Homework #6

Name: ______________________ Sec: ______

Questions:	Answers:

1. Construct the truth table for each of the following expressions. Indicate for each expression whether it is a tautology, a contradiction, or neither (meaning that it is contingent).

a) $(P \wedge (P \Rightarrow Q)) \wedge \neg Q$

b) $(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$

c) $(Q \wedge (P \Rightarrow Q)) \Rightarrow P$

Answers:

a)

P	Q	P⇒Q	P∧(P⇒Q)	(P∧(P⇒Q))∧¬Q
T	T	T	T	F
T	F	F	F	F
F	T	T	F	F
F	F	T	F	T

b)

P	Q	P⇒Q	¬P∨Q	(P⇒Q)⇔(¬P∨Q)
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

c)

P	Q	P⇒Q	Q∧(P⇒Q)	(Q∧(P⇒Q))⇒P
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

2. Consider the expression:
$(P \Rightarrow Q) \wedge (\neg P \Rightarrow Q) \Rightarrow Q$.

a) Use a truth table to show that this expression is a tautology.

b) If you substitute $(R \wedge \neg S)$ for P and $\neg(R \vee W)$ for Q, is the resulting expression a tautology?

3. A <u>dilemma</u> is an argument

that allows one to conclude R, given the premises: $P \vee Q$, $P \Rightarrow R$, and $Q \Rightarrow R$. a) Convert the dilemma into a logical expression that can be used to show that the argument is sound. b) Use a truth table to prove that the dilemma is a sound argument.	
4. Write an expression equivalent to the dual of $\cancel{\neg}P \wedge Q \wedge T$ using only the NAND operator.	
5. Reduce the expression $Q \vee \cancel{\neg}((P \Rightarrow Q) \wedge P)$ to T. ~~Your reduction must be algebraic and~~ you must justify every step with the law (or laws) you use for the step. Your proof must use logical equivalences (not truth tables). Recall that the textbook gives several tables of logical equivalences in section 1.3.2	De Morgan's Law: $\neg((P\Rightarrow Q)\wedge P)\equiv\neg(P\Rightarrow Q)\vee\neg P$ Material Implication: $\neg(P\Rightarrow Q)\equiv P\wedge\neg Q$ Now, substituting these equivalences into the expression: $Q\vee(P\wedge\neg Q)\vee\neg P$ Distributive Law (OR over AND): $(P\wedge\neg Q)\vee\neg P\equiv(P\vee\neg P)\wedge(\neg Q\vee\neg P)$ $Q\vee(P\vee\neg P)\wedge(\neg Q\vee\neg P)$ Identity Law: $P\vee\neg P\equiv T$ $Q\vee T\wedge(\neg Q\vee\neg P)$ Annulment Law: $Q\vee T\equiv T$ $T\wedge(\neg Q\vee\neg P)$ Identity Law: $T\wedge X\equiv X$, where X is any expression. $\neg Q\vee\neg P$ Now, the expression $\neg Q\vee\neg P$ evaluates to T

Use logical equivalences to find ...

6. ~~Algebraically~~ find the conjunctive normal form of the following expression.

$P \Rightarrow ((Q \wedge R) \Leftrightarrow S)$

Justify every step with the law (or laws) you use for the step.

7. Find the following:

a) the full disjunctive normal form of *f*

b) the full conjunctive normal form of *f*

Note: “full” means that every term has all three variables.

P	Q	R	*f*
F	F	F	F
F	F	T	T
F	T	F	F
F	T	T	F
T	F	F	T
T	F	T	F
T	T	F	F
T	T	T	T

a)

$f(P,Q,R)=(\neg P\wedge\neg Q\wedge\neg R)\vee(\neg P\wedge Q\wedge\neg R)\vee(P\wedge\neg Q\wedge\neg R)\vee(P\wedge Q\wedge R)$

b)

$f(P,Q,R)=(P\vee Q\vee R)\wedge(P\vee\neg Q\vee R)\wedge(\neg P\vee Q\vee R)\wedge(\neg P\vee\neg Q\vee\neg R)$