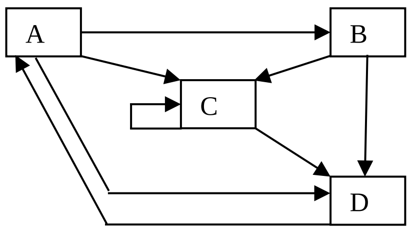
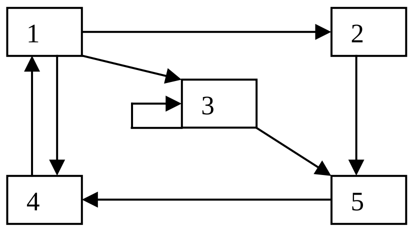
Homework #18

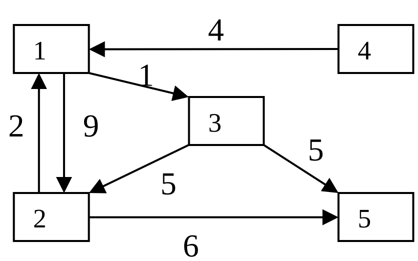
Name ____________________ Sec ______

Questions:	Answers:
1. For the graph below, use ordinary arithmetic to compute A^3. What does this matrix represent?  	
2. As directed below, prove that the following algorithm produces a matrix in which the number in the ith row and jth column is the number of paths of length n from the ith node to the jth node. initialize A as the adjacency matrix for graph G; M = A for (i = 2 to n) M = M $\times$ A (using ordinary arithmetic) a) In terms of A, what is the value of M when the loop terminates? b) Give the loop invariant. (This should be a statement based on k—the number of loops—such that when the algorithm stops, it will yield the result we want.) c) Give the basis for an induction proof. (This will be the case before going through the loop, i.e. for k = 0.) d) Prove the induction implication. (That is, prove S(k) $\Rightarrow$ S(k+1) where S(k) is your loop invariant.)	

3. Assuming Warshall's algorithm uses node 1 for its first pivot and node 2 as its second pivot, give the adjacency matrix after the first two iterations, i.e. after using both node 1 as a pivot and node 2 as a pivot.



4. Assuming Floyd's algorithm uses node 1 for its first pivot and node 2 as its second pivot, give the adjacency matrix after the first two iterations, i.e. after using both node 1 as a pivot and node 2 as a pivot.



After pivot 1:

$$
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
0 & 9 & 1 & \infty & \infty \\
2 & 0 & 3 & \infty & 6 \\
\infty & 5 & 0 & \infty & 5 \\
4 & 13 & 5 & 0 & \infty \\
\infty & \infty & \infty & \infty & 0
\end{bmatrix}
\end{array}
$$

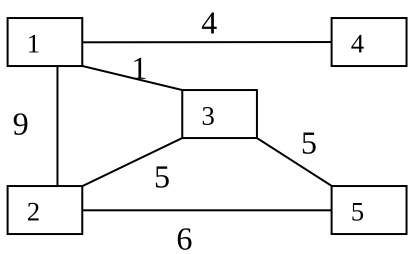
After pivot 2:

$$
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
0 & 9 & 1 & \infty & 15 \\
2 & 0 & 3 & \infty & 6 \\
7 & 5 & 0 & \infty & 5 \\
4 & 13 & 5 & 0 & 19 \\
\infty & \infty & \infty & \infty & 0
\end{bmatrix}
\end{array}
$$

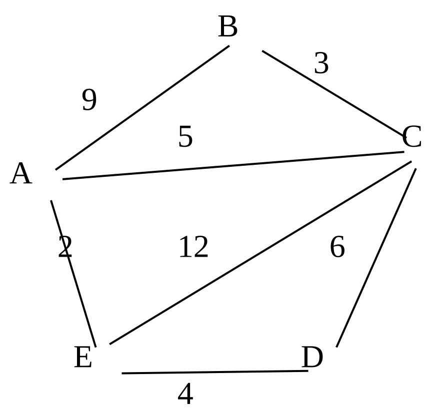
5. Recall that if we need to run Floyd's algorithm on an undirected graph, we simply replace each edge with two edges, one in each direction, and then run Floyd's algorithm as usual.

a) For the graph below, give the initial adjacency matrix so that Floyd's algorithm can be run on it.

b) Then, assuming Floyd's algorithm uses node 1 for its first pivot, give the adjacency matrix after the first iteration.



6. Consider the following weighted graph G_w.



a) List the nodes of G_w starting with A in the order in which they are settled in Dijkstra's algorithm. Show your work by giving, for each iteration, the settled

node for the iteration and the distance to each unsettled node at the end of the iteration. b) List the nodes of G_w starting with C in the order in which they are settled in Dijkstra's algorithm.	
7. Consider again graph G_w in problem #6. a) List the edges in the order added by Kruskal's algorithm. b) List the edges in the order added by Prim's algorithm when the initial chosen node is A. c) List the edges in the order added by Prim's algorithm when the initial node chosen is C.	