

# Skip Lists

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Randomized Ordered Set Data Structure

What other randomized data structures have you heard of?

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High level usage of the skip list

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- Sorted Dictionary

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Sorted Dictionary (BBSTs already can do this!)

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High level usage of the skip list

**Paralleled** Sorted Dictionary

# Linked List-Like

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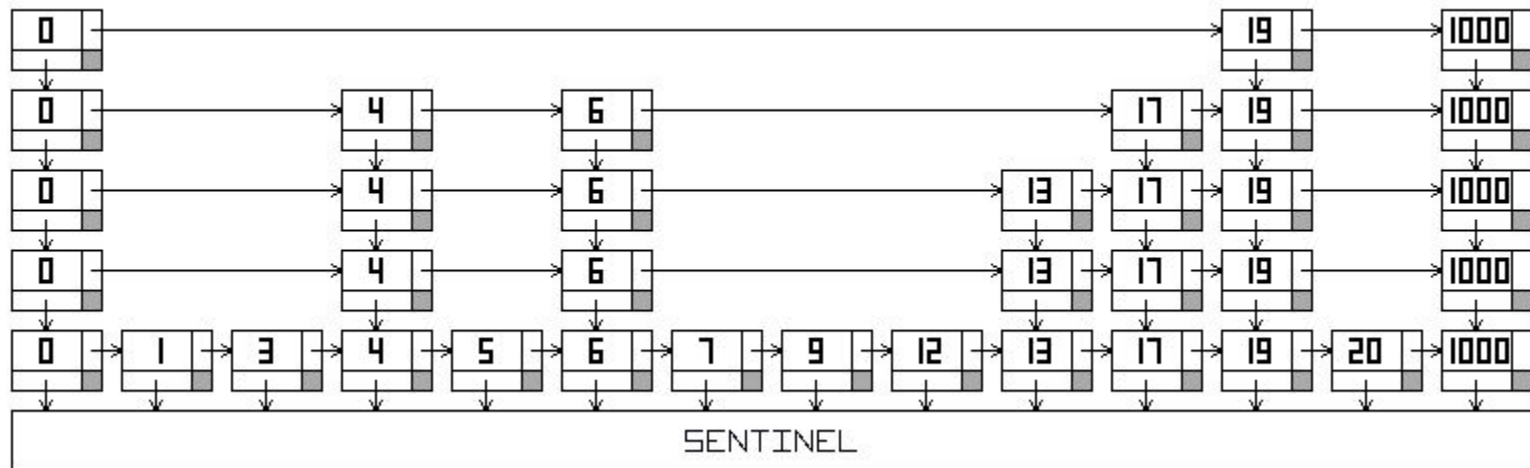
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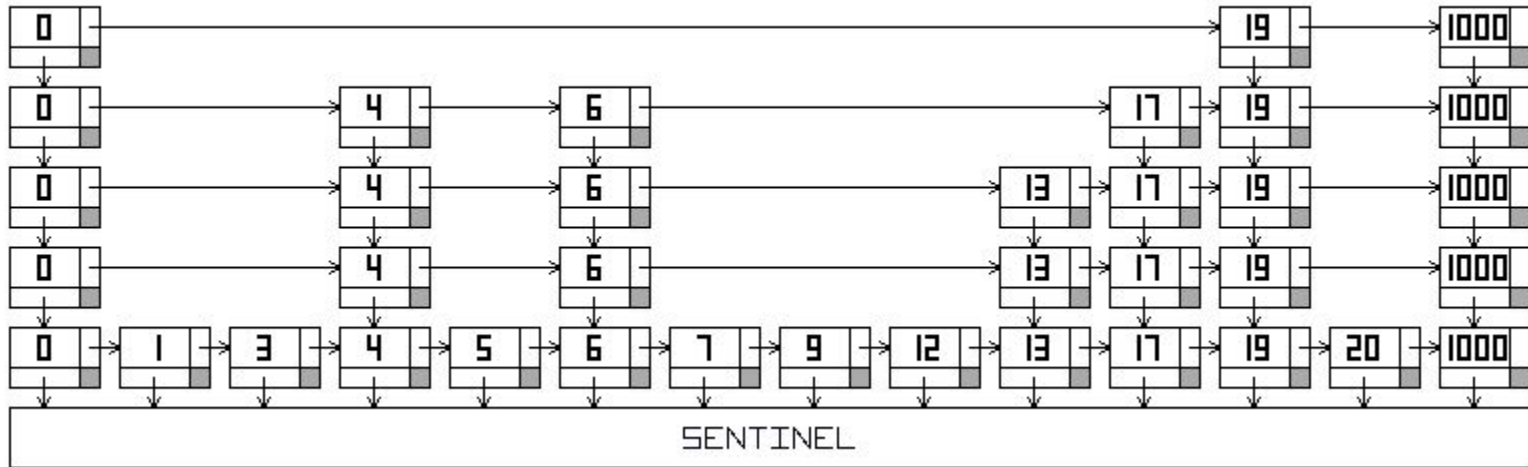
- The skip list becomes a series of lists stacked on top of each other
- If a jump is too big we move down a list

# Picture



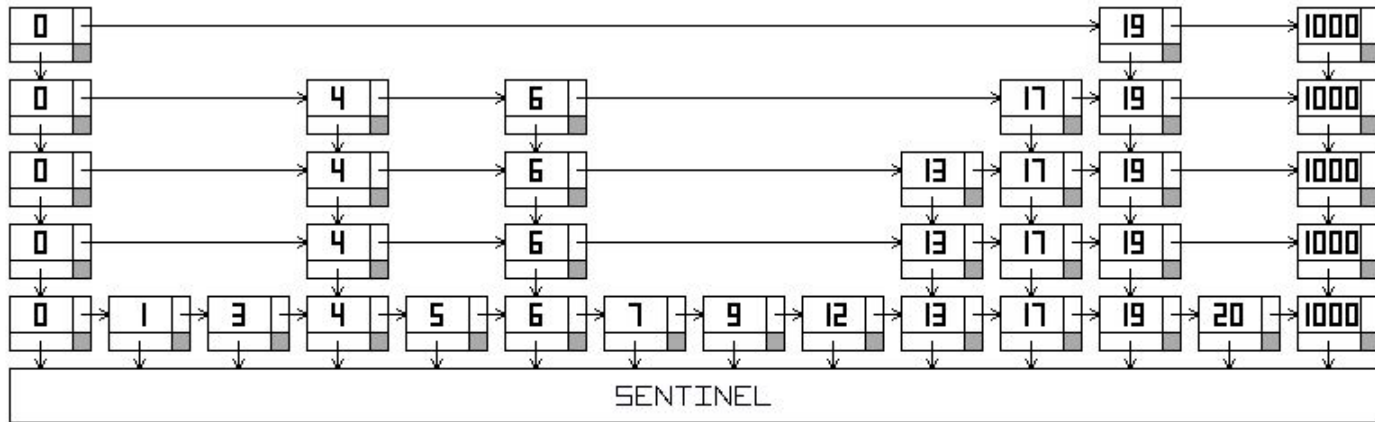
# Picture

How would we know if 14 exists or not?



# Containment

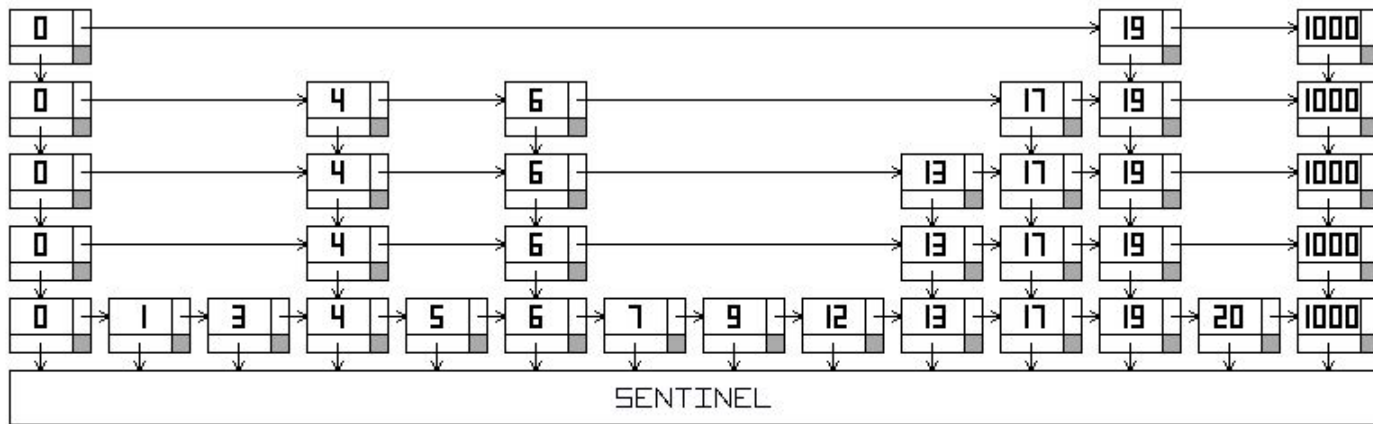
Start current node at the first node



# Containment

Start current node at the first node

- If the next value is too big step down and retry

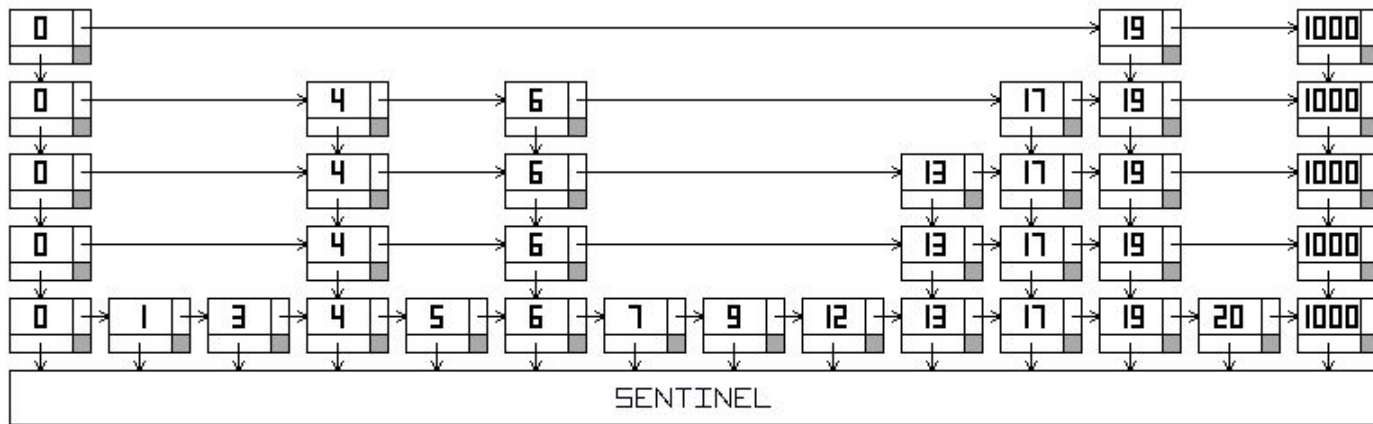




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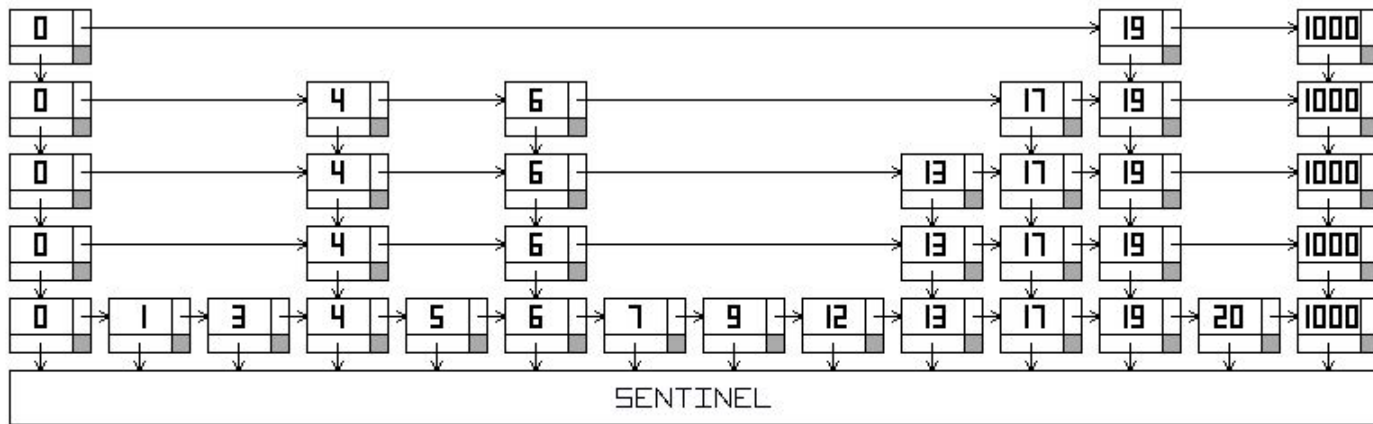
- If the next value is too big step down and retry
- If the next value is too small step to the next node and retry



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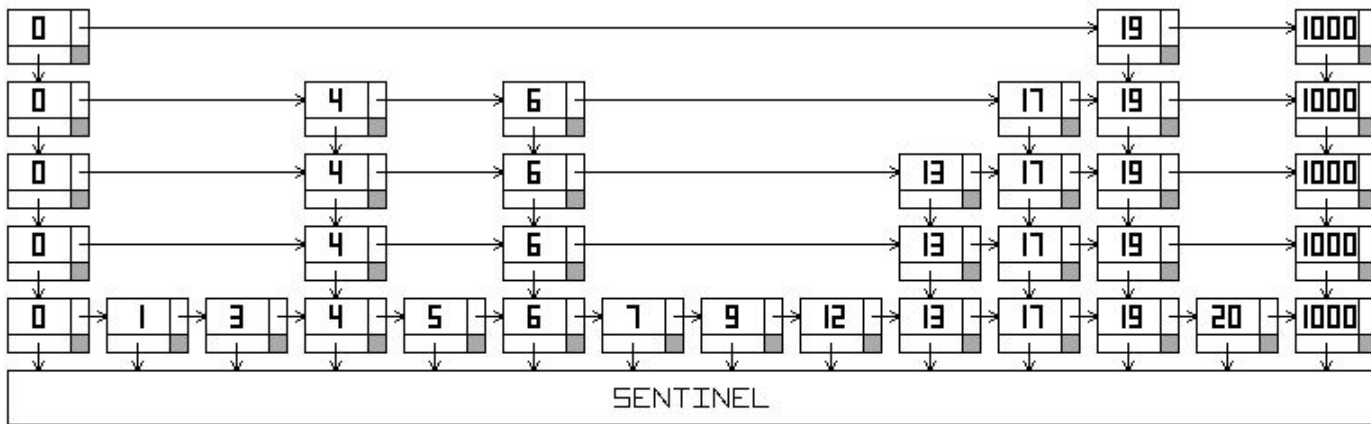
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# Containment

Start current node at the first node

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- If the next value is too small step to the next node
- If the next value is just right return that the value exists
- If the node we are at is the bottom sentinel, the value is not contained



# Creating Nodes

Determine the height

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- Deterministically

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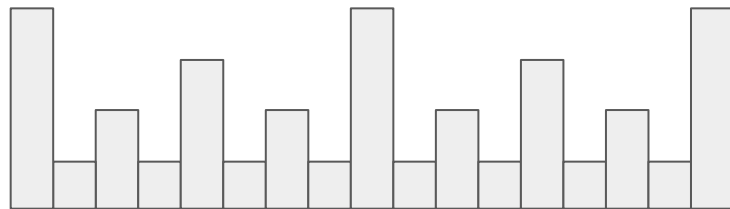
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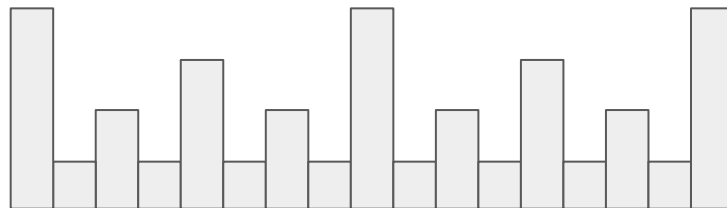
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# Creating Nodes

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  - $\frac{1}{2}$  have height 1

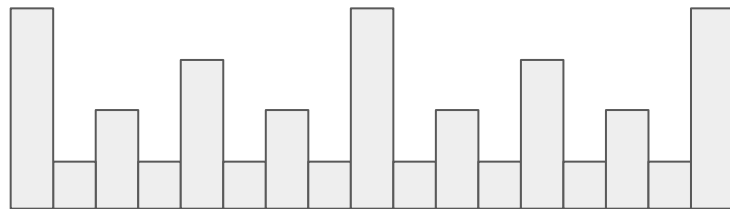




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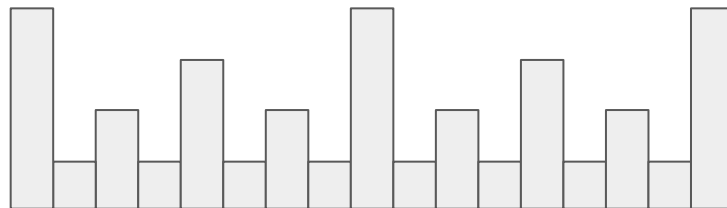
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  - $\frac{1}{2}$  have height 1
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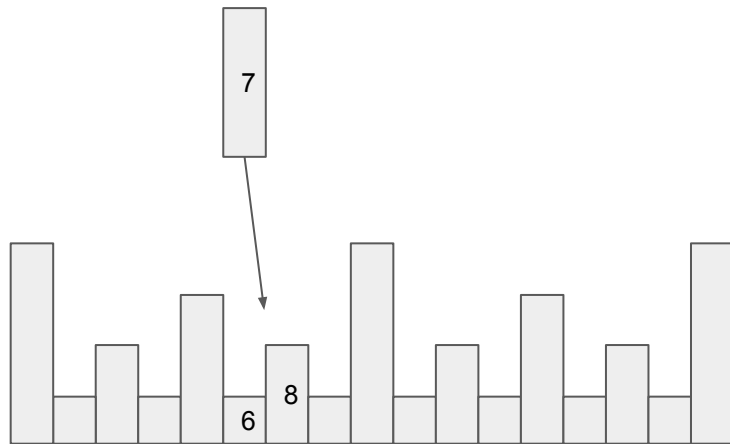
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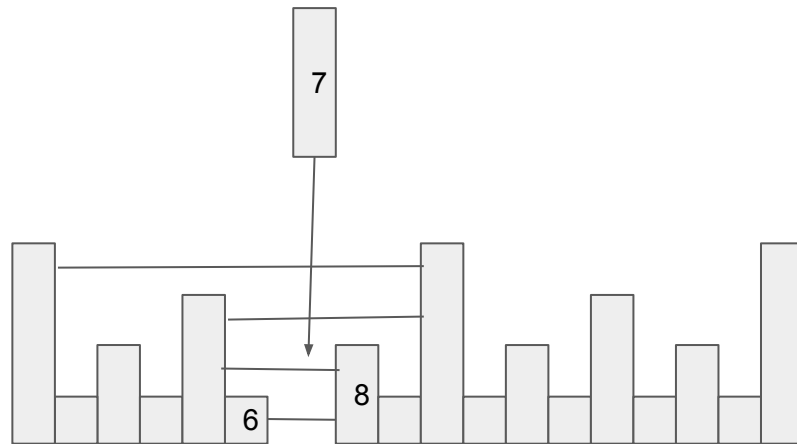


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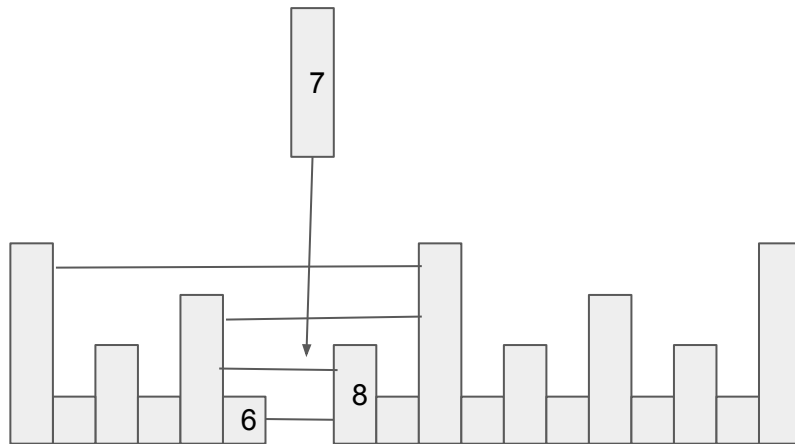


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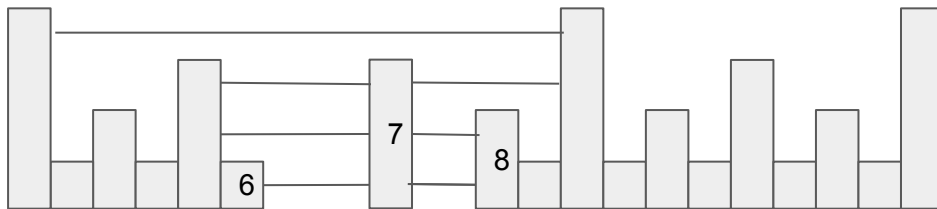
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- respect the height for each pointer

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determine if their height should increase by 1

# Runtimes

Average case?

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Worst Case?

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Deterministic height example

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