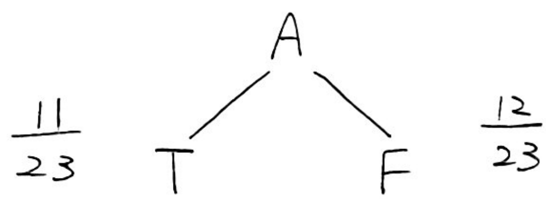
Problem 1

We can choose A, B, C for the root

A Case



Yes-World	X_1, X_2	$\frac{7}{11}$	X_5, X_7	$\frac{3}{12} = \frac{1}{4}$
No-World	X_3, X_4	$\frac{4}{11}$	X_6, X_8	$\frac{9}{12} = \frac{3}{4}$

$$ENT(T\_A) = -\frac{7}{11}\log_2\left(\frac{7}{11}\right) - \frac{4}{11}\times\log_2\left(\frac{4}{11}\right) = 0.9457$$

$$ENT(F\_A) = -\frac{1}{4}\log_2\left(\frac{1}{4}\right) - \frac{3}{4}\log_2\left(\frac{3}{4}\right) = 0.8113$$

The entropy of the state after splitting A model:

$$ENT(A) = \frac{11}{23}\times 0.9457 + \frac{12}{23}\times 0.8113$$

$$= \boxed{0.8756}$$

B case

B

$\frac{14}{23}$ T | F $\frac{9}{23}$

Yes-World	X_1, X_2, X_5	$\frac{4}{7}$	X_7	$\frac{2}{9}$
No-World	X_6	$\frac{3}{7}$	X_3, X_4, X_8	$\frac{7}{9}$

$$ENT(T\_B) = -\frac{4}{7}\log_2\left(\frac{4}{7}\right) - \frac{3}{7}\log_2\left(\frac{3}{7}\right) = 0.9852$$

$$ENT(F\_B) = -\frac{2}{9}\log_2\left(\frac{2}{9}\right) - \frac{7}{9}\log_2\left(\frac{7}{9}\right) = 0.7642$$

$$ENT(B) = \frac{14}{23}(0.9852) + \frac{9}{23}(0.7642) = \boxed{0.8987}$$

C case

C

$\frac{7}{23}$ T | F $\frac{16}{23}$

Yes-World	X_1, X_5, X_7	$\frac{4}{7}$	X_2	$\frac{3}{8} = \frac{\cancel{6}\,3}{\cancel{16}\,8}$
No-World	X_3	$\frac{3}{7}$	X_4, X_6, X_8	$\frac{5}{8} = \frac{\cancel{10}\,5}{\cancel{16}\,8}$

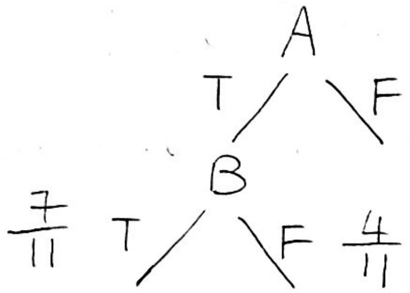
$$ENT(T\_B) = -\frac{4}{7}\log_2\left(\frac{4}{7}\right) - \frac{3}{7}\log_2\left(\frac{3}{7}\right) = 0.9852$$

$$ENT(F\_B) = -\frac{3}{8}\log_2\left(\frac{3}{8}\right) - \frac{5}{8}\log_2\left(\frac{5}{8}\right) = 0.9544$$

$$ENT(C) = \frac{7}{23}(0.9852) + \frac{16}{23}(0.9544) = \boxed{0.9638}$$

Then we pick A as the root node since it has the lowest entropy

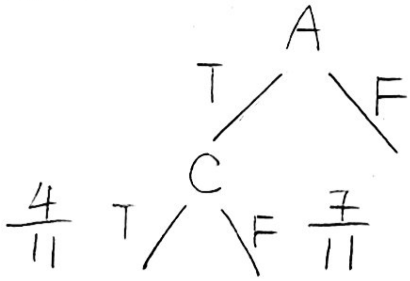
If we choose B for the next layer (A=T):



Yes-World	X_1, X_2	$\frac{7}{7}=1$	ϕ	0
No-World	ϕ	0	X_3, X_4	$\frac{4}{4}=1$

$ENT(B \mid A=T) = \boxed{0}$

If we choose C for the next layer (A=T):



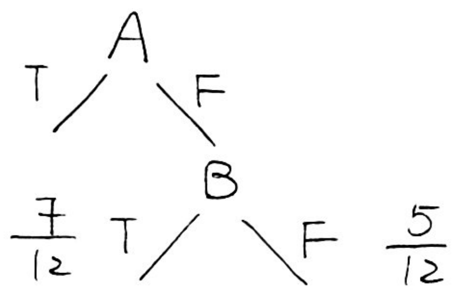
Yes-World	X_1	$\frac{1}{4}$	X_2	$\frac{6}{7}$
No-World	X_3	$\frac{3}{4}$	X_4	$\frac{1}{7}$

$$ENT((C|A)=T) = -\frac{1}{4}\log_2\left(\frac{1}{4}\right) - \frac{3}{4}\log_2\left(\frac{3}{4}\right) = 0.81127$$

$$ENT((C|A)=F) = -\frac{6}{7}\log_2\left(\frac{6}{7}\right) - \frac{1}{7}\log_2\left(\frac{1}{7}\right) = 0.59167$$

$$ENT(C|A) = \left(\frac{4}{11}\right)(0.81127) + \left(\frac{7}{11}\right)(0.59167) = \boxed{0.6715}$$

If we choose B for the next layer (A=F):



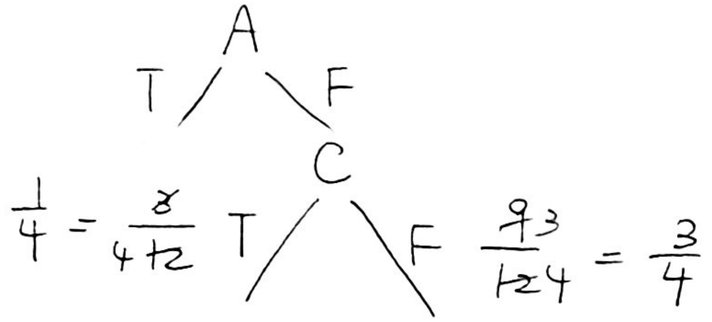
Yes-World	X_5	$\frac{1}{7}$	X_7	$\frac{2}{5}$
No-World	X_6	$\frac{6}{7}$	X_8	$\frac{3}{5}$

$$ENT((B|A)=T) = -\frac{1}{7}\log_2\left(\frac{1}{7}\right) - \frac{6}{7}\log_2\left(\frac{6}{7}\right) = 0.59167$$

$$ENT((B|A)=F) = -\frac{2}{5}\log_2\left(\frac{2}{5}\right) - \frac{3}{5}\log_2\left(\frac{3}{5}\right) = 0.97095$$

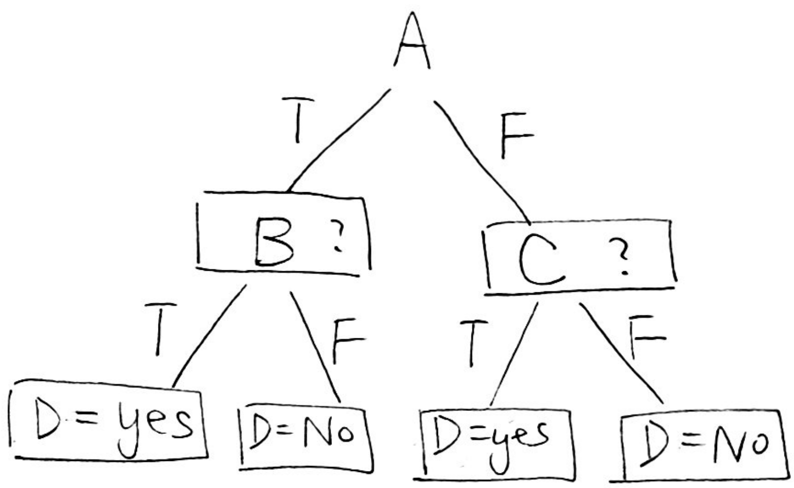
$$ENT(B|A) = \left(\frac{7}{12}\right)(0.59167) + \left(\frac{5}{12}\right)(0.97095) = \boxed{0.74970}$$

If we choose C for the next layer (A = F)



Yes - World	X_5, X_7	$\frac{3}{3} = 1$	ϕ	$\frac{0}{9} = 0$
No - World	ϕ	$\frac{0}{3}$	X_6, X_8	$\frac{9}{9} = 1$

$ENT(C \mid A = F) = \boxed{0}$



Problem 2

$(A \vee \neg B) \oplus (\neg C \vee D)$

Formula: $p \oplus q = (p \wedge \neg q) \vee (\neg p \wedge q)$

$((A \vee \neg B) \wedge \neg(\neg C \vee D)) \vee (\neg(A \vee \neg B) \wedge (\neg C \vee D))$

$((A \vee \neg B) \wedge (C \wedge \neg D)) \vee ((\neg A \wedge B) \wedge (\neg C \vee D))$

$((A \vee \neg B) \wedge C \wedge \neg D) \vee (\neg A \wedge B \wedge (\neg C \vee D))$

$((A \wedge C \wedge \neg D) \vee (\neg B \wedge C \wedge \neg D)) \vee ((\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge B \wedge D))$

Finally 4 clauses

$(A \wedge C \wedge \neg D) \vee (\neg B \wedge C \wedge \neg D) \vee (\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge B \wedge D)$

1 1 -1	-1 1 -1	-1 1 -1	-1 1 1
# < 2	# < 1	# < 1	# < 2
1.5	0.5	0.5	1.5

All four clauses weighted equally, edges pointing to the output node.

The Neural Network