Distances

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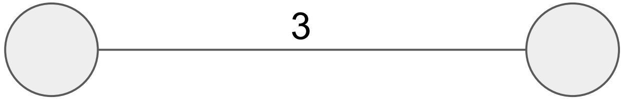

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Luckily there are three other algorithms that can be helpful.

Bellman-Ford-Moore

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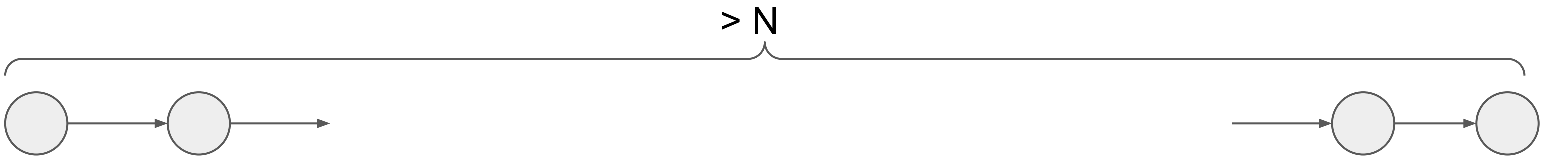
Requires an observation,

- A shortest path would consist of at most N vertices, **assuming no negative cycles exist**. Can be proven using Pigeonhole Principle and contradiction.

Proof

Assume a shortest path uses at least k (> N) vertices.



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Assume a shortest path uses at least k (> N) vertices.

Then there exists two vertices that are identical.

By Pigeonhole Principle.

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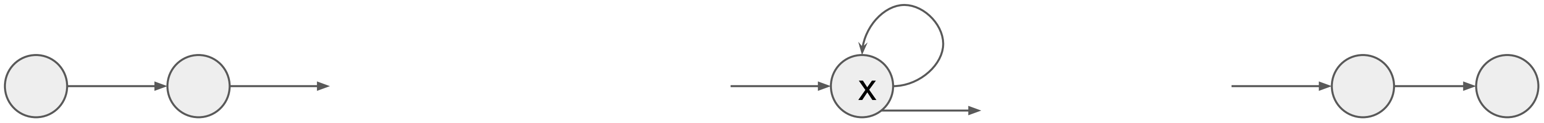

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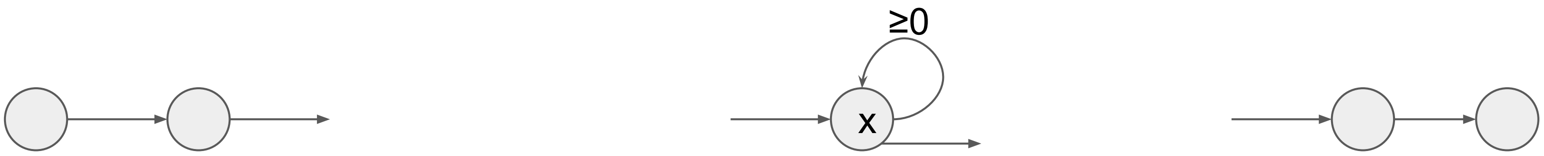
The cycle is non-negative.



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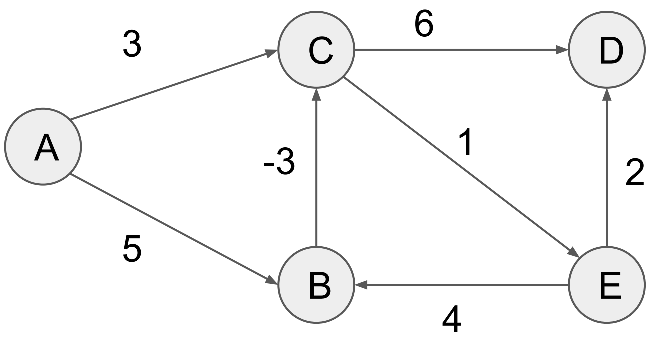

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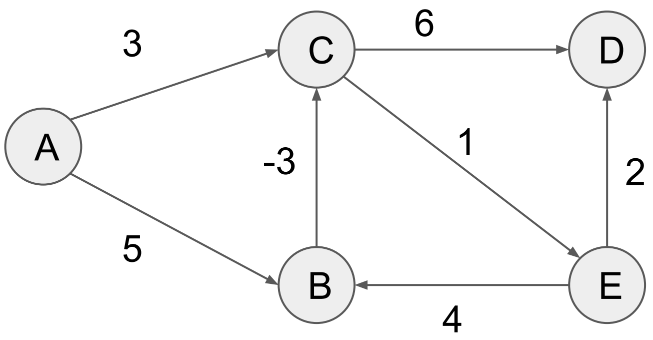

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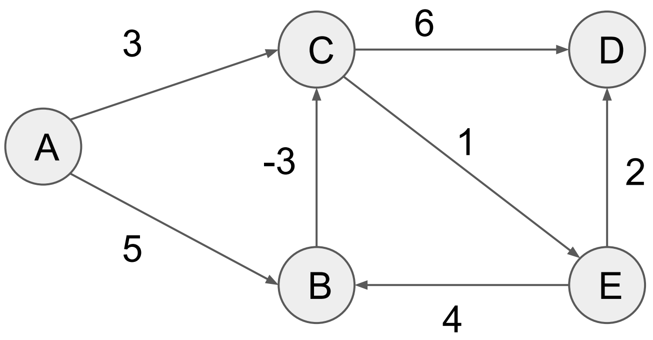

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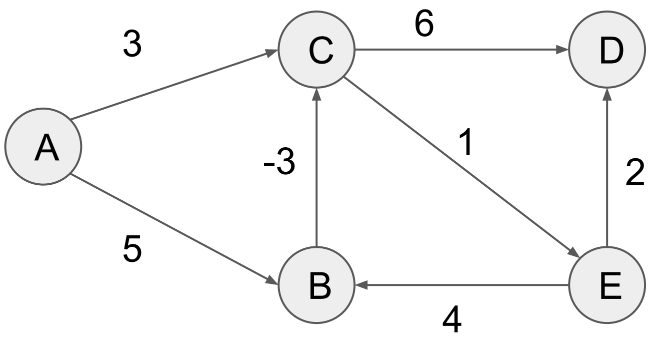

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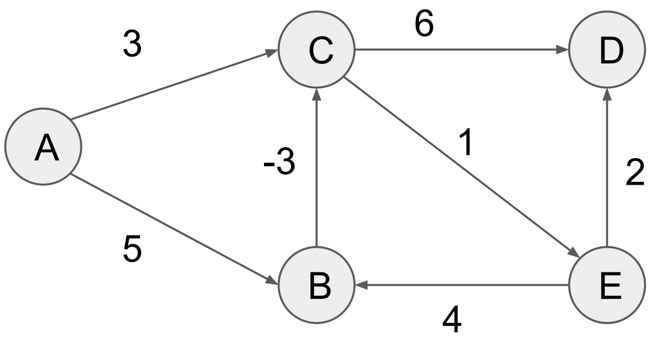
Node	A	B	C	D	E
Distance	0	5	3->2	∞->9	∞->4



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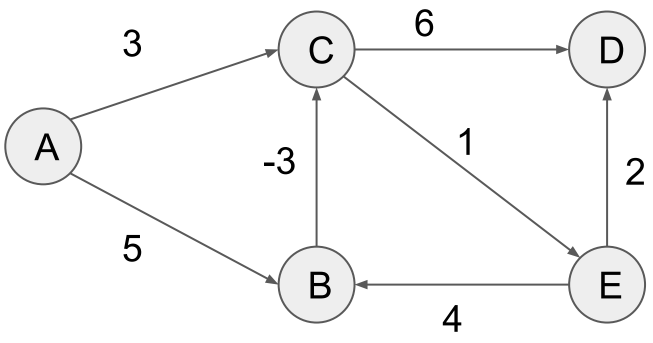

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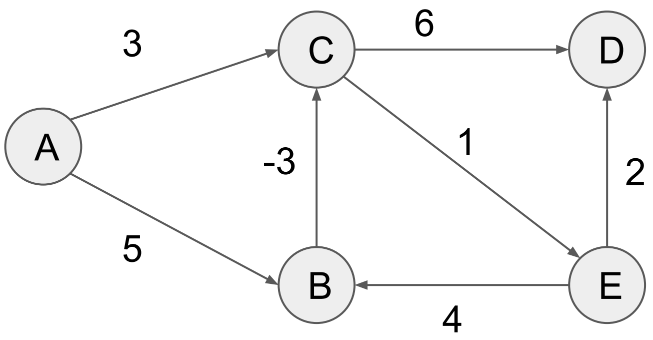

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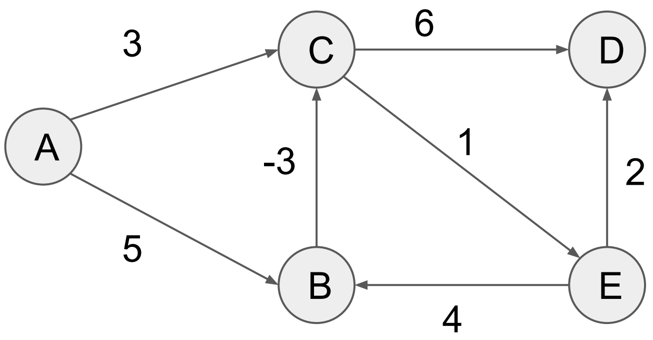

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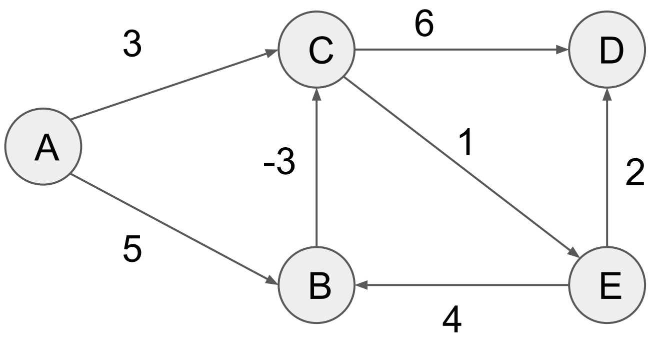

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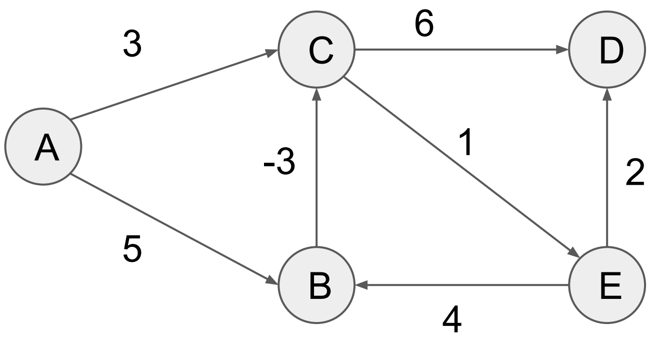

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What is the total runtime?

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After updating N times… Update again.

If any value changes, a negative cycle must exist!

Bellman-Ford Summary

Works with negative edge weights.

Can find the shortest distance from some node to all others.

Can detect negative cycles.

Reasonable runtime: $\Theta(|V||E|)$

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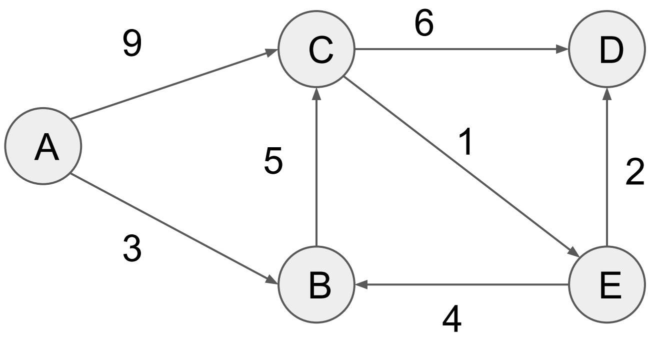

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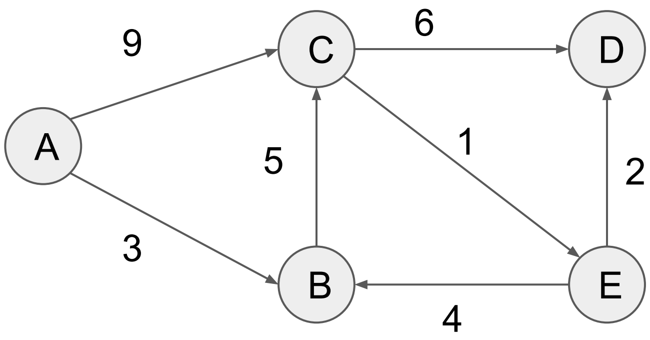

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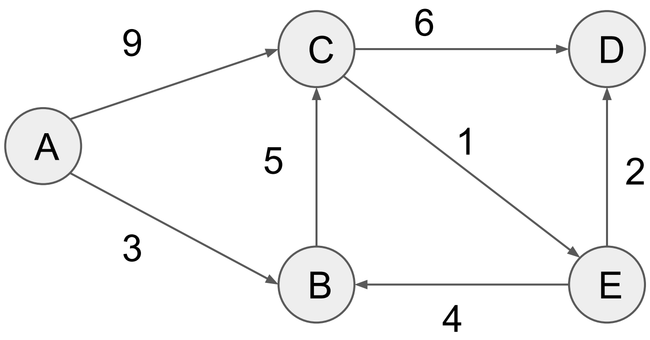

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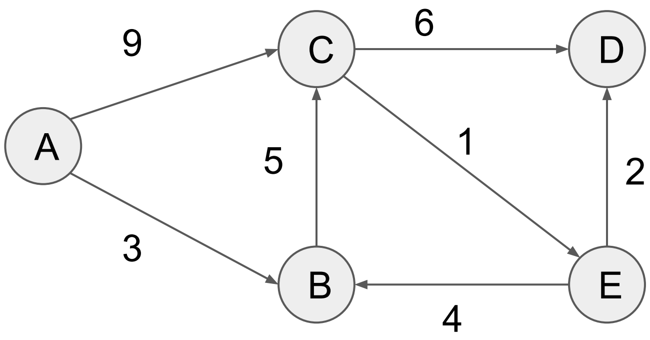

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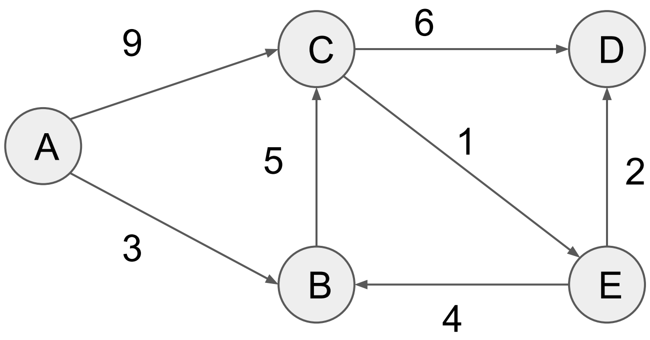

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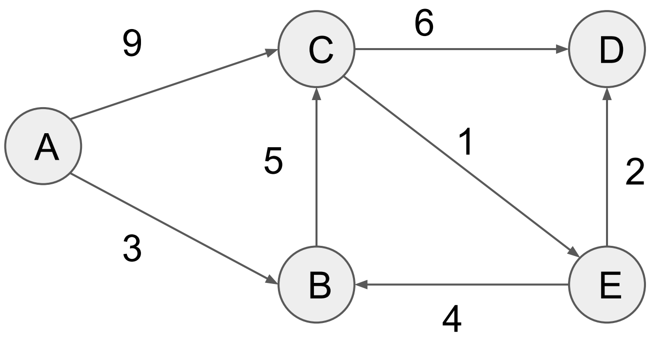

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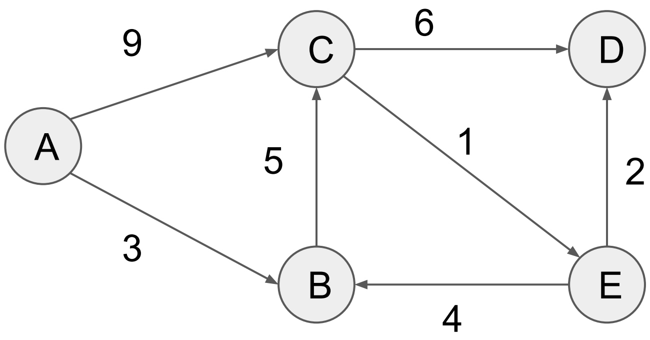

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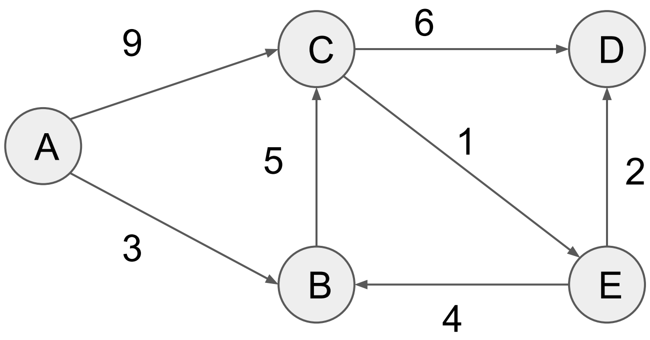

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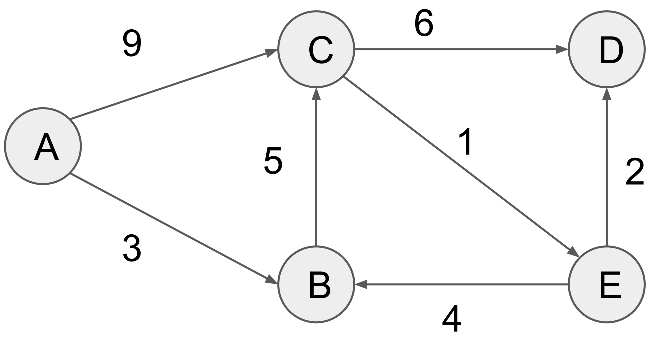

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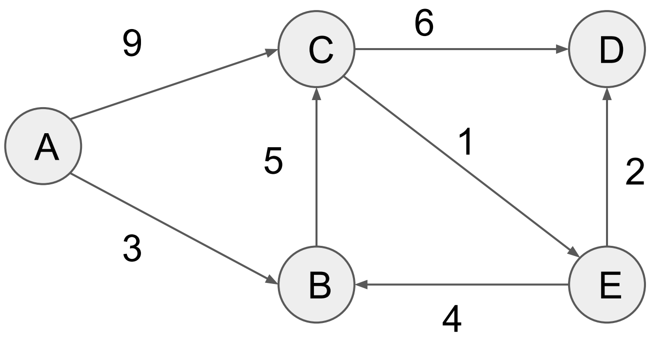
Distance	0	3	8	**14**	**9**



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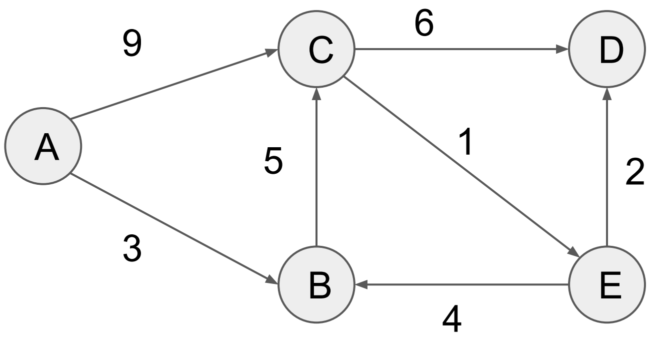

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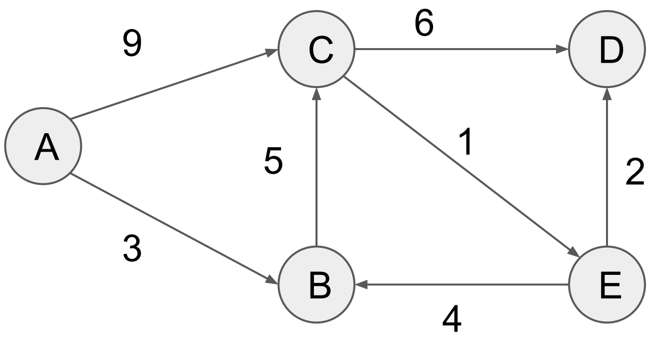

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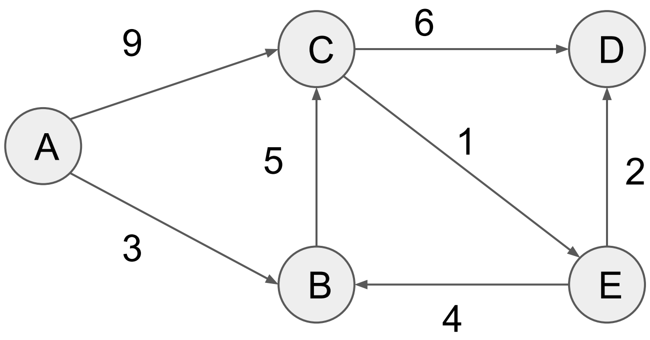

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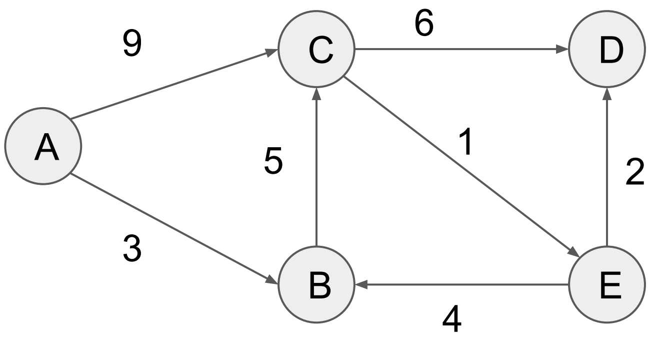

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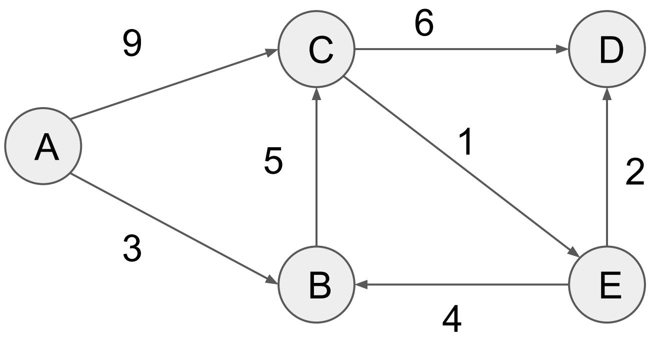

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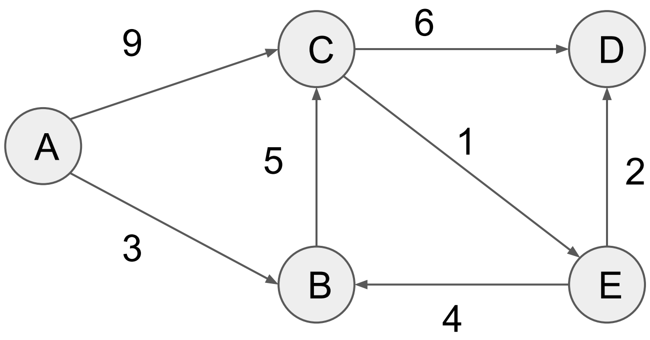
No change.



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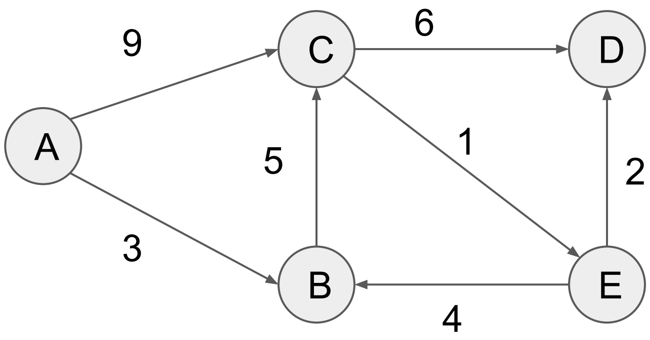

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What would be the total number of operations to insert/remove the values?

  |E|log(|E|)

Pseudo-code

```
Make a distance array of N infs
Set distance of source to 0
Add to an empty priority queue the pair (0, source)
Make a visited array of N falses
While the priority queue has elements
   Let current pair be the top of the priority queue
   Remove the top of the priority queue
   If visited the second term of the current pair
      Go to the top of the While loop
   End If
   Let visited of the second term of the current pair to true
   For all edges e leaving the second term of the current pair
      If e improves the distance for the destination of e
         Update distance for destination of e
         Add to the priority queue (new distance, destination of e)
      End If
   End For
End While
```