Floyd-Warshall

Also Bernard Roy

Floyd’s allpairspath

https://open.kattis.com/problems/allpairspath

Also big truck

https://open.kattis.com/problems/bigtruck

And import spagetti

https://open.kattis.com/problems/importspaghetti

And some ICPC problem in 2008

https://www.urionlinejudge.com.br/repository/UOJ_2130_en.html

All Pairs Shortest Path (Kattis)

Problem

We have n vertices in a graph with up to m weighted edges. There will be q queries of the form of “What is the length of the shortest path from a to b?”

Problem

We have n vertices in a graph with up to m weighted edges. There will be q queries of the form of “What is the length of the shortest path from a to b?”

Need some bounds!

Problem

We have n vertices in a graph with up to m weighted edges. There will be q queries of the form of “What is the length of the shortest path from a to b?”

Need some bounds!

- n is up to 150
- m is up to 5000
- q is up to 1000

Problem

We have n vertices in a graph with up to m weighted edges. There will be q queries of the form of “What is the length of the shortest path from a to b?”

Need some bounds!

- n is up to 150
- m is up to 5000
- q is up to 1000

Idea

Dijkstra from each node for each query

Idea

Dijkstra from each node for each query

O(m*log(m)*q)

Idea

Dijkstra from each node for each query

O(m*log(m)*q)

There is an issue!

Idea

Dijkstra from each node for each query

O(m*log(m)*q)

There is an issue!

The weights of the edges can be negative

Idea

Dijkstra from each node for each query

O(m*log(m)*q)

There is an issue!

The weights of the edges can be negative

Dijkstra’s becomes much worse when edge weights are negative

Idea

Dijkstra from each node for each query

O(m*log(m)*q)

There is an issue!

The weights of the edges can be negative

Dijkstra’s becomes much worse when edge weights are negative

Different algorithm

Idea

Dijkstra from each node for each query

O(m*log(m)*q)

There is an issue!

The weights of the edges can be negative

Dijkstra’s becomes much worse when edge weights are negative

Different algorithm

Floyd’s

Floyd’s Concept

I went to a kindergarten close to a car mechanic and pawn shop

Floyd’s Concept

I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations

Floyd’s Concept

I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

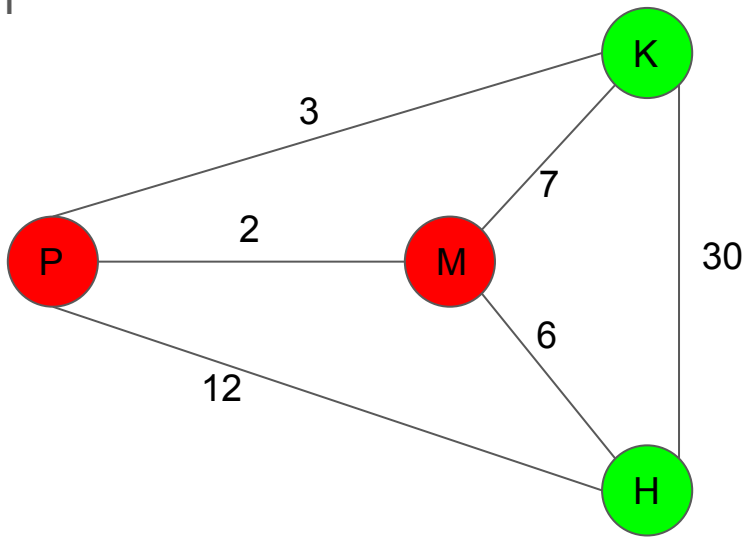
Suppose this is the graph

Floyd’s Concept

I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph



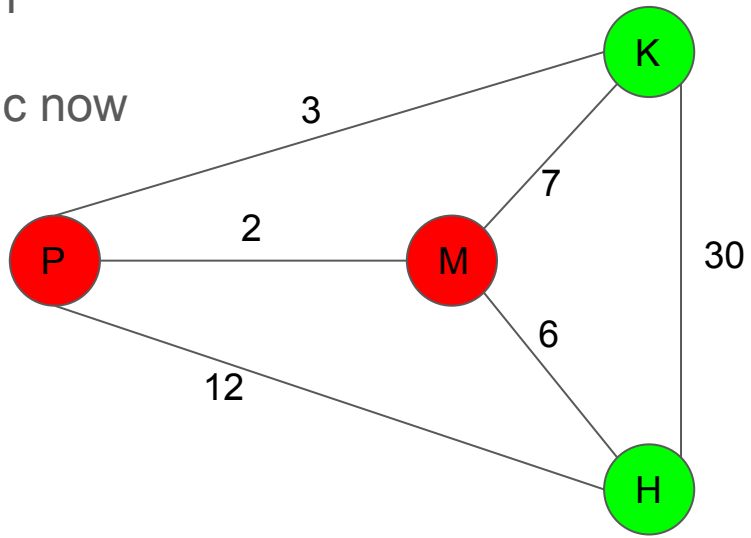
Floyd’s Concept

I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now



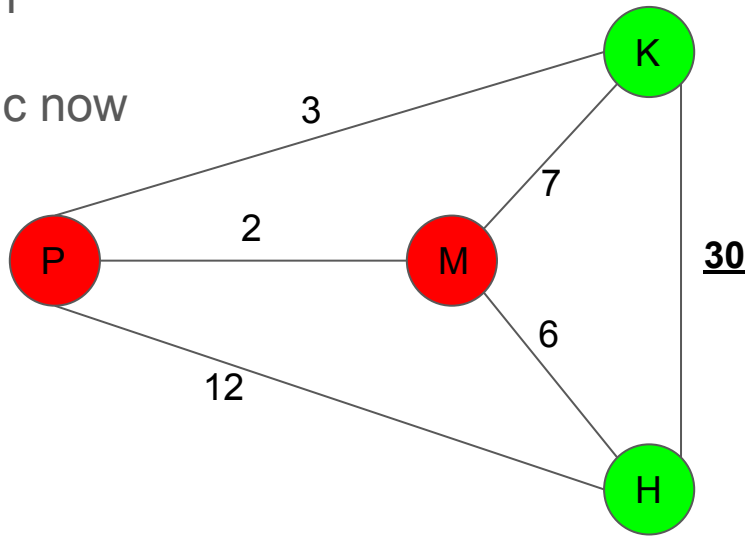
Floyd’s Concept

I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now



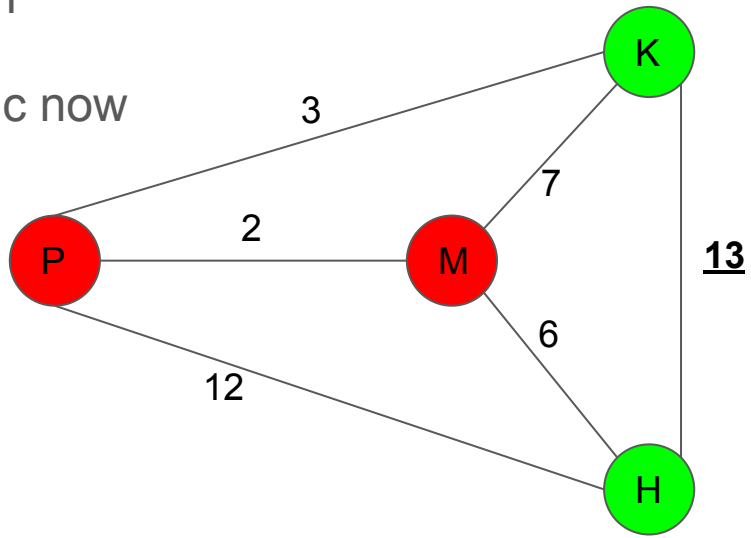
Floyd’s Concept

I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now



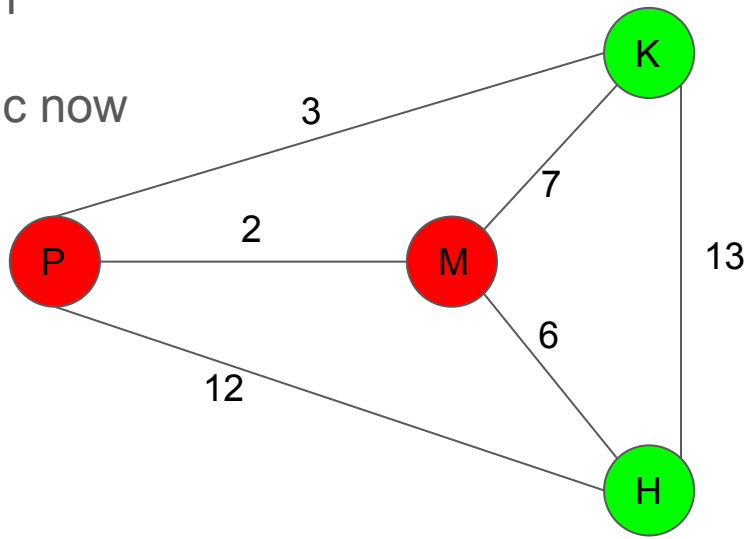
Floyd’s Concept

I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now



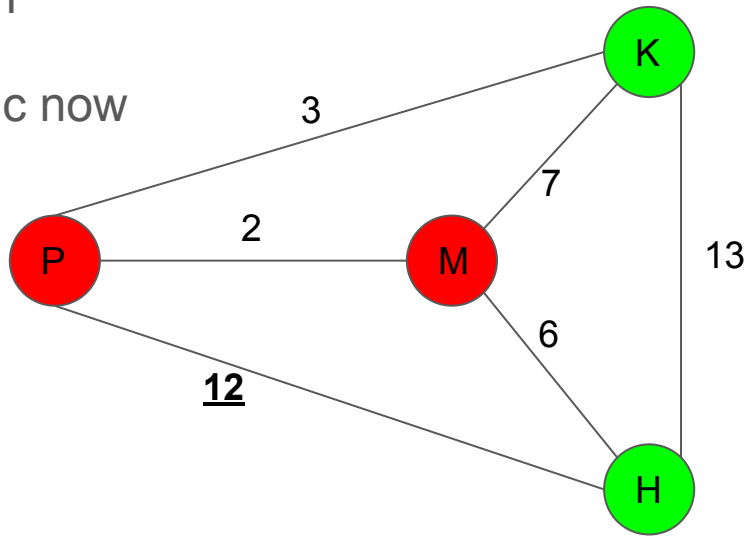
Floyd’s Concept

I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now



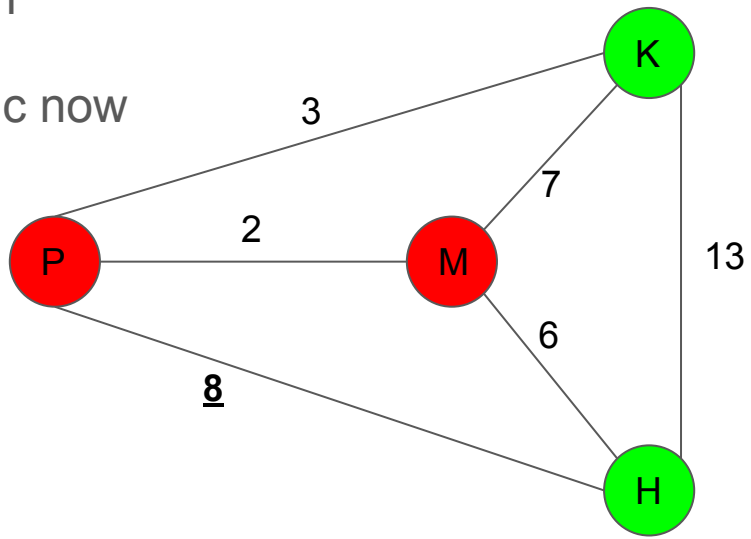
Floyd’s Concept

I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now



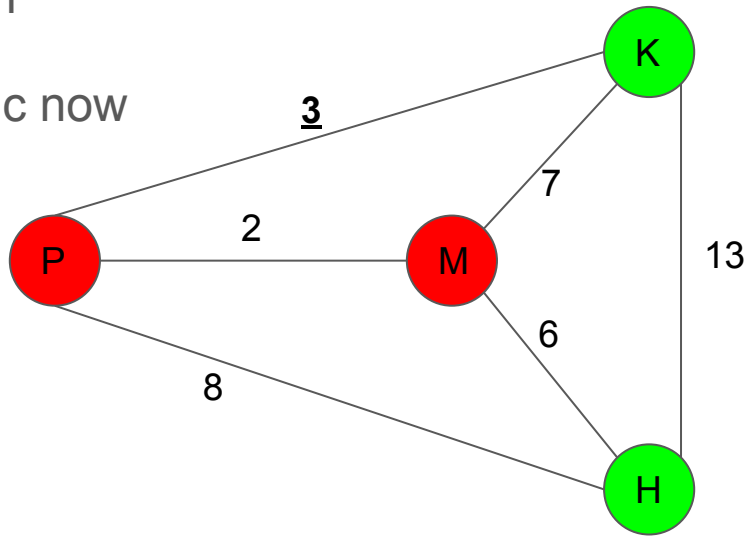
Floyd’s Concept

I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now



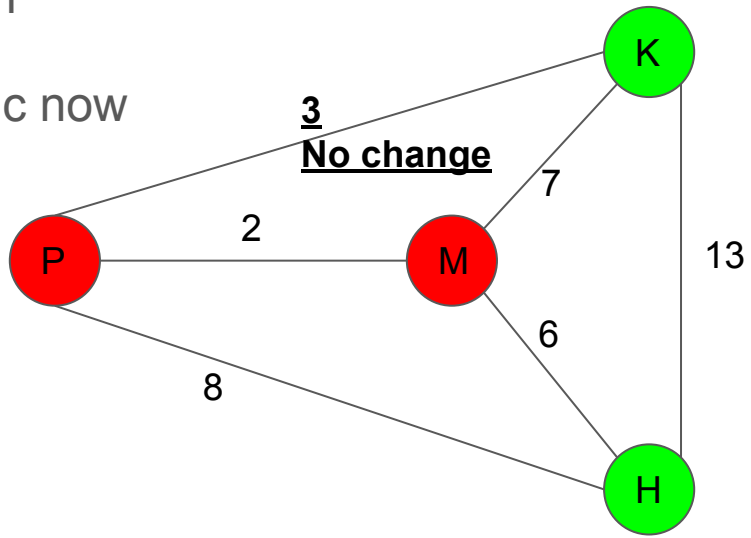
Floyd’s Concept

I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now



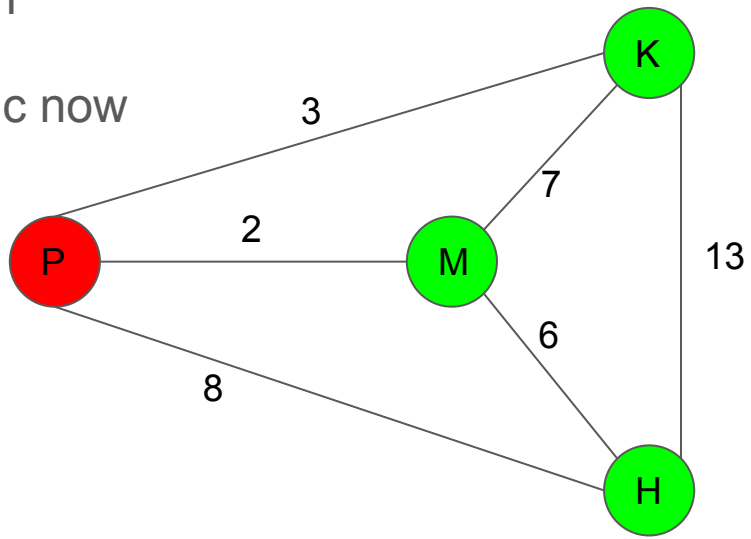
Floyd’s Concept

I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now



Floyd’s Concept

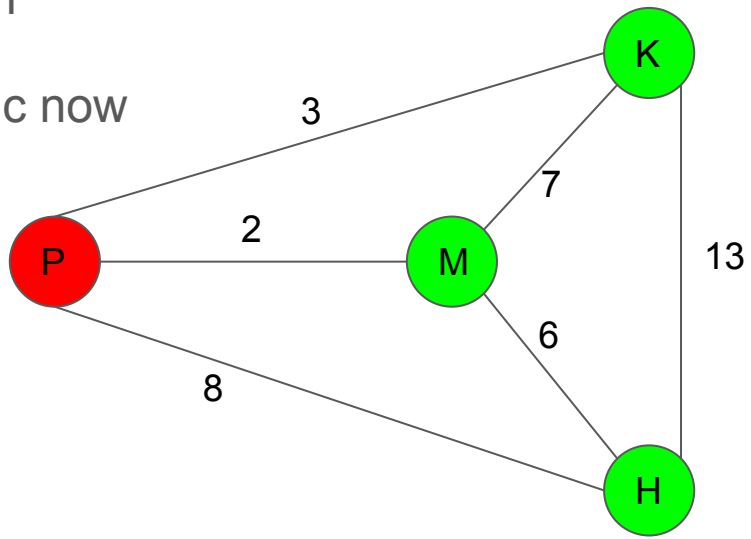
I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now

Using the pawn shop?



Floyd’s Concept

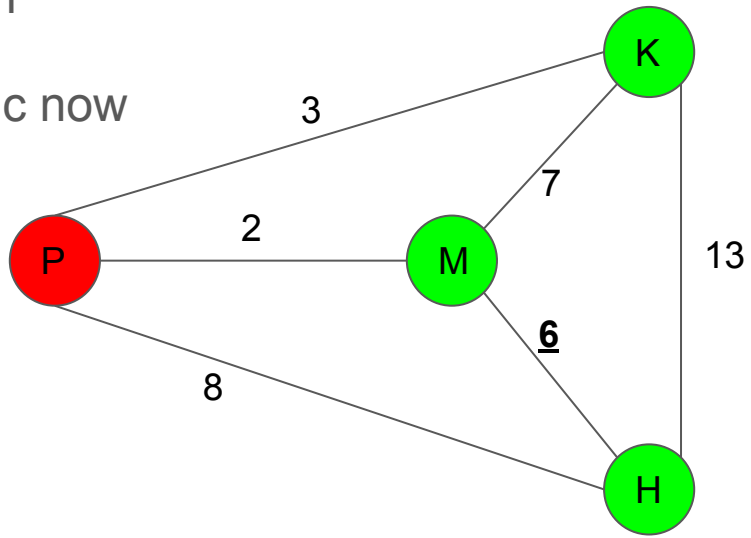
I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now

Using the pawn shop?



Floyd’s Concept

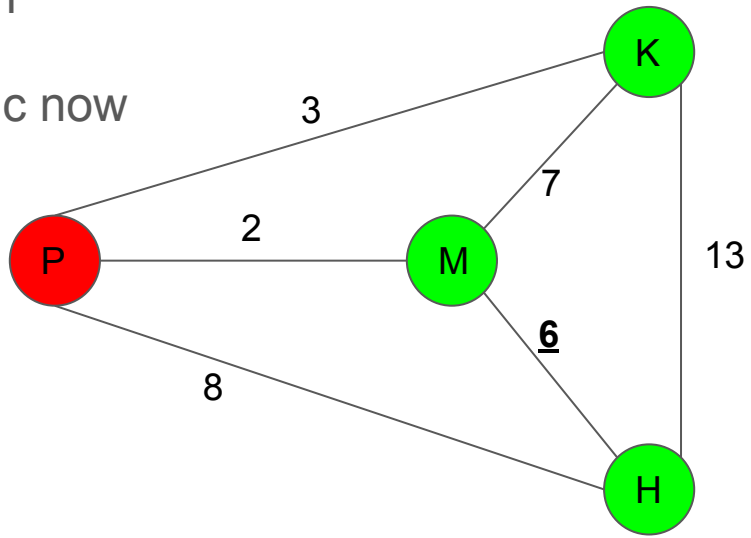
I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now

Using the pawn shop?



Floyd’s Concept

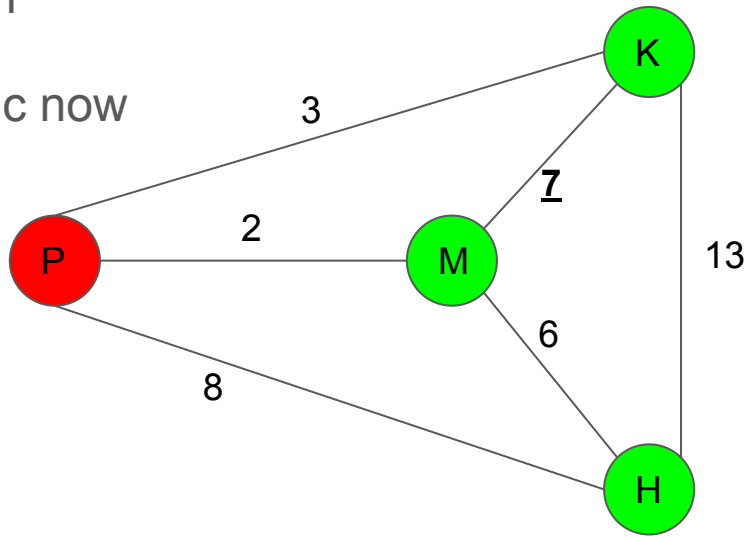
I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now

Using the pawn shop?



Floyd’s Concept

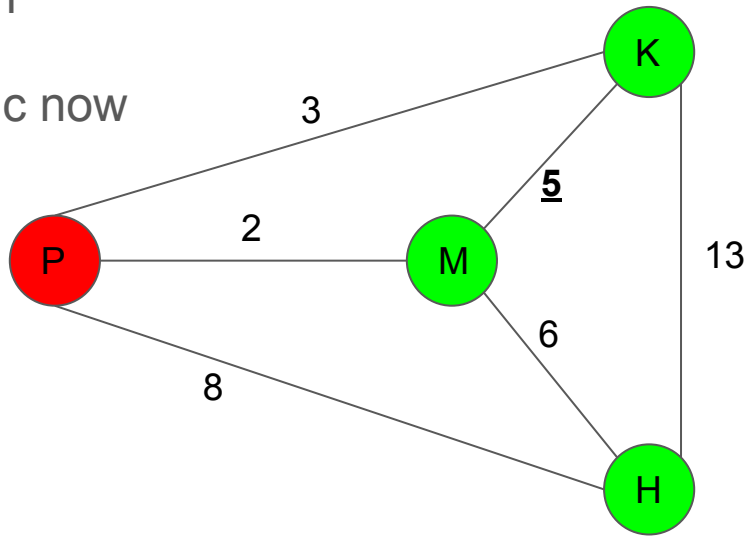
I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now

Using the pawn shop?



Floyd’s Concept

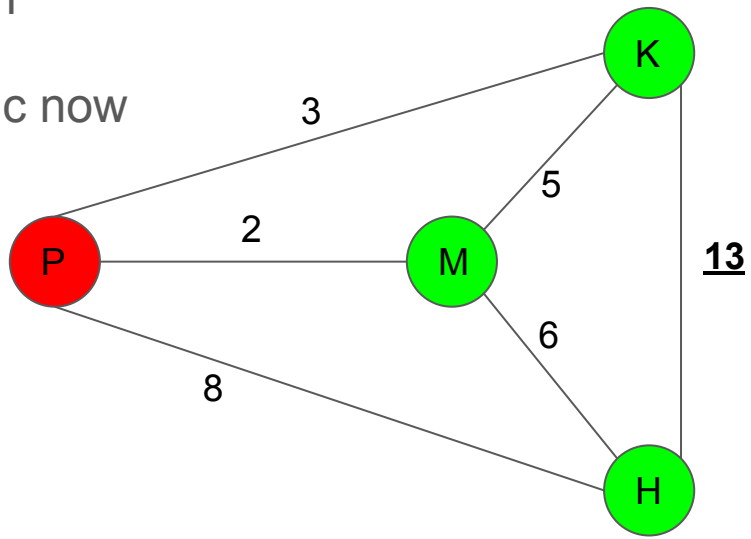
I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now

Using the pawn shop?



Floyd’s Concept

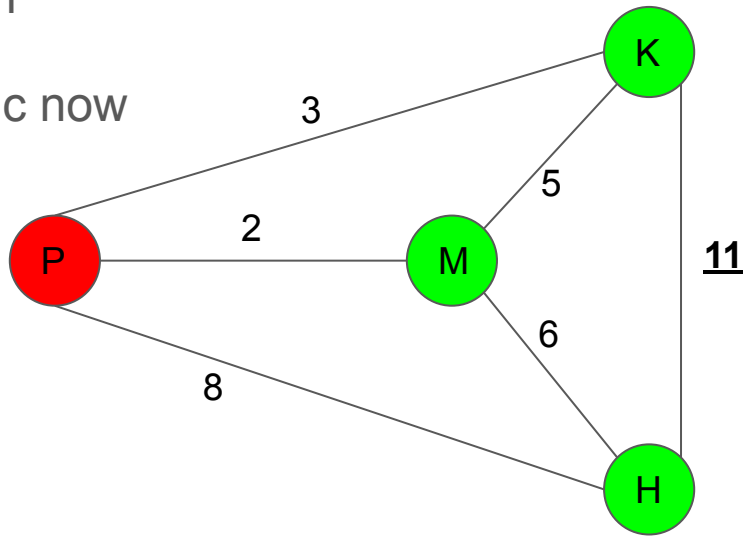
I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now

Using the pawn shop?



Floyd’s Concept

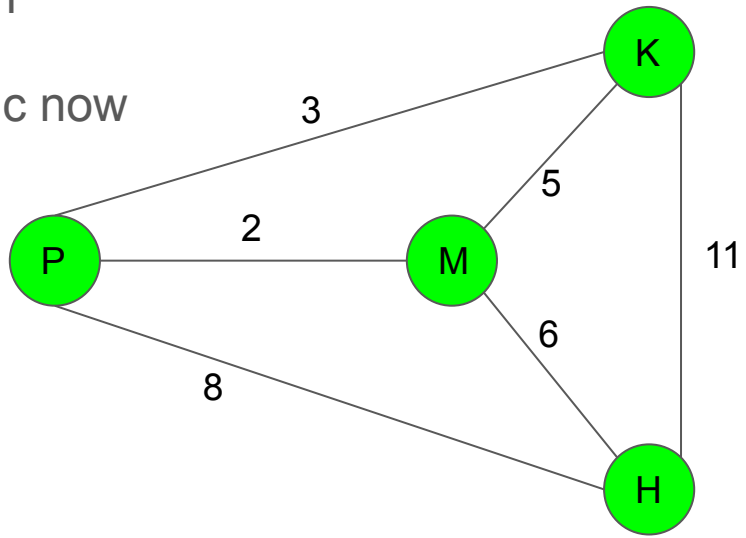
I went to a kindergarten close to a car mechanic and pawn shop

Traveling to the kindergarten was long to avoid some shady locations.

Suppose this is the graph

We can use the mechanic now

Using the pawn shop?



Floyd’s Implementation

The high level

Floyd’s Implementation

The high level

Look a every node as an intermediate point and adjust all pairs of points

Floyd's Implementation

The high level

Look a every node as an intermediate point and adjust all pairs of points

```
for (iter = 0; iter < n; iter++)
   for (st = 0; st < n; st++)
      for (en = 0; en < n; en++)
         if (path from st to en is worse than
             path from st to iter combined with
             Path from iter to en)
         Update the path from st to en
```

Floyd’s Usefulness

Can be used to find ALL pair of shortest paths on graphs with negative edges

Floyd’s Usefulness

Can be used to find ALL pair of shortest paths on graphs with negative edges

Can be used (in one pass) to find some nodes on negative cycles which can extend to finding all nodes that can

Floyd’s Usefulness

Can be used to find ALL pair of shortest paths on graphs with negative edges

Can be used (in one pass) to find some nodes on negative cycles which can extend to finding all nodes that can

- Don’t run it multiple times
- Check nodes that are on a negative cycle
- Check nodes that can reach and be reached by these negative cycles

Floyd’s Bad News

Runtime

Floyd’s Bad News

Runtime

$O(N^3)$

Floyd’s Bad News

Runtime

$O(N^3)$

Don’t use over 500 Nodes in competitive programming contests.

Floyd’s Bad News

Runtime

$O(N^3)$

Don’t use over 500 Nodes in competitive programming contests.

Up to 300 Nodes is safe.

Other Applications

Can also be used to find the transitive closure of a graph (SCCs).

Other Applications

Can also be used to find the transitive closure of a graph (SCCs).

Most people will use Tarjan’s Low Link method or Kosaraju's for SCC.