CS M182: Week 2

MATLAB/Simulink Math Modeling Basics

A Review of Solving ODEs

- Take the simple population growth ODE

$$\frac{dN(t)}{dt} = cN(t)$$

- The growth rate is proportional the to population size
- To find out the solution N(t), representing population size at any time t, we know how to solve this mathematically via integration and algebra:

(1) $$\frac{dN(t)}{dt} = cN(t)$$

(2) $$\int_{N_0}^{N} \frac{dN}{N} = \int_{0}^{t} c\,dt$$

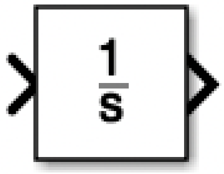
(3) $$\ln\left(\frac{N}{N_0}\right) = ct$$

(4) $$N(t) = N_0 e^{ct}$$

Solving ODEs via simulation

- Simulink makes any ODE easy to solve via an integrator, without working out the mathematics
- Useful when your system is complex or has many state variables!
- Derivative goes into the integrator, analytical solution for the state variable comes out

$$\frac{dN(t)}{dt} = cN(t) \longrightarrow$$



$$\longrightarrow N(t) = N_0 e^{ct}$$

Expression for derivative

Integrator

Scope

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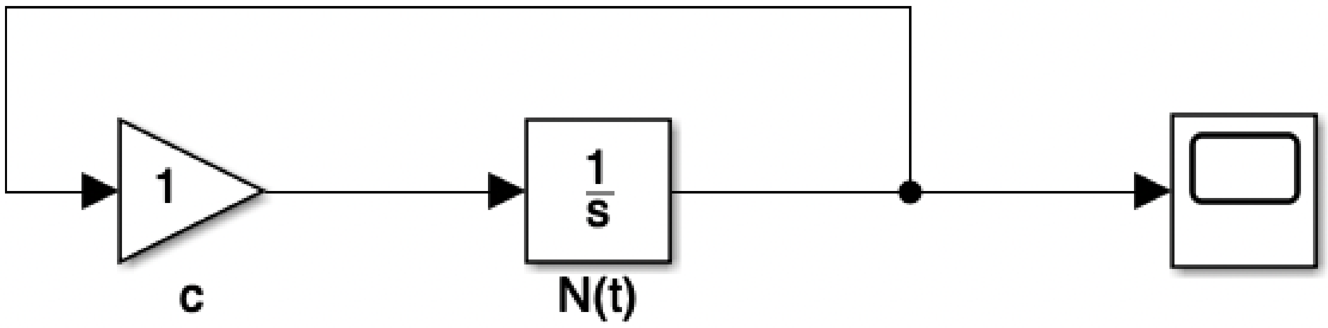


Expression for derivative

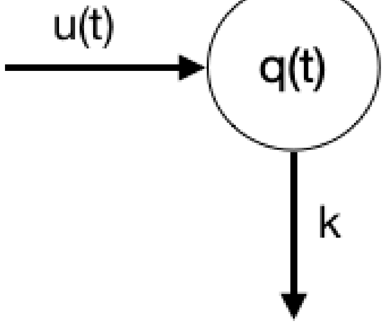
Integrator

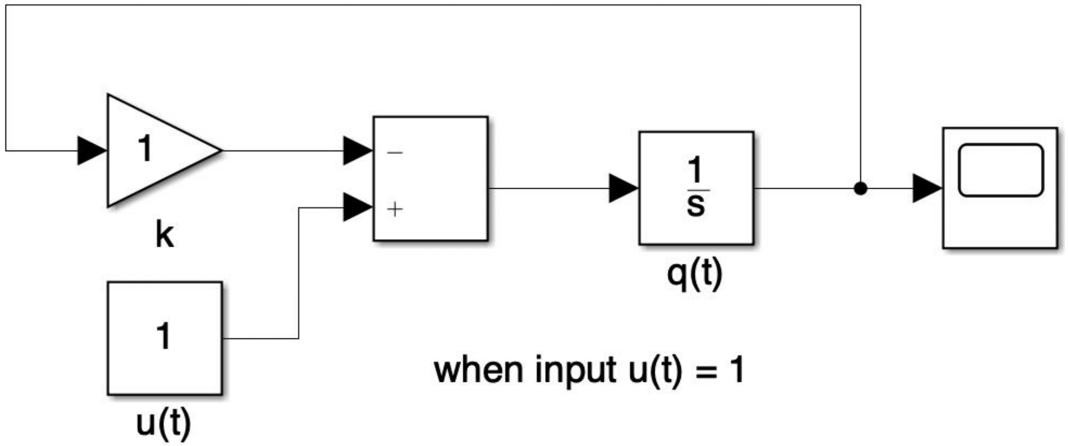
Scope

A general ODE model

- Let's include an input function u(t) to our simple model
- Here is the same ODE represented mathematically, in Simulink and as a 1-compartment model:

$$\frac{dq}{dt} = -kq + u(t)$$





Control signals as input u(t)

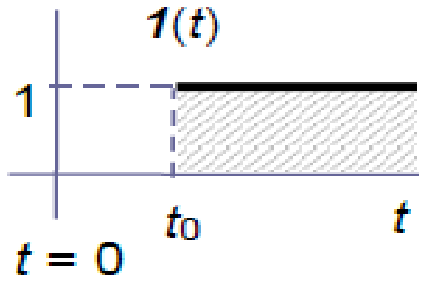
- Unit step function $1(t)$
- Finite pulse $\mathrm{F}(t)$
- Impulse function $\delta(t)$
- Ramp function $\mathrm{r}(t)$

Control signals definitions

- **Unit Step function** :

$1(t - t_0) =$ $\quad 1$ for $t > t_0$

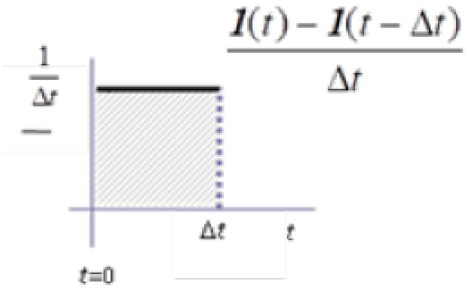
$\qquad\qquad\quad 0$ for $t <= t_0$



- **Finite Pulse** with area 1, width Δt

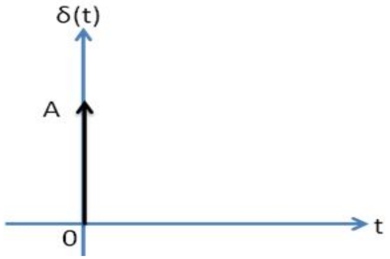
$F_{\Delta t}(t) = 1/\Delta t$ for $0 <= t <= \Delta t$

$\qquad\quad = 0$ for $t > \Delta t$



- **Unit Impulse** $\delta(t)$: $\lim_{\Delta t \to 0} F_{\Delta t}(t)$

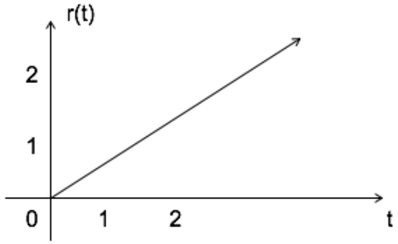
Area under the curve is 1



- **Ramp r(t):**

$r(t) =$ $\quad t$ for $t >= 0$

$\qquad\quad 0$ for $t < 0$



Note on their relationships:

$$\frac{dr(t)}{dt} = 1(t)$$

$$\frac{d1(t)}{dt} = \delta(t)$$

$$\int \delta(t) = 1(t)$$

$$\int 1(t) = r(t)$$

Unit Impulse Response and the Equivalence Property

- Input to the system u(t) = δ(t).
- Output of the system is then referred to as the impulse response.
- Equivalence Property can be represented mathematically as –

- $\frac{dx}{dt} = f(x) + c\delta(t)$
  $x(0) = x_0$

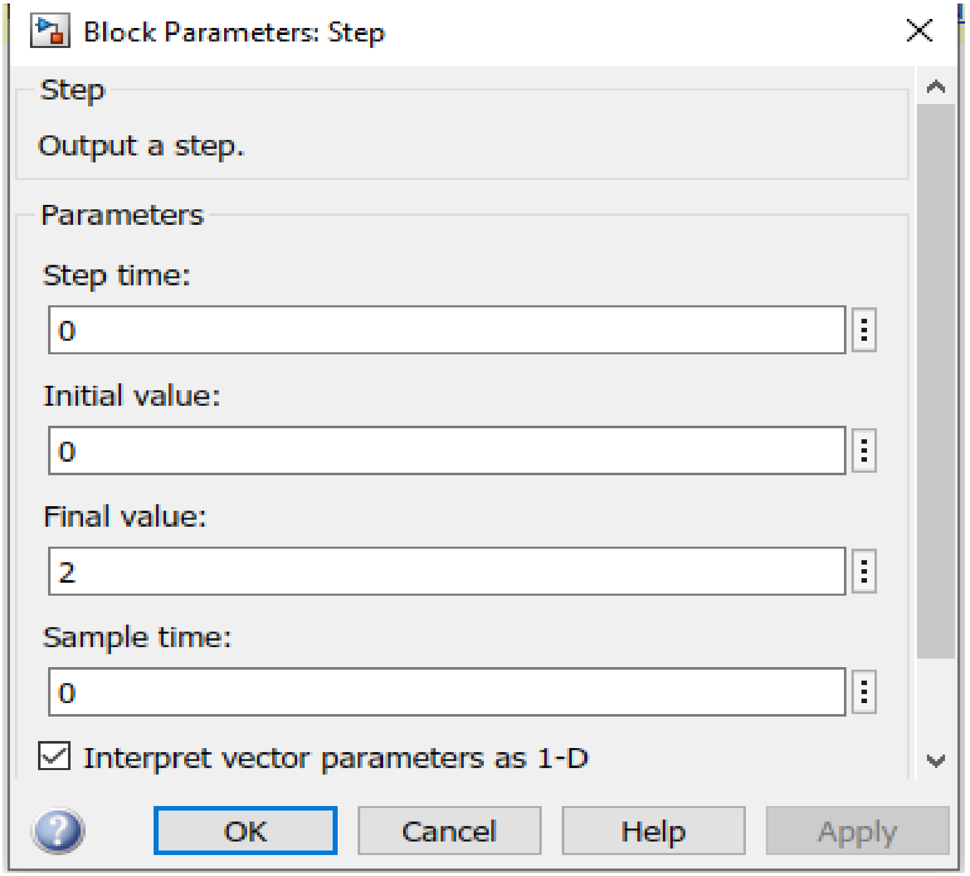
$\Longrightarrow$

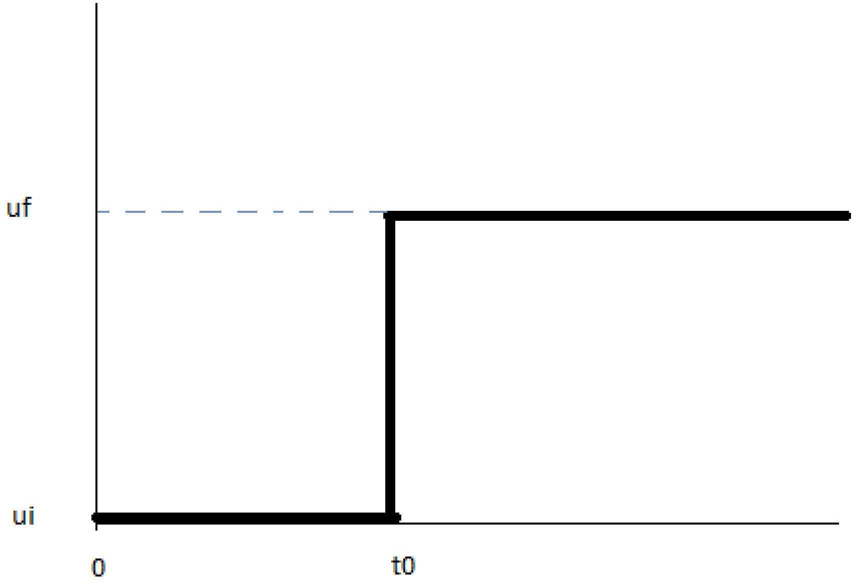
$\frac{dx}{dt} = f(x)$
$x(0) = x_0 + c$

- Impulse input can be converted to initial conditions in Simulink !

Demo

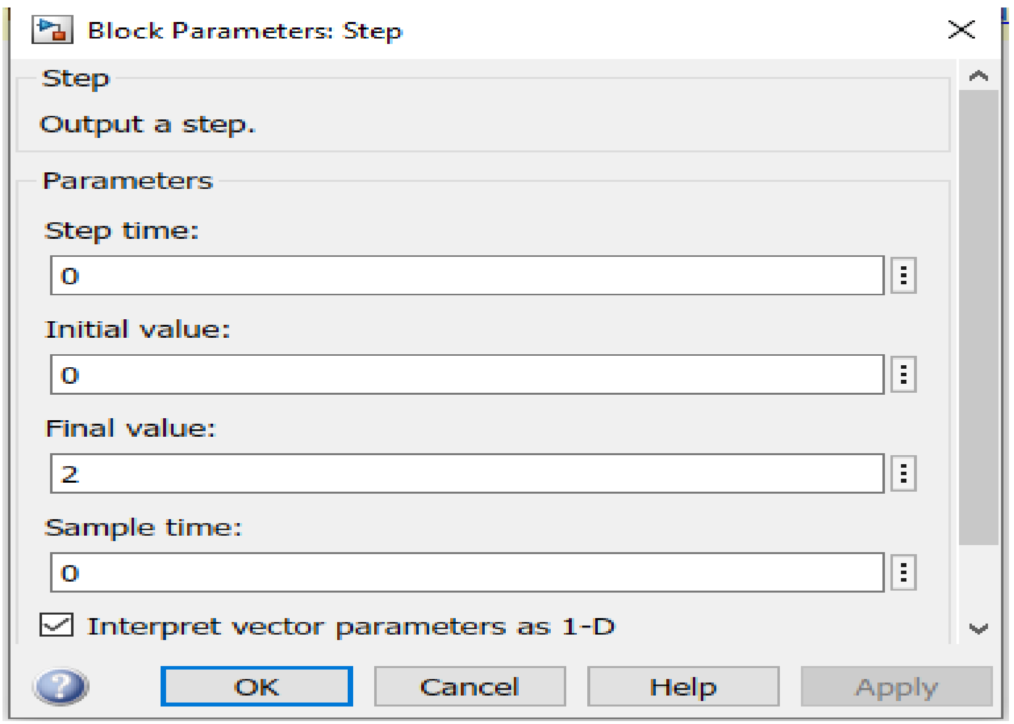
Simulink Step Block

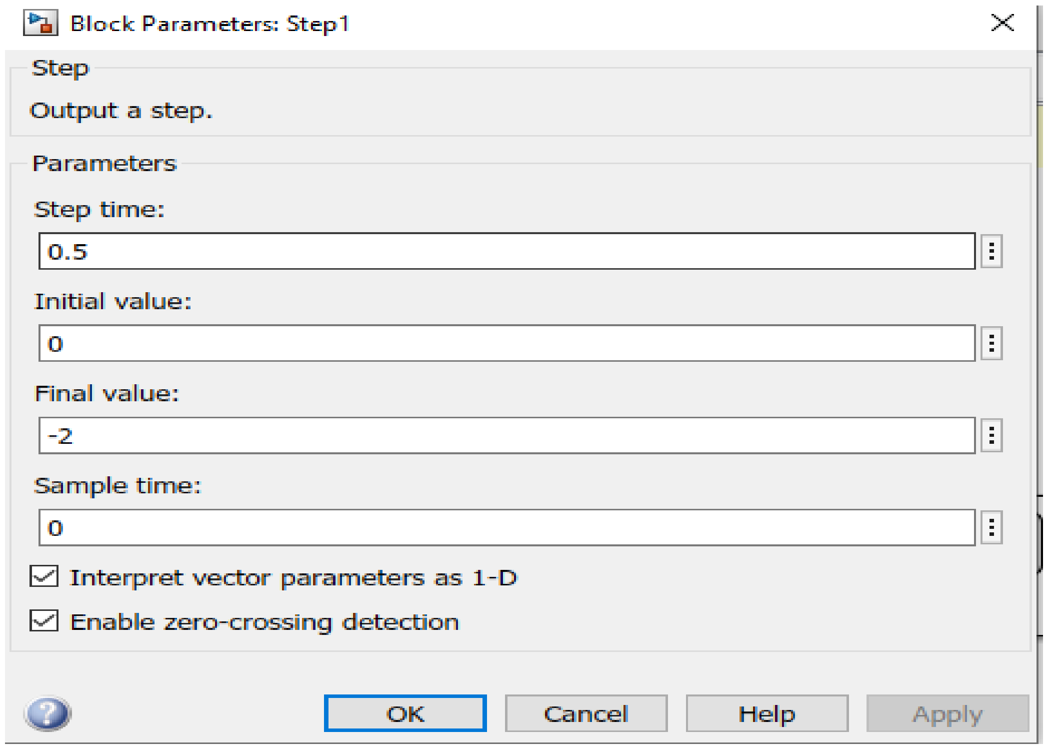


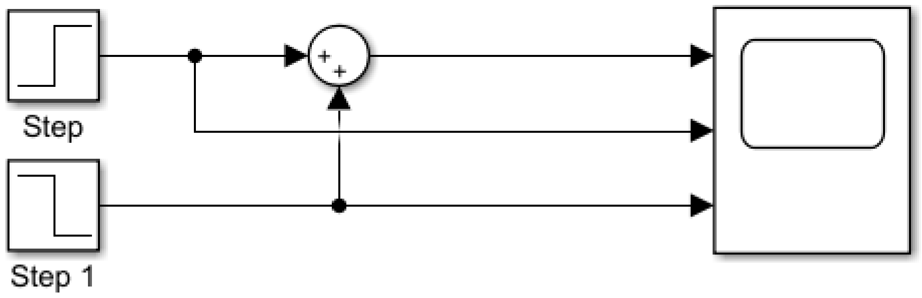


- Step Time = t0
- Initial Value = ui
- Final Value = uf
- Sample Time : Ignore !

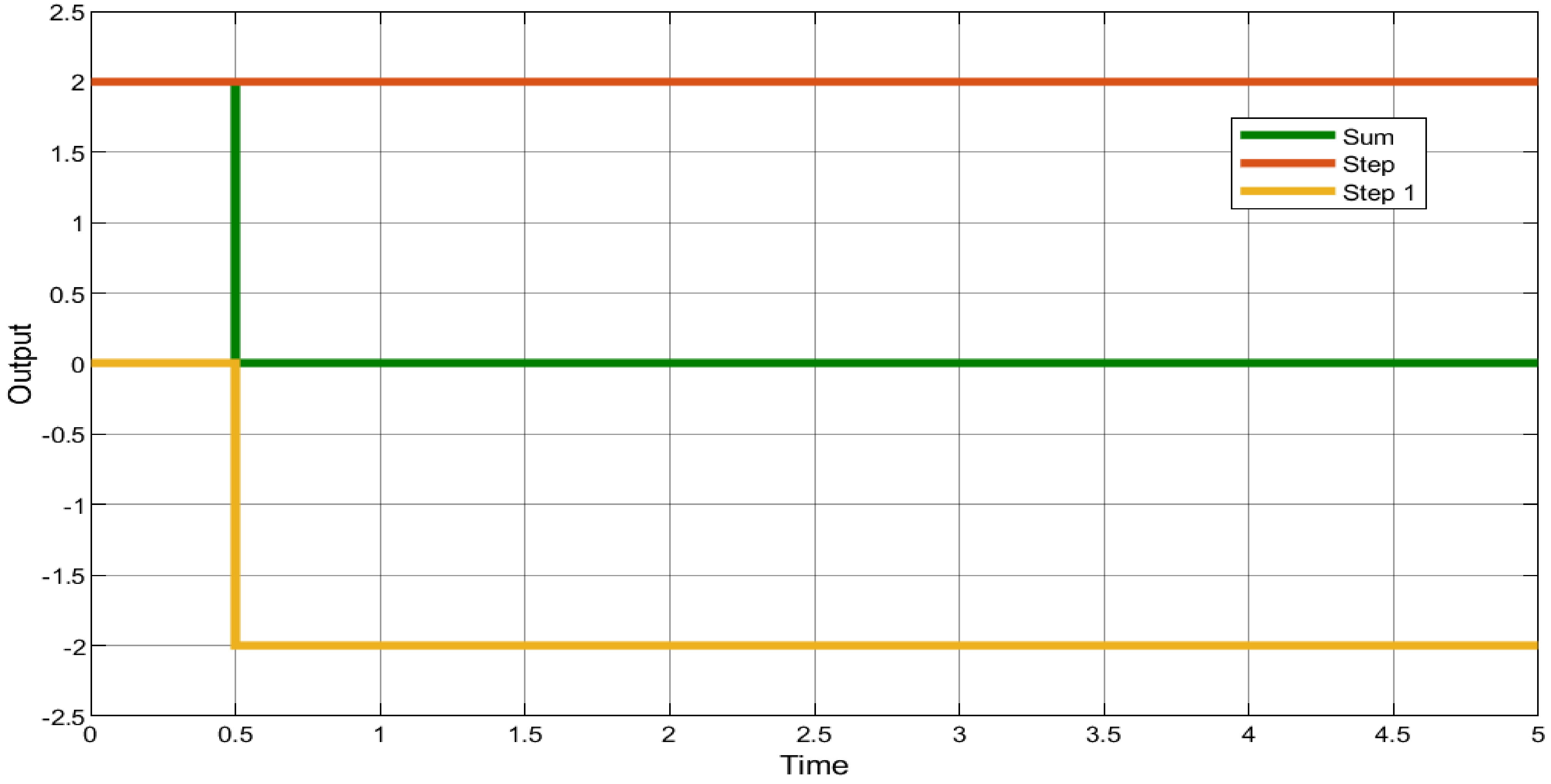
Finite Pulse in Simulink







Output Step Plots



Lab Report Submissions Checklist

- Include cover page with integrity statement
- Write your answers to the exercises
- **Include Simulink file in submission (.slx)**
- Include your models and your output graphs, via screenshots. **Label all variables and parameters in your model.**
- **Output graphs should have titles, a white background, increased line thickness, and easy-to-view colors.**
- Submit on Gradescope before Wednesday 2pm (2^N percent late policy)

Exercises

1. Create a finite pulse of area = 1 as the sum of two step inputs. The width of the pulse is 2 seconds. Plot the two step inputs and the sum on the same plot.
2. Create a finite pulse of area = 1 as the sum of two step inputs. The width of the pulse is 0.5 seconds. Plot the sum.
3. Approximate a unit impulse function $\delta(t)$ by creating a finite pulse of area = 1 with a tiny width. The width of the pulse is 0.000001 seconds. Plot the sum.

4. Now, using the finite pulse or impulse from Problems 1-3 as input u(t), plot the model $\frac{dx}{dt} = -kx + 5u(t)$, with initial condition = 0.5 and k = 2. You can create the model three times and plot the three graphs on the same scope.
   A. What do you observe?

5. Now, demonstrate the equivalence property by adding a fourth graph $\frac{dx}{dt} = -kx$. Instead of using the impulse input $5\delta(t)$, change the initial condition instead. Show that the output is the same as in Problem 4.
   A. What is your new initial condition?