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Detecting People Carrying Objects utilizing Lagrangian Dynamics

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Abstract—The availability of dense motion information in computer vision domain allows for the effective application of Lagrangian techniques that have their origin in fluid flow analysis and dynamical systems theory. A well established technique that has been proven to be useful in image- based crowd analysis are Finite Time Lyapunov Exponents (FTLE). Based on this, we present a method to detect people carrying object and describe a methodology how to apply established flow field methods onto the problem of describing individuals. Further, we reinterpret Lagrangian features in relation to the underlying motion process and show their applicability towards the appearance modeling of pedestrians. This definition allows to increase performance of state-of-the-art methods and is shown to be robust under varying parameter settings and different optical flow extraction approaches.

Keywords-people carrying objects; motion descriptor; lagrangian dynamics; optical flow; FTLE; HOG;

I. INTRODUCTION

Several approaches have been made to detect people carrying objects. In the paper we will denote the carried object or a classified person that is carrying an object as PCO. The majority of these methods are based on a spatio-temporal analysis of pedestrian silhouettes. Therefore the silhouettes have to be collected over a specified amount of time, through which these methods are dependent on the tracking quality.

Haritaoglu et al. [1] proposed the method Backpack, where they make the assumption that the silhouette of a person is symmetrical when the person is not carrying any objects. To distinguish parts representing limbs from parts representing PCO's a periodicity analysis of the non-symmetric parts is applied. An established spatio-temporal appearance model is the temporal template [2]. A temporal template is computed by accumulating the silhouette of a tracked person that is then extracted from a foreground segmentation. Tao et al. combine the temporal template with a Gabor based feature space [3] and Damen&Hogg [4] achieved convincing results by matching generated temporal templates of generic people walking on a treadmill with the temporal templates of the PCO candidates. In [5] the foreground segmentation is replaced by motion information. A statistical appearance model of the optical flow (GMMM) was applied to generate a mean template of all pedestrians observed in a video sequence. A PCO was detected as an outlier that does not fit to the template. Temporal templates and GMMM's are sensitive to tracking failures and thus an additional alignment must be applied. Therefore in [1], [4] only pedestrians that do not occlude each other are analysed and in [5] annotated tracks are used.

In contrast, Vanacloig et al. [6] proposed a blob based classification method that does not use temporal information. To detect the PCO's a blob is divided into a set of subregions. A feature set is generated by accumulating the foreground pixels of each subregion and used to apply an k-nn classifier. By ignoring the temporal behavior of the pedestrians the feature extraction of this method is not critical to the tracking precision. In the following we will demonstrate the suitability of time-dependent vector field analysis for the task of detecting people carrying objects. In the area of flow field analysis particle-based or Lagrangian approaches, that have there origins in dynamical systems theory, have been effectively applied to describe features in atmospheric phenomena and optical measurement of unsteady physical flow phenomena [7]. Besides, Lagrangian methods have also been successfully used by Ali and Shah [8] to analyze and segment crowd movement in video sequences. More recently, Mehran et al. performed a comparison on particle trajectory descriptions by means of stream lines, path lines, and streak lines for the analysis of crowded scenes [9].

In this work we will present our results on applying the Lagrangian Framework that was proposed in [10] and extending Lagrangian concepts towards detecting individual people carrying objects in video sequences. We will use forward- and backward Lagrangian Coherent Structures (LCS) described by the Finite Time Lyapunov Exponents (FTLE) field as integrated spatio-temporal feature space to analyze pedestrian motion behavior. We will present a pedestrian appearance model using the benefits of Lagrangian descriptors for video surveillance that enables temporal appearance modeling without explicit tracking. As with Vanacloig et al. our method is based on detection and offers the advantage of automatically incorporating information across a variable number of time steps into one FTLE field based on the choice of parameter τ as will be outlined in Section III.
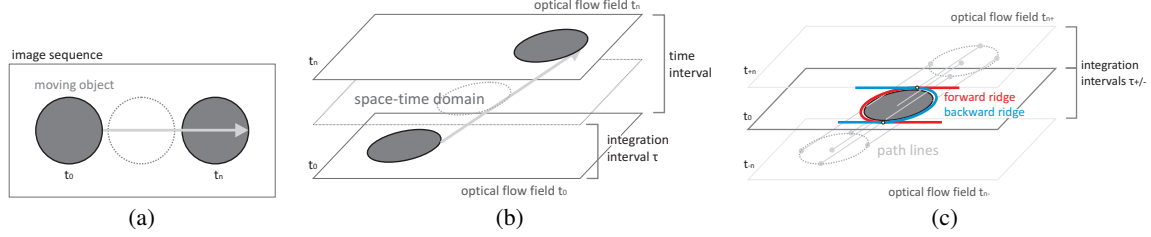
Figure 1. Conceptual representation of Lagrangian methodology in image sequence: Starting from an image sequence shown in (a) we derive a space-time domain description using the optical flow field of every time step (b). Within this domain we can integrate particle trajectories in forward and backward direction (c) over an interval τ which ideally results in a closed region for a closed moving object.

II. System overview

The PCO detection method that is based on time-dependent vector field analysis is integrated into a framework that starts by detecting pedestrians from a sequence of video frames. Therefore we apply a combination of the well-known Histogram of Gradient (HOG) descriptor and a linear SVM as proposed by Dalal&Triggs [11]. Throughout our studies stable results where achieved with the default parameters: a window size of 64×128, a block size of 16×16, a block stride and cell size of 8×8 and 9 histogram bins. The application is applied to the Pets2006 dataset that was also used by Damen&Hogg. We trained the classifier with pedestrian samples annotated from the Pets2006 sequence 2 recorded by camera 3. In order to reduce the search space and thus the computational complexity of the classification step, we integrate two post processing steps. At first a foreground blob detection is applied based on the Heras&Sikora [12] foreground segmentation method in combination with a connected component analysis. In a thinly populated sequence each blob detection would be associated with a person detection but Pets2006 contains pedestrian groups that shows the HOG detector has to be applied to each foreground blob. Secondly we use the HOG detector in a calibrated environment to deal with projective distortion. The scale of the HOG descriptor is computed directly from the calibration, while assuming a pedestrian is 1.8m. Different HOG scales are implemented by resizing the respective image content. For each bounding box we apply the spatio-temporal analysis. Therefore the optical flow of the integration interval τ has to be computed and stacked to a 3 dimensional tensor. Thus the sequence of optical flow frames is transferred into the space-time domain, which allows us to estimate the particle trajectories, flow map and finally the FTLE field of each pedestrian as described in the next Section. Finally, for each pedestrian we introduce the HOG-FTLE descriptor in combination with a linear SVM classifier to label PCO's.

III. Lagrangian features and Finite Time Lyapunov Exponents (FTLE)

In order to define a consistent theoretical methodology to work on time-dependent optical flow data, we treat the series

of such fields as time-dependent vector field. For this, we lift the fields into a higher dimension by interpreting the time as an additional axis, where we will denote this as space-time domain. In the setting of video analysis, *path lines* directly describe the evolution of pixels within the associated image data. Hence, one pixel can be reinterpreted as an particle carrying information throughout the image sequence. Formally, this can be described as follows: Given a vector field $\mathbf{v}(\mathbf{x}, t)$, at every specified space-time point $(\mathbf{x}_0, t_0) \in D$ we can start a *stream line* or a *path line* in space-time domain. This can be formulated as an autonomous system:

$$\frac{d}{dt} \begin{pmatrix} \mathbf{x} \\ t \end{pmatrix} = \begin{pmatrix} \mathbf{v}(\mathbf{x}(t), t) \\ 1 \end{pmatrix}, \quad \begin{pmatrix} \mathbf{x} \\ t \end{pmatrix}(0) = \begin{pmatrix} \mathbf{x}_0 \\ t_0 \end{pmatrix}$$

Path lines of the original vector field $\mathbf{v}$ in ordinary space now appear as stream lines of the vector field.

$$\mathbf{p}(\mathbf{x}, t) = \begin{pmatrix} \mathbf{v}(\mathbf{x}, t) \\ 1 \end{pmatrix} \quad (1)$$

in space-time. Now finding correspondences and motion characteristics within a motion flow field sequence can be described by considering $\mathbf{p}$. This concept is compatible with any optical flow field methodology that delivers a vector field for every time step. Note that the quality of FTLE features still depends on the accuracy of the underlying flow fields.

To analyze unsteady flow properties in a feature-oriented manner there has been introduced the notion of Lagrangian Coherent Structures (LCS). LCS directly describes transport features of particles moving within the flow. A general overview about recent Lagrangian methods for time-dependent flow analysis is presented by Pobitzer et al. [13]. In video analysis this corresponds to the notion of an edge of a closed moving object within the image. The choice of the time interval parameter τ is a crucial aspect and is closely associated with the temporal scale of features we are interested in. The most prominent techniques to extract LCS are Finte Time Lypunov Exponents (FTLE) presented by Haller et al. [14], while the detailed correspondence has been clarified in [15]. The numerical evaluation of FTLE based on the flow map results in a scalar field that describes the rate of separation within the flow over the considered finite time interval. FTLE has already been proven useful

to analyze flows containing divergent flow behavior from image data [8], [16].

Formally we define the *flow map* $\phi^\tau(\mathbf{x}, t) = \phi(\mathbf{x}, t, \tau)$ of $\mathbf{v}$ as the location of a particle seeded at $(\mathbf{x}, t)$ after a path line integration of $\mathbf{v}$ over a time interval τ. Let

$$\nabla(\mathbf{x}, t, \tau) = \frac{d\phi(\mathbf{x}, t, \tau)}{d\mathbf{x}}$$

denote the spatial *gradient* of the flow map ϕ and define values

$$\mu_i = \ln \sqrt{\lambda_i(\nabla^T \nabla)} \qquad (2)$$

where ∇^T is the transposed of ∇ and $\lambda_i(\nabla^T \nabla)$ denotes the i-th eigenvalue of the symmetric matrix $\nabla^T \nabla$. Then the FTLE value at $(\mathbf{x}, t)$ for integration time τ is obtained as

$$\text{FTLE}(\mathbf{x}, t, \tau) = \frac{1}{\tau} \max\{\mu_1, \mu_2\} \ . \qquad (3)$$

LCS are now in close relation to *height ridges* in the resulting FTLE scalar field [15]. Note that the ridge extraction procedure usually requires concise parameter treatment in order to obtain qualitative results. This is discussed in further detail by Eberly et al. [17]. In general FTLE can be computed in forward and backward direction resulting in the description of FTLE+ and FTLE- as described by Garth et al. [18]. Following the LCS description, features in the forward FTLE field describe regions of repelling LCS, while features in the backward FTLE describe attracting LCS structures over the considered time scope. Intersections of FTLE+ and FTLE- ridge structures segment regions of coherent movement and group invariant moving areas within the field. Using this notion, ridges can be reinterpreted as *motion boundaries* defined over a finite time scope a correspond to coherent movement with respect to a certain image region. The method parameter τ determines the length and complexity of those ridge structures. This is illustrated in Figure 2 for two practical examples using three different approaches to obtain the underlying optical flow fields.

IV. FINITE TIME LYAPUNOV EXPONENTS DESCRIPTOR

As assumed in the previous work of Damen&Hogg [4] and Senst et al. [5] a person that is carrying an object could be detected by its changing motion boundaries. These changes could be e.g. located at different outer motion boundary, the silhouette or different inner motion boundaries through the occlusion of a case. As described in Section III the FTLE field is an excellent tool to model these motion boundaries. The integration interval τ is a significant parameter as it should be aligned to the duration of the observed event. In this case an event is described by the limb motion of the observed pedestrian. Figure 2 illustrates different inner motion boundaries of a FTLE field that is computed by different integration intervals for a PCO and none PCO.

Commonly, features in the FTLE field are explicitly extracted in terms of height ridges as mentioned in Section III.

As a drawback is that, this requires the introduction of an additional ridge extraction procedure, which tend to be sensitive to noise in the underlying field [17]. In our approach we avoid the direct use of ridge structures by incorporating the FTLE field into the well-known HOG descriptor. The HOG descriptor partition a detector window into a dense grid of cells, with each cell containing a local histogram over orientation bins. We modify the HOG such that the FTLE field of the forward (FTLE+) and backward (FTLE-) integration is calculated at each pixel and converted to an angle, voting into the corresponding orientation bin with a vote weighted by the overall FTLE magnitude.

V. FINITE TIME LYAPUNOV EXPONENTS CLASSIFIER

Damen&Hogg presented in [4] ground truth data for the Pets2006 sequences of camera 3 for their PCO method. This data contains bounding boxes to describe the locations and sizes of the carried object of pedestrian, where only pedestrian that do not occluding each other are selected. We extend the ground truth data by the remaining pedestrian and label each pedestrian within a bounding box. We also label all new ground truth PCO's with a 1 and the original, previously annotated by Damen&Hogg with a 4. The PCO classification is done in a supervised manner by a linear SVM. We assume that the person detection is not perfect. For each sequence the HOG person detection is applied. Each detection is associated with the nearest ground truth bounding box of a given gating range. The gating range is set to $1/8$ of the HOG window width and to $1/16$ of the HOG window height. The HOG window is also adapted to the perspective distortion by using the calibration of the scene. If a detection is successfully associated with a ground truth annotation, it inherit the label of the respective ground truth, else it is labeled as negative sample.

VI. EXPERIMENTS

Using the concepts described in the previous Sections, we avoided explicit ridge extraction, while remain two crucial aspects: the choice of the FTLE parameter τ and suitability of the Lagrangian approach towards different optical flow field methodologies. Thus we split our experiments in two parts: First, we want to evaluate the influence of the path line integration interval τ on the classification result. The underlying motivation for these experiments is to reduce the number of required frames. Second, we want to evaluate the robustness of the proposed method related to motion estimation quality. The motion quality of a optical flow method is often indirectly related to its run time. Thus we use two different methods to compute the dense optical flow. The global optical flow method with a high quality that is proposed by Werlberger et al. [19] (Huber-L1) and a local optical flow method that has an overall lower quality but is scalable in computational complexity [20] (RLOF). To obtain a dense motion field by the RLOF we subsample
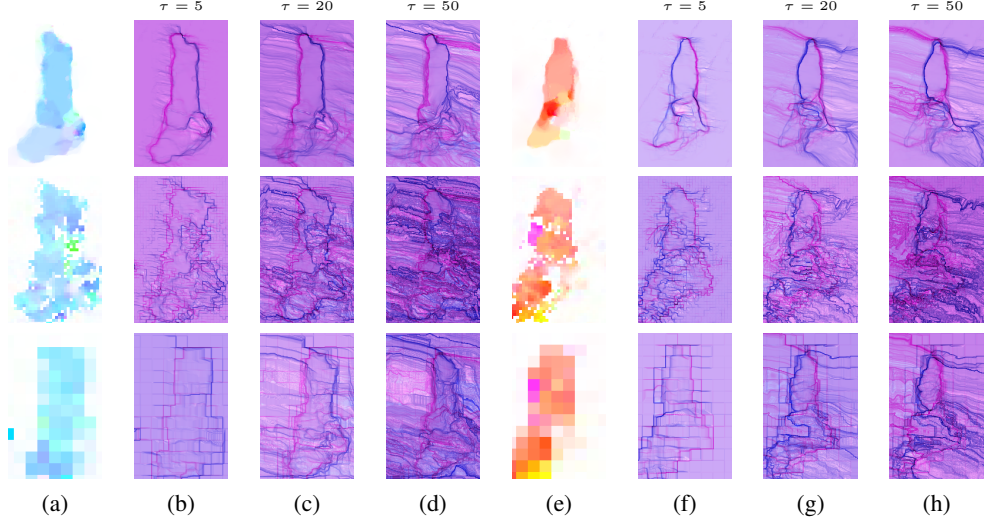
Figure 2. Examples for FTLE fields of a person without (a-d) and with carrying a bag (e-h) and different integration intervals τ for from top to bottom the Huber-L1, RLOF[1] and RLOF[2] dense optical flow.
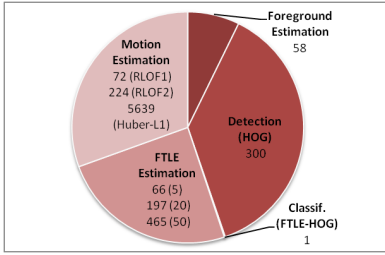

Figure 3. Partition of execution time in msec.

the image into a subset of cell with a given grid size. For each cell a motion vector is computed. We use two different grid sizes, RLOF[1] denotes a used cell size of 5 and RLOF[2] denotes a used cell size of 15. As shown in Figure 2(a,e) the Huber-L1 flow is able to preserves the silhouette of the person, while RLOF[1/2] generates a blurred and blocky vector field. Moreover the RLOF vector field could not ensure to be able to compute a dense flow as shown in the middle row of Figure 2 (a,e) where the motion of some cells could not be computed.

The proposed system was implemented using an AMD Phenom II X4 960 running at 2.99 GHz with a NVIDIA GTX 480 graphic device. The RLOF and Huber-L1 method was implemented on the GPU as well es the FTLE computation. All remaining step were performed on the CPU. Figure 3 shows the run time of the propose system. This implementation is not able to provide the PCO detection in real time but with its fastest configuration with 497ms it has a good performance and is able e.g. to support forensic analysis of video data.

Figure 4 shows the PR curves and Table I the measures of the FTLE-HOG classifier. These experiments show the

discriminative behavior of PCO and none PCO samples within the FTLE-HOG descriptor. Therefore the sample set was randomly divided into 10% trainings and 90% test data. The results of the RLOF[1/2] flows become less precise than the results of the Huber-L1 flow. However in comparison of the source motion fields in Figure 2(a,e) they are remarkably good for $\tau > 5$. The application of the path line advection is working as a deblocking filter in the space time-domain. Thus with a large integration interval the motion silhouette could be retrieve.

$\tau = 5$	Accuracy		Precision		Recall	
	μ	σ	μ	σ	μ	σ
Huber-L1	78.6%	0.01	55.3%	0.01	55.1%	0.02
RLOF[1]	75.8%	0.01	48.2%	0.02	48.8%	0.01
RLOF[2]	74.6%	0.01	46.3%	0.01	47.2%	0.02

$\tau = 20$	Accuracy		Precision		Recall	
	μ	σ	μ	σ	μ	σ
Huber-L1	81.6%	0.01	61.5%	0.01	61.5%	0.02
RLOF[1]	78.9%	0.01	55.6%	0.01	54.2%	0.02
RLOF[2]	79.2%	0.01	56.3%	0.01	54.9%	0.01

$\tau = 50$	Accuracy		Precision		Recall	
	μ	σ	μ	σ	μ	σ
Huber-L1	82.6%	0.01	63.5%	0.02	63.3%	0.02
RLOF[1]	79.9%	0.01	57.9%	0.01	55.9%	0.02
RLOF[2]	81.5%	0.01	61.3%	0.01	60.2%	0.02

Table I
CLASSIFICATION RESULTS WITH DIFFERENT INTEGRATION INTERVALS τ TRAINED ON 10% AND TESTED ON 90% RANDOMLY SELECTED SAMPLES. THE EVALUATION IS PERFORMED AT THE DETECTION LEVEL.

The results of Figure 4 and Table I are only partly representative for the overall PCO accuracy as false negatives of the HOG pedestrian detection step could induce that
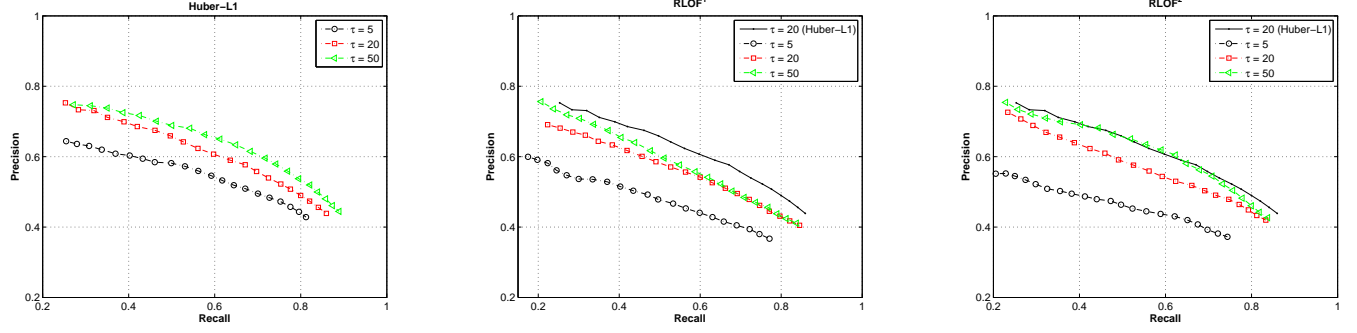
Figure 4. PR curves of the comparison of the FTLE-HOG classifier with different integration intervals τ, trained on 10% and tested on 90% of the Pets2006 labeled HOG people detections. The evaluation is performed at the person detection level i.e. for each detection a classification and comparison is done. Comparison of mean classification metrics.
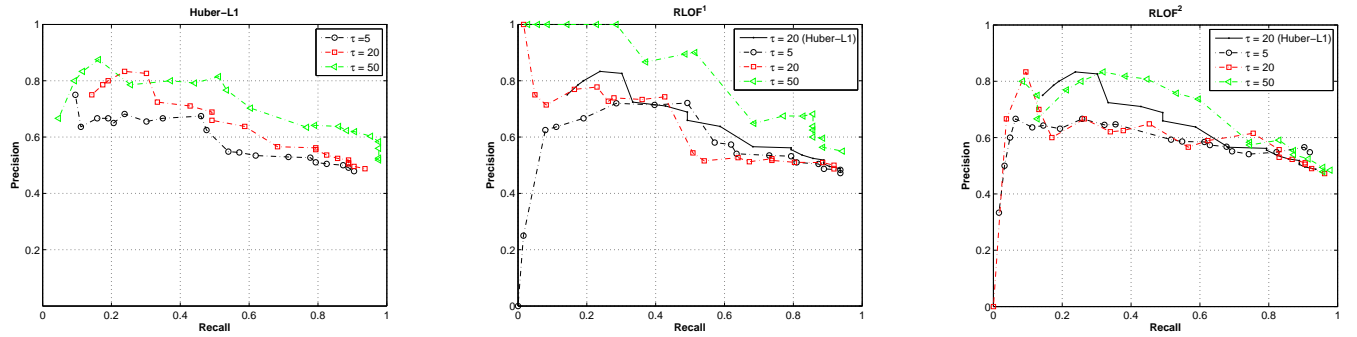

Figure 5. PR curves of the comparison of the FTLE-HOG classifier with different integration intervals τ, trained as in Figure 4. The evaluation is performed at the object level i.e. for each object a classification and comparison is done. For RLOF$^{1/2}$ the Huber-L1 curve is given as reference.

$\tau = 5$			
	Accuracy	Precision	Recall
Huber-L1	70.5%	67.4%	46.0%
RLOF^1	73.0%	72.1%	49.2%
RLOF^2	66.2%	64.7%	35.5%
$\tau = 20$			
	Accuracy	Precision	Recall
Huber-L1	70.5%	66.0%	49.2%
RLOF^1	72.5%	74.3%	42.6%
RLOF^2	69.3%	64.9%	45.3%
$\tau = 50$			
	Accuracy	Precision	Recall
Huber-L1	72.7%	76.7%	53.5%
RLOF^1	77.4%	90.0%	51.4%
RLOF^2	76.4%	73.7%	59.6%

Table II
CLASSIFICATION RESULTS WITH DIFFERENT INTEGRATION INTERVALS τ. THE EVALUATION IS PERFORMED AT THE OBJECT LEVEL.

	Precision		Recall	
	$\tau = 20$	$\tau = 50$	$\tau = 20$	$\tau = 50$
Huber-L1	60.4%	64.5%	57.1%	54.1%
RLOF^1	48.0%	54.5%	46.2%	54.5%
RLOF^2	64.8%	73.7%	45.3%	59.6%
Damen&Hogg	50.5%		55.4%	

Table III
A COMPARISON OF DIFFERENT FTLE-HOG CLASSIFIER WITH THE STATE-OF-THE-ART METHOD OF DAMEN&HOGG (MRF WITH PRIOR) FOR THE ANNOTATED DATA PROVIDED IN [4].

pedestrians get excluded from the evaluation process shown in Figure 4. Therefore Figure 5 and Table II shows the results of a pedestrian related evaluation. Therefore each annotated object is associated with a set of related classified PCO or none PCO detection. If a ground truth object is not associated to any detection it counts as false negative. An object is classified as carrying a baggage, if the major number of

related detections is classified as PCO. The results indicates that with a large τ increases the classification performance. In addition an accurate motion estimation method increases the individual or detection based performance but, in this application, within an approximated motion estimation the overall performance is barely affected. In some cases the accuracy and precision are increase, which is affected of the blurring of the RLOF methods.

It should be kept in mind that to compute the FTLE field of τ, 2τ frames are required to computed the respective motion data. The interval of $\tau = 20$ is a good compromise of classification accuracy and observation time.

To compare the FTLE-HOG classification method with

the state-of-the-art method proposed in [4] we have introduced the label 4. Only ground truth annotations with label 4 are considered for the results of Table III which demonstrates that our method outperforms the state-of-the-art.

VII. Conclusions

In summary we have proposed a method to detect people carrying objects based on Lagrangian dynamics. We introduced the notion of forward- and backward LCS towards this application case and showed that this can be used to improve the performance of state-of-the art methods. Our approach, avoids the necessity of explicitly tracking objects over multiple time-steps by extracting the FTLE field, which directly encodes complex motion information over a given time-interval. In contrast to previous approaches we avoid the extraction of explicit ridge structures by applying the HOG descriptor directly on the FTLE field. We have shown that the appearance model benefit from an accurate but run time intensive global optical flow method and that the our method achieves good results by applying local optical flow methods. The latter one are more efficient in run time, but do not preserve the silhouette accurately. For future work Lagrangian measures and LCS appears a powerful tool to describe complex motion patterns in human action recognition, without explicit object tracking. This might be exploited for the detection of gestures and repeating motion events.

References

[1] I. Haritaoglu, R. Cutler, D. Harwood, and L. S. Davis, "Backpack: Detection of people carrying objects using silhouettes," *Computer Vision and Image Understanding*, vol. 81, no. 3, pp. 385–397, 2001.

[2] J. W. Davis and A. F. Bobick, "The representation and recognition of action using temporal templates," in *Computer Vision and Pattern Recognition (CVPR 97)*, 1997, pp. 928–934.

[3] D. Tao, X. Li, X. Wu, and S. J. Maybank, "Human carrying status in visual surveillance," in *Computer Vision and Pattern Recognition (CVPR 06)*, 2006, pp. 1670–1677.

[4] D. Damen and D. Hogg, "Detecting carried objects in short video sequences," in *European Conference on Computer Vision (ECCV 08)*, 2008, pp. 154–167.

[5] T. Senst, R. Heras Evangelio, and T. Sikora, "Detecting people carrying objects based on an optical flow motion model," in *Workshop on Applications of Computer Vision (WACV 11)*, 2011, pp. 301–306.

[6] V. Atienza-Vanacloig, J. Rosell-Ortega, G. Andreu-Garcia, and J. Valiente-Gonzalez, "People and luggage recognition in airport surveillance under real-time constraints," in *International Conference Pattern Recognition (ICPR 08)*, 2008, pp. 1–4.

[7] T. Peacock and J. Dabiri, "Introduction to focus issue: Lagrangian coherent structures," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 20, no. 1, 2010.

[8] S. Ali and M. Shah, "A lagrangian particle dynamics approach for crowd flow segmentation and stability analysis," in *Computer Vision and Pattern Recognition (CVPR 07)*, 2007, pp. 1–6.

[9] R. Mehran, B. E. Moore, and M. Shah, "A streakline representation of flow in crowded scenes," in *European Conference on Computer Vision (ECCV 10)*, 2010, pp. 439–452.

[10] A. Kuhn, T. Senst, I. Keller, T. Sikora, and H. Theisel, "A lagrangian framework for video analytics," in *Workshop on Multimedia Signal Processing (MMSP 2012)*, 2012.

[11] N. Dalal and B. Triggs, "Histograms of oriented gradients for human detection," in *International Conference on Computer Vision and Pattern Recognition (ICPR 05)*, 2005, pp. 886–893.

[12] R. H. Evangelio and T. Sikora, "Complementary background models for the detection of static and moving objects in crowded environments," in *International Conference on Advanced Video and Signal-Based Surveillance (AVSS 11)*, 2011, pp. 71–76.

[13] A. Pobitzer, R. Peikert, R. Fuchs, B. Schindler, A. Kuhn, H. Theisel, K. Matkovic, and H. Hauser, "The state of the art in topology-based visualization of unsteady flow," *Computer Graphics Forum*, vol. 30, no. 6, pp. 1789–1811, 2011.

[14] G. Haller, "Distinguished material surfaces and coherent structures in three-dimensional fluid flows," *Physica D*, vol. 149, pp. 248–277, 2001.

[15] G. Haller, "A variational theory of hyperbolic lagrangian coherent structures," *Physica D*, vol. 240, pp. 574–598, 2010.

[16] S. Umair and A. M. Rizal, "Underwater crowd flow detection using lagrangian dynamics," *Indian Journal of Geo-Marine Sciences*, vol. 38, no. 3, pp. 359–364, 2009.

[17] D. Eberly, *Ridges in Image and Data Analysis*. Dordrecht: Kluwer Acadamic Publishers, 1996.

[18] C. Garth, G. Li, X. Tricoche, C. Hansen, and H. Hagen, "Visualization of coherent structures in transient 2d flows," *Topology-Based Methods in Visualization II*, pp. 1–13, 2009.

[19] M. Werlberger, W. Trobin, T. Pock, A. Wedel, D. Cremers, and H. Bischof, "Anisotropic huber-L^1 optical flow," in *British Machine Vision Conference (BMVC 09)*, 2009.

[20] T. Senst, V. Eiselein, R. Heras Evangelio, and T. Sikora, "Robust modified L2 local optical flow estimation and feature tracking," in *Workshop on Motion and Video Computing*, 2011, pp. 685–690.