Balanced
Binary Search Trees

Red Black and 2-4

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What was the average runtime for insert remove and contains?

What was the worst case runtime for insert remove and contains?

Balanced BSTs (BBSTs)

In CS I at least one BBST was covered

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Most commonly Red-Black Trees and 2-4 Trees

Red Black Trees

Personal Favorite

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Uses 5(ish) rules to ensure reduced worst case performance

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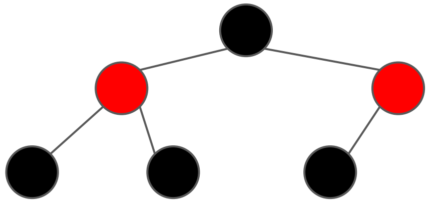
5. (optional) The root is colored black

Examples?

Assume circles are non-NULL nodes.



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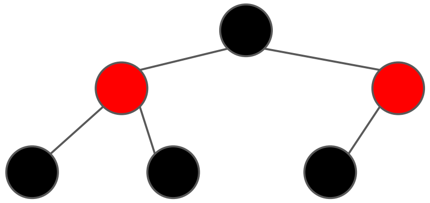

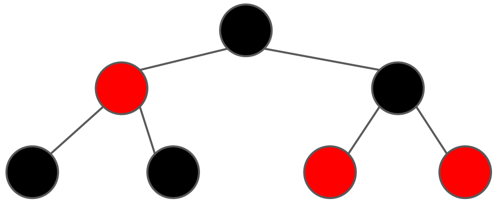

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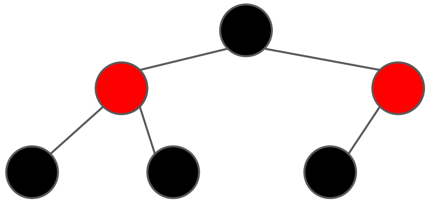

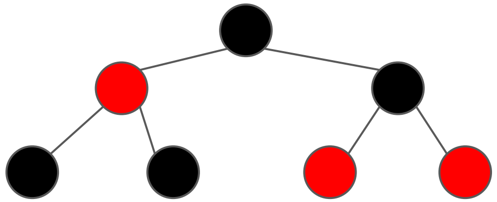

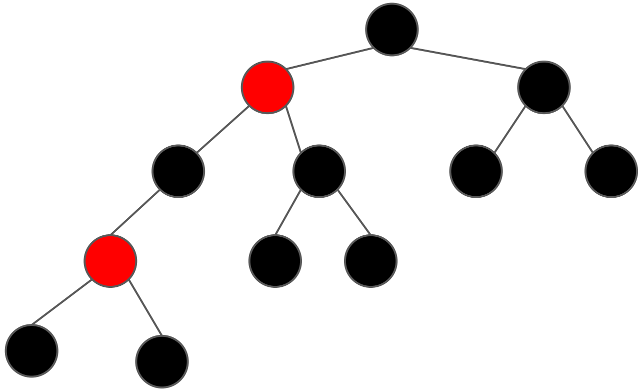

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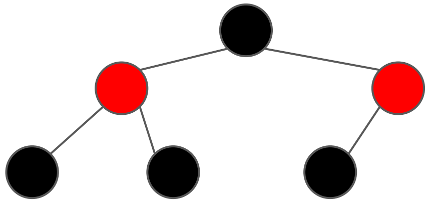

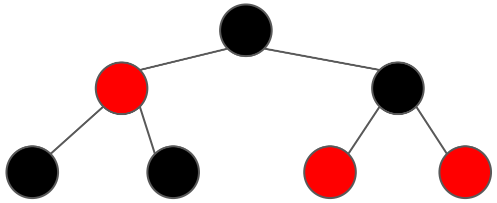

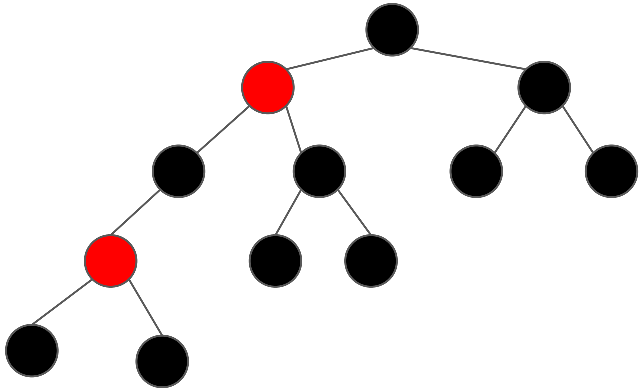

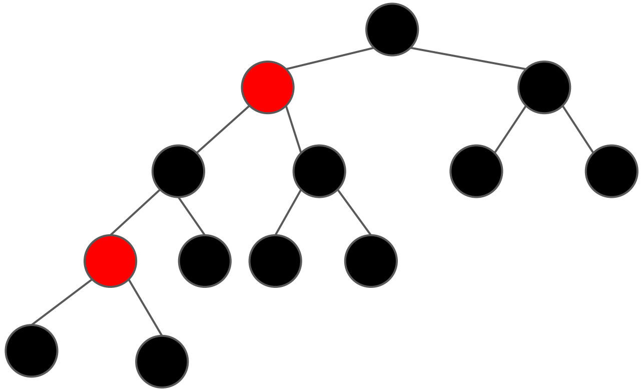

Red Black Insertion

Insertion()

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- Insert the node using normal BST insertion

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Case 3 sub cases

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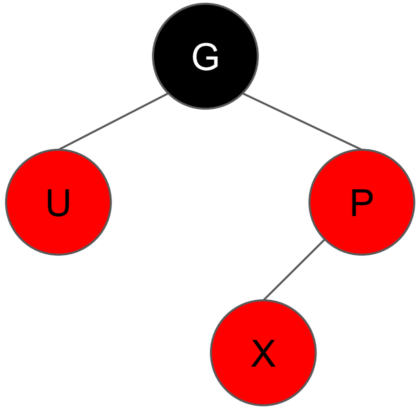

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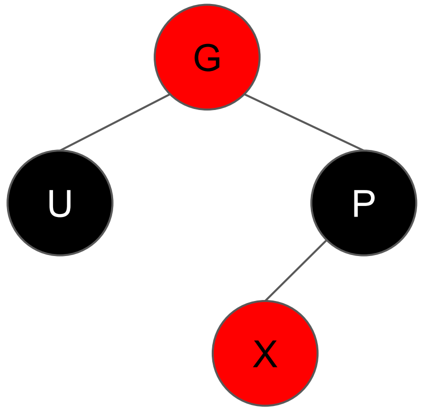

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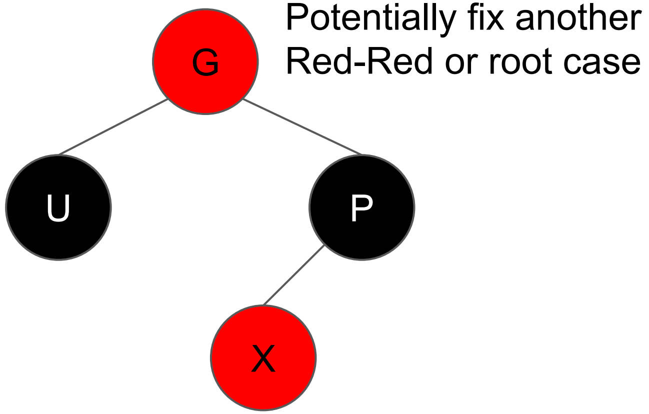

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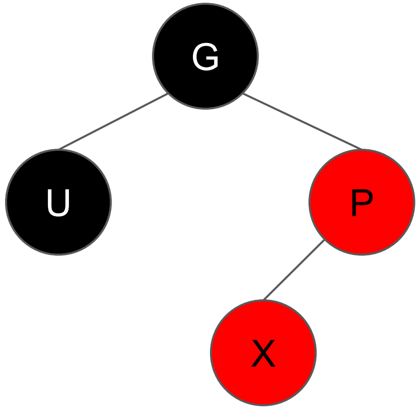

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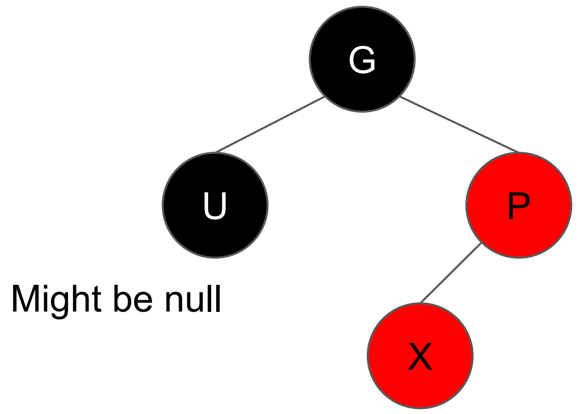

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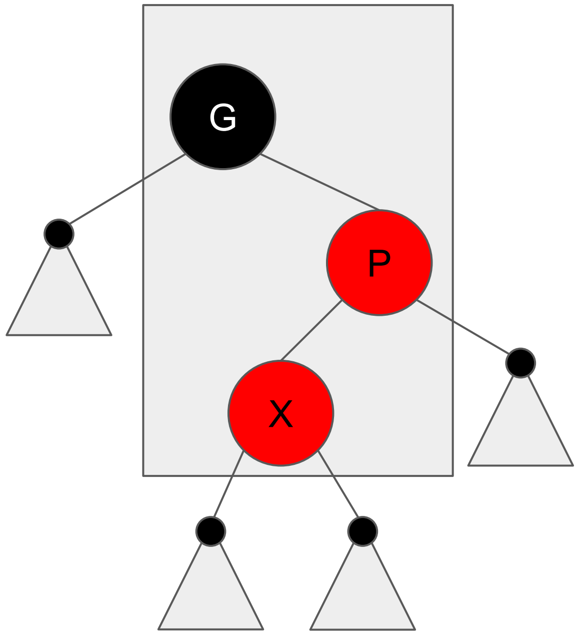

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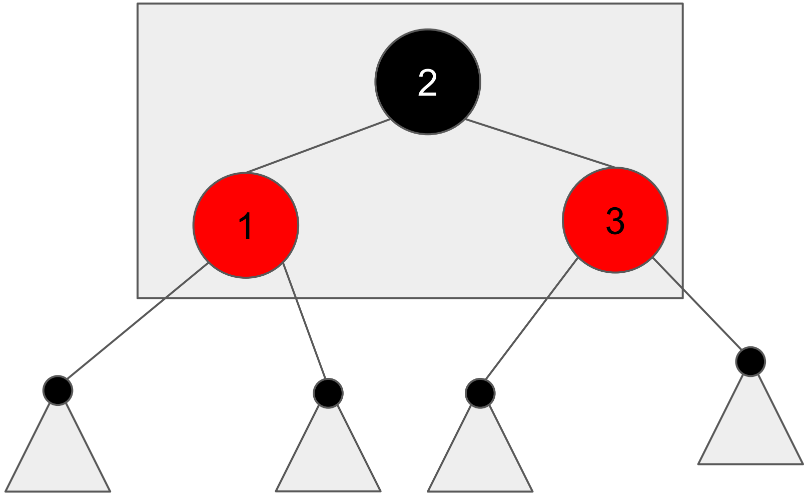

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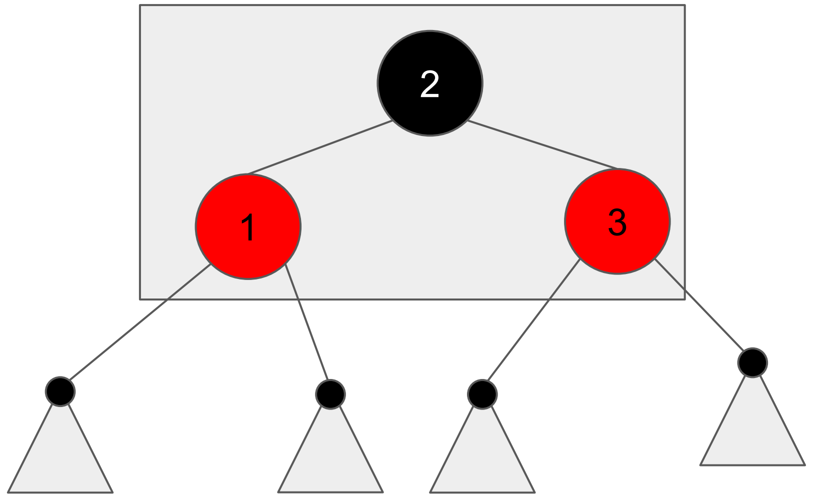
Note: Only 1 rotation is needed



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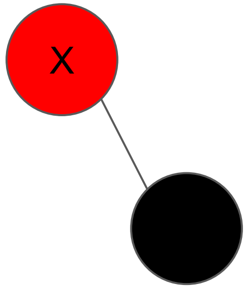

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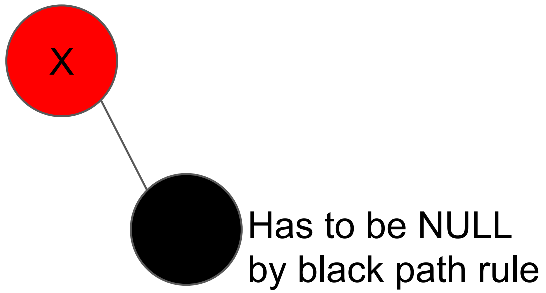

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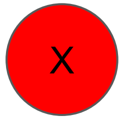

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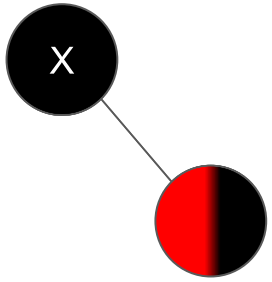
Node is black



Has to be NULL or **red** by black path rule

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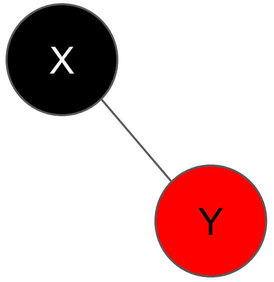

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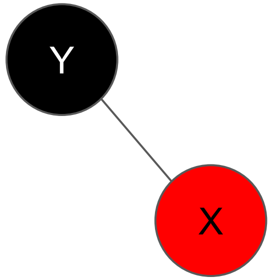

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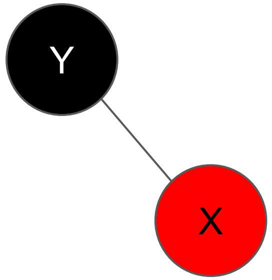

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Red Black Removal

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What cases could we encounter?

Node is red (easy removal)

Node is black (potentially not as easy removal)

Double Black Node

Caused by removing a black node with no replacement red child.

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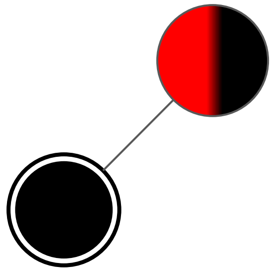

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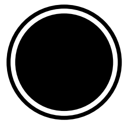
Case 1: The double black node is the root.



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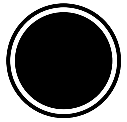

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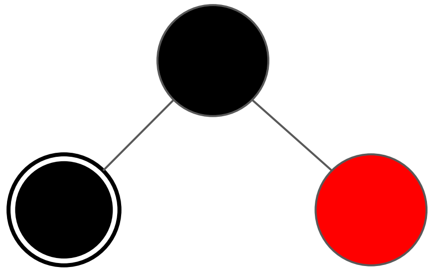
Case 2: The double black node has a red sibling



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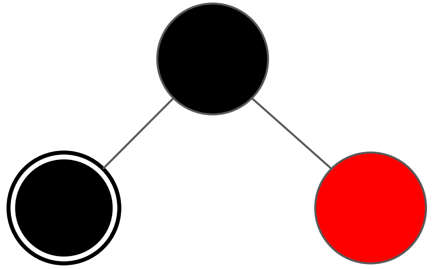
Make it black



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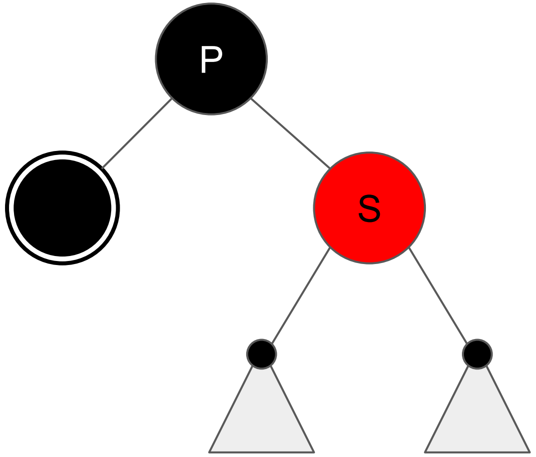
Make it black through rotation



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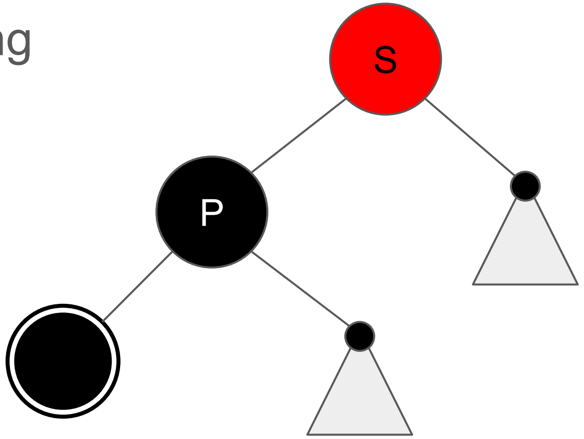

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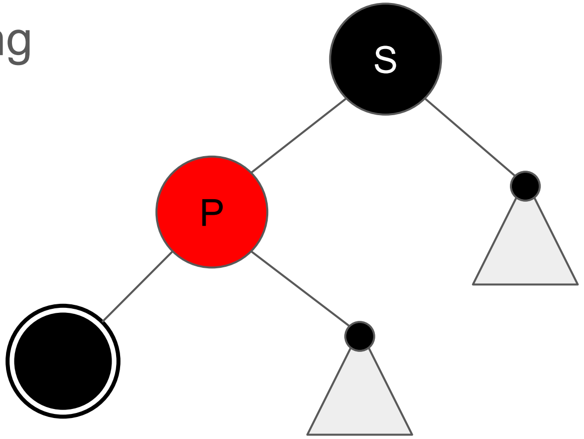

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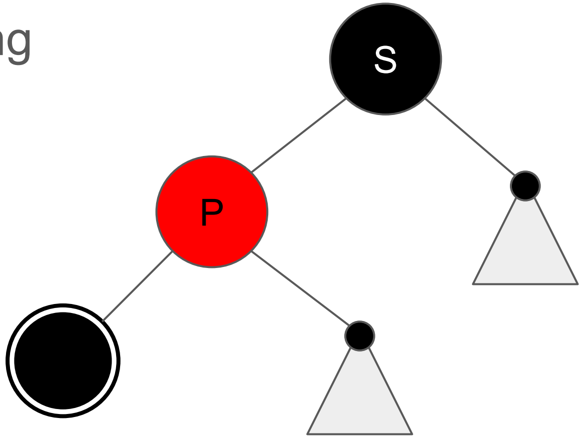
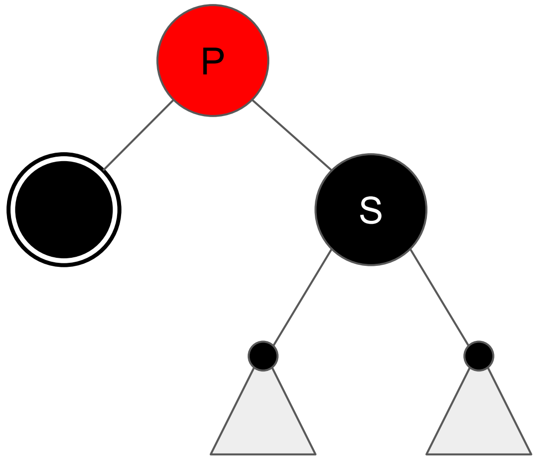


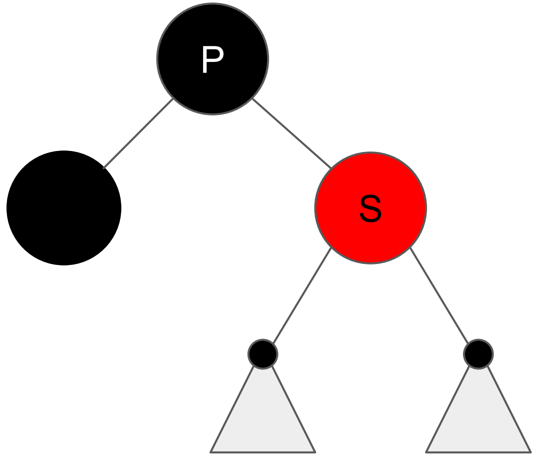
Double Black Node

Case 3: Parent is red, but nephews are black



Double Black Node

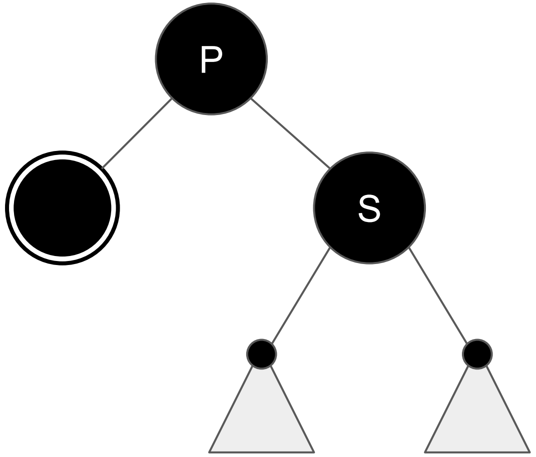
Case 3: Parent is red, but nephews are black (swap parent and sibling color)



Double Black Node

Case 3: Parent is red, but nephews are black (swap parent and sibling color)

Case 4: Parent is black and so are nephews



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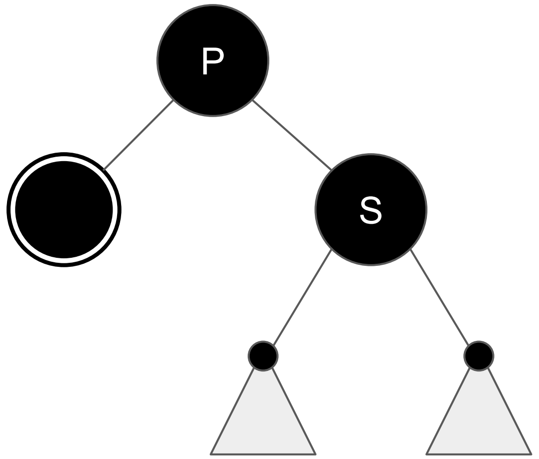
Case 3: Parent is red, but nephews are black (swap parent and sibling color)

Case 4: Parent is black and so are nephews (Let our parent inherit our problem)



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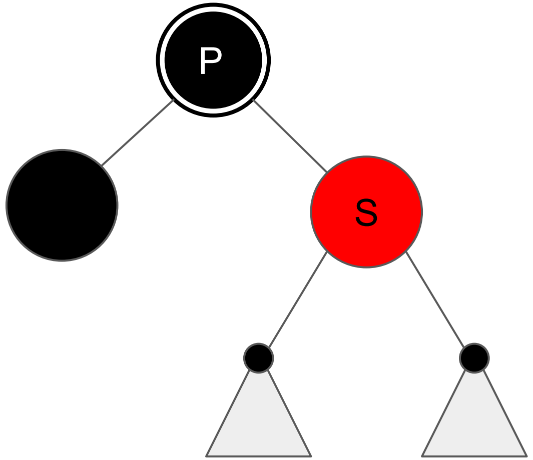

Double Black Node

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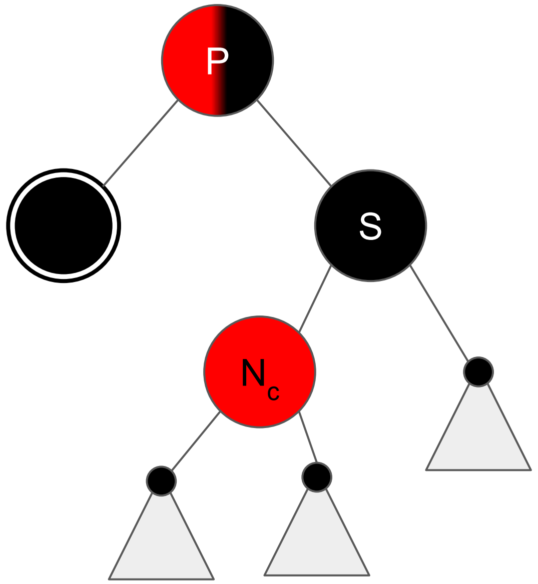
Case 5: Close nephew is red and far is black



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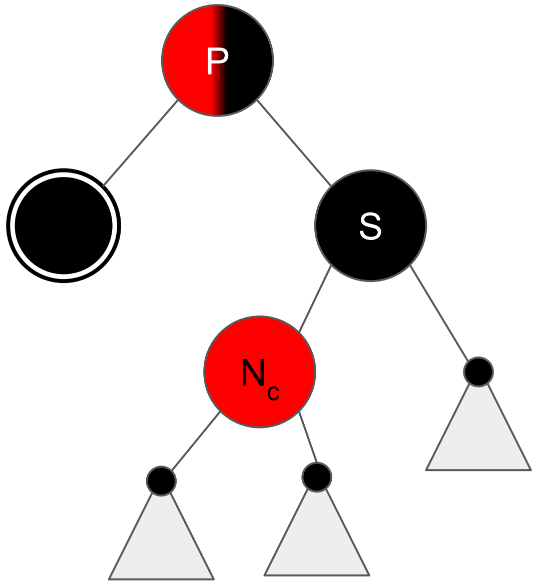
(Make far red)



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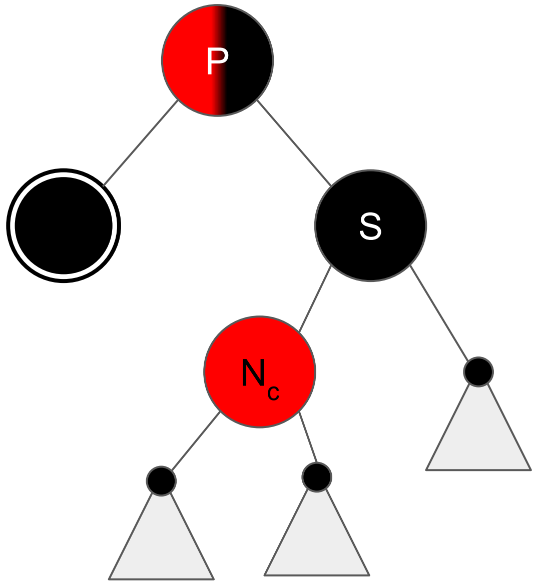
(Make far red)

Consider the sibling



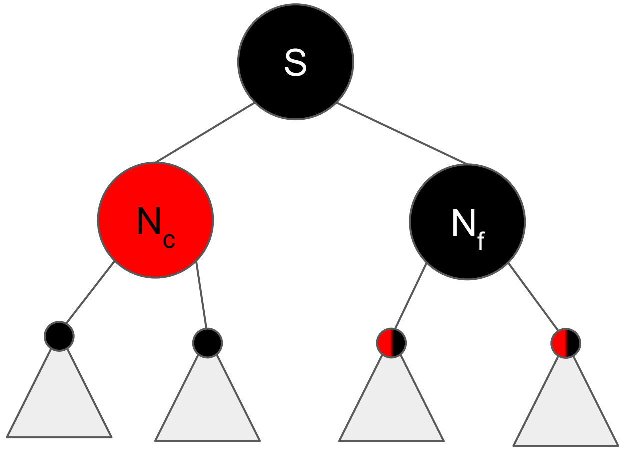
Double Black Node

Case 3: Parent is red, but nephews are black (swap parent and sibling color)

Case 4: Parent is black and so are nephews (Let our parent inherit our problem)

Case 5: Close nephew is red and far is black

(Make far red)



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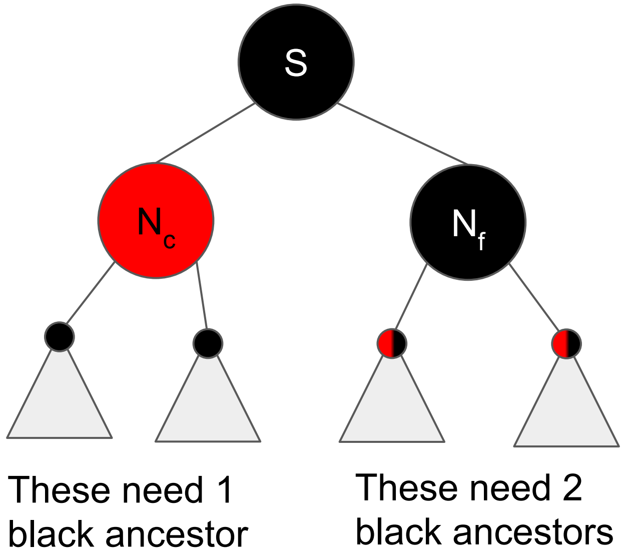

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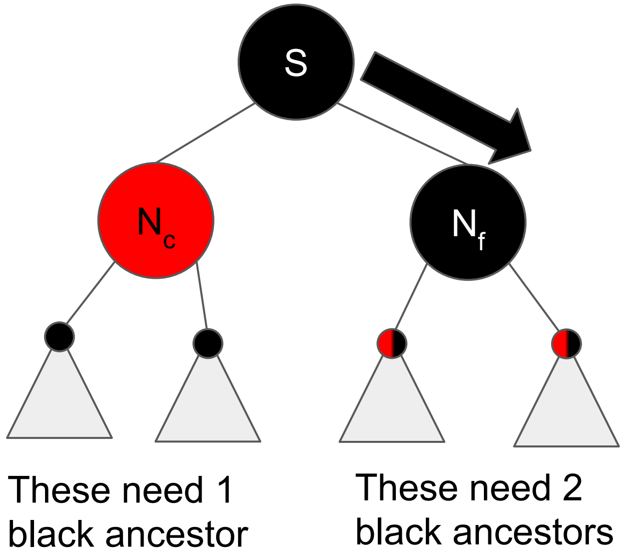

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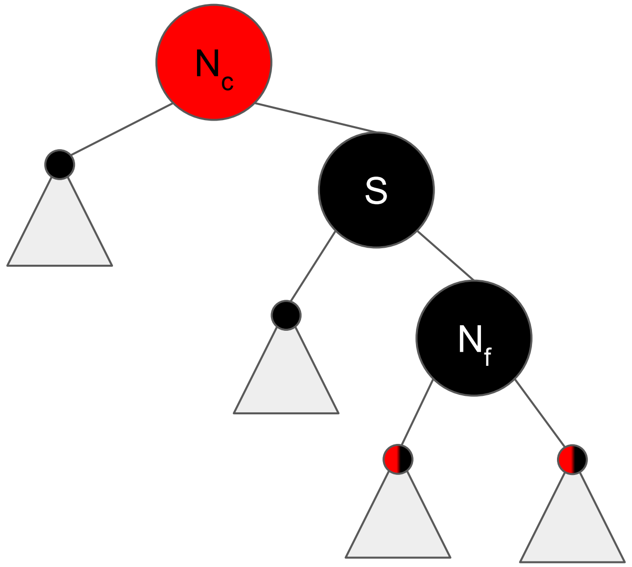

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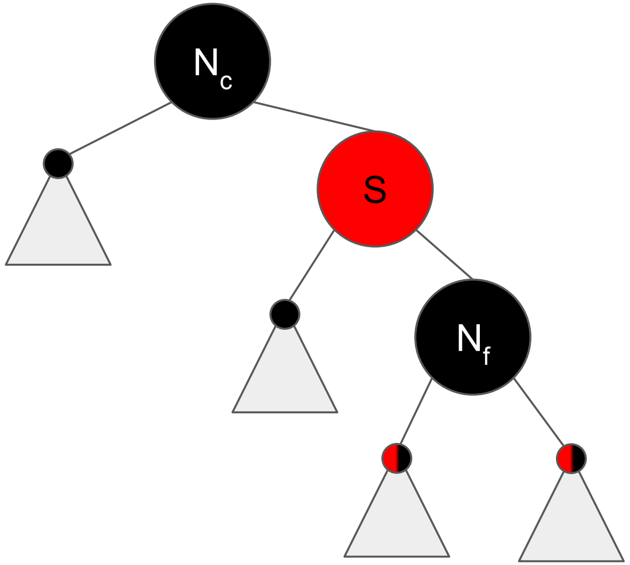

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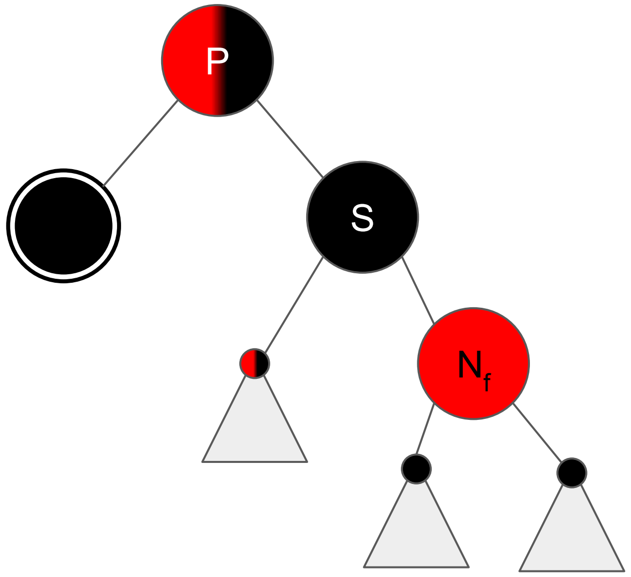
Case 5: Close nephew is red and far is black

(Make far red)

Case 6: Far nephew is red



Double Black Node

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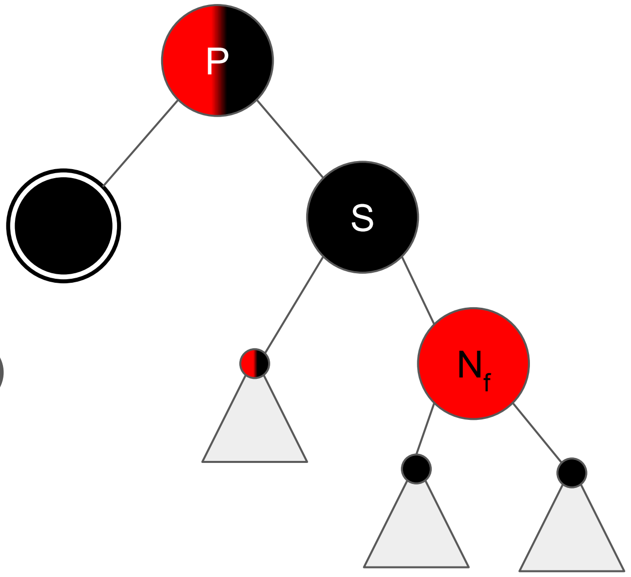
(Make far red)

Case 6: Far nephew is red

(rotate the parent towards the double black node)



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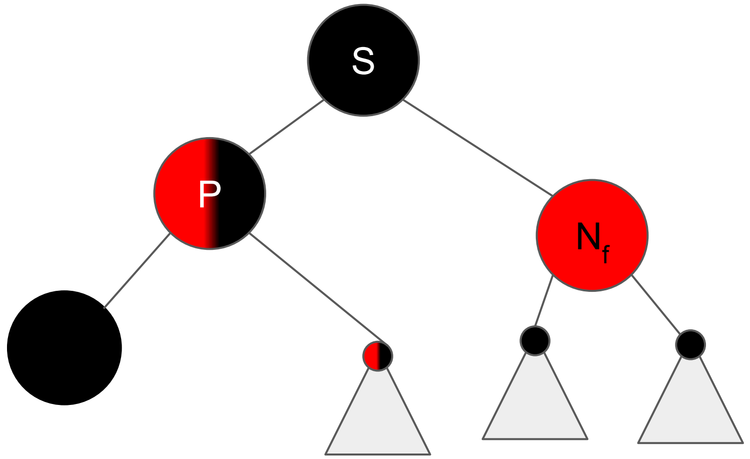
(Make far red)

Case 6: Far nephew is red



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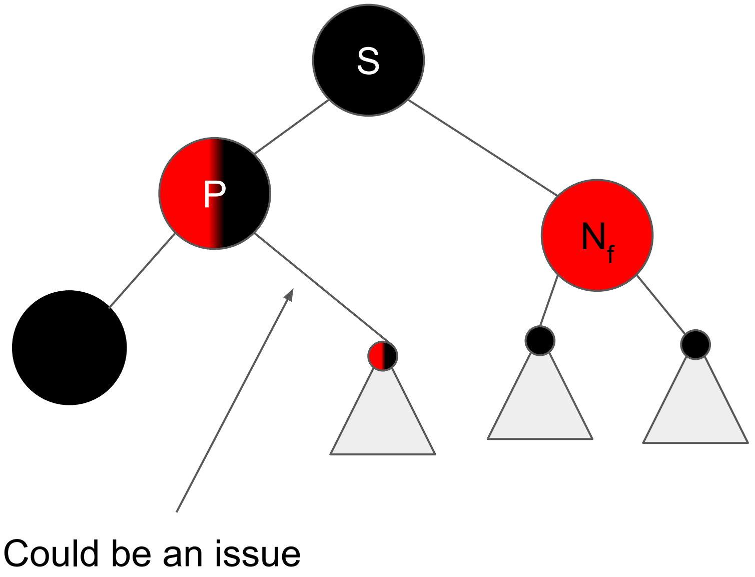
Case 6: Far nephew is red



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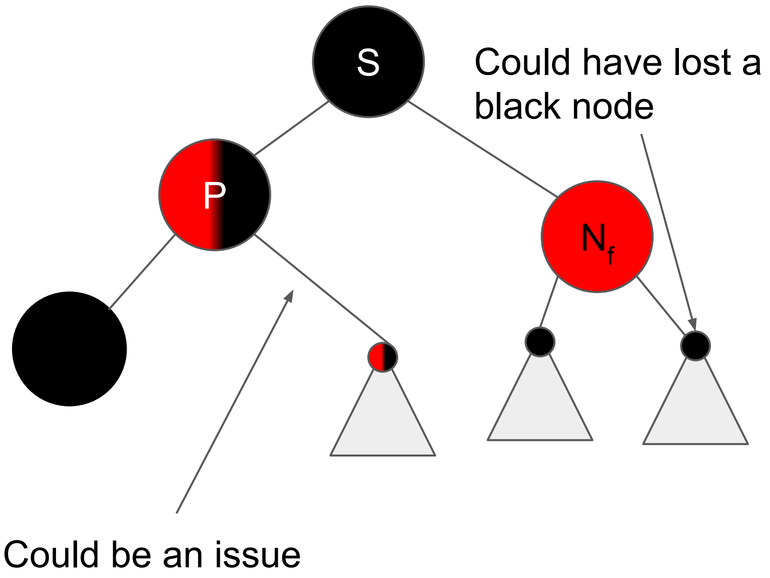
(Make far red)

Case 6: Far nephew is red



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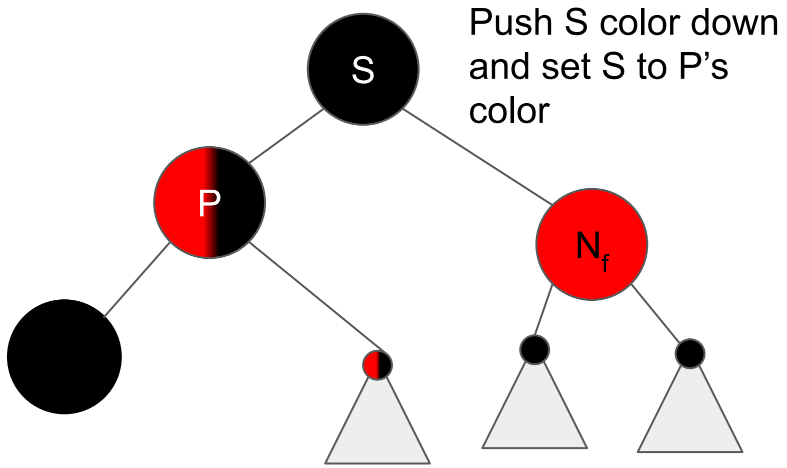

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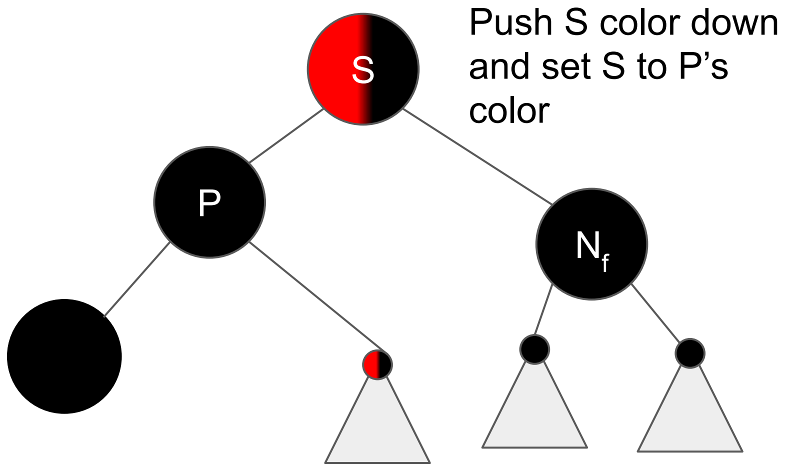

Runtime Proof

Insert when fixing the color will always work its way up the tree with each recursive call.

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Remove will perform a “constant” number adjustments before moving the double black node up the tree.

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The maximum number of operations is thus the height of the tree.