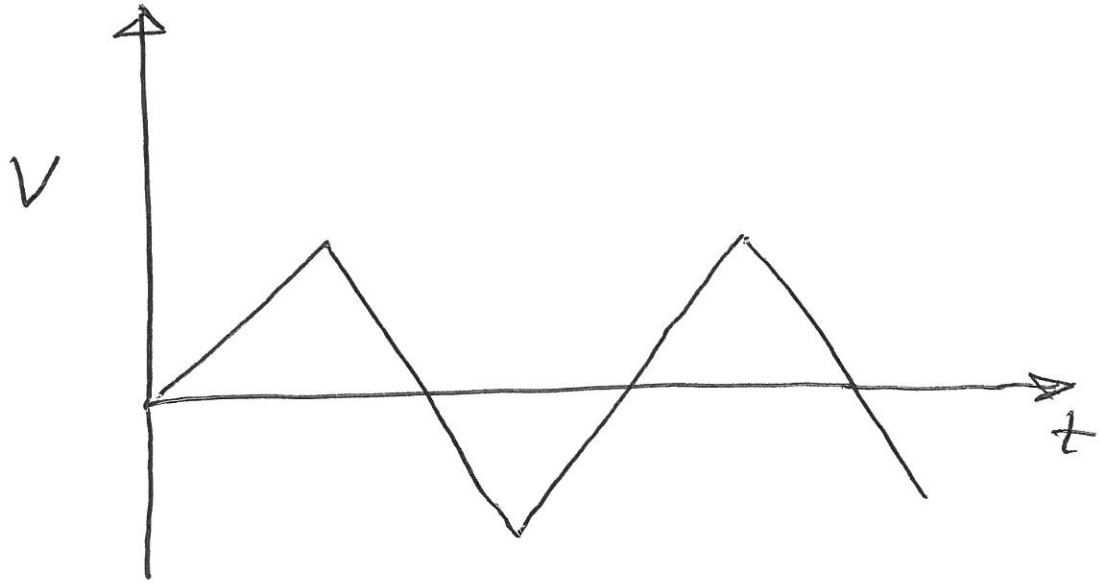
(root mean square)

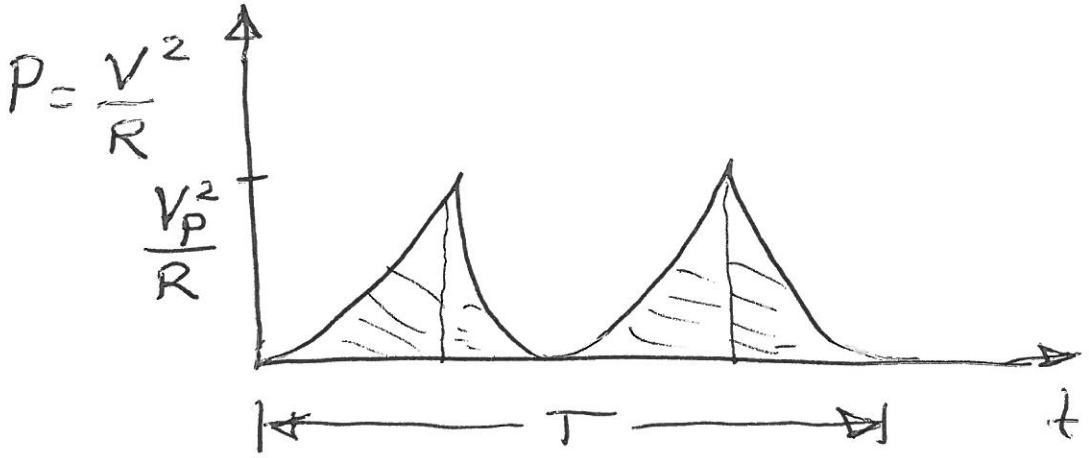


$$\frac{V_p^2}{R}\,\frac{T}{2} = \frac{V_{rms}^2}{R}\,T$$

$$V_{rms} = \frac{V_p}{\sqrt{2}} \qquad \text{Sine Wave}$$

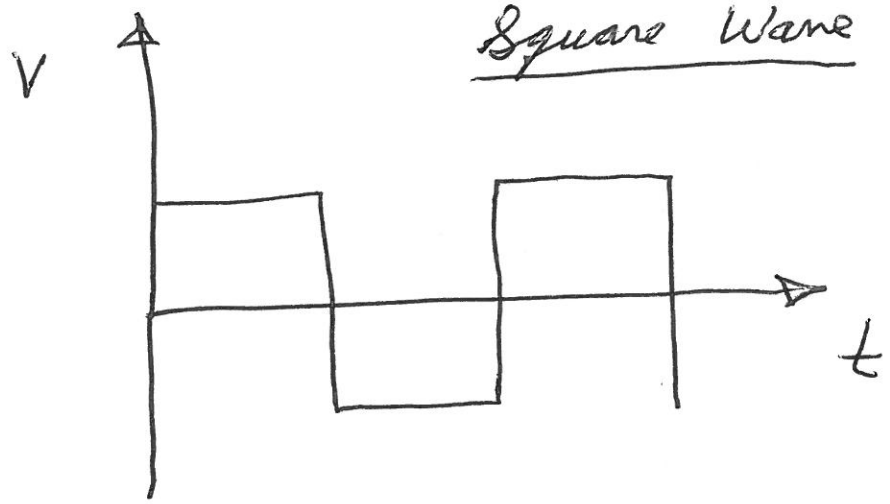
Triangle Wave





$$\cancel{4}\left(\frac{1}{3}\right)\left(\frac{\cancel{T}}{\cancel{4}}\right)\frac{V_p^2}{R} = \frac{V_{rms}^2}{R}\cancel{T}$$

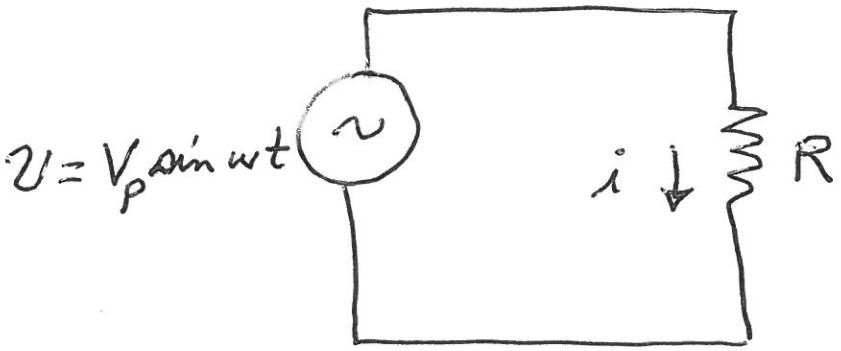
$$V_{rms} = \frac{V_p}{\sqrt{3}} \quad \text{Triangle Wave}$$



$$\frac{V_p^2}{R} T = \frac{V_{rms}^2}{R} T$$

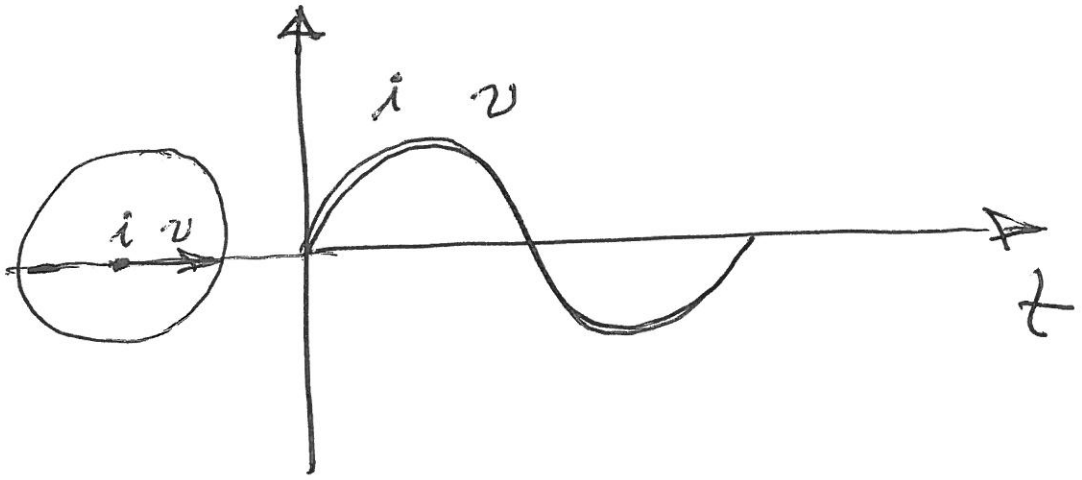
$$V_{rms} = V_p \quad \text{Square Wave}$$

Resistor

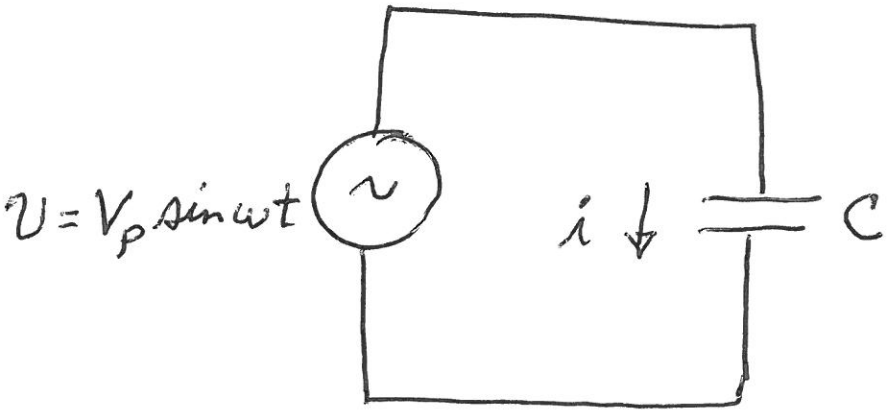


$$i = \frac{v}{R} = \frac{V_p \sin \omega t}{R}$$

$$I = \frac{V}{R} \qquad \text{rms values}$$



i and v are in phase



$$q = C v$$

$$i = \frac{dq}{dt} = C \frac{dv}{dt}$$

$$i = C V_p \omega \cos \omega t$$

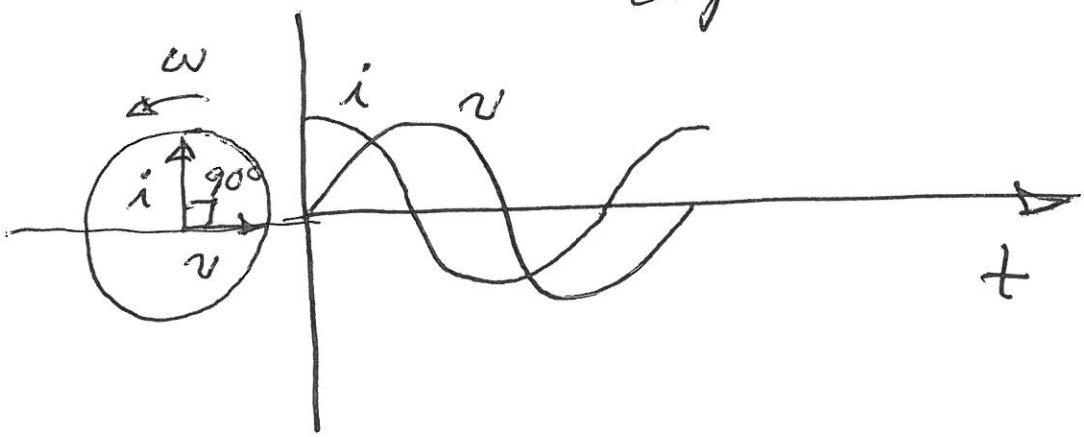
$$i = \frac{V_p \cos \omega t}{\left(\frac{1}{\omega C}\right)}$$

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$I = \frac{V}{X_c} \quad \text{rms values}$$

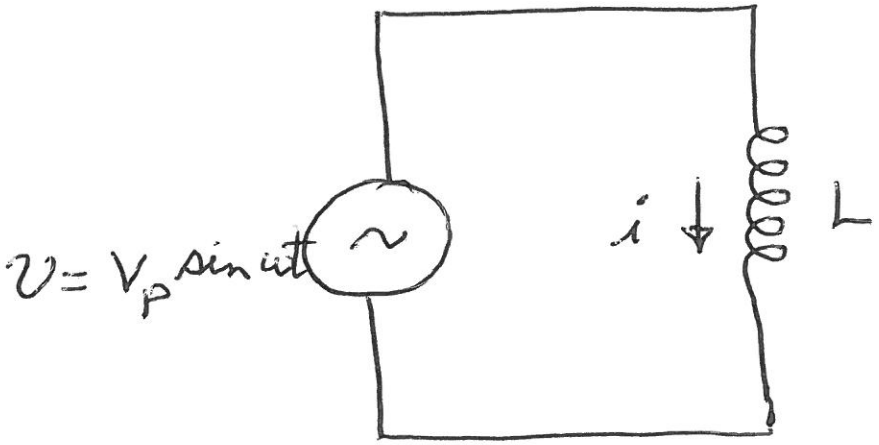
$$X_c = \frac{1}{\omega C} \quad \text{units of ohm}$$

Capacitor



current i leads voltage v by $90°$

Inductor



$$v = L \frac{di}{dt}$$

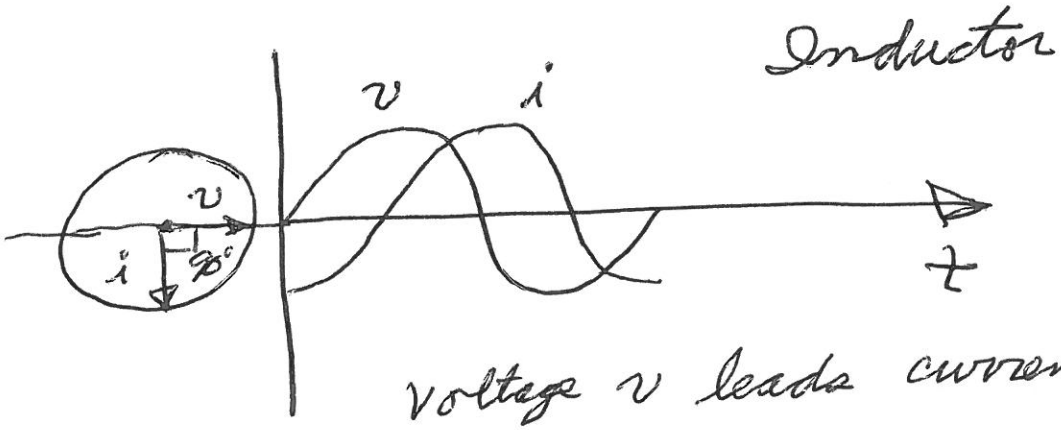
$$i = \int di = \frac{1}{L} \int v \, dt$$

$$i = \frac{1}{L} \int V_p \sin \omega t \, dt$$

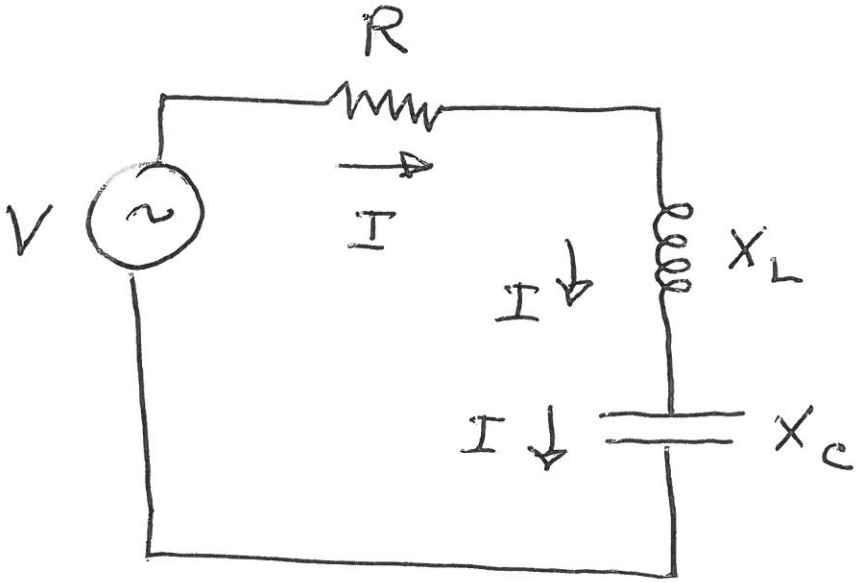
$$i = -\frac{V_p \cos \omega t}{\omega L}$$

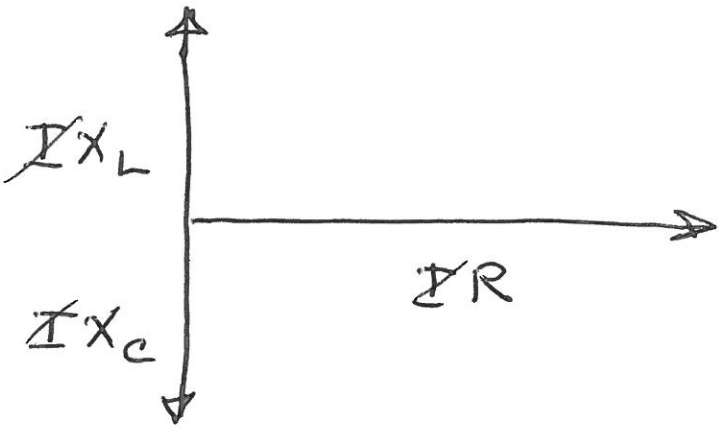
$$I = \frac{V}{\omega L} \qquad X_L = \omega L = 2\pi f L$$

$$I = \frac{V}{X_L} \qquad X_L \text{ units of ohm}$$

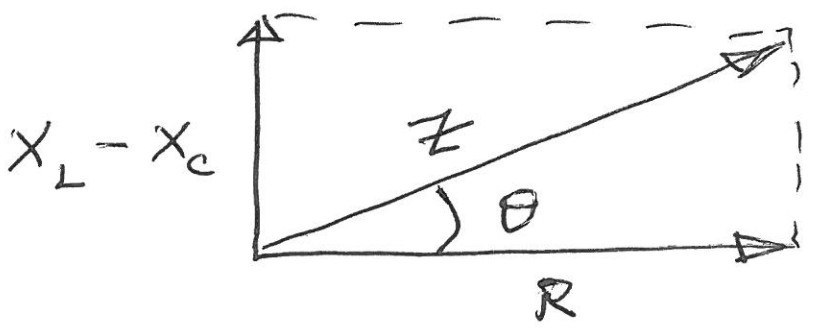


voltage v leads current i by 90°



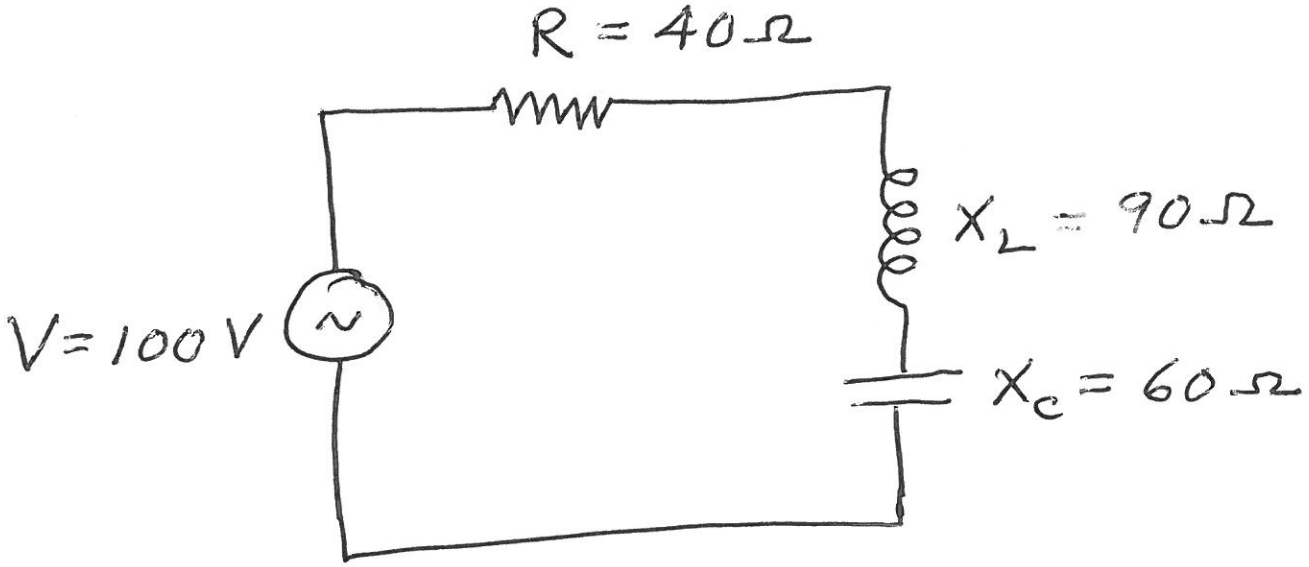


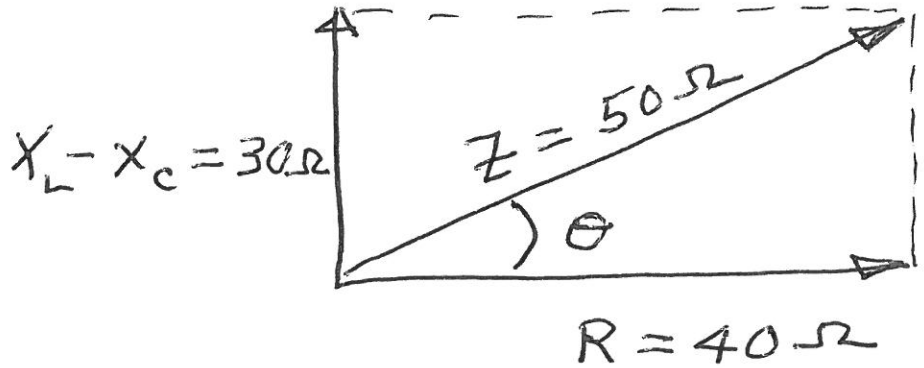
Impedance Diagram



$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$$\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$





$$Z = \sqrt{30^2 + 40^2} = \sqrt{2500} = 50\,\Omega$$

$$I = \frac{100}{50} = 2\,A \qquad \theta = \tan^{-1}\left(\frac{30}{40}\right) = 37^\circ$$