MATHEMATICAL STUDIES BINOMIAL DISTRIBUTIONS AND STANDARD NORMAL DISTRIBUTIONS

Time given: 100 minutes

Marks: /113

Name: _____

1. The equation for a normal distribution is given by $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$.

a) State the value of μ and σ for a standard normal distribution

(1)

b) Hence, simplify the equation given to suit a standard normal distribution

(2)

c) Differentiate to find the first derivative of f(x) and determine the significance of f'(x) = 0 relative to the standard normal distribution.

(3)

d) Find the second derivative of f(x) and state the significance of f''(x)=0 relative to the standard normal distribution

(4)

2. A physics teacher claims that 85% of those who study physics in year 12 believe it will be an important part of their future studies. To test this claim, 150 students we surveyed and it was found that 127 believed that physics would be an important part of their future studies.

a) State the null hypothesis

(1)

b) State the alternative hypothesis

(1)

c) What percentages of individuals believe that physics is an important part of their future studies?

(1)

d) What is the equation for Hypothesis Testing with Binomial Distributions?

(1)

e) Calculate the test statistic

(2)

f) Did the physics teacher make a valid claim? Explain how one can tell based on evidence.

(2)

g) Calculate a p value

(1)

3. Lesley is an international criminal. He robs bank vaults that contain large amounts of money or gold bars and most of the time he is successful. There are occasionally times when things don't quite go to plan for Lesley and this results in his arrest.

Today, Lesley is planning a heist in Las Vegas, where he intends to empty the contents of a large vault that is estimated to contain $50 million. On the vault there are 16 different numbers and symbols. The combination necessary to unlock the vault includes each key in a specific order.

a) Use Pascal's Triangle or some other means to determine the number of possible combinations that could be made to unlock the safe.

(1)

b) Determine the probability that if Lesley was guessing the combination, what the likelihood of him getting into the bank vault would be.

(1)

Luckily enough for Lesley, he is not guessing and knows the combination. There is however just one catch. In order to have enough time to break into the vault, Lesley must cut the correct wire in the security camera box in order to disable the system. There are three wires within the system; one red, one green and one blue. There is just one problem. Lesley is colourblind and will only be able to make two attempts at cutting the correct wire before the security arrive and he is escorted to jail.

c) What is the probability of him cutting the correct wire at random on the first attempt?

(1)

d) What is the probability that Lesley will be arrested?

(2)

e) What is the probability that Lesley will cut the correct wire on either the first or the second attempt?

(2)

After cutting the wires successfully and emptying the vault, Lesley needs to be able to get to his hideout. To do this, he has seven getaway cars outside. However after he accidentally dropped the keys to four of the cars inside the vault.

f) Assuming that he were to run to the first vehicle he could find and try to unlock it with the keys he has, what is the probability that the car would open?

(1)

g) What is the probability that it will take him exactly four cars to find one that will open?

(2)

h) What is the probability that either one of the first two cars he tries will open?

(1)

4. Pascal's Triangle includes many different patterns and secrets that can help one determine the number of possible ways that an outcome can occur. It can save time as it doesn't rely on drawing a large probability tree and can be determined on a calculator.

a) Write the first six rows of Pascal's Triangle

(4)

b) Using the triangle, explain how to determine the number of possible ways to have two successes from four trials

(2)

c) Using a calculator, determine the number of possible ways to have four successes in six trials

(1)

d) Explain why the triangle is symmetrical even if success and failure do not have symmetrical probabilities

(2)

e) What is the sum of the row of the triangle that represents having one success-failure trial?

(1)

f) Calculate the sum of the next four rows

(2)

g) Develop a conjecture by evaluating the previous two sub-questions and testing to see whether the conjecture applies to the row representing having 13 trials

(3)

5. Use the equation below to answer all sub-questions in question 5, by showing clear logical steps

$$C_r^n (p)^r (q)^{n-r}$$

a) Two successes from five trials when the probability of success is 0.43

(2)

b) Three successes from six trials when the probability of success is 0.64

(2)

c) No successes from eight trials when the probability of success is 0.03

(2)

6. Follow the proceeding instructions

a) Draw a spike graph for Bin(10, 0.1)

(3)

b) From the spike graph, determine the probability of getting 4 successes from 10 trials

(1)

7. There is a virus going around the workplace and it is causing a large number of sick days to be requested by staff. The workplace needs to have at least 1900 people working at a time to be deemed profitable and run successfully. The probability of an individual catching the disease by the end of the week is 0.78 and there are 2500 people employed at the company. Using normal approximation, determine the probability that less than 1900 people will not be sick (hence meaning that the company will have to close for next week).

(5)

8. Kevin is playing a computer coin toss simulator. On the game, he tosses a coin 14 million times.

a) Determine the mean number of successes (heads) from the simulation

(2)

b) What is the continuity correction and why is it important?

(3)

c) Determine the probability that exactly 7 million are heads

(2)

d) Determine the probability that exactly 11 million are heads

(2)

e) Determine the probability that between 6 million and 7.5 million (exclusive) of the tosses will be heads

(3)

f) Determine the probability that between 6.4 million and 6.99 million (inclusive) of the tosses will be tails

(2)

Kevin and John decide to use the computer simulator to determine which movie they should go and see at the cinema. They decide to make it the best of five tosses. Kevin chooses heads and John decides tails. John is unaware that Kevin has actually gone into the settings and made the coin biased so that the probability of the simulator getting heads is 0.80.

g) Explain whether it would be appropriate or not to use normal approximation

(2)

h) What is the probability that all five of the tosses are heads?

(1)

i) What is the probability that four tosses are heads?

(1)

j) What is the probability that three or more tosses are heads?

k) What is the probability that John will win the coin toss and they will see the movie that he wants to see?

9. The current Guinness World Record holder for being the fastest female speaker (being able to say the most eligible words in a given second) is Fran Capo, an author and TV show host. Fran is able to speak 602.32 words per minute (that is approximately 10 words per second) eligibly (when the slow motion recording is played back). Another female, Sarah Longley, attempted to break the world record and was able to speak 610 words per minute. The only problem was she was unable to consistently say this many words. From a sample of 20 attempts, it was found that she beat the world record six times.

a) Construct a 0.95 confidence interval for the proportion of times that Sarah Longley beats the record set by Fran Capo.

(2)

b) Construct a 0.99 confidence interval for the proportion of times that Sarah Longley beats the record set by Fran Capo.

(3)

c) After extensive practice, it was found that Sarah Longley was able to beat the record on 56% of her attempts (from 100 practices). Determine a 0.95 confidence interval and whether or not she could possibly beat the record 60% of the time.

(3)

10. A poll was taken to determine whether or not Julia Gillard or Tony Abbot would win the next election. From a sample size of 3000, it was found that 50.5% were in favor of Julia Gillard as the next prime minister of Australia.

a) Construct a 0.95 confidence interval for the proportion of voters who intend to vote Gillard at the next election

(2)

b) An advertisement claimed that Julia Gillard was favored as prime minister by a majority of Australians. To test this, an independent group surveyed 600 people and found that 45% of Australians were in favor of Gillard. Construct a 0.95 confidence interval for the true population proportion that wants Gillard for government.

(2)

c) What does part (b) tell you about the results in part (a)?

(2)

d) What value of p* will return the maximum number of people needed as a sample size to obtain a certain confidence interval width and why? Explain mathematically.

(3)

e) The General Voters Survey wants to know how many people must be surveyed to have a width of 0.005. Determine the maximum number they must survey.

(3)

f) It is not too long to election and the General Voters Survey committee has decided that they can only interview 400 people. Assuming a p* value of 0.5, determine what the width of the confidence interval will be.

(3)

11. Gangnam Style by PSY hit 1 billion views on December 21ˢᵗ 2012. Of this one billion views, it is claimed that 500 000 000 were unique views (hence this was the number of different IP addresses, different people, who watched the video). A small online video enthusiasts forum host a poll and find that from 5900 people, 1000 people watched Gangnam Style only once.

a) What is the null and alternate hypothesis?

(1)

b) Determine what values of the sample proportion would result in the evidence supporting the alternate hypothesis based on the sample taken (mentioned in q11)

(4)

c) Hence, state whether the null or alternate hypothesis is supported based on the sample taken.

(2)

d) Explain why the survey on the online video enthusiasts forum website may be considered biased and not a true representation of unique views.

(2)

EXTRA SPACE (IF REQUIRED)