CS CM 182 Homework 2

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I completed this written part of the homework, lab report, or exam entirely on my own.

E2.3: Variables, state variables & parameters (Examples 2.1 - 2.17)

Examples 2.1 (Einstein's mass energy-equivalence principle)

Variable : E (energy)

Parameters : M (mass), c (speed of light)

State Variables : none

Examples 2.2 (Newton's 2nd law (mechanistic theory))

Variable : F (force), v (velocity), a (acceleration), x (distance)

Parameter : x, v

State Variables : M (mass)

Examples 2.3 (Ohm's law (mechanistic theory))

Variable : V (voltage), i (electrical current), q (charge)

Parameter : R (resistance)

State Variables : q

Examples 2.4 (Malthus law - simple population growth (All Theory))

Variable : N (number of organism in the population)

Parameter : c (fractional growth rate)

State Variables : N

Examples 2.5 (Malthusian growth limited by nutrient supply: logistic population growth)

Variable : N (population)

Parameter : n (concentration nutrient in the cell culture medium)

State Variables : k1 (proportionality constant), k2 (proportionality constant), A (constant)

Examples 2.6 (Gompertz law of mortality)

Variables : N (population), c (population growth rate)

Parameter : α (constant frictional retardation rate)

State Variables : none

Examples 2.7 (Nonlinear terms and models)

Variables : y (dependent variable), u (input variable)

Parameters : none

State Variables : y (state variable)

Examples 2.8 (Newton's law as two coupled ODEs in state variable form)

Variables : F (force), v (velocity), a (acceleration), x (distance)

Parameter : M (mass)

State Variables : x , v

Examples 2.9 (Nonlinear state variables ODE model with NL parabolic & hyperbolic terms)

Variables : x1 (state variable), x2 (state variable), y (output measurement), u (input)

Parameters : c1, c2, c3, c4, c5, c6

State Variables : x1, x2

Examples 2.10 (Free (homogenous), forced & total responses)

Variables : x

Parameters : A (scalar constant), b (scalar constant)

State Variables : x

Examples 2.11 (Transient & steady state response)

Variables : y (dependent variable)

Parameters : e

State Variables : y (state variable)

Examples 2.12 (Impulse bolus input response)

Variables : q (state variable)

Parameters : Q0 (initial condition), k

State Variables : q

Examples 2.13 (Superposition)

Variables : y (output), u1 (input), u2 (input)

Parameters : none

State Variables : none

Examples 2.14 (Simple elimination dynamics from first principle)

Variables : q (dependent variables), ER (time-dependent elimination rate of X)

Parameters : k

State Variables : q (mass per time units)

Examples 2.15 (Simple gene expression dynamics, ODEs based from the 3-step modeling diagram)

Variables : mRNA, P

Parameters : DNA (constant input), k1, k2, k3, k4 (all of them are rate constants)

State Variables : mRNA, P (protein)

Examples 2.16 (A simple multiscale model, approximated and solved)

Variables : q

Parameters : k (rate of fall k), e

State Variables : r (multiscale model approximate solution)

Examples 2.17 (The immune system)

Variables : all the biochemicals in the diagram such as T cells, dendrite cell/ B-cell, antigen, MHC molecules, helper T cells, macrophages, neutrophils, lymphocytes, cytotoxic cells, cytokines, MHC-antigen complex, track down cell, infected cell,

Parameters : T cell growth rate, infected cell elimination speed, secretion speed of cytokines through T-cells

State Variable : total number of infected cells in the immune system

E2.4 : State variables and memory

(a) The memory is stored in $q(0)$, the initial condition
(b) At some time $t > 0$, the memory which is initial conditions. Different initial conditions for order differential equations yield uniquely different future motions. Order differential equations have different solution trajectories for $t > 0$, starting from different initial conditions, different starting states.

E 2.5 : Unique model solutions

In order to obtain unique solutions to ODE equations, we need initial conditions, initial state. To solve ODE uniquely, initial conditions must be explicitly supplied for each and every integrator. This means that the initial value of each state variable must be known by the user and specified in the program.

In terms of the equation, we need to know the x1, x2, x3, x4, ….., xn at time = 0

E2.6 Research Problem - model of elastic tissue based on Newton's second law

Introduction :

The purpose of the model is intended for surgical training purposes by attempting to model the process of "deformation of the soft tissue" in order to replicate the actual environment during the surgery. There are several advantages for the viscoelastic mass spring dampers model of soft tissue. First, it saves a lot of training cost for surgeries. Second, it prevents the possibility of human errors during the actual surgery by practicing with the simulated model. Third, surgeons are prepared to handle different medical emergency situations because the model is able to simulate both tearing and bleeding scenarios of the soft tissues (Xu, Lu, Liu 2018).

Picture :

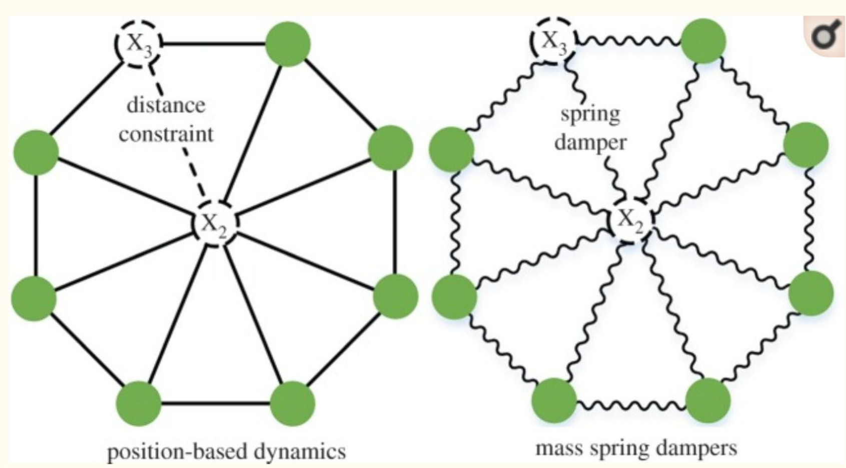

Figure 1.

The same strain limit effects of separate description in PBD by distance constraint and in MSD by a spring damper.

Description of the model :

Each node in the diagram represents masses and there exists a constraint function between nodes. The input to the constraint function is the position of the nodes which are represented as variables in this model. A "constraint iteration projection" is used to modify the position of the node. The deformation process of the soft tissue is represented as a constraint function optimization problem in this model. Based on the description of the model, both distance and the angle from the diagram are used to model as cloth while the strain energy constraint is modelled as the soft bodies. One of the challenges of this model is it is extremely difficult to the properties of soft tissue which are viscous, elastic and nonlinearity because the characteristics are time-dependent (Xu, Lu, Liu 2018).

Math explanation directly from the research paper:

The model is simulated by mass points, constraints and springs and the calculation is to mark the mass point position as times goes on. The total load of force is used to control each mass point's position. In the formula, x is the column vector for the location of the mass point while x with two dots above is the derivative of the location of the mass point with respect to time.
M is represented as the point masses's diagonal matrix. The variable $f_{constraint}$ is represented as the constraint forces column vector, the variable f_{spring} is represented as the spring forces column vector and the variable $f_{external}$ is represented as the external forces column vector.
The equation shown below is based on Newton's second law of motion.

$$\mathbf{f}_{\text{total}} = \mathbf{f}_{\text{constraint}} + \mathbf{f}_{\text{spring}} + \mathbf{f}_{\text{external}} = \mathbf{M}\ddot{\mathbf{x}},$$

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{f}_{\text{constraint}} = \begin{bmatrix} f_{c1} & f_{c2} & \cdots & f_{cm} \end{bmatrix}^{\mathrm{T}}$$

$$f_{spring} = [f_{s1}\ f_{s2}\ \cdots f_{sm}]^{T}$$

$$f_{external} = [f_{e1}\ f_{e2}\ \cdots f_{em}]^{T}$$

E2.7 Dynamic versus non-dynamic system models

(a) The gene expression model is a dynamic system model. It is because the system depends on states, memory such as the initial conditions of mRNA and protein which are mRNA(0) and P(0). A Dynamical system model is first of all a model of a system, with inputs and outputs, and equations describing system motion in space and time. Dynamic systems have memory which means the system has a system embedded in it about information at the present and past

(b) The gene expression model is linear and time invariant. It is because all the terms in the ODEs in the model are linear. Based on principles of superposition, if individual terms are linear then the summation of the individual terms must be linear also. The reason that it is time-invariant is because parameters are constants and not dependent on time t. The parameters in the model are k1, k2, k3, k4.

Citations

Xu, Lang et al. "Integrating viscoelastic mass spring dampers into position-based dynamics to simulate soft tissue deformation in real time." *Royal Society open science* vol. 5,2 171587. 14 Feb. 2018, doi:10.1098/rsos.171587