

# 1 Algebra

## 1.1 Exponent Properties

$$\frac{a^n}{a^m} = a^{n-m} \quad (1)$$

$$x^a y^a = (xy)^a \quad (2)$$

$$x^{\left(\frac{a}{b}\right)} = \sqrt[b]{x^a} \quad (3)$$

$$x^{(a-b)} = \frac{x^a}{x^b} \quad (4)$$

## 1.2 Properties of radicals

$$\sqrt[n]{a} = a^{\frac{1}{n}} \quad (5)$$

$$\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b} \quad (6)$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a} \quad (7)$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad (8)$$

$$\sqrt[n]{a^n} = |a|, \text{ if } n \text{ is even} \quad (9)$$

## 1.3 Complex numbers

$$(a + bi)(c + di) = ac - bd + (ad + bc)i \quad (10)$$

$$(a + bi)(a - bi) = a^2 + b^2 \quad (11)$$

$$|a + bi| = \sqrt{a^2 + b^2} \text{ Complex Modulus} \quad (12)$$

$$\overline{(a + bi)} = a - bi \quad (13)$$

## 1.4 Logarithms

$$\log_b b = 1 \quad (14)$$

$$\log_b 1 = 0 \quad (15)$$

$$\log_b(x^r) = r \log_b x \quad (16)$$

$$\log_b(xy) = \log_b(x) + \log_b(y) \quad (17)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y) \quad (18)$$

$$\log_b(x) = \log_b(c) \log_c(x) = \frac{\log_c(x)}{\log_c(b)} \quad (19)$$

## 1.5 Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{when } ax^2 + bx + c = 0 \quad (20)$$

## 2 Linear Algebra

conjugate transpose / adjugate

$$A^* = (\overline{A})^T = \overline{A^T} \quad (21)$$

Matrix addition: one by one.

Scalar multiplication: all.

Matrix "multiplication of rows into columns". Multiplication is not commutative.

$$c_{jk} = \sum_{i=1}^n a_{ji} b_{ik} \quad (22)$$

invert:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad (23)$$

Inner or dot product of Vectors

$$\langle a, b \rangle = \mathbf{a} \bullet \mathbf{b} = \mathbf{a}^T \mathbf{b} \quad (24)$$

### 2.1 Determinants

$$\det(\mathbf{A}) = \sum_{\sigma \in S_n} \text{sgn}(\sigma) \prod_{i=1}^n A_{i, \sigma_i} \quad (25)$$

For  $3 \times 3$  matrices (Sarrus rule)

$$\begin{array}{ccccc} & + & & + & & + & & \\ & a_{11} & & a_{12} & & a_{13} & & a_{11} & & a_{12} \\ & \swarrow & & \nearrow & & \swarrow & & \nearrow & & \swarrow & & \nearrow \\ a_{21} & & a_{22} & & a_{23} & & a_{21} & & a_{22} \\ & \swarrow & & \nearrow & & \swarrow & & \nearrow & & \swarrow & & \nearrow \\ a_{31} & & a_{32} & & a_{33} & & a_{31} & & a_{32} \\ & - & & - & & - & & \end{array} \quad (26)$$

$$\det(A \cdot B) = \det(A) \cdot \det(B) \quad (27)$$

$$\det(A^{-1}) = \det(A)^{-1} \quad (28)$$

$$\det(rA) = r^n \det(A) \quad \text{for all } A^{n \times n} \text{ and scalars } r \quad (29)$$

## 2.2 Transpose

$$[A^T]_{ij} = [A]_{ji} \quad (30)$$

$$(A^T)^T = A \quad (31)$$

$$(AB)^T = B^T A^T \quad (32)$$

$$\det(A^T) = \det(A) \quad (33)$$

$$(A^T)^{-1} = (A^{-1})^T \quad (34)$$

## 3 Trigonometry

### 3.1 Definitions

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad (35)$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} \quad (36)$$

### 3.2 Formulas and Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad (37)$$

$$\sin^2 \theta + \cos^2 \theta = 1 \quad (38)$$

$$\sin(-\theta) = -\sin \theta \quad (39)$$

$$\cos(-\theta) = \cos \theta \quad (40)$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \quad (41)$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \quad (42)$$

$$\sin 2\theta = 2 \sin \theta \cos \theta \quad (43)$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 \quad (44)$$

Euler's formula

$$e^{\pm i\theta} = \cos \theta \pm i \sin \theta \quad (45)$$

## 4 Calculus

### 4.1 Limits

#### 4.1.1 Properties

$$\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x) \quad (46)$$

L'Hopital's Rule

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)} \quad (47)$$

#### 4.1.2 Evaluations

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad (48)$$

$$\lim_{x \rightarrow -\infty} e^x = 0 \quad (49)$$

### 4.2 Derivatives

#### 4.2.1 Definition

$$\frac{d}{dx} f(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad (50)$$

#### 4.2.2 Properties

$$(fg)' = f'g + fg' \quad (51)$$

Power rule

$$\frac{d}{dx} x^n = nx^{n-1} \quad (52)$$

Chain rule

$$\frac{d}{dx} [f(u)] = \frac{d}{du} [f(u)] \frac{du}{dx} \quad \text{or} \quad (f(g(x)))' = f'(g(x)) g'(x) \quad (53)$$

### 4.2.3 Common Derivatives

$$\frac{d}{dx}(a^x) = a^x \ln(a) \quad (54)$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x} \quad (55)$$

$$\frac{d}{dx} x^n = nx^{(n-1)} \quad (56)$$

$$\frac{d}{dx} \cos x = -\sin x \quad (57)$$

$$\frac{d}{dx} \sin x = \cos x \quad (58)$$

## 4.3 Integrals

### 4.3.1 Fundamental Theorem of Calculus

$$\int_a^b \frac{d}{dx} F(x) dx = F(b) - F(a) \quad (59)$$

### 4.3.2 Properties

$$\int k dx = kx + C \quad (60)$$

### 4.3.3 Common Integrals

$$\int k dx = kx + C \quad (61)$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1 \quad (62)$$

$$\int \frac{1}{x} dx = \ln|x| + C \quad (63)$$

$$\int \ln u = u \ln(u) - u + C \quad (64)$$

$$\int e^x dx = e^x + C \quad (65)$$

$$\int \sin ax = -\frac{1}{a} \cos ax + C \quad (66)$$

$$\int \cos x = \sin x + C \quad (67)$$

Per partes

$$\int u \frac{dv}{dx} dx = uv - \int \frac{du}{dx} v dx \quad (68)$$

See also substitution...

## 4.4 Laplace transforms

### 4.4.1 Definition

$$X(s) = \int_0^{\infty} x(t) e^{-st} dt \quad (69)$$

### 4.4.2 Properties

$$1 \Leftrightarrow \frac{1}{s} \quad (70)$$

Kroeneker delta function

$$\delta(t) \Leftrightarrow 1 \quad (71)$$

$$K e^{-at} u(t) \Leftrightarrow \frac{K}{s+a} \quad (72)$$

$$t^n u(t) \Leftrightarrow \frac{n!}{s^{n+1}} \quad (73)$$

$$\sin(\alpha t) u(t) \Leftrightarrow \frac{\alpha}{(s^2 + \alpha^2)} \quad (74)$$

$$\cos(\alpha t) u(t) \Leftrightarrow \frac{s}{(s^2 + \alpha^2)} \quad (75)$$

$$e^{-at} \sin(\Omega t) u(t) \Leftrightarrow \frac{\Omega}{(s+a)^2 + \Omega^2} \quad (76)$$

$$e^{-at} \cos(\Omega t) u(t) \Leftrightarrow \frac{s+a}{(s+a)^2 + \Omega^2} \quad (77)$$

Time domain scaling

$$x(at) u(t) \Leftrightarrow \frac{1}{a} X\left(\frac{s}{a}\right) \quad (78)$$

Time domain shifting

$$x(t-a) u(t-a) \Leftrightarrow e^{-as} X(s+a) \quad (79)$$

Derivative

$$\frac{d^n x(t)}{dt^n} \Leftrightarrow s^n X(s) \quad (80)$$

Convolution

$$\int_0^{\infty} x_1(\tau)x_2(t-\tau)d\tau \Leftrightarrow X_1(s)X_2(s) \quad (81)$$

## 5 Greek letters

$\alpha A$	Alpha	$\nu N$	Nu
$\beta B$	Beta	$\xi \Xi$	Xi
$\gamma \Gamma$	Gamma	$o O$	Omicron
$\delta \Delta$	Delta	$\pi \Pi$	Pi
$\epsilon \varepsilon$	Epsilon	$\rho \varrho P$	Rho
$\zeta Z$	Zeta	$\sigma \Sigma$	Sigma
$\eta H$	Eta	$\tau T$	Tau
$\theta \vartheta \Theta$	Theta	$\upsilon \Upsilon$	Upsilon
$\iota I$	Iota	$\phi \varphi \Phi$	Phi
$\kappa K$	Kappa	$\chi X$	Chi
$\lambda \Lambda$	Lambda	$\psi \Psi$	Psi
$\mu M$	Mu	$\omega \Omega$	Omega

Table 1: Greek letters