MATHEMATICAL STUDIES STATISTICS

Time given: 90 minutes

Marks: /96

Name: _____

1. What happens to the width of a confidence interval as the sample size increases (when standard deviation is fixed)? Represent this mathematically.

(2)

2. Popular Potato produces a range of potato crisps. Recently, a significant power outage meant that they had to recalibrate their potato-frying machine. Before the power outage, the mean amount of canola oil used to fry one kilogram of potato crisps was 100.5mL. The machine variation is configured permanently to have a population standard deviation of 2.4mL.

   The manager, who was aggravated by the lack of contingency power supplies, threatened to fire the workers who were meant to ensure the factory could continue through a power failure if the machine had to be recalibrated. He ordered the Popular Potato statistician to evaluate the amount of oil used in twenty 1kg batches. From this, the statistician reported that the mean amount of canola oil used was 102.1mL per kilogram.

   a) What is the null hypothesis?

(1)

   b) What is the alternate hypothesis?

(1)

   c) What is the null distribution of the test statistic?

(1)

   d) Calculate the test statistic

‘

(2)

e) Determine whether, based on the test statistic alone, the factory workers in charge of power supply will be fired.

(3)

f) Calculate a p value and determine whether the null hypothesis is accepted or rejected at the 0.05 significance level

(2)

3. At Home and Office Supplies, the mean cost of a graphics calculator is $126 with a population standard deviation of $6.90. At Calculators R Us, the mean cost of a graphics calculator is $129 with a population standard deviation of $4.50.

a) Determine the proportion of calculators that would cost more than $130 at Home and Office supplies

(1)

b) Determine the proportion of calculators that cost more than $130 at Calculators R Us

(1)

c) At Home and Office Supplies, the Terrific Integrals Colour Graphics Calculator is more expensive than 80% of calculators. At Calculators R us, the Terrific Integrals Colour Graphics Calculator is less expensive than 15% of calculators. Determine where a maths teacher, who wants to spend the lowest amount of money on a Terrific Integrals Colour Graphics Calculator, should shop out of these two stores.

(3)

d) Calculators R Us has a Crazy Calculator sale and the pricing of all the graphics calculators shifted. 3% of calculators cost less than $112 and 15% of calculators cost more than $130. Determine the new mean and population standard deviation for Calculators R Us during their sale.

(4)

e) Hence, is the average price of a calculator cheaper at Home and Office Supplies or Calculators R Us during their Crazy Calculator sale?

(1)

4. The mean height of year 12 male students in a class of 26 students is 172cm. The standard deviation is 6.4cm.

a) Construct a 0.95 Confidence Interval for the population mean

(2)

b) Construct a 0.99 Confidence Interval for the population mean

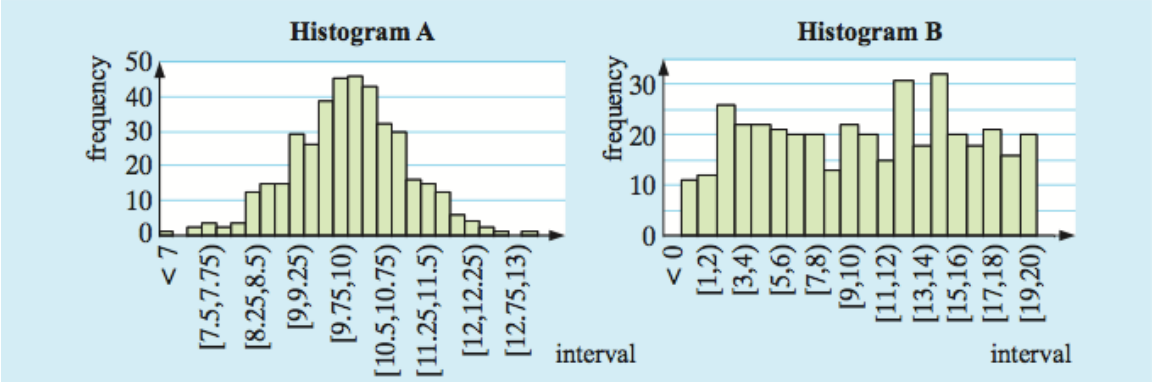
(3)

c) Suggest a conjecture as to what happens to the CI range as the level of confidence increases and show this mathematically

(3)

5. A CD reseller store sells thousands of CDs each month. X denotes the cost of an individual CD purchase. From a month of sales, the mean was found to be \$11.50 and the standard deviation was \$2.50. Let $\overline{X}_n$ denote the average CD cost from a random sample of n purchases.

The two histograms below represent the distributions of $\overline{X}_{50}$ and $\overline{X}_{220}$.



Haese and Harris Mathematical Studies textbook, Page 253

a) Identify which histogram represents $\overline{X}_{50}$ and $\overline{X}_{220}$. Explain your answer.

(2)

b) Determine the mean and standard deviation of the distribution of $\overline{X}_{220}$

(2)

c) Find an approximate value for P($\overline{X}_{220} \geq 12$)

(2)

d) Hence, or otherwise, approximate that the total purchase cost out of a random sample of 220 purchases will be \$2640 or greater.

(2)

6. The Australian Government has proposed that families who have a total income of less than \$40 000 will receive the low-income assistance benefit of \$500 a week for 1 year. However they will only go ahead with this if the total amount paid out to families will be less than \$25 000 000 000.

   According to the Bureau of Statistics, 7% of 14 000 surveyed Australian families have an income of less than \$40 000. They expect that the population standard deviation was 1.5%.

   a) Calculate the 95% Confidence Interval for the percentage of families who are entitled to the low-income assistance benefit.

(2)

   b) Based on the Confidence Interval data, from a population of 21 million, can one conclude whether the low-income assistance benefit will go ahead? Explain

(2)

c) All 21 million Australians were surveyed and it was found that 5.6% of Australians are entitled to the low-income assistance benefit. Is the low-income assistance benefit going ahead?

(3)

7. Dynamic Batteries have a mean lifespan of 31.65 hours with a standard deviation of 3.44 hours. A battery was tested from this company and found to last 36.2 hours. The competitor of Dynamic Batteries, Standardtex, claims that their batteries last 41.2 hours with a standard deviation of 3.01 hours. A battery was tested from Standardtex and it was found to last 37 hours.

a) What proportion of batteries lasted longer than the 36.2 hour Dynamic Batteries battery? Out of 1000 batteries, how many would this be?

(2)

b) What proportion of batteries lasted shorter than the 37 hour Standardtex battery? Out of 1000 batteries, how many would be this be?

(2)

c) What battery is more extreme? Explain and justify.

(3)

d) Determine how long a Standardtex battery would have to last for it to be as extreme (below the mean) as a Dynamic Batteries battery lasting the claimed mean battery life for the Standardtex battery.

(4)

e) Batteries that last for 40% of the claimed time at Dynamic Batteries are rejected. Determine what the minimum time a battery must last for in order for it to not be rejected.

(2)

8. On the opening day of the Apple Store in Adelaide, the mean amount of money spent per customer was $850 with a standard deviation of $285. There were approximately 7800 people who went to the store on the day.

a) Determine the proportion of people who spent more than $1000.

(2)

b) Determine the proportion of people who did not purchase anything

(3)

c) How many people spent exactly $500?

(2)

d) How many people spent between $100 and $200?

(2)

e) Did anyone spend more than $5000 based on this data?

(2)

f) A limitation on the data is how we cannot discern who spent the most money in the store. Explain why this is the case with the normal distribution. Does it have an upper limit?

(2)

9. A manufacturer claims that the mean battery life of one of their new mobile phones is 787 minutes of 4G LTE Internet browsing. A group of mobile phone enthusiasts tested this and found that the mean battery life was 769 minutes. Assuming that there were 35 people who tested the claim, that the data could be represented as a normal distribution and that the test was conducted appropriately, determine whether the null hypothesis should be rejected at the 0.05 significance level if standard deviation is 6.1 minutes.

a) What is the null and alternate hypothesis?

(1)

b) What is the sample error?

(1)

c) What is the null distribution?

(1)

d) Calculate the test statistic

(2)

e) Based on the test statistic should the null hypothesis be rejected? Explain why (without calculating a p value)

(2)

f) Prove your statement made in (f) by calculating a p value

(2)

g) For what values of the sample mean should the alternate hypothesis be supported?

(4)

10. The number of apps installed on mobile phones owned by teenagers is normally distributed. 17.3% of users have less than 40 apps and 0.11% have more than 115 apps. What is the mean and standard deviation? Draw a scaled distribution below with all details.

(6)

11. Bill scored 49 marks on a Science test. The mean for his class was 44 marks with a standard deviation of 2.5. He also scored 38 marks for a mathematics test when the mean for his class was 35 with a standard deviation of 1.2.

a) If the science test was out of 54 and the mathematics test was out of 42, determine Bill's percentage for both of the tests.

(2)

b) Percentage may not necessarily be a good indicator of Bill's performance relative to the rest of the class, determine using z scores whether he did better on his science test or Maths test relative to his classmates.

(2)

c) Draw a scaled set of normal distribution to show this.

(3)