In[1]:=
```
f[x_] := (x^2 - 4) / (x - 2);
point = 2;
(* Defining left limit and right limit of the function *)
leftLim[k_, x_, p_] := Limit[k[x], x → p, Direction → -1];
rightLim[k_, x_, p_] := Limit[k[x], x → p, Direction → 1];


If[leftLim[f, x, point] === f[point], "Left Continuous", "Left Discontinuous"]
If[rightLim[f, x, point] === f[point], "Right Continuous", "Right Discontinuous"]
If[leftLim[f, x, point] === rightLim[f, x, point],
 "left limit = right limit", "left limit ≠ right limit"]
(* left limit = right limit but they ≠ f(2), so discontinuous *)
If[leftLim[f, x, point] === rightLim[f, x, point] === f[point],
 "Continuous", "Discontinuous"]

DiscretePlot[f[x], {x, 0, 4, 0.05},
 AxesOrigin → {0, 0}, PlotRange → Full, PlotStyle → {Red, Thick}]
```

... Power: Infinite expression $\frac{1}{0}$ encountered.

... Infinity: Indeterminate expression 0 ComplexInfinity encountered.

Out[5]= Left Discontinuous

... Power: Infinite expression $\frac{1}{0}$ encountered.

... Infinity: Indeterminate expression 0 ComplexInfinity encountered.

Out[6]= Right Discontinuous

Out[7]= left limit = right limit

... Power: Infinite expression $\frac{1}{0}$ encountered.

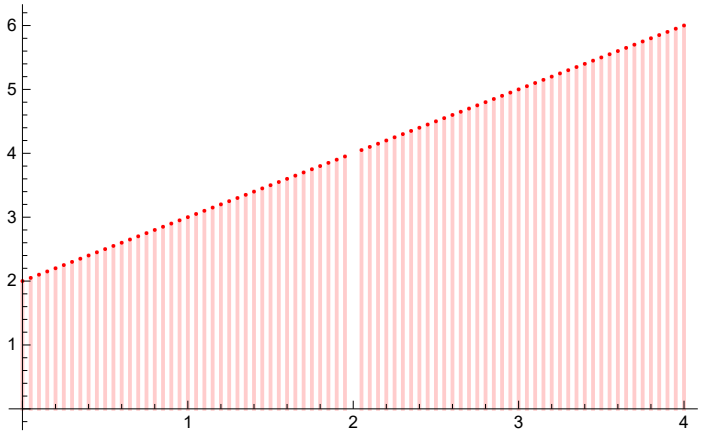
... Infinity: Indeterminate expression 0 ComplexInfinity encountered.

Out[8]= Discontinuous

... Power: Infinite expression $\frac{1}{0.}$ encountered.

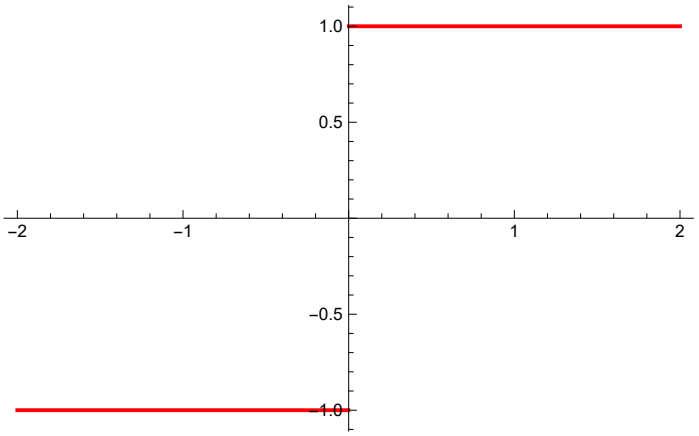
... Infinity: Indeterminate expression 0. ComplexInfinity encountered.

Out[9]=



In[10]:=
```
f[x_] := Piecewise[{{-1, x < 0}, {1, x > 0}, {0, x == 0}}];
Plot[f[x], {x, -2, 2}, PlotStyle → {Red, Thick}]
point = 0;
(* Defining left limit and right limit of the function *)
leftLim[k_, x_, p_] := Limit[k[x], x → p, Direction → -1];
rightLim[k_, x_, p_] := Limit[k[x], x → p, Direction → 1];

If[leftLim[f, x, point] === f[point], "Left Continuous", "Left Discontinuous"]
If[rightLim[f, x, point] === f[point], "Right Continuous", "Right Discontinuous"]
If[leftLim[f, x, point] === rightLim[f, x, point],
 "left limit = right limit", "left limit ≠ right limit"]
If[leftLim[f, x, point] === rightLim[f, x, point] === f[point],
 "Continuous", "Discontinuous"]
DiscretePlot[f[x], {x, 0 - 2, 2, 0.02}, AxesOrigin → {0, 0},
 PlotRange → Full, PlotStyle → {Red, Thick}]
```

Out[11]=



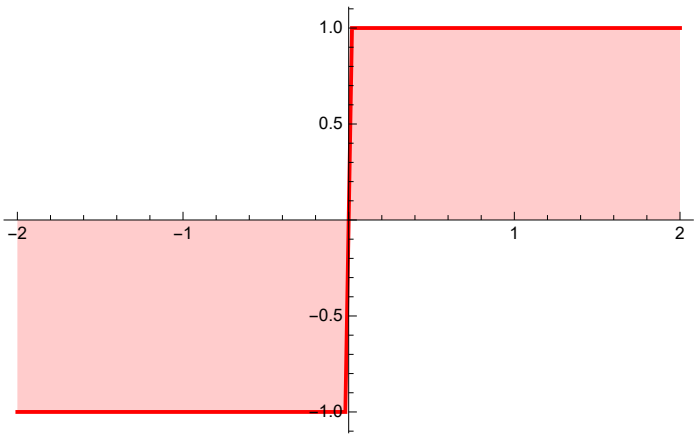
Out[15]= Left Discontinuous

Out[16]= Right Discontinuous

Out[17]= left limit ≠ right limit

Out[18]= Discontinuous

Out[19]=



In[20]:=
```
f[x_] := Piecewise[{{-x, x < 0}, {x, x > 0}}];
(* f(x)=|x| , not differenetiable at x=0 *)
g[x_] := Piecewise[{{x, x < 0}, {-x, x > 0}}];
(* g(x)=-|x| , not differentiable at x=0 *)
h[x_] := f[x] + g[x];     (* h(x)=f(x)+g(x) , differenetiable at x=0 *)

point = 0;    (* Check differentiability by changing the point *)

(* D[f[x],x]/.x→point
     D[g[x],x]/.x→point
    D[h[x],x]/.x→point *)

(*Simplify[h[x]]*)
(*leftDeriv=Limit[(h[x]-h[point])/(x-point), x→0,Direction→-1];
rightDeriv=Limit[(h[x]-h[point])/(x-point), x→0,Direction→1];
If[leftDeriv===rightDeriv, "Differentiable", "Not Differentiable"]*)

(* Defining left limit and right limit of the function *)
leftLim[k_, x_, p_] := Limit[k[x], x → p, Direction → -1];
rightLim[k_, x_, p_] := Limit[k[x], x → p, Direction → 1];

(* Defining left derivative and right derivative *)
leftDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → -1];
rightDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → 1];

If[leftLim[f, x, point] === f[point], "Left Continuous", "Left Discontinuous"]
If[rightLim[f, x, point] === f[point], "Right Continuous", "Right Discontinuous"]
If[leftLim[f, x, point] === rightLim[f, x, point],
 "left limit = right limit", "left limit ≠ right limit"]

If[leftLim[f, x, point] === rightLim[f, x, point] === f[point],
 "Continuous", "Discontinuous"]
If[leftLim[g, x, point] === rightLim[g, x, point] === g[point],
 "Continuous", "Discontinuous"]
If[leftLim[h, x, point] === rightLim[h, x, point] === h[point],
 "Continuous", "Discontinuous"]

If[leftDeriv[f, x, point] === rightDeriv[f, x, point], "Differentiable",
 "Not Differentiable"] (* continuous but has notch at x=0 *)
If[leftDeriv[g, x, point] === rightDeriv[g, x, point], "Differentiable",
 "Not Differentiable"] (* continuous but has notch at x=0 *)
If[leftDeriv[h, x, point] === rightDeriv[h, x, point], "Differentiable",
 "Not Differentiable"] (* continuous and has no notch at x=0 *)

(* (f(x) + g(x)) is a horizontal line through x-
  axis and differentiable at all points including x=
 0. Its derivative at each point is 0 *)
Plot[{f[x], g[x], (f[x] + g[x])}, {x, -3, 3},
 PlotStyle → {Directive[Green, Thick], Directive[Orange, Thick], Directive[Red, Thick]}]
```

Out[28]= Left Continuous

Out[29]= Right Continuous

Out[30]= left limit = right limit

Out[31]= Continuous

Out[32]= Continuous

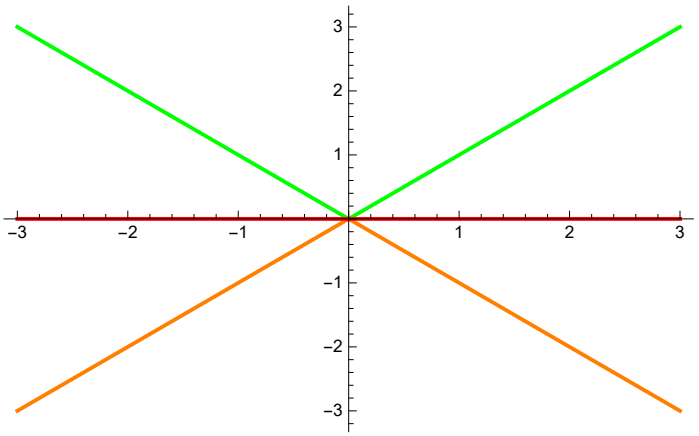
Out[33]= Continuous

Out[34]= Not Differentiable

Out[35]= Not Differentiable

Out[36]= Differentiable

Out[37]=



In[38]:=
```
f[x_] := Piecewise[{{-(x - 1), x < 1}, {(x - 1), x > 1}}]; (* f(x)=|x-1|,
decreasing below x=1 nad increasing above x>1. Not differentiable at x=1 *)
g[x_] := Piecewise[{{-(x - 5), x < 5}, {(x - 5), x > 5}}]; (* g(x)=|x-5|,
decreasing below x=5 nad increasing above x>5. Not differentiable at x=5 *)
h[x_] := f[x] + g[x];      (* h(x)=f(x)+g(x) *)

leftDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → -1];
rightDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → 1];

Check[r_, x_, p_] :=
  If[leftDeriv[r, x, p] === rightDeriv[r, x, p], "Differentiable", "Not Differentiable"];
(*Check[k_,x_,p_]:=Module[{ld,rd},
   ld=leftDeriv[k,x,p];
   rd=rightDeriv[k,x,p];
   If[NumericQ[ld]&&NumericQ[rd]&&ld==rd,"Differentiable","Not Differentiable"]];*)


(*If[leftDeriv[f,x,1]===rightDeriv[f,x,1], "Differentiable", "Not Differentiable"]
 If[leftDeriv[g,x,5]===rightDeriv[g,x,5], "Differentiable", "Not Differentiable"]
 If[leftDeriv[h,x,1]===rightDeriv[h,x,1], "Differentiable", "Not Differentiable"]
 If[leftDeriv[h,x,5]===rightDeriv[h,x,5], "Differentiable",  "Not Differentiable"]
 If[leftDeriv[h,x,3]===rightDeriv[h,x,3], "Differentiable", "Not Differentiable"]*)

(*Check[h,x,3]*)

D[f[x], x] /. x → 1
D[g[x], x] /. x → 5

D[f[x], x] /. x → 3
D[g[x], x] /. x → 3
D[h[x], x] /. x → 3
D[h[x], x] /. x → 2
D[h[x], x] /. x → 4

(* f(x) + g(x) is a horizontal line (y=4) parallel to x-axis at 1<x<5,
decreasing below x=1,
and increasing above x>5 . Its derivative at each points in between 1<x<5 is 0. *)
Plot[{f[x], g[x], (f[x] + g[x])}, {x, -3, 9},
 PlotStyle → {Directive[Green, Thick], Directive[Orange, Thick], Directive[Red, Thick]}]
```

... SetDelayed: Tag Check in Check[r_, x_, p_] is Protected.

Out[44]= Indeterminate

Out[45]= Indeterminate

Out[46]= 1

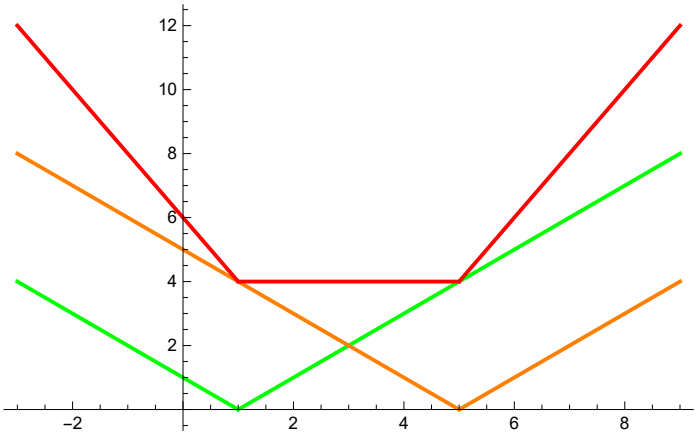
Out[47]= -1

Out[48]= 0

Out[49]= 0

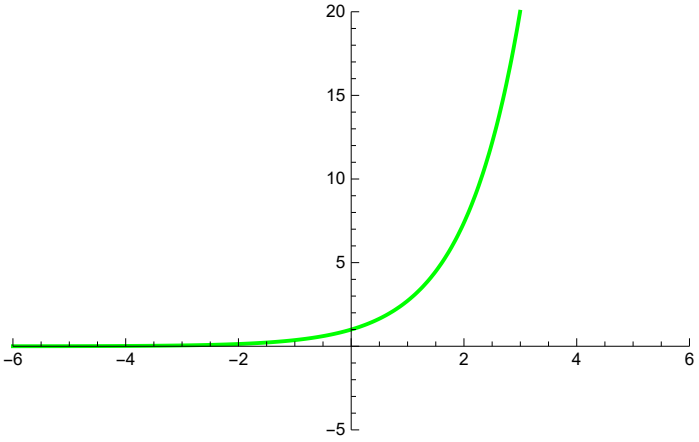
Out[50]= 0

Out[51]=

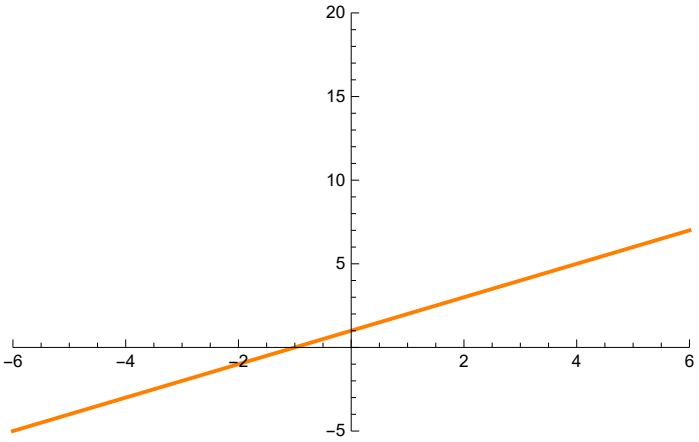


In[52]:=
```
f[x_] := Piecewise[{{Exp[x], x < 0}, {(x + 1), x ≥ 0}}];
(* continuous and has no notch at x=0 *)

Plot[Exp[x], {x, -6, 6}, PlotStyle → Directive[Green, Thick],
 PlotRange → {{-6, 6}, {-5, 20}}]
Plot[ x + 1, {x, -6, 6}, PlotStyle → Directive[Orange, Thick],
 PlotRange → {{-6, 6}, {-5, 20}}]
Plot[f[x], {x, -6, 6}, PlotStyle → Directive[Red, Thick], PlotRange → {{-6, 6}, {-5, 20}}]
Plot[{Exp[x], x + 1, f[x]}, {x, -6, 6},
 PlotStyle → {Directive[Green, Thick], Directive[Orange, Thick], Directive[Red, Thick]},
 PlotRange → {{-6, 6}, {-5, 20}}]

point = 0;

leftLim[k_, x_, p_] := Limit[k[x], x → p, Direction → -1];
rightLim[k_, x_, p_] := Limit[k[x], x → p, Direction → 1];

leftDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → -1];
rightDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → 1];

If[leftLim[f, x, point] === f[point], "Left Continuous", "Left Discontinuous"]
If[rightLim[f, x, point] === f[point], "Right Continuous", "Right Discontinuous"]
If[leftLim[f, x, point] === rightLim[f, x, point],
 "left limit = right limit", "left limit ≠ right limit"]
If[leftLim[f, x, point] === rightLim[f, x, point] === f[point],
 "Continuous", "Discontinuous"]
If[leftDeriv[f, x, point] === rightDeriv[f, x, point],
 "Differentiable", "Not Differentiable"]
```

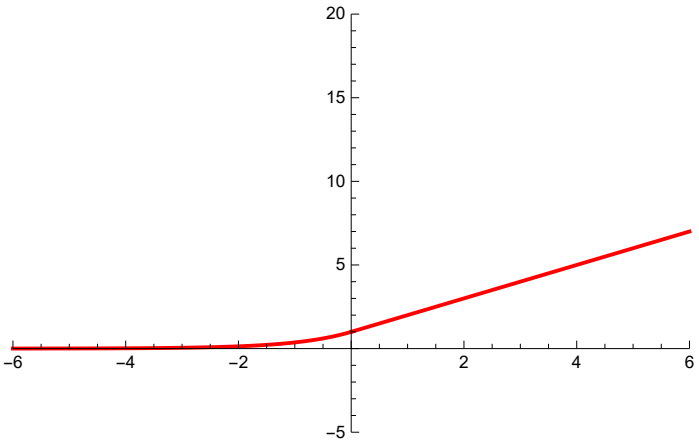
Out[53]=



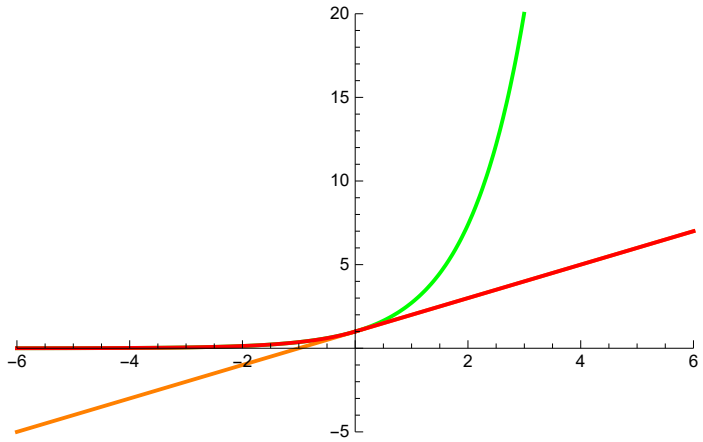
Out[54]=



Out[55]=



Out[56]=



Out[62]= Left Continuous

Out[63]= Right Continuous

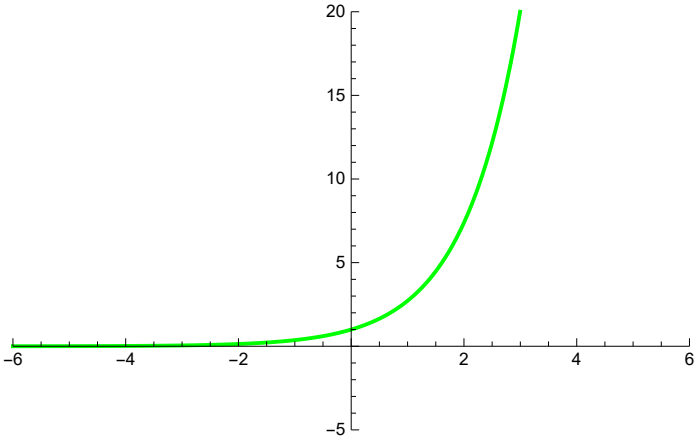
Out[64]= left limit = right limit

Out[65]= Continuous

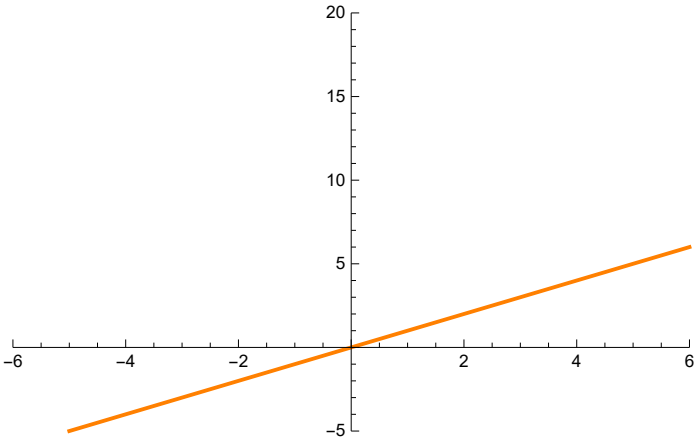
Out[66]= Differentiable

In[67]:=
```
f[x_] := Piecewise[{{Exp[x], x < 0}, {(x), x ≥ 0}}];      (* discontinuous at x=0 *)

Plot[Exp[x], {x, -6, 6},
 PlotStyle → Directive[Green, Thick], PlotRange → {{-6, 6}, {-5, 20}}]
Plot[ x, {x, -6, 6}, PlotStyle → Directive[Orange, Thick], PlotRange → {{-6, 6}, {-5, 20}}]
Plot[f[x], {x, -6, 6}, PlotStyle → Directive[Red, Thick],
 PlotRange → {{-6, 6}, {-5, 20}}]
Plot[{Exp[x], x, f[x]}, {x, -6, 6},
 PlotStyle → {Directive[Green, Thick], Directive[Orange, Thick], Directive[Red, Thick]},
 PlotRange → {{-6, 6}, {-5, 20}}]

point = 0;

leftLim[k_, x_, p_] := Limit[k[x], x → p, Direction → -1];
rightLim[k_, x_, p_] := Limit[k[x], x → p, Direction → 1];

leftDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → -1];
rightDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → 1];

If[leftLim[f, x, point] === f[point], "Left Continuous", "Left Discontinuous"]
If[rightLim[f, x, point] === f[point], "Right Continuous", "Right Discontinuous"]
If[leftLim[f, x, point] === rightLim[f, x, point],
 "left limit = right limit", "left limit ≠ right limit"]
If[leftLim[f, x, point] === rightLim[f, x, point] === f[point],
 "Continuous", "Discontinuous"]
If[leftDeriv[f, x, point] === rightDeriv[f, x, point],
 "Differentiable", "Not Differentiable"]
```

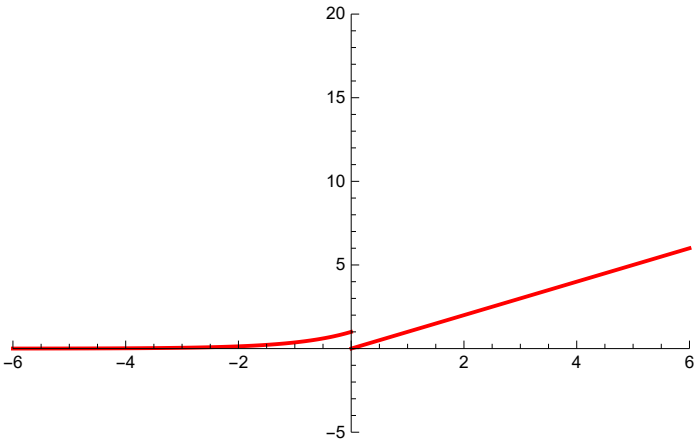
Out[68]=



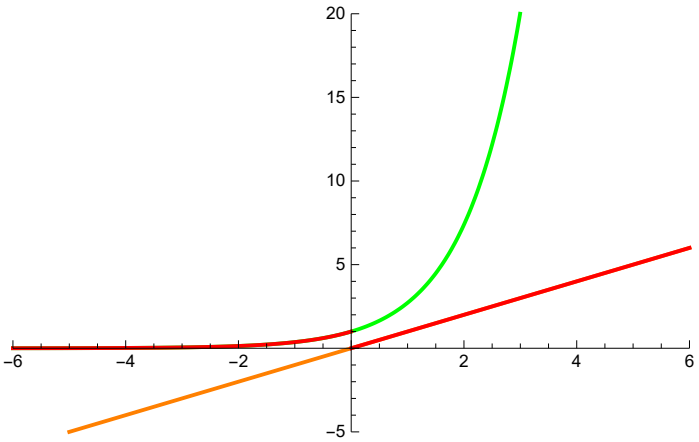
Out[69]=



Out[70]=



Out[71]=



Out[77]= Left Continuous

Out[78]= Right Discontinuous

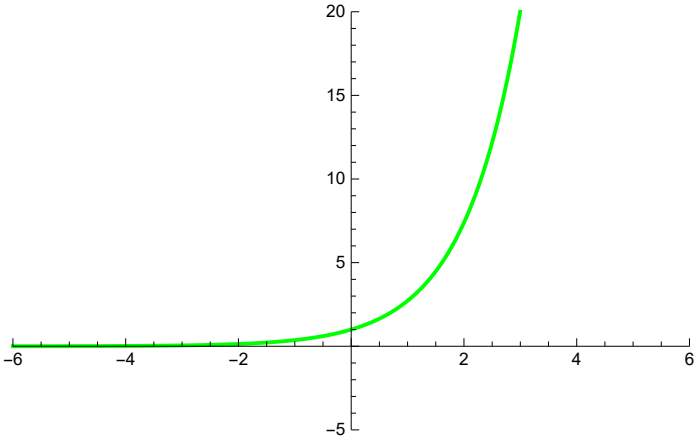
Out[79]= left limit ≠ right limit

Out[80]= Discontinuous

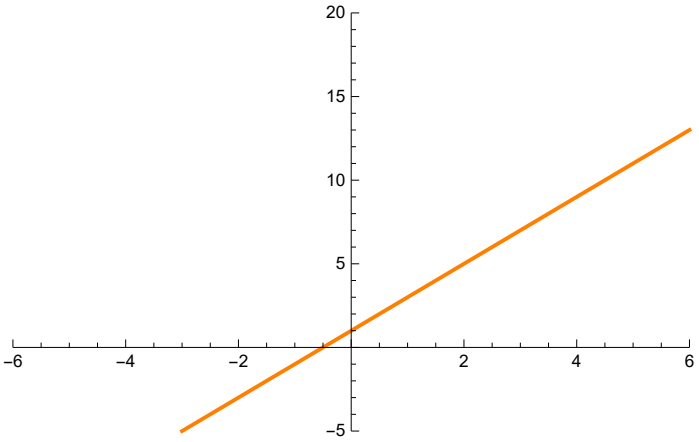
Out[81]= Not Differentiable

In[82]:=
```
f[x_] := Piecewise[{{Exp[x], x < 0}, {(2 x + 1), x ≥ 0}}];
(* continuous but has notch at x=0 *)

Plot[Exp[x], {x, -6, 6},
 PlotStyle → Directive[Green, Thick], PlotRange → {{-6, 6}, {-5, 20}}]
Plot[2 x + 1, {x, -6, 6}, PlotStyle → Directive[Orange, Thick],
 PlotRange → {{-6, 6}, {-5, 20}}]
Plot[f[x], {x, -6, 6}, PlotStyle → Directive[Red, Thick], PlotRange → {{-6, 6}, {-5, 20}}]
Plot[{Exp[x], 2 x + 1, f[x]}, {x, -6, 6},
 PlotStyle → {Directive[Green, Thick], Directive[Orange, Thick], Directive[Red, Thick]},
 PlotRange → {{-6, 6}, {-5, 20}}]

point = 0;

leftLim[k_, x_, p_] := Limit[k[x], x → p, Direction → -1];
rightLim[k_, x_, p_] := Limit[k[x], x → p, Direction → 1];

leftDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → -1];
rightDeriv[k_, x_, p_] := Limit[(k[x] - k[p]) / (x - p), x → p, Direction → 1];

If[leftLim[f, x, point] === f[point], "Left Continuous", "Left Discontinuous"]
If[rightLim[f, x, point] === f[point], "Right Continuous", "Right Discontinuous"]
If[leftLim[f, x, point] === rightLim[f, x, point],
 "left limit = right limit", "left limit ≠ right limit"]
If[leftLim[f, x, point] === rightLim[f, x, point] === f[point],
 "Continuous", "Discontinuous"]
If[leftDeriv[f, x, point] === rightDeriv[f, x, point],
 "Differentiable", "Not Differentiable"]
```

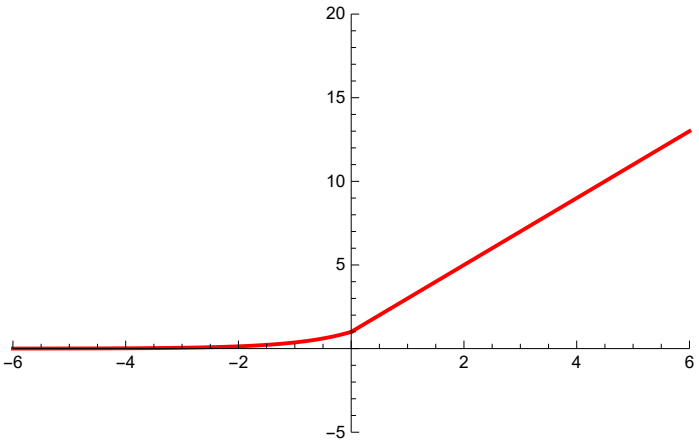
Out[83]=



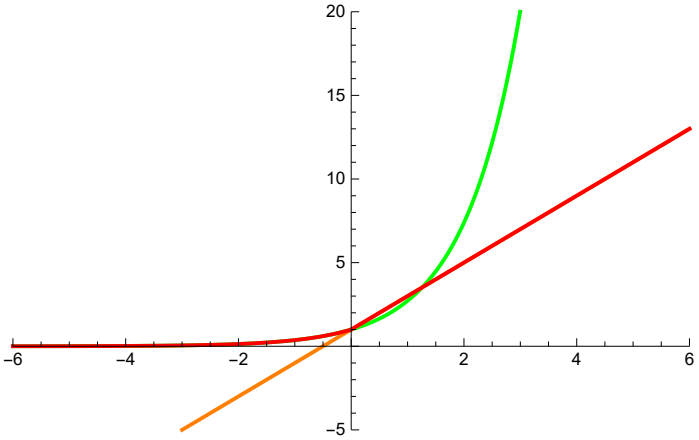
Out[84]=



Out[85]=



Out[86]=



Out[92]= Left Continuous

Out[93]= Right Continuous

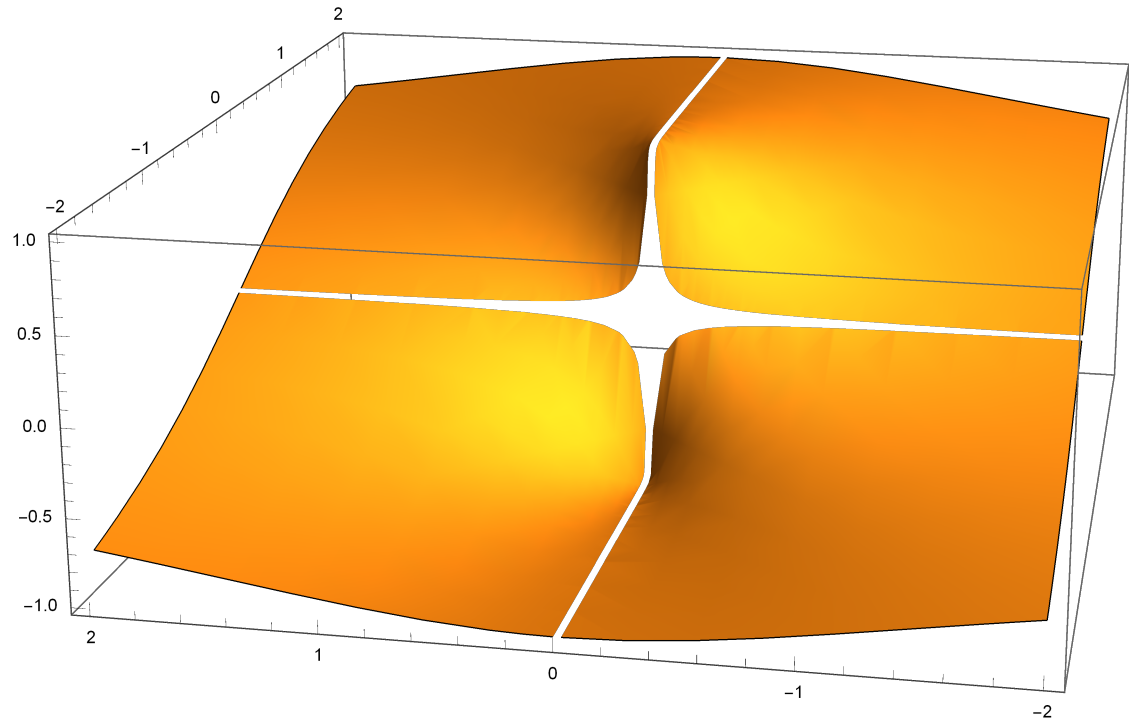
Out[94]= left limit = right limit

Out[95]= Continuous

Out[96]= Not Differentiable

In[97]:=
```
f[x_, y_] := Piecewise[{{x / √(x^2 + y^2), {x, y} ≠ {0, 0}}, {0, {x, y} === 0}}];
Plot3D[f[x, y], {x, -2, 2}, {y, -2, 2}, Mesh → None]
(* DensityPlot[f[x,y],{x,-2,2},{y,-2,2}] *)
(* ContourPlot[f[x,y],{x,-2,2},{y,-2,2}] *)
point = {0, 0};
(* Defining left limit and right limit of the function *)
Lim1[k_, x_, y_, p_] := Limit[k[x, y], {x, y} → p, Direction → {1, 1}];
(* Approaching from first first quadrant *)
Lim2[k_, x_, y_, p_] := Limit[k[x, y], {x, y} → p, Direction → {-1, 1}];
(* Approaching from second first quadrant *)
Lim3[k_, x_, y_, p_] := Limit[k[x, y], {x, y} → p, Direction → {-1, -1}];
(* Approaching from third first quadrant *)
Lim4[k_, x_, y_, p_] := Limit[k[x, y], {x, y} → p, Direction → {1, -1}];
(* Approaching from fourth first quadrant *)


If[Lim1[f, x, y, point] === f[point],
 "Right-Above Continuous", "Right-Above Discontinuous"]
If[Lim2[f, x, y, point] === f[point],
 "Left-Above Continuous", "Left-Above Discontinuous"]
If[Lim3[f, x, y, point] === f[point],
 "Left-Below Continuous", "Left-Below Discontinuous"]
If[Lim4[f, x, y, point] === f[point], "Right-Below Continuous",
 "Right-Below Discontinuous"]

If[Lim1[f, x, y, point] === Lim2[f, x, y, point],
 "Right-Above limit = Left-Above limit", "Right-Above limit ≠ Left-Above limit"]
If[Lim1[f, x, y, point] === Lim3[f, x, y, point],
 "Right-Above limit = Left-Below limit", "Right-Above limit ≠ Left-Below limit"]
If[Lim1[f, x, y, point] === Lim4[f, x, y, point],
 "Right-Above limit = Right-Below limit", "Right-Above limit ≠ Right-Below limit"]
If[Lim2[f, x, y, point] === Lim3[f, x, y, point],
 "Left-Above limit = Left-Below limit", "Left-Above limit ≠ Left-Below limit"]
If[Lim2[f, x, y, point] === Lim4[f, x, y, point],
 "Left-Above limit = Right-Below limit", "Left-Above limit ≠ Right-Below limit"]
If[Lim3[f, x, y, point] === Lim4[f, x, y, point],
 "Left-Below limit = Right-Below limit", "Left-Below limit ≠ Right-Below limit"]
If[Lim1[f, x, y, point] === Lim2[f, x, y, point] === Lim3[f, x, y, point] ===
  Lim4[f, x, y, point] === f[point], "Continuous", "Discontinuous"]
```

Out[98]=



Out[102]= Right−Above Discontinuous

Out[103]= Left−Above Discontinuous

Out[104]= Left−Below Discontinuous

Out[105]= Right−Below Discontinuous

Out[106]= Right−Above limit = Left−Above limit

Out[107]= Right−Above limit = Left−Below limit

Out[108]= Right−Above limit = Right−Below limit

Out[109]= Left−Above limit = Left−Below limit

Out[110]= Left−Above limit = Right−Below limit

Out[111]= Left−Below limit = Right−Below limit

Out[112]= Discontinuous