Pairwise Likelihood Fixed Rho

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Likelihood for the data

The bivariate likelihood for the data is given as

$$f(Y_1, Y_2) = \begin{cases} 1 - \exp\left\{-\frac{1}{z_1}\right\} - \exp\left\{-\frac{1}{z_2}\right\} + \exp\{-\vartheta(\mathbf{s}_1, \mathbf{s}_2)\} & Y_1 = 1, Y_2 = 1 \\ \exp\left\{-\frac{1}{z_2}\right\} - \exp\{-\vartheta(\mathbf{s}_1, \mathbf{s}_2)\} & Y_1 = 1, Y_2 = 0 \\ \exp\left\{-\frac{1}{z_1}\right\} - \exp\{-\vartheta(\mathbf{s}_1, \mathbf{s}_2)\} & Y_1 = 0, Y_2 = 1 \\ \exp\{-\vartheta(\mathbf{s}_1, \mathbf{s}_2)\} & Y_1 = 0, Y_2 = 0 \end{cases} \tag{1}$$

where $z_i = \left(1 - \xi \mathbf{X}_i^T \beta\right)^{1/\xi}$, and $\vartheta(\mathbf{s}_1, \mathbf{s}_2) = \sum_{l=1}^L \left[\left(\frac{w_l(\mathbf{s}_1)}{z_1}\right)^{1/\alpha} + \left(\frac{w_l(\mathbf{s}_2)}{z_2}\right)^{1/\alpha}\right]^\alpha$

Pairwise composite likelihood with fixed ρ

Based on a suggestion from Dan, we're trying to fix ρ and fit α using pairwise composite likelihood. Then we'll make predictions using MCMC. I am going to try running this for a few datasets and see how it does. At the moment, I am going to try to do something near the true ρ term.

Data settings

Right now, we're fitting $n = 1000$ observations with one replication. In the future, it would be nice to allow for multiple replications.

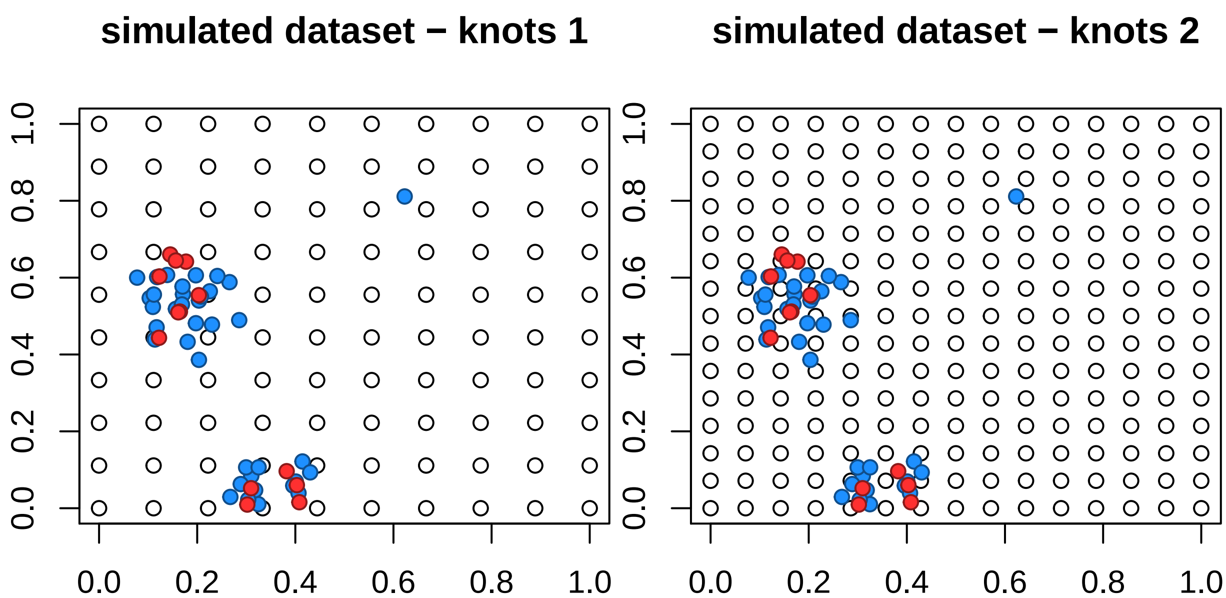
$\alpha = 0.25, \pi = 0.05, \rho = 0.1$

Our estimates are $\widehat{\alpha} = 0.55$, $\widehat{\xi} = 0.16$, and $\widehat{\beta} = -3.81$ when $\rho = 0.11$

The estimates are $\widehat{\alpha} = 0.38$, $\widehat{\xi} = 0.15$, and $\widehat{\beta} = -3.79$ when $\rho = 0.07$

Setup MCMC

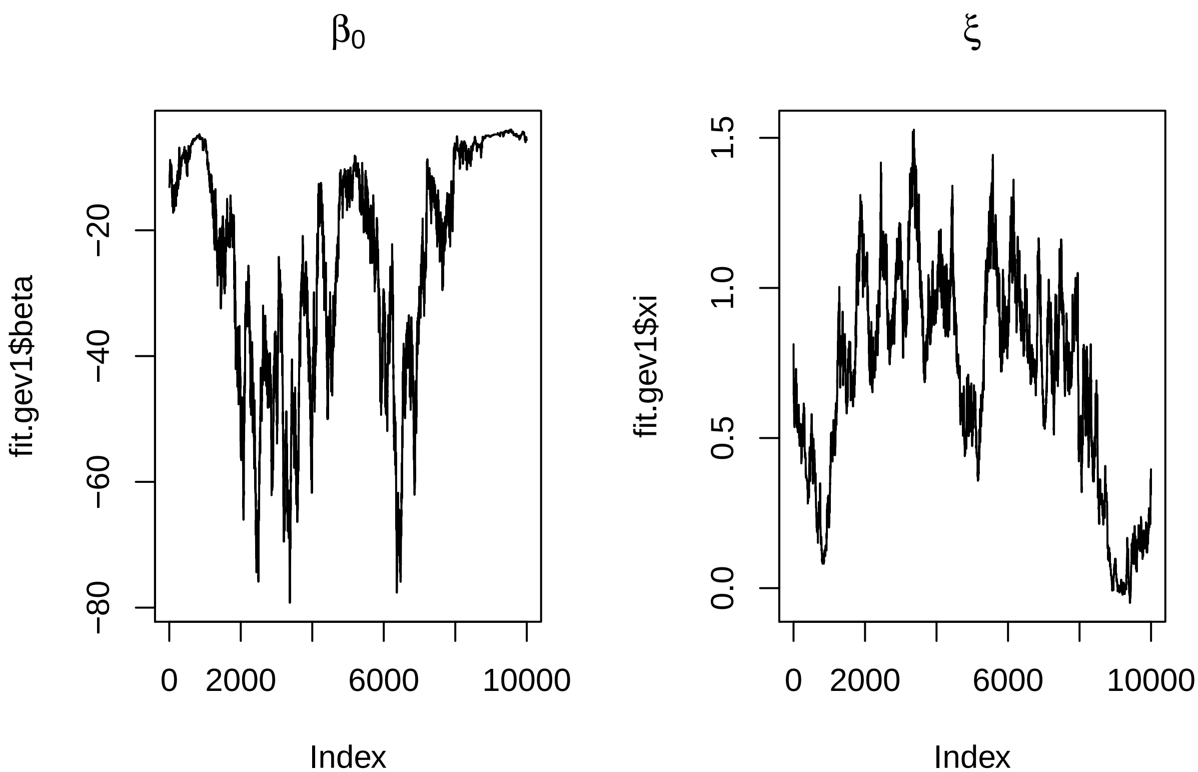
We'll start with 750 training sites, and 250 testing sites. In the following plot, the testing sites are given in red, the training sites are given in blue, and the knots are given as empty circles. The only difference between the two plots, is the knots used to fit the data. In the simulated data, the knots are placed on a 12×12 grid. In knots 1, the knots are placed on a 10×10 grid, and in knots 2, the knots are placed on a 15×15 grid.



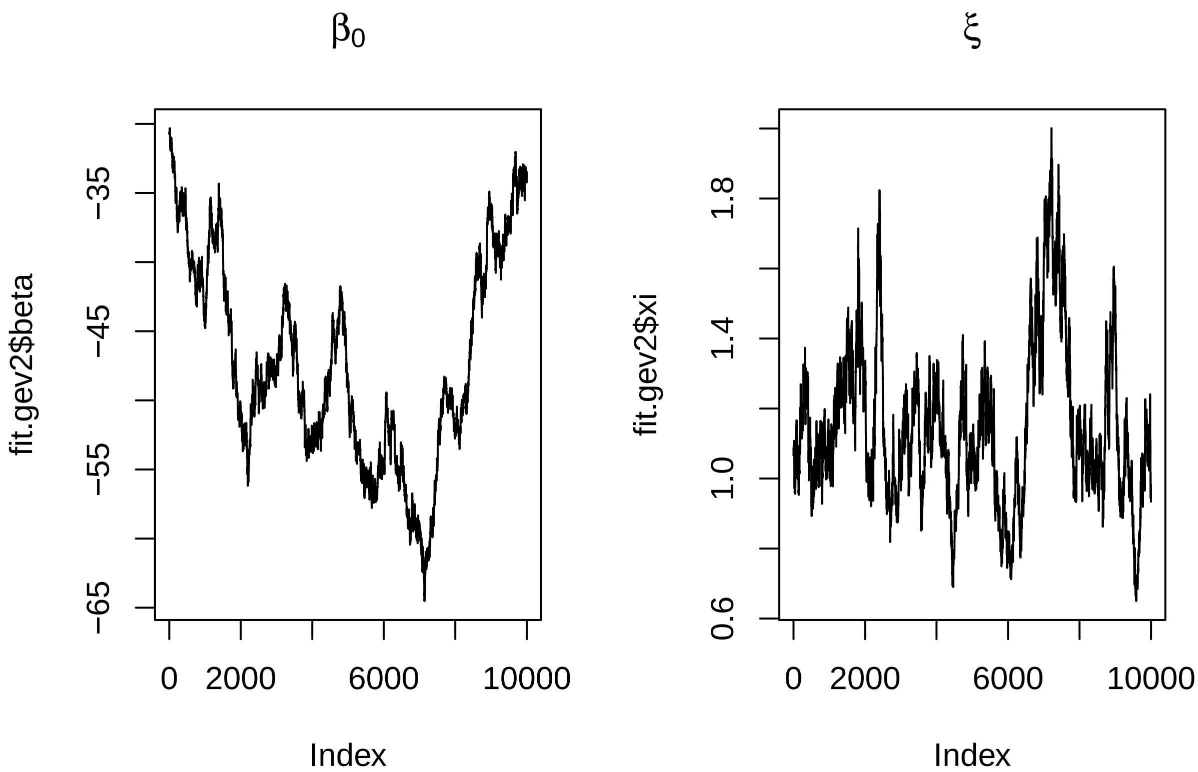
MCMC Results

Here are the iteration plots from the two GEV models. The true values are $\beta_0 = 5.7476855$, and $\xi = 0.25$.

Knot spacing 1



Knot spacing 2



Brier Scores

The brier scores are Logit 1: 0.02 Logit 2: 0.0224 Probit 1: 0.0185 Probit 2: 0.0192 GEV 1: 0.0262 GEV 2: 0.018

Generating another dataset

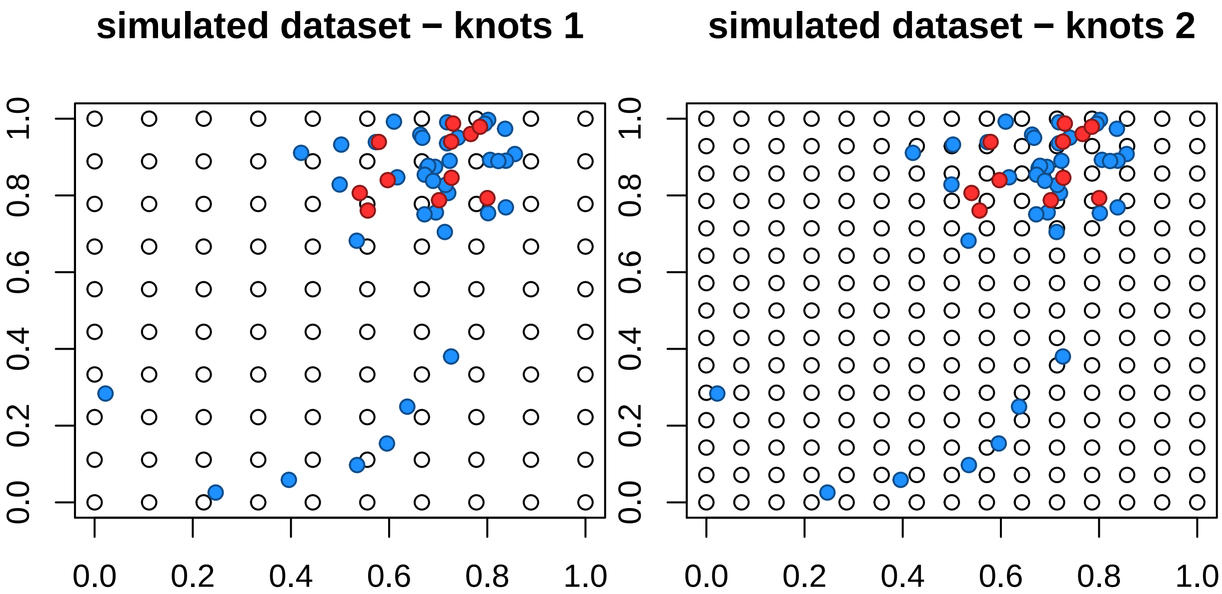
$\alpha = 0.75, \pi = 0.05, \rho = 0.1$

Our estimates are $\widehat{\alpha} = 0.6$, $\widehat{\xi} = 0.15$, and $\widehat{\beta} = -3.71$ when $\rho = 0.11$

The estimates are $\widehat{\alpha} = 0.6$, $\widehat{\xi} = 0.16$, and $\widehat{\beta} = -3.76$ when $\rho = 0.07$

Setup MCMC

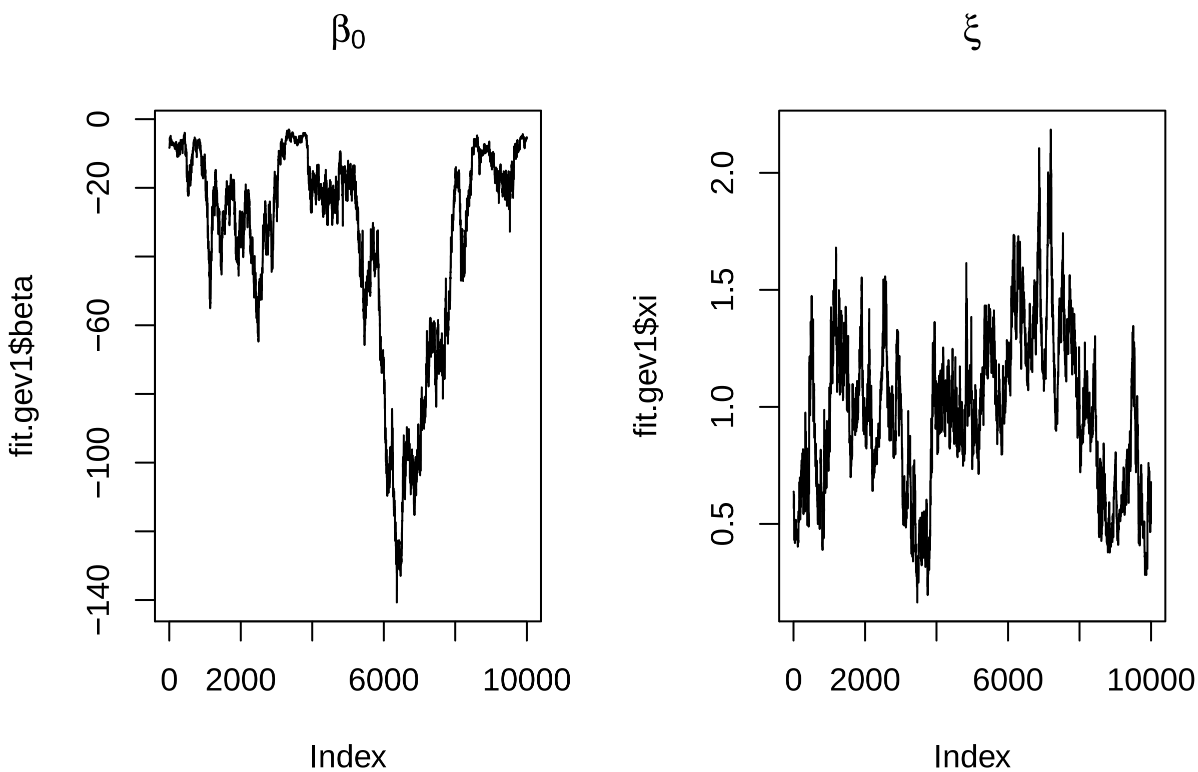
We'll start with 750 training sites, and 250 testing sites. In the following plot, the testing sites are given in red, the training sites are given in blue, and the knots are given as empty circles. The only difference between the two plots, is the knots used to fit the data. In the simulated data, the knots are placed on a 12×12 grid. In knots 1, the knots are placed on a 10×10 grid, and in knots 2, the knots are placed on a 15×15 grid.



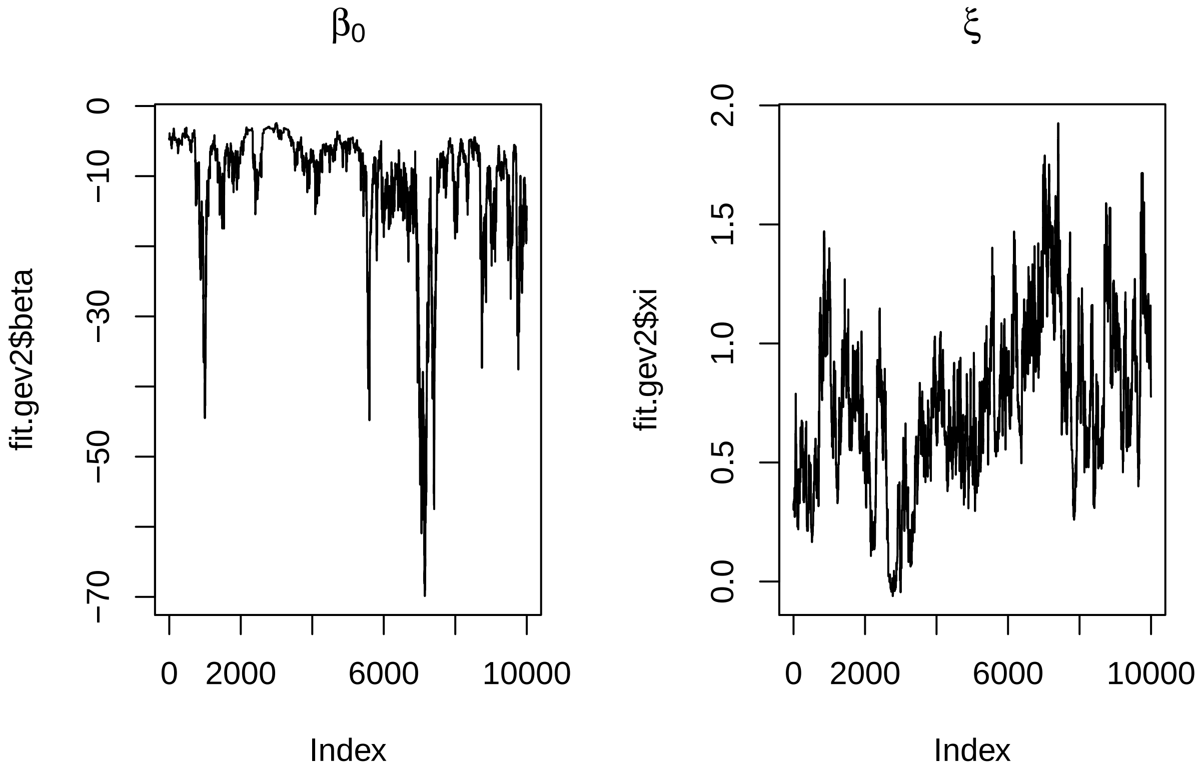
MCMC Results

Here are the iteration plots from the two GEV models. The true values are $\beta_0 = 5.2837104$, and $\xi = 0.25$.

Knot spacing 1



Knot spacing 2



Brier Scores

The brier scores are Logit 1: 0.0221 Logit 2: 0.0421 Probit 1: 0.0219 Probit 2: 0.0219 GEV 1: 0.023 GEV 2: 0.0233

Generating another dataset

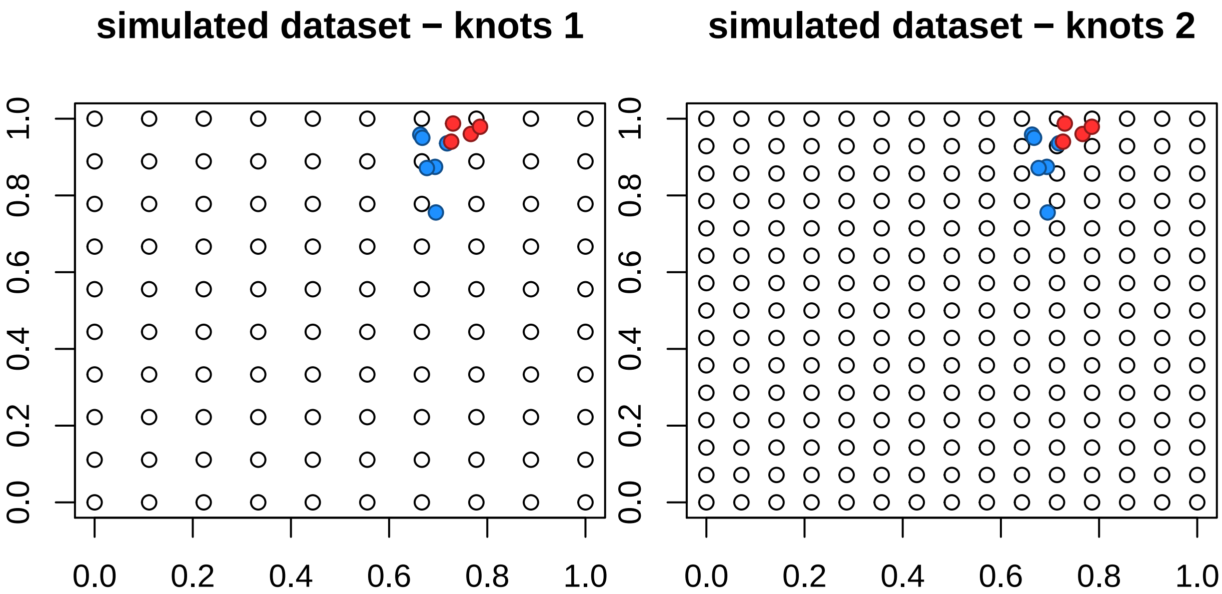
$\alpha = 0.25, \pi = 0.01, \rho = 0.1$

Our estimates are $\widehat{\alpha} = 0.82$, $\widehat{\xi} = -0.08$, and $\widehat{\beta} = -4.02$ when $\rho = 0.11$

The estimates are $\widehat{\alpha} = 0.69$, $\widehat{\xi} = -0.09$, and $\widehat{\beta} = -3.92$ when $\rho = 0.07$

Setup MCMC

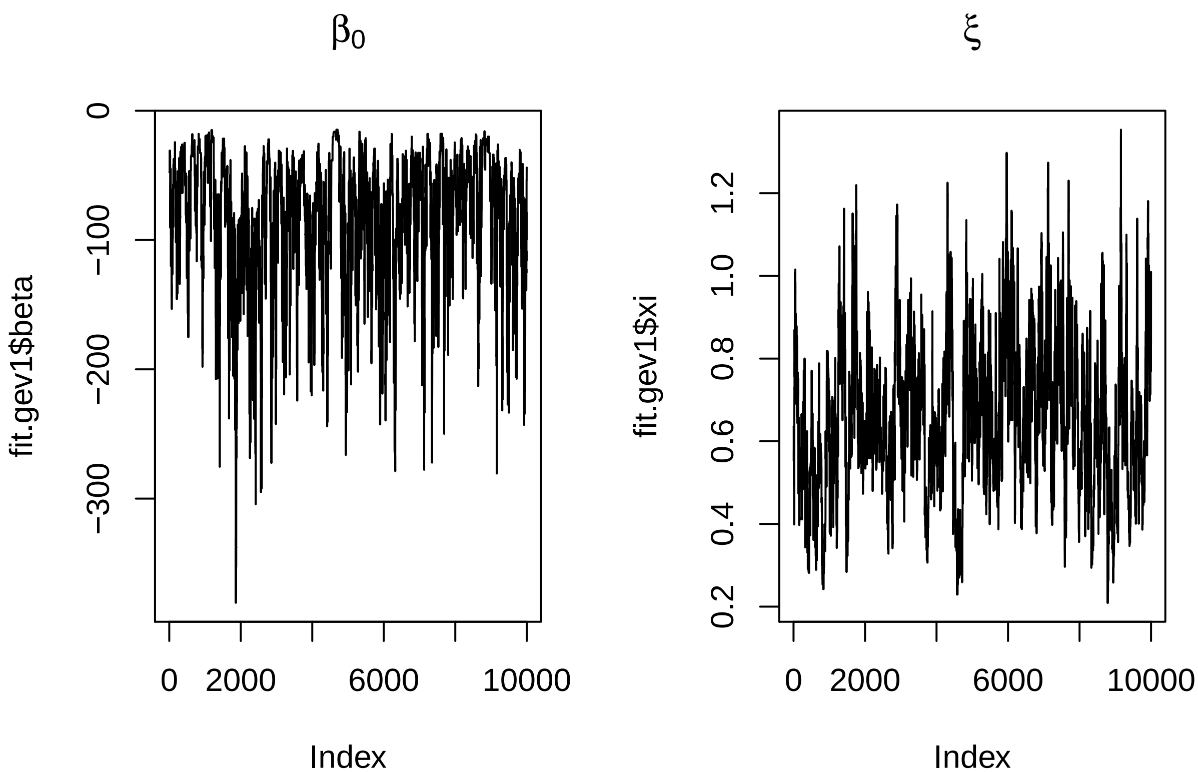
We'll start with 750 training sites, and 250 testing sites. In the following plot, the testing sites are given in red, the training sites are given in blue, and the knots are given as empty circles. The only difference between the two plots, is the knots used to fit the data. In the simulated data, the knots are placed on a 12×12 grid. In knots 1, the knots are placed on a 10×10 grid, and in knots 2, the knots are placed on a 15×15 grid.



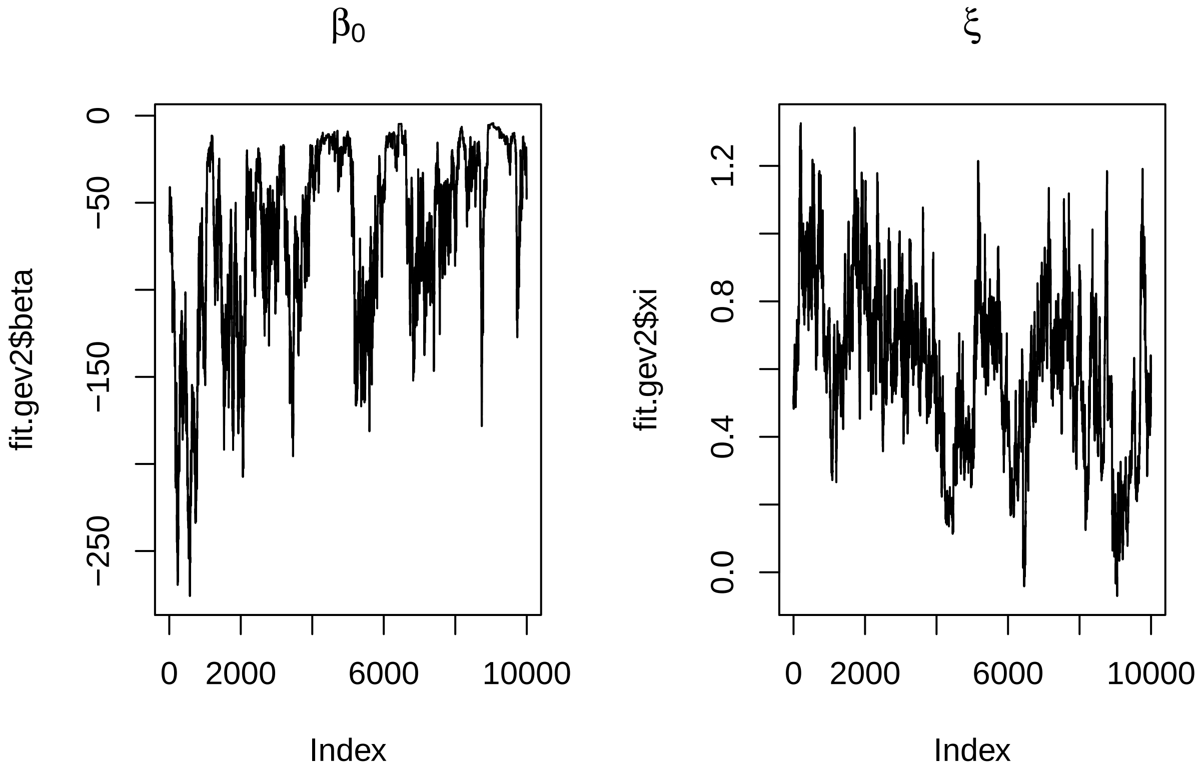
MCMC Results

Here are the iteration plots from the two GEV models. The true values are $\beta_0 = 12.4060482$, and $\xi = 0.25$.

Knot spacing 1



Knot spacing 2



Brier Scores

The brier scores are Logit 1: 0.0158 Logit 2: 0.013 Probit 1: 0.0139 Probit 2: 0.014 GEV 1: 0.0137 GEV 2: 0.0137

Generating another dataset

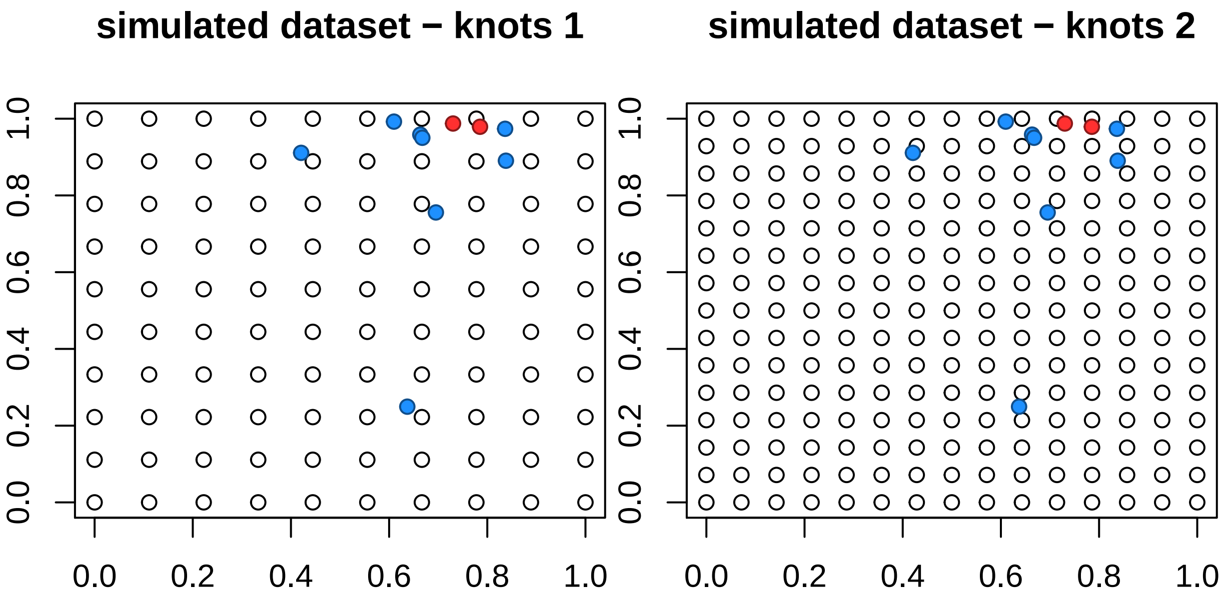
$\alpha = 0.75, \pi = 0.01, \rho = 0.1$

Our estimates are $\widehat{\alpha} = 0.91$, $\widehat{\xi} = -0.04$, and $\widehat{\beta} = -4.17$ when $\rho = 0.11$

The estimates are $\widehat{\alpha} = 0.79$, $\widehat{\xi} = -0.04$, and $\widehat{\beta} = -4.14$ when $\rho = 0.07$

Setup MCMC

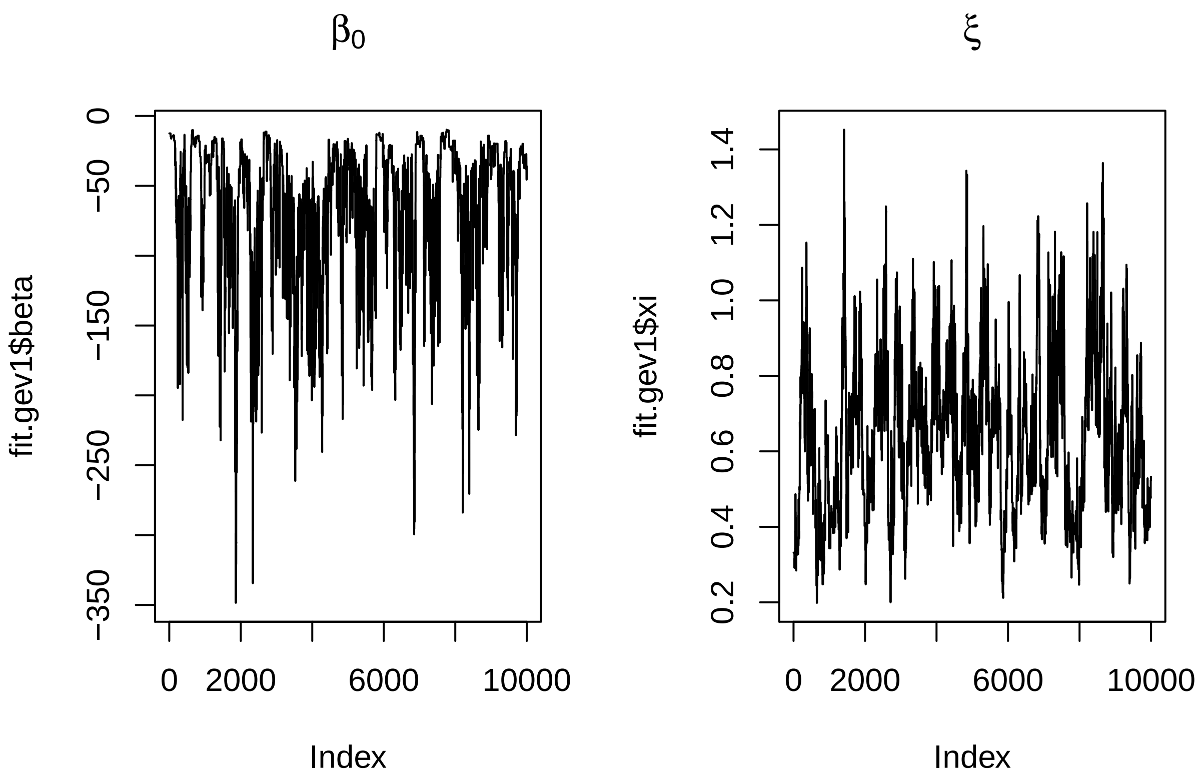
We'll start with 750 training sites, and 250 testing sites. In the following plot, the testing sites are given in red, the training sites are given in blue, and the knots are given as empty circles. The only difference between the two plots, is the knots used to fit the data. In the simulated data, the knots are placed on a 12×12 grid. In knots 1, the knots are placed on a 10×10 grid, and in knots 2, the knots are placed on a 15×15 grid.



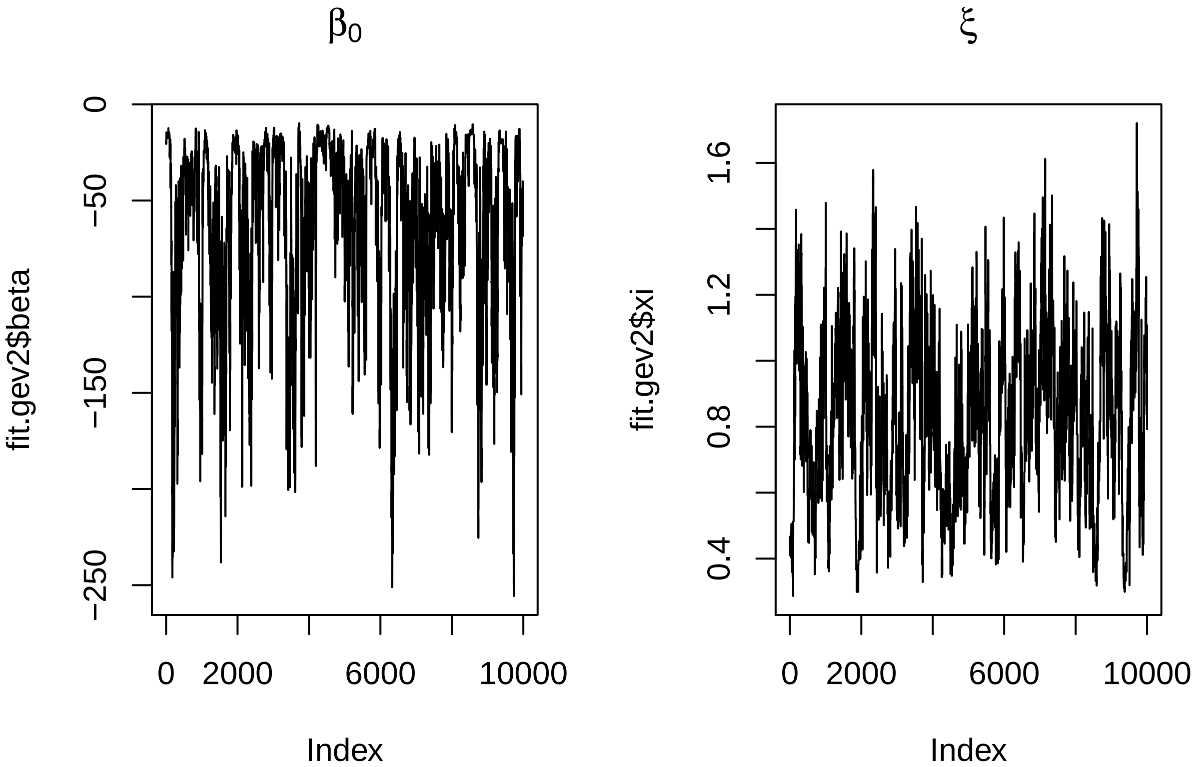
MCMC Results

Here are the iteration plots from the two GEV models. The true values are $\beta_0 = 9.9284461$, and $\xi = 0.25$.

Knot spacing 1



Knot spacing 2



Brier Scores

The brier scores are Logit 1: 0.0079 Logit 2: 0.0079 Probit 1: 0.0074 Probit 2: 0.0066 GEV 1: 0.0064 GEV 2: 0.0061