

```

In[1]:= f[x_] := Piecewise[{{-x, x < 0}, {x, x > 0}}];
(* f(x)=|x| , not differentiable at x=0 *)
g[x_] := Piecewise[{{x, x < 0}, {-x, x > 0}}];
(* g(x)=-|x| , not differentiable at x=0 *)
h[x_] := f[x] + g[x]; (* h(x)=f(x)+g(x) , differentiable at x=0 *)

point = 0; (* Check differentiability by changing the point *)

(* D[f[x],x]/.x->point
   D[g[x],x]/.x->point
   D[h[x],x]/.x->point *)

(*Simplify[h[x]]*)
(*leftDeriv=Limit[(h[x]-h[point])/(x-point), x->0,Direction->-1];
rightDeriv=Limit[(h[x]-h[point])/(x-point), x->0,Direction->1];
If[leftDeriv===rightDeriv, "Differentiable", "Not Differentiable"]*)

(* Defining left limit and right limit of the function *)
leftLim[k_, x_, p_] := Limit[k[x], x -> p, Direction -> -1];
rightLim[k_, x_, p_] := Limit[k[x], x -> p, Direction -> 1];

(* Defining left derivative and right derivative *)
leftDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x -> p, Direction -> -1];
rightDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x -> p, Direction -> 1];

If[leftLim[f, x, point] === rightLim[f, x, point], "Continuous", "Discontinuous"]
If[leftLim[g, x, point] === rightLim[g, x, point], "Continuous", "Discontinuous"]
If[leftLim[h, x, point] === rightLim[h, x, point], "Continuous", "Discontinuous"]

If[leftDeriv[f, x, point] === rightDeriv[f, x, point],
  "Differentiable", "Not Differentiable"]
If[leftDeriv[g, x, point] === rightDeriv[g, x, point],
  "Differentiable", "Not Differentiable"]
If[leftDeriv[h, x, point] === rightDeriv[h, x, point],
  "Differentiable", "Not Differentiable"]

(* (f(x) + g(x)) is a horizontal line through x-
   axis and differentiable at all points including x=
   0. Its derivative at each point is 0 *)
Plot[{f[x], g[x], (f[x] + g[x])}, {x, -3, 3},
  PlotStyle -> {Directive[Green, Thick], Directive[Orange, Thick], Directive[Red, Thick]}]

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Out[9]= Continuous

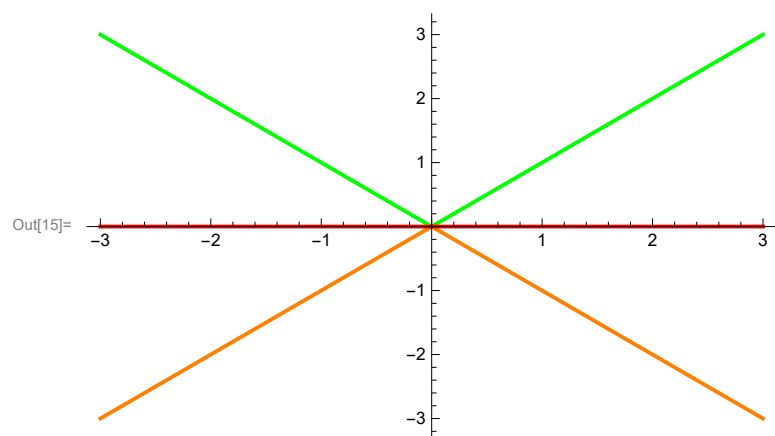
Out[10]= Continuous

Out[11]= Continuous

Out[12]= Not Differentiable

Out[13]= Not Differentiable

Out[14]= Differentiable



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In[16]:= f[x_] := Piecewise[{{-(x - 1), x < 1}, {(x - 1), x > 1}}]; (* f(x)=|x-1|,
decreasing below x=1 nad increasing above x>1. Not differentiable at x=1 *)
g[x_] := Piecewise[{{-(x - 5), x < 5}, {(x - 5), x > 5}}]; (* g(x)=|x-5|,
decreasing below x=5 nad increasing above x>5. Not differentiable at x=5 *)
h[x_] := f[x] + g[x];      (* h(x)=f(x)+g(x) *)

leftDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → -1];
rightDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → 1];

Check[r_, x_, p_] :=
  If[leftDeriv[r, x, p] === rightDeriv[r, x, p], "Differentiable", "Not Differentiable"];
(*Check[k_, x_, p_] := Module[{ld, rd},
  ld=leftDeriv[k, x, p];
  rd=rightDeriv[k, x, p];
  If[NumericQ[ld]&&NumericQ[rd]&&ld==rd, "Differentiable", "Not Differentiable"]];*)

(*If[leftDeriv[f, x, 1]===rightDeriv[f, x, 1], "Differentiable", "Not Differentiable"]
If[leftDeriv[g, x, 5]===rightDeriv[g, x, 5], "Differentiable", "Not Differentiable"]
If[leftDeriv[h, x, 1]===rightDeriv[h, x, 1], "Differentiable", "Not Differentiable"]
If[leftDeriv[h, x, 5]===rightDeriv[h, x, 5], "Differentiable", "Not Differentiable"]
If[leftDeriv[h, x, 3]===rightDeriv[h, x, 3], "Differentiable", "Not Differentiable"]*)

(*Check[h, x, 3]*)

D[f[x], x] /. x → 1
D[g[x], x] /. x → 5

D[f[x], x] /. x → 3
D[g[x], x] /. x → 3
D[h[x], x] /. x → 3
D[h[x], x] /. x → 2
D[h[x], x] /. x → 4

(* f(x) + g(x) is a horizontal line (y=4) parallel to x-axis at 1<x<5,
decreasing below x=1,
and increasing above x>5 . Its derivative at each points in between 1<x<5 is 0. *)
Plot[{f[x], g[x], (f[x] + g[x])}, {x, -3, 9},
  PlotStyle → {Directive[Green, Thick], Directive[Orange, Thick], Directive[Red, Thick]}]

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SetDelayed: Tag Check in Check[r_, x_, p_] is Protected.

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Out[22]= Indeterminate

Out[23]= Indeterminate

Out[24]= 1

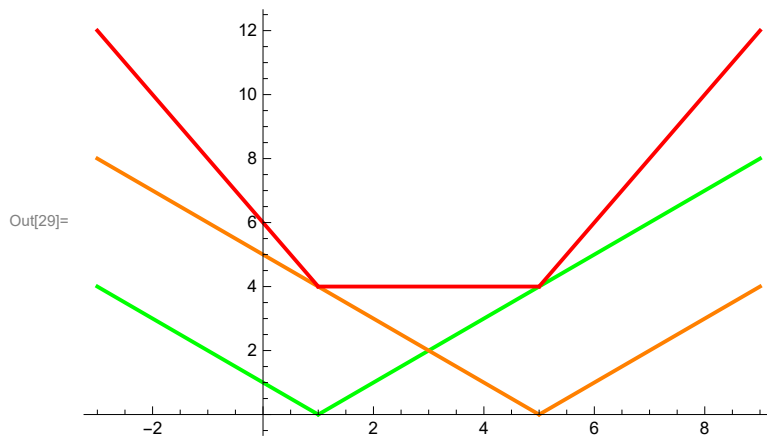
Out[25]= -1

Out[26]= 0

Out[27]= 0

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Out[28]= 0



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In[30]:= f[x_] := Piecewise[{{Exp[x], x < 0}, {(x + 1), x ≥ 0}}]; (* continuous at x=0 *)

Plot[Exp[x], {x, -6, 6}, PlotStyle → Directive[Green, Thick],
  PlotRange → {{-6, 6}, {-5, 20}}]
Plot[x + 1, {x, -6, 6}, PlotStyle → Directive[Orange, Thick],
  PlotRange → {{-6, 6}, {-5, 20}}]
Plot[f[x], {x, -6, 6}, PlotStyle → Directive[Red, Thick], PlotRange → {{-6, 6}, {-5, 20}}]
Plot[{Exp[x], x + 1, f[x]}, {x, -6, 6},
  PlotStyle → {Directive[Green, Thick], Directive[Orange, Thick], Directive[Red, Thick]},
  PlotRange → {{-6, 6}, {-5, 20}}]

point = 0;

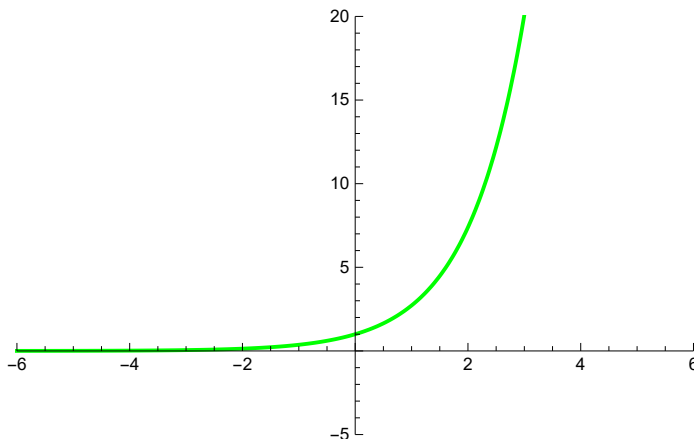
leftLim[k_, x_, p_] := Limit[k[x], x → p, Direction → -1];
rightLim[k_, x_, p_] := Limit[k[x], x → p, Direction → 1];

leftDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → -1];
rightDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → 1];

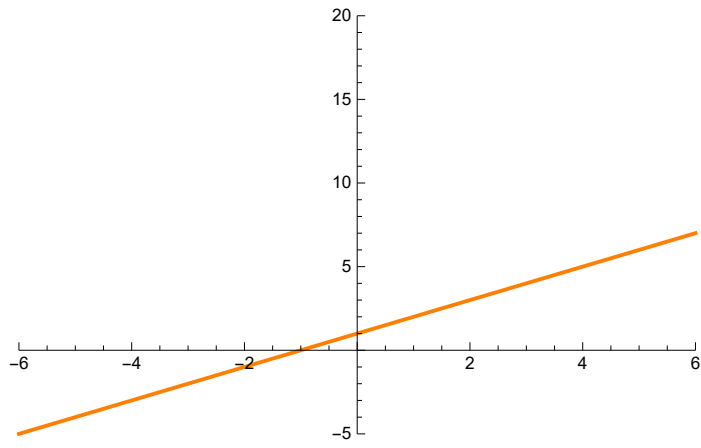
If[leftLim[f, x, point] === rightLim[f, x, point], "Continuous", "Discontinuous"]
If[leftDeriv[f, x, point] === rightDeriv[f, x, point],
  "Differentiable", "Not Differentiable"]

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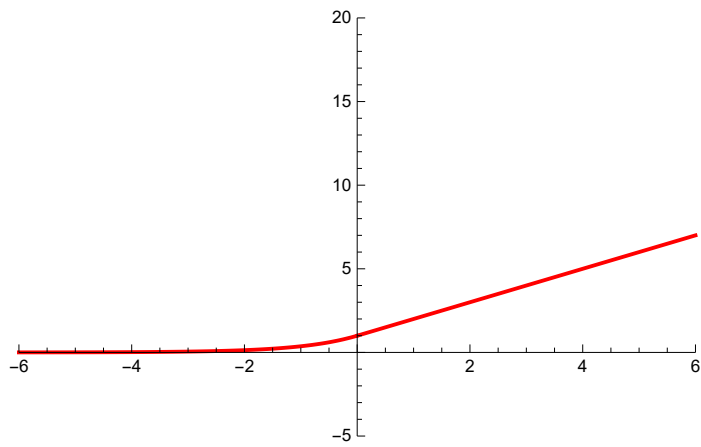
Out[31]=



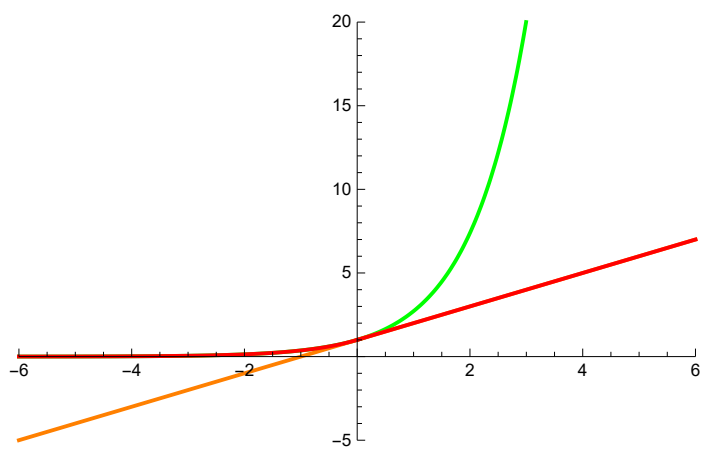
Out[32]=



Out[33]=



Out[34]=



Out[40]= Continuous

Out[41]= Differentiable

```
In[42]:= f[x_] := Piecewise[{{Exp[x], x < 0}, {(x), x ≥ 0}}]; (* discontinuous at x=0 *)
```

```
Plot[Exp[x], {x, -6, 6},
  PlotStyle → Directive[Green, Thick], PlotRange → {{-6, 6}, {-5, 20}}]
Plot[x, {x, -6, 6}, PlotStyle → Directive[Orange, Thick], PlotRange → {{-6, 6}, {-5, 20}}]
Plot[f[x], {x, -6, 6}, PlotStyle → Directive[Red, Thick],
  PlotRange → {{-6, 6}, {-5, 20}}]
Plot[{Exp[x], x, f[x]}, {x, -6, 6},
  PlotStyle → {Directive[Green, Thick], Directive[Orange, Thick], Directive[Red, Thick]},
  PlotRange → {{-6, 6}, {-5, 20}}]
```

```
point = 0;
```

```
leftLim[k_, x_, p_] := Limit[k[x], x → p, Direction → -1];
```

```
rightLim[k_, x_, p_] := Limit[k[x], x → p, Direction → 1];
```

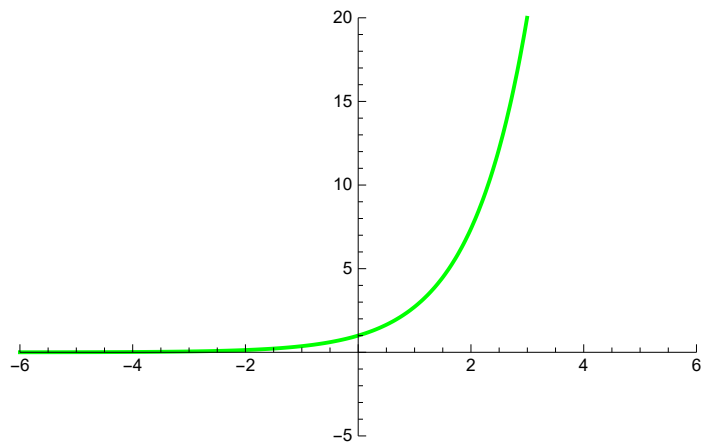
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leftDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → -1];
```

```
rightDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → 1];
```

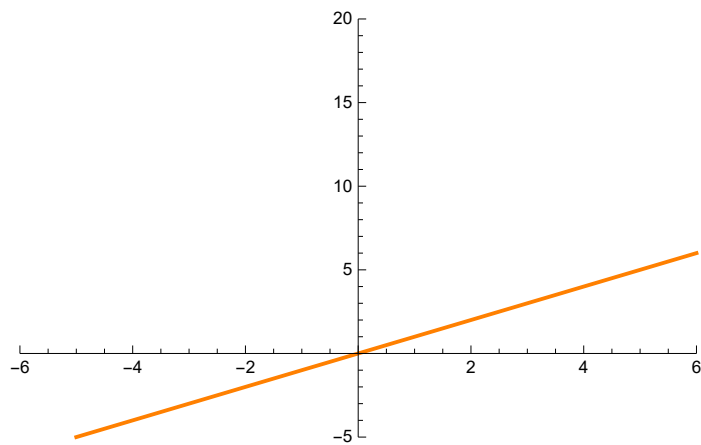
```
If[leftLim[f, x, point] === rightLim[f, x, point], "Continuous", "Discontinuous"]
```

```
If[leftDeriv[f, x, point] === rightDeriv[f, x, point],
  "Differentiable", "Not Differentiable"]
```

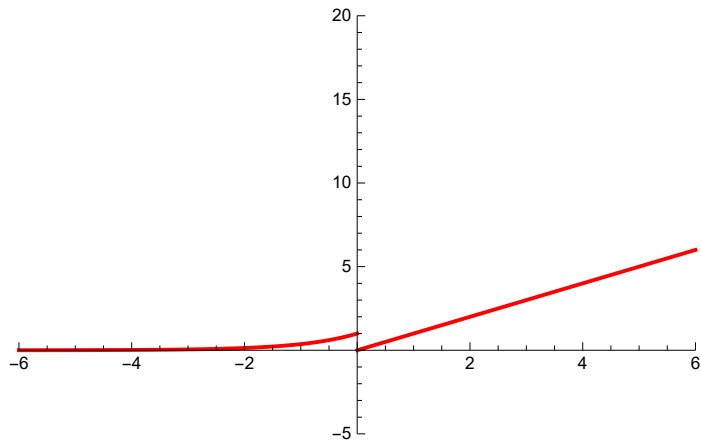
Out[43]=



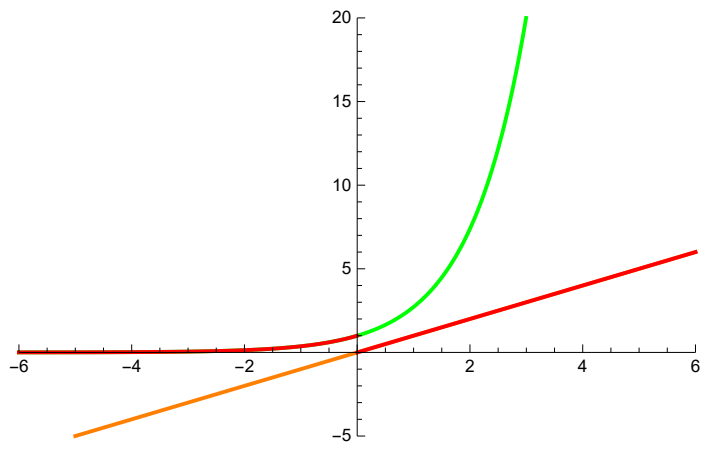
Out[44]=



Out[45]=



Out[46]=



Out[52]= **Discontinuous**

Out[53]= **Not Differentiable**

```

In[54]:= f[x_] := Piecewise[{{Exp[x], x < 0}, {(2 x + 1), x ≥ 0}}]; (* notch at x=0 *)

Plot[Exp[x], {x, -6, 6},
  PlotStyle → Directive[Green, Thick], PlotRange → {{-6, 6}, {-5, 20}}]
Plot[2 x + 1, {x, -6, 6}, PlotStyle → Directive[Orange, Thick],
  PlotRange → {{-6, 6}, {-5, 20}}]
Plot[f[x], {x, -6, 6}, PlotStyle → Directive[Red, Thick], PlotRange → {{-6, 6}, {-5, 20}}]
Plot[{Exp[x], 2 x + 1, f[x]}, {x, -6, 6},
  PlotStyle → {Directive[Green, Thick], Directive[Orange, Thick], Directive[Red, Thick]},
  PlotRange → {{-6, 6}, {-5, 20}}]

point = 0;

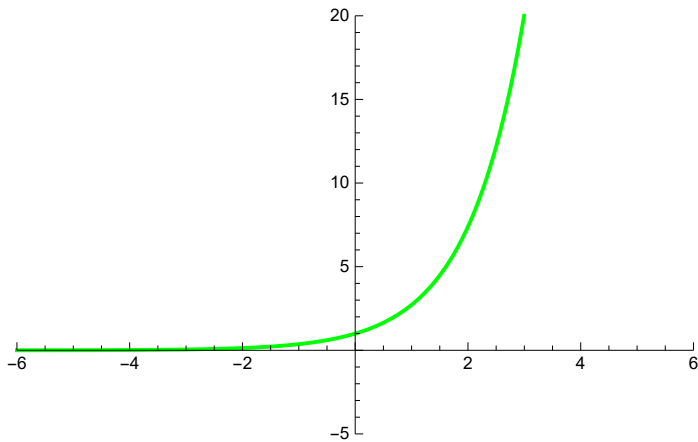
leftLim[k_, x_, p_] := Limit[k[x], x → p, Direction → -1];
rightLim[k_, x_, p_] := Limit[k[x], x → p, Direction → 1];

leftDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → -1];
rightDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → 1];

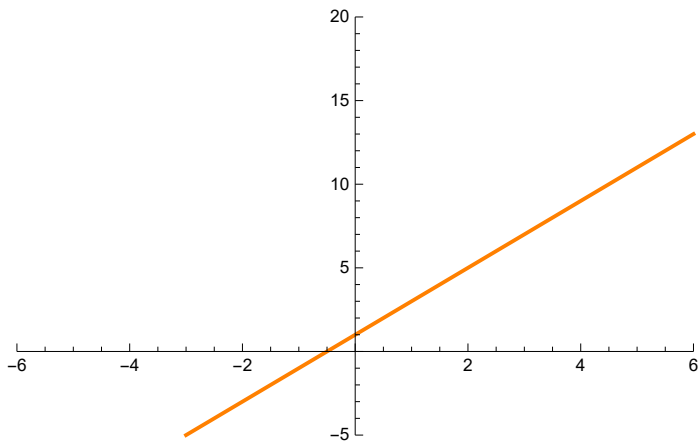
If[leftLim[f, x, point] === rightLim[f, x, point], "Continuous", "Discontinuous"]
If[leftDeriv[f, x, point] === rightDeriv[f, x, point],
  "Differentiable", "Not Differentiable"]

```

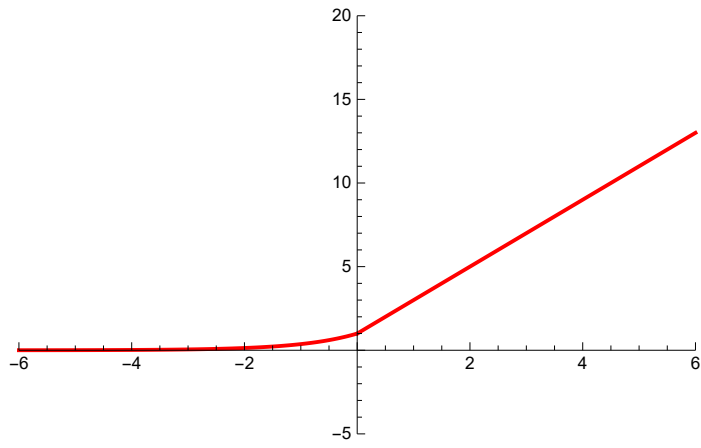
Out[55]=



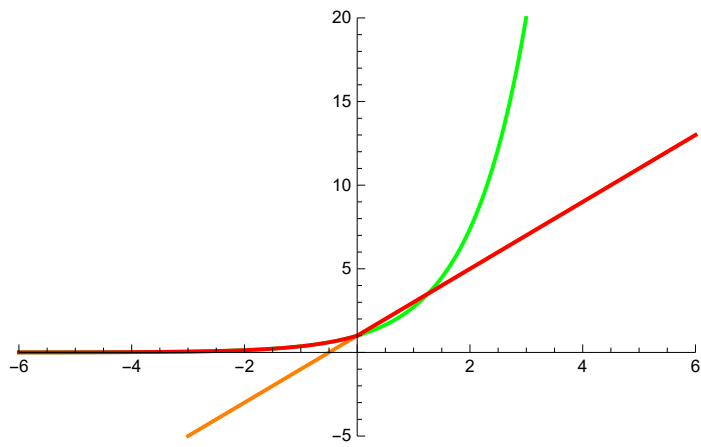
Out[56]=



Out[57]=



Out[58]=



Out[64]= Continuous

Out[65]= Not Differentiable