

CS 270 - Deduction Homework

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This homework is worth 100 points.

Complete each of the following proofs using **Natural Deduction Proof Editor** from the Open Logic Project at <http://proofs.openlogicproject.org>. To submit the assignment do the following:

1. Create a word document *drexeluser_id_hw2.doc*.
2. Complete each proof using the website and take a screenshot of the proof. You **may** only use the Basic Rules of Deduction.
3. Copy-Paste each screenshot into the Word Document. Make sure your screenshot includes the part that says either **Congratulations! This proof is correct** or **Sorry there were errors**. If you cannot get your answer to work, partial credit will be given. You **must** include all errors generated by the checker to earn partial credit.
4. Export the Word document as a *single* PDF and submit *drexeluser_id_hw2.pdf* to GradeScope.

Question 1 : 5 points

Write the following English Sentence as formal logic.

"I'll get into outerspace only if I'm abducted by aliens."

Let S mean "I'll get into outerspace" and Let A mean "I'm abducted by aliens".

$A \leftrightarrow S$

Question 2 : 5 points

Write the following English Sentence as formal logic.

"If there are no eggs left, then I will need to stop at the store."

Let E mean "there are eggs left" and Let S mean "I need to stop at the store".

$\neg E \rightarrow S$

E	S	$\neg E \rightarrow S$	EVS	$Q_2 \rightarrow Q_3$	$Q_3 \rightarrow Q_2$
T	T	T	T	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	T	T

Question 3 : 5 points

Write the following English Sentence as formal logic.

"There are eggs left or I need to stop at the store."

Let E mean "there are eggs left" and Let S mean "I need to stop at the store".

$E \vee S$

Question 4 : 5 points

Prove by Truth Table that your answer to Question 2 and your answer to Question 3 are logically equivalent. ($Q_2 \iff Q_3$)

Question 5 : 10 points

$(A \wedge B) \therefore \neg(\neg A \vee \neg B)$

Question 6 : 10 points

$A \vee (B \wedge C) \therefore (A \vee B) \wedge (A \vee C)$

Question 7 : 10 points

$(A \vee B) \wedge (A \vee C) \therefore A \vee (B \wedge C)$

Question 8 : 15 points

$\neg(A \wedge B) \therefore \neg A \vee \neg B$

Construct a proof for the argument: $A \wedge B \therefore \neg(\neg A \vee \neg B)$

1	$A \wedge B$	
2	A	$\wedge E$ 1
3	B	$\wedge E$ 1
4	$\neg A \vee \neg B$	
5	$\neg A$	
6	\bot	$\neg E$ 5, 2
7	$\neg B$	
8	\bot	$\neg E$ 7, 3
9	\bot	$\vee E$ 4 5-6, 7-8
10	$\neg(\neg A \vee \neg B)$	$\neg I$ 4-9

Congratulations! This proof is correct.

Construct a proof for the argument: $A \vee (B \wedge C) \therefore (A \vee B) \wedge (A \vee C)$

1	$A \vee (B \wedge C)$	
2	A	
3	$A \vee B$	$\vee I$ 2
4	$B \wedge C$	
5	B	$\wedge E$ 4
6	$A \vee B$	$\vee I$ 5
7	$A \vee C$	$\vee I$ 1, 2-3, 4-6
8	A	
9	$A \vee C$	$\vee I$ 8
10	$B \wedge C$	
11	C	$\wedge E$ 10
12	$A \vee C$	$\vee I$ 11
13	$A \vee C$	$\vee E$ 1, 8-9, 10-12
14	$(A \vee B) \wedge (A \vee C)$	$\wedge I$ 7, 13

Congratulations! This proof is correct.

Construct a proof for the argument: $(A \vee B) \wedge (A \vee C) \therefore A \vee (B \wedge C)$

1	$(A \vee B) \wedge (A \vee C)$	
2	$A \vee B$	$\wedge E$ 1
3	$A \vee C$	$\wedge E$ 1
4	A	
5	$A \vee (B \wedge C)$	$\vee I$ 4
6	B	
7	A	
8	$A \vee (B \wedge C)$	$\vee I$ 7
9	C	
10	$B \wedge C$	$\wedge I$ 6, 9
11	$A \vee (B \wedge C)$	$\vee I$ 10
12	$A \vee (B \wedge C)$	$\vee E$ 3, 7-8, 9-11
13	$A \vee (B \wedge C)$	$\vee E$ 2, 4-5, 6-12

Congratulations! This proof is correct.

Question 9 : 15 points

 $(\neg A \vee \neg B) \therefore \neg(A \wedge B)$

Question 10 : 20 points

 $\therefore (\neg A \implies B) \iff (A \vee B)$ Construct a proof for the argument: $\neg A \vee \neg B \therefore \neg(A \wedge B)$

1	$\neg A \vee \neg B$	
2	$\neg A$	
3	$A \wedge B$	
4	A	$\wedge E$ 3
5	\perp	$\neg E$ 2, 4
6	$\neg(A \wedge B)$	$\neg I$ 3-5
7	$\neg B$	
8	$A \wedge B$	
9	B	$\wedge E$ 8
10	\perp	$\neg E$ 7, 9
11	$\neg(A \wedge B)$	$\neg I$ 8-10
12	$\neg(A \wedge B)$	$\vee E$ 1, 2-6, 7-11

NEW LINE

NEW SUBPROOF

👍 Congratulations! This proof is correct.

Question 8

Construct a proof for the argument: $\neg(A \wedge B) \therefore \neg A \vee \neg B$

1	$\neg(A \wedge B)$	
2	$\neg(\neg A \vee \neg B)$	
3	$\neg A$	
4	$(\neg A \vee \neg B)$	$\vee I$ 3
5	\perp	$\neg E$ 2, 4
6	A	IP 3-5
7	$\neg B$	
8	$\neg A \vee \neg B$	$\vee I$ 7
9	\perp	$\neg E$ 2, 8
10	B	IP 7-9
11	$A \wedge B$	$\wedge I$ 6, 10
12	\perp	$\neg E$ 1, 11
13	$\neg A \vee \neg B$	IP 2-12

NEW LINE

NEW SUBPROOF

👍 Congratulations! This proof is correct.

Construct a proof for the argument: $\therefore (\neg A \rightarrow B) \iff (A \vee B)$

1	$\neg A \rightarrow B$	
2	$\neg(A \vee B)$	
3	A	
4	$A \vee B$	$\vee I$ 3
5	\perp	$\neg E$ 2, 4
6	$\neg A$	$\neg I$ 3-5
7	B	$\rightarrow E$ 1, 6
8	$A \vee B$	$\vee I$ 7
9	\perp	$\neg E$ 2, 8
10	$A \vee B$	IP 2-9
11	$A \vee B$	
12	$\neg A$	
13	A	
14	\perp	$\neg E$ 12, 13
15	B	X 14
16	B	
17	$B \wedge B$	$\wedge I$ 16, 16
18	B	$\wedge E$ 17
19	B	$\vee E$ 11, 13-15, 16-18
20	$\neg A \rightarrow B$	$\rightarrow I$ 12-19
21	$(\neg A \rightarrow B) \iff (A \vee B)$	$\leftrightarrow I$ 1-10, 11-20

NEW LINE

NEW SUBPROOF

👍 Congratulations! This proof is correct.