Comparison Sorts

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Comparison sorts are theoretically limited.

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Best case is the split is even.

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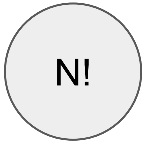
Pictorially



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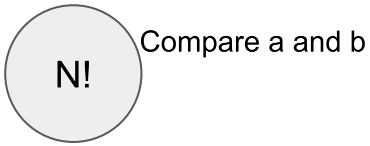

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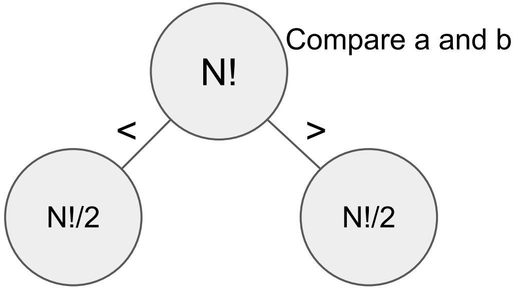

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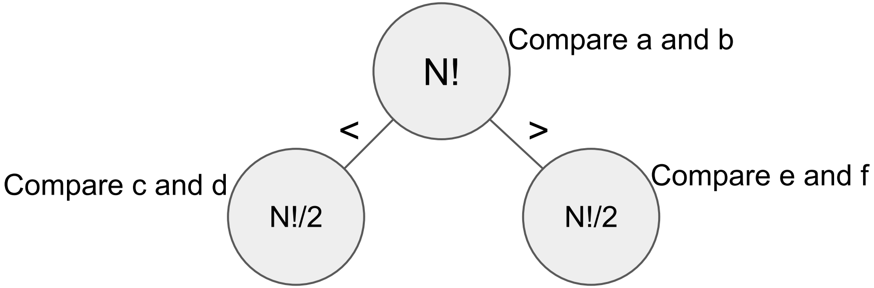

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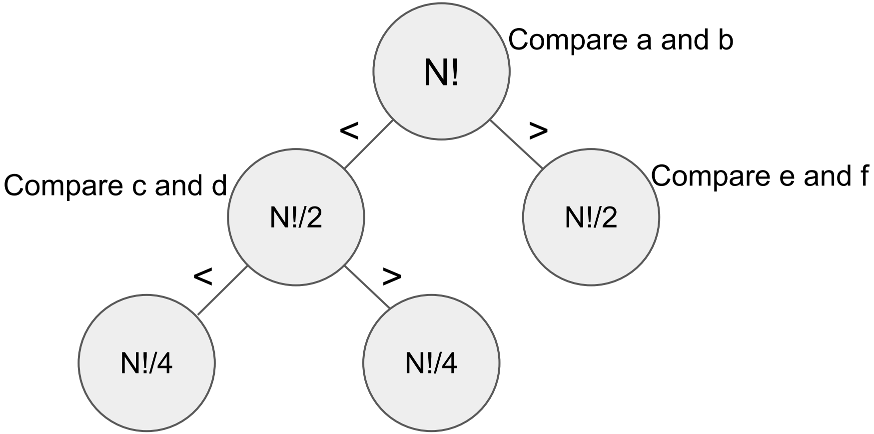

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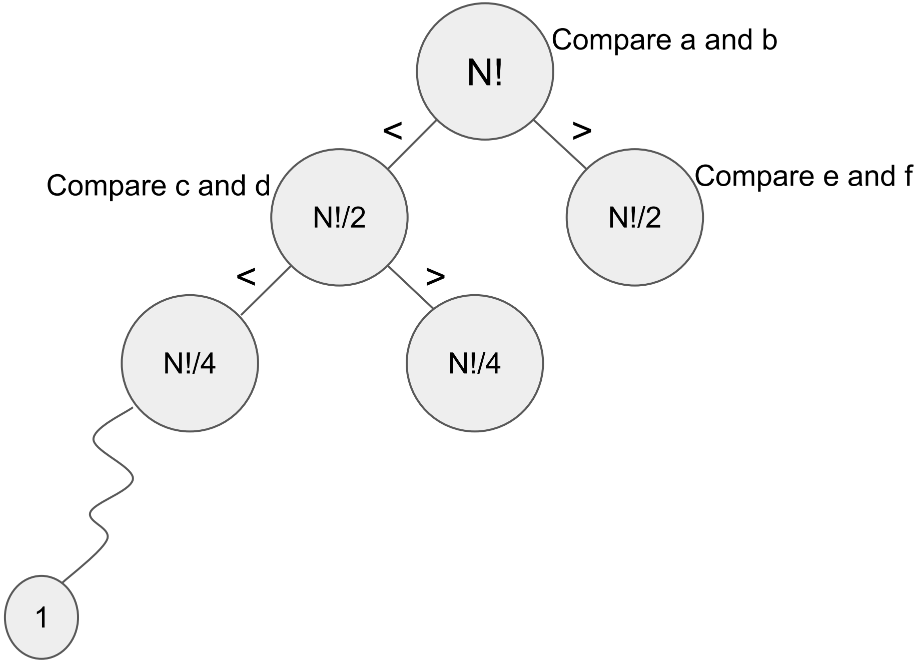

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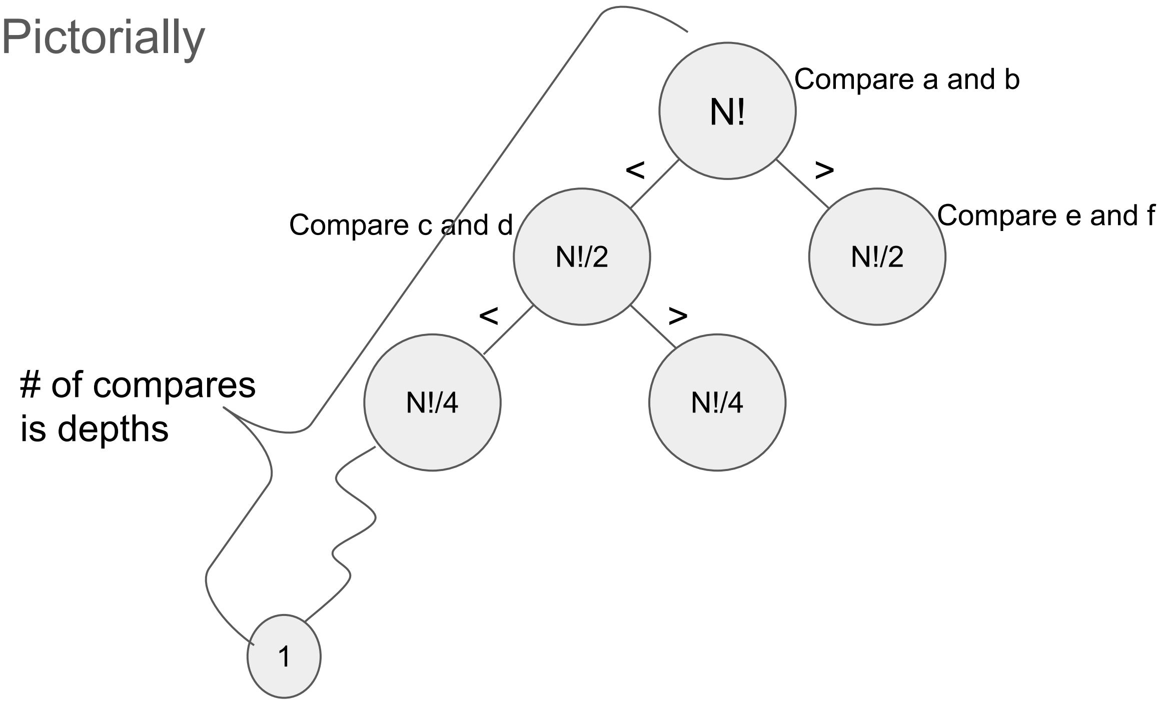

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Thus, it's in $\Theta(N\log(N))$