Rare Binary Regression

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Motivation

Binary regression

Generalized extreme value

- Link function is defined as

$$G(z_i) = 1 - \exp(-z_i)$$

where

$$z_i = \begin{cases} (1 - \xi \mathbf{X}_i \boldsymbol{\beta})^{1/\xi} & \xi \neq 0 \\ \exp(-\mathbf{X}_i \boldsymbol{\beta}) & \xi = 0 \end{cases}$$

is standardized to give unit Fréchet distribution.

- Note: The cloglog link is a special case when $\xi = 0$

Spatial setting: Logit and probit

Spatial setting: GEV

Max-stable processes

Random effects representation

- Problem: n is very large, and computationally challenging to work with
- Consider a set of $L << n$ knots $\mathbf{v}_1, \ldots, \mathbf{v}_L$
- At each knot, there is a random effect
  - Logit and probit methods use Gaussian random effects
  - GEV method uses Positive stable random effects

Random effects representation

- Logit and probit use kriging

$$z_i = \mathbf{X}_i\boldsymbol{\beta} + \theta_i$$

- The random effect impacts the marginal distribution for the GEV

$$\theta_i = \left[\sum_{l=1}^{L} A_l w_l(\mathbf{s}_i)^{1/\alpha}\right]^{\alpha}$$

Method

- Two-main steps for model fitting
- First, fit a pairwise composite likelihood
  - Pairwise composite likelihood estimates used for initial values in MCMC
- Then, fit a hierarchical random effects model using MCMC

Pairwise composite likelihood

- We use a censored pairwise composite likelihood
  - Marginalizes out random effects
- Common in extremes
  - Bivariate distributions are computationally tractible
  - Only latent values where $Y = 1$ inform likelihood

MCMC

- $Y|\ldots \overset{ind}{\sim} \text{Bernoulli}(\pi_i)$
- $\pi_i = 1 - \exp\left\{\sum_{l=1}^{L} A_l \left[\frac{w_l(\mathbf{s}_i)}{z_i}\right]\right\}$
- $z_i = \begin{cases} (1 - \xi \mathbf{X}_i \boldsymbol{\beta})^{1/\xi} & \xi \neq 0 \\ \exp(-\mathbf{X}_i \boldsymbol{\beta}) & \xi = 0 \end{cases}$
- $A_l \overset{iid}{\sim} \text{PS}(\alpha)$
- $w_l(\mathbf{s}_i)$ is a scaled Gaussian kernel

Questions

- Questions?
- Thank you for your attention.

References