Problem 1

Part 1 $P \Rightarrow Q$, $\neg Q \Rightarrow \neg P$

P	Q	$P \Rightarrow Q$	$\neg Q$	$\neg P$	$\neg Q \Rightarrow \neg P$	$P \Rightarrow Q \stackrel{?}{=} \neg Q \Rightarrow \neg P$
t	t	t	f	f	t	✓
t	f	f	t	f	f	✓
f	t	t	f	t	t	✓
f	f	t	t	t	t	✓

Part 2 $P \Leftrightarrow \neg Q$, $((P \wedge \neg Q) \vee (\neg P \wedge Q))$

P	Q	$\neg Q$	$P \Leftrightarrow \neg Q$	$\neg P$	$(P \wedge \neg Q)$	$(\neg P \wedge Q)$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
t	t	f	f	f	f	f	f
t	f	t	t	f	t	f	t
f	t	f	t	t	f	t	t
f	f	t	f	t	f	f	f

$$P \Leftrightarrow \neg Q \stackrel{?}{=} ((P \wedge \neg Q) \vee (\neg P \wedge Q))$$

✓
✓
✓
✓

Problem 2 Let S = Smoke, F = Fire, H = Heat

Part 1 (Smoke $\Rightarrow$ Fire) $\Rightarrow$ ($\neg$Smoke $\Rightarrow$ $\neg$Fire)

$$(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)$$

S	F	$(S \Rightarrow F)$	$\neg S$	$\neg F$	$(\neg S \Rightarrow \neg F)$	$(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)$
t	t	t	f	f	t	t
t	f	f	f	t	t	t
f	t	t	t	f	f	f
f	f	t	t	t	t	t

Neither — Not all paths lead to true, so it is not valid, but there are instances where the statement is true, so satisfiable.

Part 2 (Smoke $\Rightarrow$ Fire) $\Rightarrow$ ((Smoke $\vee$ Heat) $\Rightarrow$ Fire)

$$(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$$

S	F	H	$(S \Rightarrow F)$	$(S \vee H)$	$(S \vee H) \Rightarrow F$	$(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$
t	t	t	t	t	t	t
t	t	f	t	t	t	t
t	f	t	f	t	f	t
t	f	f	f	t	f	t
f	t	t	t	t	t	t
f	t	f	t	f	t	t
f	f	t	t	t	f	f
f	f	f	t	f	t	t

Neither

Not all paths lead to true, so it is not valid, but there are instances where the statement is true, so satisfiable

Problem 2

Part 3 $((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$

$((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$

S	H	F	$(S\wedge H)$	$(S\wedge H) \Rightarrow F$	$(S \Rightarrow F)$	$(H \Rightarrow F)$	$(S\Rightarrow F) \vee (H \Rightarrow F)$	
t	t	t	t	t	t	t	t	t
t	t	f	t	f	f	f	f	t
t	f	t	f	t	t	t	t	t
t	f	f	f	t	f	t	t	t
f	t	t	f	t	t	t	t	t
f	t	f	f	t	t	f	t	t
f	f	t	f	t	t	t	t	t
f	f	f	f	t	t	t	t	t

$\boxed{\text{Valid}}$

Problem 3

part (a)

Assumption :

Immortal = I ; Mortal = $\neg I$

Horned = H

Mythical = Myt

Magical = Mag

Mammal = Mam

1) $Myt \Rightarrow I$ If the unicorn is mythical, then it is immortal.

2) $\neg Myt \Rightarrow (\neg I \wedge Mam)$ if it is not mythical, then it is a mortal mammal.

3) $(I \vee Mam) \Rightarrow H$ if the unicorn is either immortal or a mammal, then it is horned.

4) $H \Rightarrow Mag$ The unicorn is magical if it is horned.

Problem 3

part (b) Convert the knowledge base into CNF

1) Get rid of all connectives for the propositional knowledge base

① $Myt \Rightarrow I$ equivalent to $\boxed{\neg Myt \vee I}$

② $\neg Myt \Rightarrow (\neg I \wedge Mam)$ equivalent to

$\neg\neg Myt \vee (\neg I \wedge Mam)$ equivalent to

$\boxed{Myt \vee (\neg I \wedge Mam)}$

③ $(I \vee Mam) \Rightarrow H$ equivalent to

$\boxed{\neg(I \vee Mam) \vee H}$

④ $H \Rightarrow Mag$ equivalent to

$\boxed{\neg H \vee Mag}$

2) Use Demorgan Laws to push negation inward

① $\boxed{\neg Myt \lor I}$

② $\boxed{Myt \lor (\neg I \land Mam)}$

③ $\neg(I \lor Mam) \lor H$ equivalent to
$\boxed{(\neg I \land \neg Mam) \lor H}$

④ $\boxed{\neg H \lor Mag}$

3) Distribute $\lor$ over $\land$

① $\boxed{\neg Myt \lor I}$

② $Myt \lor (\neg I \land Mam)$ equivalent to
$(\neg I \land Mam) \lor Myt$ equivalent to
$\boxed{(\neg I \lor Myt) \land (Mam \lor Myt)}$

③ $\boxed{(\neg I \lor H) \land (\neg Mam \lor H)}$

④ $\boxed{\neg H \lor Mag}$

Problem 3

Combined together for all knowledge base

$$\boxed{(\neg Myt \lor I) \land (\neg I \lor Myt) \land (Mam \lor Myt) \land (\neg I \lor H) \land (\neg Mam \lor H) \land (\neg H \lor Mag)}$$

part (c) Prove mythical: $\Delta \land \neg \alpha$ is unsatisfiable

Proof by contradiction

1) $\neg Myt \lor I$
2) $(\neg I \lor Myt) \land (Mam \lor Myt)$
3) $(\neg I \lor H) \land (\neg Mam \lor H)$
4) $\neg H \lor Mag$
5) $\neg I \lor Myt$ (extracted from 2)
6) $Mam \lor Myt$ (extracted from 2)
7) $\neg I \lor H$ (extracted from 3)
8) $\neg Mam \lor H$ (extracted from 3)

---

9) $\neg Myt$ (Assumption)
10) $\neg I$ (5 & 9)
11) Mam (6 & 9)
12) H (8 & 11)
13) Mag (4 & 12)

No contradiction

Due to the fact that $\Delta \neg \alpha$ is satisfiable, we cannot use Δ to prove that the unicorn is mythical.

part c) cont. Proved Magical & Horned

Prove magical :

$\Delta \wedge \neg Mag$ is unsatisfiable

1) $\neg Myt \vee I$
2) $(\neg I \vee Myt) \wedge (Mam \vee Myt)$
3) $(\neg I \vee H) \wedge (\neg Mam \vee H)$
4) $\neg H \vee Mag$
5) $\neg I \vee Myt$ (extracted from 2)
6) $Mam \vee Myt$ (extracted from 2)
7) $\neg I \vee H$ (extracted 3)
8) $\neg Mam \vee H$ (extracted 3)

---

9) $\neg Mag$ (Assumption)
10) $\neg H$ (4 & 9)
11) $\neg I$ (7 & 10)
12) $\neg Myt$ (1 & 11)
13) Mam (6 & 12)
14) H (8 & 13)

contradiction

$\therefore$ Proved it is magical

part c) cont.

Prove horned: $\Delta \wedge \neg H$ is unsatisfiable.

1) $\neg Myt \vee I$

2) $(\neg I \vee Myt) \wedge (Mam \vee Myt)$

3) $(\neg I \vee H) \wedge (\neg Mam \vee H)$

4) $\neg H \vee Mag$

5) $\neg I \vee Myt$ (extracted from 2)

6) $Mam \vee Myt$ (extracted from 2)

7) $\neg I \vee H$ (extracted from 3)

8) $\neg Mam \vee H$ (extracted from 3)

---

9) $\neg H$ (Assumption)

10) $\neg I$ (7 & 9)

11) $\neg Myt$ (1 & 10)

12) Mam (6 & 11)

13) H (8 & 12)

contraction

$\therefore$ Proved it is horned

Problem 4

Figure 1

Decomposable: Figure 1 is decomposable because each AND gate in the graph does not share circuit variable. This means that for each AND gate has different circuit variable on either side of the AND gate.

Deterministic: Figure 1 is deterministic because each side of all the OR gate has at most one high circuit input which each OR gate is mutually exclusive.

Smooth: Figure 1 is not smooth because two of the OR gates at the 2nd level does not have the same variables on either side of the OR gate.

1) The OR gate that point to C, AND($\neg D \wedge \neg C$) do not have the same variable on both side.

2) The OR gate that point to A, AND($\neg A \wedge \neg B$) do not have the same variable on both side.

Figure 2 (Problem 4)

Decomposable: Figure 2 is decomposable because each AND gate in the graph does not share circuit variable. This means that for each AND gate has different circuit variable on either side of the AND gate.

Deterministic: Figure 2 is not deterministic because one of the OR gate in the graph does not have at most one high circuit input. The OR gate is not mutually exclusive

1) The OR gate that point to ($\neg A \wedge B$), ($\neg A \wedge B$), each side of the OR gate have $B = 1$, more than one high circuit input from the OR gate.

Smooth: Figure 2 is smooth because the OR gates in the graph have the same variables on either side of the OR gates.

Problem 5

Part (a) $(\neg A \wedge B) \vee (\neg B \wedge A)$

Models $= \{\neg A, B\}, \{\neg B, A\}$

$= w(\neg A)w(B) + w(\neg B)w(A)$

$= (0.8)(0.4) + (0.6)(0.2)$

$= 0.32 + 0.12 = \boxed{0.44}$

Part (b)

$(\neg A \wedge B) \vee (\neg B \wedge A)$

$(\neg A * B) + (\neg B * A)$

$= (0.8)(0.4) + (0.6)(0.2) = \boxed{0.44}$

The count on the root is the same with the Weighted Model Count for the formula.

Problem 5

Part (c)

Bottom 1st OR Gate

$(\neg A \wedge B) \vee (\neg B \wedge A)$

$= (0.8)(0.4) + (0.6)(0.2) = 0.32 + 0.12 = 0.44$

Bottom 2nd OR Gate

$(C \wedge D) \vee (\neg D \wedge \neg C)$

$= (0.6)(0.8) + (0.2)(0.4) = 0.48 + 0.08 = 0.56$

Bottom 3rd OR Gate

$(\neg A \wedge \neg B) \vee (B \wedge A)$

$= (0.8)(0.6) + (0.4)(0.2) = 0.48 + 0.08 = 0.56$

Bottom 4th OR Gate

$(C \wedge \neg D) \vee (D \wedge \neg C)$

$= (0.6)(0.2) + (0.8)(0.4) = 0.12 + 0.32 = 0.44$

$(((\neg A \wedge B) \vee (\neg B \wedge A)) \wedge ((C \wedge D) \vee (\neg D \wedge \neg C)) \vee$

$((\neg A \wedge \neg B \vee (B \wedge A)) \wedge ((C \wedge \neg D) \vee (D \wedge \neg C)))$

$= (0.44)(0.56) + (0.56)(0.44) = 0.2464 + 0.2464$

$= \boxed{0.4928}$