1. Basic Logarithm

Q: Find $\log_2 8$.

Sol:

Ask: "2 to what power gives 8?"

$$2^3 = 8$$
, so $\log_2 8 = \boxed{3}$.

2. Log of 1

Q: Solve $\log_5 1$.

Sol:

Any number to power 0 is 1.

So,
$$\log_5 1 = 0$$
.

3. Same Base & Number

Q: What is $\log_7 7$?

Sol:

"7 to what power gives 7?"

$$7^1=7$$
, so $\log_7 7=\boxed{1}$.

4. Logarithm with Power

Q: Simplify $\log_3 9^2$.

Sol:

Step 1: $\log_3 81$ (because \(9^2 = 81\).

Step 2: $3^4 = 81$, so answer is $\boxed{4}$.

5. Adding Logs (Product Rule)

Q: Solve $\log_2 4 + \log_2 8$.

Sol:

$$\log_2(4\times 8) = \log_2 32.$$

Now, $2^5 = 32$, so 5.

6. Subtracting Logs (Quotient Rule)

Q: Simplify $\log_{10} 1000 - \log_{10} 10$.

Sol:

$$\log_{10}\left(\frac{1000}{10}\right) = \log_{10} 100.$$

$$10^2 = 100$$
, so $\boxed{2}$.

7. Change of Base

Q: Find $\log_4 16$ using base 2.

Sol:

$$\frac{\log_2 16}{\log_2 4} = \frac{4}{2} = \boxed{2}.$$

8. Solving for x

Q: If $\log_5 x = 3$, find x.

Sol:

$$x = 5^3 = 125$$
, so $\begin{bmatrix} 125 \end{bmatrix}$.

Quick Rules to Remember

1.
$$\log_a b = x \to a^x = b$$
.

2.
$$\log_a 1 = 0$$
.

3.
$$\log_a a = 1$$
.

4.
$$\log_a(xy) = \log_a x + \log_a y$$
.

5.
$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$
.

Tip: Always rewrite logs in exponential form if stuck!