

# Homework #7

Name \_\_\_\_\_

Sec \_\_\_\_\_

Questions:	Answers:																																																																																																																																																																																																																																																																																								
<p>1) Use the truth table method to show that the following arguments are sound.</p> <p>a)</p> <p><math>P \Rightarrow Q</math></p> <p><math>P \Rightarrow R</math></p> <p>-----</p> <p><math>P \Rightarrow Q \wedge R</math></p> <p>b)</p> <p><math>P \vee Q</math></p> <p><math>P \Rightarrow R</math></p> <p><math>Q \Rightarrow R</math></p> <p>-----</p> <p>R</p>	<table><tr><td> </td><td>P</td><td> </td><td>Q</td><td> </td><td>R</td><td> </td><td><math>P \Rightarrow Q</math></td><td> </td><td><math>P \Rightarrow R</math></td><td> </td><td><math>P \Rightarrow (Q \wedge R)</math></td><td> </td></tr><tr><td> </td><td>---</td><td> </td><td>---</td><td> </td><td>---</td><td> </td><td>-----</td><td> </td><td>-----</td><td> </td><td>-----</td><td> </td></tr><tr><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td></tr><tr><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>F</td><td> </td><td>F</td><td> </td></tr><tr><td> </td><td>T</td><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>F</td><td> </td></tr><tr><td> </td><td>T</td><td> </td><td>F</td><td> </td><td>F</td><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>F</td><td> </td></tr><tr><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td></tr><tr><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td></tr><tr><td> </td><td>F</td><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td></tr><tr><td> </td><td>F</td><td> </td><td>F</td><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td></tr></table> <table><tr><td> </td><td>P</td><td> </td><td>Q</td><td> </td><td>R</td><td> </td><td><math>P \vee Q</math></td><td> </td><td><math>P \Rightarrow R</math></td><td> </td><td><math>Q \Rightarrow R</math></td><td> </td><td>R</td><td> </td></tr><tr><td> </td><td>---</td><td> </td><td>---</td><td> </td><td>---</td><td> </td><td>-----</td><td> </td><td>-----</td><td> </td><td>-----</td><td> </td><td>---</td><td> </td></tr><tr><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td></tr><tr><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>F</td><td> </td><td>F</td><td> </td><td>F</td><td> </td></tr><tr><td> </td><td>T</td><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td></tr><tr><td> </td><td>T</td><td> </td><td>F</td><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>F</td><td> </td></tr><tr><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td></tr><tr><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>F</td><td> </td><td>F</td><td> </td></tr><tr><td> </td><td>F</td><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>T</td><td> </td></tr><tr><td> </td><td>F</td><td> </td><td>F</td><td> </td><td>F</td><td> </td><td>F</td><td> </td><td>T</td><td> </td><td>T</td><td> </td><td>F</td><td> </td></tr></table>		P		Q		R		$P \Rightarrow Q$		$P \Rightarrow R$		$P \Rightarrow (Q \wedge R)$			---		---		---		-----		-----		-----			T		T		T		T		T		T			T		T		F		T		F		F			T		F		T		F		T		F			T		F		F		F		T		F			F		T		T		T		T		T			F		T		F		T		T		T			F		F		T		T		T		T			F		F		F		T		T		T			P		Q		R		$P \vee Q$		$P \Rightarrow R$		$Q \Rightarrow R$		R			---		---		---		-----		-----		-----		---			T		T		T		T		T		T		T			T		T		F		T		F		F		F			T		F		T		T		T		T		T			T		F		F		T		F		T		F			F		T		T		T		T		T		T			F		T		F		T		T		F		F			F		F		T		F		T		T		T			F		F		F		F		T		T		F	
	P		Q		R		$P \Rightarrow Q$		$P \Rightarrow R$		$P \Rightarrow (Q \wedge R)$																																																																																																																																																																																																																																																																														
	---		---		---		-----		-----		-----																																																																																																																																																																																																																																																																														
	T		T		T		T		T		T																																																																																																																																																																																																																																																																														
	T		T		F		T		F		F																																																																																																																																																																																																																																																																														
	T		F		T		F		T		F																																																																																																																																																																																																																																																																														
	T		F		F		F		T		F																																																																																																																																																																																																																																																																														
	F		T		T		T		T		T																																																																																																																																																																																																																																																																														
	F		T		F		T		T		T																																																																																																																																																																																																																																																																														
	F		F		T		T		T		T																																																																																																																																																																																																																																																																														
	F		F		F		T		T		T																																																																																																																																																																																																																																																																														
	P		Q		R		$P \vee Q$		$P \Rightarrow R$		$Q \Rightarrow R$		R																																																																																																																																																																																																																																																																												
	---		---		---		-----		-----		-----		---																																																																																																																																																																																																																																																																												
	T		T		T		T		T		T		T																																																																																																																																																																																																																																																																												
	T		T		F		T		F		F		F																																																																																																																																																																																																																																																																												
	T		F		T		T		T		T		T																																																																																																																																																																																																																																																																												
	T		F		F		T		F		T		F																																																																																																																																																																																																																																																																												
	F		T		T		T		T		T		T																																																																																																																																																																																																																																																																												
	F		T		F		T		T		F		F																																																																																																																																																																																																																																																																												
	F		F		T		F		T		T		T																																																																																																																																																																																																																																																																												
	F		F		F		F		T		T		F																																																																																																																																																																																																																																																																												
<p>2) Do the following proofs deductively. Justify each step in your proof with a law or inference rule.</p> <p>a) If <math>P \Rightarrow Q</math>, <math>\neg R \Rightarrow \neg Q</math>, and P then prove R.</p> <p>b) If <math>P \Rightarrow (Q \wedge R)</math> and <math>\neg R \wedge Q</math> then prove <math>\neg P</math>.</p>	<p>a) <u>Given:</u></p> <p>1. <math>P \Rightarrow Q</math></p> <p>2. <math>\neg R \Rightarrow \neg Q</math></p> <p>3. P</p> <p><u>Proof:</u></p> <p>1. <math>Q (P \Rightarrow Q \text{ and } P)</math></p> <p>2. <math>R (\neg R \Rightarrow \neg Q \text{ and } Q)</math></p> <p>So, if <math>P \Rightarrow Q</math>, <math>\neg R \Rightarrow \neg Q</math>, and P are true, then R must also be true</p> <p>b) <u>Given:</u></p> <p>1. <math>P \Rightarrow (Q \wedge R)</math></p> <p>2. <math>\neg R \wedge Q</math></p> <p><u>Proof:</u></p> <p>1. <math>\neg R (\neg R \wedge Q)</math></p> <p>2. <math>\neg P (P \Rightarrow (Q \wedge R) \text{ and } \neg R)</math></p> <p>So, if <math>P \Rightarrow (Q \wedge R)</math> and <math>\neg R \wedge Q</math> are true, then <math>\neg P</math> must also be true</p>																																																																																																																																																																																																																																																																																								

<p>3. Prove deductively:</p> <p>if <math>A \Rightarrow B</math> then <math>A \vee B \Rightarrow B</math>.</p> <p>Justify each step in your proof with a law or inference rule.</p> <p>Once you have proven an if-then statement, you can use it in a future proof. To reference a proven if-then statement, we often give it a name. Give the if-then law in this problem your own first name (e.g. Dorothy's Law). This law may be beneficial in the proof of Problem 4.</p>	
<p>4. Prove deductively:</p> <p>If <math>P \vee Q</math>, <math>P \Rightarrow Q</math>, and <math>Q \Rightarrow P</math> then <math>P \wedge Q</math>.</p> <p>Justify each step in your proof with a law, an inference rule, or the law proved in Problem 3.</p>	
<p>⑤ Given the following premises:</p> <p>“if cows fly then there is a man in the moon”</p> <p>“if there is a man in the moon then the earth is flat”</p> <p>“cows fly”</p> <p>Prove the following conclusion:</p> <p>“the earth is flat.”</p> <p>Justify each step in your proof with a law or an inference rule.</p>	<p>P: Cows fly</p> <p>Q: There is a man in the moon</p> <p>R: The earth is flat</p> <div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p><u>Given:</u></p> <ol style="list-style-type: none"> <li>1. <math>P \Rightarrow Q</math></li> <li>2. <math>Q \Rightarrow R</math></li> <li>3. <math>P</math></li> </ol> </div> <div style="width: 45%;"> <p><u>Proof:</u></p> <ol style="list-style-type: none"> <li>1. <math>Q</math> (<math>P \Rightarrow Q</math> and <math>P</math>)</li> <li>2. <math>R</math> (<math>Q \Rightarrow R</math> and <math>Q</math>)</li> </ol> </div> </div>

6 For each of the following arguments, indicate which of the rules of inference are used (Modus ponens, Disjunctive syllogism, etc.).

a) If Mr. Smith or Mrs. Smith earns more than \$30,000 per year, the Smith family can afford holidays in Hawaii. Since I know that either Mr. Smith or his wife earns more than \$30,000, I conclude that the family can afford a holiday in Hawaii.

b) If John was at the party yesterday, he will sleep in. John did not sleep in. Consequently, he was not at the party.

c) If Susan studies hard, she'll get an A and if she gets an A, she'll be happy. Hence, if Susan studies hard, then she'll be happy.

a) Disjunctive Syllogism

b) Modus Tollens

c) Hypothetical Syllogism

7. Prove the following:

If  $x$  is a positive integer,  $y$  is a positive integer, and  $x+y = z$ , then  $z$  is a positive integer larger than either  $x$  or  $y$ .

Justify each step in your proof with a law or inference rule. As part of the proof, you must use modus ponens with the following fact: "if  $x$  and  $y$  are positive integers, then  $x+y$  is a positive integer larger than either  $x$  or  $y$ ."

Recall that a fact may be introduced into a deductive proof as a true statement at any time.

Also, you should know that substitution of one thing for another when the two are equal is always a valid inference rule.