

# Integration using u-substitution

1. Integrate using u substitution to find  $F(x)$  with respect to  $x$ :  $f(x) = x(x-2)^{5/2}$

(3 marks)

2. Integrate using u substitution to find  $F(x)$  with respect to  $x$ :  $f(x) = x\sqrt{x+10}$

(2 marks)

3. Integrate using u substitution to find  $F(x)$  with respect to  $x$ :  $f(x) = (\sin(x))^5 (\cos(x))$

(2 marks)

4. Integrate using u substitution to find  $F(x)$  with respect to  $x$ :  $f(x) = (14-2x)\sqrt{x^2-14x+1}$

(3 marks)

5. Integrate using u substitution to find  $F(x)$  with respect to  $x$ :  $f(x) = x^2(2x)$

(1 mark)

6. Integrate using u substitution to find  $F(x)$  with respect to  $x$ :  $f(x) = (1 + x^2)(1 + 3x + x^3)^5$

(4 marks)

7. Integrate using u substitution to find  $f'(x)$  with respect to  $x$ :  $f''(x) = \frac{x}{(x^2 + \sqrt[3]{2\pi})^{7/2}}$

(5 marks)

8. Integrate using u-substitution to find  $F(x)$  with respect to  $x$ , if the  $f(x) = 4x(x^2)$

(2 marks)

## U-Substitution Using Integration

1. Integrate using u substitution to find F(x) with respect to x:  $f(x) = x(x-2)^{5/2}$

$$u = x-2$$

$$du = dx$$

$$\therefore \int (u+2)(u)^{5/2} du = \int u^{7/2} + 2u^{5/2} du = \frac{2}{9} u^{9/2} + \frac{4}{7} u^{7/2} + C$$

$$\therefore F(x) = \frac{2}{9} (x-2)^{9/2} + \frac{4}{7} (x-2)^{7/2} + C$$

(3 marks)

2. Integrate using u substitution to find F(x) with respect to x:  $f(x) = x\sqrt{x+10}$

$$u = x+10$$

$$du = dx$$

$$\therefore \int (u-10)(u)^{1/2} du = \int u^{3/2} - 10u^{1/2} du = \frac{2}{5} u^{5/2} - 15u^{3/2} + C$$

$$\therefore F(x) = \frac{2}{5} (x+10)^{5/2} - 15(x+10)^{3/2} + C$$

(2 marks)

3. Integrate using u substitution to find F(x) with respect to x:  $f(x) = (\sin(x))^5 (\cos(x))$

$$u = \sin(x) \quad \text{as} \quad f'(\sin(x)) = \cos(x)$$

$$\therefore du = \cos(x) dx$$

$$\therefore \int u^5 du = \frac{1}{6} u^6 + C$$

$$\therefore F(x) = \frac{1}{6} (\sin(x))^6 + C$$

(2 marks)

4. Integrate using u substitution to find F(x) with respect to x:  $f(x) = (14-2x)\sqrt{x^2-14x+1}$

$$u = x^2 - 14x + 1$$

$$du = 2x - 14 dx$$

$$\therefore -1 du = 14 - 2x dx$$

$$\therefore \int -1 du \cdot u^{1/2} = -1 \times \frac{2}{3} (u)^{3/2} + C$$

$$\therefore F(x) = -\frac{2}{3} (x^2 - 14x + 1)^{3/2} + C$$

(3 marks)

5. Integrate using u substitution to find F(x) with respect to x:  $f(x) = x^2(2x)$

$$u = x^2$$

$$\therefore du = 2x dx$$

$$\therefore \int u du = \int 2x^3 dx \therefore f(x) = \frac{1}{2} x^4 + C$$

(1 mark)

6. Integrate using u substitution to find  $F(x)$  with respect to  $x$ :  $f(x) = (1+x^2)(1+3x+x^3)^5$

$$u = 1+3x+x^3$$

$$\therefore du = 3x^2 + 3$$

$$\therefore \frac{1}{3} du = x^2 + 1$$

$$\therefore \int \frac{1}{3} du \cdot u^5 = \frac{1}{3} \int u^5 du = \frac{1}{3} \times \frac{1}{6} u^6 + C$$

$$\therefore F(x) = \frac{1}{18} (1+3x+x^3)^6 + C$$

(4 marks)

7. Integrate using u substitution to find  $f(x)$  with respect to  $x$ :  $f'(x) = \frac{x}{(x^2 + \sqrt[3]{2\pi})^{7/2}}$

$$u = (x^2 + \sqrt[3]{2\pi})$$

$$\therefore du = 2x dx \quad \therefore \frac{1}{2} du = x dx$$

$$\therefore \frac{1}{2} \int \frac{du}{u^{7/2}} = \frac{1}{2} \int u^{-7/2} du = \frac{1}{2} \times -\frac{2}{5} u^{-5/2} + C$$

$$= -\frac{2}{10} u^{-5/2} + C = -\frac{1}{5} u^{-5/2} + C$$

$$\therefore F(x) = -\frac{1}{5} (x^2 + \sqrt[3]{2\pi})^{-5/2} + C$$

$$\therefore f(x) = -\frac{1}{5 (x^2 + \sqrt[3]{2\pi})^{5/2}} + C$$

(5 marks)

8. Integrate using u-substitution to find  $F(x)$  with respect to  $x$ , if the  $f(x) = 4x(x^2)$

$$u = x^2$$

$$\therefore du = 2x dx$$

$$\therefore 2 du = 4x dx$$

$$\therefore 2 \int u dx = 2 \times \frac{1}{2} u^2 + C$$

$$\therefore F(x) = x^4 + C$$

(2 marks)