Homework #9 ______________________ Name ______ Sec

Questions:	Answers:
1. Translate the following into predicate calculus. For each answer, also state your assumed universe of discourse. a) “Anyone who was an ancient Roman and tried to kill Caesar was not loyal to Caesar.” b) “All cats which are calico, are female.” c) “Some Texans have never left the state of Texas.”	a) Let A(x) represent "x was an ancient Roman," and K(x) represent "x tried to kill Caesar." Let L(x) represent "x is loyal to Caesar." $\forall x(A(x)\land K(x)\rightarrow\neg L(x))$ Assumed universe of discourse: All individuals. b) Let C(x) represent "x is a cat" and F(x) represent "x is female." Let H(x) represent "x is calico." $\forall x((C(x)\land H(x))\rightarrow F(x))$ Assumed universe of discourse: All animals. c) Let $T(x)$ represent "x is a Texan" and $L(x)$ represent "x left the state of Texas." $\exists x(T(x)\land\neg L(x))$ Assumed universe of discourse: All individuals.
2. A universe contains the three individuals a, b, and c. For these individuals, a predicate Q(x, y) is defined, and its truth values are given by the following table: $\begin{array}{c\|ccc} x\backslash y & a & b & c \\ \hline a & T & F & T \\ b & F & T & F \\ c & F & T & T \end{array}$ Write each of the following expressions without quantifiers (i.e. convert them to expressions with ANDs and ORs or both) and then evaluate the expression. a) $\forall x\exists yQ(x, y)$ b) $\forall yQ(y, b)$ c) $\forall yQ(y,y)$	a) $\forall x\exists yQ(x,y)$ $Q(a,a)\lor Q(a,b)\lor Q(a,c)$ $Q(b,a)\lor Q(b,b)\lor Q(b,c)$ $Q(c,a)\lor Q(c,b)\lor Q(c,c)$ $Q(a,a)\lor Q(a,b)\lor Q(a,c)$ $T\lor F\lor T=T$ $Q(b,a)\lor Q(b,b)\lor Q(b,c)$ $F\lor T\lor F=T$ $Q(c,a)\lor Q(c,b)\lor Q(c,c)$ $F\lor T\lor T=T$ $\forall x\exists yQ(x,y)$ evaluates to T b) $\forall yQ(y,b)$ $Q(a,b)\land Q(b,b)\land Q(c,b)$ $Q(a,b)\land Q(b,b)\land Q(c,b)$ $F\land T\land T=F$ $\forall yQ(y,b)$ evaluates to F c) $\forall yQ(y,y)$ $Q(a,a)\land Q(b,b)\land Q(c,c)$ $Q(a,a)\land Q(b,b)\land Q(c,c)$ $T\land T\land T=T$ $\forall yQ(y,y)$ evaluates to T

5. Using the facts in the class university database in Discussion #15, write predicate logic statements to answer the following questions. a) What are the names of students who live at 12 Apple St.? b) What are the names of students who are getting an A in CS101? Your predicate logic statements must answer these questions for any state of the database, not just the one in the discussion slides.	
6. Algebraically transform: $\leftarrow\forall x(P(x) \wedge Q(y) \Rightarrow \exists zR(z))$ to $\exists x\forall z(P(x) \wedge Q(y) \wedge \leftarrow R(z))$ Justify each step with one or more laws.	De Morgan's Law: $\neg\forall x(P(x)\wedge Q(y)\Rightarrow\exists zR(z))$ becomes $\exists x\neg(P(x)\wedge Q(y)\Rightarrow\exists zR(z))$ Implication Law: $\neg(P\Rightarrow Q)$ is equivalent to $QP\wedge\neg Q$ Distributive Law: $\exists x(P(x)\wedge Q(y)\wedge\neg\exists zR(z))$ is equivalent to $\exists x(P(x)\wedge Q(y)\wedge\forall z\neg R(z))$ Putting these together: $\neg\forall x(P(x)\wedge Q(y)\Rightarrow\exists zR(z))\equiv\exists x\neg(P(x)\wedge Q(y)\Rightarrow\exists zR(z))$ (De Morgan's Law) $\exists x\neg(P(x)\wedge Q(y)\Rightarrow\exists zR(z))\equiv\exists x(P(x)\wedge Q(y)\wedge\neg\exists zR(z))$ (Implication Law) $\exists x(P(x)\wedge Q(y)\wedge\neg\exists zR(z))\equiv\exists x(P(x)\wedge Q(y)\wedge\forall z\neg R(z))$ (Distributive Law) $\neg\forall x(P(x)\wedge Q(y)\Rightarrow\exists zR(z))$ algebraically transforms to $\exists x\forall z(P(x)\wedge Q(y)\wedge\neg R(z))$
7. Consider the following expression: $\forall x\exists y(P(y, x) \wedge \exists xQ(y, x)) \vee R(x) \wedge \exists yR(y)$. a) Identify the subexpression in the scope of $\forall x$. b) Identify the free variables. c) Identify each bound variable and the quantifier to which it is bound. d) Rectify the expression. (“Rectification is also called “standardizing variables apart.”)	