

1.

- a) Stack: D T# Input: 1,0#
- b) Stack: T# Input: ,0#
- c) Stack: ', ' L# Input: ,0#
- d) Stack: L# Input: 0#

2.

- $E \rightarrow O$
- $O \rightarrow O '==' A \mid A$
- $A \rightarrow A '!=' X \mid X$
- $X \rightarrow X '^' U \mid U$
- $U \rightarrow '!' U \mid P$
- $P \rightarrow '(' E ')' \mid 'x'$

3.

- a) $Z \rightarrow '++' Z$
- b) $C \rightarrow '(' E ')'$
- c) $E \rightarrow A X$
- d) $X \rightarrow \text{epsilon}$

4.

- a) (E/E)
- b) $((E/E)*E)$
- c) $((x/E)*E)$
- d) $((x/2)*E)$

5.

- a) Yes
- b) No
- c) Yes
- d) No

6.

a) Logic expressions

- $(J \vee L) \Rightarrow C$ (Premise)
- $\sim T$ (Premise)
- $C \Rightarrow T$ (Premise)
- $\sim J$ (Conclusion)

b) Clauses

- $(\sim J \vee C) \wedge (\sim L \vee C)$
- $\sim T$
- $(\sim C \vee T)$

c) Justification

Prove: $\sim J$

- J, Assume the negation of the conclusion
- J and $(J \vee L) \Rightarrow C$ resolves to $L \Rightarrow C$
- From $\sim T$ and $C \Rightarrow T$ we have $\sim C$
- But $L \Rightarrow C$ and $\sim C$ contradict because L and $\sim L$ can't resolve
- So assumption of J is true must be incorrect, and $\sim J$ is proven

7.

a) Rules as clauses

- $\text{cousin}(C1, C2) :- \text{grandchild}(C1, GP), \text{grandchild}(C2, GP).$
- $\text{grandchild}(C, GP) :- \text{child}(C, P), \text{child}(P, GP).$

b) Proof

Premises:

- $\text{cousin}(C1, C2) :- \text{grandchild}(C1, GP), \text{grandchild}(C2, GP).$
- $\text{grandchild}(C, GP) :- \text{child}(C, P), \text{child}(P, GP).$
- $\text{child}('Tim', 'Bea').$
- $\text{child}('Sue', 'Ned').$
- $\text{child}('Ned', 'Bea').$
- $\text{child}('Ann', 'Tim').$

Query:

- $\text{cousin}('Ann', 'Sue')?$

Proof:

- $\text{cousin}('Ann', 'Sue') :- \text{grandchild}('Ann', GP), \text{grandchild}('Sue', GP)$ (From premise 1)
- $\text{grandchild}('Ann', GP)$ (Instantiate 1 with $C1 = 'Ann'$)
- $\text{child}('Ann', P), \text{child}(P, GP)$ (From premise 2)
- $\text{child}('Ann', P), \text{child}(P, GP), \text{child}('Sue', GP)$ (From premise 4 and instantiation of 2 with $GP = 'Ned'$)
- $\text{child}('Ann', P), \text{child}(P, 'Ned'), \text{child}('Sue', 'Ned')$ (From premise 5 and instantiation of 4 with $GP = 'Bea'$)
- $\text{child}('Ann', 'Tim'), \text{child}('Tim', 'Bea'), \text{child}('Sue', 'Ned')$ (From premise 6 and instantiation of 5 with $P = 'Tim'$).
- $'Tim' \neq 'Sue'$ (Contradiction from premise 6).
- Therefore, $\text{cousin}('Ann', 'Sue')$ is false

8.

a) Inductive hypothesis

- Assume that $P(n)$ is true for some integer $n \geq 1$,
- $1*2*3 + 2*3*4 + \dots + n(n+1)(n+2) = (n(n+1)(n+2)(n+3))/4$

b) To be proven in the inductive step

- Prove that $P(n+1)$ is true,
- $1*2*3 + 2*3*4 + \dots + n(n+1)(n+2)(n+3) = (n(n+1)(n+2)(n+3)(n+4))/4$

c) Result of using the inductive hypothesis

- $P(n+1) = (n(n+1)(n+2)(n+3))/4 + (n+1)(n+2)(n+3)$

d) Algebraic manipulation

- $= P(n+1) = (n(n+1)(n+2)(n+3))/4 + (n+1)(n+2)(n+3)$
- $= (n(n+1)(n+2)(n+3))/4 + (4(n+1)(n+2)(n+3))/4$
- $= (n(n+1)(n+2)(n+3) + 4(n+1)(n+2)(n+3))/4$
- $= (n(n+1)(n+2)(n+3)(n+4))/4$

9.

- a) True
- b) False
- c) True
- d) False

10.

- a) Applying the implication law
 - $(\sim A \wedge \sim B) \Rightarrow \sim B = \sim(\sim A \wedge \sim B) \vee \sim B$
- b) Applying DeMorgan's law
 - $\sim(\sim A \wedge \sim B) \vee \sim B = (A \vee B) \vee \sim B$
- c) Applying idempotent law
 - $(A \vee B) \vee \sim B = A \vee B$
- d) Applying idempotent law again
 - $A \vee B = \text{True}$