Max. Time: 90 min.   Day & Date: Monday, March 06, 2017   Max. Marks: 105

Note: Define events and random variables as and when required. Answer each sub-part of a question together.

1 (a) A letter is known to have come either from the city *TATANAGAR* or from the city *KOLKATA* with equal probabilities. On the envelope of the letter just two consecutive alphabets *TA* are visible. What is the probability that the letter has come from *KOLKATA*?   [10]

(b) If a day is dry, the conditional probability that the next day will also be dry is p. If a day is wet, the conditional probability that the next day will be dry is q. If W_n is the probability that n^{th} day will be dry, then prove that
$$W_n - (p-q)W_{n-1} - q = 0; (n \geq 2).$$
If the first day is sure to be dry and $p = 0.75, q = 0.25$, find W_n.   [16]

2 (a) A shipment of 8 similar microcomputers to a retail outlet contains 3 defectives. If a school makes a random purchase of 2 of these computers, find the probability distribution for number of defectives. Also compute the cumulative distribution function and moment generating function.   [15]

(b) A biased six-faced die is such that the probability that it gives 3 even numbers in 5 throws is twice the probability that it gives 2 even numbers in 5 throws. Let Y be number of sets of 5 throws out of 2500 sets in which no even number appears. Find $E[Y]$.   [12]

3 (a) Let X have Poisson distribution with parameter k. Derive $E[X]$ and $Var[X]$ using definition (without using moment generating function).   [14]

(b) The time of order of shirts from a manufacturer is known to have a gamma distribution with a mean of 20 days and a standard deviation of 10 days. Determine the probability of receiving an order within 5 days of placement date.   [12]

4 (a) Let X be a normal random variable with parameters μ and σ and let $Y = e^X$. (i) Derive the density of Y. (ii) If $\mu = 0.8$ and $\sigma = 0.1$, find $P[2.7 < Y < 3]$.   [14]

(b) A prescription drug made by a pharma company recovers a particular disease 70% of times. A drug regulation agency takes a feedback from 60 random patients using that prescription drug and decides to ban the drug if less than 45 of these patients report the recovery from the particular disease, and warn the company if at least 45 and at most 50 of these patients report the recovery from the particular disease. Find the probability that pharma company will be warned by the drug regulation agency using normal approximation. Justify your approximation.   [12]

*** Please write END at the end of your work in the answer sheet***

Standard normal table:

$z_{0.8009}$ = -0.845	$z_{0.0122}$ = 2.25	$z_{0.9732}$ = -1.93	$z_{0.0019}$ = 2.9	$z_{0.0084}$ = 2.39	$z_{0.7580}$ = -0.70