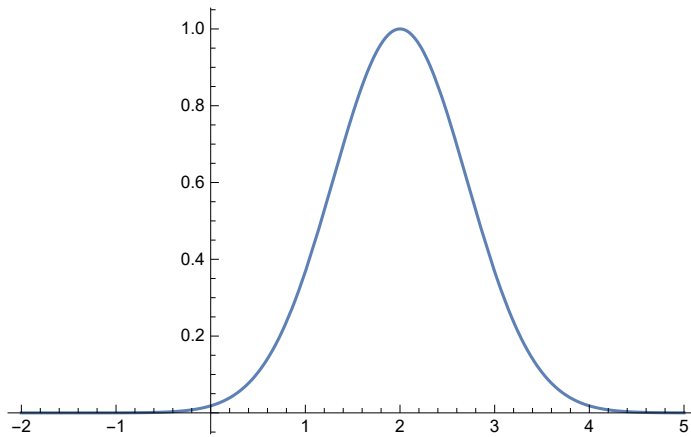


```

In[ ]:= f[x_] := Exp[- (x - a) ^2];          (*Not normalized*)
a = 2;
Plot[f[x], {x, -2, 5}, PlotRange -> Full, PlotLegends -> "Expressions"]
NIntegrate[f[x], {x, -∞, ∞}]
(*Area under Exp[- (x-a)^2] is  $\sqrt{\pi}$  *)
N[Sqrt[Pi], 8]

```

Out[ ]:=



Out[ ]:= 1.77245

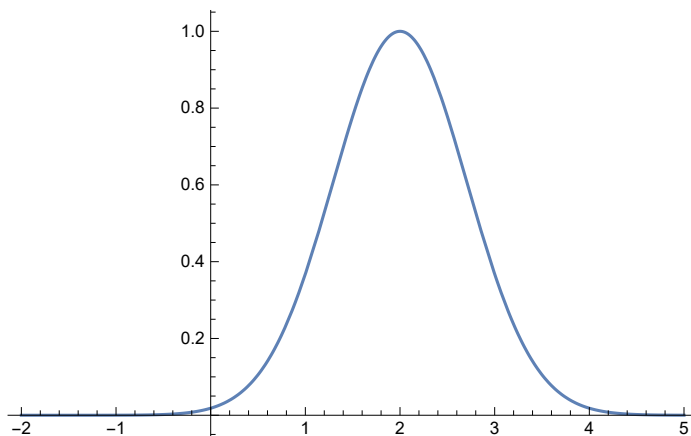
Out[ ]:= 1.7724539

```

In[ ]:= f[x_] := 2.71828^(- (x - a) ^2);    (*Not normalized*)
a = 2;
Plot[f[x], {x, -2, 5}, PlotRange -> Full, PlotLegends -> "Expressions"]
NIntegrate[f[x], {x, -∞, ∞}]
(*Area under Exp[- (x-a)^2] is  $\sqrt{\pi}$  *)
N[Sqrt[Pi], 8]

```

Out[ ]:=



Out[ ]:= 1.77245

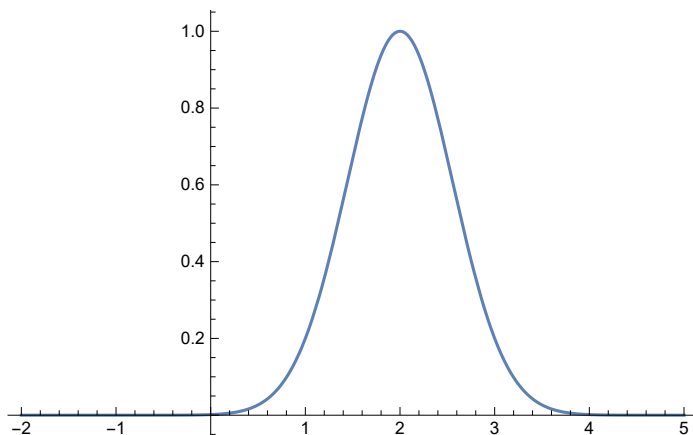
Out[ ]:= 1.7724539

```

In[ ]:= f[x_] := 5^(-(x-a)^2);                      (*Not normalized*)
a = 2;
Plot[f[x], {x, -2, 5}, PlotRange -> Full, PlotLegends -> "Expressions"]
NIntegrate[f[x], {x, -∞, ∞}]
(*Area under 5^(-(x-a)^2 *)

```

Out[ ]:=



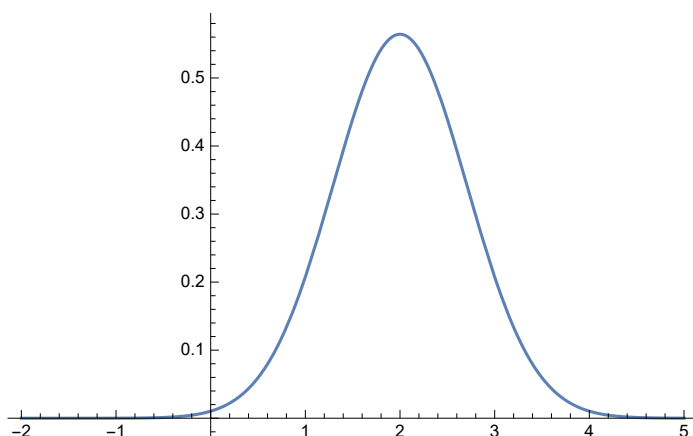
Out[ ]:= 1.39713

```

In[ ]:= f[x_] := (1/Sqrt[π]) * Exp[-(x-a)^2];      (*Normalized*)
a = 2;
Plot[f[x], {x, -2, 5}, PlotRange -> Full, PlotLegends -> "Expressions"]
NIntegrate[f[x], {x, -∞, ∞}]

```

Out[ ]:=

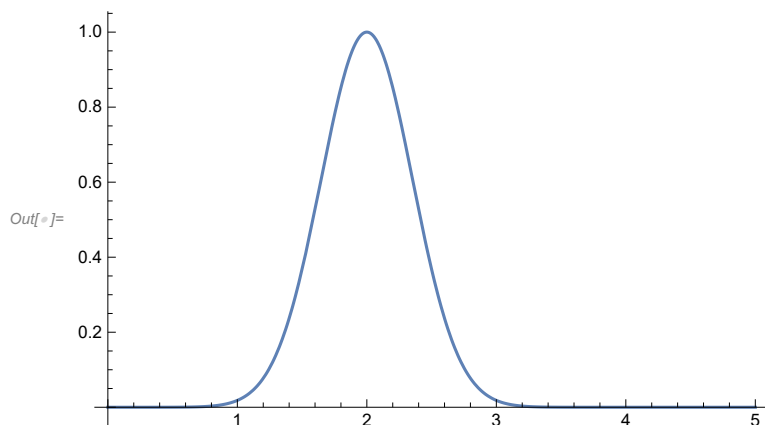


Out[ ]:= 1.

```

In[ ]:= f[x_, k_] := Exp[- (x - a)^2 / k^2];
(*Not normalized*) (*What does k do in the exponential*)
a = 2;
Plot[f[x, k = 0.5], {x, 0, 5}, PlotRange -> Full, PlotLegends -> "Expressions"]
NIntegrate[f[x, k = 0.5], {x, -∞, ∞}]
2 * NIntegrate[f[x, k = 0.5], {x, -∞, ∞}]

```



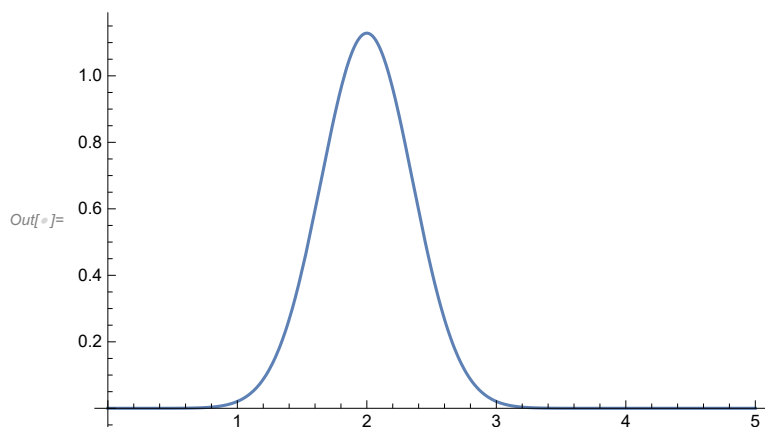
Out[ ]:= 0.886227

Out[ ]:= 1.77245

```

In[ ]:= f[x_, k_] := (1 / (k Sqrt[π])) Exp[- (x - a)^2 / k^2]; (*Normalized*)
a = 2;
Plot[f[x, k = 0.5], {x, 0, 5}, PlotRange -> Full, PlotLegends -> "Expressions"]
NIntegrate[f[x, k = 0.5], {x, -∞, ∞}]
NIntegrate[f[x, k = 0.05], {x, -∞, ∞}]

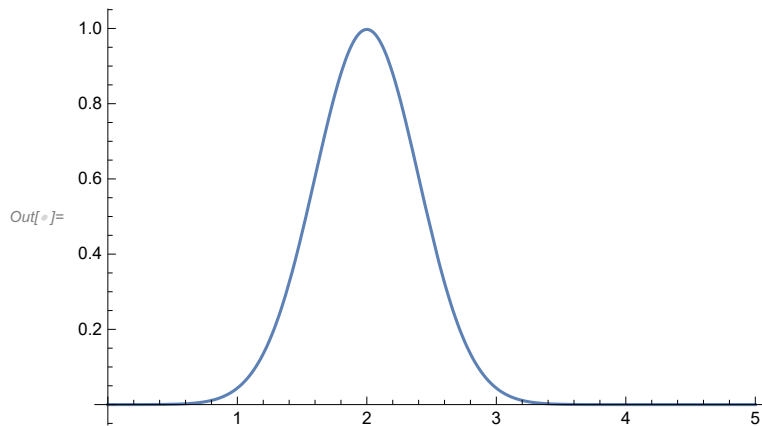
```



Out[ ]:= 1.

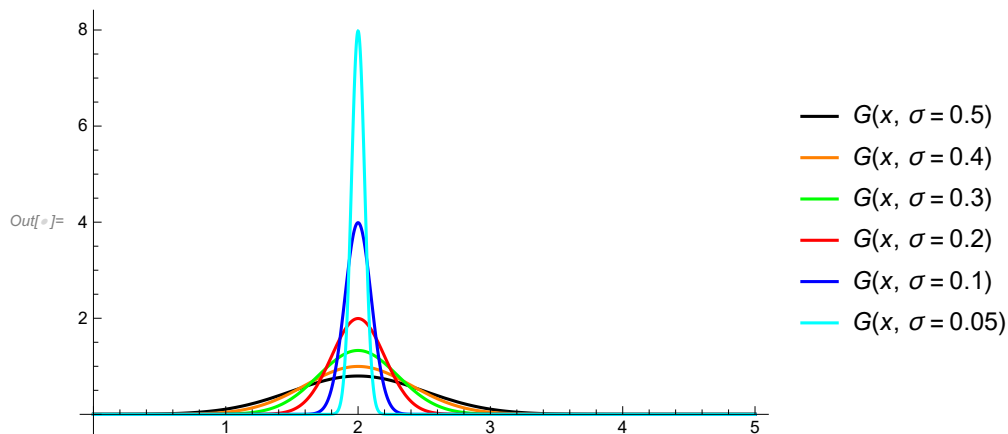
Out[ ]:= 1.

```
In[ ]:= G[x_] := (1 / (σ * Sqrt[2 Pi])) Exp[- (x - a)^2 / (2 σ^2)];
(*What do σ do in the exponenetial*)
a = 2; σ = 0.4;
Plot[G[x], {x, 0, 5}]
```



```
In[ ]:= G[x_, σ_] := (1 / (σ * Sqrt[2 Pi])) Exp[- (x - a)^2 / (2 σ^2)];
(*What do σ do in the exponenetial*)
a = 2;
Plot[{G[x, σ = 0.5], G[x, σ = 0.4], G[x, σ = 0.3],
      G[x, σ = 0.2], G[x, σ = 0.1], G[x, σ = 0.05]}, {x, 0, 5}, PlotRange → Full,
      PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
NIntegrate[G[x, σ = 0.5], {x, -∞, ∞}]
NIntegrate[G[x, σ = 0.05], {x, -∞, ∞}]
```

General: Exp[-799.918] is too small to represent as a normalized machine number; precision may be lost.



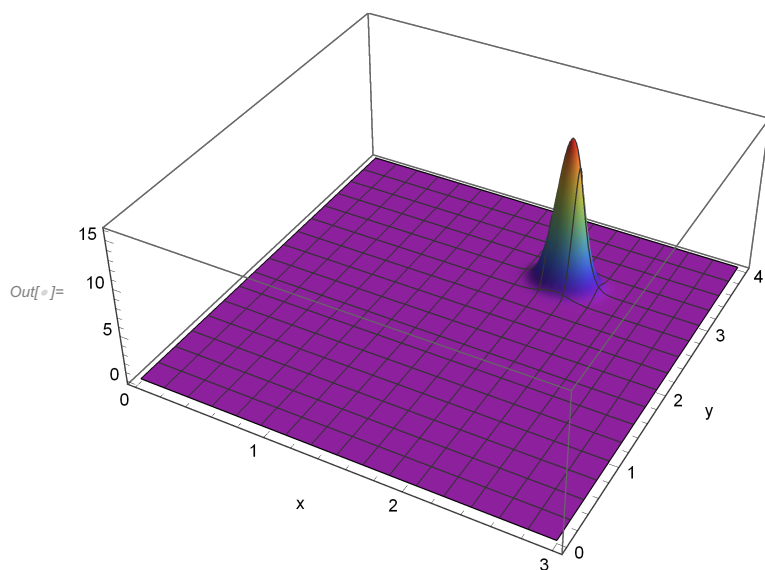
Out[ ]:= 1.

Out[ ]:= 1.

```

In[ ]:= G2[x_, y_, σ_] := (1 / (σ^2 * 2 Pi)) Exp[-((x - a)^2 + (y - b)^2) / (2 σ^2)];
a = 2; b = 3;
Plot3D[G2[x, y, σ = 0.1], {x, 0, 3}, {y, 0, 4}, PlotPoints → 100,
  PlotRange → Full, AxesLabel → {"x", "y"}, ColorFunction → "Rainbow"]

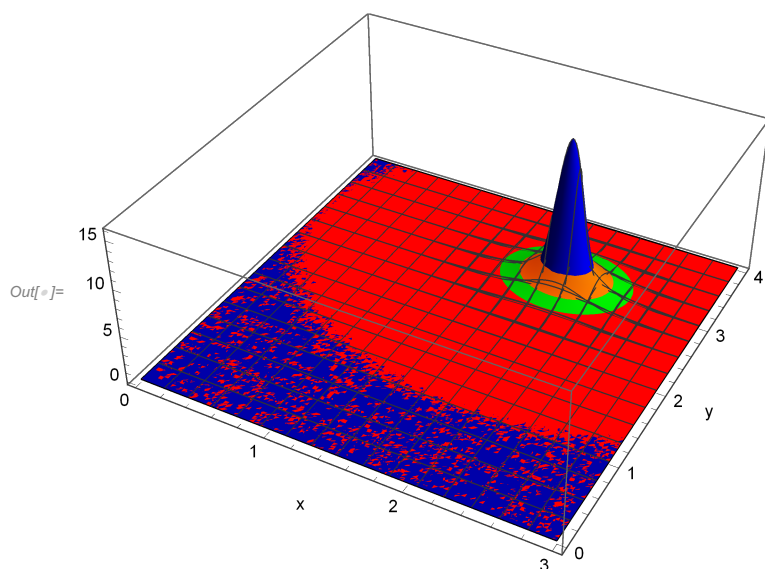
```



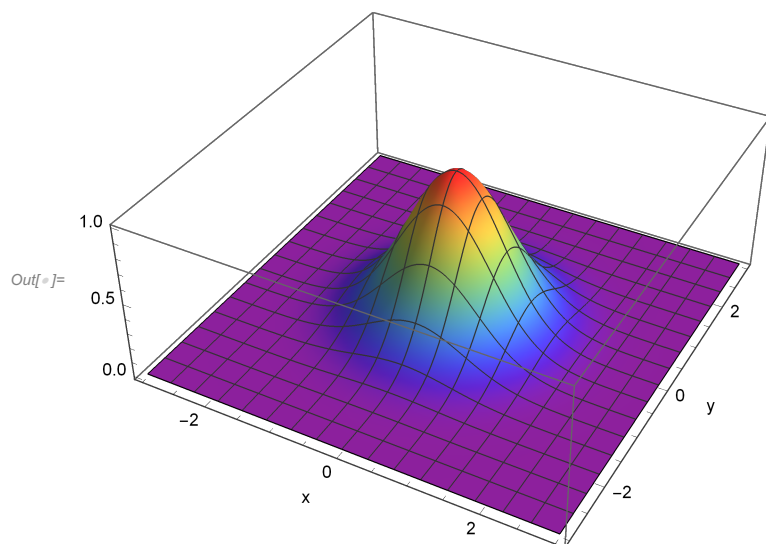
```

In[ ]:= G2[x_, y_, σ_] := (1 / (σ^2 * 2 Pi)) Exp[-((x - a)^2 + (y - b)^2) / (2 σ^2)];
a = 2; b = 3;
Plot3D[{G2[x, y, σ = 0.4], G2[x, y, σ = 0.3], G2[x, y, σ = 0.2], G2[x, y, σ = 0.1]},
  {x, 0, 3}, {y, 0, 4}, PlotPoints → 100, PlotRange → Full,
  AxesLabel → {"x", "y"}, PlotStyle → {Red, Green, Orange, Blue, Opacity[0.3]}]

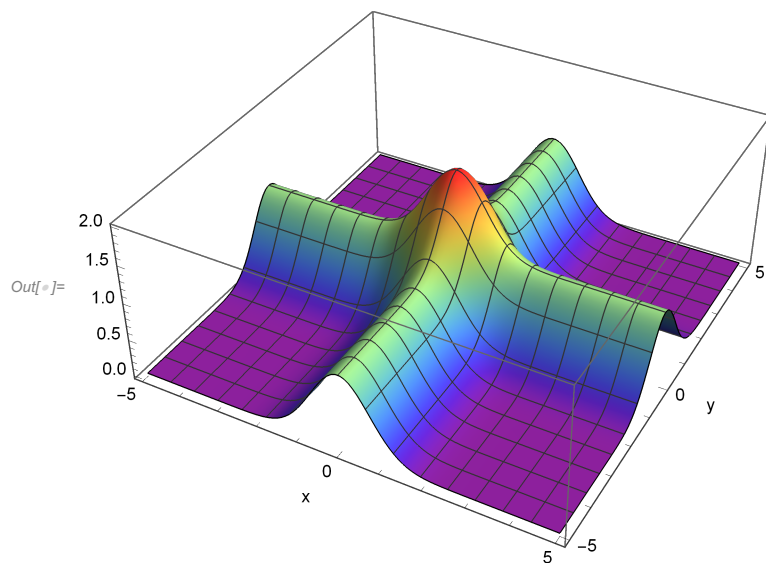
```



```
In[ ]:= g2[x_, y_] := Exp[-((x^2) + (y^2))];  
Plot3D[g2[x, y], {x, -3, 3}, {y, -3, 3}, PlotPoints -> 100,  
PlotRange -> Full, AxesLabel -> {"x", "y"}, ColorFunction -> "Rainbow"]
```



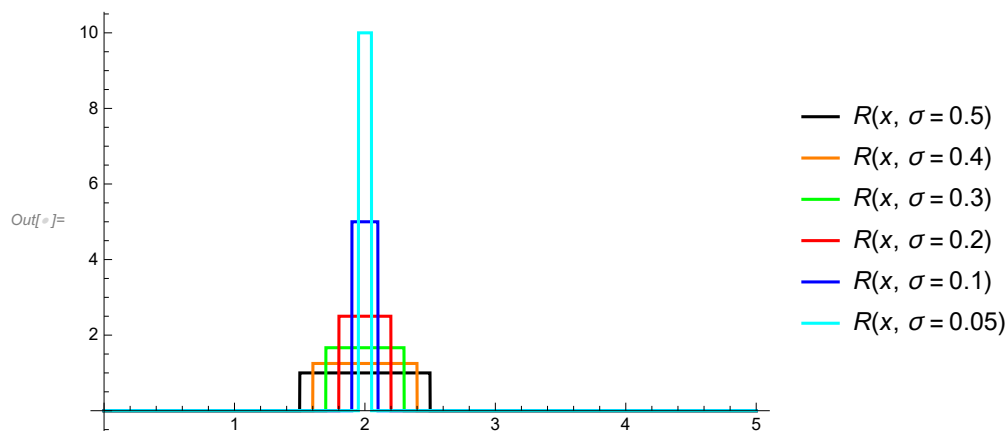
```
In[ ]:= g2[x_, y_] := Exp[-x^2] + Exp[-y^2];  
Plot3D[g2[x, y], {x, -5, 5}, {y, -5, 5}, PlotPoints -> 100,  
PlotRange -> Full, AxesLabel -> {"x", "y"}, ColorFunction -> "Rainbow"]
```



```

In[ ]:= R[x_, σ_] := Piecewise[{{1/(2 σ), -σ < x - a < σ}, {0, Modulus[x - a] > σ}}];
(*Rectangular function*)
a = 2;
Plot[{R[x, σ = 0.5], R[x, σ = 0.4], R[x, σ = 0.3],
      R[x, σ = 0.2], R[x, σ = 0.1], R[x, σ = 0.05]}, {x, 0, 5}, PlotRange → Full,
      PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
NIntegrate[R[x, σ = 0.5], {x, -∞, ∞}]
NIntegrate[R[x, σ = 0.05], {x, -∞, ∞}]

```



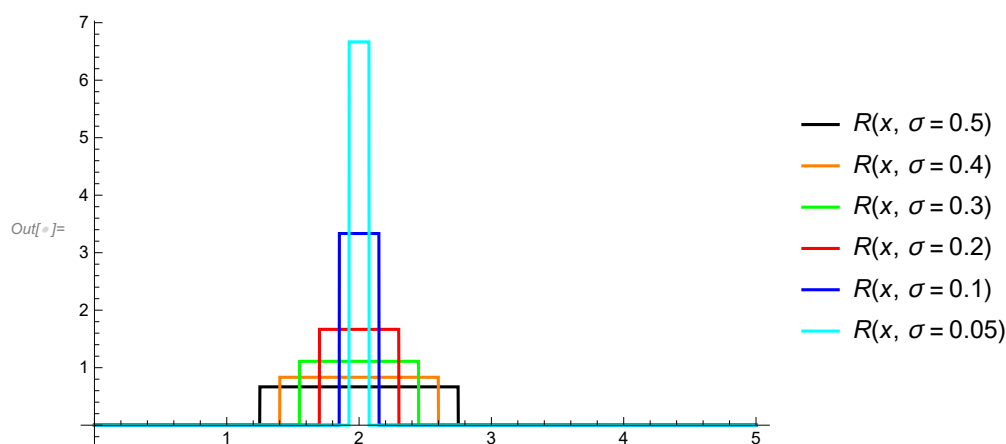
Out[ ]:= 1.

Out[ ]:= 1.

```

In[ ]:= R[x_, σ_] := Piecewise[{{1/(3 σ), -3 σ/2 < x - a < 3 σ/2}, {0, Modulus[x - a] > 3 σ/2}}];
a = 2;
Plot[{R[x, σ = 0.5], R[x, σ = 0.4], R[x, σ = 0.3],
      R[x, σ = 0.2], R[x, σ = 0.1], R[x, σ = 0.05]}, {x, 0, 5}, PlotRange → Full,
      PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
NIntegrate[R[x, σ = 0.5], {x, -∞, ∞}]
NIntegrate[R[x, σ = 0.05], {x, -∞, ∞}]

```



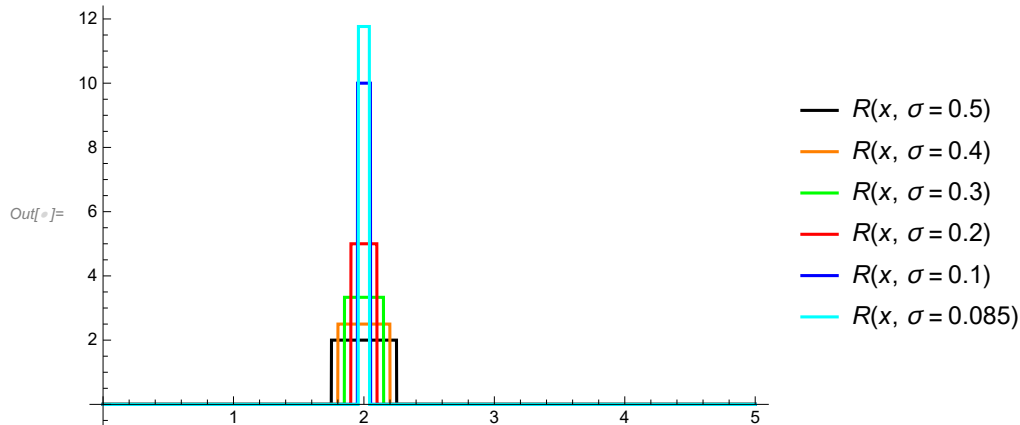
Out[ ]:= 1.

Out[ ]:= 1.

```

In[ ]:= R[x_, σ_] := Piecewise[{{1/σ, -σ/2 < x - a < σ/2}, {0, Modulus[x - a] > σ/2}}];
a = 2;
Plot[{R[x, σ = 0.5], R[x, σ = 0.4], R[x, σ = 0.3],
      R[x, σ = 0.2], R[x, σ = 0.1], R[x, σ = 0.085]}, {x, 0, 5}, PlotRange → Full,
      PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
NIntegrate[R[x, σ = 0.5], {x, -∞, ∞}]
NIntegrate[R[x, σ = 0.05], {x, -∞, ∞}]

```



Out[ ]:= 1.

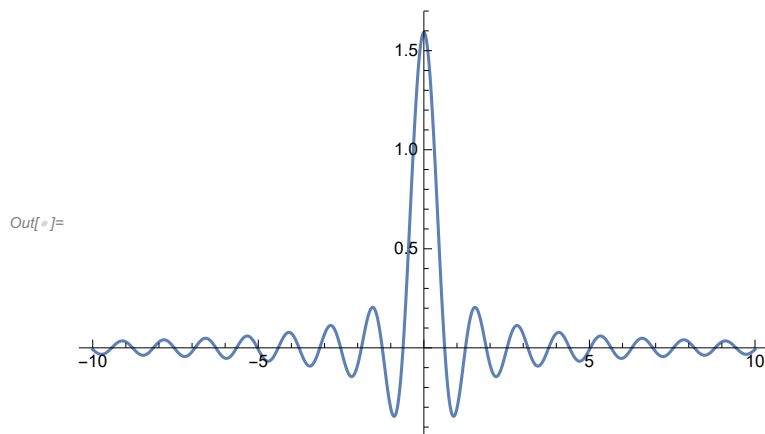
Out[ ]:= 1.

```

In[ ]:= s[x_] := Sin[g x] / (π x);
Limit[s[x], x → 0]
g = 5;
Plot[s[x], {x, -10, 10}, PlotRange → Full]

```

Out[ ]:=  $\frac{g}{\pi}$



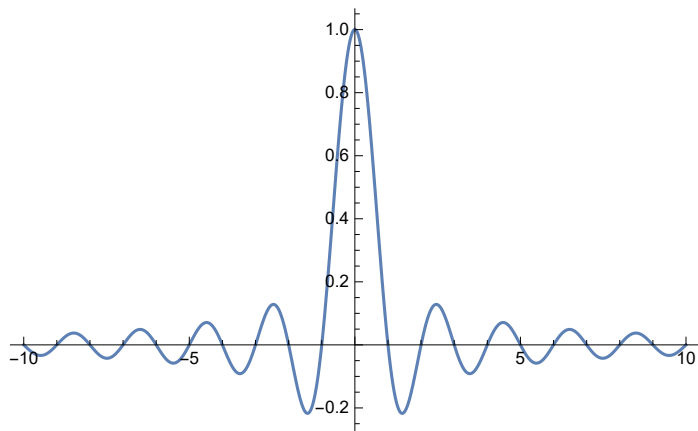


```

In[ ]:= s[x_] := Sin[π x] / (π x);
Limit[s[x], x → 0]
Plot[s[x], {x, -10, 10}, PlotRange → Full]

```

Out[ ]:= 1

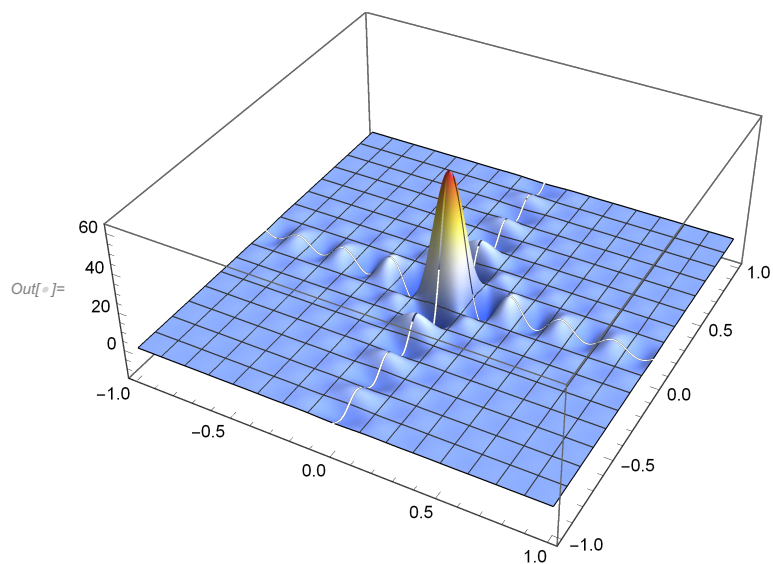


```

In[ ]:= s[x_, y_] := (Sin[g x] / (π x)) (Sin[g y] / (π y));
Limit[s[x], x → 0]
g = 25;
Plot3D[s[x, y], {x, -1, 1}, {y, -1, 1}, PlotPoints → 100,
  PlotRange → Full, ColorFunction → "TemperatureMap"]

```

Out[ ]:= 1



```

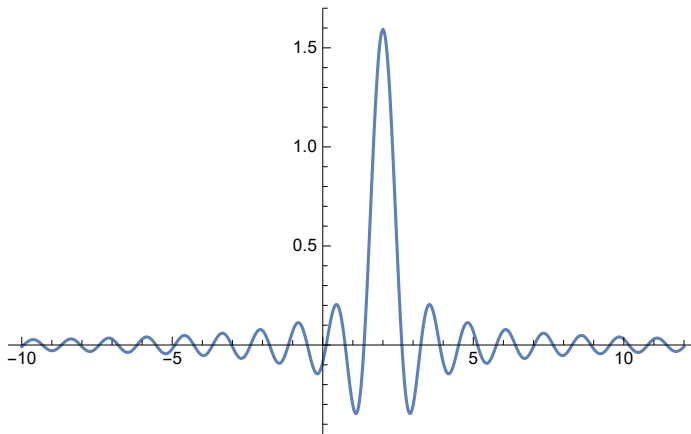
In[ ]:= s[x_] := Sin[g (x - a)] / (π (x - a));
Limit[s[x], x → 0]
a = 2; g = 5;
Plot[s[x], {x, -10, 10 + a}, PlotRange → Full, PlotLegends → "Expressions"]

```

Out[ ]:=  

$$\frac{\sin[50]}{2\pi}$$

Out[ ]:=



```

In[ ]:= S[x_, p_] := Sin[p (x - b)] / (π (x - b));
Limit[S[x, p], x → 0]
Limit[S[x, p], p → ∞]

```

Out[ ]:=  

$$\frac{\sin[3p]}{3\pi}$$

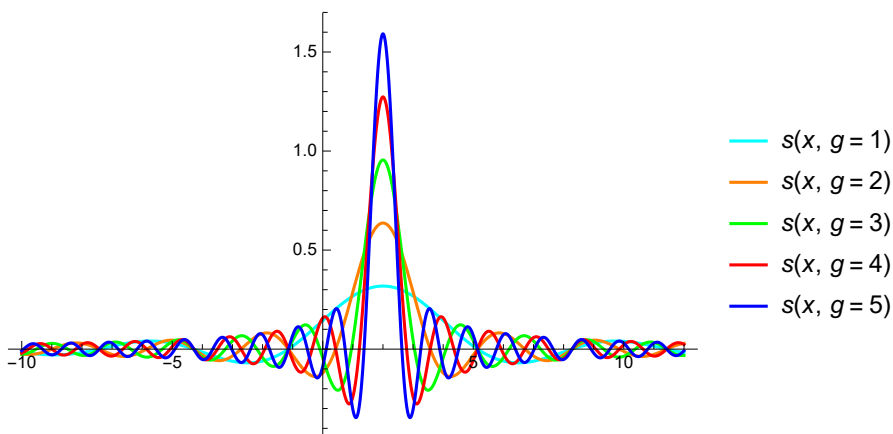
Out[ ]:= ConditionalExpression[Indeterminate, x ∈ ℝ]

```

In[ ]:= s[x_, g_] := Sin[g (x - a)] / (π (x - a));
a = 2;
Plot[{s[x, g = 1], s[x, g = 2], s[x, g = 3], s[x, g = 4], s[x, g = 5]}, {x, -10, 10 + a},
PlotRange → Full, PlotStyle → {Cyan, Orange, Green, Red, Blue}, PlotLegends → "Expressions"]

```

Out[ ]:=

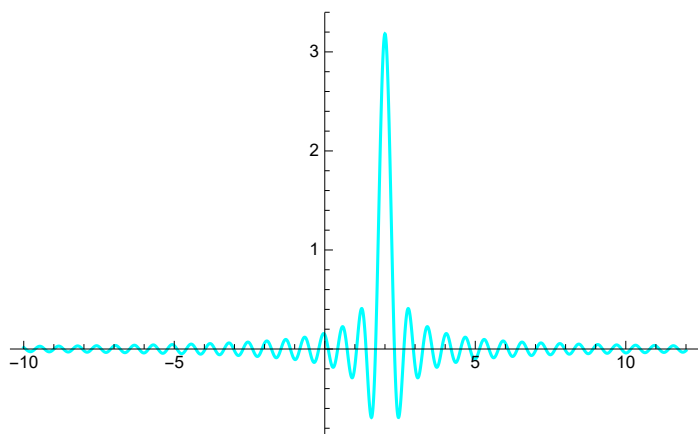


```

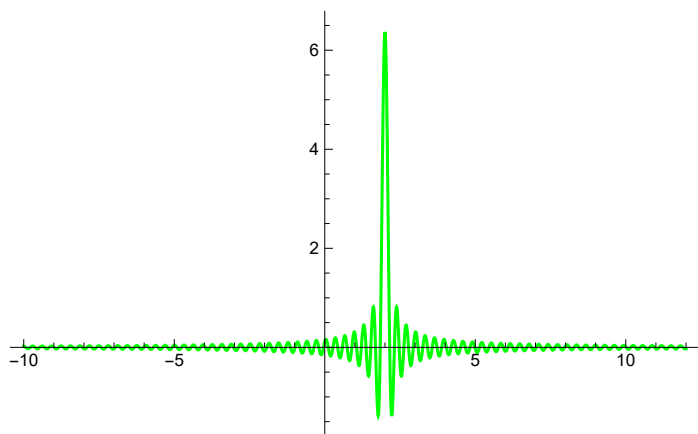
In[ ]:= s[x_, g_] := Sin[g (x - a)] / (π (x - a));
a = 2;
Plot[s[x, g = 10], {x, -10, 10 + a}, PlotRange → Full,
  PlotStyle → Cyan, PlotLegends → "Expressions"]
Plot[s[x, g = 20], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Green]
Plot[s[x, g = 50], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Blue]
Plot[s[x, g = 100], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Orange]
Plot[s[x, g = 1000], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Purple]
Plot[s[x, g = 1000], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Red]

```

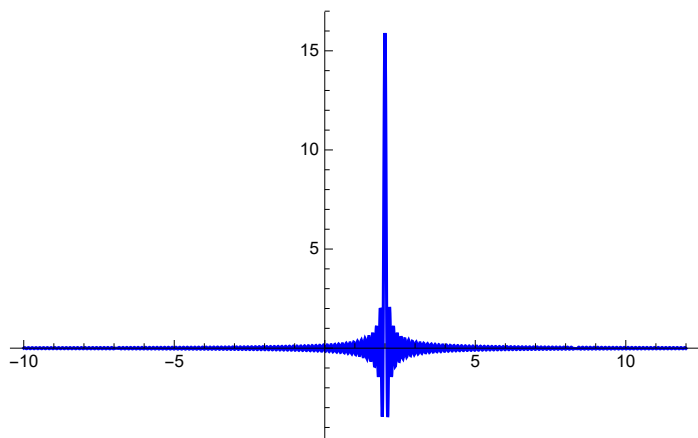
Out[ ]:=

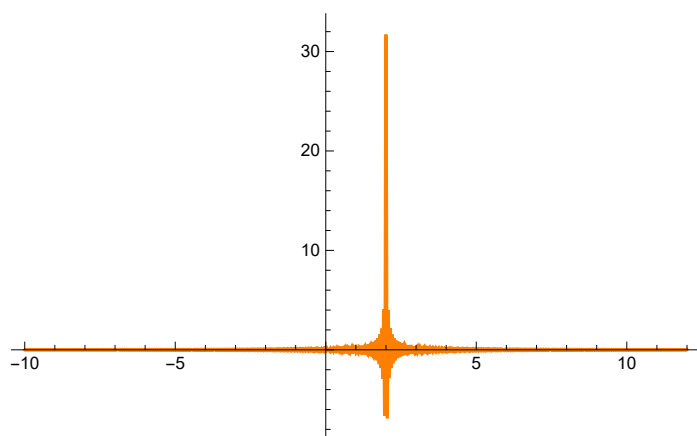
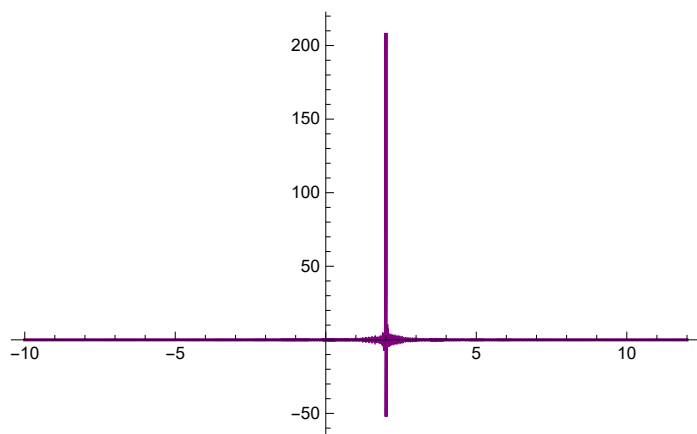
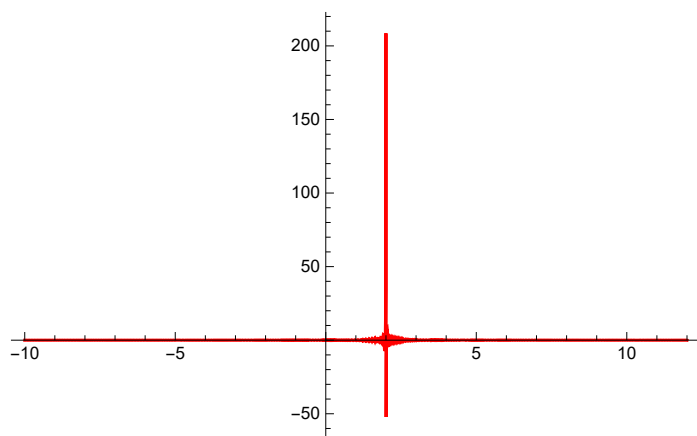


Out[ ]:=

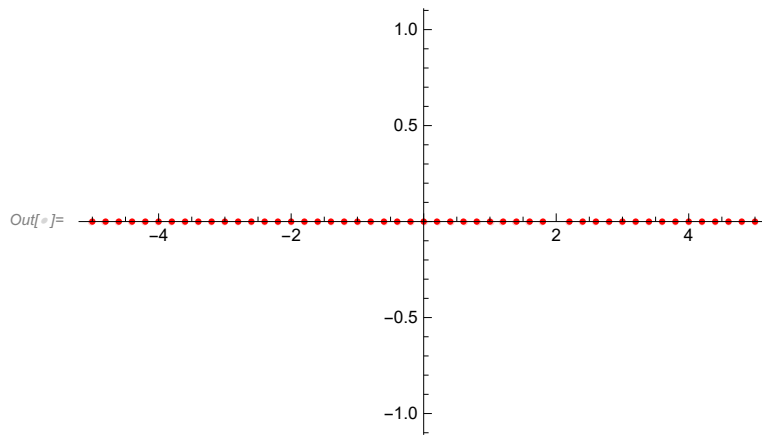


Out[ ]:=

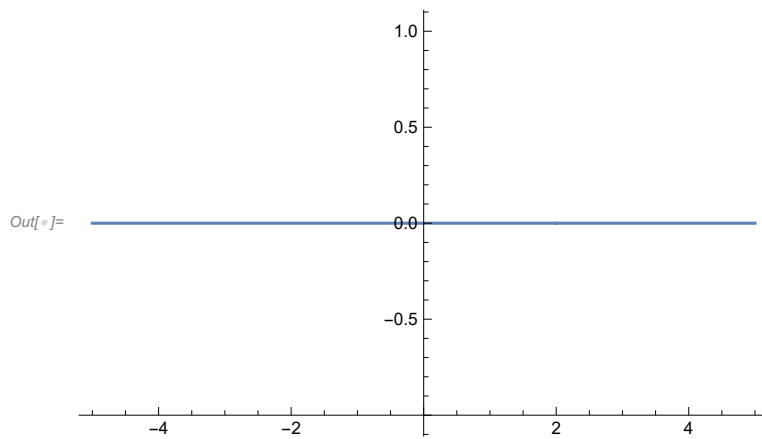


$Out[ ]=$  $Out[ ]=$  $Out[ ]=$ 

```
In[ ]:= f[x_] := DiracDelta[x - a];
a = 2;
DiscretePlot[f[x], {x, -5, 5, 0.2}, PlotStyle -> {Red, Thick}]
```



```
In[ ]:= f[x_] := DiracDelta[x - a];
a = 2;
Plot[f[x], {x, -5, 5}, AxesOrigin -> {0, -1}]
```

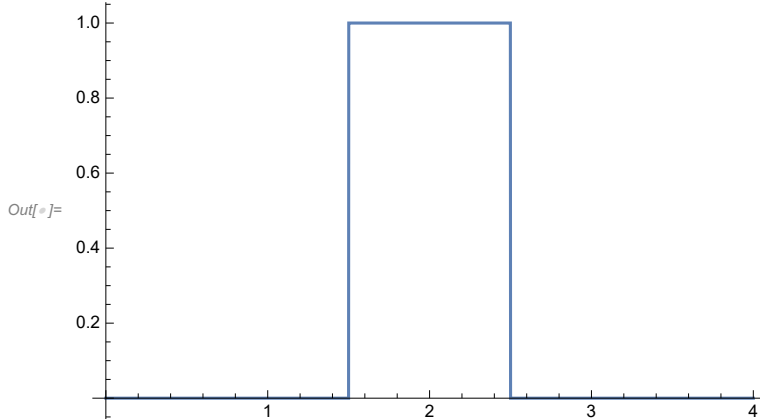
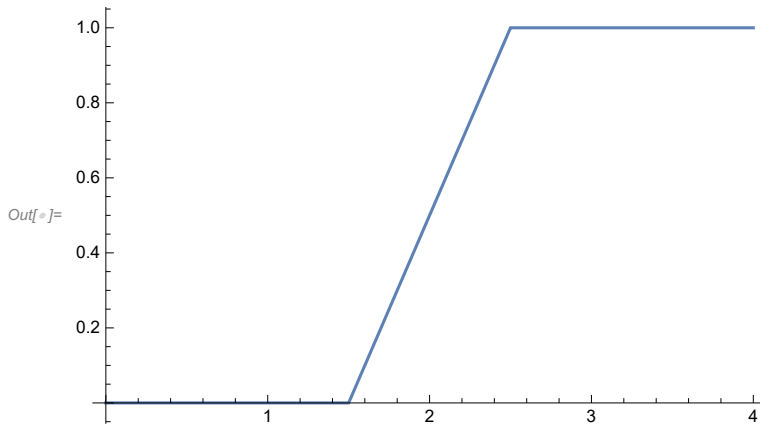


```
In[ ]:= Exit
```

```

In[ ]:= F[x_, σ_] := Piecewise[{{0, x < a - σ}, {{1/(2 σ)} (x - a + σ), -σ < x - a < σ}, {1, x > a + σ}}];
(*Ramp function*)
der[x_, σ_] =
  D[F[x, σ], x];
(*Derivative of Ramp function is Rectangular function R*)
a = 2;
Plot[F[x, σ = 0.5], {x, 0, 4}, PlotRange → Full]
Plot[der[x, σ = 0.5], {x, 0, 4}, PlotRange → Full]

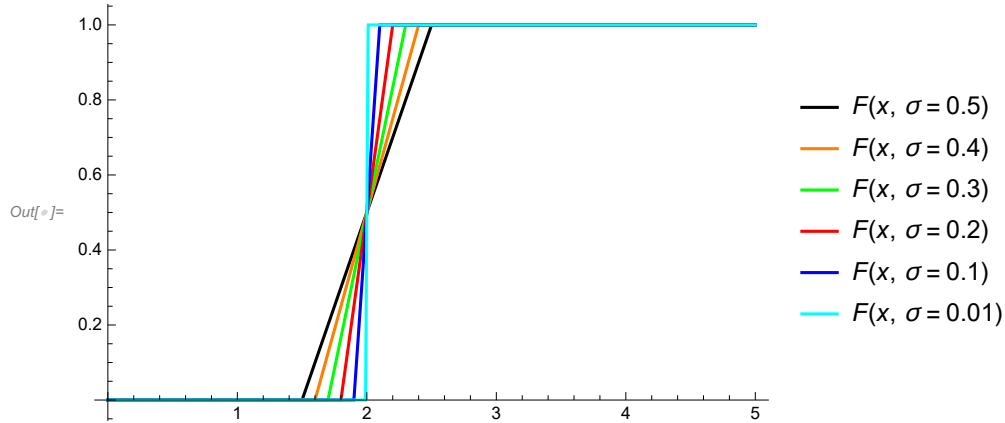
```



```

In[ ]:= F[x_, σ_] := Piecewise[{{0, x < a - σ}, {(1/(2 σ)) (x - a + σ), -σ < x - a < σ}, {1, x > a + σ}}];
a = 2;
Plot[{F[x, σ = 0.5], F[x, σ = 0.4], F[x, σ = 0.3],
      F[x, σ = 0.2], F[x, σ = 0.1], F[x, σ = 0.01]}, {x, 0, 5}, PlotRange → Full,
      PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]

```



```

In[ ]:= Exit

```

```

In[ ]:= F[x_, σ_] := Piecewise[{{0, x < a - σ}, {(1/(2 σ)) (x - a + σ), -σ < x - a < σ}, {1, x > a + σ}}];
H[x_, a_] =
  Limit[F[x, σ], σ → 0];
(*For limit σ→0 Ramp function becomes Heaviside unit step function*)
H[x, a]
delta[x_, a_] =
  D[H[x, a], x];
(*Derivative of discontinuous Heaviside unit step function is Dirac delta function*)
delta[x, a]
Integrate[delta[x, a = 2], {x, -5, 5}]

```

```

Out[ ]:= {
  1      a < x
  0      a > x
  Indeterminate True
}

```

```

Out[ ]:= {
  0      a - x < 0 || a - x > 0
  Indeterminate True
}

```

```

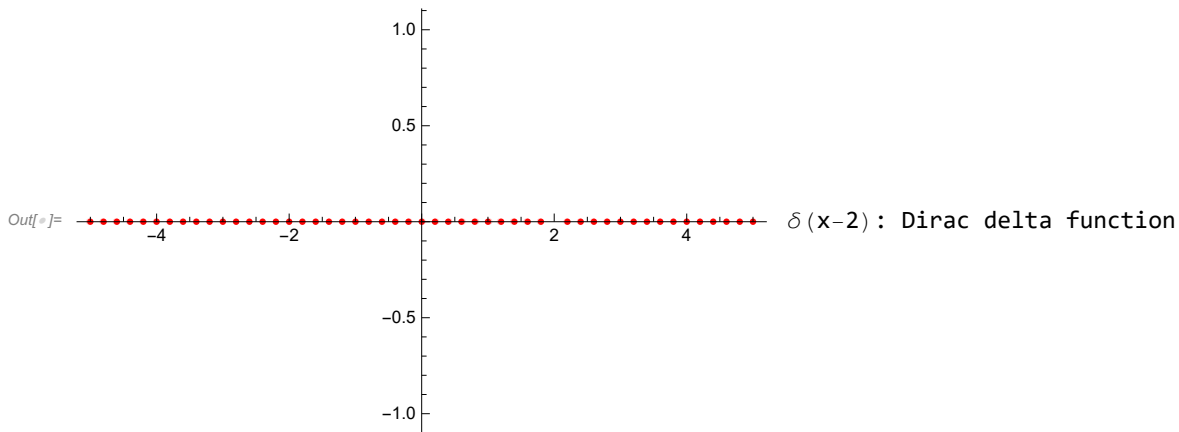
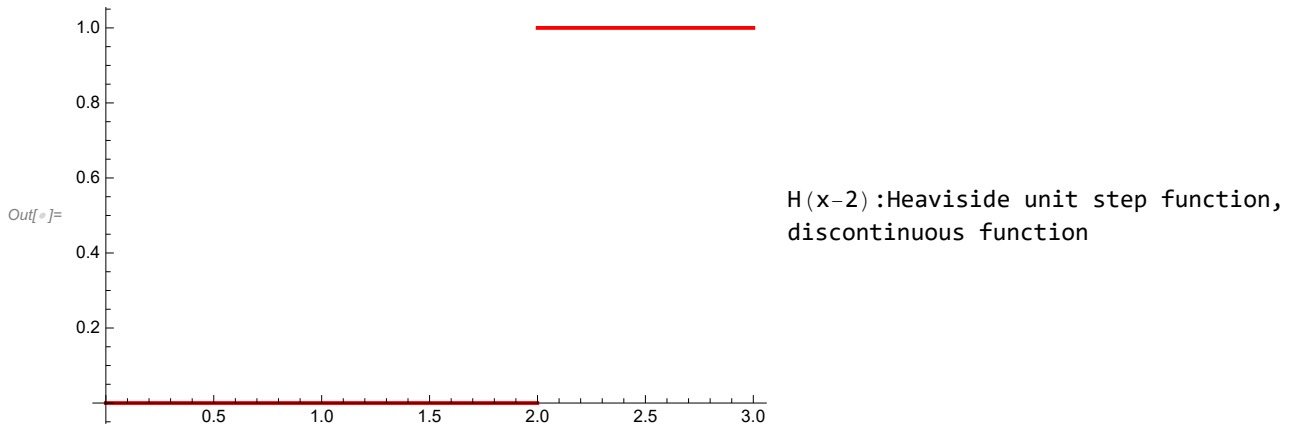
Out[ ]:= 0

```

```

In[ ]:= a = 2;
Plot[H[x, a], {x, 0, 3}, PlotStyle -> {Red, Thick},
PlotLegends -> "H(x-2):Heaviside unit step function,\ndiscontinuous function"]
DiscretePlot[delta[x, a], {x, -5, 5, 0.2}, PlotStyle -> {Red, Thick},
PlotLegends -> "\delta(x-2): Dirac delta function"]

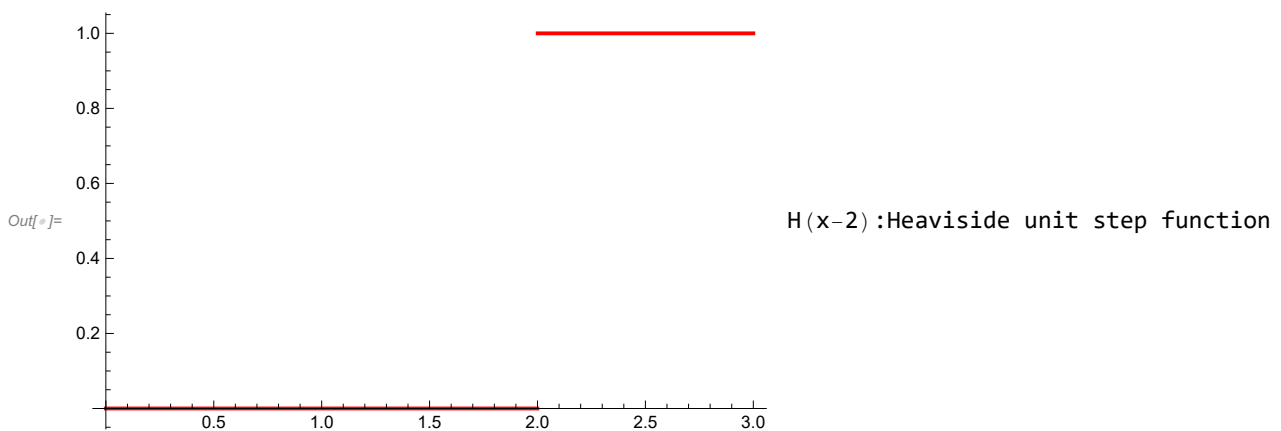
```



```

In[ ]:= Plot[UnitStep[x - 2], {x, 0, 3}, PlotStyle -> {Red, Thick},
PlotLegends -> "H(x-2):Heaviside unit step function"]

```



```

In[ ]:= Exit

```



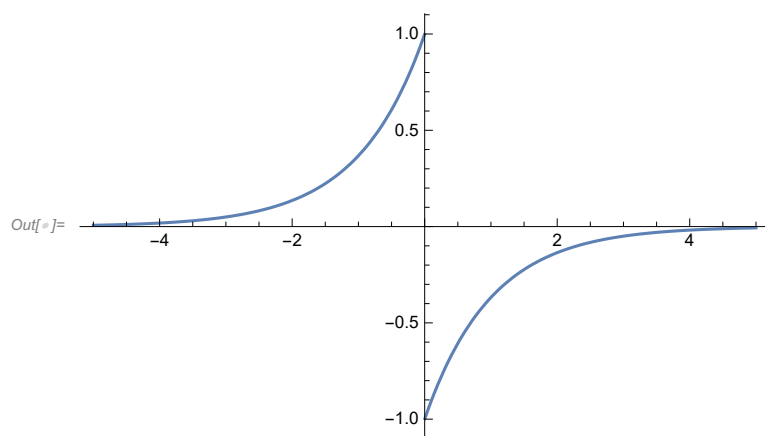
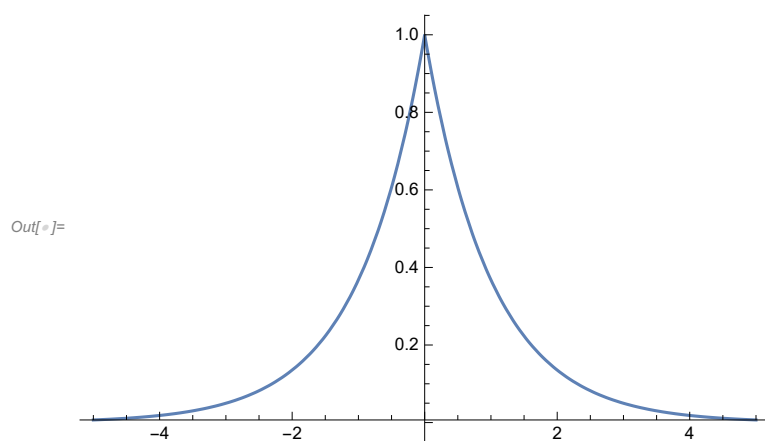
```

In[ ]:= (*psi[x_]:=Exp[-Abs[x]];*);
psi[x_] := Piecewise[{{Exp[-(x)], x >= 0}, {Exp[-(-x)], x < 0}}];
derpsi[x_] = D[psi[x], x]
derderpsi[x_] = D[derpsi[x], x]
Plot[psi[x], {x, -5, 5}, PlotRange -> Full]
Plot[derpsi[x], {x, -5, 5}, PlotRange -> Full]
Plot[derderpsi[x], {x, -5, 5}, PlotRange -> Full, PlotStyle -> {Red, Thick}]
    
```

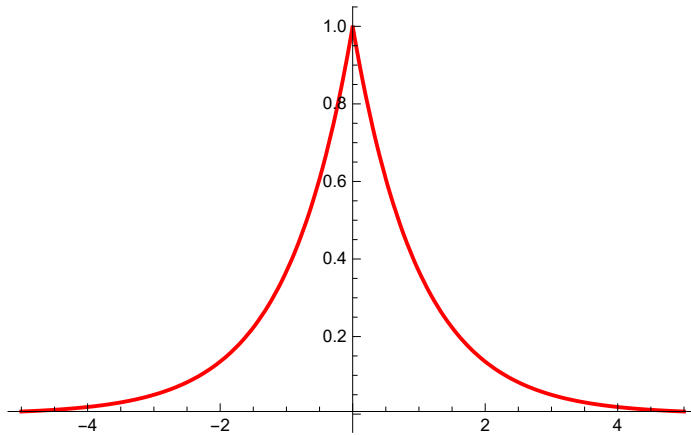
Out[ ]:=

$$\begin{cases} e^x & x < 0 \\ -e^{-x} & x > 0 \\ \text{Indeterminate} & \text{True} \end{cases}$$

Out[ ]:=

$$\begin{cases} e^x & x < 0 \\ e^{-x} & x > 0 \\ \text{Indeterminate} & \text{True} \end{cases}$$


Out[ ]=



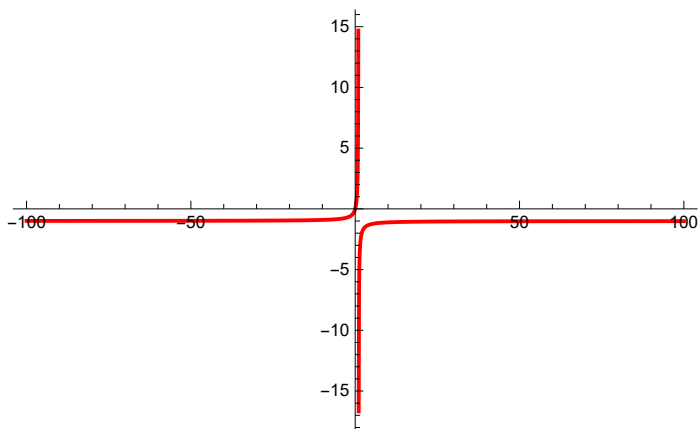
```

In[ ]:= f[z_] := z / (1 - z);
der2[z_] = D[f[z], z]
Plot[f[z], {z, -100, 100}, PlotRange -> Full, PlotStyle -> {Red, Thick}]
Plot[der2[z], {z, -100, 100}, PlotRange -> Full, PlotStyle -> {Red, Thick}]
Plot[der2[z], {z, -1, 1}, PlotRange -> Full, PlotStyle -> {Red, Thick}]
Plot[der2[z], {z, -0.1, 0.1}, PlotRange -> Full, PlotStyle -> {Red, Thick}]

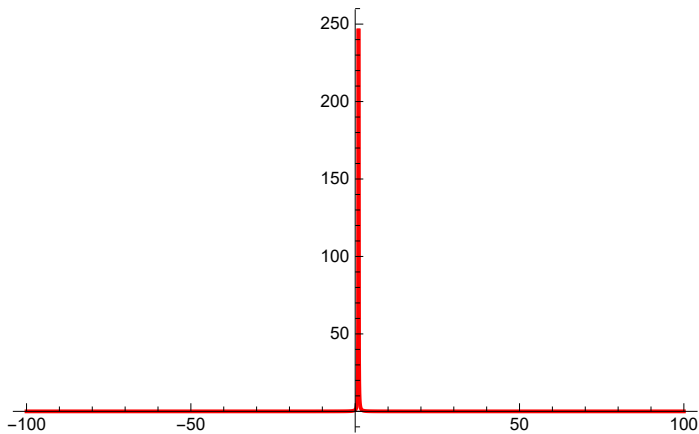
```

Out[ ]=  $\frac{1}{1-z} + \frac{z}{(1-z)^2}$

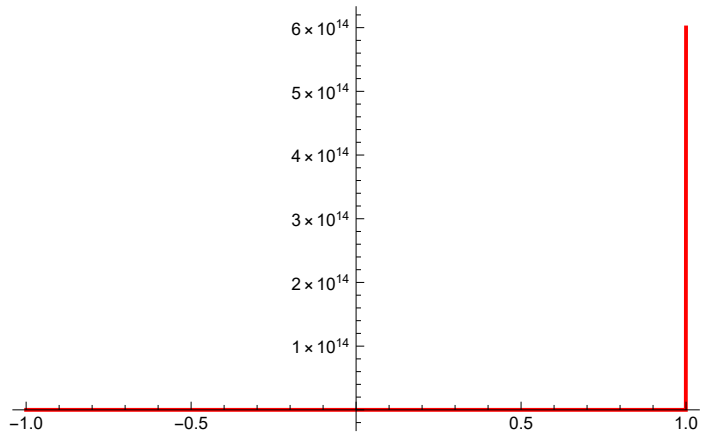
Out[ ]=



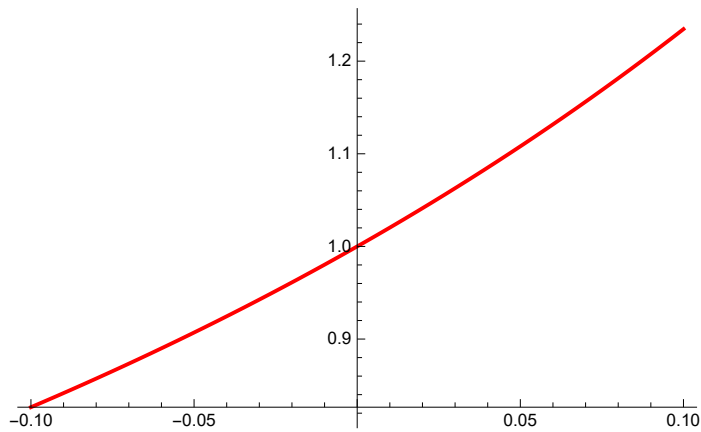
Out[ ]=



Out[ ]=



Out[ ]=



```

In[ ]:= h[x_] := Piecewise[{{-x, x < 0}, {x, x ≥ 0}}];
derh[x_] = D[h[x], x]
derderh[x_] = D[derh[x], x]
Plot[h[x], {x, -5, 5}]
Plot[derh[x], {x, -5, 5}, PlotRange → Full, PlotStyle → {Red, Thick}]
DiscretePlot[derderh[x], {x, -5, 5, 0.2},
  AxesOrigin → {0, -0.2}, PlotRange → Full, PlotStyle → {Red, Thick}]

```

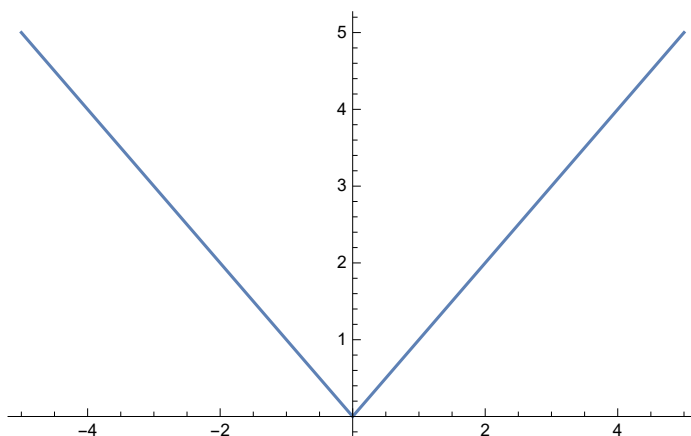
Out[ ]=

$$\begin{cases} -1 & x < 0 \\ 1 & x > 0 \\ \text{Indeterminate} & \text{True} \end{cases}$$

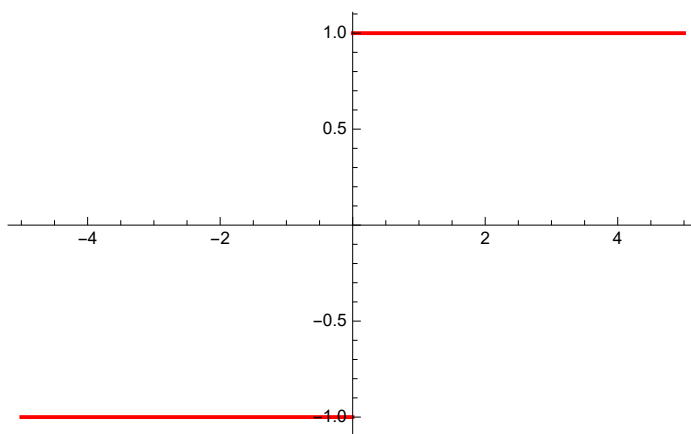
Out[ ]=

$$\begin{cases} 0 & x < 0 \mid x > 0 \\ \text{Indeterminate} & \text{True} \end{cases}$$

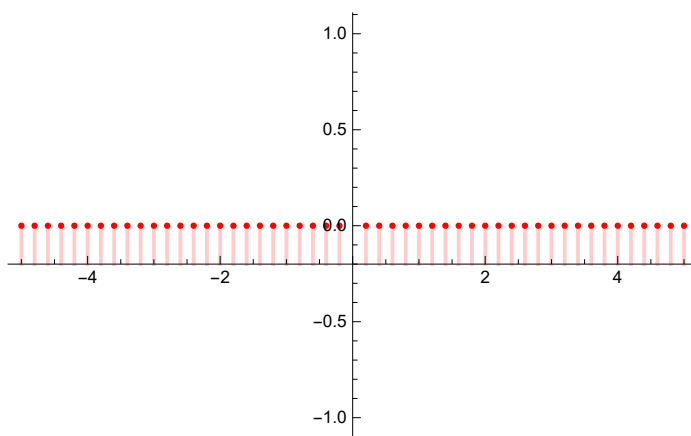
Out[ ]=



Out[ ]=



Out[ ]=



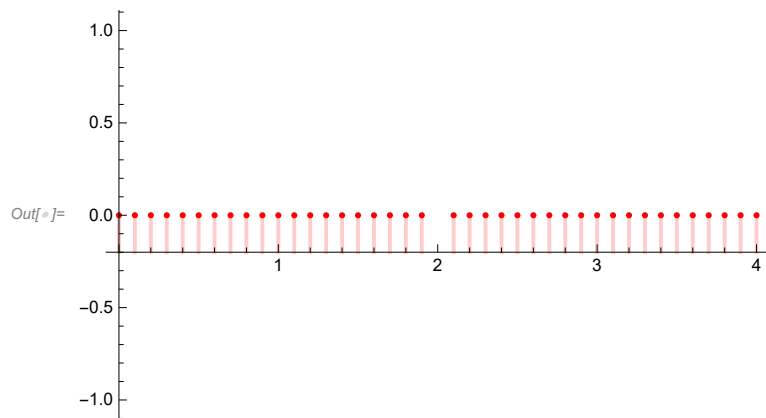
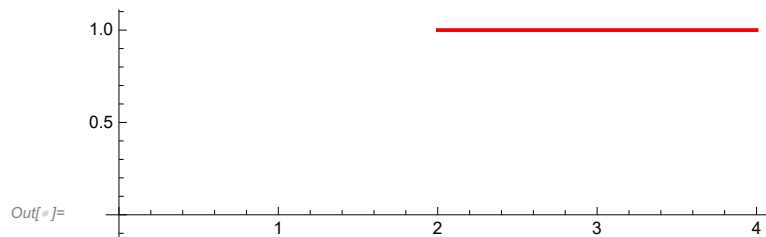
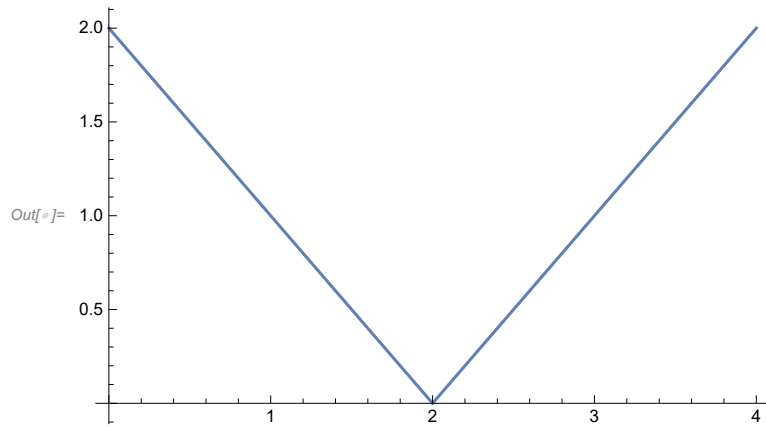
```

In[ ]:= h[x_] := Piecewise[{{-x + a, x < a}, {x - a, x ≥ a}}];
a = 2;
derh[x_] = D[h[x], x]
derderh[x_] = D[derh[x], x]
Plot[h[x], {x, 0, 4}]
Plot[derh[x], {x, 0, 4}, PlotRange → Full, PlotStyle → {Red, Thick}]
DiscretePlot[derderh[x], {x, 0, 4, 0.1},
  AxesOrigin → {0, -0.2}, PlotRange → Full, PlotStyle → {Red, Thick}]

```

Out[ ]:= 
$$\begin{cases} -1 & x < 2 \\ 1 & x > 2 \\ \text{Indeterminate} & \text{True} \end{cases}$$

Out[ ]:= 
$$\begin{cases} 0 & x < 2 \mid x > 2 \\ \text{Indeterminate} & \text{True} \end{cases}$$



In[ ]:= **Exit**

In[ ]:= **f[x\_] := x^2;**  
**Integrate[f[x] × DiracDelta[x - 2], {x, -5, 5}]**

Out[ ]:= **4**

```

In[ ]:= G3[x_, y_, z_] :=
  (1 / ((σ^3) * (2 Pi)^(3/2))) Exp[-((x - a)^2 + (y - b)^2 + (z - x)^2) / (2 σ^2)];
a = 2; b = 3; c = 2, σ = 0.1;
ContourPlot3D[G3[x, y, z], {x, 0, 5},
  {y, 0, 5}, {z, 0, 5}, PlotPoints → 100, PlotRange → Full]

f[x_] := Piecewise[{{-x, x < 0}, {x, x > 0}}];
(* f(x) = |x| , not differentiable at x=0 *)
g[x_] := Piecewise[{{x, x < 0}, {-x, x > 0}}];
(* g(x) = -|x| , not differentiable at x=0 *)
h[x_] := f[x] + g[x]; (* h(x) = f(x) + g(x) , differentiable at x=0 *)

point = 0; (* Check differentiability by changing the point *)

D[f[x], x] /. x → point
D[g[x], x] /. x → point
D[h[x], x] /. x → point

(*Simplify[h[x]]*)
(*leftDeriv=Limit[(h[x]-h[point])/(x-point), x→0,Direction→-1];
rightDeriv=Limit[(h[x]-h[point])/(x-point), x→0,Direction→1];
If[leftDeriv===rightDeriv, "Differentiable", "Not Differentiable"]*)

(* Defining left derivative and right derivative *)
leftDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → -1];
rightDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → 1];

If[leftDeriv[f, x, point] === rightDeriv[f, x, point],
  "Differentiable", "Not Differentiable"]
If[leftDeriv[g, x, point] === rightDeriv[g, x, point],
  "Differentiable", "Not Differentiable"]
If[leftDeriv[h, x, point] === rightDeriv[h, x, point],
  "Differentiable", "Not Differentiable"]

(* (f(x) + g(x)) is a horizontal line through x-
  axis and differentiable at all points including x=
  0. Its derivative at each point is 0 *)
Plot[{f[x], g[x], (f[x] + g[x])}, {x, -3, 3},
  PlotStyle → {Directive[Green, Thick], Directive[Orange, Thick], Directive[Red, Thick]}]

Out[ ]:= Indeterminate

Out[ ]:= Indeterminate

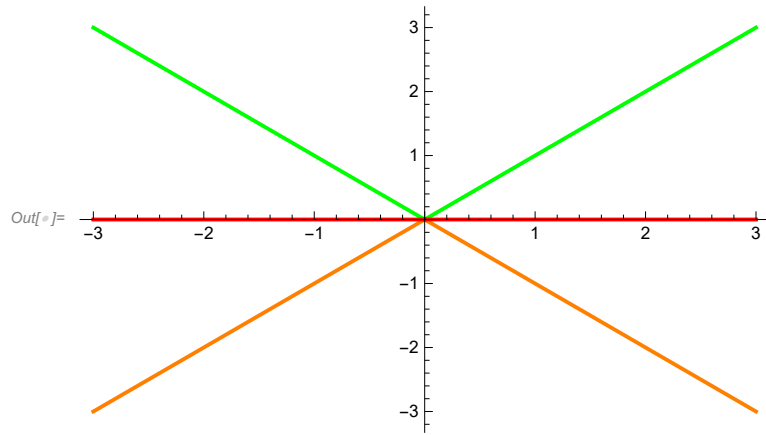
Out[ ]:= Indeterminate

Out[ ]:= Not Differentiable

Out[ ]:= Not Differentiable

Out[ ]:= Differentiable

```



```

f[x_] := Piecewise[{{-(x - 1), x < 1}, {(x - 1), x > 1}}]; (* f(x)=|x-1|,
decreasing below x=1 nad increasing above x>1. Not differentiable at x=1 *)
g[x_] := Piecewise[{{-(x - 5), x < 5}, {(x - 5), x > 5}}]; (* g(x)=|x-5|,
decreasing below x=5 nad increasing above x>5. Not differentiable at x=5 *)
h[x_] := f[x] + g[x];      (* h(x)=f(x)+g(x) *)

leftDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → -1];
rightDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → 1];

Check[r_, x_, p_] :=
  If[leftDeriv[r, x, p] === rightDeriv[r, x, p], "Differentiable", "Not Differentiable"];
(*Check[k_, x_, p_] := Module[{ld, rd},
  ld=leftDeriv[k, x, p];
  rd=rightDeriv[k, x, p];
  If[NumericQ[ld]&&NumericQ[rd]&&ld==rd, "Differentiable", "Not Differentiable"]];*)

(*If[leftDeriv[f, x, 1]===rightDeriv[f, x, 1], "Differentiable", "Not Differentiable"]
If[leftDeriv[g, x, 5]===rightDeriv[g, x, 5], "Differentiable", "Not Differentiable"]
If[leftDeriv[h, x, 1]===rightDeriv[h, x, 1], "Differentiable", "Not Differentiable"]
If[leftDeriv[h, x, 5]===rightDeriv[h, x, 5], "Differentiable", "Not Differentiable"]
If[leftDeriv[h, x, 3]===rightDeriv[h, x, 3], "Differentiable", "Not Differentiable"]*)

(*Check[h, x, 3] *)

D[f[x], x] /. x → 1
D[g[x], x] /. x → 5

D[f[x], x] /. x → 3
D[g[x], x] /. x → 3
D[h[x], x] /. x → 3
D[h[x], x] /. x → 2
D[h[x], x] /. x → 4

(* f(x) + g(x) is a horizontal line (y=4) parallel to x-axis at 1<x<5,
decreasing below x=1,
and increasing above x>5 . Its derivative at each points in between 1<x<5 is 0. *)
Plot[{f[x], g[x], (f[x] + g[x])}, {x, -3, 9},
  PlotStyle → {Directive[Green, Thick], Directive[Orange, Thick], Directive[Red, Thick]}]

```

... **SetDelayed:** Tag Check in Check[r\_, x\_, p\_] is Protected.

... **Message:** Message name 3 is not of the form symbol::name or symbol::name::language.

Out[ ]= Check[h, x, 3]

Out[ ]= Indeterminate

Out[ ]= Indeterminate



Out[ ]= 1

Out[ ]= -1

Out[ ]= 0

Out[ ]= 0

Out[ ]= 0

