(Azzalini, 1996)

Consider $\mathbf{Y} \in \mathcal{R}^k$ that is distributed as MVN and is independent from $Y_0 \sim N(0,1)$. Then for

$$\begin{pmatrix} Y_0 \\ \mathbf{Y} \end{pmatrix} \sim N_{k+1}\left\{0, \begin{pmatrix} 1 & 0 \\ 0 & \mathbf{\Psi} \end{pmatrix}\right\} \tag{1}$$

Then, $Z_j = \delta_j |Y_0| + \sqrt{1-\delta_j^2} Y_j, j = 1, \ldots, k$ is skewed normal and its density is

$$f_k(z) = 2\phi_k(z; \mathbf{\Omega})\Phi(\boldsymbol{\alpha}^t \mathbf{z}) \quad \mathbf{z} \in \mathcal{R}^k \tag{2}$$

(Azzalini and Capitanio, 1999)

(Branco and Dey, 2001)

Modeling distributions that can account for skewness and heavy tails.

Multivariate elliptical

Notation: $\mathbf{X} \sim El_k(\boldsymbol{\mu}, \mathbf{\Sigma}; \phi)$ means that $\mathbf{X} \in \mathcal{R}^k$ follows an elliptical distribution with location vector $\boldsymbol{\mu} \in \mathcal{R}^k$, a dispersion matrix $\mathbf{\Sigma} \in \mathcal{R}^{k \times k}$ and characteristic function ϕ. If the density exists, then it is given by

$$f(\mathbf{x} \mid \boldsymbol{\mu}, \mathbf{\Sigma}) = |\mathbf{\Sigma}|^{-1/2} g^{(k)}[(\mathbf{x}-\boldsymbol{\mu})^T \mathbf{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})] \tag{3}$$

Multivariate skew elliptical

Notation: $\mathbf{Y} \sim SE_k(\boldsymbol{\mu}, \mathbf{\Omega}, \boldsymbol{\delta}; \phi)$ means that $\mathbf{Y} \in \mathcal{R}^k$ follows a skew-elliptical distribution with location vector $\boldsymbol{\mu} \in \mathcal{R}^k$, a dispersion matrix $\mathbf{\Sigma} \in \mathcal{R}^{k \times k}$, characteristic function ϕ and skewness parameter $\boldsymbol{\delta}$. If the density exists, then it is given by

$$f_{\mathbf{Y}}(\mathbf{y}) = 2 f_{g^{(k)}}(\mathbf{y}) F_{g_{q(\mathbf{y})}}(\boldsymbol{\lambda}^T(\mathbf{y}-\boldsymbol{\mu})) \tag{4}$$

Multivariate skew normal distribution

For a multivariate skew normal distribution, the density function is

$$f_{\mathbf{Y}}(\mathbf{y}) = 2\phi_k(\mathbf{y}; \boldsymbol{\mu}, \mathbf{\Omega})\Phi(\boldsymbol{\lambda}^T(\mathbf{y}-\boldsymbol{\mu})) \tag{5}$$

where

$$\boldsymbol{\lambda}^T = \frac{\boldsymbol{\delta}^T \mathbf{\Omega}^{-1}}{(1-\boldsymbol{\delta}^T \mathbf{\Omega}^{-1}\boldsymbol{\delta})^{1/2}} \tag{6}$$

Multivariate skew t distribution

For a multivariate skew t distribution, the density function is

$$f_{\mathbf{Y}}(\mathbf{y}) = 2 f_{\nu,\tau}(\mathbf{y}; \boldsymbol{\mu}; \mathbf{\Omega}) F_{\nu^*,\tau^*}(\boldsymbol{\lambda}^T(\mathbf{y}-\boldsymbol{\mu})) \tag{7}$$

Other densities mentioned

- Skew logistic
- Skew stable distribution
- Skew exponential power distribution
- Skew Pearson Type II distribution

(Sahu et al., 2003)

(Gupta et al., 2004)

p. 189 The general multivariate skew normal distribution has density function

$$2\phi_k(z;\Omega)\Phi(\alpha^{\mathrm{T}}z) \qquad (z \in \mathcal{R}^k) \tag{8}$$

where $\phi_k(z;\Omega)$ is a k-dimensional process with mean zero, and correlation matrix Ω, $\Phi(\cdot)$ is the $N(0,1)$ distribution, and $\alpha \in \mathcal{R}^k$ is a shape term.

(Allard and Naveau, 2007)

(Zhang and ElShaarawi, 2010)

(Minozzo and Ferracuti, 2012)

p. 164 Let $U_t \sim N(0,1)$ and $V_t(\mathbf{s}) \sim MVN$ with mean 0, and variance 1, then

$$\mathbf{Y}_t(\mathbf{s}) = \sigma\delta|U_t| + \sigma\sqrt{1-\delta^2}V_t(\mathbf{s}) \tag{9}$$

follows a

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