

1 Algebra

1.1 Exponent Properties

$$\frac{a^n}{a^m} = a^{n-m} \quad (1)$$

$$x^a y^a = (xy)^a \quad (2)$$

$$x^{\left(\frac{a}{b}\right)} = \sqrt[b]{x^a} \quad (3)$$

1.2 Properties of radicals

$$\sqrt[n]{a} = a^{\frac{1}{n}} \quad (4)$$

$$\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b} \quad (5)$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a} \quad (6)$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad (7)$$

$$\sqrt[n]{a^n} = |a|, \text{ if } n \text{ is even} \quad (8)$$

1.3 Complex numbers

$$(a + bi)(c + di) = ac - bd + (ad + bc)i \quad (9)$$

$$(a + bi)(a - bi) = a^2 + b^2 \quad (10)$$

$$|a + bi| = \sqrt{a^2 + b^2} \quad \text{Complex Modulus} \quad (11)$$

$$\overline{(a + bi)} = a - bi \quad (12)$$

$$z_1 \cdot z_2 = r_1 \cdot r_2 e^{i(\theta_1 + \theta_2)} \quad (13)$$

$$z^{\frac{1}{n}} = \sqrt[n]{r} \cdot e^{i\left(\frac{\phi}{n} + \frac{2k\pi}{n}\right)}; \quad k = 0, 1, \dots, n-1 \quad (14)$$

$$e^{ni\theta} = \cos n\theta + i \sin n\theta \quad \text{De Moivre's Formula} \quad (15)$$

1.4 Logarithms

$$\log_b b = 1 \quad (16)$$

$$\log_b 1 = 0 \quad (17)$$

$$\log_b(x^r) = r \log_b x \quad (18)$$

$$\log_b(xy) = \log_b(x) + \log_b(y) \quad (19)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y) \quad (20)$$

$$\log_b(x) = \log_b(c) \log_c(x) = \frac{\log_c(x)}{\log_c(b)} \quad (21)$$

1.5 Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{when } ax^2 + bx + c = 0 \quad (22)$$

2 Linear Algebra

Matrix addition: one by one. (commutative, associative)

Scalar multiplication: all.

Matrix "multiplication of rows into columns". Multiplication is not commutative ($AB \neq BA$).

$$c_{jk} = \sum_{i=1}^n a_{ji} b_{ik} \quad (23)$$

Inner or dot product of Vectors

$$\langle a, b \rangle = \mathbf{a} \bullet \mathbf{b} = \mathbf{a}^T \mathbf{b} \quad (24)$$

Matrix to the power $A^0 = I$

Inverse:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad (25)$$

Identities

$$(AB)^T = B^T A^T \quad (26)$$

$$(A + B)^T = A^T + B^T \quad (27)$$

$$(AB)^{-1} = B^{-1} A^{-1} \quad (28)$$

$$A^k B^l = A^{k+l} \quad (29)$$

Conjugate transpose / adjugate

$$A^* = (\overline{A})^T = \overline{A^T} \quad (30)$$

Determinants

$$\det(\mathbf{A}) = \sum_{\sigma \in S_n} \text{sgn}(\sigma) \prod_{i=1}^n A_{i, \sigma_i} \quad (31)$$

For 3×3 matrices (Sarrus rule)

$$\begin{array}{ccccc} & + & & + & & + & & \\ & a_{11} & & a_{12} & & a_{13} & & \\ & \swarrow & & \nearrow & & \nearrow & & \\ a_{21} & & a_{22} & & a_{23} & & a_{21} & & a_{22} \\ & \swarrow & & \nearrow & & \nearrow & & \\ a_{31} & & a_{32} & & a_{33} & & a_{31} & & a_{32} \\ & - & & - & & - & & \end{array} \quad (32)$$

$$\det(A \cdot B) = \det(A) \cdot \det(B) \quad (33)$$

$$\det(A^{-1}) = \det(A)^{-1} \quad (34)$$

$$\det(rA) = r^n \det(A) \quad \text{for all } A^{n \times n} \text{ and scalars } r \quad (35)$$

The determinant of a triangular matrix equals the product of the diagonal entries. Since for any triangular matrix A the matrix $\lambda I - A$, whose determinant is the characteristic polynomial of A , is also triangular, the diagonal entries of A in fact give the multiset of eigenvalues of A (an eigenvalue with multiplicity m occurs exactly m times as diagonal entry)

2.1 Transpose

$$[A^T]_{ij} = [A]_{ji} \quad (36)$$

$$(A^T)^T = A \quad (37)$$

$$(AB)^T = B^T A^T \quad (38)$$

$$\det(A^T) = \det(A) \quad (39)$$

$$(A^T)^{-1} = (A^{-1})^T \quad (40)$$

3 Trigonometry

3.1 Definitions

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad (41)$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} \quad (42)$$

Θ	0°	30°	45°	60°	90°
$\sin \Theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \Theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \Theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	/

Table 1: Trigonometric functions standard values

3.2 Formulas and Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad (43)$$

Pythagorean identities

$$\sin^2 \theta + \cos^2 \theta = 1 \quad (44)$$

Odd/Even formulas

$$\sin(-\theta) = -\sin \theta \quad (45)$$

$$\cos(-\theta) = \cos \theta \quad (46)$$

Sum and difference formulas

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \quad (47)$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \quad (48)$$

Double angle formulas

$$\sin 2\theta = 2 \sin \theta \cos \theta \quad (49)$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 \quad (50)$$

Half angle formulas

$$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos(\theta)}{1 + \cos(\theta)}} \quad (51)$$

Euler's theorem

$$e^{\pm i\theta} = \cos \theta \pm i \sin \theta \quad (52)$$

$$\cos \theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta}) \quad (53)$$

$$\sin \theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta}) \quad (54)$$

Miscellaneous formulas

$$\sin^2 \theta = \frac{1 - \cos(2\theta)}{2} \quad (55)$$

$$\cos^2 \theta = \frac{1 + \sin(2\theta)}{2} \quad (56)$$

4 Calculus

4.1 Limits

4.1.1 Properties

$$\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x) \quad (57)$$

L'Hopital's Rule

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)} \quad (58)$$

4.1.2 Evaluations

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad (59)$$

$$\lim_{x \rightarrow -\infty} e^x = 0 \quad (60)$$

4.2 Derivatives

4.2.1 Definition

$$\frac{d}{dx} f(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad (61)$$

4.2.2 Properties

Product rule

$$(fg)' = f'g + fg' \quad (62)$$

Chain rule

$$\frac{d}{dx} [f(u)] = \frac{d}{du} [f(u)] \frac{du}{dx} \quad \text{or} \quad (f(g(x)))' = f'(g(x))g'(x) \quad \text{or} \quad (f \circ g)' = (f' \circ g) \cdot g' \quad (63)$$

Quotient Rule

$$\left[\frac{f(x)}{g(x)} \right]' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}. \quad (64)$$

4.2.3 Common Derivatives

$$\frac{d}{dx} (a^x) = a^x \ln(a) \quad (65)$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}, \quad x > 0 \quad (66)$$

Power rule

$$\frac{d}{dx} x^n = nx^{(n-1)} \quad (67)$$

$$\frac{d}{dx} \cos x = -\sin x \quad (68)$$

$$\frac{d}{dx} \sin x = \cos x \quad (69)$$

4.3 Integrals

4.3.1 Fundamental Theorem of Calculus

$$\int_a^b \frac{d}{dx} F(x) dx = F(b) - F(a) \quad (70)$$

4.3.2 Properties

$$\int k dx = kx + C \quad (71)$$

4.3.3 Common Integrals

$$\int k dx = kx + C \quad (72)$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1 \quad (73)$$

$$\int \frac{1}{x} dx = \ln |x| + C \quad (74)$$

$$\int \ln u = u \ln(u) - u + C \quad (75)$$

$$\int e^x dx = e^x + C \quad (76)$$

$$\int \sin ax = -\frac{1}{a} \cos ax + C \quad (77)$$

$$\int \cos x = \sin x + C \quad (78)$$

Per partes

$$\int u \frac{dv}{dx} dx = uv - \int \frac{du}{dx} v dx \quad (79)$$

Substitution Rule

$$\int f(u) \frac{du}{dx} dx = f(u) du \quad (80)$$

4.4 Laplace transforms

4.4.1 Definition

$$X(s) = \int_0^{\infty} x(t) e^{-st} dt \quad (81)$$

4.4.2 Properties

$$1 \Leftrightarrow \frac{1}{s} \quad (82)$$

Kroeneker delta function

$$\delta(t) \Leftrightarrow 1 \quad (83)$$

$$K e^{-at} u(t) \Leftrightarrow \frac{K}{s+a} \quad (84)$$

$$t^n u(t) \Leftrightarrow \frac{n!}{s^{n+1}} \quad (85)$$

$$\sin(\alpha t)u(t) \Leftrightarrow \frac{\alpha}{(s^2 + \alpha^2)} \quad (86)$$

$$\cos(\alpha t)u(t) \Leftrightarrow \frac{s}{(s^2 + \alpha^2)} \quad (87)$$

$$e^{-at} \sin(\Omega t)u(t) \Leftrightarrow \frac{\Omega}{(s + a)^2 + \Omega^2} \quad (88)$$

$$e^{-at} \cos(\Omega t)u(t) \Leftrightarrow \frac{s + a}{(s + a)^2 + \Omega^2} \quad (89)$$

$$e^{at}x(t) \Leftrightarrow X(s - a) \quad (90)$$

Time domain scaling

$$x(at)u(t) \Leftrightarrow \frac{1}{a}X\left(\frac{s}{a}\right) \quad (91)$$

Time domain shifting

$$x(t - a)u(t - a) \Leftrightarrow e^{-as}X(s + a) \quad (92)$$

Derivative

$$\frac{d^n x(t)}{dt^n} \Leftrightarrow s^n X(s) \quad \text{or} \quad \mathcal{L}[\dot{x}] = sX(s) - x(0+) \quad (93)$$

Integral

$$\int x(t)dt \Leftrightarrow \frac{X(s)}{s} \quad (94)$$

Convolution

$$\int_0^\infty x_1(\tau)x_2(t - \tau)d\tau \Leftrightarrow X_1(s)X_2(s) \quad (95)$$

5 Greek letters

αA	Alpha	νN	Nu
βB	Beta	$\xi \Xi$	Xi
$\gamma \Gamma$	Gamma	$o O$	Omicron
$\delta \Delta$	Delta	$\pi \Pi$	Pi
$\epsilon \varepsilon$	Epsilon	$\rho \varrho P$	Rho
ζZ	Zeta	$\sigma \Sigma$	Sigma
ηH	Eta	τT	Tau
$\theta \vartheta \Theta$	Theta	$\upsilon \Upsilon$	Upsilon
ιI	Iota	$\phi \varphi \Phi$	Phi
κK	Kappa	χX	Chi
$\lambda \Lambda$	Lambda	$\psi \Psi$	Psi
μM	Mu	$\omega \Omega$	Omega

Table 2: Greek letters