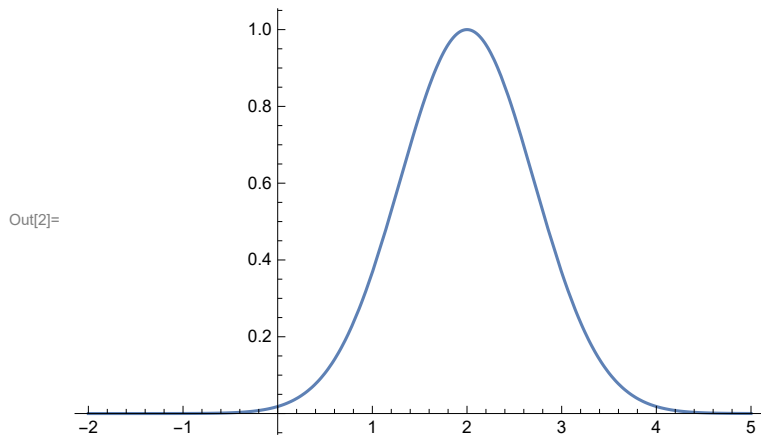In[1]:= `f[x_] := Exp[- (x - a) ^2];`   (*Not normalized*)
`a = 2;`
`Plot[f[x], {x, -2, 5}, PlotRange → Full, PlotLegends → "Expressions"]`
`NIntegrate[f[x], {x, -∞, ∞}]`

Out[2]=

Out[3]= 1.77245

In[4]:= $f[x\_] := \left(1\big/\text{Sqrt}[\pi]\right) * \text{Exp}[-(x-a)^2];$ (*Normalized*)
`a = 2;`
`Plot[f[x], {x, -2, 5}, PlotRange → Full, PlotLegends → "Expressions"]`
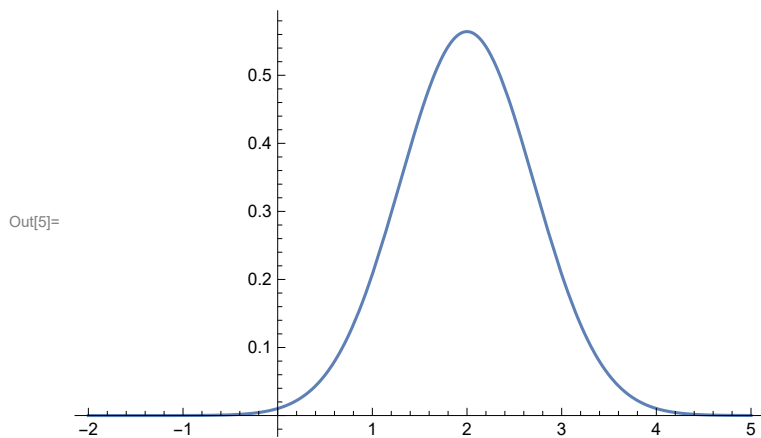`NIntegrate[f[x], {x, -∞, ∞}]`
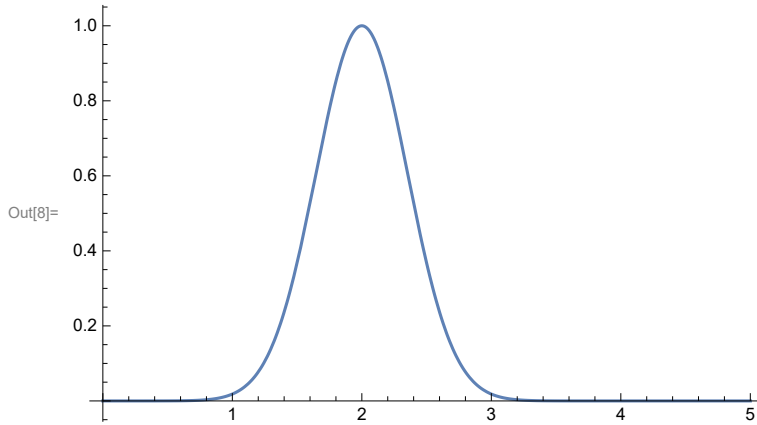
Out[5]=

Out[6]= 1.

In[7]:= `f[x_, k_] := Exp[-((x - a)^2)/k^2];`   `(*Not normalized*)`
`a = 2;`
`Plot[f[x, k = 0.5], {x, 0, 5}, PlotRange → Full, PlotLegends → "Expressions"]`
`NIntegrate[f[x, k = 0.5], {x, -∞, ∞}]`
`2 * NIntegrate[f[x, k = 0.5], {x, -∞, ∞}]`

Out[8]=

Out[9]= `0.886227`

Out[10]= `1.77245`

In[11]:= `f[x_, k_] := (1/(k Sqrt[π])) Exp[-((x - a)^2)/k^2];`   `(*Normalized*)`
`a = 2;`
`Plot[f[x, k = 0.5], {x, 0, 5}, PlotRange → Full, PlotLegends → "Expressions"]`
`NIntegrate[f[x, k = 0.5], {x, -∞, ∞}]`
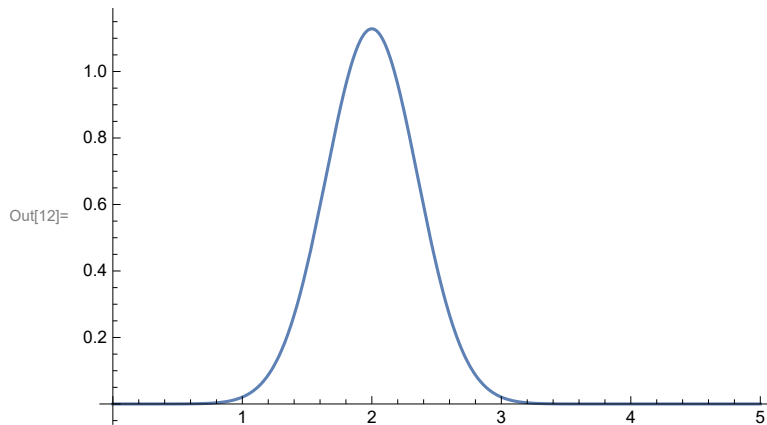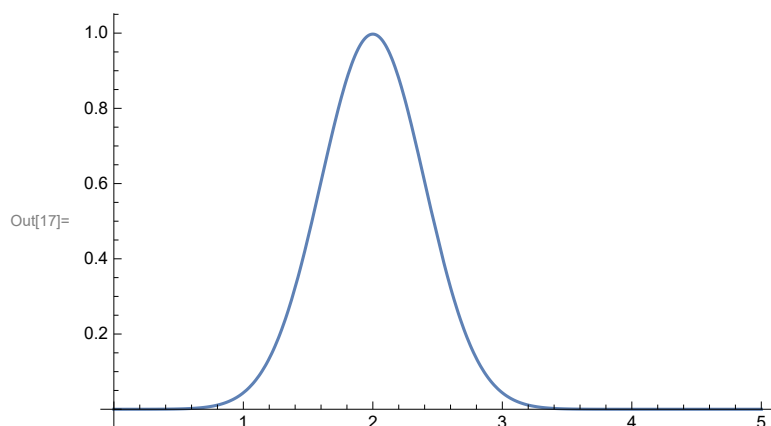`NIntegrate[f[x, k = 0.05], {x, -∞, ∞}]`

Out[12]=

Out[13]= `1.`

Out[14]= `1.`

In[15]:= `G[x_] := (1 / (σ * Sqrt[2 Pi])) Exp[- ((x - a) ^2) / (2 σ^2)];`
`a = 2; σ = 0.4;`
`Plot[G[x], {x, 0, 5}]`

Out[17]=

In[18]:= `G[x_, σ_] := (1 / (σ * Sqrt[2 Pi])) Exp[- ((x - a) ^2) / (2 σ^2)];`
`a = 2;`
`Plot[{G[x, σ = 0.5], G[x, σ = 0.4], G[x, σ = 0.3],`
`  G[x, σ = 0.2], G[x, σ = 0.1], G[x, σ = 0.05]}, {x, 0, 5}, PlotRange → Full,`
` PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]`
`NIntegrate[G[x, σ = 0.5], {x, -∞, ∞}]`
`NIntegrate[G[x, σ = 0.05], {x, -∞, ∞}]`

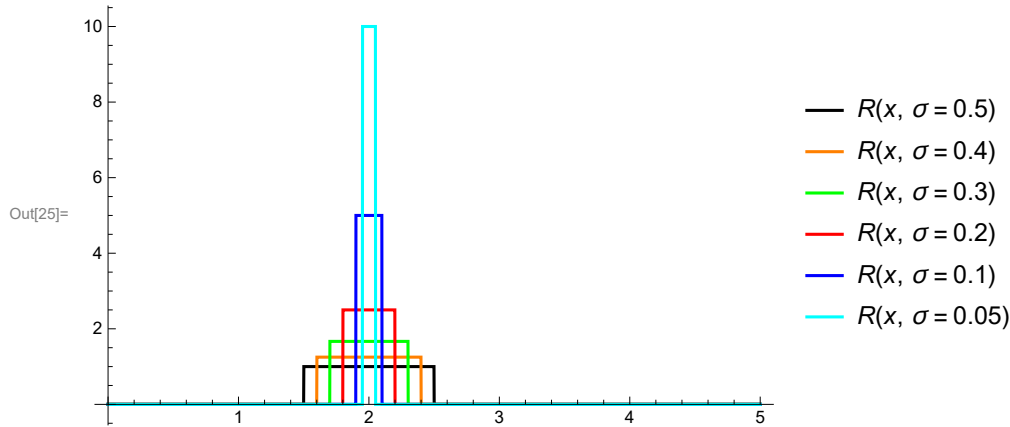⋯ General: Exp[−799.918] is too small to represent as a normalized machine number; precision may be lost.

Out[20]=
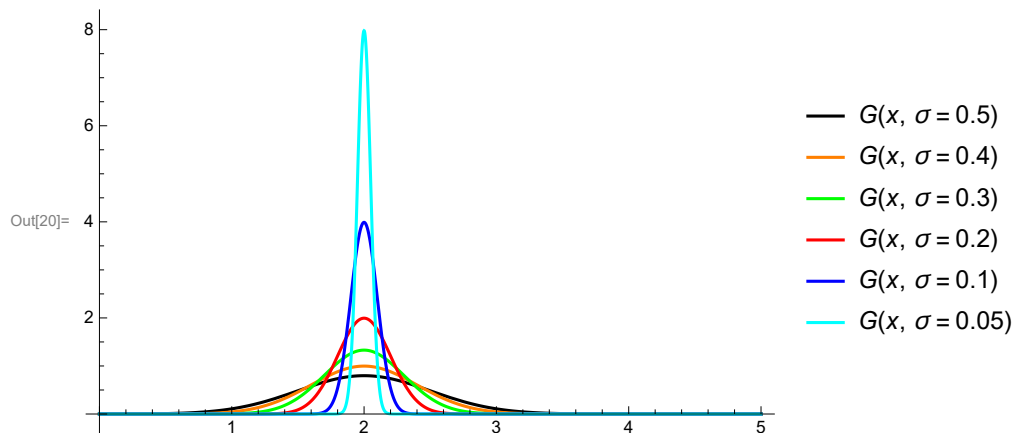
Out[21]= `1.`

Out[22]= `1.`

In[23]:= `R[x_, σ_] := Piecewise[{{1 / (2 σ), -σ < x - a < σ}, {0, Modulus[x - a] > σ}}];`
`a = 2;`
`Plot[{R[x, σ = 0.5], R[x, σ = 0.4], R[x, σ = 0.3],`
`    R[x, σ = 0.2], R[x, σ = 0.1], R[x, σ = 0.05]}, {x, 0, 5}, PlotRange → Full,`
`  PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]`
`NIntegrate[R[x, σ = 0.5], {x, -∞, ∞}]`
`NIntegrate[R[x, σ = 0.05], {x, -∞, ∞}]`

Out[25]=

Out[26]= 1.

Out[27]= 1.

In[28]:= `R[x_, σ_] := Piecewise[{{1 / (3 σ), -3 σ / 2 < x - a < 3 σ / 2}, {0, Modulus[x - a] > 3 σ / 2}}];`
`a = 2;`
`Plot[{R[x, σ = 0.5], R[x, σ = 0.4], R[x, σ = 0.3],`
`    R[x, σ = 0.2], R[x, σ = 0.1], R[x, σ = 0.05]}, {x, 0, 5}, PlotRange → Full,`
`  PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]`
`NIntegrate[R[x, σ = 0.5], {x, -∞, ∞}]`
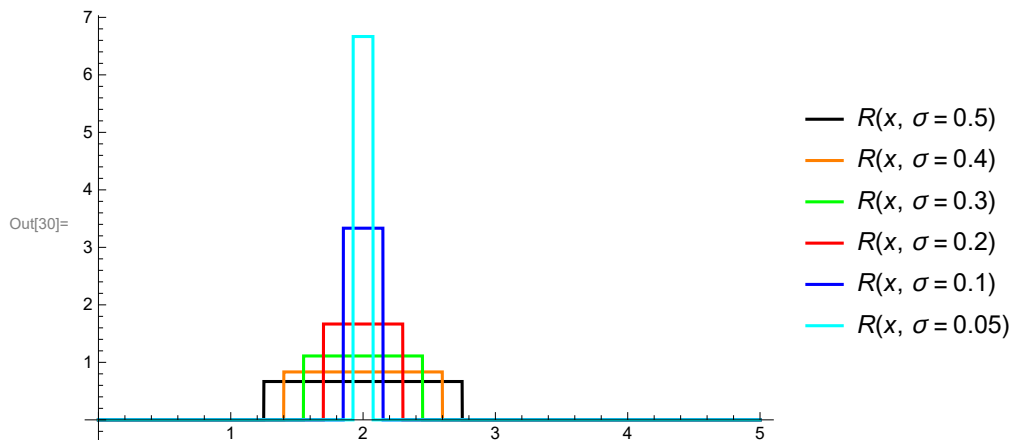`NIntegrate[R[x, σ = 0.05], {x, -∞, ∞}]`

Out[30]=

Out[31]= 1.

Out[32]= 1.

In[33]:= `R[x_, σ_] := Piecewise[{{1/σ, -σ/2 < x - a < σ/2}, {0, Modulus[x - a] > σ/2}}];`
`a = 2;`
`Plot[{R[x, σ = 0.5], R[x, σ = 0.4], R[x, σ = 0.3],`
`  R[x, σ = 0.2], R[x, σ = 0.1], R[x, σ = 0.085]}, {x, 0, 5}, PlotRange → Full,`
` PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]`
`NIntegrate[R[x, σ = 0.5], {x, -∞,  ∞}]`
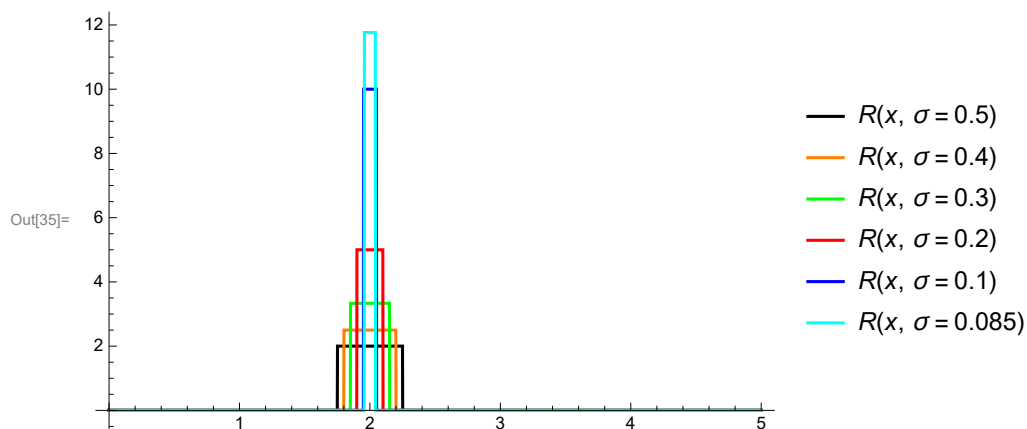`NIntegrate[R[x, σ = 0.05], {x, -∞,  ∞}]`

Out[35]=

Out[36]= `1.`

Out[37]= `1.`