

Algorithm:

Input: A connected weighted graph which contained with positive weight.

output: $L(z)$, the length of a shortest path from a to z .

step 1: set $L(a) = 0$

step 2: for all vertices $x \neq a$, do $L(x) = \infty$

step 3: set $T = V$, where T = set of vertices having temporary vertex.

and V = set of vertex of G .

step 4: Let ' v ' be a vertex in ' T '.

i.e. ($v \in T$) for which $L(v)$ is minimum & hence the permanent label of v .

step 5: $T = T - \{v\}$

step 6: for every edge, $e = (v, u)$ adjacent to v ,
change $L(u)$ to $\min \{ \text{old } L(u), L(v) + w(e) \}$

step 7: If $v = z$ stop, otherwise go to step 2.

step 8: End.

Solution: It's initial labelling is given by:

Vertex v	A	B	C	D	E
$L(v)$	0	∞	∞	∞	∞
T	{A, B, C, D, E}				

Iteration 1: Let $U = A$ has $L(U) = 0$

$$\therefore T = T - \{a\}$$

since there are three edges adjacent with A,
i.e., AB, AC, AE where $B, C, E \in T$

$$\begin{aligned} \therefore L(B) &= \min \{ \text{old } L(B), L(A) + w(AB) \} \\ &= \min \{ \infty, 0 + 9 \} \\ &= 9 \end{aligned}$$

$$\begin{aligned} \therefore L(C) &= \min \{ \text{old } L(C), L(A) + w(AC) \} \\ &= \min \{ \infty, 0 + 5 \} \\ &= 5 \end{aligned}$$

$$\begin{aligned} \therefore L(E) &= \min \{ \text{old } L(E), L(A) + w(AE) \} \\ &= \min \{ \infty, 0 + 2 \} \\ &= 2 \end{aligned}$$

Hence the minimum label is $L(E) = 2$
Now we have to modify the table,

Vertex v	A	B	C	D	E
$L(v)$	0	9	5	∞	2
T	{	B	C	D	E}

Iteration 2 : Let $U = E$ has $L(U) = 2$

$$\therefore T = T - \{E\}$$

Since there are two edges adjacent with E

i.e., EC and ED where $C, D \in T$

$$\begin{aligned}\therefore L(C) &= \min \{ \text{old } L(C), L(E) + w(EC) \} \\ &= \min \{ 5, 2 + 12 \} \\ &= 5\end{aligned}$$

$$\begin{aligned}\therefore L(D) &= \min \{ \text{old } L(D), L(E) + w(ED) \} \\ &= \min \{ \infty, 2 + 4 \} \\ &= 6\end{aligned}$$

Hence the minimum label is $L(C) = 5$

Now we have to modify the table :

vertex (v)	A	B	C	D	E
$L(v)$	0	9	5	6	2
T	{	B	C	D	}

Iteration 3: Let $U = C$ has $L(U) = 5$

$$\therefore T = T - \{C\}$$

Since there are two edges adjacent with C ,

i.e., CB , CD where $B, D \in T$

$$\begin{aligned}\therefore L(B) &= \min \{ \text{old } L(B), L(C) + w(CB) \} \\ &= \min \{ 9, 5 + 3 \} \\ &= 8\end{aligned}$$

$$\begin{aligned}\therefore L(D) &= \min \{ \text{old } L(D), L(C) + w(CD) \} \\ &= \min \{ 6, 5 + 6 \} \\ &= 6\end{aligned}$$

Hence the minimum label is $L(D) = 6$

Now we have to modify the table:

vertex (v)	A	B	C	D	E
$L(v)$	0	8	5	6	2
T	{	B		D	}

Iteration 4: Let $U = D$ has $L(U) = 6$

$$\therefore T = T - \{D\}$$

Since D doesn't have any non visited neighbours, so, we don't need to check anything. We mark it as visited and modify the table:

vertex(v)	A	B	C	D	E
$L(v)$	0	8	5	6	2
T	{	B			}

Iteration 5: Let $U = B$ has $L(U) = 8$

$$\therefore T = T - \{B\}$$

Since B doesn't have any non-visited neighbours, so we don't need to check anything.

Therefore we stop the iteration. Finally we get the following shortest path tree.

