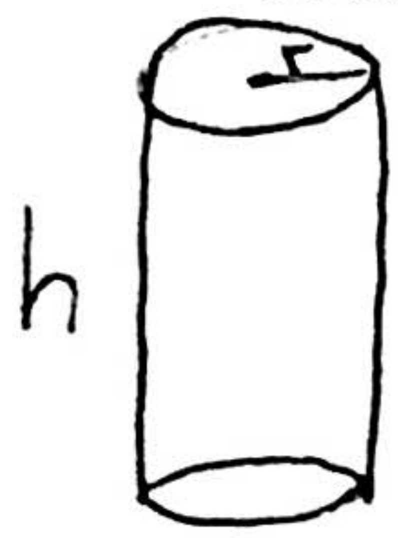
**Midterm 3**

#1. (7 points) The radius of a cylinder is increasing at a rate of 1 meter per hour, and the height of the cylinder is decreasing at a rate of 4 meters per hour. At a certain instant, the base radius is 5 meters and the height is 8 meters. What is the rate of change of the volume of the cylinder at the instant?




$\frac{dr}{dt} = 1 m/hr \quad \frac{dh}{dt} = -4 m/hr$ when $r=5$ and $h=8$, find $\frac{dV}{dt}$

$V = \pi r^2 h$

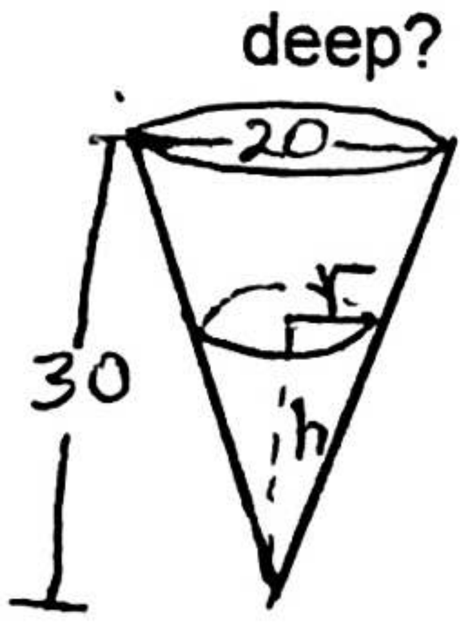
$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$

$= 2\pi(5)(8)(1) + \pi(5)^2(-4)$

$= 80\pi - 100\pi$

$\frac{dV}{dt} = -20\pi \ m^3/hr$

#2. (7 points) A funnel in the shape of an inverted cone is 30 cm deep and has a diameter across the top of 20 cm. Liquid is flowing out of the funnel at a rate of $12\ cm^3\ per\ second$. At what rate is the height of liquid decreasing at the instant that the liquid in the cone is 20 cm deep?




$\frac{h}{r} = \frac{30}{10} \Rightarrow r = \frac{h}{3} \qquad \frac{dV}{dt} = -12\ cm^3/sec \quad h = 20\ cm$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$

$V = \frac{1}{27}\pi h^3$

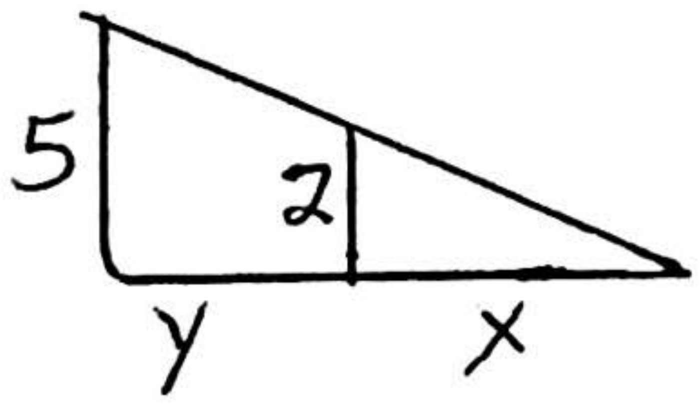
$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt} \Rightarrow \frac{1}{9}\pi(20)^2 \frac{dh}{dt} = -12$

$\frac{dh}{dt} = \frac{-27}{100\pi}$

#3. (7 points) A person 2 meters tall walks toward a lamppost on level ground at a rate of 0.75 m/sec. The lamp on the post is 5 meters high. Find the rate at which the length of the shadow is changing when the person is 3 meters from the post.

$\frac{dy}{dt} = -.75$

x = length of shadow




$\frac{2}{5} = \frac{x}{y+x} \Rightarrow 2y + 2x = 5x$

$2y = 3x$

$2\frac{dy}{dt} = 3\frac{dx}{dt}$

$2(-.75) = 3\frac{dx}{dt}$

$\frac{-1.50}{3} = \frac{dx}{dt}$

$\frac{dx}{dt} = -.5\ m/sec$

#4. (5 points) Find the equation for a linear approximation to the function $f(x) = 3xe^{2x-10}$, at the point where x = 5.

$f'(x) = 3x \cdot e^{2x-10} \cdot 2 + 3e^{2x-10}$

$f'(5) = 3(5)\, e^{0} \cdot 2 + 3e^{0} = 33$

$m = 33 \quad f(5) = 15$

$L = 33(x-5) + 15$

$\underline{L = 33x - 150}$

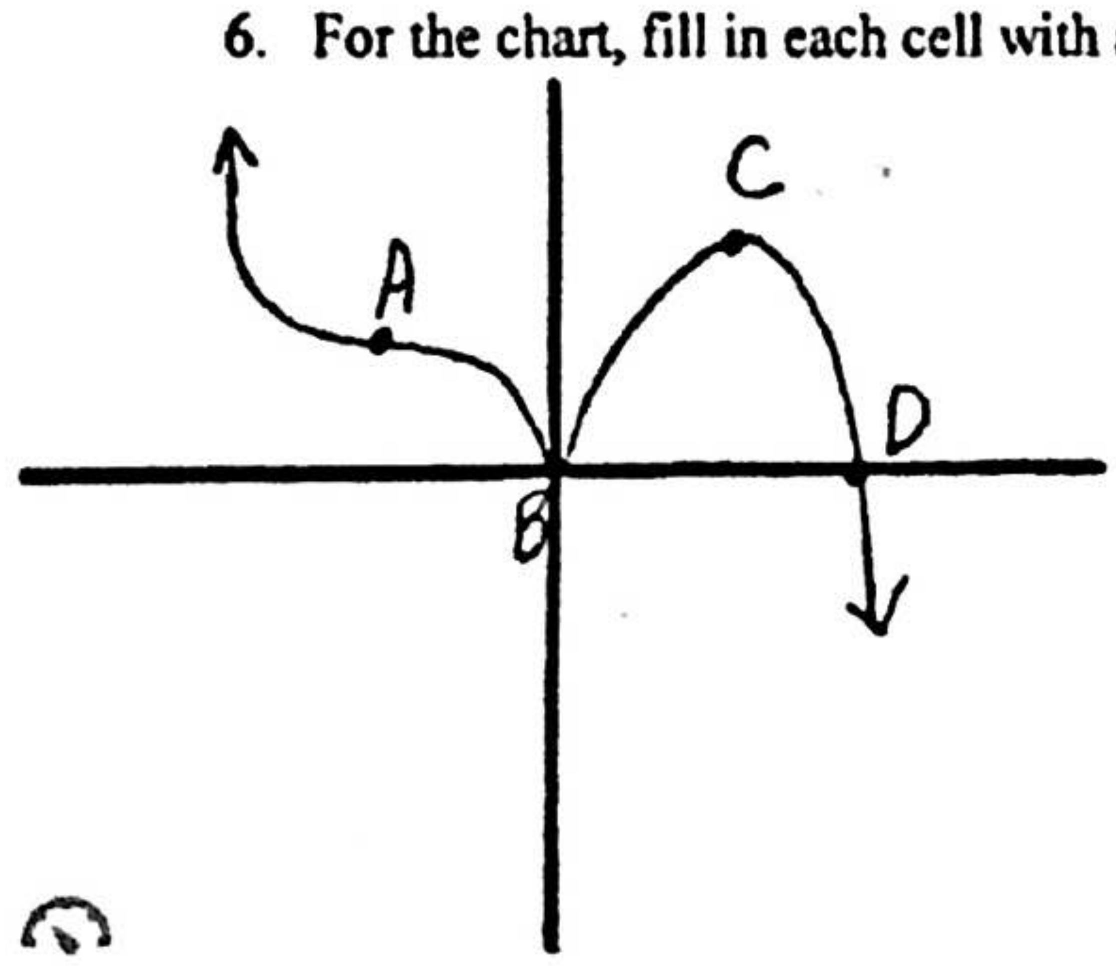
#5. (5 points) Find dy for $y = e^{x^2}$.

$dy = e^{x^2} \cdot 2x\, dx$

$\underline{dy = 2xe^{x^2}\, dx}$

#6. (6 points)

6. For the chart, fill in each cell with a +, -, 0, or DNE after examining the graph.




| X | F(x) | F'(x) | F"(x) |
|---|---|---|---|
| A | + | 0 | 0 |
| B | 0 | DNE | DNE |
| C | + | 0 | — |
| D | 0 | — | — |

**#7. (5 points) Estimate the error in the computed volume of a cube if the side length is measured to be 6 cm with an accuracy of 0.20 cm.**

Show work using the calculus of differentials. A calculator answer will not be accepted.

$V = x^3$

$x = 6$

$dx = .2$

$dV = 3x^2 dx$

$dV = 3(6)^2(.2)$

$dV = \frac{108}{5}$

$dV = 21.6 \text{ cm}^3$ OR $\frac{108}{5} \text{ cm}^3$

**#8. (5 points) Use the linear approximation method to approximate √8.5**

Answer must be written exactly and all work must be shown. A calculator answer will not be accepted.

$y = \sqrt{x}$ use $x = 9$ $dx = -.5$, $y = 3$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{6}$

$L = \frac{1}{6}(x-9)+3$

$L(8.5) = \frac{1}{6}(8.5-9)+3 = -\frac{1}{12}+3 = \frac{35}{12}$

**#9. (5 points) Find y' and simplify completely.**

$y = \ln(x^2 + y^2)$.

$y' = \frac{2x+2yy'}{x^2+y^2} = \frac{2x}{x^2+y^2} + \frac{2y}{x^2+y^2}y'$

$y' - \left(\frac{2y}{x^2+y^2}\right)y' = \frac{2x}{x^2+y^2}$

$y'\left(1 - \frac{2y}{x^2+y^2}\right) = \frac{2x}{x^2+y^2}$

$y'\left(\frac{x^2+y^2-2y}{x^2+y^2}\right) = \frac{2x}{x^2+y^2}$

$y' = \frac{2x}{x^2+y^2-2y}$

#10. (5 points) If $f(x) = x^3 - 12x + 5$ satisfies the Mean Value Theorem on the interval [1, 6], find the exact value(s) of all c values that satisfy the conclusion of the Mean Value Theorem.

$f(1) = -6 \quad f(6) = 149$

$\frac{149 - -6}{6-1} = \frac{155}{5} = 31$

find where $f'(c) = 31$

$f' = 3x^2 - 12$

$31 = 3x^2 - 12$

$43 = 3x^2$

$\frac{43}{3} = x^2$

$x = \pm\sqrt{\frac{43}{3}}$

$-\sqrt{\frac{43}{3}}$ is not in interval

$c = \sqrt{\frac{43}{3}}$

#11 (5 points) Differentiate and simplify:

$y = (\ln x)^{\sin x}$

$\ln y = \ln\left[(\ln x)^{\sin x}\right]$

$\ln y = \sin x \ln[\ln x]$

$\frac{1}{y}\frac{dy}{dx} = \cos x\left[\ln(\ln x)\right] + \sin x \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$

$\frac{dy}{dx} = y\left[\cos x\left[\ln(\ln x)\right] + \frac{\sin x}{x \ln x}\right]$

$\frac{dy}{dx} = (\ln x)^{\sin x}\left[\cos x\left[\ln(\ln x)\right] + \frac{\sin x}{x \ln x}\right]$

OR $y = e^{\ln\left[(\ln x)^{\sin x}\right]}$

$y = e^{\sin x \cdot \ln(\ln x)}$

$y' = e^{\sin x \cdot \ln(\ln x)}\left[\cos x\left[\ln(\ln x)\right] + \frac{\sin x}{x \ln x}\right]$

$y' = (\ln x)^{\sin x}\left[\cos x\left[\ln(\ln x)\right] + \frac{\sin x}{x \ln x}\right]$

#12 (6 points) Find the absolute maximum and absolute minimum on the interval [1, 9] for the function $f(x) = x + \frac{9}{x}$.

critical values where $f' = 0$ or $f'$ is undefined

Show all work.

$f' = 1 - \frac{9}{x^2}$

$f'$ is undefined when $x = 0$
however $x = 0$ is not $[1, 9]$

$0 = 1 - \frac{9}{x^2}$

Max + mins occur at critical values and end points

$\frac{9}{x^2} = 1$

$x^2 = 9$

$x = \pm 3$

only $+3$ is $[1, 9]$

(3, 6)

(1, 10)

(9, 10)

Thus (3, 6) is abs min.
(1, 10) and (9, 10) are abs max

#13 (6 points) Find the limit: $lim_{x \to \frac{\pi}{2}} \frac{1 - sinx}{1 + \cos(2x)}$ $\frac{0}{0}$

$$\lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{2 \sin 2x} = \frac{-0}{0}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\sin x}{-4 \cos 2x} \quad \boxed{\frac{1}{4}}$$

#14 (6 points) Find the limit: $lim_{x \to 0} \frac{e^x - 1}{3x - x^2}$ $= \frac{0}{0}$

$$\lim_{x \to 0} \frac{e^x}{3 - 2x} = \boxed{\frac{1}{3}}$$

#15 (6 points) Find the exact limit: $lim_{\alpha \to 0} \frac{\tan(7\alpha)}{\sin(3\alpha)}$ $\frac{0}{0}$

$$\lim_{\alpha \to 0} \frac{7 \sec^2 7\alpha}{3 \cos 3\alpha} = \boxed{\frac{7}{3}}$$

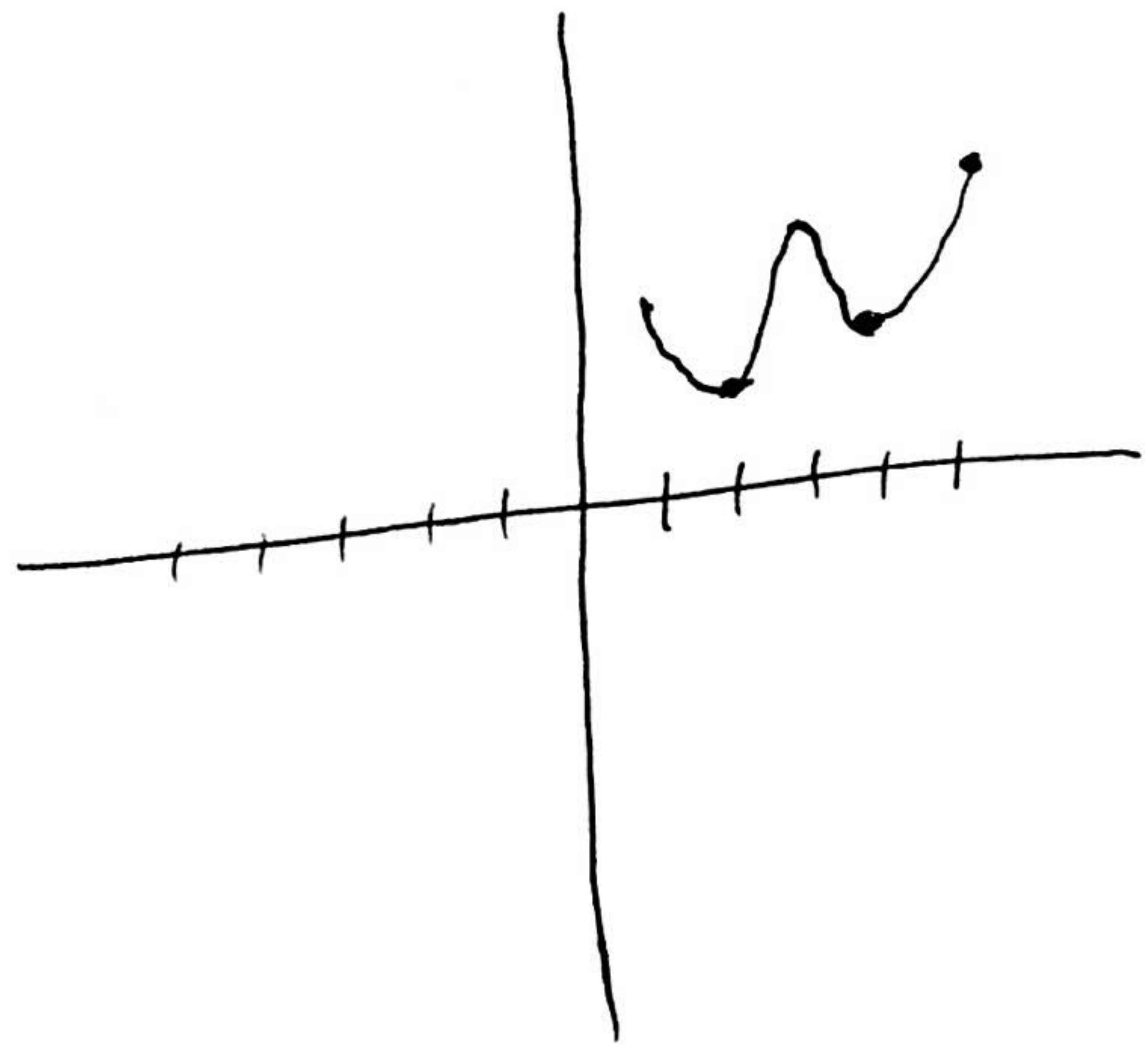
#16 (7 points) Sketch the graph of a function $f$ that is continuous on the interval [1, 5] and has the given properties:

Absolute maximum at x = 5

Absolute minimum at x = 2

Local maximum at x = 3

Local minima at x = 2 and at x = 4



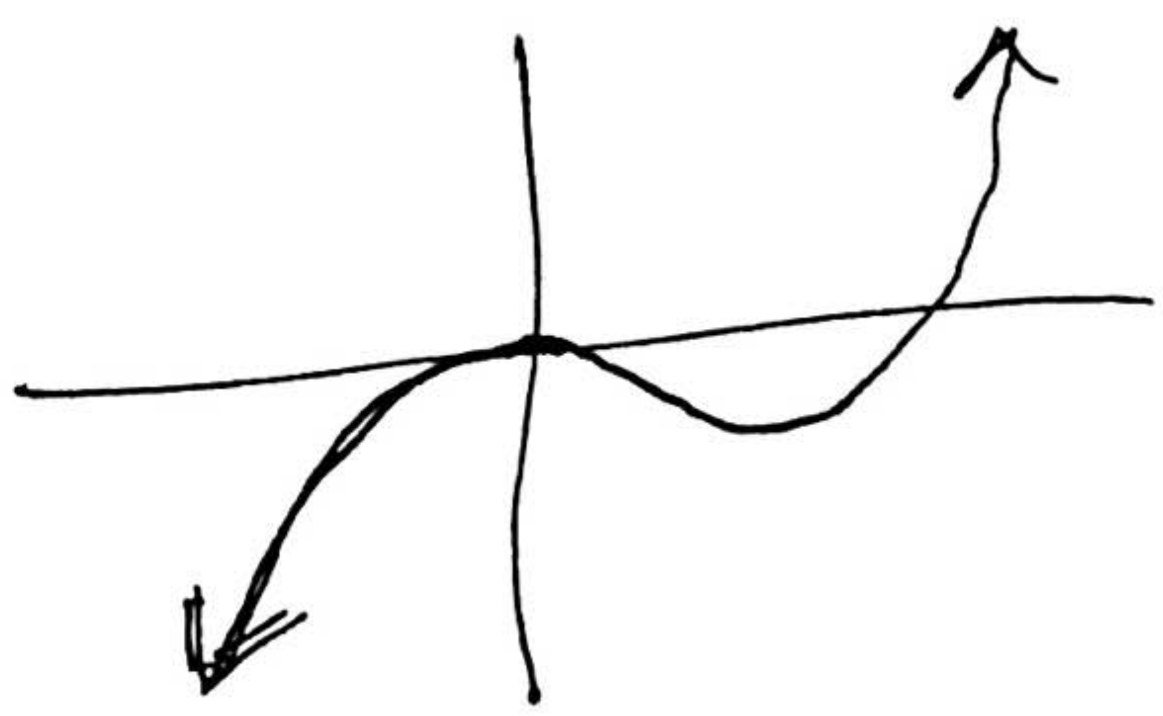
#17 (7 points) Sketch the function $f(x) = 2x^5 - 3x^4$ on the axes provided. Label points on the graph. Also, show the intervals where the function is increasing/decreasing and concave up/concave down. Also list any minimum values, maximum values, and inflection points. You must give the exact value for all of these points - no decimal approximations.

$f' = 10x^4 - 12x^3$ $\qquad 2x^3(5x-6) \qquad x=0 \quad x=\frac{6}{5}$

$f'' = 40x^3 - 36x^2$ $\qquad 4x^2(10x-9) \qquad x=0 \quad x=\frac{9}{10}$

$f'$: $+$ | $0$ | $-$ | $\frac{6}{5}$ | $+$

$f''$: $-$ | $0$ | $-$ | $\frac{9}{10}$ | $+$



increasing : $(-\infty, 0) \cup (\frac{6}{5}, \infty)$

decreasing : $(0, \frac{6}{5})$

rel min $(\frac{6}{5}, \frac{-3888}{3125})$
$\approx -1.24416$

concave up : $(\frac{9}{10}, \infty)$

concave down : $(-\infty, \frac{9}{10})$

inflection : $(0,0)$ $(\frac{9}{10}, \frac{-19,683}{25,000})$
$-.78732$

rel max $(0,0)$