CS CM 182 Homework 9

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I completed this written part of the homework, lab report, or exam entirely on my own.

Problem 1 - SI for linear models

Problem 1 SI for linear model

a) Derive the TF for the model given

$$\dot{x}(t) = -p_1 x(t) + p_2 u(t)$$

$$y(t) = p_3 x(t)$$

Show $H(s, p_1, p_2, p_3) = \dfrac{p_2 p_3}{(s+p_1)} = \dfrac{Y(s)}{U(s)}$ $\quad p_i > 0$

$$\frac{dx(t)}{dt} = -p_1 x(t) + p_2 u(t)$$

$$\frac{dx(t)}{dt} + p_1 x(t) = p_2 u(t)$$

$$y(t) = p_3 x(t)$$

$$x(t) = \frac{y(t)}{p_3}$$

$$\frac{dx(t)}{dt} = \frac{dy(t)}{dt}\frac{1}{p_3} \quad \text{(plug into } \dot{x}(t)\text{)}$$

$$\frac{dy(t)}{dt}\frac{1}{p_3} + p_1\left(\frac{y(t)}{p_3}\right) = p_2 u(t)$$

$$\frac{1}{p_3}\frac{dy(t)}{dt} + \frac{p_1}{p_3} y(t) = p_2 u(t)$$

$$\frac{1}{p_3}\mathcal{L}\left[\frac{dy(t)}{dt}\right] + \frac{p_1}{p_3}\mathcal{L}[y(t)] = p_2 \mathcal{L}[u(t)]$$

$$\frac{1}{p_3} s Y(s) + \frac{p_1}{p_3} Y(s) = p_2 U(s)$$

$$\frac{Y(s)}{p_3}(s+p_1) = p_2 U(s)$$

$$\frac{Y(s)}{p_3} = \frac{p_2 U(s)}{(s+p_1)}$$

$$\frac{Y(s)}{U(s)} = \frac{p_2 p_3}{(s+p_1)} = H(s, p_1, p_2, p_3)$$

all $p_i > 0$

Part (b) Evaluate the SI of this model from the TF

partb) continue

$$\dot{x}(t) = -p_1 x(t) + p_2 u(t) \qquad y(t) = p_3 x(t)$$

$$y(t) = p_3 \int_0^t \dot{x}(t)\,dt$$

$$y(t) = p_3 \int_0^t -p_1 x(t) + p_2 u(t)\,dt$$

$$\frac{dx(t)}{dt} = -p_1 x(t) + p_2 u(t) \quad \text{defined over } (t_0, t_{max})$$

$$(t_0, t_{max}) = (\tau, t)$$

$$\frac{dx(\tau)}{d\tau} = (t-\tau)(-p_1 x(\tau) + p_2 u(\tau))$$

$$\frac{dx(\tau)}{d\tau} = -(t-\tau)p_1 x(\tau) + (t-\tau)p_2 u(\tau)$$

$$y(t) = p_3 \int_0^t -(t-\tau)p_1 x(\tau) + (t-\tau)p_2 u(\tau)\,d\tau$$

$$y(t) = p_2 p_3 \int_0^t e^{-p_1(t-\tau)} u(\tau)\,d\tau$$

$$y(t) = p_2 p_3\, e^{-p_1 t}$$

unknown proportionality constant relate bioassay result to the true drug concentration in blood.

$x(t) \equiv$ mass of a drug in blood

$y(t) \equiv$ measured drug concentration in blood

$p_1 \equiv$ fractional rate of elimination of the drug from blood

$p_2 \equiv$ unknown constant fraction of drug entering blood from the IP cavity

Part (c) - Which individual or combinations of parameters are SI?

part (c)

A semi-logarithmic plot of the data $y(t) = p_2 p_3 e^{-p_1 t}$ is a straight line, yielding the coefficient $A \equiv p_2 p_3$ from the intercept and the exponent $\lambda \equiv -p_1$ from the slope.
Thus, only p_1 and the product $p_2 p_3$ can be determined are SI and not p_2 or p_3 individually. The model is unidentifiable. For any $u(t)$, it is because neither p_2 nor p_3 can be separated from the product $p_2 p_3$ in the solution $y(t)$ for any input u.

Problem 2 - Sensitivity ODEs

Problem 2

$$v = \frac{dx}{da}$$

$$\dot{v} = \frac{dv}{dt} = \frac{d}{dt}\frac{dx}{da} = \frac{dx}{dt} \cdot \frac{d}{da} = \frac{d}{da} \times \dot{x} = \frac{d\dot{x}}{da}$$

$$\dot{v} = \frac{d}{da}(-ax(t,a) + u(t))$$

$$\dot{v} = -(x(t,a) + av) + 0$$

$$\dot{v} = -(av + x) + 0$$

$$\boxed{\dot{v} = -av - x}$$

$$\dot{v} = \left(\frac{df}{dx}\right)v + \frac{df}{dp}$$

$$\dot{v} = \frac{d(-ax+u)}{dx}v + \frac{d(-ax+u)}{da}$$

$$\dot{v} = \frac{d(-ax+u)}{dx}\,v + \frac{d(-ax+u)}{da}$$

$$\dot{v} = (-a)v + (-x) = \boxed{-av - x}$$

Problem 3 - App for SI analysis

Part (a) - 1 compartment model with M-M elimination and show that all 3 parameters are uniquely SI

COMBOS: Web App for Finding Identifiable Parameter Combinations in...

ODEs: *dx/dt = f(x,p,u)*, with Outputs: *y = g(x,p)* Parameters *p* and *f, x, p, y* and *u* vectors

Structurally Identifiable Parameters & Parameter Combinations

All Solutions
A is uniquely identifiable
B is uniquely identifiable
V1 is uniquely identifiable
COMBOS Runtime = 0.37 seconds

Model Entered

dx[1](t)/dt = -(A*x[1](t)) / (B + x[1](t))+u[1](t)
y[1](t) = x[1](t)/V1

Model in Copy/Paste Format

dx1/dt = -(A*x1) / (B + x1)+u1; y1 = x1/V1;

Mapped Parameters

A	p1
B	p2
V1	p3

(Process another model with COMBOS!)

Part (b) 3-compartment model for SI analysis

Problem 3 b)

First, list out the equations

$$\dot{q}_2 = u_2 - k_{12} q_2 + k_{21} q_1$$

$$\dot{q}_1 = u_1 - k_{01} q_1 + k_{12} q_2 - k_{21} q_1 - k_{31} q_1 + k_{13} q_3$$

$$\dot{q}_3 = k_{31} q_1 - k_{13} q_3$$

$$y = q_1$$

Parameters p and I, x, p, y and u vectors

Structurally Identifiable Parameters & Parameter Combinations

All Solutions
k1,2 is uniquely identifiable
k2,1 is uniquely identifiable
k0,1 is uniquely identifiable
k3,1 is uniquely identifiable
k1,3 is uniquely identifiable
COMBOS Runtime = 0.65 seconds

Model Entered

dx[1](t)/dt = u[2](t)-(k1,2*x[2](t))+(k2,1*x[1](t))
dx[2](t)/dt = u[1](t)-(k0,1*x[1](t))+(k1,2*x[2](t))-(k2,1*x[1](t))-(k3,1*x[1](t))+(k1,3*x[3](t))
dx[3](t)/dt = (k3,1*x[1](t)) - (k1,3*x[3](t))
y[1](t) = x[1](t)

Model in Copy/Paste Format

dx1/dt = u2-(k1,2*x2)+(k2,1*x1); dx2/dt = u1-(k0,1*x1)+(k1,2*x2)-(k2,1*x1)-(k3,1*x1)+(k1,3*x3); dx3/dt = (k3,1*x1) - (k1,3*x3); y1 = x1;

Mapped Parameters

k1,2	p1
k2,1	p2
k0,1	p3
k3,1	p4
k1,3	p5

(Process another model with COMBOS!)