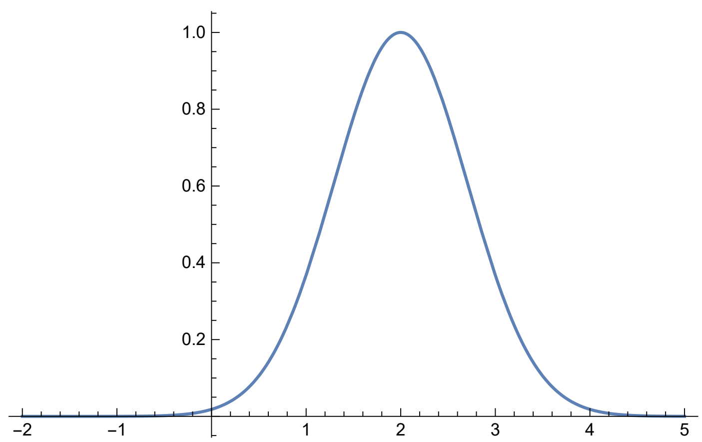
```
In[]:= f[x_] := Exp[- (x - a) ^2];                    (*Not normalized*)
a = 2;
Plot[f[x], {x, -2, 5}, PlotRange → Full, PlotLegends → "Expressions"]
NIntegrate[f[x], {x, -∞, ∞}]
(*Area under Exp[-(x-a)^2] is √π *)
N[Sqrt[Pi], 8]
```

Out[]=

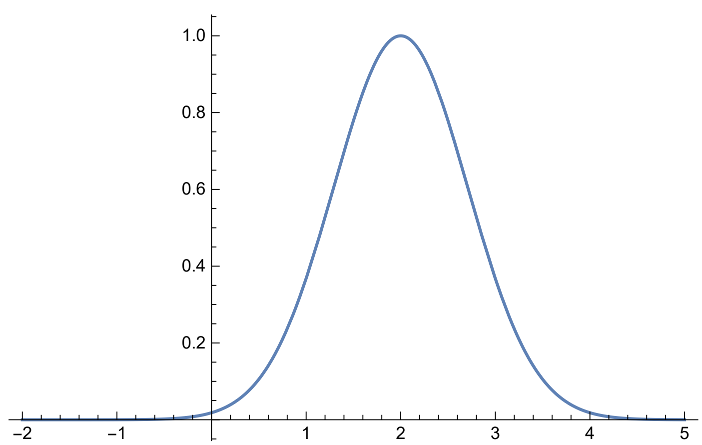


Out[]= 1.77245

Out[]= 1.7724539

```
In[]:= f[x_] := 2.71828^(- (x - a) ^2);                    (*Not normalized*)
a = 2;
Plot[f[x], {x, -2, 5}, PlotRange → Full, PlotLegends → "Expressions"]
NIntegrate[f[x], {x, -∞, ∞}]
(*Area under Exp[-(x-a)^2] is √π *)
N[Sqrt[Pi], 8]
```

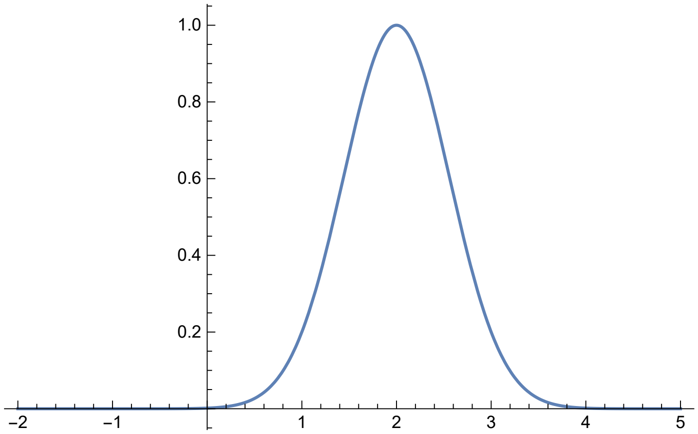
Out[]=



Out[]= 1.77245

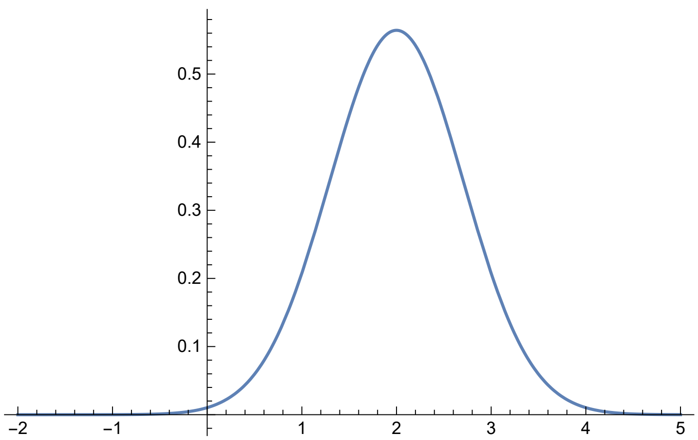
Out[]= 1.7724539

```
In[ ]:= f[x_] := 5^(-(x - a)^2);                    (*Not normalized*)
a = 2;
Plot[f[x], {x, -2, 5}, PlotRange → Full, PlotLegends → "Expressions"]
NIntegrate[f[x], {x, -∞, ∞}]
(*Area under 5^(-(x-a)^2 *)
```

Out[ ]=


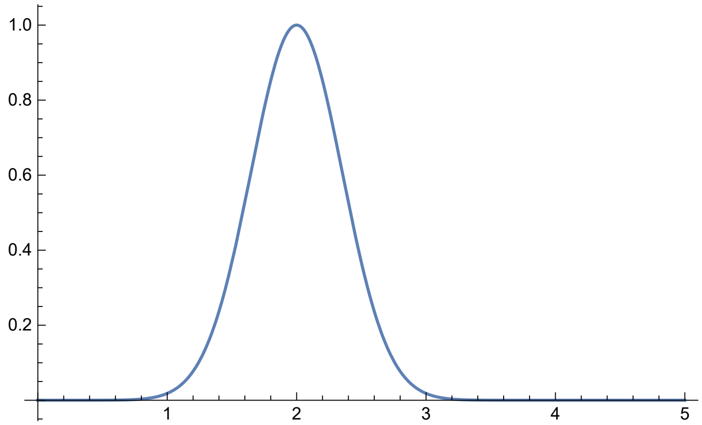

```
Out[ ]= 1.39713
```

```
In[ ]:= f[x_] := (1/Sqrt[π]) * Exp[-(x - a)^2];          (*Normalized*)
a = 2;
Plot[f[x], {x, -2, 5}, PlotRange → Full, PlotLegends → "Expressions"]
NIntegrate[f[x], {x, -∞, ∞}]
```

Out[ ]=



```
Out[ ]= 1.
```

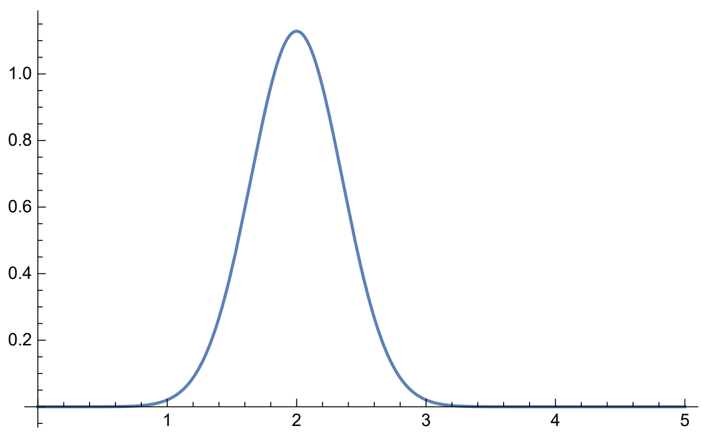
In[•]:=
```
f[x_, k_] := Exp[-((x - a)^2) / k^2];
(*Not normalized*) (*What do k do in the exponential*)
a = 2;
Plot[f[x, k = 0.5], {x, 0, 5}, PlotRange → Full, PlotLegends → "Expressions"]
NIntegrate[f[x, k = 0.5], {x, -∞, ∞}]
2 * NIntegrate[f[x, k = 0.5], {x, -∞, ∞}]
```

Out[•]=


Out[•]= 0.886227

Out[•]= 1.77245

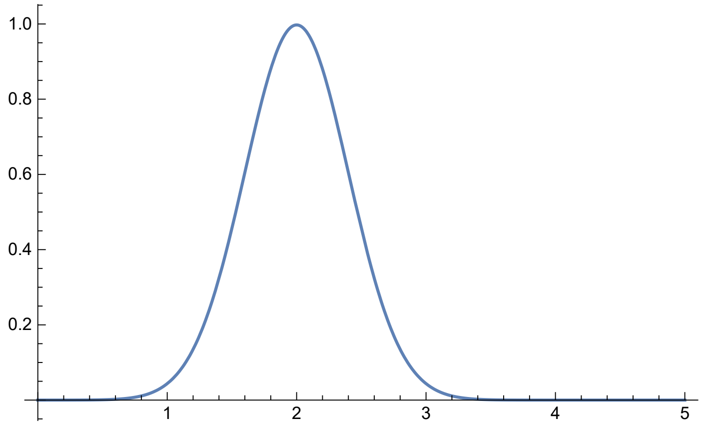
In[•]:=
```
f[x_, k_] := (1 / (k Sqrt[π])) Exp[-((x - a)^2) / k^2];            (*Normalized*)
a = 2;
Plot[f[x, k = 0.5], {x, 0, 5}, PlotRange → Full, PlotLegends → "Expressions"]
NIntegrate[f[x, k = 0.5], {x, -∞, ∞}]
NIntegrate[f[x, k = 0.05], {x, -∞, ∞}]
```

Out[•]=


Out[•]= 1.

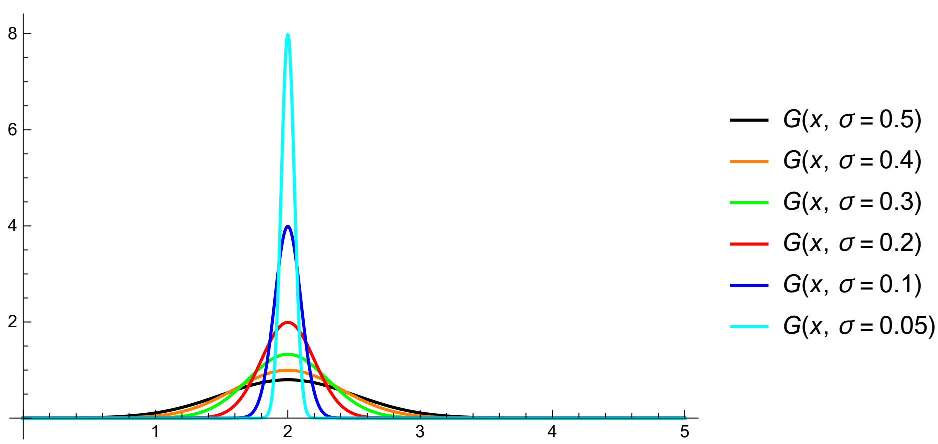
Out[•]= 1.

In[◦]:=
```
G[x_] := (1/(σ*Sqrt[2 Pi])) Exp[-((x - a)^2)/(2 σ^2)];
(*What do σ do in the exponenetial*)
a = 2; σ = 0.4;
Plot[G[x], {x, 0, 5}]
```

Out[◦]=


In[◦]:=
```
G[x_, σ_] := (1/(σ*Sqrt[2 Pi])) Exp[-((x - a)^2)/(2 σ^2)];
(*What do σ do in the exponenetial*)
a = 2;
Plot[{G[x, σ = 0.5], G[x, σ = 0.4], G[x, σ = 0.3],
  G[x, σ = 0.2], G[x, σ = 0.1], G[x, σ = 0.05]}, {x, 0, 5}, PlotRange → Full,
 PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
NIntegrate[G[x, σ = 0.5], {x, -∞, ∞}]
NIntegrate[G[x, σ = 0.05], {x, -∞, ∞}]
```

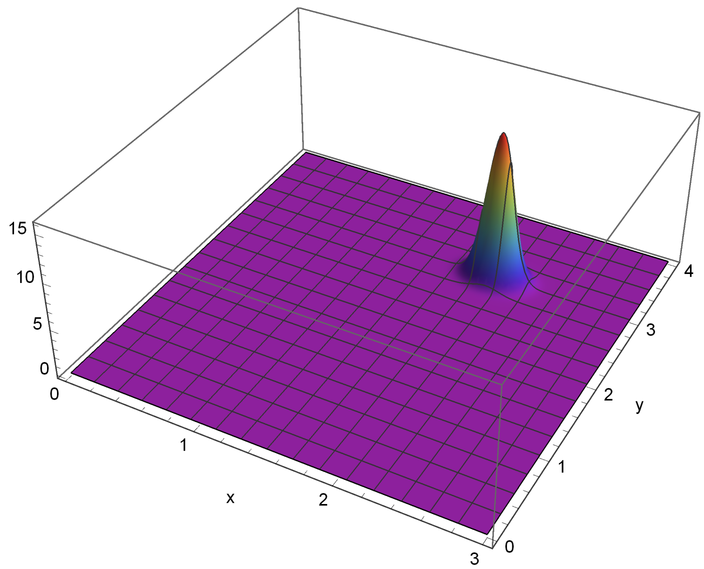
General: Exp[−799.918] is too small to represent as a normalized machine number; precision may be lost.

Out[◦]=


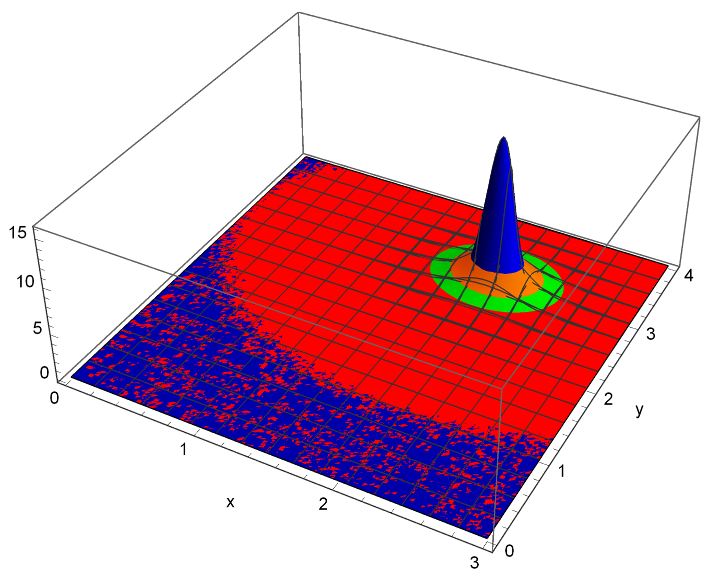
Out[◦]= 1.

Out[◦]= 1.

In[ ]:=
```
G2[x_, y_, σ_] := (1/((σ^2) * 2 Pi)) Exp[-(((x - a) ^2) + ((y - b) ^2)) / (2 σ^2)];
a = 2; b = 3;
Plot3D[G2[x, y, σ = 0.1], {x, 0, 3}, {y, 0, 4}, PlotPoints → 100,
 PlotRange → Full, AxesLabel → {"x", "y"}, ColorFunction → "Rainbow"]
```

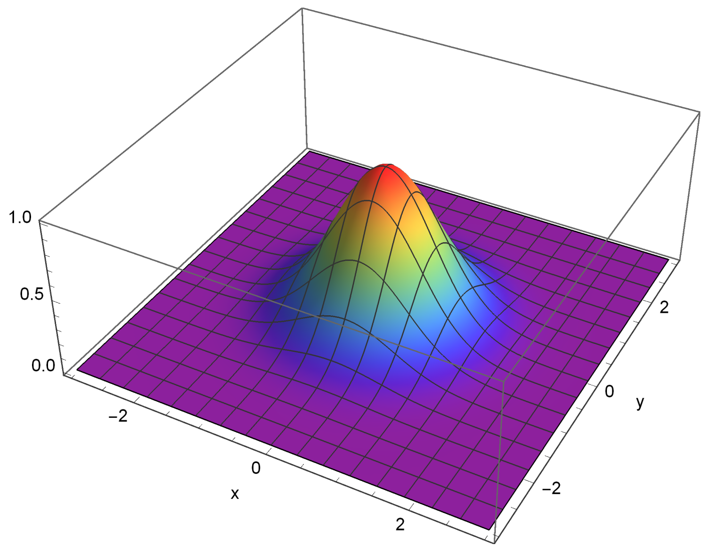
Out[ ]=


In[ ]:=
```
G2[x_, y_, σ_] := (1/((σ^2) * 2 Pi)) Exp[-(((x - a) ^2) + ((y - b) ^2)) / (2 σ^2)];
a = 2; b = 3;
Plot3D[{G2[x, y, σ = 0.4], G2[x, y, σ = 0.3], G2[x, y, σ = 0.2], G2[x, y, σ = 0.1]},
 {x, 0, 3}, {y, 0, 4}, PlotPoints → 100, PlotRange → Full,
 AxesLabel → {"x", "y"}, PlotStyle → {Red, Green, Orange, Blue, Opacity[0.3]}]
```

Out[ ]=


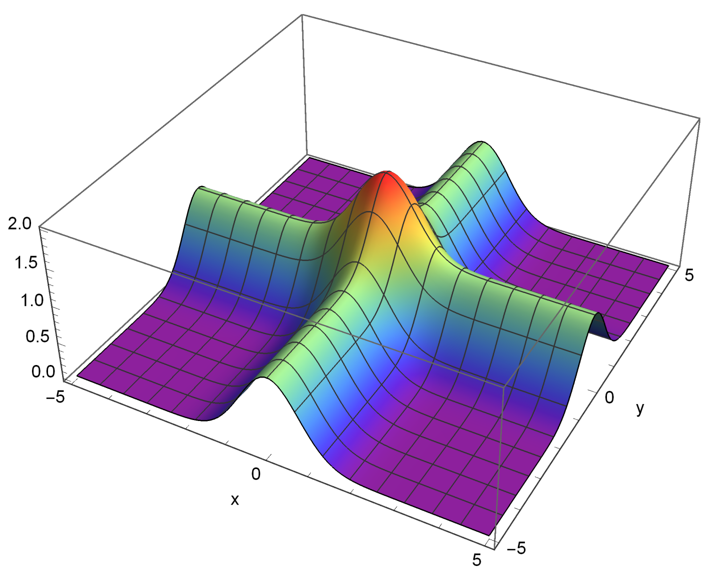
In[◦]:=
```
g2[x_, y_] := Exp[-((x^2) + (y^2))];
Plot3D[g2[x, y], {x, -3, 3}, {y, -3, 3}, PlotPoints → 100,
 PlotRange → Full, AxesLabel → {"x", "y"}, ColorFunction → "Rainbow"]
```

Out[◦]=



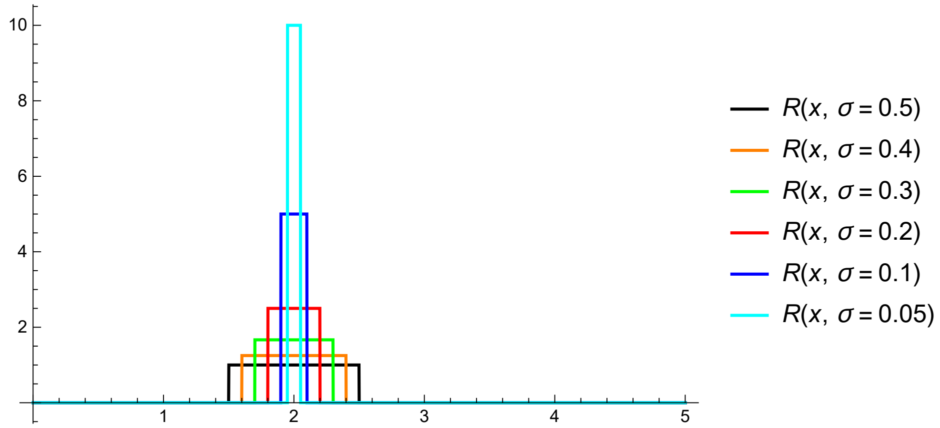
In[◦]:=
```
g2[x_, y_] := Exp[-x^2] + Exp[-y^2];
Plot3D[g2[x, y], {x, -5, 5}, {y, -5, 5}, PlotPoints → 100,
 PlotRange → Full, AxesLabel → {"x", "y"}, ColorFunction → "Rainbow"]
```

Out[◦]=



In[∘]:=
```
R[x_, σ_] := Piecewise[{{1/(2 σ), -σ < x - a < σ}, {0, Modulus[x - a] > σ}}];
(*Rectangular function*)
a = 2;
Plot[{R[x, σ = 0.5], R[x, σ = 0.4], R[x, σ = 0.3],
  R[x, σ = 0.2], R[x, σ = 0.1], R[x, σ = 0.05]}, {x, 0, 5}, PlotRange → Full,
 PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
NIntegrate[R[x, σ = 0.5], {x, -∞, ∞}]
NIntegrate[R[x, σ = 0.05], {x, -∞, ∞}]
```

Out[∘]=

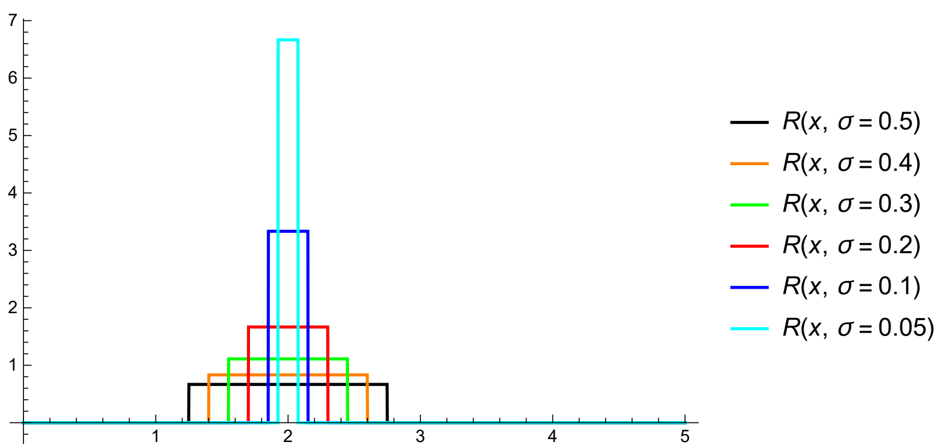


Out[∘]= 1.

Out[∘]= 1.

In[∘]:=
```
R[x_, σ_] := Piecewise[{{1/(3 σ), -3 σ/2 < x - a < 3 σ/2}, {0, Modulus[x - a] > 3 σ/2}}];
a = 2;
Plot[{R[x, σ = 0.5], R[x, σ = 0.4], R[x, σ = 0.3],
  R[x, σ = 0.2], R[x, σ = 0.1], R[x, σ = 0.05]}, {x, 0, 5}, PlotRange → Full,
 PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
NIntegrate[R[x, σ = 0.5], {x, -∞, ∞}]
NIntegrate[R[x, σ = 0.05], {x, -∞, ∞}]
```

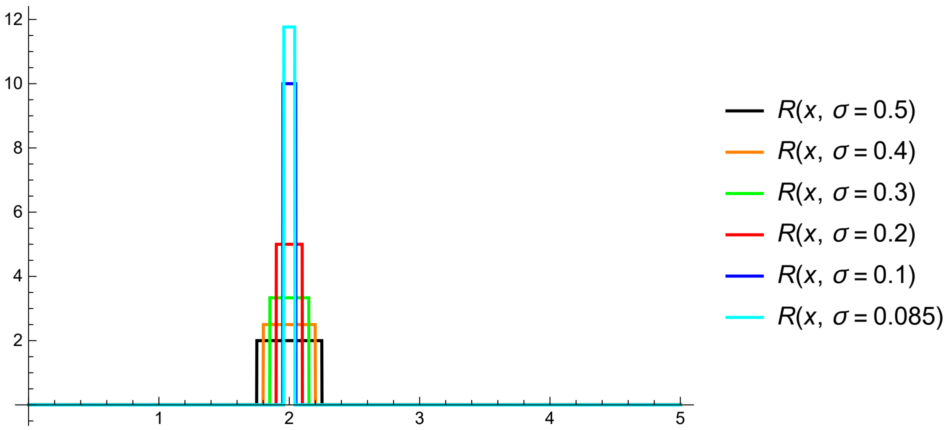
Out[∘]=



Out[∘]= 1.

Out[∘]= 1.

In[ ]:=
```
R[x_, σ_] := Piecewise[{{1/σ, -σ/2 < x - a < σ/2}, {0, Modulus[x - a] > σ/2}}];
a = 2;
Plot[{R[x, σ = 0.5], R[x, σ = 0.4], R[x, σ = 0.3],
   R[x, σ = 0.2], R[x, σ = 0.1], R[x, σ = 0.085]}, {x, 0, 5}, PlotRange → Full,
  PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
NIntegrate[R[x, σ = 0.5], {x, -∞, ∞}]
NIntegrate[R[x, σ = 0.05], {x, -∞, ∞}]
```

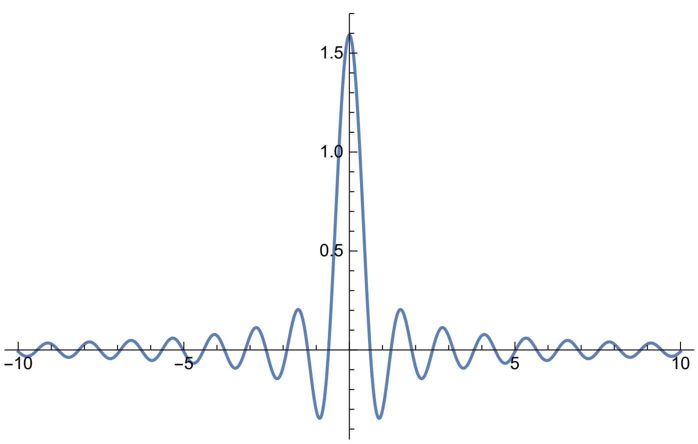
Out[ ]=


Out[ ]= 1.

Out[ ]= 1.

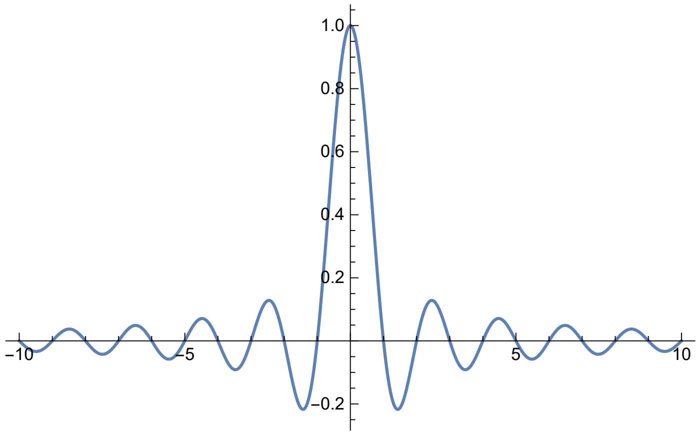
In[ ]:=
```
s[x_] := Sin[g x] / (π x);
Limit[s[x], x → 0]
g = 5;
Plot[s[x], {x, -10, 10}, PlotRange → Full]
```

Out[ ]= $\frac{g}{\pi}$

Out[ ]=


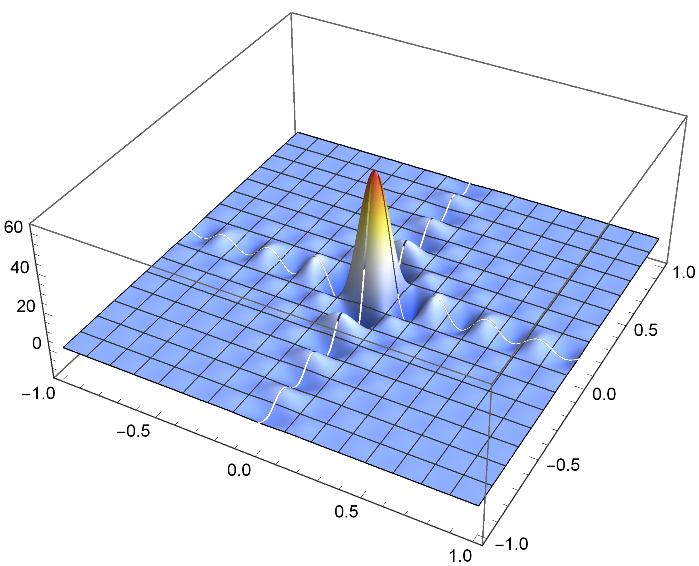
In[∘]:=
```
s[x_] := Sin[π x] / (π x);
Limit[s[x], x → 0]
Plot[s[x], {x, -10, 10}, PlotRange → Full]
```

Out[∘]= 1

Out[∘]=


In[∘]:=
```
s[x_, y_] := (Sin[g x] / (π x)) (Sin[g y] / (π y));
Limit[s[x], x → 0]
g = 25;
Plot3D[s[x, y], {x, -1, 1}, {y, -1, 1}, PlotPoints → 100,
 PlotRange → Full, ColorFunction → "TemperatureMap"]
```

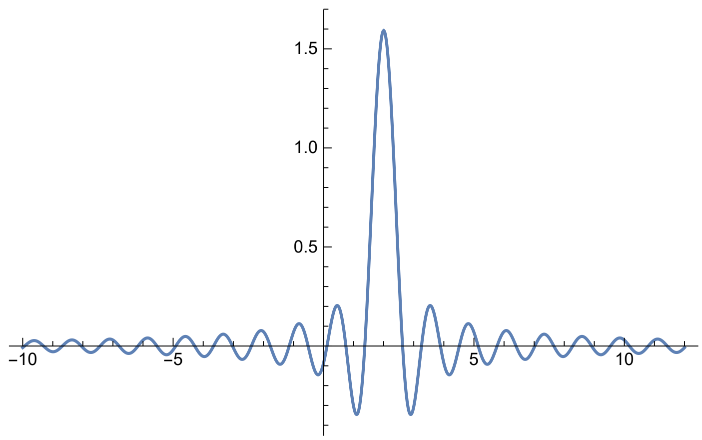
Out[∘]= 1

Out[∘]=


In[ ]:=
```
s[x_] := Sin[g (x - a)] / (π (x - a));
Limit[s[x], x → 0]
a = 2; g = 5;
Plot[s[x], {x, -10, 10 + a}, PlotRange → Full, PlotLegends → "Expressions"]
```

Out[ ]= $\frac{\text{Sin}[50]}{2\pi}$

Out[ ]=



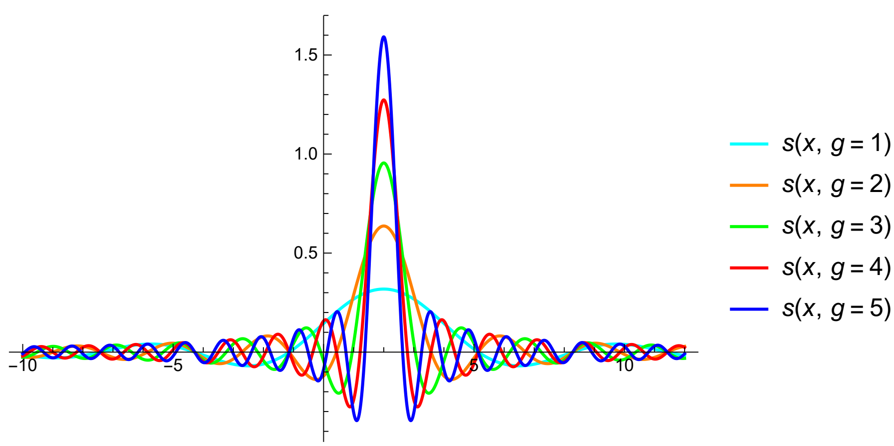
In[ ]:=
```
S[x_, p_] := Sin[p (x - b)] / (π (x - b));
Limit[S[x, p], x → 0]
Limit[S[x, p], p → ∞]
```

Out[ ]= $\frac{\text{Sin}[3\,p]}{3\pi}$

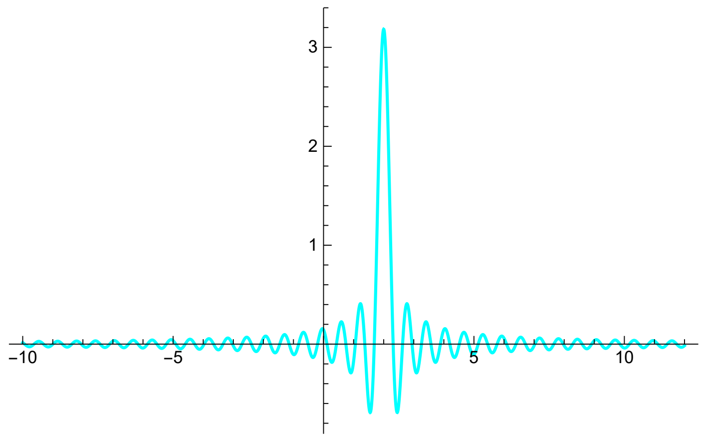
Out[ ]= ConditionalExpression[Indeterminate, x ∈ ℝ]

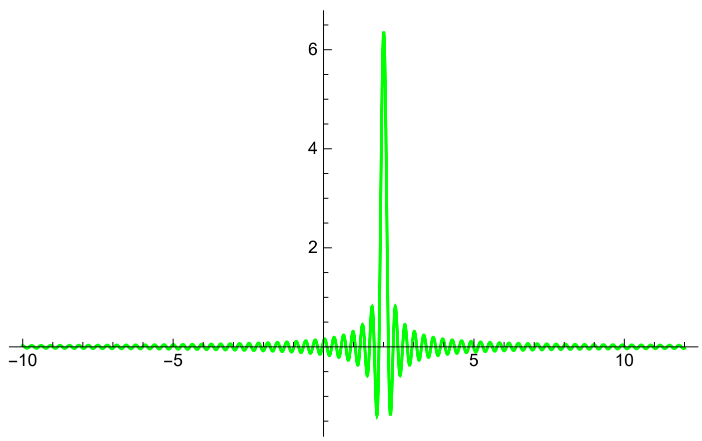
In[ ]:=
```
s[x_, g_] := Sin[g (x - a)] / (π (x - a));
a = 2;
Plot[{s[x, g = 1], s[x, g = 2], s[x, g = 3], s[x, g = 4], s[x, g = 5]}, {x, -10, 10 + a},
 PlotRange → Full, PlotStyle → {Cyan, Orange, Green, Red, Blue}, PlotLegends → "Expressions"]
```

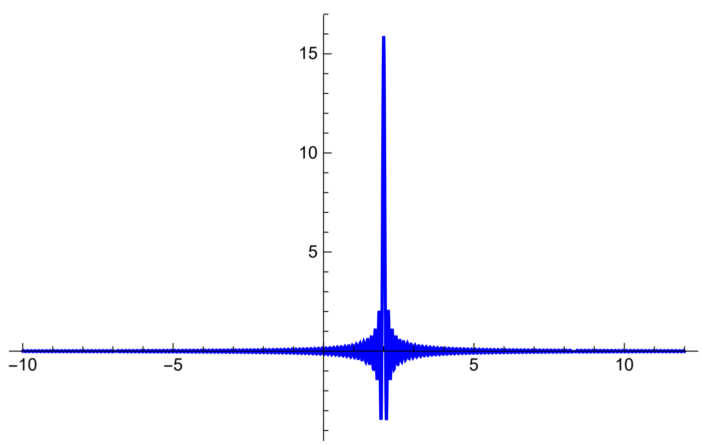
Out[ ]=



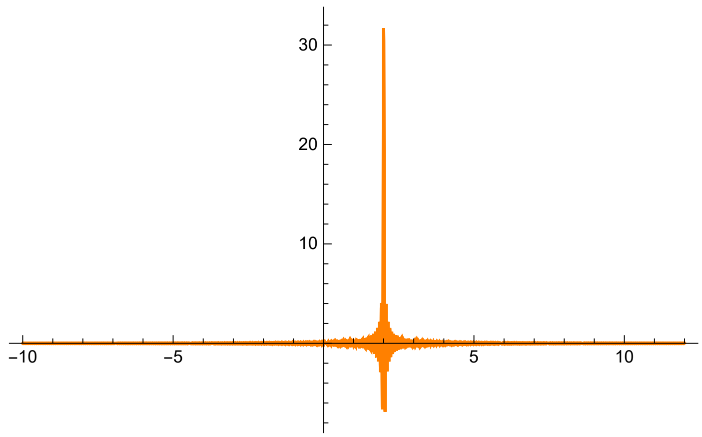
In[•]:=
```
s[x_, g_] := Sin[g (x - a)] / (π (x - a));
a = 2;
Plot[s[x, g = 10], {x, -10, 10 + a}, PlotRange → Full,
 PlotStyle → Cyan, PlotLegends → "Expressions"]
Plot[s[x, g = 20], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Green]
Plot[s[x, g = 50], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Blue]
Plot[s[x, g = 100], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Orange]
Plot[s[x, g = 1000], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Purple]
Plot[s[x, g = 1000], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Red]
```

Out[•]=


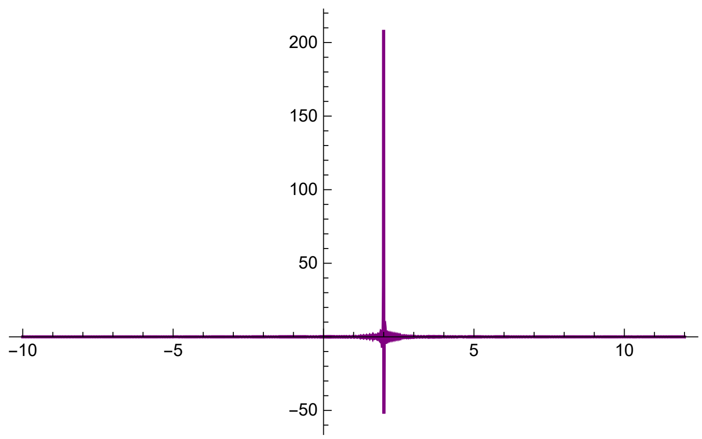
Out[•]=


Out[•]=


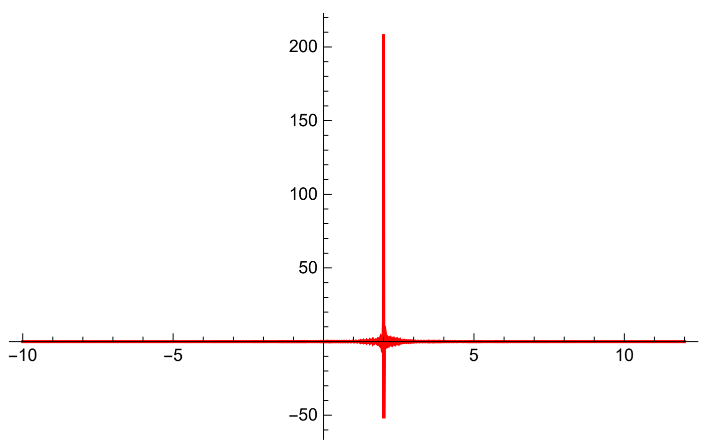
Out[∘]=


30
20
10
–10
–5
5
10

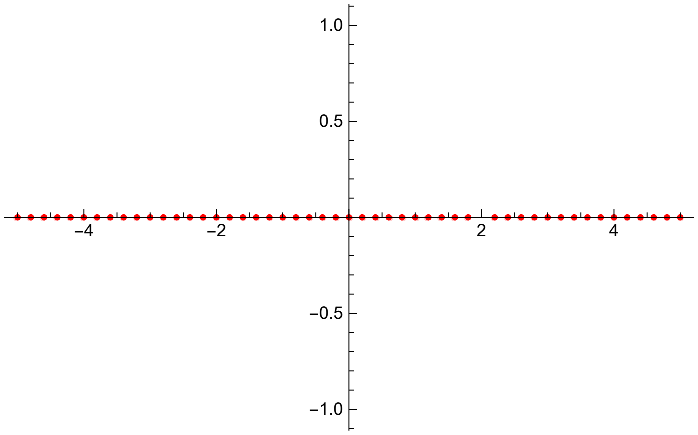
Out[∘]=


200
150
100
50
–10
–5
5
10
–50

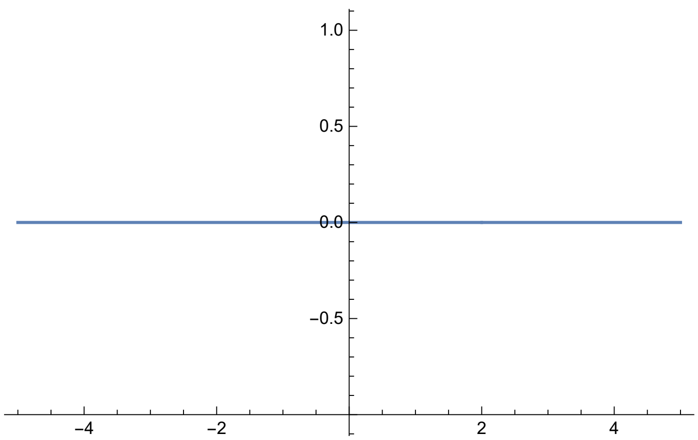
Out[∘]=


200
150
100
50
–10
–5
5
10
–50

In[ ]:=
```
f[x_] := DiracDelta[x - a];
a = 2;
DiscretePlot[f[x], {x, -5, 5, 0.2}, PlotStyle → {Red, Thick}]
```

Out[ ]=


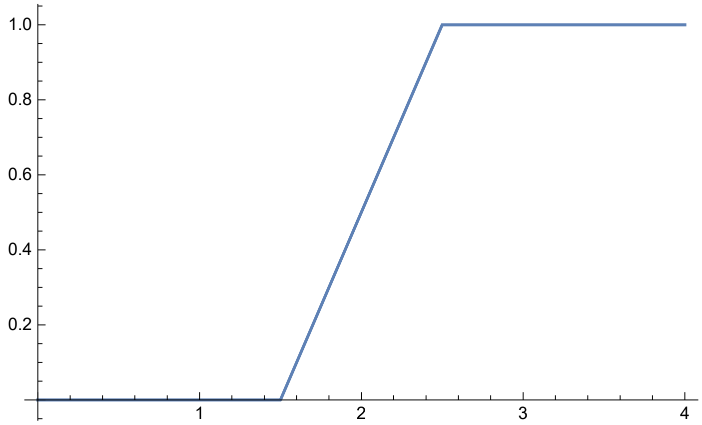
In[ ]:=
```
f[x_] := DiracDelta[x - a];
a = 2;
Plot[f[x], {x, -5, 5}, AxesOrigin → {0, -1}]
```

Out[ ]=


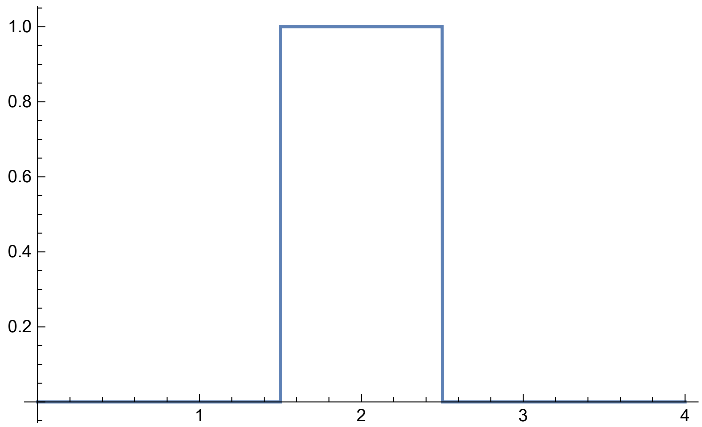
In[ ]:=
```
Exit
```

In[∘]:=
```
F[x_, σ_] := Piecewise[{{0, x < a - σ}, {(1/(2 σ)) (x - a + σ), -σ < x - a < σ}, {1, x > a + σ}}];
(*Ramp function*)
der[x_, σ_] =
  D[F[x, σ], x];
(*Derivative of Ramp function is Rectangular function R*)
a = 2;
Plot[F[x, σ = 0.5], {x, 0, 4}, PlotRange → Full]
Plot[der[x, σ = 0.5], {x, 0, 4}, PlotRange → Full]
```

Out[∘]=

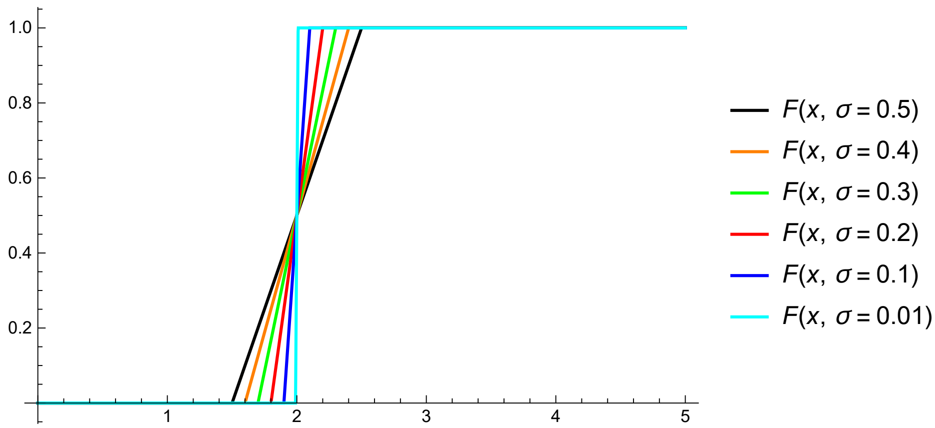


Out[∘]=



In[∘]:=
```
F[x_, σ_] := Piecewise[{{0, x < a - σ}, {(1/(2 σ)) (x - a + σ), -σ < x - a < σ}, {1, x > a + σ}}];
a = 2;
Plot[{F[x, σ = 0.5], F[x, σ = 0.4], F[x, σ = 0.3],
  F[x, σ = 0.2], F[x, σ = 0.1], F[x, σ = 0.01]}, {x, 0, 5}, PlotRange → Full,
 PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
```

Out[∘]=



In[∘]:=
```
Exit
```

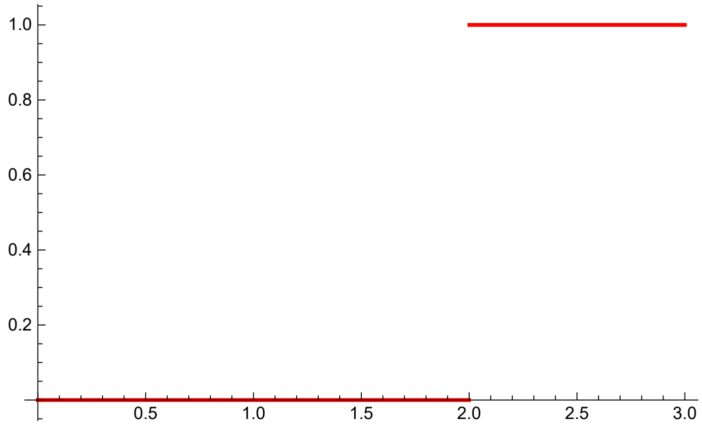
In[∘]:=
```
F[x_, σ_] := Piecewise[{{0, x < a - σ}, {(1/(2 σ)) (x - a + σ), -σ < x - a < σ}, {1, x > a + σ}}];
H[x_, a_] =
  Limit[F[x, σ], σ → 0];
(*For limit σ→0 Ramp function becomes Heaviside unit step function*)
H[x, a]
delta[x_, a_] =
  D[H[x, a], x];
(*Derivative of dicontinuous Heaviside unit step function is Dirac delta function*)
delta[x, a]
Integrate[delta[x, a = 2], {x, -5, 5}]
```

Out[∘]=
$$\begin{cases} 1 & a < x \\ 0 & a > x \\ \text{Indeterminate} & \text{True} \end{cases}$$

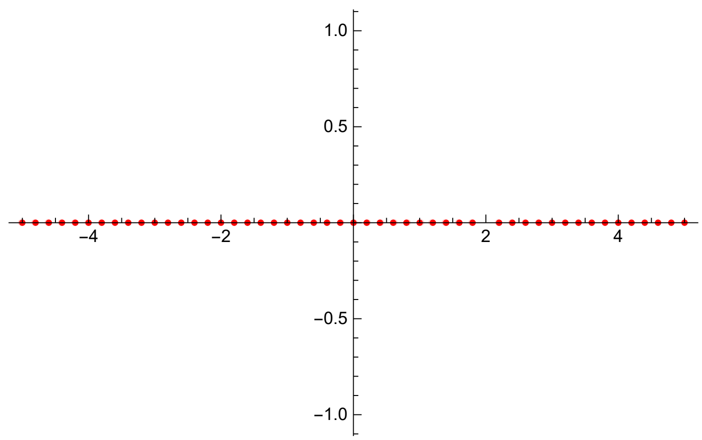
Out[∘]=
$$\begin{cases} 0 & a - x < 0 \,||\, a - x > 0 \\ \text{Indeterminate} & \text{True} \end{cases}$$

Out[∘]= 0

In[∘]:=
```
a = 2;
Plot[H[x, a], {x, 0, 3}, PlotStyle → {Red, Thick},
 PlotLegends → "H(x-2):Heaviside unit step function,\ndiscontinuous function"]
DiscretePlot[delta[x, a], {x, -5, 5, 0.2}, PlotStyle → {Red, Thick},
 PlotLegends → "δ(x-2): Dirac delta function"]
```

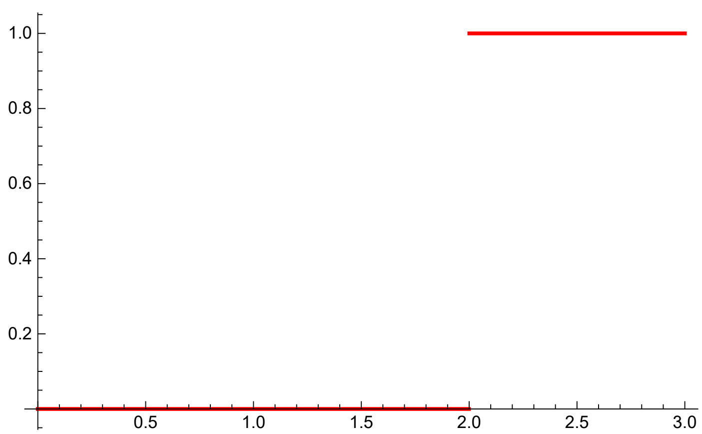
Out[∘]=


H(x-2):Heaviside unit step function,
discontinuous function

Out[∘]=


δ(x-2): Dirac delta function

In[∘]:=
```
Plot[UnitStep[x - 2], {x, 0, 3}, PlotStyle → {Red, Thick},
 PlotLegends → "H(x-2):Heaviside unit step function"]
```

Out[∘]=


H(x-2):Heaviside unit step function

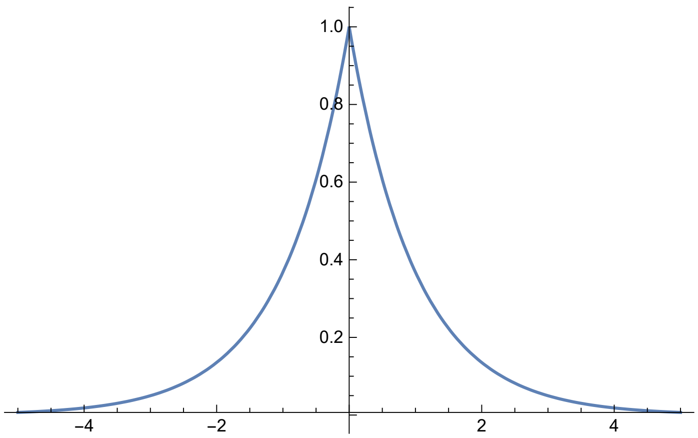
In[∘]:=
```
Exit
```

In[◦]:=
```
(*psi[x_]:=Exp[-Abs[x]];*)
psi[x_] := Piecewise[{{Exp[- (x)], x >= 0}, {Exp[- (-x)], x < 0}}];
derpsi[x_] = D[psi[x], x]
derderpsi[x_] = D[derpsi[x], x]
Plot[psi[x], {x, -5, 5}, PlotRange → Full]
Plot[derpsi[x], {x, -5, 5}, PlotRange → Full]
Plot[derderpsi[x], {x, -5, 5}, PlotRange → Full, PlotStyle → {Red, Thick}]
```

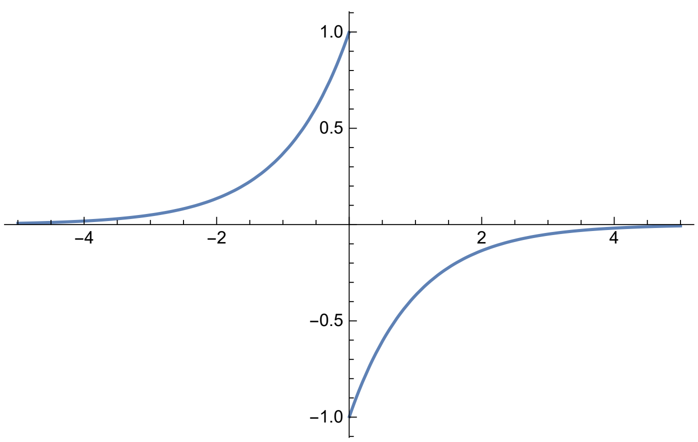
Out[◦]=
$$\begin{cases} e^{x} & x < 0 \\ -e^{-x} & x > 0 \\ \text{Indeterminate} & \text{True} \end{cases}$$

Out[◦]=
$$\begin{cases} e^{x} & x < 0 \\ e^{-x} & x > 0 \\ \text{Indeterminate} & \text{True} \end{cases}$$

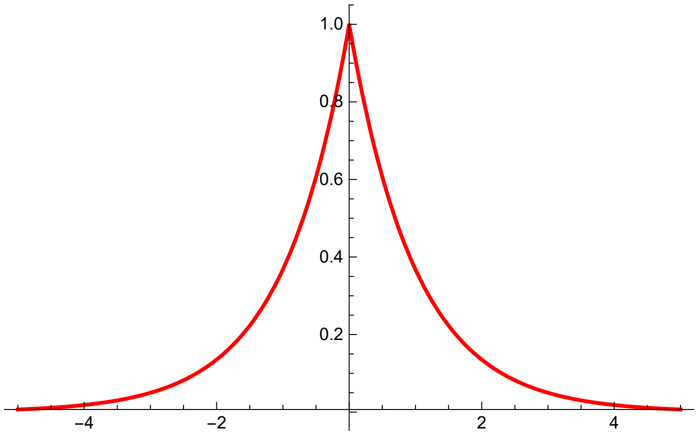
Out[◦]=



Out[◦]=



Out[∘]=

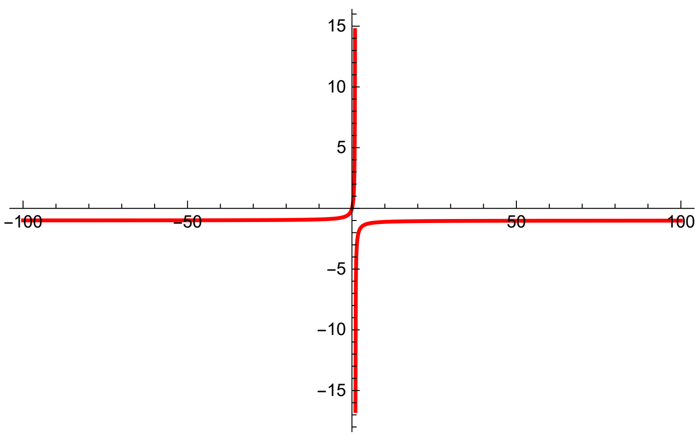


In[∘]:=

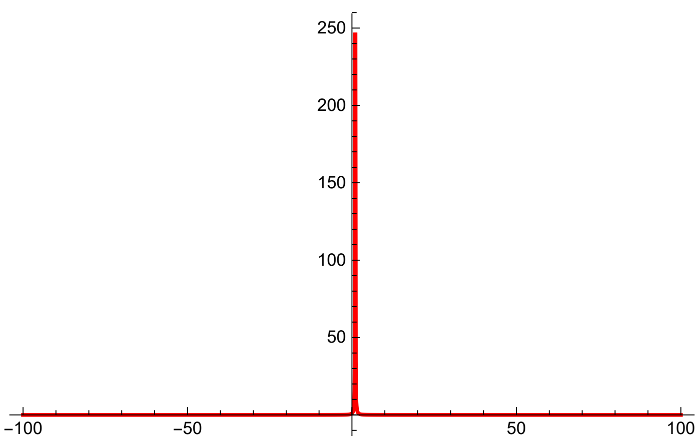
```
f[z_] := z / (1 - z);
der2[z_] = D[f[z], z]
Plot[f[z], {z, -100, 100}, PlotRange → Full, PlotStyle → {Red, Thick}]
Plot[der2[z], {z, -100, 100}, PlotRange → Full, PlotStyle → {Red, Thick}]
Plot[der2[z], {z, -1, 1}, PlotRange → Full, PlotStyle → {Red, Thick}]
Plot[der2[z], {z, -0.1, 0.1}, PlotRange → Full, PlotStyle → {Red, Thick}]
```

Out[∘]= $\frac{1}{1-z}+\frac{z}{(1-z)^2}$

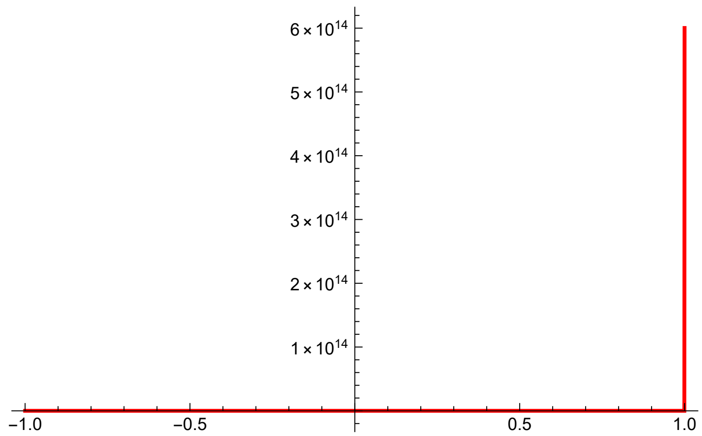
Out[∘]=



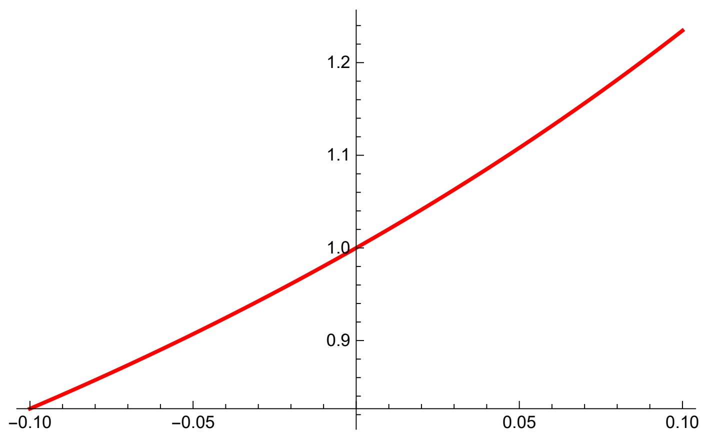
Out[∘]=



Out[∘]=



Out[∘]=



In[∘]:=

```
h[x_] := Piecewise[{{-x, x < 0}, {x, x ≥ 0}}];
derh[x_] = D[h[x], x]
derderh[x_] = D[derh[x], x]
Plot[h[x], {x, -5, 5}]
Plot[derh[x], {x, -5, 5}, PlotRange → Full, PlotStyle → {Red, Thick}]
DiscretePlot[derderh[x], {x, -5, 5, 0.2},
 AxesOrigin → {0, -0.2}, PlotRange → Full, PlotStyle → {Red, Thick}]
```

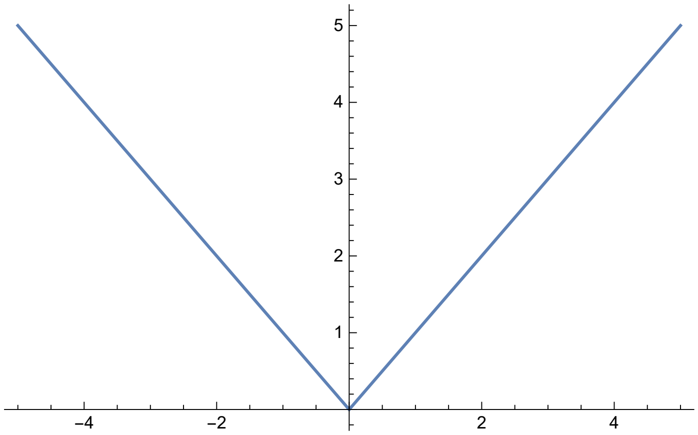
Out[∘]=

$$\begin{cases} -1 & x < 0 \\ 1 & x > 0 \\ \text{Indeterminate} & \text{True} \end{cases}$$

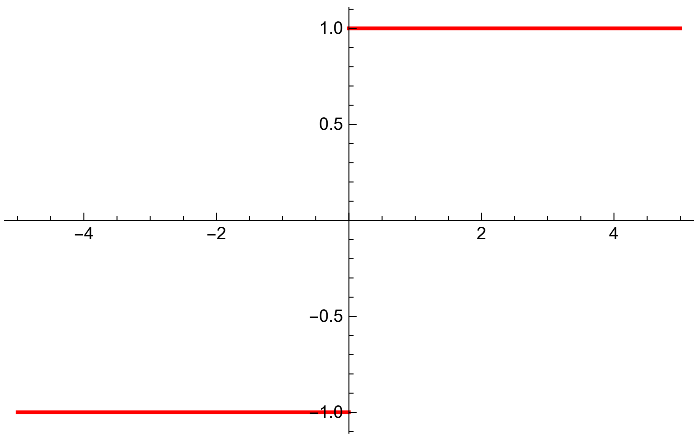
Out[∘]=

$$\begin{cases} 0 & x < 0 \mid\mid x > 0 \\ \text{Indeterminate} & \text{True} \end{cases}$$

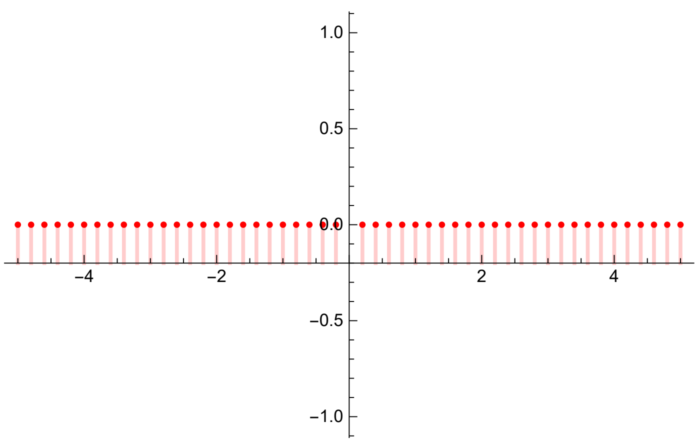
Out[ ]=



Out[ ]=



Out[ ]=

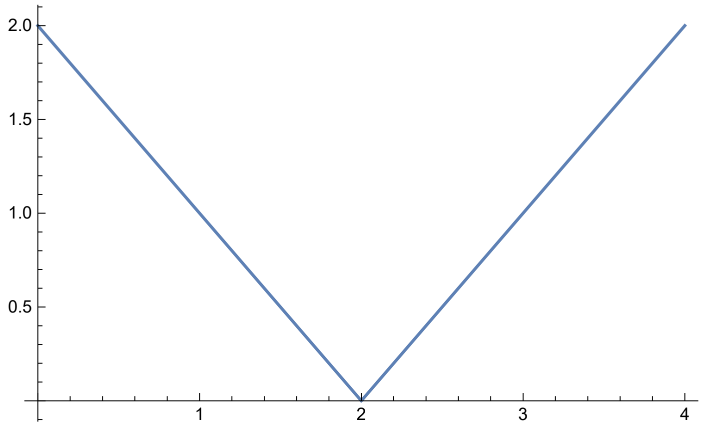


In[ ]:=
```
h[x_] := Piecewise[{{-x + a, x < a}, {x - a, x ≥ a}}];
a = 2;
derh[x_] = D[h[x], x]
derderh[x_] = D[derh[x], x]
Plot[h[x], {x, 0, 4}]
Plot[derh[x], {x, 0, 4}, PlotRange → Full, PlotStyle → {Red, Thick}]
DiscretePlot[derderh[x], {x, 0, 4, 0.1},
 AxesOrigin → {0, -0.2}, PlotRange → Full, PlotStyle → {Red, Thick}]
```

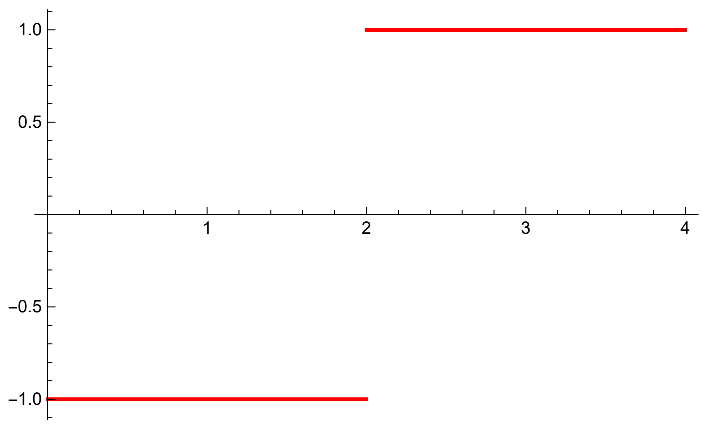
Out[∘]= $\begin{cases} -1 & x < 2 \\ 1 & x > 2 \\ \text{Indeterminate} & \text{True} \end{cases}$

Out[∘]= $\begin{cases} 0 & x < 2 \,||\, x > 2 \\ \text{Indeterminate} & \text{True} \end{cases}$

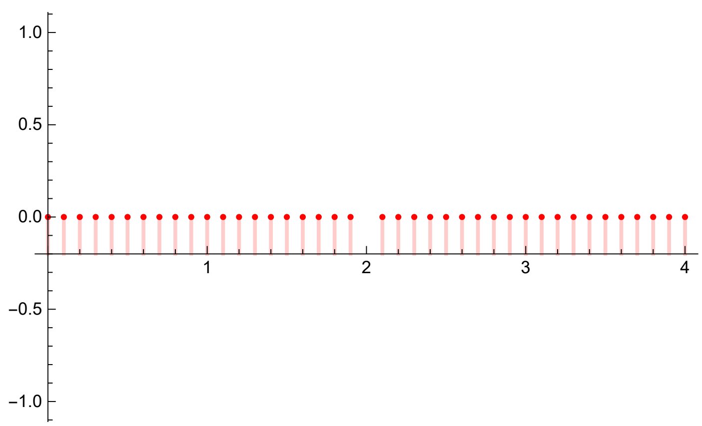
Out[∘]=



Out[∘]=



Out[∘]=



In[∘]:=
```
Exit
```

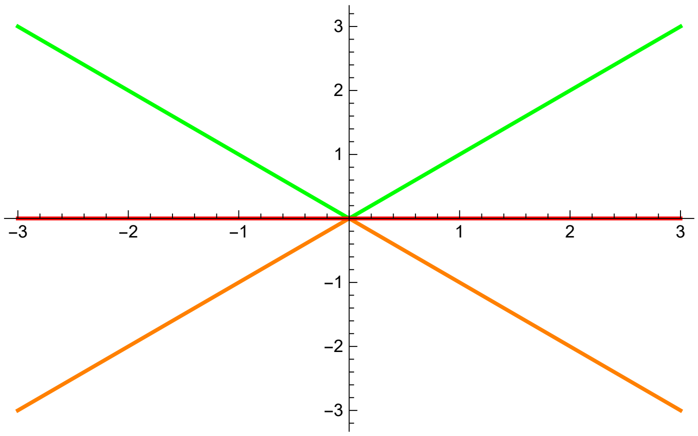
In[∘]:=
```
f[x_] := x^2;
Integrate[f[x] × DiracDelta[x - 2], {x, -5, 5}]
```

Out[∘]= 4

In[16]:=
```
G3[x_, y_, z_] :=
  (1/((σ^3) * (2 Pi)^(3/2))) Exp[-(((x - a)^2) + ((y - b)^2) + ((z - x)^2)) / (2 σ^2)];
a = 2; b = 3; c = 2, σ = 0.1;
ContourPlot3D[G3[x, y, z], {x, 0, 5},
 {y, 0, 5}, {z, 0, 5}, PlotPoints → 100, PlotRange → Full]

f[x_] := Piecewise[{{-x, x < 0}, {x, x > 0}}];
(* f(x)=|x| , not differenetiable at x=0 *)
g[x_] := Piecewise[{{x, x < 0}, {-x, x > 0}}];
(* g(x)=-|x| , not differentiable at x=0 *)
(* f(x) + g(x) is a horizontal line through x-
  axis and differentiable at all points including x=
 0. Its derivative at each point is 0 *)
Plot[{f[x], g[x], (f[x] + g[x])}, {x, -3, 3},
 PlotStyle → {Directive[Green, Thick], Directive[Orange, Thick], Directive[Red, Thick]}]
```

Out[19]=



In[20]:=
```
f[x_] := Piecewise[{{- (x - 1), x < 1}, {(x - 1), x > 1}}]; (* f(x)=|x-1|,
decreasing below x=1 nad increasing above x>1. Not differentiable at x=1 *)
g[x_] := Piecewise[{{- (x - 5), x < 5}, {(x - 5), x > 5}}]; (* g(x)=|x-5|,
decreasing below x=5 nad increasing above x>5. Not differentiable at x=5 *)
(* f(x) + g(x) is a horizontal line (y=4) parallel to x-axis at 1<x<5,
decreasing below x=1,
and increasing above x>5 . Its derivative at each points in between 1<x<5 is 0. *)
Plot[{f[x], g[x], (f[x] + g[x])}, {x, -3, 9},
 PlotStyle → {Directive[Green, Thick], Directive[Orange, Thick], Directive[Red, Thick]}]
```

Out[21]=