Backtracking

Optimized Bruteforce

General Idea

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- Example
  - Sudoku

Sudoku

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	2		4	5				
	7			6		2		4
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6								
	8		7		3			
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8	5				9			
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Sudoku

- What decisions do we have?

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Sudoku

- What decisions do we have?
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- What decisions do we have?
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  - In our case 9^{58}

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  - ~2e55

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Solving!

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Solving!

- Let’s try the first decision

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Solving!

- Let's try the first decision
- And the second...

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- We could keep trying decisions

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Solving!

- Let's try the first decision
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  - Probably a bad idea

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- Let's try the first decision
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  - So we won't

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- Represent our choices graphically

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- Let's try the first decision
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- And the second…
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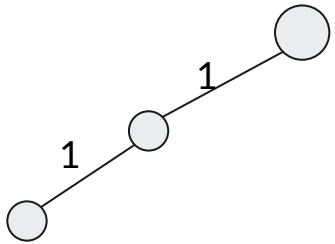

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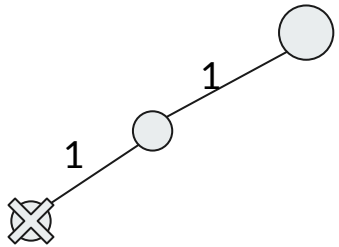

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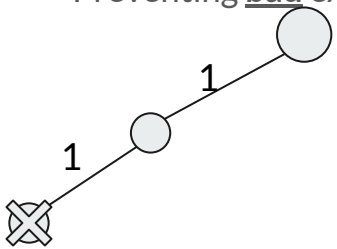
- Represent our choices graphically
  - Decision tree
- Preventing **bad** explorations reduces runtime



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- Solve Recursively

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  - Recurse (decision)
    - IF decision is at the end THEN Do Base Case
    - FOR each choice in decision
      - Try choice
      - Recurse (decision + 1)

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  - **IF choices are not valid THEN Do first base case**
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- The earlier we make an invalid decision the better!

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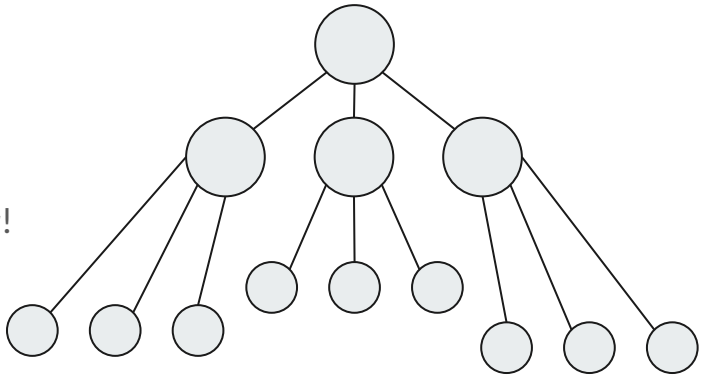

Sudoku Coding Time!

Other Problems

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  - Place N queens on a N by N chess board

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  - Select the position for the queen in a given row starting with the first
- No two queens have the same column
  - They form a permutation
- How do we check for validity?
  - A validity function is easiest
- Note this exact problem will be part of the first lab group