

Key Matrix Concepts

1. **Addition/Subtraction:** Same dimensions, element-wise.
2. **Multiplication:** Rows \times columns (dot product).
3. **Determinant:** For 2×2 : $ad - bc$.
4. **Inverse:** $A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.
5. **Applications:** Solving systems, transformations, data organization.

1. Matrix Addition

Q: Add $\begin{bmatrix} 2 & 5 \\ -1 & 3 \end{bmatrix}$ and $\begin{bmatrix} 4 & -2 \\ 0 & 7 \end{bmatrix}$.

Solution:

$$\begin{bmatrix} 2 + 4 & 5 + (-2) \\ -1 + 0 & 3 + 7 \end{bmatrix} = \boxed{\begin{bmatrix} 6 & 3 \\ -1 & 10 \end{bmatrix}}$$

2. Scalar Multiplication

Q: Multiply $\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$ by 4.

Solution:

$$4 \times \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} 12 & -4 \\ 8 & 16 \end{bmatrix}}$$

3. Matrix Multiplication

Q: Multiply $\begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$ by $\begin{bmatrix} 4 & -1 \\ 2 & 5 \end{bmatrix}$.

Solution:

$$\begin{bmatrix} (1 \times 4 + 3 \times 2) & (1 \times -1 + 3 \times 5) \\ (2 \times 4 + 0 \times 2) & (2 \times -1 + 0 \times 5) \end{bmatrix} = \boxed{\begin{bmatrix} 10 & 14 \\ 8 & -2 \end{bmatrix}}$$

4. Determinant of a 2×2 Matrix

Q: Find the determinant of $\begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}$.

Solution:

$$\det = (5 \times 4) - (2 \times 3) = 20 - 6 = \boxed{14}$$

5. Solving Equations with Matrices

Q: Solve for x and y :

$$\begin{cases} 2x + 3y = 8 \\ 4x - y = 6 \end{cases} \quad \text{using matrices.}$$

Solution:

Convert to matrix form $AX = B$:

$$\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

Find A^{-1} and multiply:

$$X = A^{-1}B = \boxed{\begin{bmatrix} 2 \\ 1 \end{bmatrix}} \quad (\text{Solution : } x = 2, y = 1)$$

6. Transpose of a Matrix

Q: Find the transpose of $\begin{bmatrix} 1 & -2 & 3 \\ 0 & 4 & 5 \end{bmatrix}$.

Solution:

$$\text{Transpose} = \boxed{\begin{bmatrix} 1 & 0 \\ -2 & 4 \\ 3 & 5 \end{bmatrix}}$$

7. Identity Matrix

Q: Multiply $\begin{bmatrix} 2 & 7 \\ -1 & 3 \end{bmatrix}$ by the 2×2 identity matrix.

Solution:

$$\begin{bmatrix} 2 & 7 \\ -1 & 3 \end{bmatrix} \times I = \boxed{\begin{bmatrix} 2 & 7 \\ -1 & 3 \end{bmatrix}} \quad (\text{Identity matrix leaves})$$

8. Real-World Application (Inventory)

Q: A store sells apples (A) and bananas (B). Matrix $P = \begin{bmatrix} 20 & 30 \end{bmatrix}$ shows stock, and matrix $Q = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ shows price per item. Calculate total value.

Solution:

$$PQ = [20 \times 2 + 30 \times 1] = \boxed{70 \text{ dollars}}$$

9. Inverse of a 2×2 Matrix

Q: Find the inverse of $\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$.

Solution:

$$A^{-1} = \frac{1}{(3 \times 4 - 1 \times 2)} \begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix} = \boxed{\begin{bmatrix} 0.4 & -0.1 \\ -0.2 & 0.3 \end{bmatrix}}$$

10. System of Equations (3 Variables)

Q: Use matrices to solve:

$$\begin{cases} x + y + z = 6 \\ 2y + 5z = -4 \\ 2x + 5y - z = 27 \end{cases}$$

Solution:

Convert to $AX = B$:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 2 & 5 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 27 \end{bmatrix}$$

Using row reduction or inverse:

$$X = \boxed{\begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}} \quad (\text{Solution : } x = 5, y = 3, z = -2)$$