CS CM 182 HW 8

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I completed this written part of the homework, lab report, or exam entirely on my own.

Problem 1 - Feedback control system simulation of simple linear constitutive production and elimination

Part (a)

Problem 1

(a) $\dot{y} = u - ky$ Normalize the simulation interval $1 \le t \le 10$ to $0 < \tau < 1$

$$\frac{dy(t)}{dt} = u(t) - k\,y(t)$$

Based from the textbook :

$$\frac{dx}{dt} = f[x(t), u(t), p] \qquad \frac{dx(\tau)}{d\tau} = f'[x(\tau), u(\tau), p]$$
$$= (t_{max} - t_0) f[x(\tau, u(\tau), p]$$

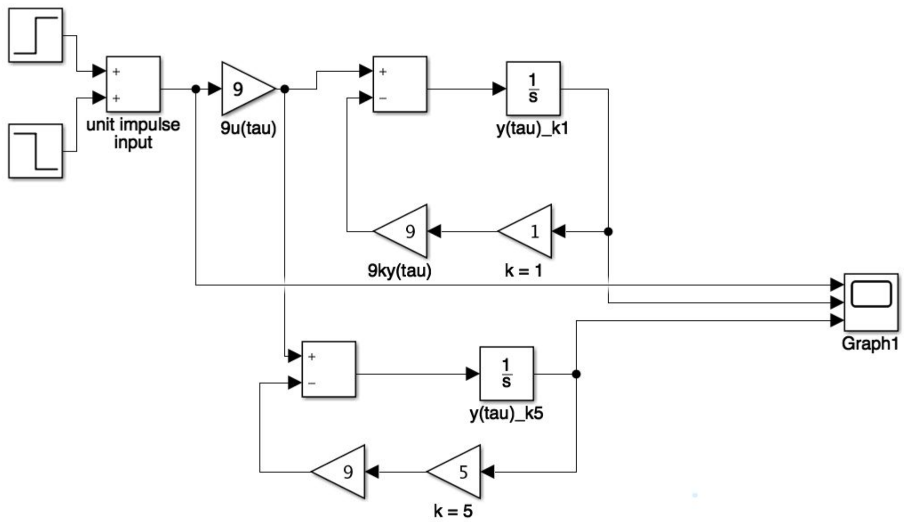
$$t_{max} - t_0 = 10 - 1 = 9$$

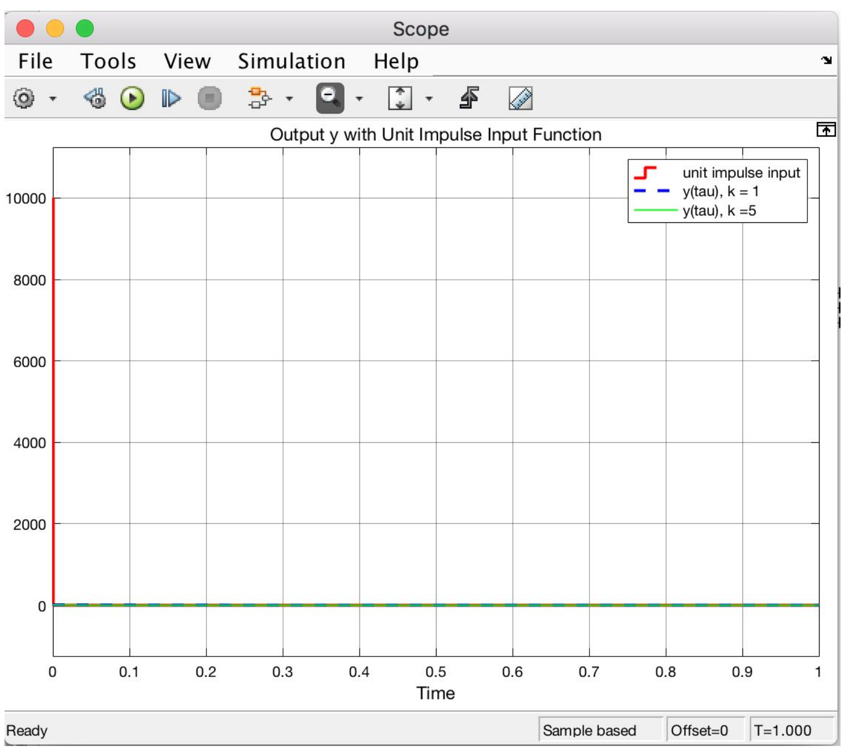
$$\frac{dy(\tau)}{d\tau} = (t_{max} - t_0)[u(\tau) - ky(\tau)]$$
$$= (9)[u(\tau) - ky(\tau)] = \boxed{9u(\tau) - 9ky(\tau)}$$

on the interval $0 < \tau < 1$

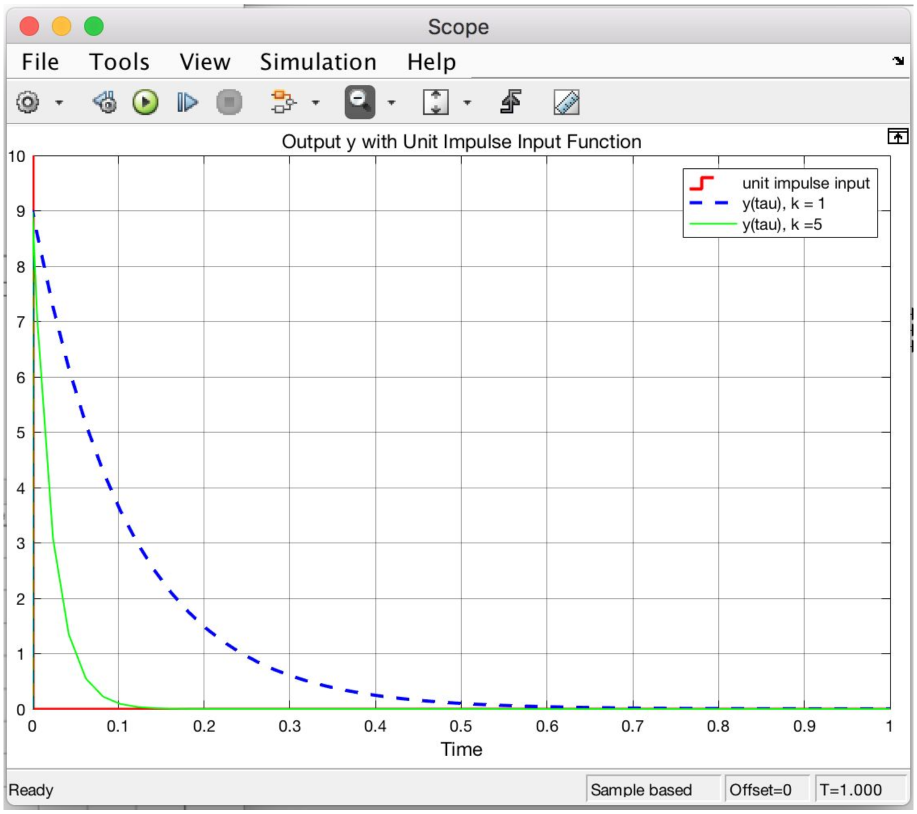
Problem 1 - Part (b)

<u>**Unit Impulse Function Input (k = 1)**</u>
<u>**Unit Impulse Function Input (k = 5)**</u>



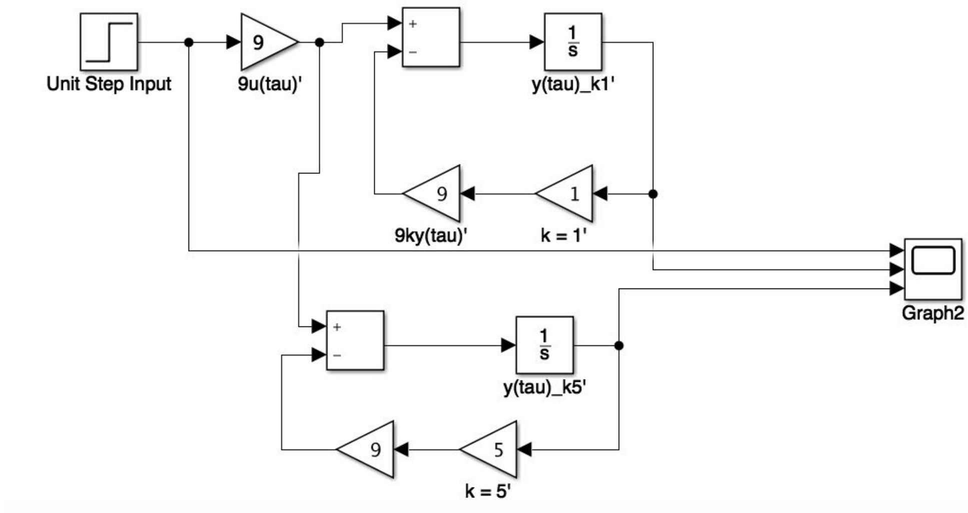


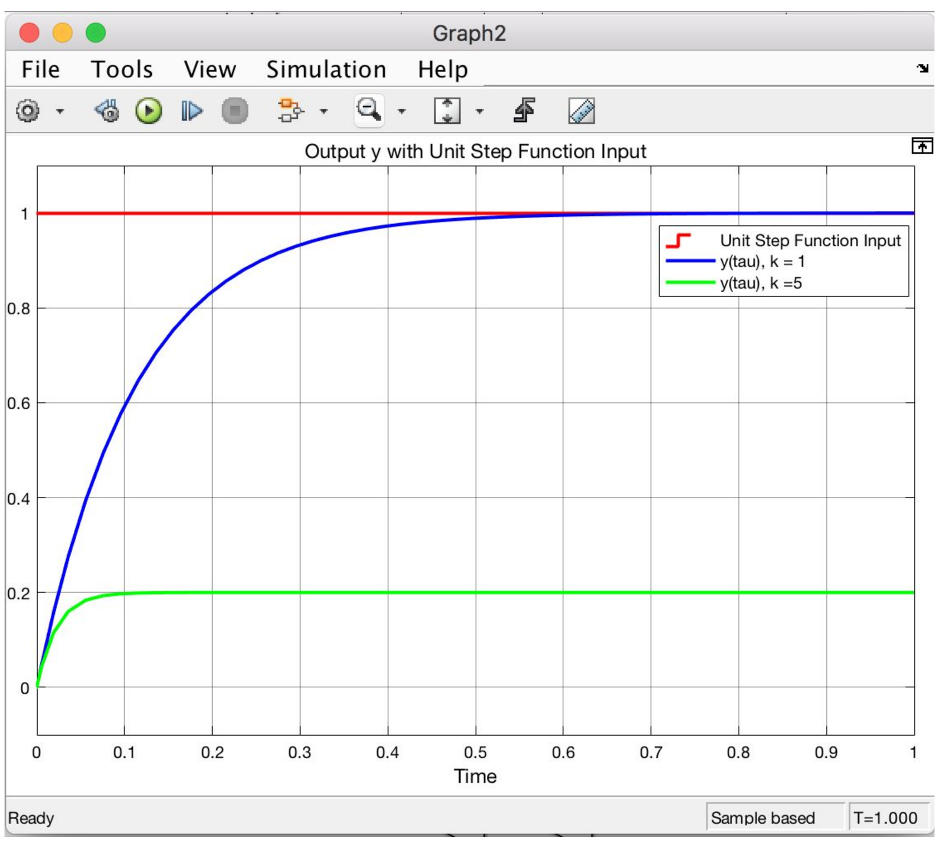
<u>**(zoom out)**</u>


Scope
File Tools View Simulation Help
Output y with Unit Impulse Input Function
unit impulse input
y(tau), k = 1
y(tau), k =5
10
9
8
7
6
5
4
3
2
1
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Time
Ready
Sample based
Offset=0
T=1.000

(zoom out)

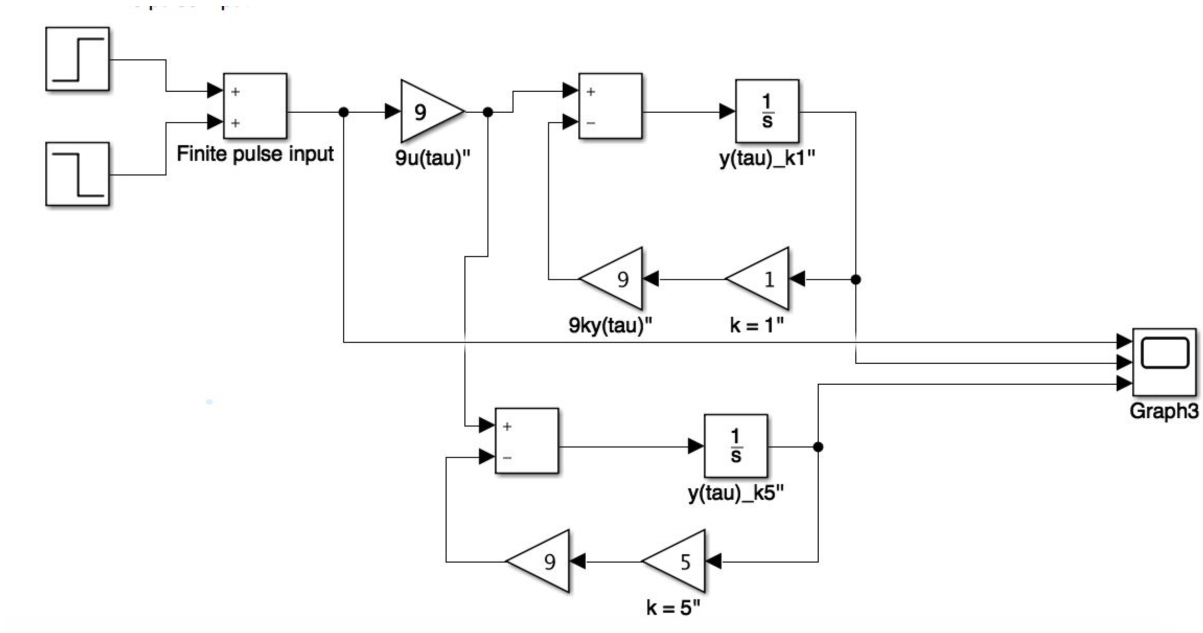
Unit Step Function Input (k = 1)
Unit Step Function Input (k = 5)

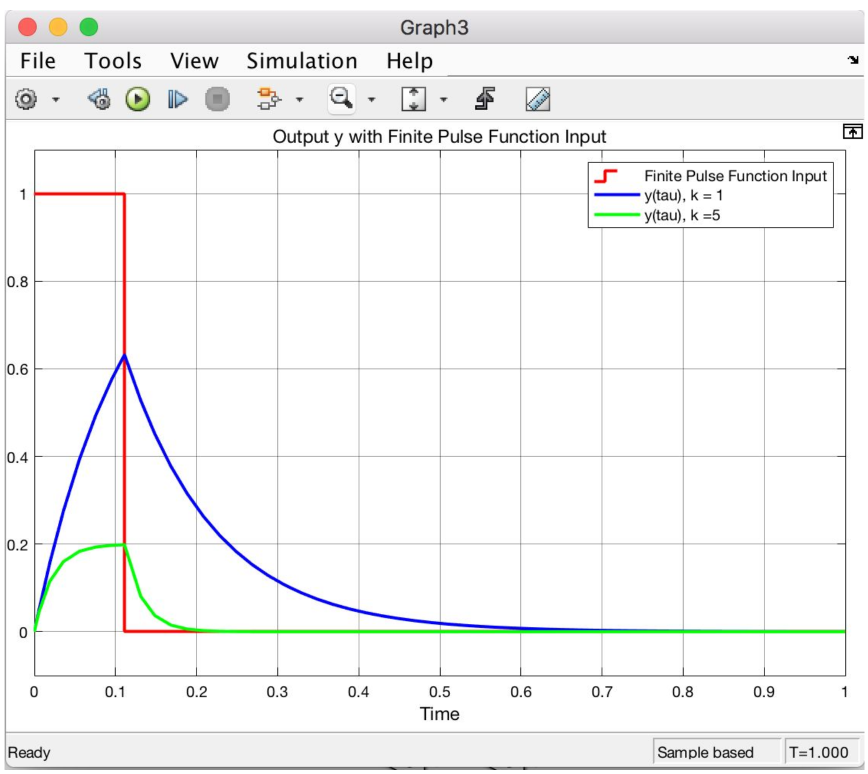




Finite Impulse Input (k = 1)

Finite Impulse Input (k = 5)





Problem 1 - Part (c)

How does the value of k affect the responses?

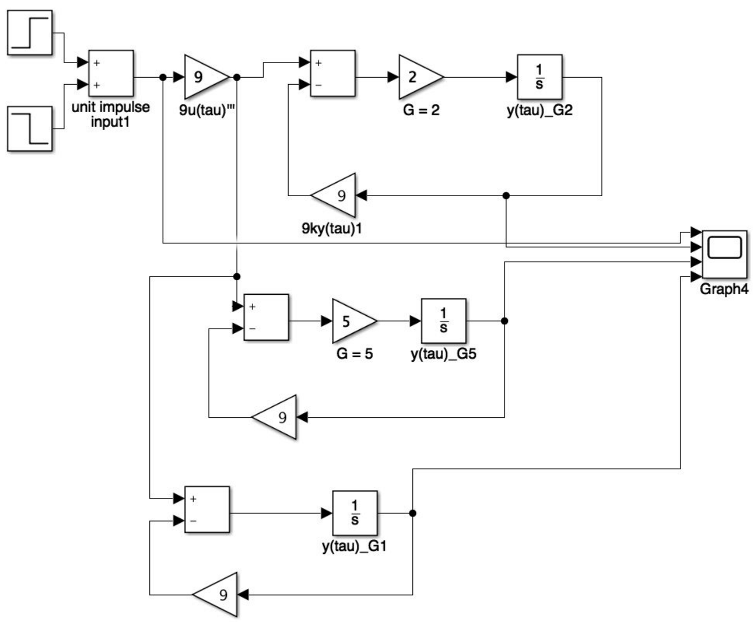
When k =1 from time = 0 to time = 1, the output response from the graph reaches the steady state value at time = 0.5 seconds. When k = 5 from time = 0 to time = 1, the output response from the graph reaches the steady state value at time = 0.1 seconds. The model with k = 5 reaches the steady state value 5 times quicker than the model with k = 1. As a result, the model with k = 5, a higher value in k, has a higher value in negative feedback than model with k =1.
On the other hand, the steady state value of a model with k = 5 is smaller than the steady state value of the model with k = 1 by 5 times.
As k becomes larger, the maximum output value becomes smaller and takes less time to fall back to zero as k increases.

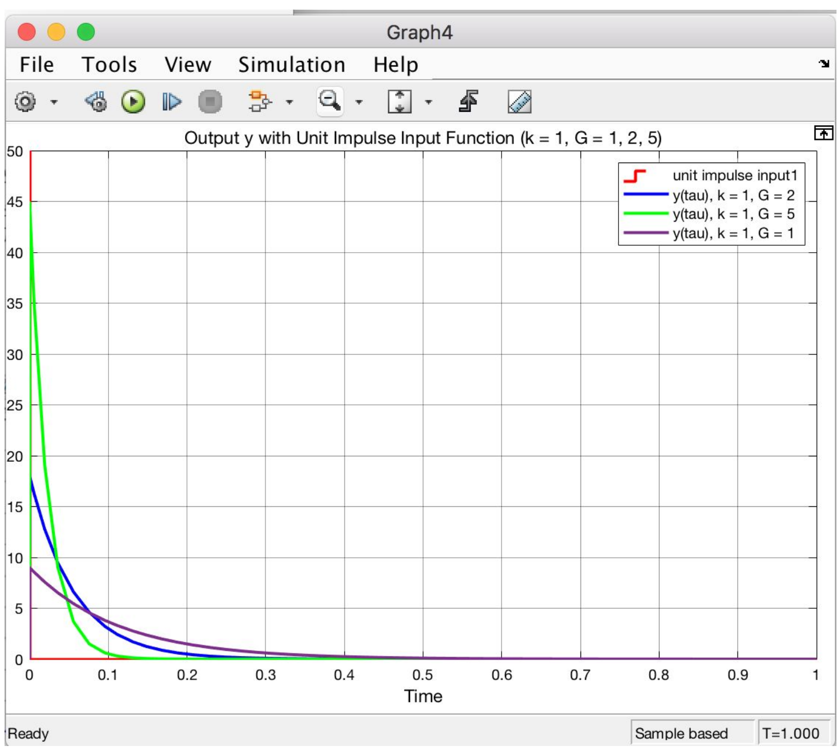
Problem 1 - part (d)

Unit impulse function input, k = 1, G = 1

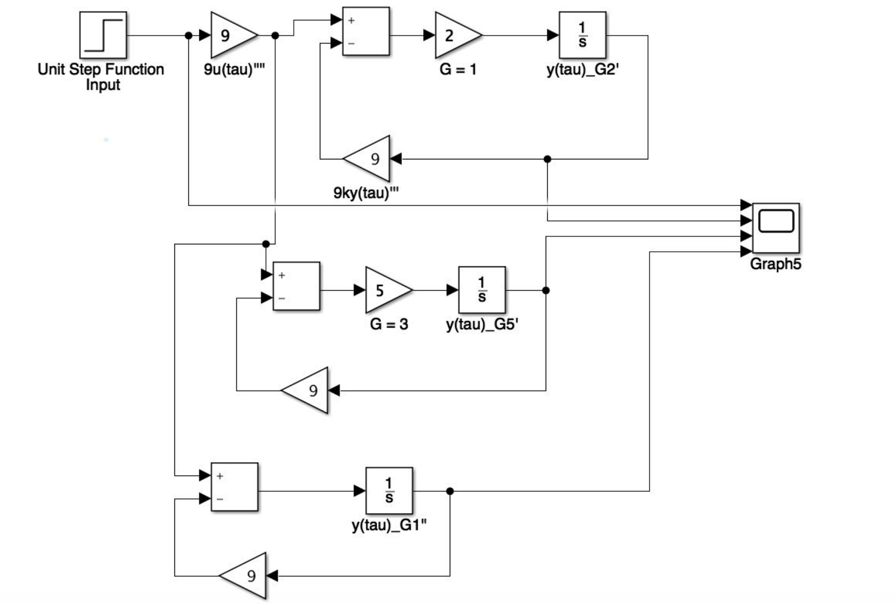
Unit impulse function input, k = 1, G = 2

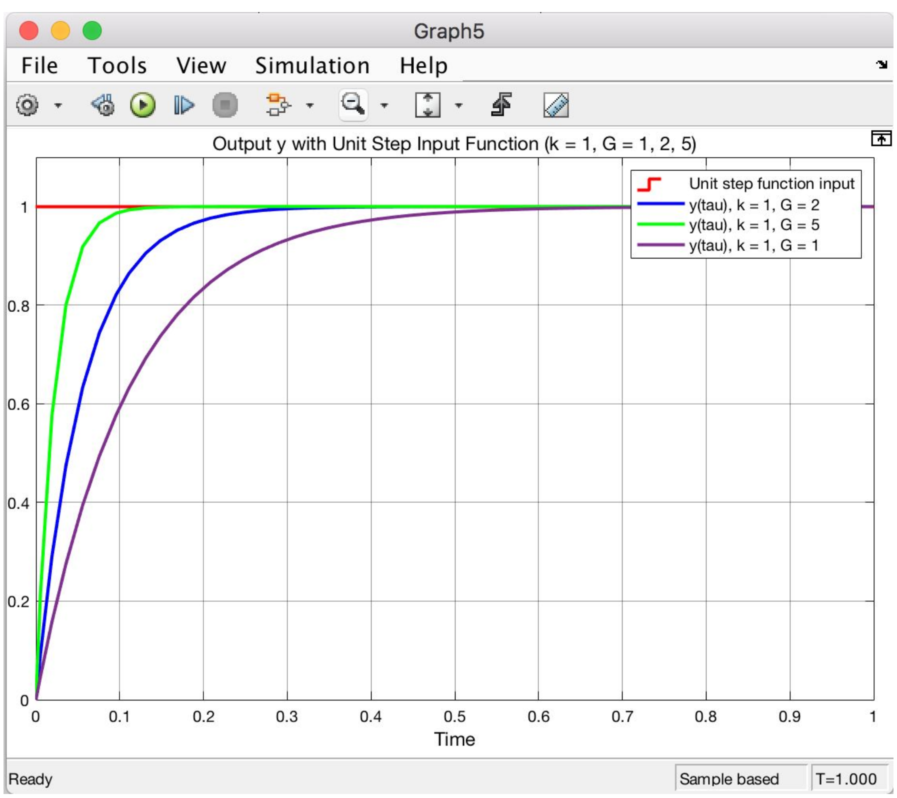
Unit impulse function input, k = 1, G = 5



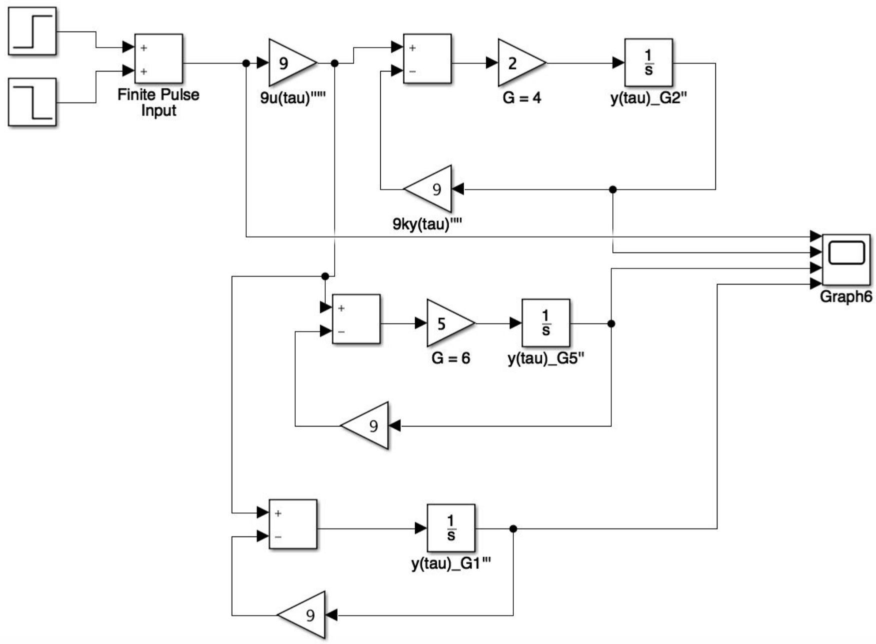


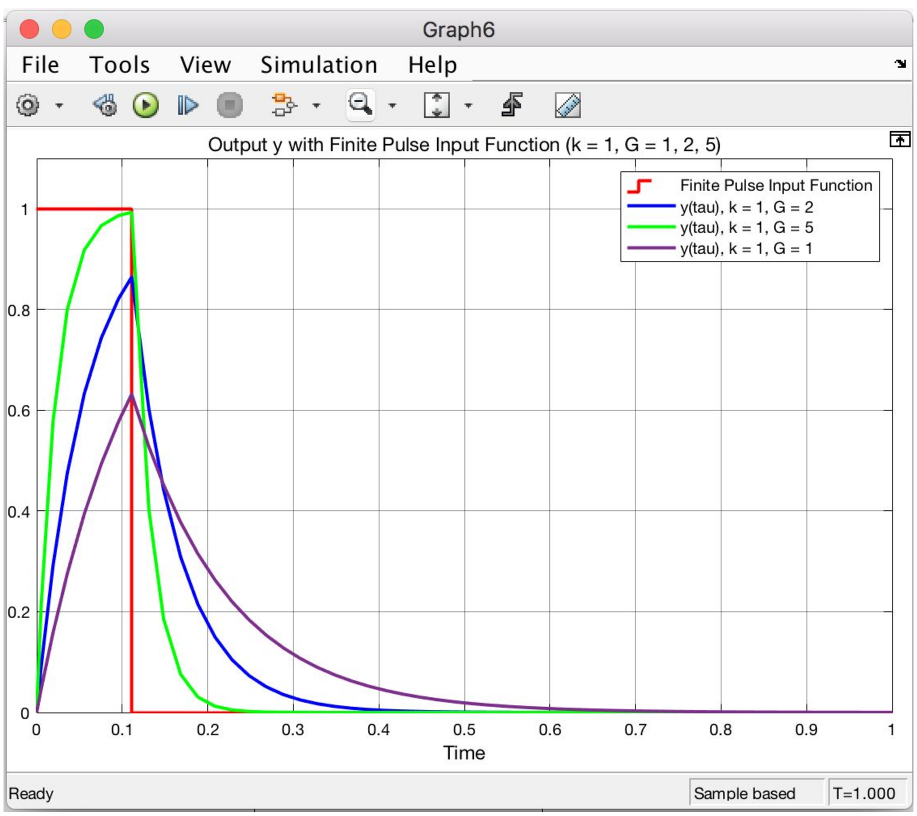
Unit step function input, k = 1, G = 1
Unit step function input, k = 1, G = 2
Unit step function input, k = 1, G = 5





Finite pulse input, k = 1, G = 1
Finite pulse input, k = 1, G = 2
Finite pulse input, k = 1, G = 5





Problem 1 - Part (e)
How do these gain changes (with k fixed) affect the dynamic responses?

In terms of unit impulse input with $k = 1$ fixed from time = 0 to time = 1, output at $G = 5$ starts off at a highest value, output at $G = 2$ starts off at a middle value and output $G = 1$ starts off at a lowest value. As G increases, the output graph falls quicker back to zero with a shorter amount of time. As G decreases, the output graph takes longer time to fall back to zero.

In terms of unit step function input with $k = 1$ fixed from time = 0 to time = 1, the response steady value stays the same for $G = 1, 2, 5$. As G increases, it takes a shorter time to reach the steady state value. As G decreases, it takes longer time to reach the same steady state value.

In terms of finite pulse input from time = 0 to time = 1, $k = 1$ and $G = 1$, It reaches the maximum value 0.65 at time = 1/9. At time = 1/9, the graph falls slowly to approach 0.
At time = 0.6, it reaches zero and the output value stays at zero afterwards.
In terms of finite pulse input from time = 0 to time = 1, $k = 1$ and $G = 2$, It reaches the maximum value 0.85 at time = 1/9. At time = 1/9, the graph falls slowly to approach 0.
At time = 0.4, it reaches zero and the output value stays at zero afterwards.
In terms of finite pulse input from time = 0 to time = 1, $k = 1$ and $G = 5$, It reaches the maximum value 1 at time = 1/9. At time = 1/9, the graph falls slowly to approach 0.
At time = 0.3, it reaches zero and the output value stays at zero afterwards.
As G becomes larger and fixed k, the maximum output value becomes smaller and it takes the same amount of time to reach the max value in the output graph. It also takes less time to fall back to zero as G increases.

Problem 2 - Normalization

Part (a) - Normalization of the state variable

Problem 2 - Normalization

(a) Normalize the state variable of the 2 ODEs

1st ODE

$$\frac{dq_1(t)}{dt} = -\left(k_{21} + \frac{C}{A+q_1}\right) q_1 + k_{12} q_2 + u$$

$$= -k_{21} q_1 - \frac{C q_1}{A+q_1} + k_{12} q_2 + u$$

$$\frac{d\frac{q_1(t)}{q_{10}}}{dt} = -k_{21}\frac{q_1(t)}{q_{10}} - \frac{C}{A+q_1(t)}\frac{q_1(t)}{q_{10}} + \frac{k_{12} q_2(t)}{q_{10}} + \frac{u}{q_{10}}$$

$$= -k_{21}\frac{q_1(t)}{q_{10}} - \frac{\frac{C}{q_{10}}}{\frac{A}{q_{10}} + \frac{q_1(t)}{q_{10}}}\frac{q_1(t)}{q_{10}} + \frac{k_{12} q_{20}}{q_{10}}\frac{q_2(t)}{q_{20}} + \frac{u}{q_{10}}$$

Let the following be the substitutions for some of the terms from $\frac{d\frac{q_1(t)}{q_{10}}}{dt}$

① $q_1(t)' = \frac{q_1(t)}{q_{10}}$ ③ $k_{12}' = \frac{k_{12} q_{20}}{q_{10}}$

② $q_2(t)' = \frac{q_2(t)}{q_{20}}$ ④ $u' = \frac{u}{q_{10}}$

⑤ $C' = \frac{C}{q_{10}}$ ⑥ $A' = \frac{A}{q_{10}}$

After substitution :

$$\frac{dq_1(t)'}{dt} = -k_{21}\, q_1(t)' - \frac{C'}{A' + q_1(t)'}\, q_1(t)' + k_{12}'\, q_2(t)' + u'$$

$$\boxed{\frac{dq_1(t)'}{dt} = -\left[k_{21} + \frac{C'}{A' + q_1(t)'}\right] q_1(t)' + k_{12}'\, q_2(t)' + u'}$$

2nd ODE

$$\frac{dq_2}{dt} = k_{21}\, q_1 + k_{22}\, q_2$$

$$\frac{\frac{dq_2(t)}{q_{20}}}{dt} = \frac{k_{21}\, q_1(t)}{q_{20}} + \frac{k_{22}\, q_2(t)}{q_{20}}$$

$$= \frac{k_{21}\, q_{10}}{q_{20}} \frac{q_1(t)}{q_{10}} + k_{22} \frac{q_2(t)}{q_{20}}$$

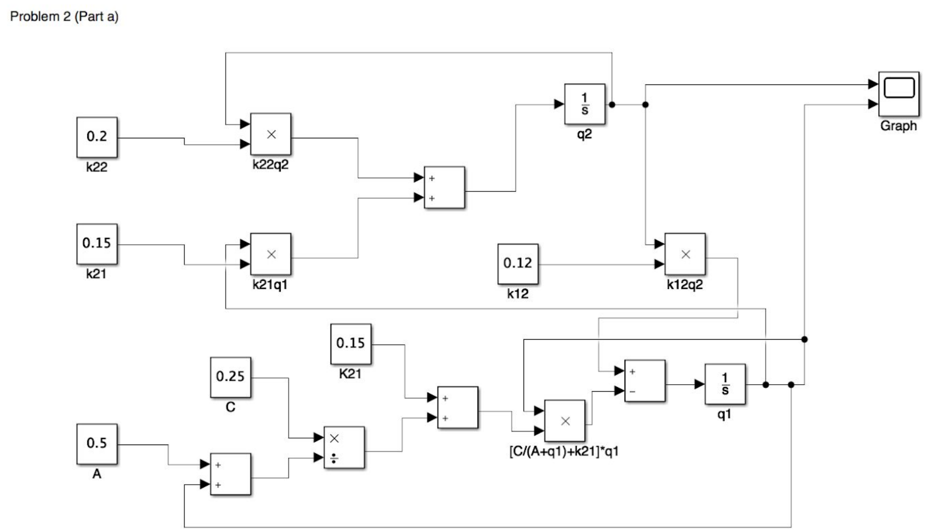
Assume the following substitutions

① $q_1(t)' = \frac{q_1(t)}{q_{10}}$ ② $q_2(t)' = \frac{q_2(t)}{q_{20}}$

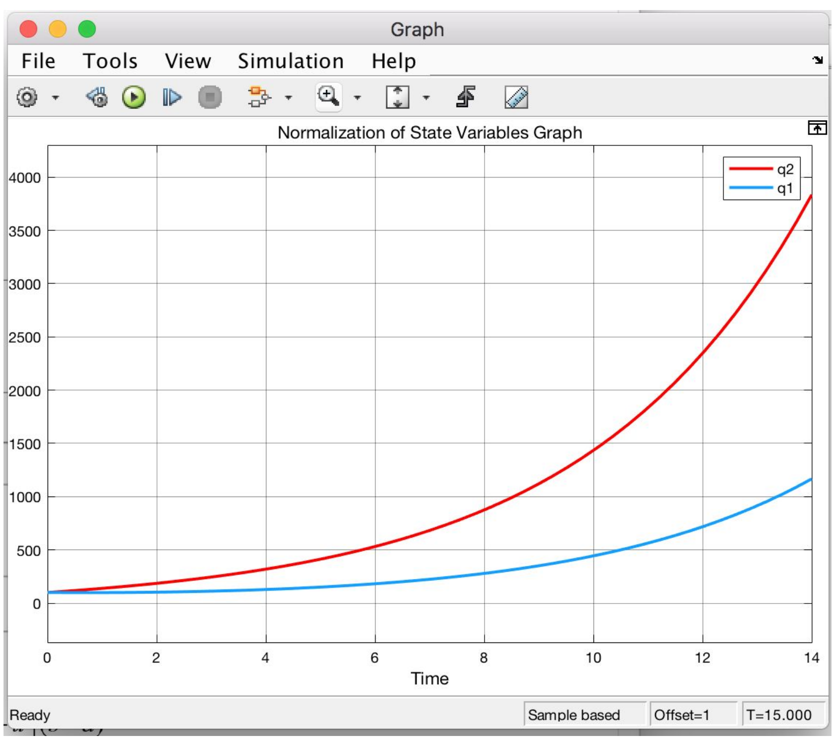
③ $k_{21}' = \frac{q_{10}\, k_{21}}{q_{20}}$

Normalization Process

Simulink



Graph

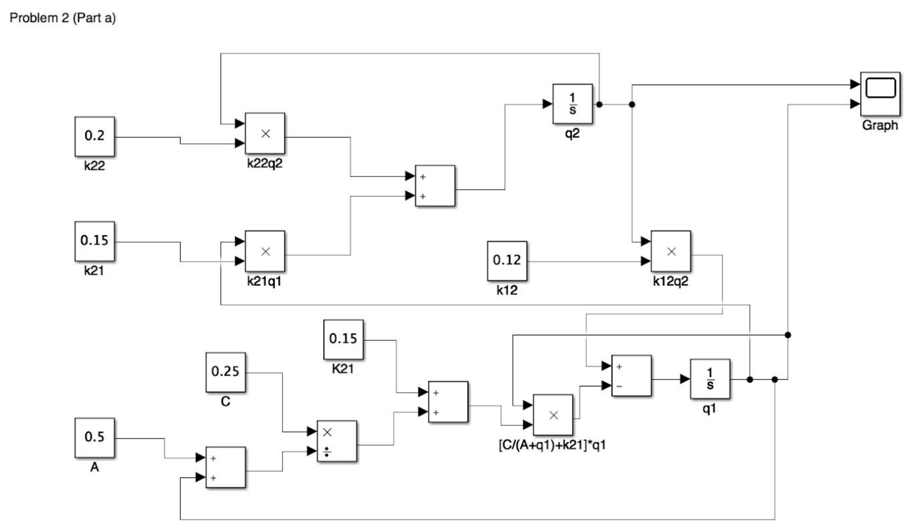


We cannot simulate the model because q1(0), q2(0) and input u(t) are missing. Therefore, the model might not be the most accurate. I try my best to simulate the model as close as possible.

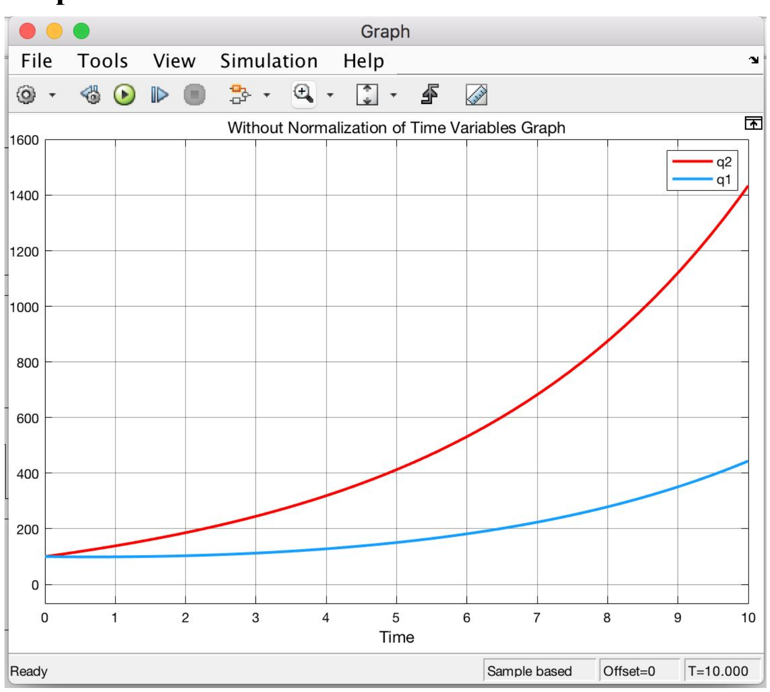
Problem 2 Part (b) - Normalize the time variable

Without normalization of the time variable

Simulink (reuse from the above simulink diagram since it is already without normalization of the time variable)



Graph



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Problem 2 Part (b) - Normalize the time variable

Problem 2b)

Normalize the time variable so the computation from normalized time $\tau = 0$ to $\tau = 1$

1st ODE

$$\dot{q}_1 = -\left(k_{21} + \frac{C}{A + q_1(t)}\right) q_1(t) + k_{21} q_2(t) + u$$

$$\boxed{\frac{dq_1(\tau)}{d\tau} = \left[-\left(k_{21} + \frac{C}{A + q_1(t)}\right) q_1(t) + k_{21} q_2(t) + u\right] \overbrace{\underbrace{(t_f - t_i)}_{15-1}}^{14}}$$

2nd ODE

$$\dot{q}_2 = k_{21} q_1(t) + k_{22} q_2(t)$$

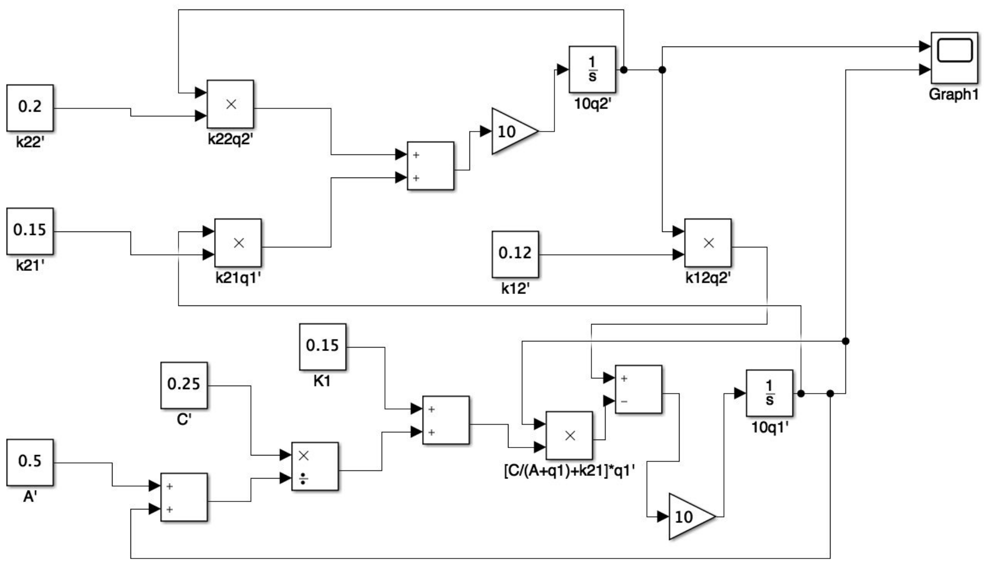
$$\boxed{\frac{dq_2(\tau)}{d\tau} = [k_{21} q_1(\tau) + k_{22} q_2(\tau)] \underbrace{\overset{15-1}{(t_f - t_i)}}_{14}}$$

$$\left.\begin{aligned} t_i &= 1 \\ t_f &= 15 \end{aligned}\right\} \text{ from the problem statement}$$

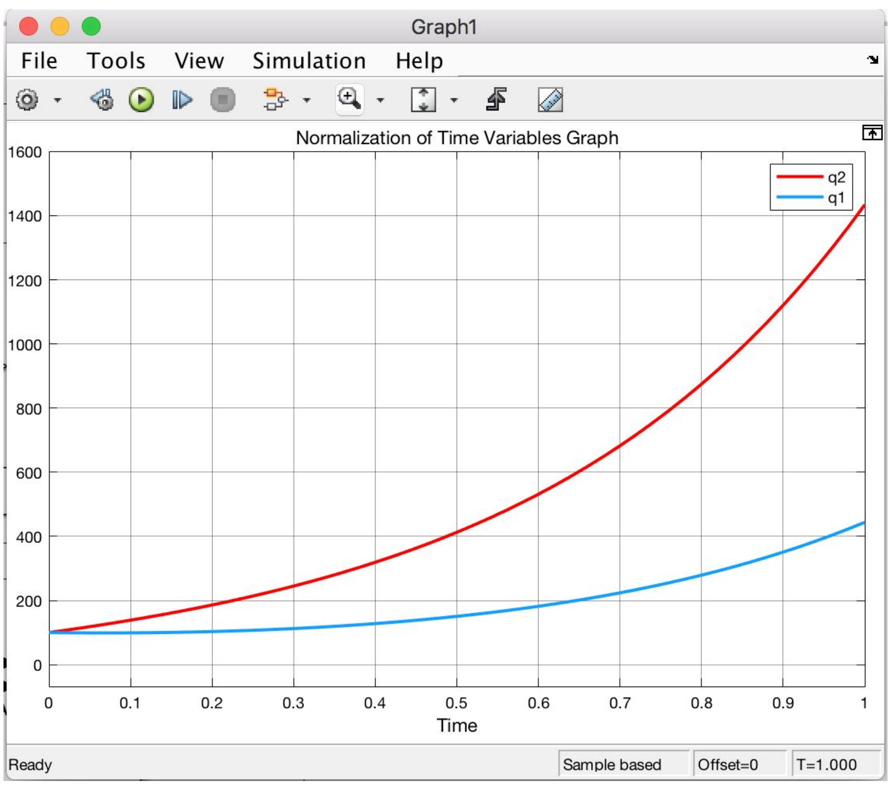
After substitution :

$$\boxed{\frac{dq_2(t)'}{dt} = k_{21}' q_1(t)' + k_{22} q_2(t)'}$$

With normalization of the time variable
Simulink



Graph



We cannot simulate the model because q1(0), q2(0) and input u(t) are missing. Therefore, the model might not be the most accurate. I try my best to simulate the model as close as possible.

Problem 2 - Part (b)

How does this change simulation results?

The shape of the graph stays the same between the with normalization of time variables and without normalization of the time variables. Therefore, the time-scaled dynamics does not change that much between the two graphs.
The graph without normalization of time variables spans over the time interval 0 to 10 units of time while the graph with normalization of time variables spans over the time interval 0 to 1 units of time.

Problem 3 - Discrete-time model solution with an ODE solver

Problem 3

The difference equations for the compound interest

$$P(t+1) = (1+\text{Int})\,P(t)$$

Let $P(t) \equiv$ the total amount of money after t units of time

$\text{Int} \equiv$ the interest rate

Apply Euler's method with $\Delta t = 1$

$$\frac{dP}{dt} = (1+\text{Int})P - P = \cancel{P} + \text{Int}\cdot P - \cancel{P} = \text{Int}\cdot P$$

Given $\text{Int} = \frac{0.05}{\text{week}}$ (5% compounded weekly)

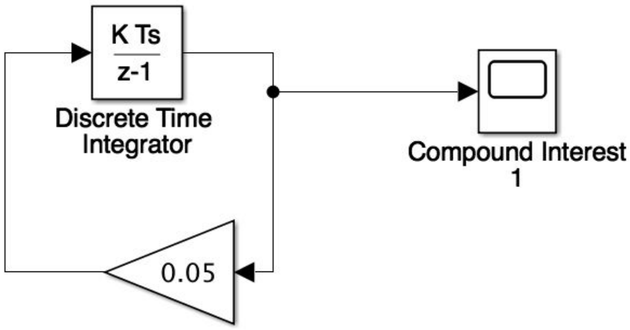
$P(0) = 1000$ (initial investment of \$1000)

time period = 5 years means 260 weeks

$12 \times 5 \times 4.333$

estimate week per month

Simulink (Initial condition = 1000 inside the integrator)



Graph