① $1+3+6+\ldots+\frac{k(k+1)}{2}=\frac{k(k+1)(k+2)}{6}$

$$\frac{k(k+1)(k+2)}{6}+\frac{(k+1)(k+2)}{2}=\frac{(k+1)(k+2)(k+3)}{6}$$

$$\frac{k(k+1)(k+2)+3(k+1)(k+2)}{6}=\frac{(k+1)(k+2)(k+3)}{6}$$

$$\frac{(k+1)(k+2)(k+3)}{6}=\frac{(k+1)(k+2)(k+3)}{6}$$

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② $1\cdot 1!+2\cdot 2!+\ldots+n\cdot n!=(n+1)!-1$

$$(k+1)!-1+(k+1)\cdot(k+1)!=((k+1)+1)!-1$$

$$(k+1)!+(k+1)\cdot(k+1)!=(k+2)!-1$$

$$(k+1)!(1+k+1)=(k+2)!-1$$

$$(k+1)!(k+2)=(k+2)!-1$$

$$(k+1)!(k+2)=(k+2)(k+1)!-1$$

$$(k+1)!(k+2)=(k+1)!(k+2)-1$$