Valuing American Options using Monte-carlo Methods

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American Options

- An American option is one in which the holder has the right to exercise the option on or before the expiration date.
- American options allow option holders to exercise the option at any time prior to and including its maturity date, thus increasing the value of the option to the holder relative to European options, which can only be exercised at maturity.
- Exercise time “t” can be represented as a stopping time American options are an example of optimal stopping time problem.

Bermudan Options

- Bermudan options are a combination of both American and European options.
- A Bermudan Option is a type of nonstandard American option with early exercise restricted to certain dates during the life of the option.
- Bermuda option is exotic option that can be exercised on a predetermined date typically every month.
- When the no of intervals of bermuda option is increased then it approximates american option.

Monte Carlo Methods

- Valuation of American-Options is one of the most challenging problems in derivatives finance.This is because when more than one value depends on the value of the option then the finite-difference and binomial methods become impractical because there will be so many paths corresponding factors that the tree will be huge for small values of exipary time.
- There is no closed-form available for valuation of american options. There are approximations but a single approximation does not work for all values of interest rate or volatality.(i.e as rate and volatality changes the error increases).
- Simulation based method give good approximations for the valuation of american options

Least Square Method

This is a technique used to fit a function to given set of data (similarly to regression).So let the function be a kth degree polynomial

$$y = a_0 + a_1 x + a_2 x^2 + \ldots . a_k x^k$$

And we have a set of data points (x_1,y_1), (x_2 , y_2) ... (x_n,y_n) where n>=3.

The best fitting function should have least mean square error.Given below is the error function.

$$\prod = \sum_{i=1}^{n} [y_i - f(x_i)]^2$$

- Here $a_0, a_1 \ldots\ldots a_k$ are unknown constants which we have to determine such that mean square error is minimized.To obtain least square error the first derivative of the error function w.r.t a_i should be zero.

$$\frac{\partial \prod}{\partial a_0} = 2\sum_{i=1}^{n}[y_i - (a_0 + a_1 x_i + a_2 x_i^2 + ... + a_k x_i^k)] = 0$$

$$\frac{\partial \prod}{\partial a_1} = 2\sum_{i=1}^{n} x_i[y_i - (a_0 + a_1 x_i + a_2 x_i^2 + ... + a_k x_i^k)] = 0$$

....

$$\frac{\partial \prod}{\partial a_k} = 2\sum_{i=1}^{n} x_i^k[y_i - (a_0 + a_1 x_i + a_2 x_i^2 + ... + a_k x_i^k)] = 0$$

$$\sum_{i=1}^{n} y_i = a_0 \sum_{i=1}^{n} 1 + a_1 \sum_{i=1}^{n} x_i + a_2 \sum_{i=1}^{n} x_i^2 + \ldots.a_k \sum_{i=1}^{n} x_i^k.$$

$$\sum_{i=1}^{n} x_i y_i = a_0 \sum_{i=1}^{n} x_i + a_1 \sum_{i=1}^{n} x_i^2 + a_2 \sum_{i=1}^{n} x_i^3 + \ldots.a_k \sum_{i=1}^{n} x_i^{k+1}.$$

....

$$\sum_{i=1}^{n} x_i^k y_i = a_0 \sum_{i=1}^{n} x_i^k + a_1 \sum_{i=1}^{n} x_i^{k+1} + a_2 \sum_{i=1}^{n} x_i^{k+2} + \ldots.a_k \sum_{i=1}^{n} x_i^{2k}.$$

LSM applied to Bermuda option.

So let's consider a bermudan put option where exercise is possible now and at three future dates t= 1, 2, 3. For the put option S_0=1, X=1.1, r=0.06.

So now we will simulate some paths that the stock follows with Geometric brownian motion.So basically at t=0 the price of the stock is S0=1 anf after that we will put the above values in to GBM equation $S_1 = S_0\, e^{\wedge}(\mu-\sigma^{\wedge}2)/2*t+\sigma*W_t$

And simulate it for three time intervals.

Stock price simulation table

Path	t=0	t=1	t=2	t=3
1	1.00	1.09	1.08	1.34
2	1.00	1.16	1.26	1.54
3	1.00	1.22	1.07	1.03
4	1.00	0.93	0.97	0.92
5	1.00	1.11	1.56	1.52
6	1.00	0.76	0.77	0.90
7	1.00	0.92	0.84	1.01
8	1.00	0.88	1.22	1.34

Cash flow for t=3

Path	t=1	t=2	t=3
1			0.00
2			0.00
3			0.07
4			0.18
5			0.00
6			0.20
7			0.09
8			0.00

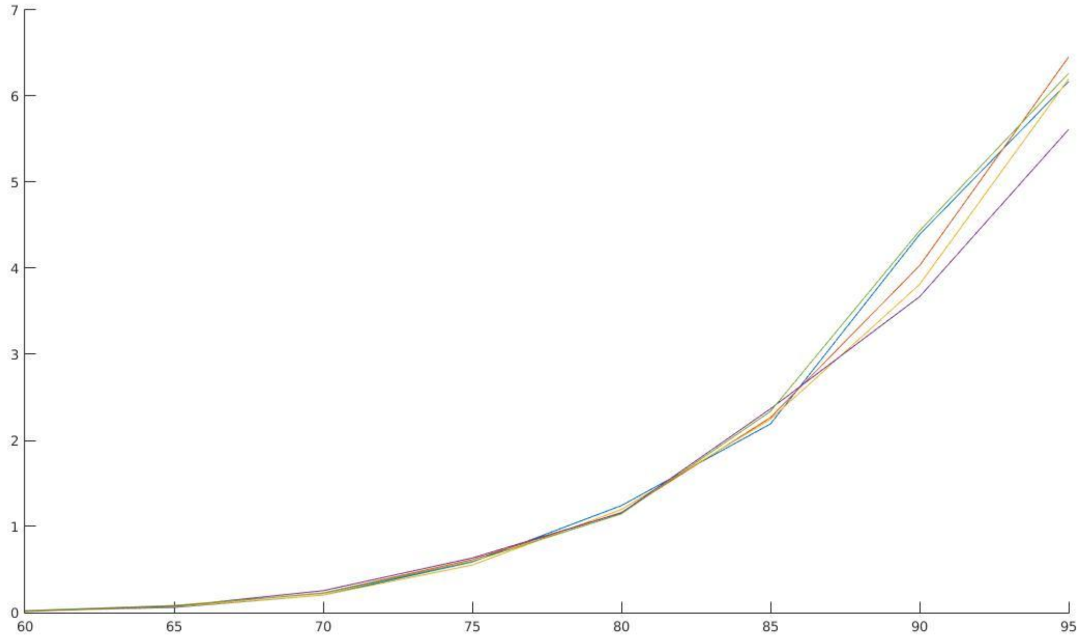
Cash flow at time t=2 generated recursively

Path	t=1	t=2	t=3
1		0.00	0.00
2		0.00	0.00
3		0.00	0.07
4		0.13	0.00
5		0.00	0.00
6		0.33	0.00
7		0.26	0.00
8		0.00	0.00

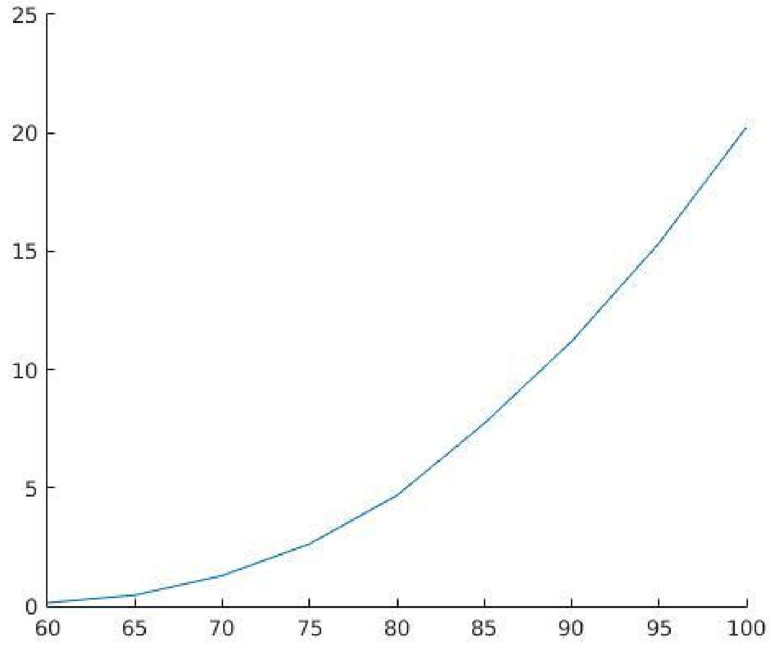
Similarly we can generate for t=1.

Path	t=1	t=2	t=3
1	0.00	0.00	0.00
2	0.00	0.00	0.00
3	0.00	0.00	0.07
4	0.17	0.00	0.00
5	0.00	0.00	0.00
6	0.34	0.00	0.00
7	0.18	0.00	0.00
8	0.22	0.00	0.00

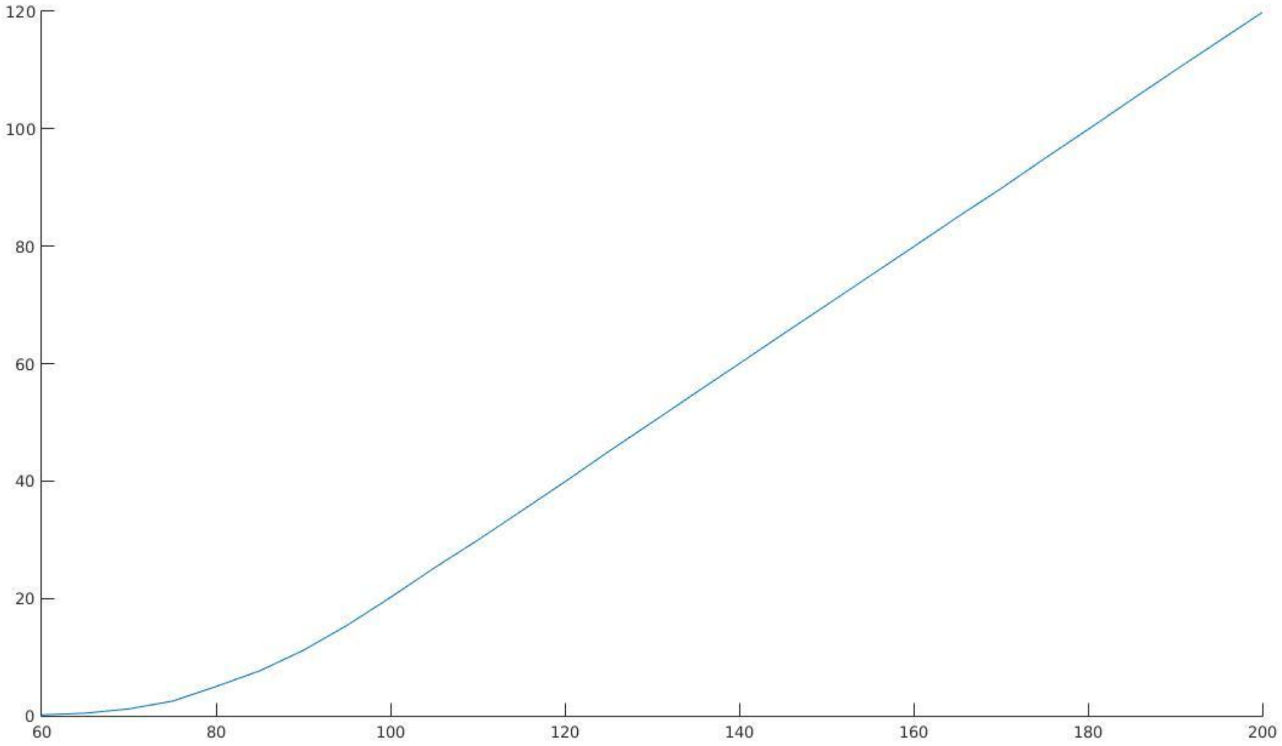
Simulation Results



S01=60 , S02=80 and strike price is varied from 60 to 95. As the maturity time is increased the value of the option increases by a little bit.



S01=60,S02=80 ,Stirke=100 time-to-mature 1



Parameters are same as the previous slide just strike price range is increased

References

1) Francis A. Longstaff Eduardo S. Schwartz ,*Valuing American Options by Simulation:A Simple Least-Squares Approach* , Review of Financial Studies.