

Summary of Key Formulas

Figure	Formula
Regular Pentagon	$\frac{1}{2} \times \text{Perimeter} \times \text{Apothem}$
Regular Hexagon	$\frac{3\sqrt{3}}{2} \times \text{side}^2$
Prism Surface Area	$2 \times \text{Base Area} + \text{Lateral Area}$
Sector ($^\circ$)	$\frac{\theta}{360^\circ} \times \pi r^2$
Sector (rad)	$\frac{1}{2} r^2 \theta$
Segment	$\text{Sector Area} - \text{Triangle Area}$

1. Area of a Regular Pentagon

Question:

What is the area of a regular pentagon with a side length of 6 cm and an apothem of 4.13 cm?

Solution:

$$\text{Area} = \frac{1}{2} \times \text{Perimeter} \times \text{Apothem}$$

$$\text{Perimeter} = 5 \times 6 = 30 \text{ cm}$$

$$\text{Area} = \frac{1}{2} \times 30 \times 4.13 = \boxed{61.95 \text{ cm}^2}$$

2. Area of a Regular Hexagon

Question:

Calculate the area of a regular hexagon with a side length of 8 cm. Use $\sqrt{3} = 1.732$.

Solution:

$$\text{Area} = \frac{3\sqrt{3}}{2} \times \text{side}^2$$

$$= \frac{3 \times 1.732}{2} \times 64 = 2.598 \times 64 = \boxed{166.27 \text{ cm}^2}$$

3. Surface Area of a Rectangular Prism

Question:

Find the surface area of a rectangular prism with dimensions 5 cm (length), 3 cm (width), and 10 cm (height).

Solution:

$$\text{Surface Area} = 2(lw + lh + wh)$$

$$= 2[(5 \times 3) + (5 \times 10) + (3 \times 10)] = 2(15 + 50 + 30) = 2 \times 95 = \boxed{190 \text{ cm}^2}$$

4. Area of a Sector (Degrees)

Question:

Determine the area of a sector in a circle of radius 7 cm and a central angle of 60°. Use $\pi = \frac{22}{7}$.

Solution:

$$\text{Area} = \frac{\theta}{360^\circ} \times \pi r^2 = \frac{60}{360} \times \frac{22}{7} \times 49 = \frac{1}{6} \times 154 = \boxed{25.67 \text{ cm}^2}$$

5. Area of a Segment (Degrees)

Question:

Find the area of a circular segment created by a 90° angle in a circle with a radius of 14 cm. Use $\pi = \frac{22}{7}$.

Solution:

- **Sector Area:**

$$\frac{90}{360} \times \frac{22}{7} \times 14^2 = \frac{1}{4} \times 616 = 154 \text{ cm}^2$$

- **Triangle Area:**

$$\frac{1}{2} \times 14 \times 14 = 98 \text{ cm}^2$$

- **Segment Area:**

$$154 - 98 = \boxed{56 \text{ cm}^2}$$

6. Lateral Surface Area of a Triangular Prism

Question:

A triangular prism has a base with sides 3 cm, 4 cm, and 5 cm, and a prism height of 10 cm. Calculate the lateral surface area.

Solution:

$$\text{Lateral SA} = \text{Base Perimeter} \times \text{Height} = (3 + 4 + 5) \times 10 = 12 \times 10 = \boxed{120 \text{ cm}^2}$$

7. Area of an Irregular Pentagon

Question:

An irregular pentagon is split into a rectangle (4 cm × 6 cm) and a triangle (base = 4 cm, height = 3 cm). Find the total area.

Solution:

- Rectangle:

$$4 \times 6 = 24 \text{ cm}^2$$

- Triangle:

$$\frac{1}{2} \times 4 \times 3 = 6 \text{ cm}^2$$

- Total Area:

$$24 + 6 = \boxed{30 \text{ cm}^2}$$

8. Surface Area of a Hexagonal Prism

Question:

Find the total surface area of a regular hexagonal prism with side length 5 cm and height 12 cm. Use $\sqrt{3} = 1.732$.

Solution:

- Base Area:

$$\frac{3\sqrt{3}}{2} \times 25 = 64.95 \text{ cm}^2$$

- Lateral Area:

$$6 \times 5 \times 12 = 360 \text{ cm}^2$$

- Total Surface Area:

$$2 \times 64.95 + 360 = \boxed{489.9 \text{ cm}^2}$$

9. Sector Area (Radians)

Question:

Calculate the area of a sector with radius 10 cm and angle $\frac{\pi}{3}$ radians.

Solution:

$$\text{Area} = \frac{1}{2}r^2\theta = \frac{1}{2} \times 100 \times \frac{\pi}{3} = \frac{50\pi}{3} \approx \boxed{52.36 \text{ cm}^2}$$

10. Segment Area from Chord (Challenge)

Question:

A chord of length 12 cm creates a segment in a circle with radius 10 cm. Find the segment's area. Use $\pi = 3.14$.

Solution:

- **Central Angle (θ):**

$$\sin\left(\frac{\theta}{2}\right) = \frac{6}{10} \Rightarrow \theta \approx 73.74^\circ$$

- **Sector Area:**

$$\frac{73.74}{360} \times 3.14 \times 100 \approx 64.37 \text{ cm}^2$$

- **Triangle Area:**

$$\frac{1}{2} \times 10 \times 10 \times \sin(73.74^\circ) \approx 48 \text{ cm}^2$$

- **Segment Area:**

$$64.37 - 48 = \boxed{16.37 \text{ cm}^2}$$