

<p>6. Prove the following if $(\neg P \vee R) \wedge (\neg Q \vee R) \wedge (P \vee Q)$ then R</p> <p>Prove by contradiction, using resolution.</p>	<p>Given:</p> <ol style="list-style-type: none"> 1. $\neg P \vee R$ 2. $\neg Q \vee R$ 3. $P \vee Q$ <p>To prove: R</p> <p>Assume the negation of the conclusion: $\neg R$</p> <p>resolve $\neg P \vee R$ and $\neg R$ to derive P</p> <p>resolve $P \vee Q$ and P to derive Q</p> <p>resolve $\neg Q \vee R$ and $\neg R$ to derive Q</p> <p>Now we have derived both Q and R, which contradicts the original assumption $\neg R$. Therefore, our assumption was incorrect, and R must be true</p>
<p>7. Use resolution to prove:</p> <p>if $(\neg P \vee Q) \wedge (\neg(R \Rightarrow \neg Q) \vee \neg R)$ then $\neg P$</p> <p>Construct your proof as follows.</p> <p>a) Convert the premise to conjunctive normal form.</p> <p>b) Using the re-written premises, prove by contradiction, using resolution.</p>	<p>a) Given: $(\neg P \vee Q) \wedge \neg(\neg(R \Rightarrow \neg Q) \vee \neg R)$</p> <p>Step 1: Apply De Morgan's Laws: $\neg(R \Rightarrow \neg Q) \equiv \neg(\neg R \vee \neg Q) \equiv R \wedge Q$</p> <p>Step 2: Substitute back: $(\neg P \vee Q) \wedge \neg(R \wedge Q \vee \neg R)$</p> <p>Step 3: Apply De Morgan's Laws: $(\neg P \vee Q) \wedge (\neg(R \wedge Q) \wedge R)$</p> <p>Step 4: Distribute the negation: $(\neg P \vee Q) \wedge ((\neg R \vee \neg Q) \wedge R)$</p> <p>Step 5: Distribute again: $(\neg P \vee Q) \wedge (\neg R \wedge R \vee \neg Q)$</p> <p>Step 6: Simplify $\neg R \wedge R$ to False: $(\neg P \vee Q) \wedge (\text{False} \vee \neg Q)$</p> <p>Step 7: Simplify $\text{False} \vee \neg Q$ to $\neg Q$: $(\neg P \vee Q) \wedge \neg Q$</p> <p>b) Given: $(\neg P \vee Q) \wedge \neg Q$</p> <p>To prove: $\neg P$</p> <p>Assume the negation of the conclusion: P</p> <p>We have:</p> <p>$\neg P \vee Q$</p> <p>$\neg Q$</p> <p>P (negation of conclusion)</p> <p>resolution:</p> <p>resolve $\neg P \vee Q$ and P to derive Q</p> <p>resolve $\neg Q$ and Q to derive False</p> <p>Since we derived a contradiction (False), our assumption that P was true must be incorrect. $\neg P$ must be true.</p>