1 Algebra

1.1 Exponent Properties

$$\frac{a^n}{a^m} = a^{n-m} \tag{1}$$

$$x^a y^a = (xy)^a \tag{2}$$

$$x^{\left(\frac{a}{b}\right)} = \sqrt[b]{x^a} \tag{3}$$

$$x^{(a-b)} = \frac{x^a}{x^b} \tag{4}$$

1.2 Properties of radicals

$$\sqrt[n]{a} = a^{\frac{1}{n}} \tag{5}$$

$$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b} \tag{6}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a} \tag{7}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \tag{8}$$

$$\sqrt[n]{a^n} = |a|, \quad \text{if } n \text{ is even} \tag{9}$$

1.3 Complex numbers

$$(a+bi)(c+di) = ac - bd + (ad+bc)i \tag{10}$$

$$(a+bi)(a-bi) = a^2 + b^2 \tag{11}$$

$$|a+bi| = \sqrt{a^2+b^2} \quad \text{Complex Modulus} \tag{12}$$

$$\overline{(a+bi)} = a - bi \tag{13}$$

1.4 Logarithms

$$\log_b b = 1 \tag{14}$$

$$\log_b 1 = 0 \tag{15}$$

$$\log_b(x^r) = r \log_b x \tag{16}$$

$$\log_b(xy) = \log_b(x) + \log_b(y) \tag{17}$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y) \tag{18}$$

$$\log_b(x) = \log_b(c)\log_c(x) = \frac{\log_c(x)}{\log_c(b)} \tag{19}$$

1.5 Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{when } ax^2 + bx + c = 0 \tag{20}$$

2 Linear Algebra

conjungate transpose / adjugate

$$A^* = (\overline{A})^{\mathrm{T}} = \overline{A^{\mathrm{T}}} \tag{21}$$

Matrix addition: one by one.

Scalar multiplication: all.

Matrix "multiplication of rows into columns". Multiplication is not commutative.

$$c_{jk} = \sum_{i=1}^{n} a_{ji} b_{ik} \tag{22}$$

invert:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \tag{23}$$

Inner or dot product of Vectors

$$\langle a, b \rangle = \mathbf{a} \bullet \mathbf{b} = \mathbf{a}^T \mathbf{b} \tag{24}$$

2.1 Determinants

$$\det(\mathbf{A}) = \sum_{\sigma \in S_n} \operatorname{sgn}(\sigma) \prod_{i=1}^{n} A_{i,\sigma_i} \tag{25}$$

For 3×3 matrices (Sarrus rule)

$$\begin{array}{ccccccc} + & & + & & + & & \\ & \diagdown & & \diagdown & & \diagdown & \\ a_{11} & & a_{12} & & a_{13} & & a_{11} & & a_{12} \\ & \diagdown & & \diagdown & & \diagdown & & \diagdown \\ a_{21} & & a_{22} & & a_{23} & & a_{21} & & a_{22} \\ & \diagdown & & \diagdown & & \diagdown & & \diagdown \\ a_{31} & & a_{32} & & a_{33} & & a_{31} & & a_{32} \\ & \diagdown & & \diagdown & & \diagdown & \\ - & & - & & - & & \end{array} \tag{26}$$

$$\det(A \cdot B) = \det(A) \cdot \det(B) \tag{27}$$

$$\det(A^{-1}) = \det(A)^{-1} \tag{28}$$

$$\det(rA) = r^n \det(A) \quad \text{for all } A^{n \times n} \text{ and scalars } r \tag{29}$$

2.2 Transpose

$$[A^{\mathrm{T}}]_{ij} = [A]_{ji} \tag{30}$$

$$(A^T)^T = A \tag{31}$$

$$(AB)^T = B^T A^T \tag{32}$$

$$det(A^T) = det(A) \tag{33}$$

$$(A^T)^{-1} = (A^{-1})^T \tag{34}$$

3 Trigonometry

3.1 Definitions

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \tag{35}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \tag{36}$$

3.2 Formulas and Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \tag{37}$$

$$\sin^2\theta + \cos^2\theta = 1 \tag{38}$$

$$\sin(-\theta) = -\sin\theta \tag{39}$$

$$\cos(-\theta) = \cos\theta \tag{40}$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \tag{41}$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \tag{42}$$

$$\sin 2\theta = 2\sin\theta\cos\theta \tag{43}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 \tag{44}$$

Euler's formula

$$e^{\pm i\theta} = \cos\theta \pm i\sin\theta \tag{45}$$

4 Calculus

4.1 Limits

4.1.1 Properties

$$\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x) \tag{46}$$

L'Hopital's Rule

$$\lim_{x \to c} \frac{f\left(x\right)}{g\left(x\right)} = \lim_{x \to c} \frac{f'\left(x\right)}{g'\left(x\right)} \tag{47}$$

4.1.2 Evaluations

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \tag{48}$$

$$\lim_{x \to -\infty} e^x = 0 \tag{49}$$

4.2 Derivatives

4.2.1 Definition

$$\frac{d}{dx} f\left(x\right) = \lim_{h \to 0} \frac{f\left(x+h\right) - f\left(x\right)}{h} \tag{50}$$

4.2.2 Properties

$$(fg)' = f'g + fg' \tag{51}$$

Power rule

$$\frac{d}{dx} x^n = nx^{n-1} \tag{52}$$

Chain rule

$$\frac{d}{dx}\left[f\left(u\right)\right] = \frac{d}{du}\left[f\left(u\right)\right]\frac{du}{dx} \quad \text{or} \quad \left(f(g(x)\right)' = f'\left(g(x)\right)g'(x) \tag{53}$$

4.2.3 Common Derivatives

$$\frac{d}{dx}\left(a^x\right) = a^x \ln(a) \tag{54}$$

$$\frac{d}{dx}\ln\left(x\right) = \frac{1}{x} \tag{55}$$

$$\frac{d}{dx}x^n = nx^{(n-1)} \tag{56}$$

$$\frac{d}{dx}\cos x = -\sin x \tag{57}$$

$$\frac{d}{dx}\sin x = \cos x \tag{58}$$

4.3 Integrals

4.3.1 Fundamental Theorem of Calculus

$$\int_a^b \frac{d}{dx}F\left(x\right)dx = F\left(b\right) - F\left(a\right) \tag{59}$$

4.3.2 Properties

$$\int kdx = kx + C \tag{60}$$

4.3.3 Common Integrals

$$\int kdx = kx + C \tag{61}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C, \quad \text{n} \neq \text{-1} \tag{62}$$

$$\int \frac{1}{x}dx = \ln|x| + C \tag{63}$$

$$\int \ln u = u\ln(u) - u + C \tag{64}$$

$$\int e^x dx = e^x + C \tag{65}$$

$$\int \sin ax = -\frac{1}{a}\cos ax + C \tag{66}$$

$$\int \cos x = \sin x + C \tag{67}$$

Per partes

$$\int u\frac{dv}{dx}dx = uv - \int \frac{du}{dx}vdx \tag{68}$$

See also substitution...

4.4 Laplace transforms

4.4.1 Definition

$$X(s) = \int_0^\infty x(t)e^{-st}dt \tag{69}$$

4.4.2 Properties

$$1 \Leftrightarrow \frac{1}{s} \tag{70}$$

Kroeneker delta function

$$\delta(t) \Leftrightarrow 1 \tag{71}$$

$$Ke^{-at}u(t) \Leftrightarrow \frac{K}{s+a} \tag{72}$$

$$t^n u(t) \Leftrightarrow \frac{n!}{s^{n+1}} \tag{73}$$

$$\sin(\alpha t)u(t) \Leftrightarrow \frac{\alpha}{(s^2+\alpha^2)} \tag{74}$$

$$\cos(\alpha t)u(t) \Leftrightarrow \frac{s}{(s^2+\alpha^2)} \tag{75}$$

$$e^{-at}\sin(\Omega t)u(t) \Leftrightarrow \frac{\Omega}{(s+a)^2+\Omega^2} \tag{76}$$

$$e^{-at}\cos(\Omega t)u(t) \Leftrightarrow \frac{s+a}{(s+a)^2+\Omega^2} \tag{77}$$

Time domain scaling

$$x(at)u(t) \Leftrightarrow \frac{1}{a}X\left(\frac{s}{a}\right) \tag{78}$$

Time domain shifting

$$x(t-a)u(t-a) \Leftrightarrow e^{-as}X(s+a) \tag{79}$$

Derivative

$$\frac{d^n x(t)}{dt^n} \Leftrightarrow s^n X(s) \tag{80}$$

Convolution

$$\int_0^\infty x_1(\tau)x_2(t-\tau)d\tau \Leftrightarrow X_1(s)X_2(s) \tag{81}$$