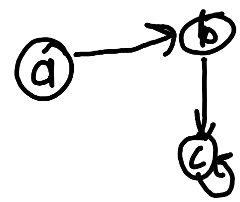
Homework #15

______________________ Name ____ Sec

Questions:	Answers:
1. Give the following closures, as specified. a) Let R be the relation {(a, b), (b, c), (c, c)} on {a, b, c}. Give the reflexive closure as a graph. b) Let R be the relation {(a, b), (a, c), (b, a), (c, b), (c, c)} on{a, b, c}. Give the symmetric closure as a matrix. c) Let R be the relation {(a, a), (a, b), (c, d), (d, a)} on{a, b, c, d}. Give the transitive closure as a set of ordered pairs. d) Let R be the relation {(a, d), (a, b), (c, d), (d, a)} on{a, b, c, d, e}. Give the reflexive-transitive closure as a matrix.	a)  b) $\begin{array}{c c} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$ c) {(a,a), (a,b), (c,a), (c,d), (d,a), (d,b)} d) $\begin{array}{c c} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} & \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$
2. For each of the following relations, i) state whether the relation is an equivalence relation and ii) if it is an equivalence relation then list the equivalence classes of its corresponding partition (as a set of sets), and if it is not then list all of the properties (reflexive, symmetric, transitive) it violates. In all cases, the relations are on the set A: A = {1, 2, 3, 4}. a) {(1, 1), (1, 4), (2, 2), (3, 3), (4, 1), (4, 4)} b) A × A c) {(1, 1), (2, 2), (3, 1), (1, 3), (4, 4)} d) ({2, 3} × A) ∪ {(1, 1), (4, 4)}	

3. For each of the following relations:

i) Determine if the relation is a weak poset, a strict poset, a total ordering, and/or a well founded poset. List all that apply.
ii) Draw the Hasse diagram, if possible.
iii) Give a descending sequence based on the relation, if possible.

For each relation, assume the set on which the relation is built is the set consisting of the elements mentioned in the ordered pairs.

a) transitive closure of R where R = {(a, s), (a, z), (z, x), (x, c), (s, x), (d, c), (s, d)}

b) reflexive-transitive closure of R where R= {(a, b), (b, c), (c, d)}

c) {(1,2), (2,3), (1,3), (3,4), (2,4), (4,1)}

d) The "has as a factor" relation on the set of positive integers. (For example, 6 "has as a factor" 2; 6 also "has as a factor" 3, 1, and 6 but not 4 and 5. Note that for this question, 6 "has as a factor" 6 and 6 "has as a factor" 1)

4. For the poset R = {(a, c), (a, d), (a, e), (a, f), (a, g), (b, d), (b, e), (b, g), (c, f), (d, e), (d, g)} on {a, b, c, d, e, f, g}, give the following.

a) the upper bounds of e

b) the lower bounds of b

c) the maximal elements of R

d) the minimal elements of R

e) the least upper bounds of f and g

f) the greatest lower bounds of b and c	
5. Classify each of the following as a total function, a partial function, or not a function. a) f = {(1, a)} for domain space {1} and range space {a, b}. b) f(x) = 1/x when both domain and range space are the **R**eal numbers (**R**). c) f(x) = sin(x) when domain space is the **N**atural numbers (**N**, the positive integers) and range space is **R.** d) $f(x) = x^{½}$ when both domain and range space are **R**. (Recall that the square root of a positive number n is ±n.) e) f(x, y) = max(x, y) for domain space **R** × **R** and range space **R.** f) f = {((1, 1), a), ((2, 1), b), ((1, 2), b)} for domain space {1, 2} × {1, 2} and range space {a, b}. g) f = {((1, 1), a), ((1, 1), b), ((1, 2), b)} for domain space {1} × {1, 2} and range space {a, b}.	
6. Using the functions f = {(a, 1), (b, 2), (c, 2), (d, 3)}, g = {(1, z), (2 ,z), (3, x)}, and h = {(1, a), (2, b), (3, c)}, give the following. (Assume that the domain and range spaces are the values shown.) a) f(b) b) g(f(d)) c) g restricted to the domain space {1, 2}	

d) $h \circ f$ e) h^{-1} f) $h^{-1}(h(2))$	
7. Classify each of the following functions as: injection, surjection, bijection, or none. Give the most specific answer. a) f ={(a, 1), (b, 2), (c, 2), (d, 3)} for domain space {a, b, c, d} and range space {1, 2, 3, 4} b) f ={(a, 1), (b, 2), (d, 3), (c, 5)} for domain space {a, b, c, d} and range space {1, 2, 3, 4, 5} c) f ={(a, 1), (b, 2), (c, 4), (d, 3)} for domain space {a, b, c, d} and range space {1, 2, 3, 4} d) f ={(a, 1), (b, 2), (c, 4), (d, 3), (e, 1)} for domain space {a, b, c, d, e} and range space {1, 2, 3, 4} e) $f(x) = \cos(x)$ when both domain and range space are **R**. f) $f(x) = 3x+2$ when both domain and range space are **R**. g) $f(x) = x^2+1$ when both domain and range space are positive **R**.	