Calculus and Mathematics Cheatsheet

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Contents

1 Algebra

1.1 Exponent Properties

$$\frac{a^n}{a^m} = a^{n-m}$$

$$x^a y^a = (xy)^a$$

$$x^{\left(\frac{a}{b}\right)} = \sqrt[b]{x^a}$$

1.2 Properties of radicals

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[n]{a^n} = |a|, \quad \text{if } n \text{ is even}$$

1.3 Complex numbers

$$(a+bi)(c+di) = ac - bd + (ad+bc)i$$

$$(a+bi)(a-bi) = a^2 + b^2$$

$$|a+bi| = \sqrt{a^2+b^2} \quad \text{Complex Modulus}$$

$$\overline{(a+bi)} = a - bi$$

$$z_1 \cdot z_2 = r_1 \cdot r_2 e^{i(\theta_1+\theta_2)}$$

$$z^{\frac{1}{n}} = \sqrt[n]{r} \cdot e^{i\left(\frac{\phi}{n}+\frac{2k\pi}{n}\right)}; \quad k = 0, 1, .., n-1$$

$$e^{ni\theta} = \cos n\theta + \sin n\theta \quad \text{De Moivre's Formula}$$

1.4 Logarithms

$$\log_b b = 1$$

$$\log_b 1 = 0$$

$$\log_b(x^r) = r \log_b x$$

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

$$\log_b(x) = \log_b(c)\log_c(x) = \frac{\log_c(x)}{\log_c(b)}$$

1.5 Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{when } ax^2 + bx + c = 0$$

2 Linear Algebra

Matrix addition: one by one. (commutative, associative)

Scalar multiplication: all.

Matrix "multiplication of rows into columns". Multiplication is not commutative ($AB \neq BA$).

$$c_{jk} = \sum_{i=1}^{n} a_{ji} b_{ik}$$

Inner or dot product of Vectors

$$\langle a, b\rangle = \mathbf{a} \bullet \mathbf{b} = \mathbf{a}^T \mathbf{b}$$

Matrix to the power $A^0 = I$

Inverse:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(\mathbf{A})}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Identities

$$(AB)^T = B^T A^T$$

$$(A + B)^T = A^T + B^T$$

$$(AB)^{-1} = B^{-1} + A^{-1}$$

$$A^k B^l = A^{k+l}$$

Conjugate transpose / adjugate

$$A^* = (\overline{A})^{\mathrm{T}} = \overline{A^{\mathrm{T}}}$$

Determinants

$$\det(\mathbf{A}) = \sum_{\sigma \in S_n} \operatorname{sgn}(\sigma) \prod_{i=1}^{n} A_{i,\sigma_i}$$

For 3×3 matrices (Sarrus rule):

$$\begin{array}{ccc|cc} + & + & + & & \\ \hline a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \\ \hline - & - & - & & \end{array} \tag{1}$$

$$\det(A \cdot B) = \det(A) \cdot \det(B)$$

$$\det(A^{-1}) = \det(A)^{-1}$$

$$\det(rA) = r^n \det(A) \quad \text{for all } A^{n \times n} \text{ and scalars } r$$

The determinant of a triangular matrix equals the product of the diagonal entries. Since for any triangular matrix A the matrix $\lambda I - A$, whose determinant is the characteristic polynomial of A, is also triangular, the diagonal entries of A in fact give the multiset of eigenvalues of A (an eigenvalue with multiplicity m occurs exactly m times as diagonal entry)

2.1 Transpose

$$[A^{\mathrm{T}}]_{ij} = [A]_{ji}$$

$$(A^T)^T = A$$

$$(AB)^T = B^T A^T$$

$$\det(A^T) = \det(A)$$

$$(A^T)^{-1} = (A^{-1})^T$$

3 Trigonometry

3.1 Definitions

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Table 1: Trigonometric functions standard values

Θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\Theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\Theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\Theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	/

3.2 Formulas and Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Pythagorean identities

$$\sin^2\theta + \cos^2\theta = 1$$

Odd/Even formulas

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

Sum and difference formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

Double angle formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$$

Half angle formulas

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}}$$

Euler's theorem

$$e^{\pm i\theta} = \cos\theta \pm i\sin\theta$$

$$\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$$

$$\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$$

Miscellaneous formulas

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$\cos^2\theta = \frac{1+\sin(2\theta)}{2}$$

4 Calculus

4.1 Limits

4.1.1 Properties

$$\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$$

L'Hopital's Rule

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

4.1.2 Evaluations

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to -\infty} e^x = 0$$

4.2 Derivatives

4.2.1 Definition

$$\frac{d}{dx} f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

4.2.2 Properties

Product rule

$$(fg)' = f'g + fg'$$

Chain rule

$$\frac{d}{dx} [f(u)] = \frac{d}{du} [f(u)] \frac{du}{dx}$$

or $(f(g(x))' = f'(g(x))g'(x)$ or $(f \circ g)' = (f' \circ g) \cdot g'$

Quotient Rule

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

4.2.3 Common Derivatives

$$\frac{d}{dx} (a^x) = a^x \ln(a)$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}, \quad x > 0$$

Power rule

$$\frac{d}{dx} x^n = nx^{(n-1)}$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \sin x = \cos x$$

4.3 Integrals

4.3.1 Fundamental Theorem of Calculus

$$\int_a^b \frac{d}{dx} F\left(x\right) dx = F\left(b\right) - F\left(a\right)$$

4.3.2 Properties

$$\int k dx = kx + C$$

4.3.3 Common Integrals

$$\int k dx = kx + C$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \ln u du = u \ln(u) - u + C$$

$$\int e^x dx = e^x + C$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax + C$$

$$\int \cos x dx = \sin x + C$$

Per partes (Integration by parts)

$$\int u \frac{dv}{dx} dx = uv - \int \frac{du}{dx} v dx$$

Substitution Rule

$$\int f(u) \frac{du}{dx} dx = \int f(u) du$$

4.4 Laplace transforms

4.4.1 Definition

$$X(s) = \int_0^\infty x(t) e^{-st} dt$$

4.4.2 Properties

$$1 \Leftrightarrow \frac{1}{s}$$

Kronecker delta function

$$\delta(t) \Leftrightarrow 1$$

$$Ke^{-at}u(t) \Leftrightarrow \frac{K}{s+a}$$

$$t^n u(t) \Leftrightarrow \frac{n!}{s^{n+1}}$$

$$\sin(\alpha t)u(t) \Leftrightarrow \frac{\alpha}{(s^2+\alpha^2)}$$

$$\cos(\alpha t)u(t) \Leftrightarrow \frac{s}{(s^2+\alpha^2)}$$

$$e^{-at}\sin(\Omega t)u(t) \Leftrightarrow \frac{\Omega}{(s+a)^2+\Omega^2}$$

$$e^{-at}\cos(\Omega t)u(t) \Leftrightarrow \frac{s+a}{(s+a)^2+\Omega^2}$$

$$e^{at}x(t) \Leftrightarrow X(s-a)$$

Time domain scaling

$$x(at)u(t) \Leftrightarrow \frac{1}{a}X\left(\frac{s}{a}\right)$$

Time domain shifting

$$x(t-a)u(t-a) \Leftrightarrow e^{-as}X(s+a)$$

Derivative

$$\frac{d^n x(t)}{dt^n} \Leftrightarrow s^n X(s)$$

or $\mathcal{L}[\dot{x}] = sX(s) - x(0+)$

Integral

$$\int x(t)dt \Leftrightarrow \frac{X(s)}{s}$$

Convolution

$$\int_0^\infty x_1(\tau)x_2(t-\tau)d\tau \Leftrightarrow X_1(s)X_2(s)$$

Symbol	Name	Symbol	Name

5 Greek letters

Symbol	Name	Symbol	Name
αA	Alpha	νN	Nu
βB	Beta	$\xi\Xi$	Xi
$\gamma\Gamma$	Gamma	$o O$	Omicron
$\delta\Delta$	Delta	$\pi\Pi$	Pi
$\epsilon\varepsilon$	Epsilon	$\rho\varrho P$	Rho
ζZ	Zeta	$\sigma\Sigma$	Sigma
ηH	Eta	τT	Tau
$\theta\vartheta\Theta$	Theta	$\upsilon\Upsilon$	Upsilon
ιI	Iota	$\phi\varphi\Phi$	Phi
κK	Kappa	χX	Chi
$\lambda\Lambda$	Lambda	$\psi\Psi$	Psi
μM	Mu	$\omega\Omega$	Omega