CS CM 182 Homework 4

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I completed this written part of the homework, lab report, or exam entirely on my own.

Exercise 6.1 - M-M ODE model reduction

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Derive $\frac{dC_S}{dt} = -k_1 C_E(0) C_S + (k_1 C_S + k_{-1}) C_{ES}$

$$\frac{dC_{ES}}{dt} = k_1 C_E(0) C_S - (k_1 C_S + k_{-1} + k_2) C_{ES}$$

by $C_E(t) + C_{ES}(t) = C_E(0)$

Derivations $C_E(t) = C_E(0) - C_{ES}(t)$

substitute $C_E(t)$ into ODEs for $\frac{dC_S}{dt}$

$$\frac{dC_S}{dt} = -V_1 + V_{-1} = -k_1 C_E C_S + k_{-1} C_{ES}$$

$$= -k_1 [C_E(0) - C_{ES}(t)] C_S + k_{-1} C_{ES}$$

$$= -k_1 C_E(0) C_S + k_1 C_{ES}(t) C_S + k_{-1} C_{ES}$$ same as C_{ES}

$$= -k_1 C_E(0) C_S + (k_1 C_S + k_{-1}) C_{ES}$$

$$\frac{dC_{ES}}{dt} = V_1 - V_{-1} - V_2 = k_1 C_E C_S - (k_{-1} + k_2) C_{ES}$$

substitute $C_E(t)$ into ODE for $\frac{dC_{ES}}{dt}$

$$\frac{dC_{ES}}{dt} = k_1 [C_E(0) - C_{ES}(t)] C_S - (k_{-1} + k_2) C_{ES}$$

$$\frac{dC_{ES}}{dt} = k_1 C_E(0) C_S - k_1 C_{ES} C_S - k_{-1} C_{ES} - k_2 C_{ES}$$

$$\frac{dC_{ES}}{dt} = k_1 C_E(0) C_S - (k_1 C_S + k_{-1} + k_2) C_{ES}$$

Exercise 6.2 - M-M algebraic model derivation

Exercise 6.2 : M−M algebraic model Derivation.

Given these equations :

$$C_E(t) + C_{ES}(t) = C_E(0)$$

$$C_E(0) - C_{ES} = C_E$$

$$\frac{dC_S}{dt} = -k_1 C_E C_S + k_{-1} C_{ES}$$

$$\frac{dC_E}{dt} = -k_1 C_E C_S + k_{-1} C_{ES} + k_2 C_{ES}$$

$$\frac{dC_{ES}}{dt} = k_1 C_E C_S - (k_{-1} + k_2) C_{ES}$$

$$\frac{dC_P}{dt} = k_2 C_{ES}$$

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① Substitute for C_E first

$$\frac{dC_S}{dt} = -k_1 (C_E(0) - C_{ES}) C_S + k_{-1} C_{ES}$$

$$\frac{dC_E}{dt} = -k_1 (C_E(0) - C_{ES}) C_S + k_{-1} C_{ES} + k_2 C_{ES}$$

$$\frac{dC_{ES}}{dt} = k_1 (C_E(0) - C_{ES}) C_S - (k_{-1} + k_2) C_{ES} = \frac{-dC_E}{dt}$$

$$\frac{dC_P}{dt} = k_2 C_{ES}$$

(c) set $\frac{dC_{ES}}{dt} = 0 = -\frac{dC_E}{dt}$ & solve for C_{ES}

$$k_1(C_E(0) - C_{ES})C_S - (k_{-1} + k_2)C_{ES} = 0$$

$$k_1 C_E(0) C_S - \underline{k_1 C_{ES} C_S} - \underline{k_{-1} C_{ES}} - \underline{k_2 C_{ES}} = 0$$

$$\cancel{-}C_{ES}(k_1 C_S + k_{-1} + k_2) = \cancel{-}k_1 C_E(0) C_S$$

$$C_{ES} = \frac{k_1 C_E(0) C_S}{k_1 C_S + k_{-1} + k_2}$$

$$\frac{dC_P}{dt} = k_2 C_{ES} = \frac{k_2 k_1 C_E(0) C_S}{k_1 C_S + k_{-1} + k_2} = \frac{\overbrace{k_2 C_E(0)}^{=V_{max}} k_1 C_S}{k_1 C_S + k_{-1} + k_2}$$

$$= \frac{V_{max} k_1 C_S}{k_1 C_S + k_{-1} + k_2} = \frac{V_{max} C_S k_1}{k_{-1} + k_2 + k_1 C_S}$$

$$= \frac{V_{max} C_S(t)}{\frac{k_{-1} + k_2 + k_1 C_S(t)}{k_1}} = \frac{V_{max} C_S(t)}{\underbrace{\frac{k_{-1} + k_2}{k_1}}_{= K_m} + \frac{\cancel{k_1} C_S(t)}{\cancel{k_1}}}$$

$$= \frac{V_{max} C_S(t)}{K_m + C_S(t)} = -\frac{dC_S}{dt} \quad \text{for all } t > 0$$

Ex 6.2 cont.

show $\frac{dC_P}{dt} = -\frac{dC_S}{dt}$

$$-\frac{dC_S}{dt} = k_1 C_E C_S - k_{-1} C_{ES} = k_1 C_E(0) C_S - (k_1 C_S + k_{-1}) C_{ES}$$

$$= \frac{dC_P}{dt} = k_2 C_{ES}$$

We have

$$k_1 C_E(0) C_S - (k_1 C_S + k_{-1}) C_{ES} = 0$$

$$k_1 C_E(0) C_S - (k_1 C_S + k_{-1}) C_{ES} - k_2 C_{ES} = 0$$

$$k_1 C_E(0) C_S - (k_1 C_S + k_{-1}) C_{ES} - \frac{dC_P}{dt} = 0$$

$$\frac{dC_P}{dt} = k_1 C_E(0) C_S - (k_1 C_S + k_{-1}) C_{ES}$$

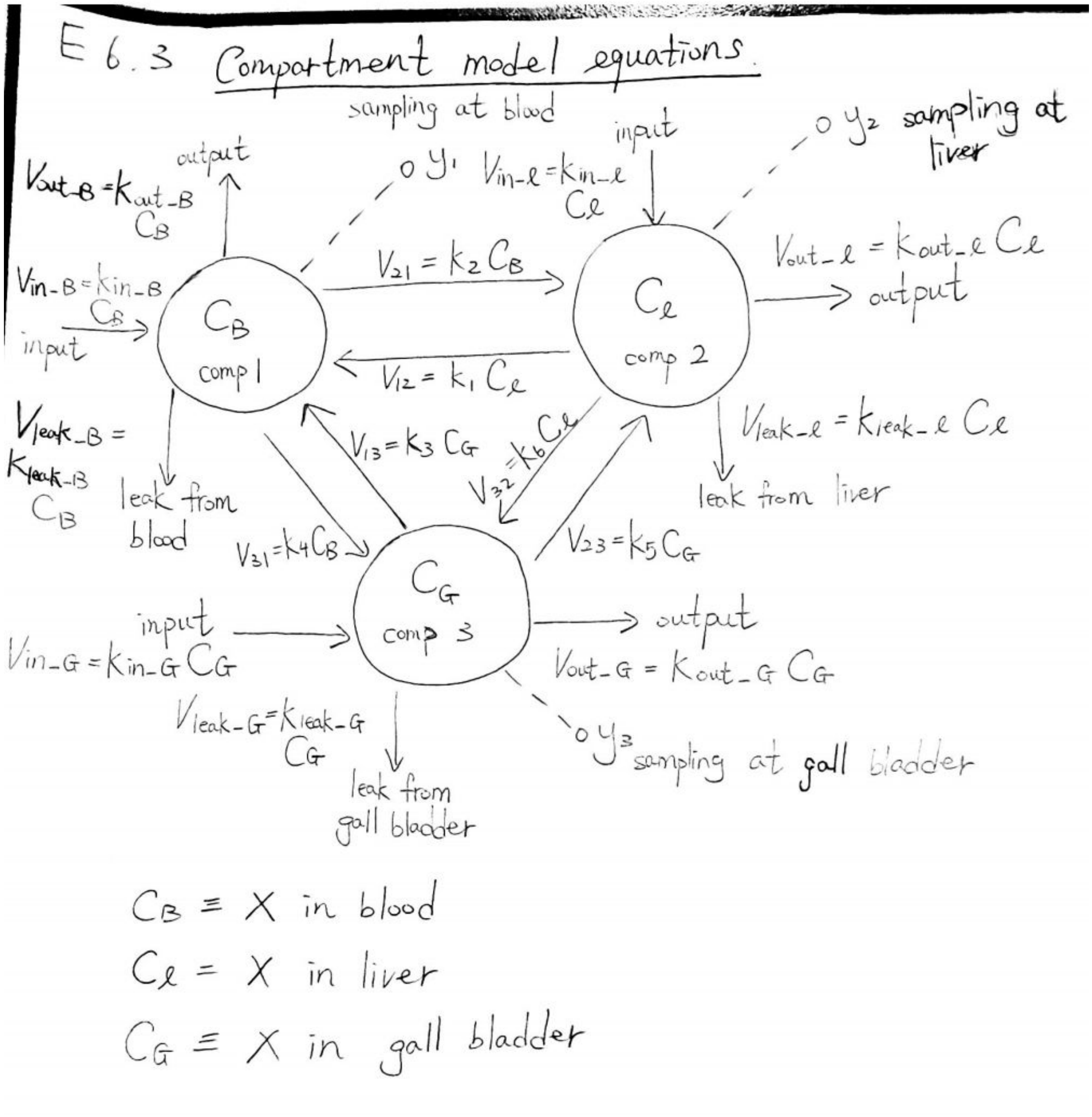
$$= -(-k_1 C_E(0) C_S + (k_1 C_S + k_{-1}) C_{ES})$$

$$= -\frac{dC_S}{dt}$$

$$\frac{dC_P}{dt} = [k_2 C_E(0) C_S] / [C_S + (k_{-1} + k_2)/k_1]$$

$$= -\frac{dC_S}{dt} \blacksquare$$

Exercise 6.3 : Compartmental model equations



Let q1 = Cb = X in blood
Let q2 = Cl = X in liver
Let q3 = Cg = X in gallbladder

Here are equations for the sample y1, y2, y3 (volumes)
y1 = q1 / v1
y2 = q2 / v2
y3 = q3 / v3

ODE for this model

① $\frac{dC_B}{dt} = V_{in\_B} - V_{out\_B} - V_{leak\_B} - V_{21} + V_{12} - V_{31} + V_{13}$

$= \boxed{k_{in\_B} C_B - k_{out\_B} C_B - k_{leak\_B} C_B - k_2 C_B + k_1 C_\ell - k_4 C_B + k_3 C_G}$

② $\frac{dC_\ell}{dt} = V_{in\_\ell} - V_{out\_\ell} - V_{leak\_\ell} - V_{12} + V_{21} - V_{32} + V_{23}$

$= \boxed{k_{in\_\ell} C_\ell - k_{out\_\ell} C_\ell - k_{leak\_\ell} C_\ell - k_1 C_\ell + k_2 C_B - k_6 C_\ell + k_5 C_G}$

③ $\frac{dC_G}{dt} = V_{in\_G} - V_{out\_G} - V_{leak\_G} - V_{23} + V_{32} - V_{13} + V_{31}$

$= \boxed{k_{in\_G} C_G - k_{out\_G} C_G - k_{leak\_G} C_G - k_5 C_G + k_6 C_\ell - k_3 C_G + k_4 C_B}$

Exercise 6.7 : Feedback & parameter sensitivity (feedback compensation)

(a)

Exercise 6.7 : Feedback & parameter sensitivity (feedback compensation)

(a) Negative feedback system model,
disturbance $D = 0$.

Show $y = \frac{Gu}{1+FG}$

Given

$$y = Gu \pm FGy + D$$

$$y = Gu \pm FGy + 0$$

$$y \pm FGy = Gu$$

$$y(1 \pm FG) = Gu$$

$$y = \frac{Gu}{1 \pm FG}$$

According to textbook equation (6.41)

$$\frac{1}{1 \pm FG} = \begin{cases} < 1 & \text{for negative feedback (+ sign)} \\ > 1 & \text{for positive feedback (- sign)} \end{cases}$$

So $y = \frac{Gu}{1+FG}$ ∎

(b)

(b) show that I-O become independent of G if FG is approaching ∞.

Given $y = \frac{Gu}{1+FG}$, Divide the numerator & denominator by FG

$$\frac{\frac{\cancel{G}u}{F\cancel{G}}}{\frac{1+FG}{FG}} = \frac{\frac{u}{F}}{\underbrace{\frac{1}{FG}}_{0}+1}$$

As FG become large, $\frac{1}{FG}$ approach 0

so $y = \frac{\frac{u}{F}}{0+1} = \boxed{\frac{u}{F}}$ independent of G

(c)

(c) Since I-O become independent of G if FG is large enough, the sensitivity of the output is not as sensitive to variations in G.

This means the output does not change or vary a lot even when G changes a lot.

For example, if G has changed by 5%, the output will be changed by less than 5%.

The effects of output disturbances D are reduced by negative feedback system and the error e, the difference between input and output is reduced by negative feedback system. The negative feedback generally reduces relative output sensitivity to parameter and disturbances D variations.

(d)
Based from part b), assume FG is large enough to use the same expression from part b)

(d) If $y = 10u$, $F = ??$ retains desired I-O relation.

From part b) $y = \frac{u}{F}$

$$10u = \frac{u}{F}$$

$$10uF = u$$

$$F = \frac{\cancel{u}}{10\cancel{u}} = \boxed{\frac{1}{10}}$$ value of F

(e)

(e) Given $G = 1000$

$F = \frac{1}{10}$ (based from d)

$$y = \frac{Gu}{1+FG}$$

$$y = \frac{(1000)u}{1+\left(\frac{1}{10}\right)(1000)} = \boxed{\frac{1000u}{101} \approx 9.90099u}$$

(f)

(f) $G \pm 10\%$

① $G + 10\%$ case $G = 1000$

$G(1+10\%) = (1000)(1+10\%) = 1100$

$$y = \frac{Gu}{1+FG} = \frac{(1100)u}{1+(\frac{1}{10})(1100)} = \boxed{\frac{1100u}{111}}$$

② $G - 10\%$ case

$G(1-10\%) = (1000)(1-10\%) = 900$

$$y = \frac{Gu}{1+FG} = \frac{(900)u}{1+(\frac{1}{10})(900)} = \boxed{\frac{900u}{91}}$$

% change in relative sensitivity

1) Between $G = 1100$ & $G = 1000$

2) Between $G = 900$ & $G = 1000$

3) Between $G = 1100$ & $G = 900$

next page

(f) cont.

1) % change in between $G = \overset{(\text{new})}{1100}$ & $G = \overset{(\text{old})}{1000}$

$$\frac{\left(\frac{1100u}{111}\right) - \left(\frac{1000u}{101}\right)}{\left(\frac{1000u}{101}\right)} \times 100\%$$

$$= \frac{(0.0089198109u)}{(9.9009900 99u)} \times 100\%$$

$$\approx \boxed{0.0900900\%}$$

2) % change in between $G = \overset{(\text{new})}{900}$ & $G = \overset{(\text{old})}{1000}$

$$\frac{\left(\frac{900u}{91}\right) - \left(\frac{1000u}{101}\right)}{\left(\frac{1000u}{101}\right)} \times 100\%$$

$$= \frac{(-0.010880208 9u)}{(9.9009900 99u)} \times 100\%$$

$$\approx \boxed{-0.1098901 0\%}$$

3) % change between $G = 1100$ & $G = 900$
(new) (old)

$$\frac{\left(\frac{1100u}{111}\right) - \left(\frac{900u}{91}\right)}{\left(\frac{900u}{91}\right)} \times 100\%$$

$$= \frac{(0.0198000198u)}{(9.89010989u)} \times 100\%$$

$$\approx \boxed{0.2002002\%}$$

Based on the three differences in relative sensitivity, each of them are less than 1%. Therefore, the output y does not really depend on G despite variations in the value of G.

(g)

Yes. The negative feedback system is more robust than the equivalent open-loop system. It is because the negative feedback system is more consistent even when there are large errors or noise. According to part (g), the calculated output y in change if G varies by plus or minus 10% is really low which is less than 1 % for both cases. When G varies by +10%, the change relative sensitivity in % is roughly 0.0900900%. When G varies by -10%, the change relative sensitivity in % is roughly 0.10989010%. Both of them show that input output relation becomes independent of G if FG is large enough.

The effects of output disturbances D are reduced by negative feedback system and the error e, the difference between input and output is reduced by negative feedback system. The negative feedback generally reduces relative output sensitivity to parameter and disturbances D variations.