PORTFOLIO SELECTION AND PERFORMANCE

Ethan Cating, Faizan Mahmood, Nicole Richardson, Ted Samore

Rose-Hulman Institute of Technology
{catingew, mahmoof, richarnj, samoretc}@rose-hulman.edu

ABSTRACT

Stock prices behave highly stochastically. Combining multiple stocks into a portfolio diminishes the impact of any stochastic behavior of a single stock to diminish risk. Using historical data from a portfolio of 24 selected stocks, future values of the stocks were simulated for six month intervals. After each six month interval, the performance was evaluated to re-estimate the drift and volatility to update the portfolio as necessary, and stocks were reinvested. In addition a transaction fee was assessed.

Keywords— Portfolio, Stock, Price, Investment, Stochastic, Computational Finance

1. INTRODUCTION

Using computational modeling and historical data available from nasdaq.com, the behavior of a collection of stocks, called a portfolio, was predicted. Brownian motion is used to model a stock's price as a function of time so that a standard stochastic differential equation defines the stock price in terms of the drift rate, μ, and volatility, σ. From creating a portfolio of 24 stocks and compiling historical data of each stock, a model was created to assess the portfolio's performance. Investment decisions can then be made based on this assessment.

2. METHODOLOGY

Twenty-four unique stocks, as seen in Table 1, were selected from nasdaq.com. These stocks were selected either by entering random letters in the search bar until a ticker symbol was found or by selecting large well-known companies.

Table 1: Stock selection for the analyzed portfolio.

Ticker Symbol	Full Name
GOOG	Google, Inc.
MSFT	Microsoft Corporation
YHOO	Yahoo! Inc.
AMZN	Amazon.com, Inc.
TSLA	Tesla Motors, Inc.
EA	Electronic Arts Inc.
LUV	Southwest Airlines Company
PAG	Penske Automotive Group, Inc.
UNP	Union Pacific Corporation
VZ	Verizon Communication Inc.
YUM	Yum! Brands, Inc.
BRK/A	Berkshire Hathaway Inc.
MON	Monsanto Company
HES	Hess Corporation
CAKE	The Cheesecake Factory Incorporated
ADM	Archer-Daniels-Midland Company
COP	ConocoPhillips
OCR	Omnicare, Inc.
TMH	Team Health Holdings, Inc.
NEO	NeoGenomics, Inc.
ECTY	Ecotality, Inc.
NSR	Neustar, Inc.
USMO	USA Mobility, Inc.

The previous year's data was downloaded for each stock, and the data was compiled into a single Excel book with each stock on a separate sheet. The data in this Excel book was then read into MATLAB for analysis.

The basic theory for how these stocks were analyzed begins with the standard, stochastic differential equation,

$$dP = \mu P dt + \sigma P dz,$$

$$\text{where } dz = \phi\sqrt{dt}.$$

In this equation, μ is the drift rate, σ is the volatility and the component containing the volatility is a factor of randomness or variation. Additionally, ϕ represents a random variable taken from a standard normal (curve centered at zero with a standard deviation of one). The solution of the solved equation is the price of a stock.

Using Ito's Lemma,

$$df = \left(\frac{df}{dt} + \mu P \frac{\delta f}{\delta P} + \frac{\sigma^2 P^2}{2}\frac{\delta^2 f}{\delta P^2}\right) dt + \sigma P \frac{\delta f}{\delta P} dz\,,$$

a solution to the differential equation can be found of the form

$$P(t) = P(0) e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\sqrt{t}z}.$$

For optimization of our portfolio, a certain amount of money V is used to invest in N stocks. The amount of money allocated to each stock at time t is $x_i(t)$ and leads to the relation for the overall amount of money allocated for the portfolio where,

$$V(t) = \sum_{i=1}^{N} P_i(t)x_i(t).$$

The overall return $R(t)$ is of the form,

$$R(t) = \sum_{i=1}^{N} \omega_i(t)R_i(t)$$

where $\omega_i(t)$ is the weight given to each stock and $R_i(t)$ is the rate of return for each individual stock. The overall goal for optimizing the portfolio becomes earning the highest amount of return with the least amount of risk. Portfolio selection is often based on the weighted difference,

$$\omega^T C\omega - \alpha r^T \omega,$$

where $\alpha \geq 0$ measures the investor's emphasis on risk. The first term is the risk term that uses the covariance matrix C. The second term is the return term. Minimizing this difference with respect to the weights produces an efficient solution for that particular value of α.

Using these principles and the historical data of the stocks, future stock prices were simulated to assess the performance of the portfolio. This was repeated for a two year period assessing the performance every six months. Each six month simulation then was assumed to be part of the historical data. After each simulation, the drift and volatility parameters were re-estimated along with the sample mean and variances.

$$\mu_i = \frac{1}{5}\sum_{k=0}^{4} R_i(t_k) \; and \; \sigma_i^2 = \frac{1}{4}\sum_{k=0}^{4}(R_i(t_k) - \mu_i)^2$$

Once these values and the covariance matrix were updated, Matlab's Optimization toolbox optimizes the risk versus return equation with `quadprog` to give a risk-return curve as seen in Figure 2. A sample of this function is given below,

```
quadprog(C,-alpha(k)*r,[],[],ones(1,S),
[1],zeros(S,1),Inf*ones(S,1).
```

3. RESULTS

All simulation results were plotted to obtain a visual understanding of the stock behavior. Figure 1 shows a year of historical data for the portfolio, as well as the average simulated portfolio value fort six months. The simulation was run one-hundred times and averaged to find the average value. Figure 2 shows the risk return generated from the first six months.

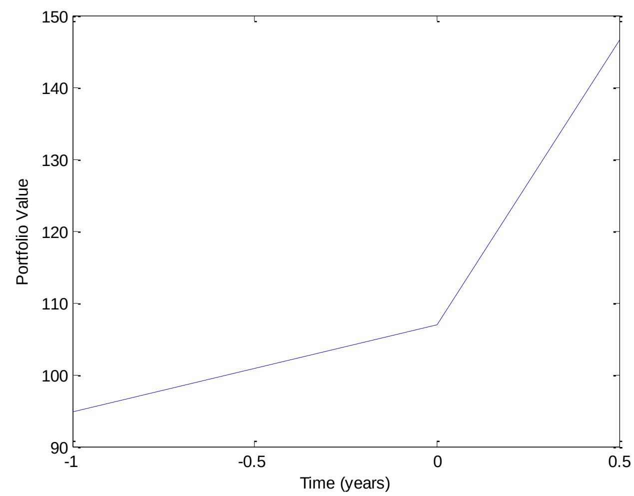

Figure 1: The historical value of the portfolio over the previous year and the average simulated value for 6 months.

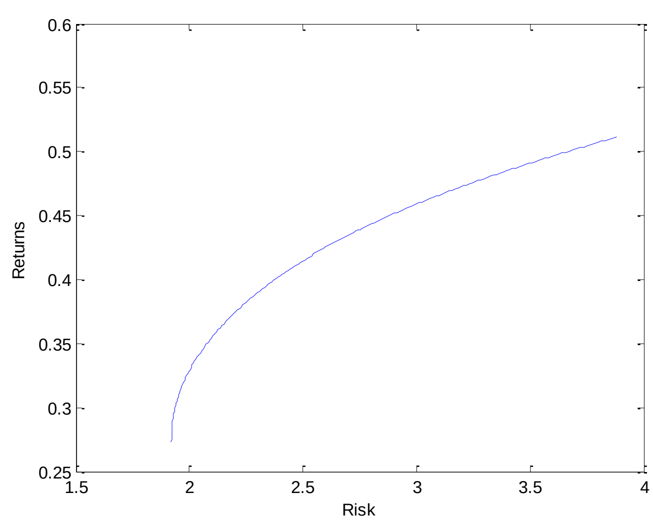

Figure 2: The risk-return curve for the first 6 months of the simulated portfolio values.

In Figure 3, the portfolio's average performance of a hundred simulations for two years is shown.

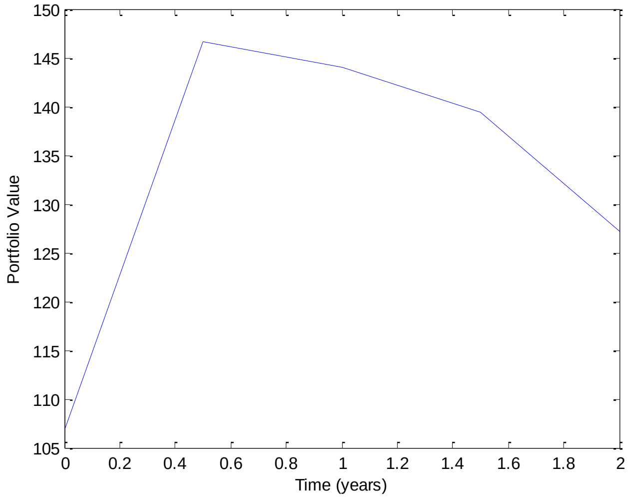

Figure 3: Average portfolio value for first two years.

To better understand the variance in the results of the simulation, the simulation was run ten times and plotted together as shown in Figure 4. Figure 5 depicts this simulation with standard deviation bars.

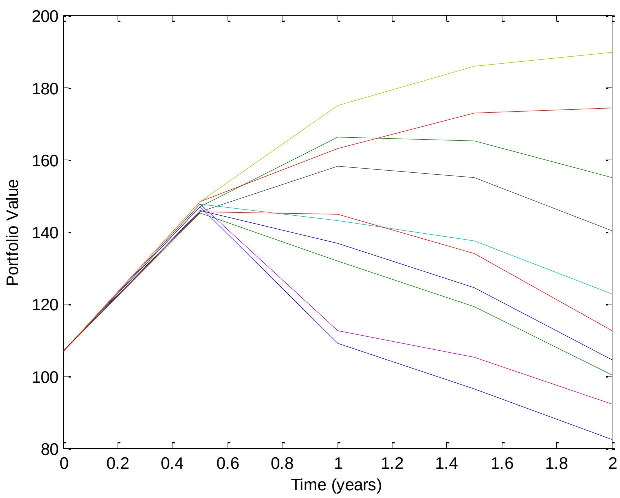

Figure 4: Ten iterations of the portfolio value for two years.

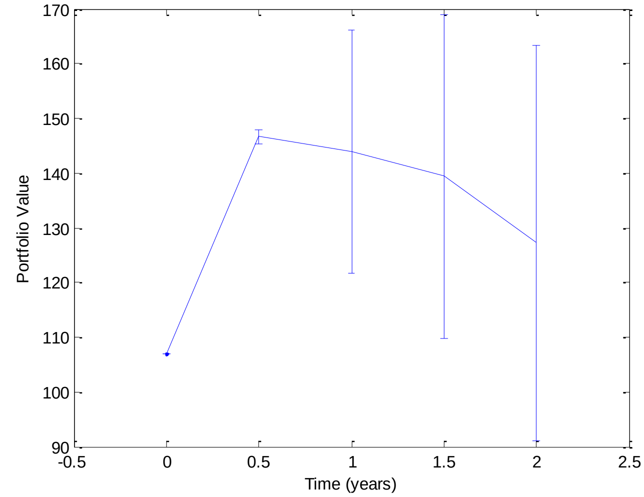

Figure 5: Average portfolio value over ten iterations with standard deviation bars. Note the very small error at six months and the enormous variation for the next three intervals.

The general risk-return curve in Figure 2 was then extended to include the risk-return curves for all of the six month periods as seen in Figure 6.

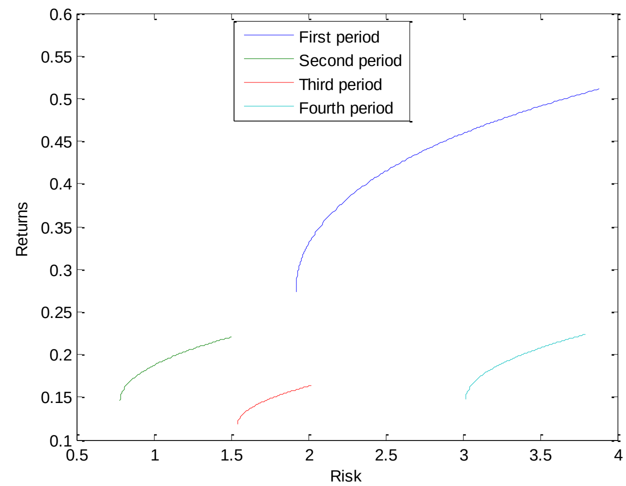

Figure 6: The risk-return curve for the each of the six month intervals of the simulated portfolio values.

Once a 5% transaction fee was introduced into the portfolio analysis, the risk-return curve changed dramatically over the two year simulation period as seen in Figure 5.

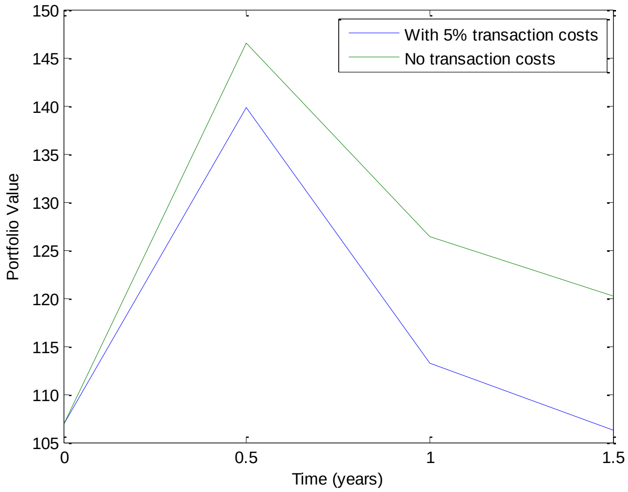

Figure 7: The average portfolio value for the each of intervals when considering a 5% transaction cost.

4. DISCUSSION

4.1 Portfolio Selection

Our portfolio was selected from a range of industries including fast food, agriculture, and technology. Having stocks from a diverse range of industries minimizes risk; however, we had a slightly disproportionately high amount of technology stocks including Yahoo, Google, Microsoft, and Amazon.

4.2.1 Model Simulation for six months

The stocks true data for was used to determine the volatility and drift parameters. Upon executing the optimization routine to determine the risk-return curve (as seen in Figure 2), we found the ideal value of alpha that balances risk and return. To do this, the slope of the risk-return curve was calculated using Euler's method. The value of alpha where the slope was one was deemed the optimal value because it is the point where assuming additional risk yields less returns. As seen in Figure 1, the model predicts a substantial increase in money of more than forty percent over the six month interval. Moreover, the variance for the first six month is very small, as seen in Figure 5 and Figure 6.

4.2.2 Model Simulation for two years

The model was simulated for two years in six month increments. At each six month interval, parameters were re-estimated using data from the previous increment.

As seen in Figure 3, the model predicted a loss of money of about half of the gains seen in the first six month interval. This is secondary to the fact that model behaves much more stochastically as time increases. This can be illustrated in Figure 4 and Figure 5. Simulations maintain a tight path and small variation for six months before diverging sharply and experiencing a large variation. In short, the model is not robust enough to make meaningful predictions after more than six months.

The diminished returns can be visualized in Figure 6. The first period - when the model experiences forty percent gains - returns are the highest. The other three risk-returns curves are significantly lower which means less returns.

4.3 Transactional Cost

To account for transaction cost, a five percent fee was added to the model. The transaction cost was found by multiplying the transaction cost (5%) with the transaction volume. The transaction volume was considered to be the differences in allocations between time intervals. As illustrated in Figure 7, the model predicted a significantly worse outcome when transaction costs where included.

5. CONCLUSION

The present model can make meaningful predictions for six months as it consistently predicts significant gains. After six months, the model behaves very stochastically, and, on average the portfolio loses money. In order to make better long term predictions, the model must be augmented.