Algorithm Analysis

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Main Question:

> How does resource consumption **scale** with input?

Comparing Algorithms (cont.)

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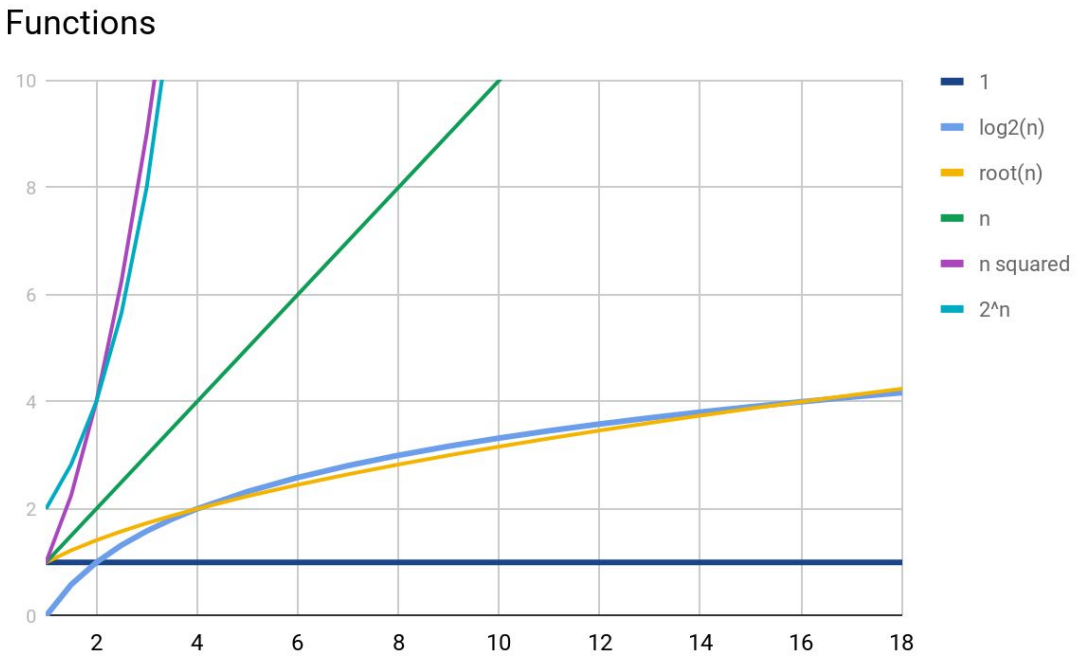
Formally we would say that

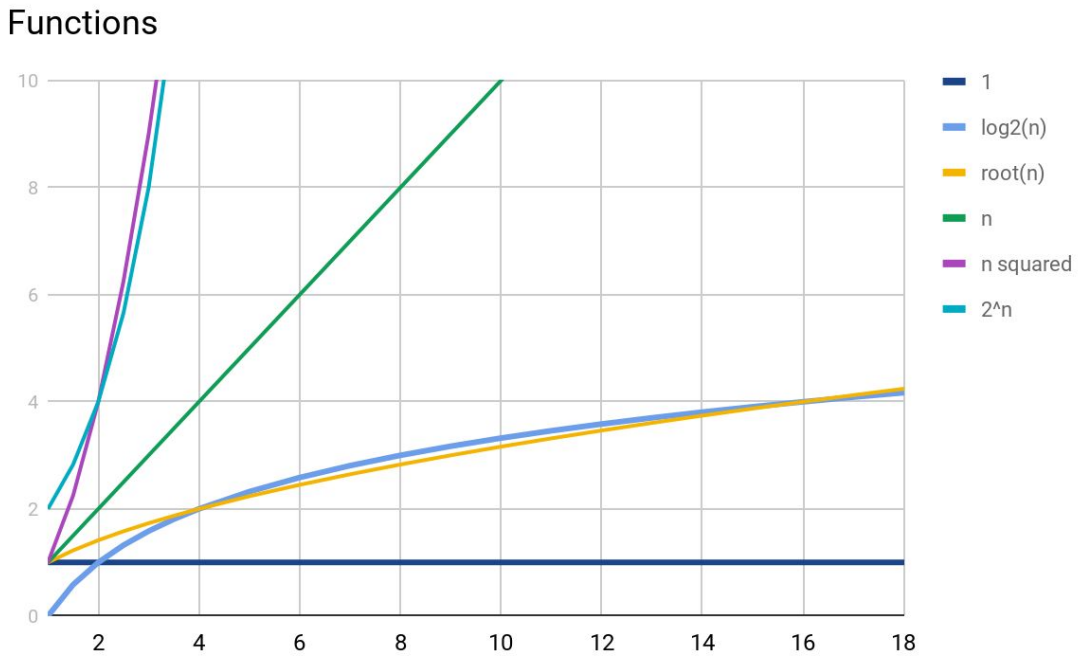
$$f(x) \in \mathrm{O}(g(x)) \equiv \exists\ n_0, c\ \forall\ n > n_0\ (f(n) < cg(n))$$

The Big O Picture



The Big O Picture

Which function is above us for large values of n?



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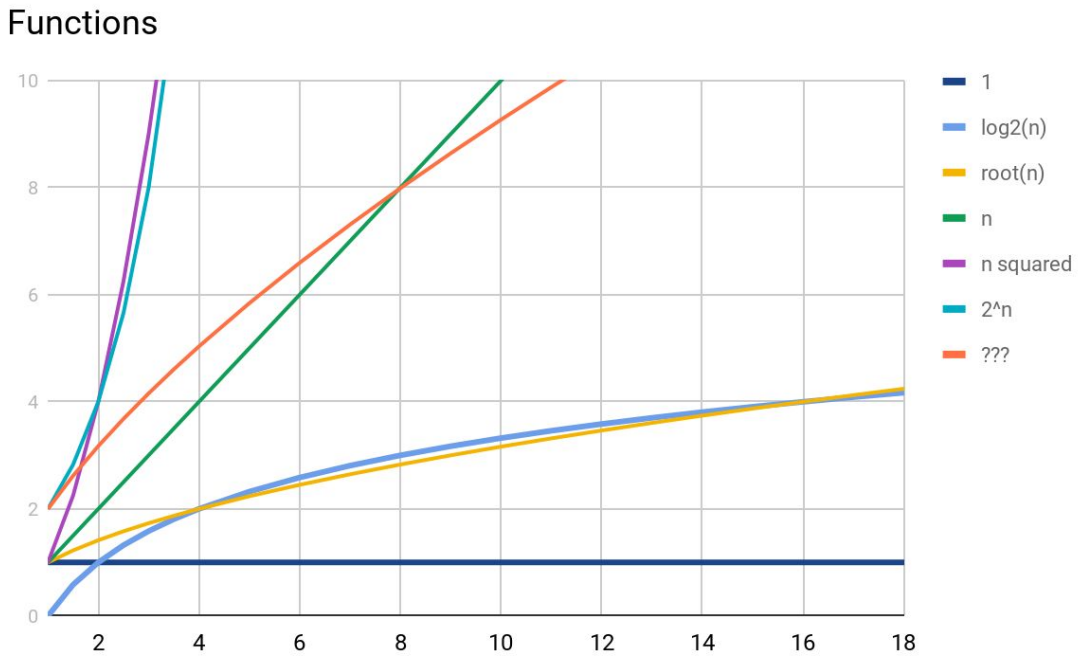

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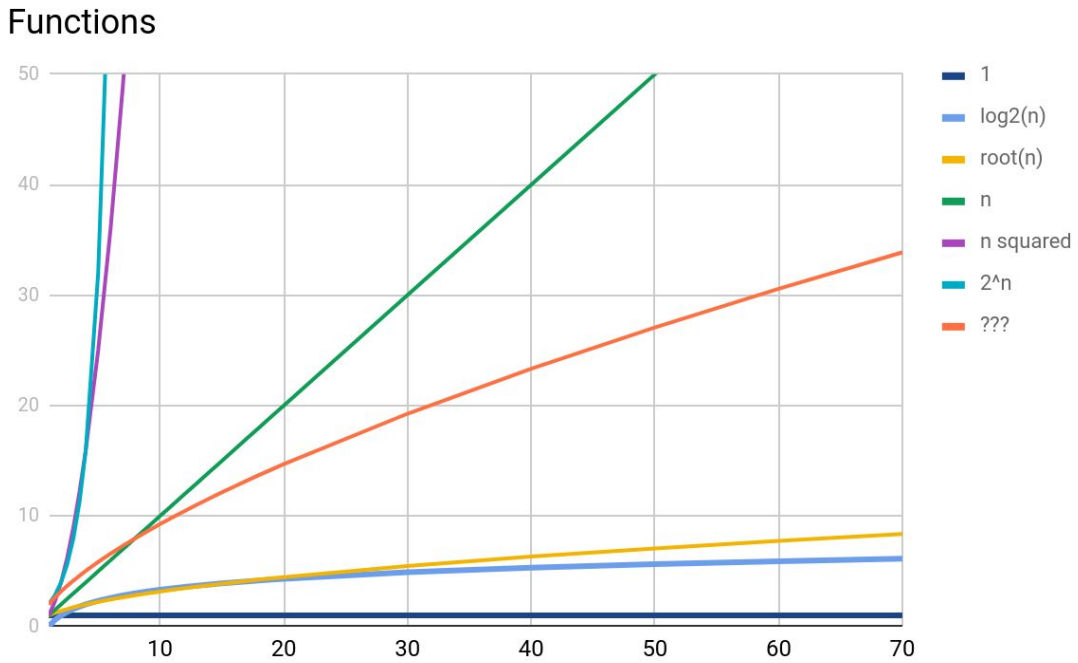

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```
int function(int N, int[] a) {
   int x = 0;
   for (int i = 0; i < N; i++)
      x+=a[i];
   return x;
}
```

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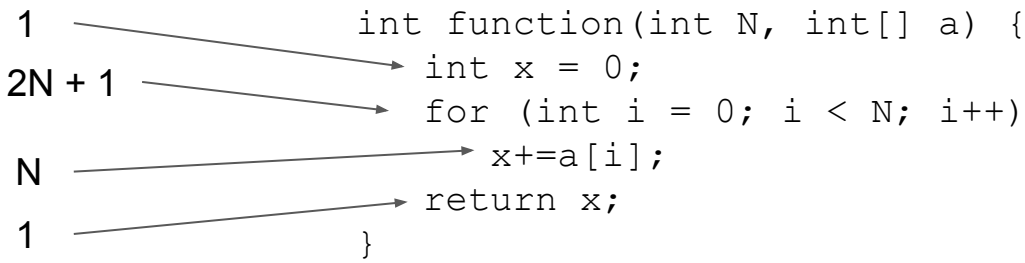

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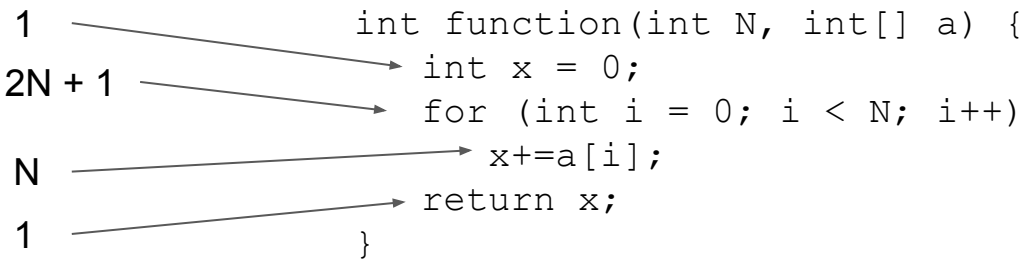

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THAT is what the Big-Oh can be represented as.

3**N**+3 total $\in$ O(N)

Comparing Algorithms Example

Suppose a program takes N^2 solutions when N is less than 10, but Nlog(N) when N is greater than 10. What would you say the Big-Oh runtime is in terms of N?