BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI (RAJASTHAN)

I Sem./II Sem./Summer Term 20 - 20

Comprehensive Examination

Sec. No.

Instructor's Name

ID No.

Name

Course No. PHY F-111

Course Title Mechanical Osscillation and Waves.

Date 14/10/17

Day Saturday

No. of Supplementary copies attached :

Question No.	Marks Obtained	Student's request for rechecking with remarks	Examiner's remarks
1.	8	Q.5 not corrected.	+12
2.	15	Q4 Outward and inward forces are depicted by	
3.	7	the direction of $\hat{r}$.	
4.	13		
5.	00	+12	
6.	30 V. good!		
7.			
8.			
Total	73 (in figures) +12 = 85 changed (in words)		

INSTRUCTIONS TO CANDIDATES

Examiner's Signature

(1) Write clearly and legibly. (2) Enter all the required details on the cover of every answer book. (3) The question number given in the answersheet by the student while answering the question should be the same as in the question paper. (4) Start answering every question from a new page. (5) Write on both sides of the sheet in the answer book. Rough work, if any, should be done at the bottom of the page. Finally cross it out and draw a horizontal line to separate it from the rest of the material on the page. (6) Any answer crossed out by the student will not be examined by the examiner. (7) A supplementary answer book should not be asked for until the first answer book is filled up. (8) No sheet should be torn from the answer book. (9) Use of any unfair means will make the candidate liable to disciplinary action. (10) No paper should be brought in the examination hall for scribbling on. (11) A student should not leave the examination hall without handing over the answer book to invigilator on duty.

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Ans 1:- $a = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$

$a_\theta = 0$

$T = m_A a_r$

$T - m_B g = ma$

$T_{min} = m_B g$.

$2\dot{r}\dot{\theta} = -r\ddot{\theta}$

$\int -\frac{\ddot{\theta}}{\dot{\theta}} = \int 2\frac{\dot{r}}{r}$

$-\ln\dot{\theta} = 2\ln r + c$

$\therefore r^2\omega = k = r_0^2\omega_0$

At that instant $\ddot{r} = 0$ $\because$ We need T_{min}.

$T_{min} = m_A r\dot{\theta}^2 = m_B g$

$m_A r_0 \omega_0^2 = m_B g$

$\omega_0^2 = \frac{m_B g}{r_0 m_A}$

✓ 8

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Ans 2 :- $-mg\sin\theta - \mu mg\cos\theta = m\frac{dv}{dt} - (-v_r)\frac{dm}{dt}$

$-mg(\sin\theta + \mu\cos\theta) = m\frac{dv}{dt} + v_r(-b)$

$\int_0^t \left(-gk + \frac{v_r b}{m}\right) dt = \int_0^v dv$

Let initial velocity be 0.

$m = M_0 - bt$

Let $k = \sin\theta + \mu\cos\theta$.

$-gkt + \frac{v_r b}{-b}\left[\ln(M_0 - bt)\right]_0^t = v$

$v + gkt = +v_r \ln\frac{M_0}{M_0 - bt}$.

$v = -v_r \ln\left(\frac{M_0 - bt}{M_0}\right) - gkt$.

$= v_r \ln\left(\frac{M_0}{M_0 - bt}\right) - gkt$.

$a = \frac{dv}{dt} = -gk$.

$M_0 - bt = \frac{M_0}{2}$

$bt = \frac{M_0}{2}$.

$v = v_r \ln\left(\frac{M_0}{\frac{M_0}{2}}\right) - gk\frac{M_0}{2b}$

$v_{max} = v_r \ln 2 - gk\frac{M_0}{2b}$.

15

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Ans 3 :- Conservation of energy till 10 metres.

$$\frac{1}{2}\cancel{m}(20)^2 - \frac{1}{2}mv^2 = \cancel{m}g(10)$$

$$400 - v^2 = 200$$

$$v = \sqrt{200} = 10\sqrt{2}$$

Conservation of momentum $\because$ Perfectly inelastic collision.

$$50(10\sqrt{2}) = 65(v_f)$$

$$v_f = \frac{100\sqrt{2}}{13}$$

$$h = \frac{v_f^2}{2g} = \frac{10^4 \cdot 2}{2(10)13^2} = 5.9172\ m$$

$\therefore$ Total height $= 15.9172\ m$.

Ans 4 :- In the rotating frame.

$$a_\theta = 0$$

$$a_r = -\frac{v^2}{r}$$

$$a_{in} = a_{rot} + \vec{\Omega}\times\vec{v}_{rot} + \vec{\Omega}\times(\vec{\Omega}\times\vec{r})$$

$$= 0 - \frac{v^2}{r}(\hat{r}) + \Omega v(-\hat{r}) + \Omega^2 r(-\hat{r})$$

$$a_{in} = \left(\frac{v^2}{r} + \Omega v + \Omega^2 r\right)(-\hat{r})$$

$$\frac{d(a_{in})}{dr} = -\frac{v^2}{r^2} + 0 + \Omega^2 = 0 \qquad \therefore r^2 = \frac{v^2}{\Omega^2} = \frac{v}{\Omega}$$

8

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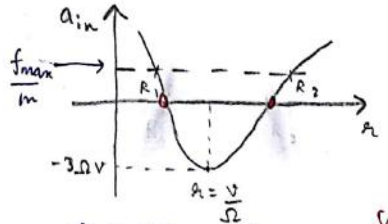
At $r = \frac{v}{\Omega}$

$a_{in} = \left(\frac{v}{\cancel{r}}\Omega + \Omega v + \Omega v\right)(-\hat{r}) = -3\Omega v\,\hat{r}$ which is negative.

$\therefore$ The graph a_{in} vs r has a minima at $r = \frac{v}{\Omega}$

$\because$ friction will always be +ve.
It will create an annular region.



$\therefore$ For $a_{in} < \frac{f_{max}}{m}$ it wont skid.

$\therefore$ Finite R_1 & R_2 exist.

R_1 & R_2 are solutions to the equation

$\frac{f_{max}}{m} = \frac{v^2}{r} + \Omega v + \Omega^2 r$.

Fictitious forces $= \underbrace{-2m\Omega v\hat{r}}_{\text{Coriolis force}} \underbrace{- m\Omega^2 r\hat{r}}_{\text{Centrifugal force}}$ (out ward) 2

Fictitious forces in frame of car $= \underbrace{-2m\Omega v\hat{r}}_{\text{coriolis force}} \underbrace{- m\Omega^2 r\hat{r} - \frac{v^2}{r}\hat{r}}_{\text{Centrifugal force}}$. (out-ward) 2

Fictitious forces for clockwise $= \underbrace{2m\Omega v\hat{r}}_{\text{Coriolis force}} \underbrace{- m\Omega^2 r\hat{r}}_{\text{Centrifugal force}}$. outward

Inward 1

8/4

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Ans 5 :- a) The scale will read correcting reading of the fallen part + the rate of change of momentum of the falling part.

Let $\lambda = \frac{M}{L}$

Reading $= (\lambda x) g + \frac{dP}{dt}$

The part (dx) of the chain just at 'x' distance from lower end initially, falls through distance x.

$\therefore$ Loss in PE = Gain in KE

$$(\lambda\, dx)\, g\, x = \frac{1}{2} (\lambda\, dx)\, v^2$$

$$\therefore v = \sqrt{2 g x}$$

$\therefore$ Loss in momentum at the scale $= (\lambda dx)\, v$ $\quad \because v_{final} = 0$. Inelastic collision.

$$\frac{dP}{dt} = \lambda v \left(\frac{dx}{dt}\right) = \lambda v^2$$

$$\therefore \text{Reading} = \lambda x g + \lambda v^2 = \lambda x g + \lambda 2 g x = 3 \lambda g x .$$

12

x can range from 0 to L $\therefore$ Reading ranges from 0 to $3\lambda g L$ — $\because$ It is a linear function

$\therefore$ Maximum reading $= 3\lambda g L$ when entire chain has fallen.

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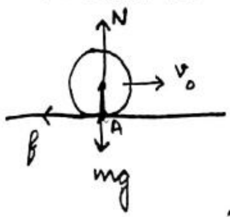
Ans 6 :- Friction acts on it.



$f = ma_x$

$v = v_0 - a_x t = v_0 - \frac{f t}{m}$

$N = mg$

τ about centre of mass

$\tau_{com} = fR = I\alpha$

$\omega = \alpha t = \frac{fR t}{I}$

Assuming friction is sufficiently large to make it pure roll.

$v = \omega R$

$v_0 - \frac{ft}{m} = \frac{fR^2 t}{I} = \frac{f t}{\beta m}$

$v_0 = \frac{ft}{m}\left(1 + \frac{1}{\beta}\right)$

$v = v_0 - \frac{ft}{m} = v_0 - \frac{v_0 \beta}{1+\beta} = \frac{v_0}{1+\beta}$ ✓ 7

$\omega = \frac{\beta m v_0 R}{(1+\beta)\beta m R^2} = \frac{v_0}{(1+\beta)R}$.

τ about A - point of contact.

$\tau_A = 0$ $\because$ All 3 force pass through 'A'.

$\therefore$ Angular momentum about A is conserved.

$L_{i_z} = L_{f_z}$

$$m v_0 R = m v R + I\omega$$

In pure rolling $v = R\omega$

$$m v_0 R = m v R + \beta m R^2 \frac{v}{R}$$ 5

$$v_0 = v + \beta v$$

$$v = \frac{v_0}{1+\beta}$$

$$\omega = \frac{v_0}{(1+\beta)R}.$$

$$KE_i = \frac{1}{2} m v_0^2$$

$$KE_f = \frac{1}{2} m v^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} m v^2 + \frac{1}{2}\beta m R^2 \frac{v^2}{R^2} = \frac{1}{2} m (1+\beta) v^2 = \frac{1}{2}\frac{m v_0^2}{(1+\beta)}$$

$$\Delta KE = \frac{1}{2} m v_0^2 \left(1 - \frac{1}{1+\beta}\right) = \frac{1}{2} m v_0^2 \left(\frac{\beta}{1+\beta}\right).$$ 3

Till it attains pure rolling, the bottommost point is never at rest.

$\therefore$ Kinetic friction.

$$v = v_0 - \frac{f t}{m}$$

$$\omega = \frac{f R t}{I} = \frac{f t}{\beta m R}$$

$$f = \mu m g$$

$$v = \frac{v_0}{1+\beta} = v_0 - \mu g t$$

$$\mu g t = v_0 \left(1 - \frac{1}{1+\beta}\right) = \frac{\beta v_0}{1+\beta}$$

$$t = \frac{\beta v_0}{(1+\beta)\mu g}.$$

$$s = \frac{v^2 - u^2}{2a} = \frac{V_0^2\left(\frac{1}{(1+\beta)^2} - 1\right)}{-2\mu g} = \frac{V_0^2(\beta^2 + 2\beta)}{2\mu g(1+\beta)^2}$$

7

$$W_1 = \oint (\mu m \vec{g}) \cdot \vec{dr} = \mu m g s = \frac{m V_0^2 (\beta^2 + 2\beta)}{2(1+\beta)^2}$$

$$\theta = \frac{\omega^2 - 0}{2\alpha} = \frac{\frac{V_0^2}{(1+\beta)^2 R^2}}{\frac{2\mu g}{\beta R}} = \frac{V_0^2 \beta}{(1+\beta)^2 R (2\mu g)}$$

$$W_2 = \oint \vec{\tau} \cdot \vec{d\theta} = f R \int d\theta = \frac{\mu m g R V_0^2 \beta}{(1+\beta)^2 R (2\mu g)}$$

$$W_{Total} = W_1 - W_2$$

$$= \frac{m V_0}{2} \frac{(\beta^2 + 2\beta - \beta)}{(1+\beta)^2}$$

8

$$= \frac{m v_0^2 \beta}{2(1+\beta)}$$