

Pairwise Composite Likelihood

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Likelihood for the data

The bivariate likelihood for the data is given as

$$f(Y_1, Y_2) = \begin{cases} 1 - \exp\left\{-\frac{1}{z_1}\right\} - \exp\left\{-\frac{1}{z_2}\right\} + \exp\{-\vartheta(\mathbf{s}_1, \mathbf{s}_2)\} & Y_1 = 1, Y_2 = 1 \\ \exp\left\{-\frac{1}{z_2}\right\} - \exp\{-\vartheta(\mathbf{s}_1, \mathbf{s}_2)\} & Y_1 = 1, Y_2 = 0 \\ \exp\left\{-\frac{1}{z_1}\right\} - \exp\{-\vartheta(\mathbf{s}_1, \mathbf{s}_2)\} & Y_1 = 0, Y_2 = 1 \\ \exp\{-\vartheta(\mathbf{s}_1, \mathbf{s}_2)\} & Y_1 = 0, Y_2 = 0 \end{cases} \quad (1)$$

where $z_i = \left(1 - \xi \mathbf{X}_i^T \beta\right)^{1/\xi}$, and $\vartheta(\mathbf{s}_1, \mathbf{s}_2) = \sum_{l=1}^L \left[\left(\frac{w_l(\mathbf{s}_1)}{z_1}\right)^{1/\alpha} + \left(\frac{w_l(\mathbf{s}_2)}{z_2}\right)^{1/\alpha} \right]^\alpha$

Pairwise composite likelihood

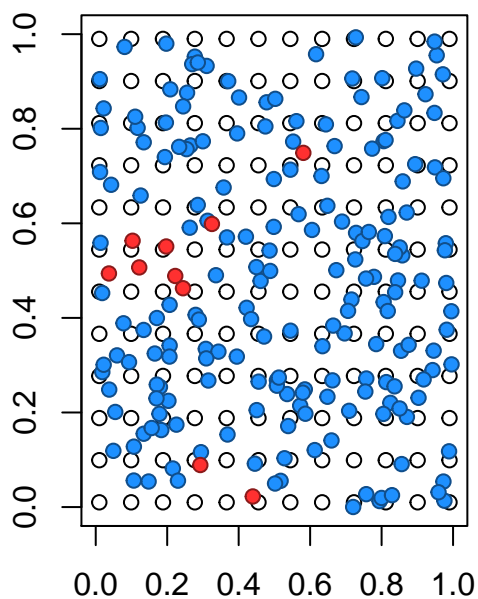
We initially tried using `optim` on the likelihood with four parameters: a) α , b) ρ , c) ξ , and d) β , but it appears to be challenging for `optim` to find the true values. So, we decided to make a grid of α and ρ and run `optim` to only fit ξ and β

Data settings

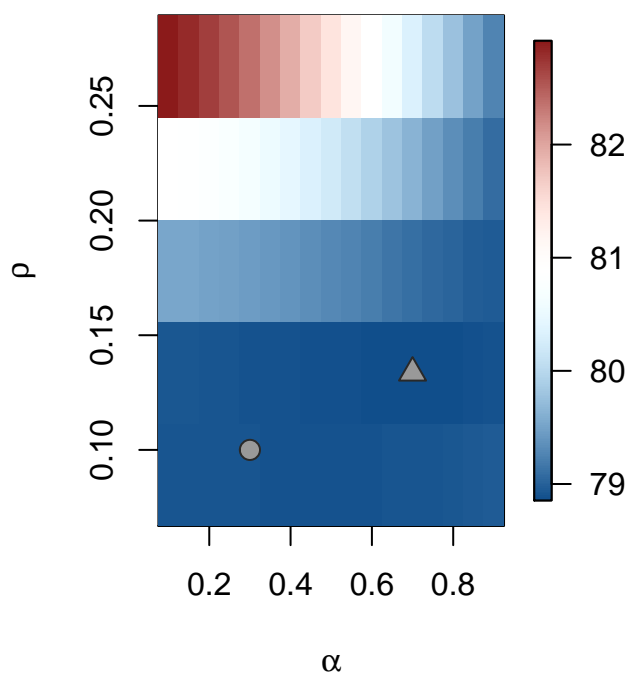
Right now, we're fitting $n = 200$ observations with one replication. In the future, we plan to allow for multiple replications. In each plot, we give the optimized value for the negative log likelihood when conducting a grid search. To examine accuracy, the true values for α and ρ are given by the circle, and the optimized values for α and ρ are given by the triangle.

$\alpha = 0.3, \pi = 0.05, \rho = 0.1$

simulated dataset

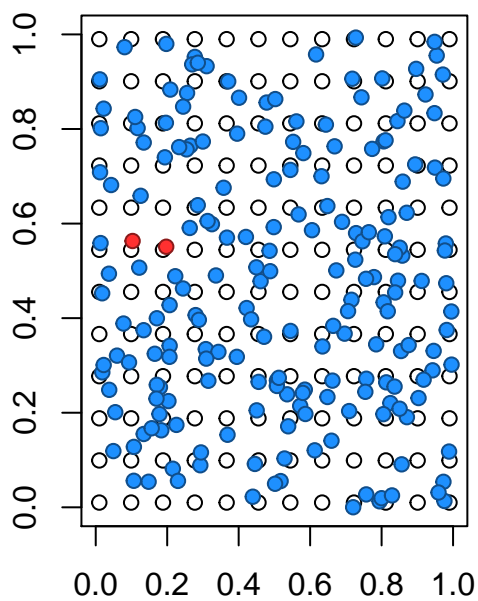


nll grid search ($\pi = 0.05$)

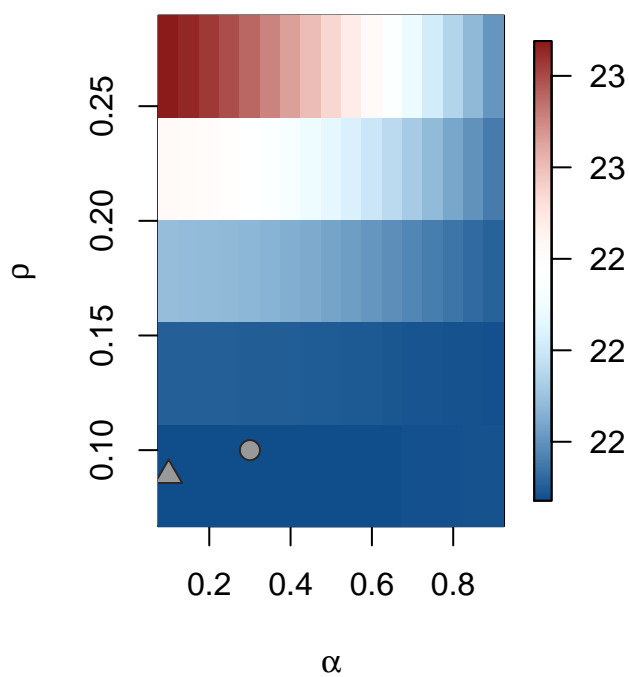


$\alpha = 0.3, \pi = 0.01, \rho = 0.1$

simulated dataset

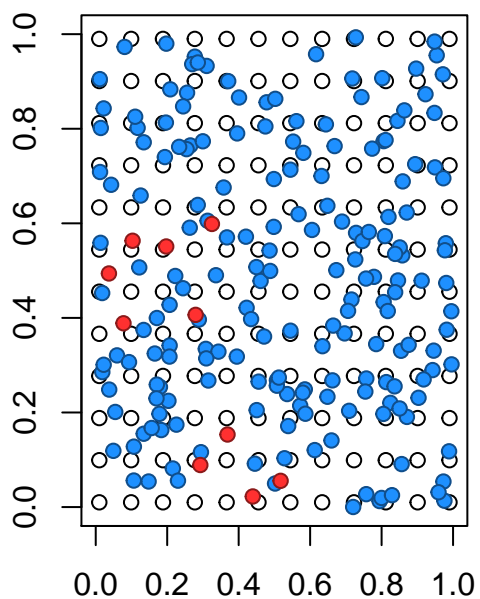


nll grid search ($\pi = 0.01$)

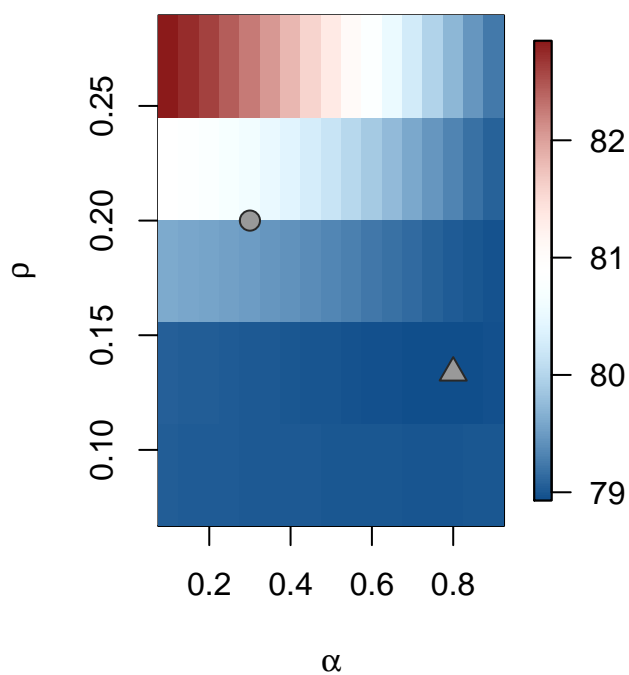


$\alpha = 0.3, \pi = 0.05, \rho = 0.2$

simulated dataset

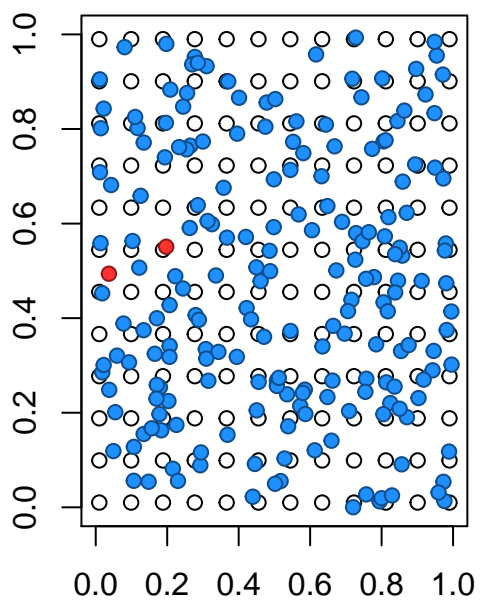


nll grid search ($\pi = 0.05$)

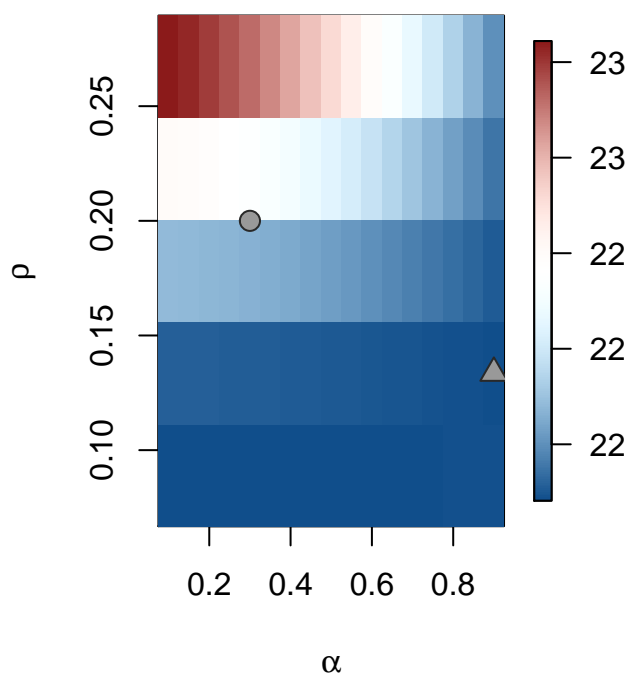


$\alpha = 0.3, \pi = 0.01, \rho = 0.2$

simulated dataset

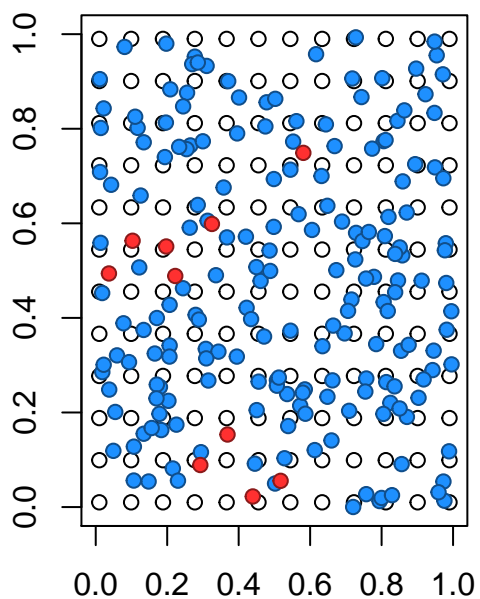


nll grid search ($\pi = 0.01$)

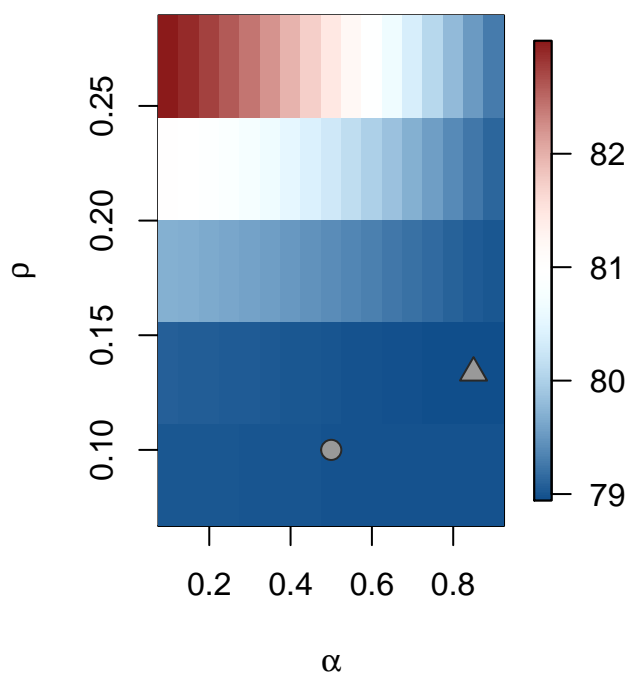


$\alpha = 0.5, \pi = 0.05, \rho = 0.1$

simulated dataset

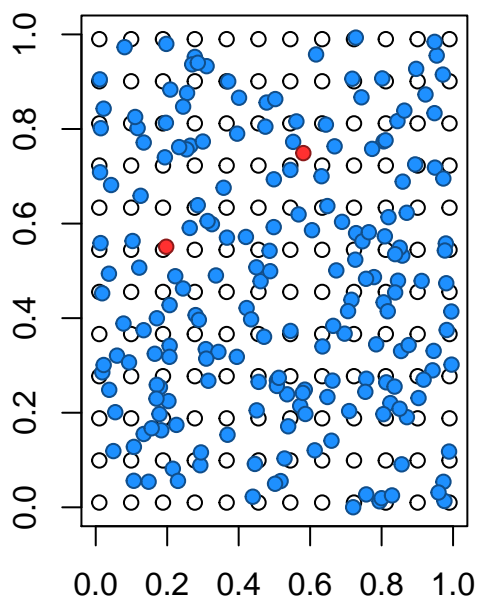


nll grid search ($\pi = 0.05$)

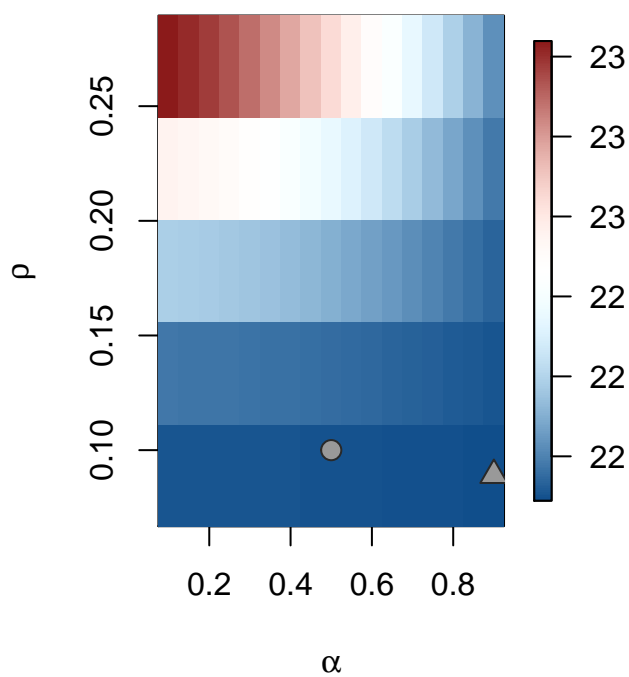


$\alpha = 0.5, \pi = 0.01, \rho = 0.1$

simulated dataset

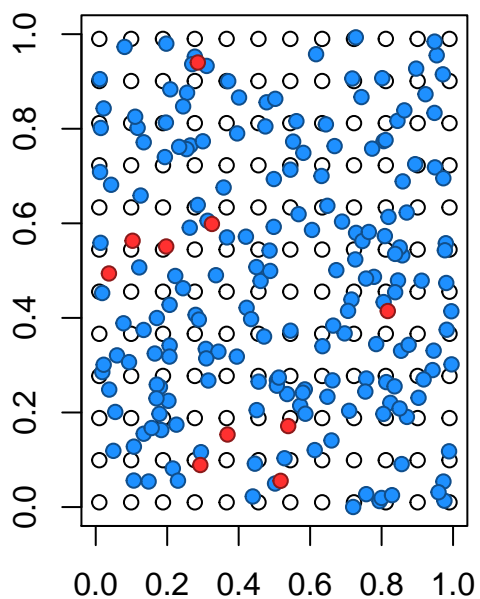


nll grid search ($\pi = 0.01$)

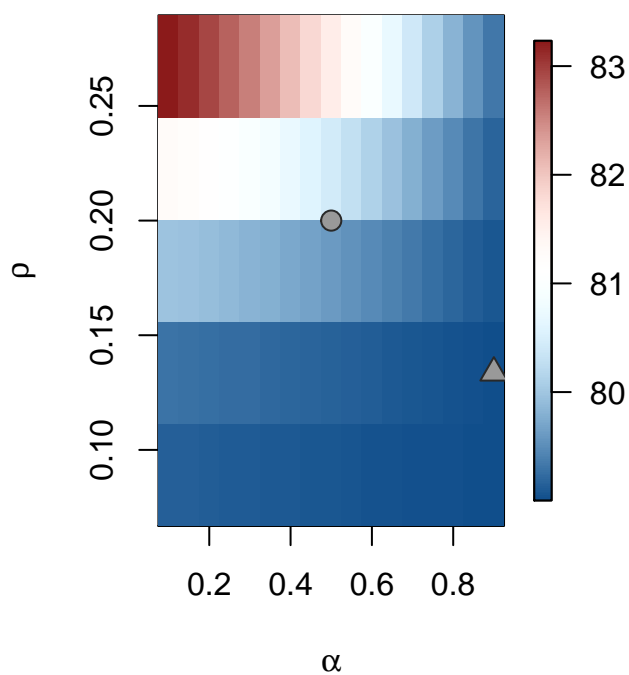


$$\alpha = 0.5, \pi = 0.05, \rho = 0.2$$

simulated dataset

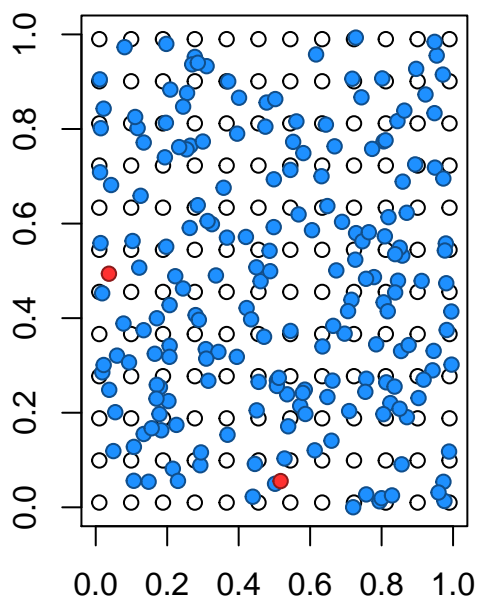


nll grid search ($\pi = 0.05$)

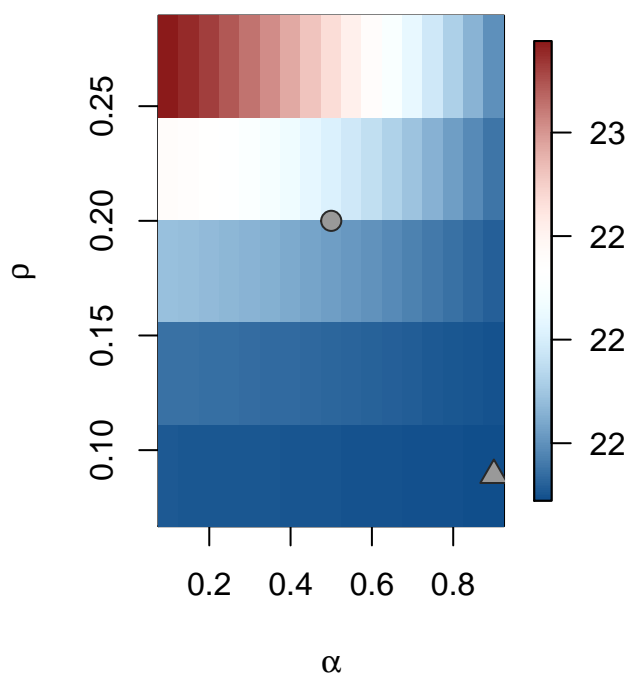


$$\alpha = 0.5, \pi = 0.01, \rho = 0.2$$

simulated dataset

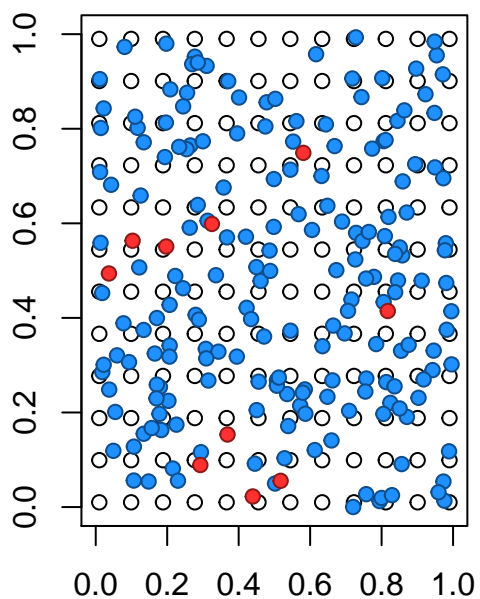


nll grid search ($\pi = 0.01$)

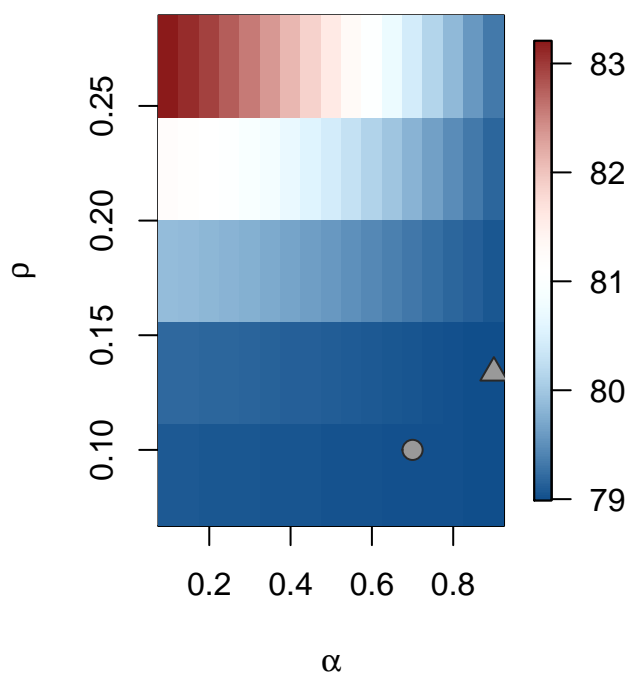


$$\alpha = 0.7, \pi = 0.05, \rho = 0.1$$

simulated dataset

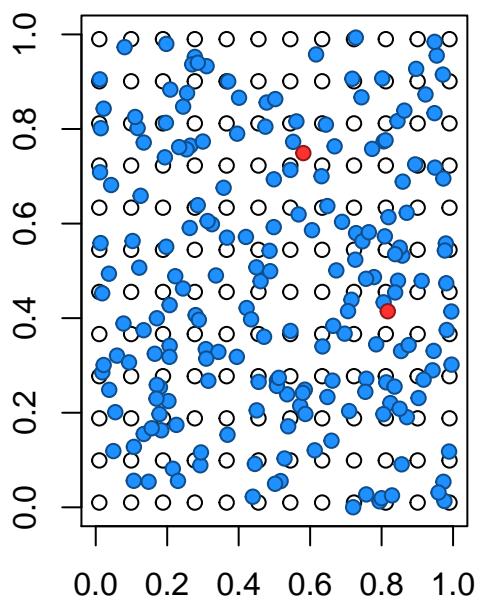


nll grid search ($\pi = 0.05$)

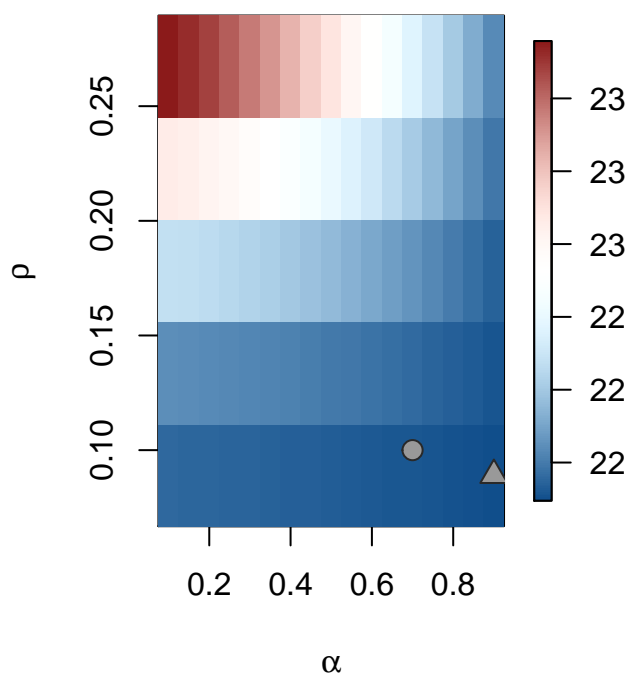


$$\alpha = 0.7, \pi = 0.01, \rho = 0.1$$

simulated dataset

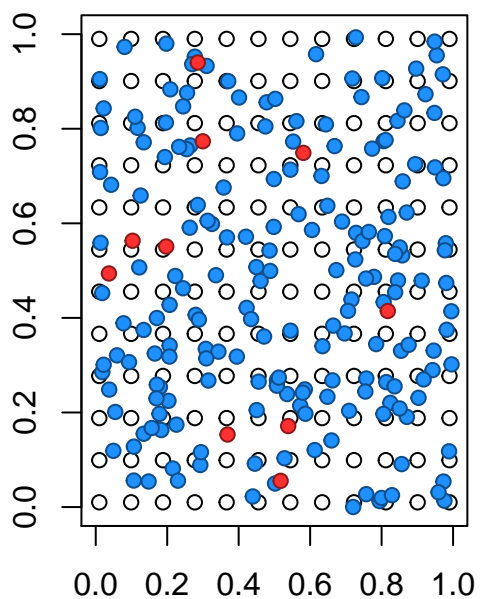


nll grid search ($\pi = 0.01$)

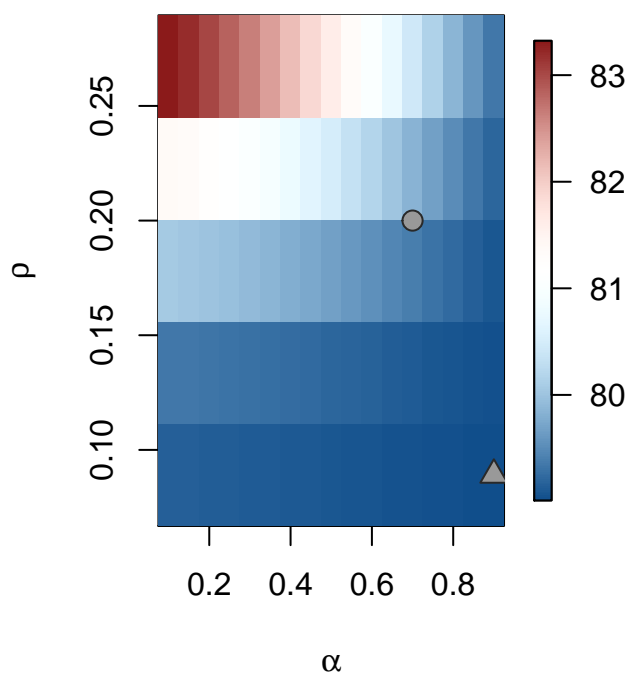


$\alpha = 0.7, \pi = 0.05, \rho = 0.2$

simulated dataset

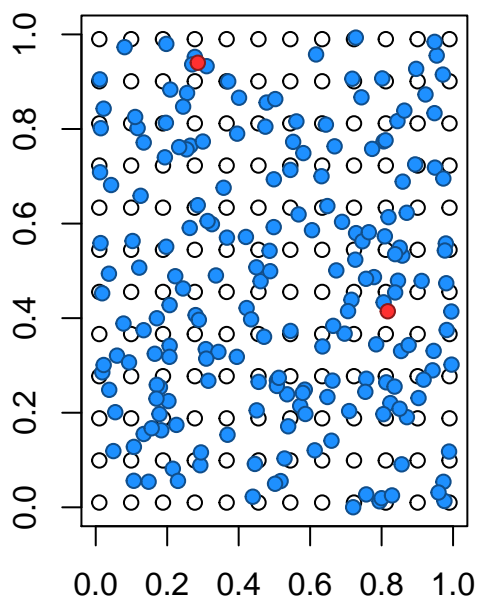


nll grid search ($\pi = 0.05$)



$\alpha = 0.7, \pi = 0.01, \rho = 0.2$

simulated dataset



nll grid search ($\pi = 0.01$)

