Modeling Chaos in the Heart

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Abstract: *Chaos theory describes the dynamics of a deterministic system whose dynamics show extreme dependence on initial conditions, due to which long-term prediction is impossible. Chaos theory has found its applications in diverse fields like, fluid dynamics, lasers, chemical reactions, population dynamics, circuit analysis, stock exchange predictions etc. This research is aimed at the modeling of chaotic behavior in the heart where it results in fibrillation. Fibrillation is the uncoordinated contraction of the heart muscles that results due to the chaotic transmission of signals, which are generated, in the Sino-atrial node. This research is aimed at analyzing one of the models presented for transmission of signals in the heart and how a chaotic behavior may result.*

Keywords: *Chaos, Modeling, Simulation, Biomedical*

1. INTRODUCTION

1.1 Chaos Theory

Water flowing smoothly in a stream and turning turbulent at times or places, and smoke rising smoothly upward from a fire and then breaking into a disordered, turbulent pattern are examples of chaotic systems that have been intriguing the observers. Many such observations are made in our daily life when an ordered system turns into a chaotic system by showing an aperiodic behavior that appears to be random to a casual observer. The most important feature of such systems is their extreme dependence on the initial conditions. Chaos Theory is the theory that describes the complex and unpredictable motion or dynamics of such systems. Due to this extreme dependence on initial conditions, long-term prediction is not possible and a little change can result in a huge difference in the final output.

The most useful property of chaotic systems is that these systems are deterministic and their dynamics are governed by precise mathematical laws. Although work on chaos theory started in the late nineteenth century, lack of computational power hampered the pace of its development as most of the problems involve complex mathematical equations coupled with the extreme sensitivity on initial conditions. Rapid progress has been made possible with the advent of digital computer and scientists are working on modeling such chaotic systems. Chaos theory has found its applications in many diverse fields like electric circuits, epidemic outbreaks, lasers, heart rhythms, brain activity, fluid dynamics, population studies, chemical reactions, and even stock exchanges.

1.2 Our Heart

Our heart forms one of the most vital organs of the body that pumps blood to each and every cell of our bodies. This blood is required to supply nutrients and oxygen to all the cells and to remove the waste materials and carbon dioxide from them. The pumping of blood is the result of coordinated contraction of the heart muscles that is controlled by its magnificent electrical system. An electrical oscillator called the Sino-Atrial node generates the electrical impulses that are conducted by the heart's conduction system to each of its muscles. The Sino-Atrial node is also termed as the natural pacemaker of the heart although Allah Almighty has provided the provision that in case the Sino-Atrial node fails to generate the pace, cells in the conduction system can take that responsibility and if they also fail, any other muscle can start that duty. The conduction system of the heart consists of the Atrio-Ventricular node, the bundle of His, and the Purkinje fibers. The electrical impulses first reach the muscles of the two upper chambers of the heart that are termed as Atria which contract to pump blood to the lower chambers termed as Ventricles. Impulses are transmitted through muscle fibers of the two atria to the Atrio-Ventricular node, located between the atria and the ventricles. This node acts as a delay line that slows the transmission of impulses to the lower chambers to allow blood pumped by the upper chambers to completely fill the lower chambers before the muscles contract. Another interesting thing is that there is no other path for the electrical signals to pass from the upper chambers to the lower chambers, as the fibrous layer that divides the upper and lower chambers is an electrical insulator. From the Atrio-Ventricular node, the electrical signals travel along the bundle of His and the Purkinje fibers that are specialized conducting cells with low resistance that quickly distribute the signal to all the muscles so that coordinated contraction occurs to effectively pump blood out of the heart.

1.3 Signal generation and transmission

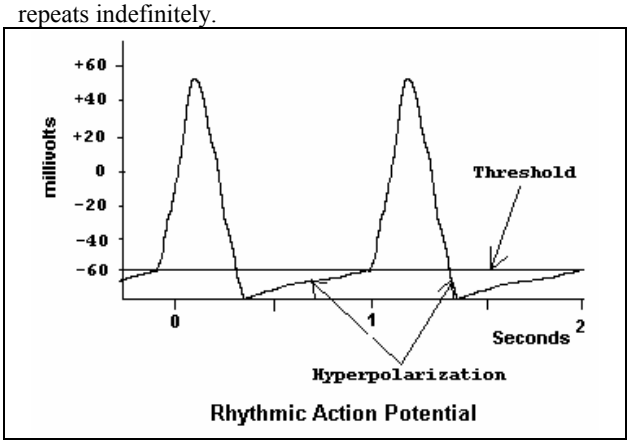
Cell membrane plays an important role in the generation and transmission of electrical potentials. The cell membrane is a lipid-bilayer that has different channels that allow passage of selective ions or molecules inside or outside. These channels are voltage-controlled, chemically controlled or mechanically controlled. This selective nature of the cell membrane results in an imbalance of ions that results in a negative potential inside the cell that is termed as the resting membrane potential and the membrane is termed as polarized. This resting membrane potential in most of our cells is –90mV. The major role is played by Sodium, Potassium and Calcium ions. Sodium and Calcium ions have a higher concentration outside the

cell as compared to the inside while Potassium has a higher concentration in the inside as compared to the outside. If the membrane potential is somehow raised above a certain level, termed as the threshold, the membrane suddenly becomes permeable to sodium ions that cause a tremendous flow of sodium ions from the outside to inside. This influx of positive ions raises the inside potential and the potential often overshoots beyond zero and becomes positive. The cell is termed as depolarized. The threshold in many cells has been observed as about –65mV. Within a few 10,000ths of a second, the sodium channels begin to close and potassium channels open more than the normal. The rapid diffusion of potassium restores the normal membrane potential. This is called repolarization. Potassium channels are however slow to open and close and their outflow sometimes decreases the potential even below the normal resting potential. This is termed as hyperpolarization. The complete process is termed as the action potential. It should also be noted that other ions also affect the action potential. Calcium ions in some cases have a dominant role as compared to Sodium ions and their concentration even affects the potential at which Sodium channels work After the repolarization process is complete, different channels actively start to restore the concentration of ions that have been slightly disturbed in the process [7].

In some instances, the excitable membrane does not repolarize immediately after depolarization but it remains on a plateau near the peak of the spike for many milliseconds and after that repolarization starts. Such type of action potential is observed in the heart muscles where the plateau lasts for about 2/10 to 3/10 second and the muscle contracts for the same period. This plateau is the result of slow Calcium channels that open slowly and take time to close while maintaining the plateau. The Potassium channels in this case are also slow to open and the potential remains high for come period.

1.4 Generation of Rhythm in certain cells

Repetitive self-induced discharge or rhythmic behavior occurs in the heart and in most of the smooth muscles, and in many neurons of the central nervous system. This behavior is made possible if their threshold potential for stimulation is reduced low enough. If it is reduced enough, the membrane becomes permeable for sodium or calcium & sodium even at the normal resting potential. If the normal resting potential is brought to about –60mV, rhythmic behavior can occur. Sodium and calcium channels will open and the ions flow inward. This further increases the potential and the gates are fully open, resulting in the sharp rise in potential. The potassium channels also open slowly and begin the repolarization process. As the conductivity of potassium has been increased very much, the membrane will hyperpolarize. At this time, the sodium and calcium channels will be closed. After some time, the hyperpolarization is finished and once again the potential is high enough to open the calcium and sodium channels. The cycle, therefore, repeats indefinitely.



Rhythmic Action Potential

1.5 Refractory Period

Once an action potential has started and the calcium & sodium channels are open, a stimulus applied at this time cannot re-excite the cell due to the fact that the calcium & sodium channels are also inactivated at the same time they are opened. They can be turned on only after the potential has returned to its normal resting value. During this time, any stimulus, however strong it may be, cannot re-excite the channel. This period is known as absolute refractory period. After the absolute refractory period, there is an interval known as relative refractory period. During this time, a stimulus greater than the normal value can excite the cell. The reason is that during this time, most of the potassium channels are still wide open and making the inner potential more negative [7].

2. BASIS OF A HEART MODEL

2.1 Action potentials in cardiac muscles

The resting membrane potential of normal cardiac muscles has been observed as about –85 to –95mV while the resting potential of specialized conducting fibers, Purkinje fibers is –90 to –100mV. The action potential causes the potential to overshoot to about +20mV. The membrane remains depolarized for about 0.2 second in atrial muscles and 0.3 seconds in ventricular muscles. Thus, a plateau is observed that cause the muscles to contract for that time which is about 3 to 15 times that observed in the normal skeletal muscles. This plateau is the result of slow Calcium channels along with the fast Sodium channels. After this plateau is an abrupt repolarization that is caused by the Potassium outflow. The velocity of conduction of action potential in atrial and ventricular muscle fibers is about 0.4m/sec. The velocity of conduction in Purkinje fibers may reach up to 4m/sec.

The transmission of electrical potential within the cell is the result of a complex interaction of ions traveling across the membrane through voltage and time varying channels as well as metabolic pumps and chemical reactions.

Different models that have emerged are based on the working of these channels.

2.2 Refractory period of heart

The refractory period of heart is the time interval during which a normal cardiac impulse cannot re-excite an already excited area of cardiac muscle. The normal refractory period of the ventricle is 0.25 to 0.3 seconds, which is about the duration of the action potential. There is an additional refractory period of about 0.05 seconds during which the muscle is more difficult than normal to excite. The refractory period for the atrial muscles is less than that of the ventricular muscles. The normal refractory period is about 0.15 seconds and the relative refractory period is about 0.03 seconds. Therefore, the rhythmical contraction of the atrial muscles may be faster than the contraction of the ventricular muscles.

2.3 Chaos In Heart

The loss of rhythm in the heart is termed as Arrhythmia. Different types of Arrhythmia have been described in the medical literature. One of the common types of Arrhythmias is Ventricular Fibrillation. Ventricular fibrillation is the devastating condition in which the ventricular muscles start to fibrillate without any coordination and pumping of blood seizes causing death within few minutes. The process is so rapid that most of the victims die before any medical attention is paid to them. It has been observed that sudden cardiac death is the leading cause of death in the industrialized world with the majority of such tragedies due to ventricular fibrillation [1]. Fibrillation is another example of a chaotic system exhibiting chaos all of a sudden after a very peaceful behavior.

3. THE HEART MODEL

The model of signal transmission through heart is based upon the working of the different ionic channels that cause the cells to excite. Many different models have been presented to account for the events taking place during the action potential. One of the simplest model developed by Richard Fitzhugh is based on only two state variables: a fast excitation variable analogous to membrane potential and a much slower recovery variable to characterize the average outward current. The two variables are given in the form of parabolic partial differential equations and have been used extensively to simulate the dynamic characteristics of cardiac tissues. The model is also based upon the facts that the cells are excitable and exhibit a refractory period during which excitation does not take place.

The model has been described as under:

$$\partial e/\partial t = (\partial/\partial x_i)d_{ij}\,(\partial e/\partial x_j) - f(e) - g. \quad (1)$$

$$\partial g/\partial t = \varepsilon(e,g)(ke - g). \quad (2)$$

Where,
e is the electric potential variable &
g is the recovery variable
d_{ij} is a conductivity tensor accounting for the anisotropy of cardiac tissue. In case of an isotropic medium d_{ij} is taken as a unity matrix and in that case $(\partial/\partial x_i)d_{ij}(\partial e/\partial x_j) = \nabla^2 e$

$f(e) = C_1 * e$ when $e < e_1$;
$f(e) = -C_2 * e + a$ when $e_1 \le e \le e_2$;
$f(e) = C_3 * (e-1)$ when $e > e_2$;

$\varepsilon(e,g) = \varepsilon_1$ when $e < e_2$;
$\varepsilon(e,g) = \varepsilon_2$ when $e > e_2$;
$\varepsilon(e,g) = \varepsilon_3$ when $e < e_1$ and $g < g_1$;

The parameters determining the shape of the function f(e) are:
$e_1 = 0.0026$, $e_2 = 0.837$, $C_1 = 20$, $C_2 = 3$, $C_3 = 15$, $a = 0.06$ and $k = 3$.
The parameters determining the shape of the function g are:
$\varepsilon_1^{-1} = 75$, $\varepsilon_2^{-1} = 1$, $g_1 = 1.8$, and $0.5 < \varepsilon_3^{-1} < 10$. [2]

4. IMPORTANT CHARACTERISTICS OF THE MODEL

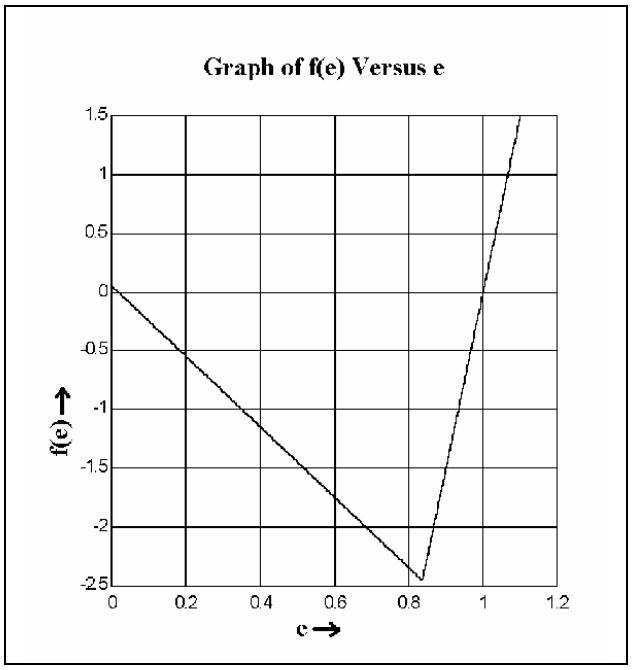
The model consists of two variables: e & g. e is the electric potential variable while g is termed as the recovery variable.

4.1 $(\partial/\partial x_i)d_{ij}(\partial e/\partial x_j)$ or $\nabla^2 e$

The above term describes the electrical coupling between adjacent cells. In most of our body cells, there is no mutual coupling and the potential in one cell does not affect the other but the presence of gap junctions in the heart provide a low resistance path between adjacent cells due to which a potential in one cell causes excitation in the adjacent cells and the electrical signal is propagated causing the normal working of the heart. Taking 'd_{ij}' as unity results in isotropic propagation of signals in which the signals propagate equally well in all directions although anisotropy has also been observed but for simplicity it may be taken as unity.

4.2 f(e)

The shape of the function f(e) specifies fast processes such as the initiation of the action potential.[2]



It can be seen from the plot of f(e) versus e that f(e) has a small positive value for very small values of e for e less than e_1 (0.0026). Thus e_1 serves as a threshold that inhibits the initiation of action potential as long as the variable e is below it. As e rises above e_1, the value of f(e) begins to decrease and becomes negative. This causes the action potential to start. e_2 serves as another threshold that results in a rapid rise in f(e) that causes inhibition of the action potential. Even if all other factors are neglected, f(e) will ultimately inhibit excessive increase in the value of the action potential.

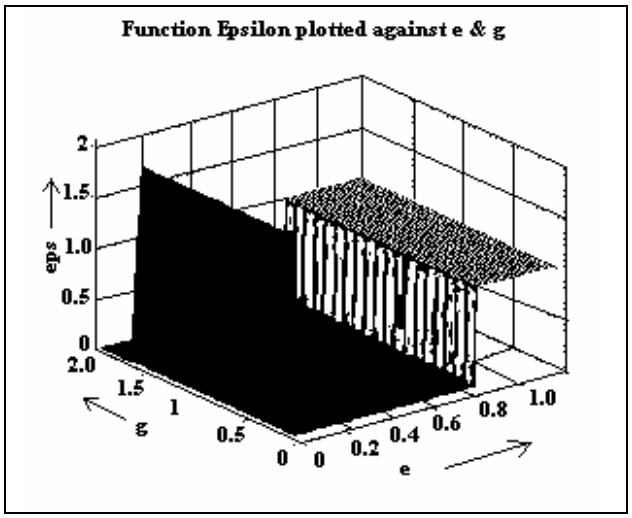
4.3 g in e

The variable g contained in the equation of e shows its importance as a recovery variable by inhibiting rapid rise in the value of e.

4.4 ε (e,g)

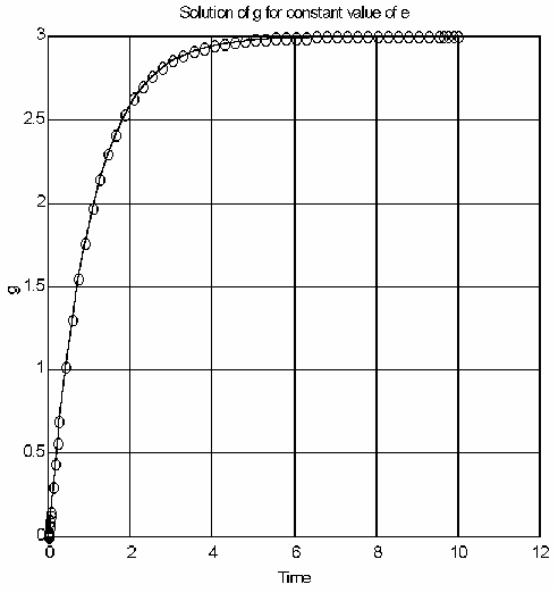
The dynamics of the recovery variable g is determined by the function ε(e,g).

The function has a very high value for low values of e. This causes the function 'g' to inhibit the initiation of action potential when the potential is below the threshold. For intermediate values of 'e', the value of 'g' is negligible as compared to the value of f(e). Thus it is mainly the value of f(e) that is responsible for the initiation of action potential. For large values of 'e', it is mainly 'g' that is responsible for bringing the potential back to its resting value. It is because of these reasons that 'g' is termed as the recovery variable that recovers the resting potential.



4.5 (ke – g)

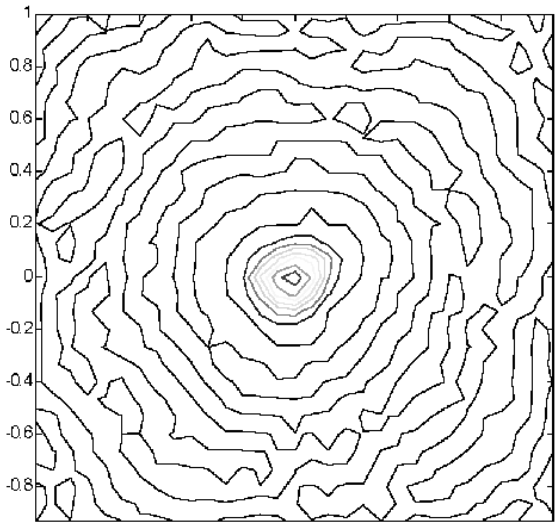
The term (ke – g) takes into account the dependence of 'g' on itself and 'e'. It shows the increase in g with e while k is the constant of proportionality that describes how quickly g will increase with rise in e. Value of k greater than one describes a rapid rise in g to ultimately recover from the action potential. The dependence of g on itself results in a self-restraining factor.



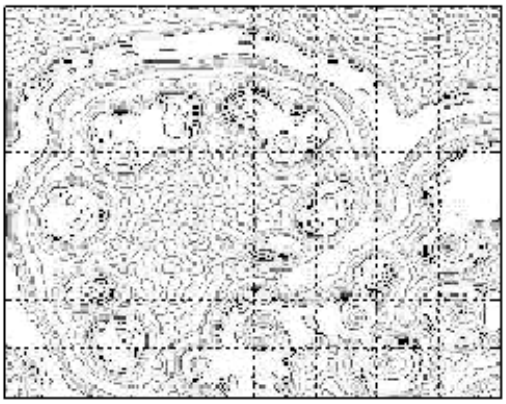
4.6 Solving Altogether

The two equations of electric potential variable and recovery variable may be solved with different values of parameters and initial conditions to observe their effect on the transmission of signals through the heart cells. The two equations of e & g have been solved using the PDE Toolbox of MATLAB. A mesh has been formed consisting of small triangles on which the two equations

have been solved. The time step has been taken as 1msec. Neumann boundary conditions were taken. The solutions were observed using contour plots and surface plots.



Regular transmission of signals (a)



Chaotic Transmission of Signals

5. CONCLUSIONS

From the solutions of the two equations with different values of the given parameters and initial conditions, it is observed that the refractoriness of heart causes regular transmission of signals even at abnormal initial conditions but decreasing the effect of refractoriness results in chaos with abnormal initial conditions.

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