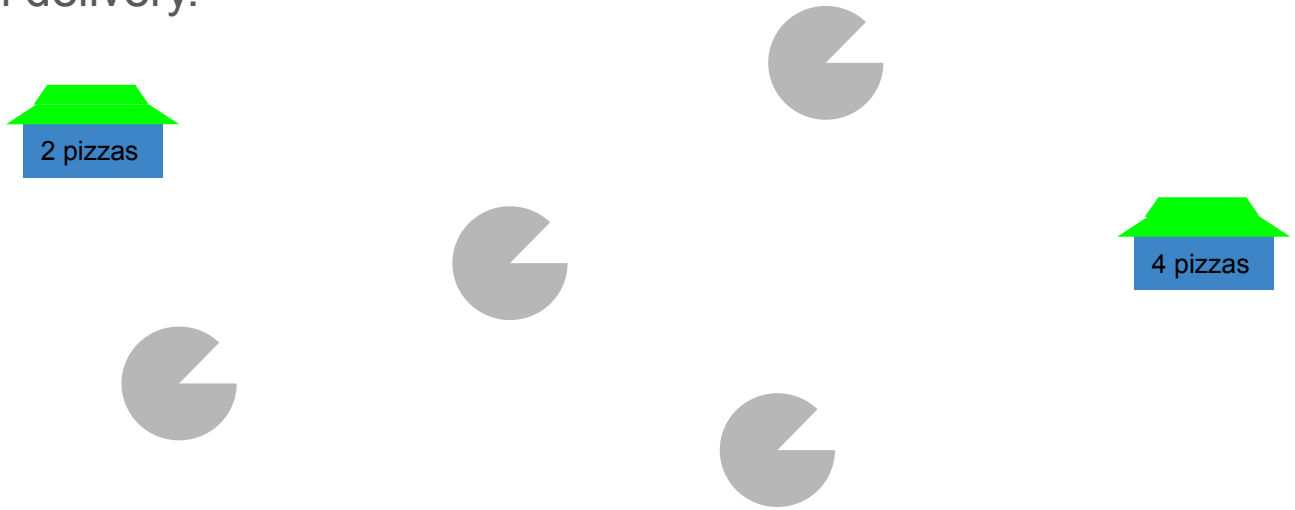
Other Greedy Problems

Pizza Delivery

You own two pizza places. Each place has some number of pizzas. There are people that have ordered pizzas. Find the least amount of gas needed to deliver all the pizzas assume you drive from the pizza joint to the person and back with each delivery.



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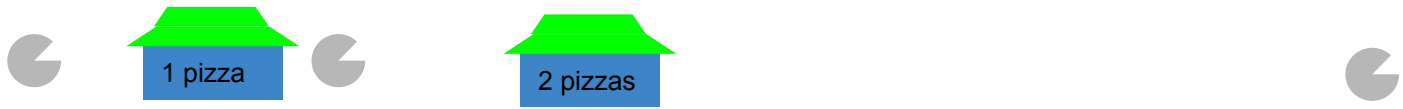

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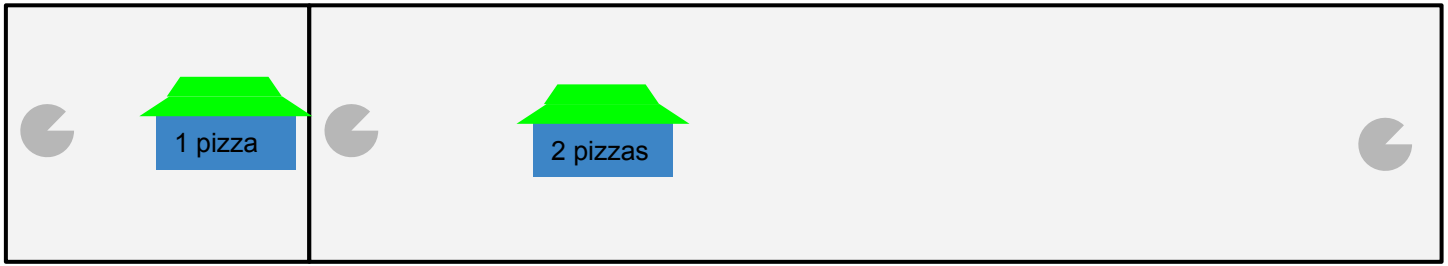

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If a location runs out of pizza, everyone else goes to the other location.

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What if some characters occurred ***a lot*** more often than other characters?

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In general no code can be the prefix of another code.

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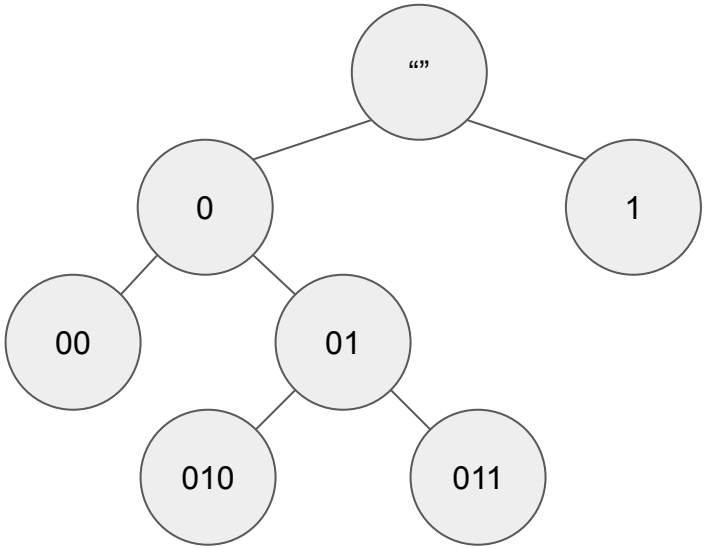

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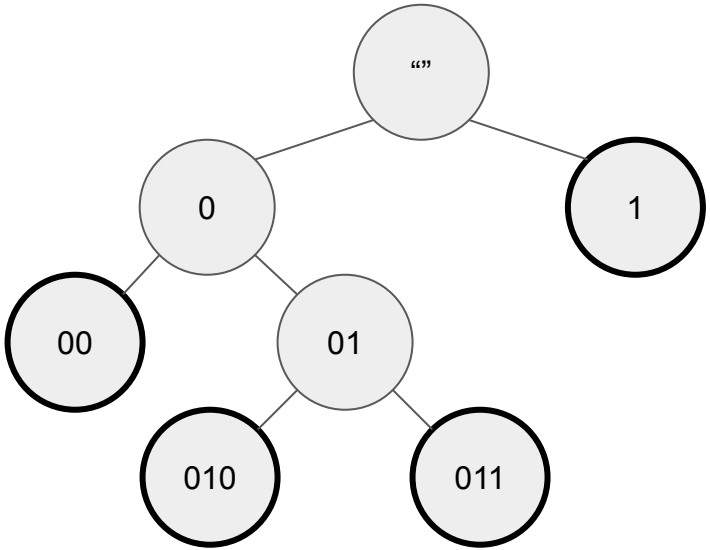

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(Originally 2 * (30 + 100 + 10 + 5) = 290)

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Biggest first is not a good idea...

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Occs.	384	67	109	212	581	125	94

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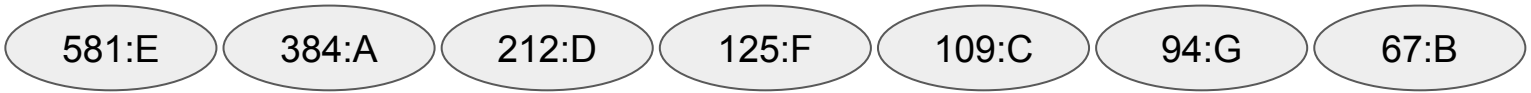

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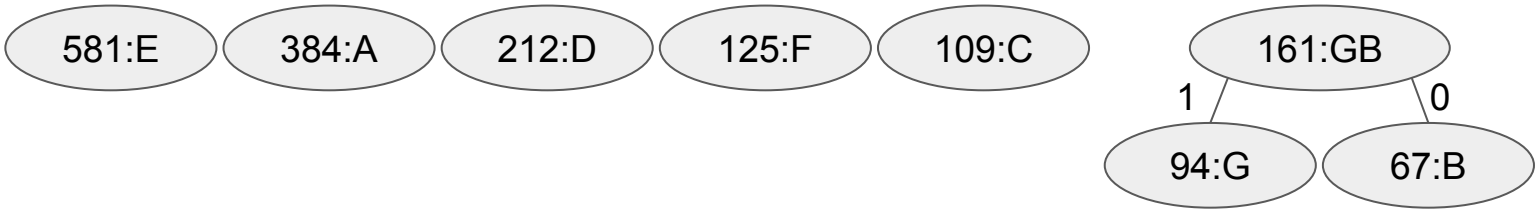

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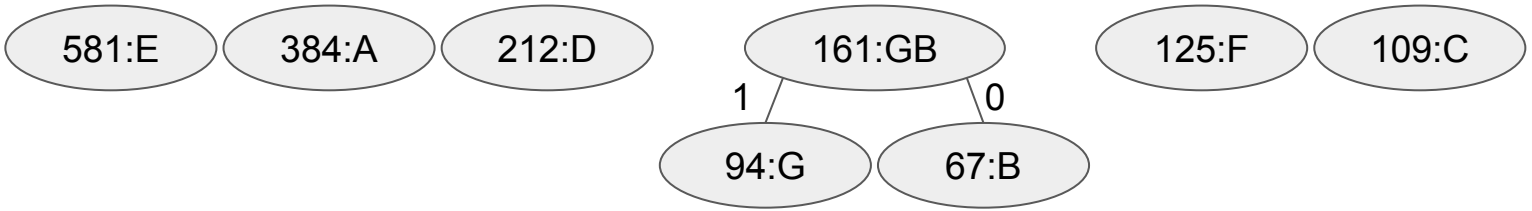

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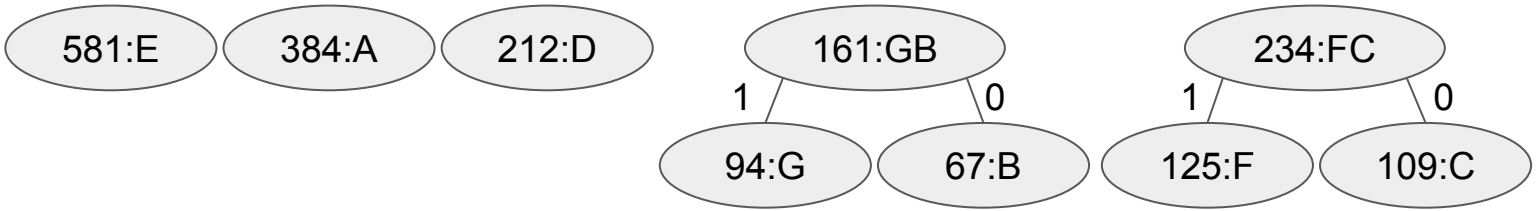

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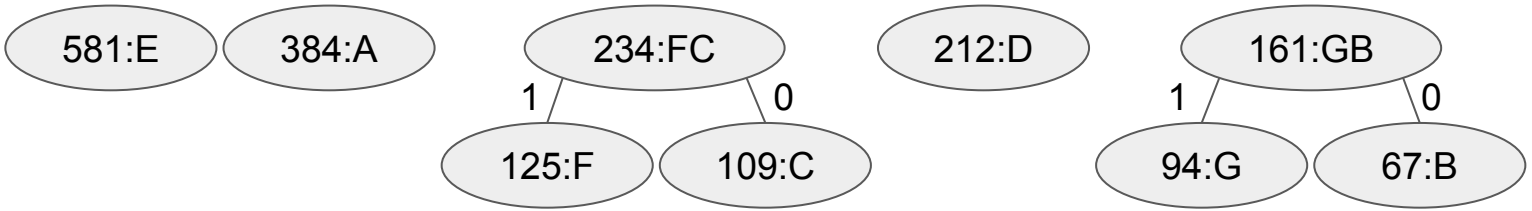

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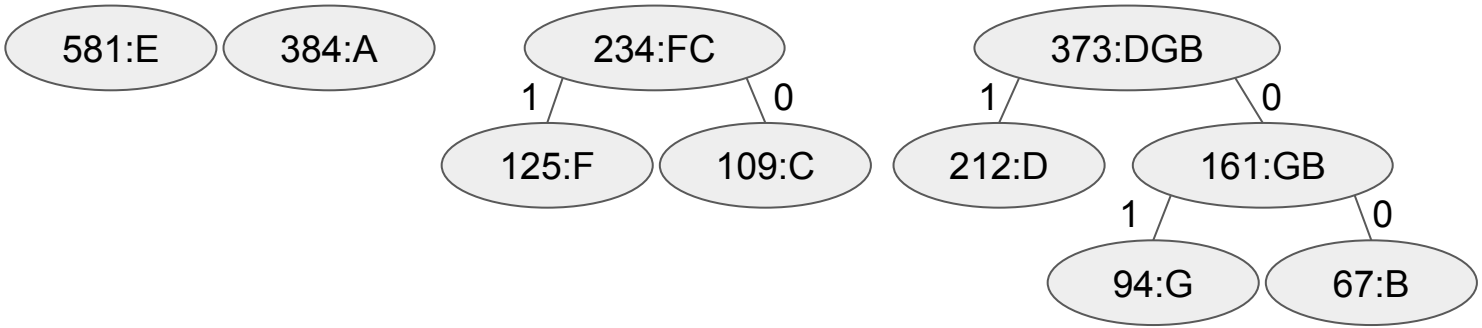

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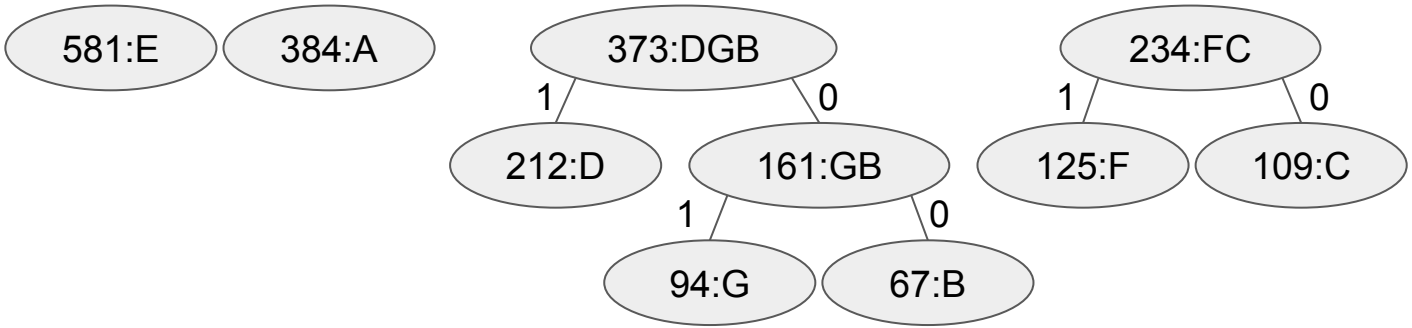

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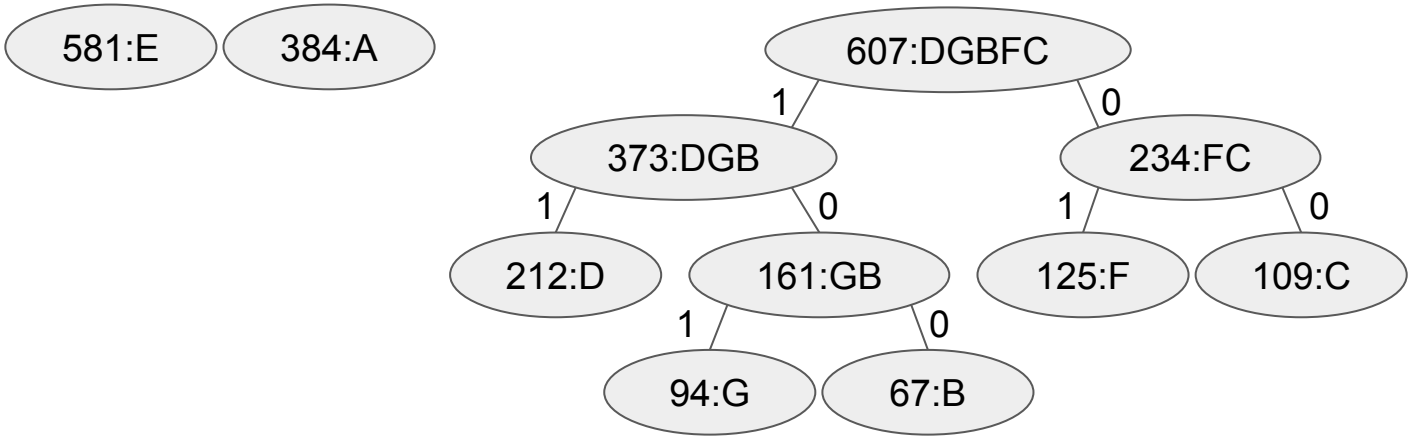

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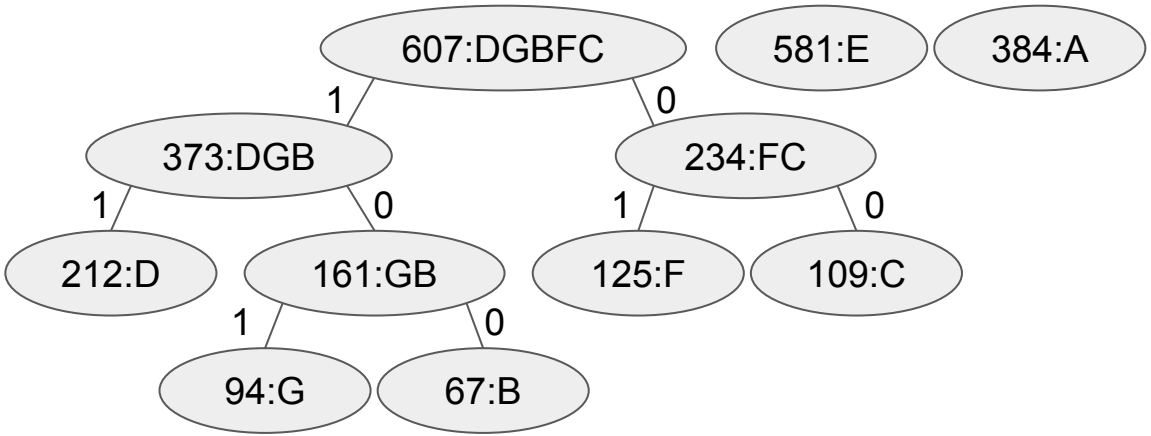

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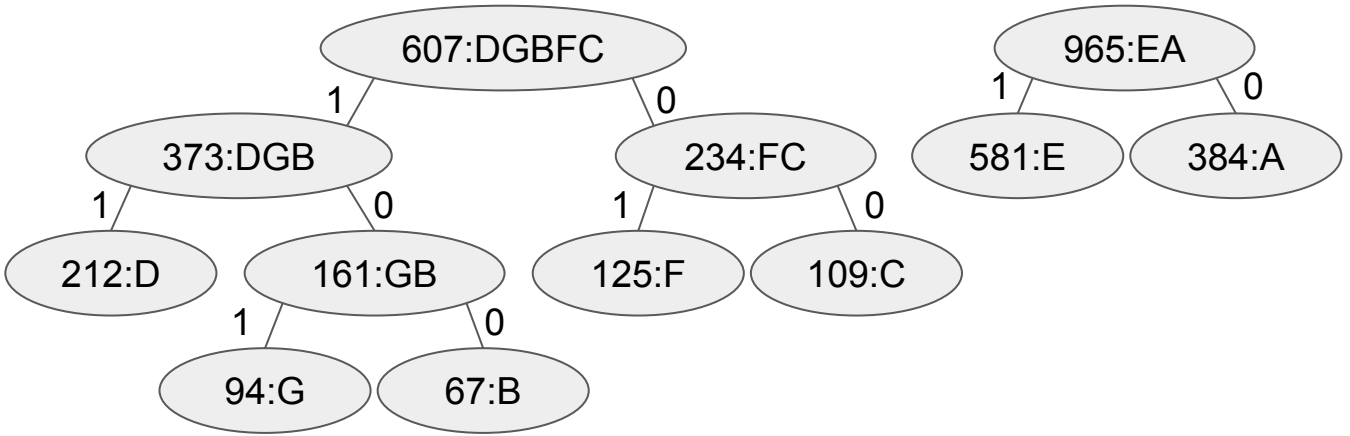

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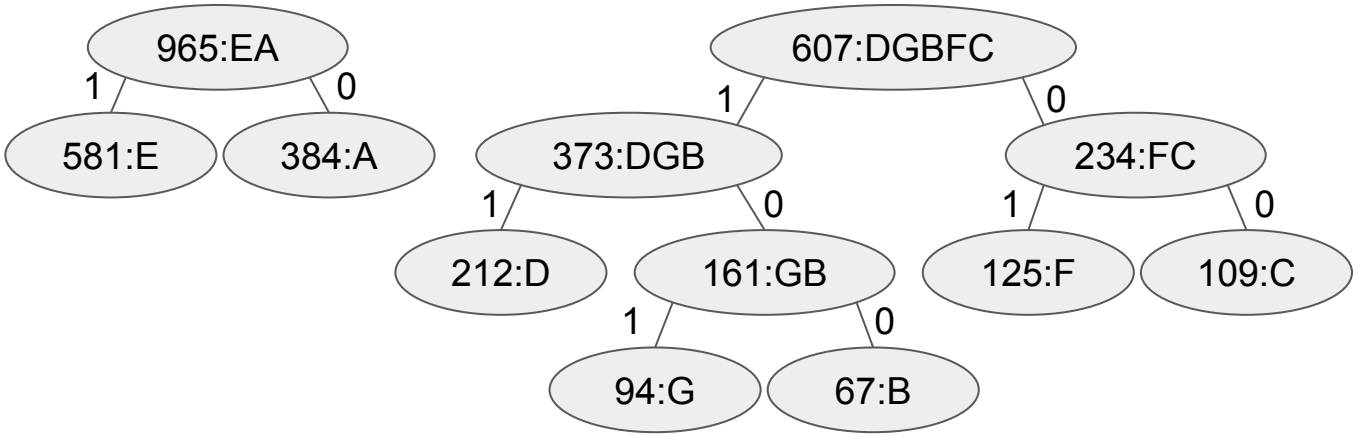

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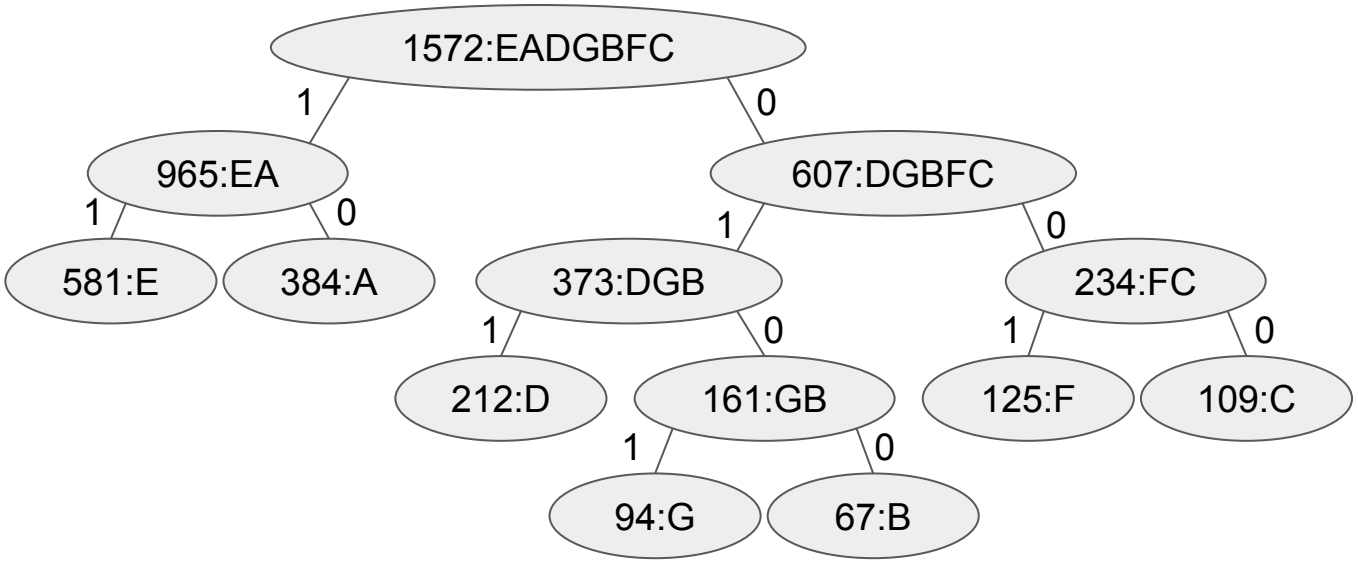

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