CS CM 182 Homework 7

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I completed this written part of the homework, lab report, or exam entirely on my own.

Problem 1

Problem 1: TF

Given $\frac{d^3y}{dt^3} + 12\frac{d^2y}{dt^2} + 22\frac{dy}{dt} + 20y = 20u$

$$s^3Y + 12s^2Y + 22sY + 20Y = 20U$$

$$(s^3 + 12s^2 + 22s + 20)Y = 20U$$

$$\boxed{H(s) = \frac{Y(s)}{U(s)} = \frac{20}{s^3 + 12s^2 + 22s + 20}}$$

Problem 2

Problem 2 Partial Fraction Expansion

$$Y(s) = \frac{12s(s+4)}{(s+2)(s+3)}$$

Let $m = n = 2$, $b_2 = \frac{b_m}{a_n} = \frac{12}{1} = 12$

$$Y(s) = b_2 + \frac{C_1}{s+2} + \frac{C_2}{s+3} = 12 + \frac{C_1}{s+2} + \frac{C_2}{s+3}$$

$$C_1 = (s+2)Y(s)\Big|_{s=-2} = \frac{\cancel{(s+2)}\,12s(s+4)}{\cancel{(s+2)}(s+3)}\Bigg|_{s=-2}$$

$$= \frac{12s^2+48s}{s+3}\Bigg|_{s=-2} = -48$$

$$C_2 = (s+3)Y(s)\Big|_{s=-3} = \frac{\cancel{(s+3)}\,12s(s+4)}{(s+2)\cancel{(s+3)}}\Bigg|_{s=-3}$$

$$= \frac{12s^2+48s}{s+2}\Bigg|_{s=-3} = 36$$

$$\boxed{Y(s) = 12 - \frac{48}{s+2} + \frac{36}{s+3}}$$

Problem 3

Problem 3 ODE solution by LT

$$Y(s) = \frac{12s(s+4)}{(s+2)(s+3)} = \frac{12s^2+48s}{s^2+5s+6}$$

Long division to separate the constant

$$\begin{array}{r} 12 \\ s^2+5s+6 \overline{\smash{)}\, 12s^2+48s+0} \\ -\ \underline{12s^2+60s+72} \\ -12s-72 \end{array}$$

According to Problem 2

$$Y(s) = 12 - \frac{48}{s+2} + \frac{36}{s+3}$$

Based from the table of LT-ILT pairs, apply the lapalace transform

$$\boxed{y(t) = 12\delta(t) - 48e^{-2t} + 36e^{-3t}} \quad \text{for } t > 0$$

Problem 4

Part(a)

Problem 4 Time - delay transit delay modeling using LTs

part(a)

when $i=1$ case for $\dot{q}_1$

$$\dot{q}_1 = u - kq = D\,\delta(t) - kq_1$$

$$L(\dot{q}_1) = L\,(D \times \delta(t) - kq_1)$$

$$sQ_1(s) = D - kQ_1(s)$$

$$\therefore Q_1(s) = \frac{D}{s+k} \quad \& \quad q_1(t) = De^{-kt}$$

when $i=2$ case for $\dot{q}_2$

$$\dot{q}_2 = k(q_1 - q_2) = k\,(De^{-kt} - q_2)$$

$$L(\dot{q}_2) = L(k\,(De^{-kt} - q_2))$$

$$sQ_2(s) = k\left(\frac{D}{s+k} - Q_2(s)\right)$$

$$= \frac{kD}{s+k} - kQ_2(s)$$

$$\therefore Q_2(s) = \frac{kD}{(s+k)^2} \quad \& \quad q_2(t) = ktDe^{-kt}$$

Part (a) continue

when $i = 3$ case for $\dot{q}_3$

$$\dot{q}_3 = k(q_2 - q_3) = k(kDte^{-kt} - q_3)$$

$$L(\dot{q}_3) = L(k(kDte^{-kt} - q_3))$$

$$sQ_3(s) = k\left(\frac{kD}{(s+k)^2} - Q_3(s)\right)$$

$$= \frac{k^2 D}{(s+k)^2} - kQ_3(s)$$

$$\therefore\ Q_3(s) = \frac{k^2 D}{(s+k)^3} \quad \& \quad q_3(t) = \frac{k^2 D t^2}{2} e^{-kt}$$

$$\boxed{Q_n(s) = \frac{k^{n-1} D}{(s+k)^n}}$$

$$\boxed{q_n(t) = \frac{D(kt)^{n-1}}{(n-1)!} e^{-kt}}$$

Problem 4

Part (b)

Problem 4 part (b)

$$H_{n\text{-DELAY}} = \frac{Q_n}{U} = \frac{k^{n-1}\cancel{U}}{(s+k)^n} \times \frac{1}{\cancel{U}}$$

$$Q_n = \frac{k^{n-1}U}{(s+k)^n}$$

$$\boxed{H_{n\text{-DELAY}} = \frac{k^{n-1}}{(s+k)^n}}$$

Problem 4

Part(c)

Problem 4 part (c)

Find t_p for which $q_n(t)$ peaks (POMTT)

the maximum point

so find t when $\frac{dq_n(t)}{dt} = 0$

From part(a)

$$q_n(t) = \frac{Dose\,(kt)^{n-1}}{(n-1)!} e^{-kt}$$

$$= \frac{Dose}{(n-1)!} (kt)^{n-1} e^{-kt}$$

apply chain rule

$$\frac{dq_n(t)}{dt} = \frac{Dose}{(n-1)!}(n-1)(kt)^{n-2}(k)e^{-kt} - \frac{Dose}{(n-1)!}(kt)^{n-1}e^{-kt}k = 0$$

$$\cancel{\frac{Dose}{(n-1)!}}(n-1)(kt)^{n-2}\cancel{(k)}\cancel{e^{-kt}} = \cancel{\frac{Dose}{(n-1)!}}(kt)^{n-1}\cancel{e^{-kt}}\cancel{k}$$

$$n-1 = \frac{(kt)^{n-1}}{(kt)^{n-2}}$$

$$n-1 = \frac{k^{n-1}t^{n-1}}{k^{n-2}t^{n-2}}$$

$$n-1 = \frac{\cancel{k^{n-1}}\cancel{t^{n-1}}}{\cancel{k^{n-1}}k^{-1}\cancel{t^{n-1}}t^{-1}}$$

$$n-1 = \frac{1}{k^{-1}t^{-1}} = kt$$

$$\boxed{t_p = \frac{n-1}{k}}$$