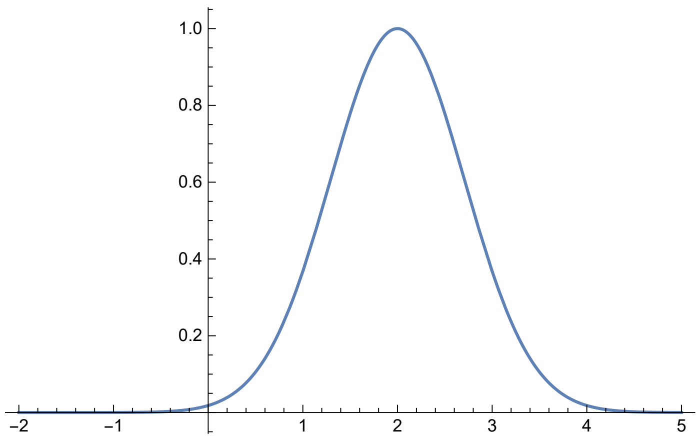
In[◦]:=
```
f[x_] := Exp[- (x - a) ^2];                              (*Not normalized*)
a = 2;
Plot[f[x], {x, -2, 5}, PlotRange → Full, PlotLegends → "Expressions"]
NIntegrate[f[x], {x, -∞, ∞}]
```

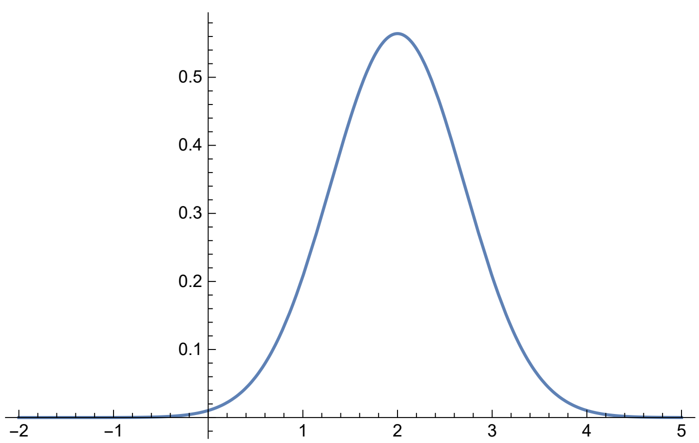
Out[◦]=




Out[◦]= 1.77245

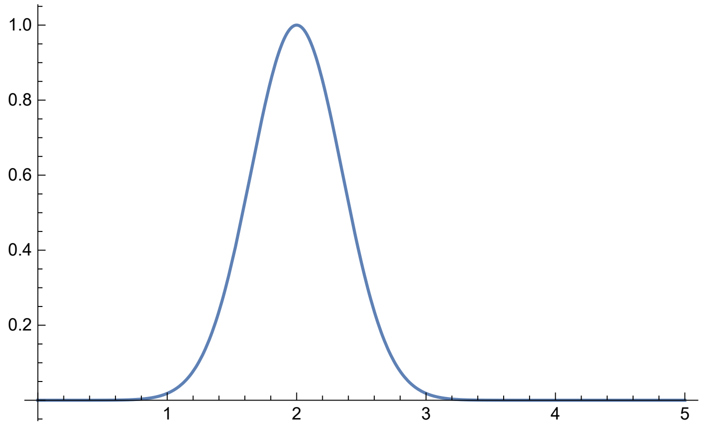
In[◦]:=
```
f[x_] := (1 / Sqrt[π]) * Exp[- (x - a) ^2];              (*Normalized*)
a = 2;
Plot[f[x], {x, -2, 5}, PlotRange → Full, PlotLegends → "Expressions"]
NIntegrate[f[x], {x, -∞, ∞}]
```

Out[◦]=




Out[◦]= 1.

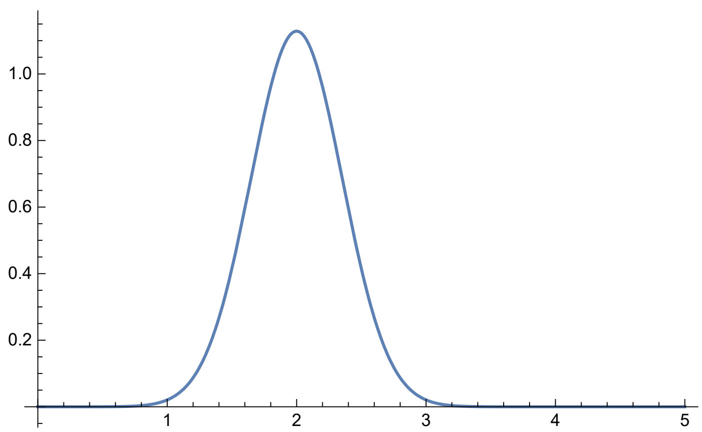
In[ ]:=
```
f[x_, k_] := Exp[-((x - a)^2)/k^2];
(*Not normalized*) (*What do k do in the exponential*)
a = 2;
Plot[f[x, k = 0.5], {x, 0, 5}, PlotRange → Full, PlotLegends → "Expressions"]
NIntegrate[f[x, k = 0.5], {x, -∞, ∞}]
2 * NIntegrate[f[x, k = 0.5], {x, -∞, ∞}]
```

Out[ ]=



Out[ ]= 0.886227

Out[ ]= 1.77245

In[ ]:=
```
f[x_, k_] := (1/(k Sqrt[π])) Exp[-((x - a)^2)/k^2];          (*Normalized*)
a = 2;
Plot[f[x, k = 0.5], {x, 0, 5}, PlotRange → Full, PlotLegends → "Expressions"]
NIntegrate[f[x, k = 0.5], {x, -∞, ∞}]
NIntegrate[f[x, k = 0.05], {x, -∞, ∞}]
```

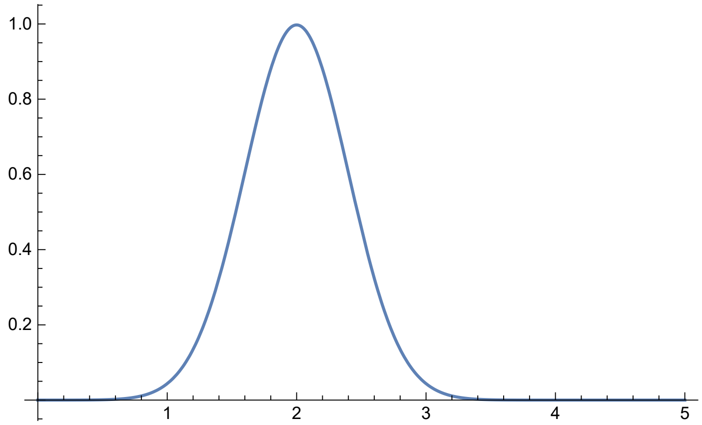
Out[ ]=



Out[ ]= 1.

Out[ ]= 1.

In[ ]:=
```
G[x_] := (1/(σ * Sqrt[2 Pi])) Exp[-((x - a)^2)/(2 σ^2)];
(*What do σ do in the exponenetial*)
a = 2; σ = 0.4;
Plot[G[x], {x, 0, 5}]
```

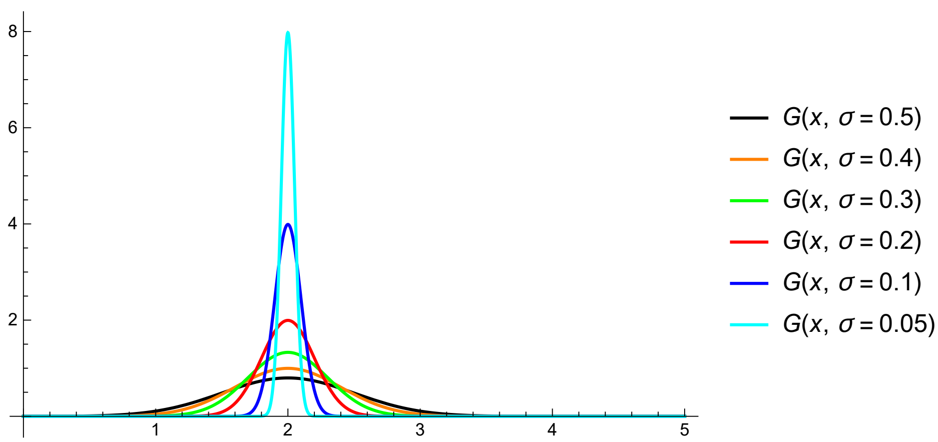
Out[ ]=



In[ ]:=
```
G[x_, σ_] := (1/(σ * Sqrt[2 Pi])) Exp[-((x - a)^2)/(2 σ^2)];
(*What do σ do in the exponenetial*)
a = 2;
Plot[{G[x, σ = 0.5], G[x, σ = 0.4], G[x, σ = 0.3],
  G[x, σ = 0.2], G[x, σ = 0.1], G[x, σ = 0.05]}, {x, 0, 5}, PlotRange → Full,
 PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
NIntegrate[G[x, σ = 0.5], {x, -∞, ∞}]
NIntegrate[G[x, σ = 0.05], {x, -∞, ∞}]
```

... General: Exp[−799.918] is too small to represent as a normalized machine number; precision may be lost.

Out[ ]=

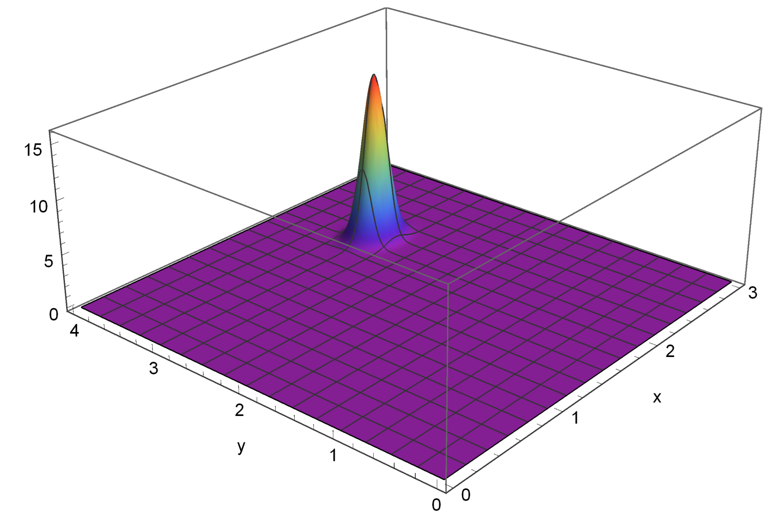



Out[ ]= 1.

Out[ ]= 1.

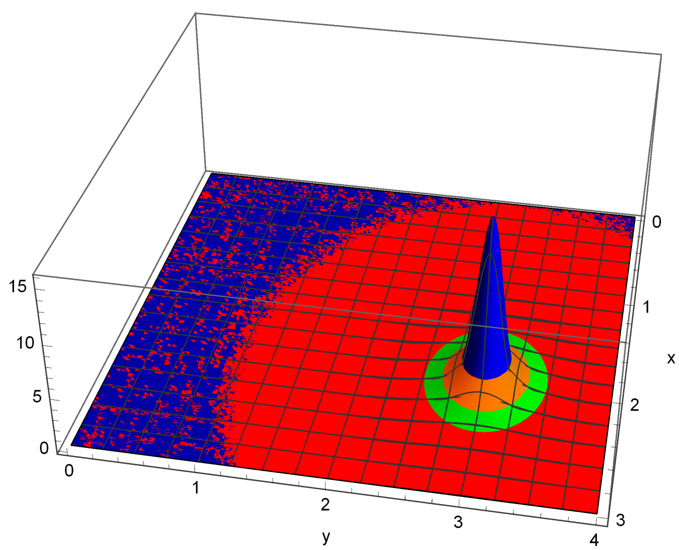
In[ ]:=
```
G2[x_, y_, σ_] := (1/((σ^2) * 2 Pi)) Exp[-(((x - a)^2) + ((y - b)^2)) / (2 σ^2)];
a = 2; b = 3;
Plot3D[G2[x, y, σ = 0.1],  {x,  0, 3}, {y, 0, 4}, PlotPoints → 100,
 PlotRange → Full, AxesLabel → {"x", "y"}, ColorFunction → "Rainbow"]
```

Out[ ]=



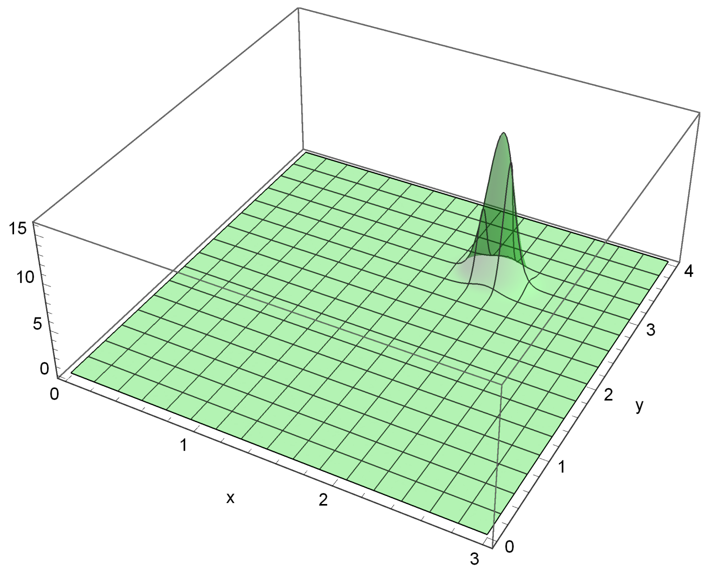

In[ ]:=
```
G2[x_, y_, σ_] := (1/((σ^2) * 2 Pi)) Exp[-(((x - a)^2) + ((y - b)^2)) / (2 σ^2)];
a = 2; b = 3;
Plot3D[{G2[x, y, σ = 0.4], G2[x, y, σ = 0.3], G2[x, y, σ = 0.2], G2[x, y, σ = 0.1]},
 {x,  0, 3}, {y, 0, 4}, PlotPoints → 100, PlotRange → Full,
 AxesLabel → {"x", "y"}, PlotStyle → {Red, Green, Orange, Blue, Opacity[0.3]}]
```

Out[ ]=



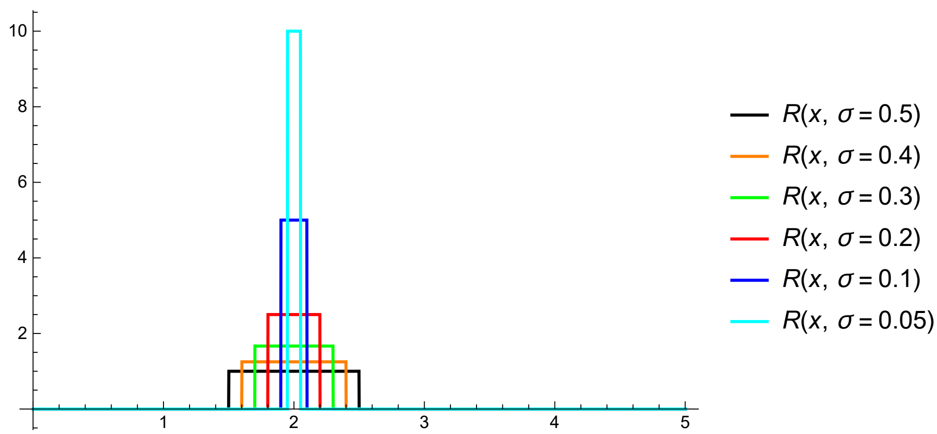
In[•]:=
```
G2[x_, y_] := (1/((σ^2) * 2 Pi)) Exp[-(((x - a)^2) + ((y - b)^2)) / (2 σ^2)];
a = 2; b = 3; σ = 0.1;
Plot3D[G2[x, y], {x, 0, 3}, {y, 0, 4}, PlotPoints → 100,
 PlotRange → Full, AxesLabel → {"x", "y"}, PlotStyle → {Green, Opacity[0.3]}]
```

Out[•]=




In[•]:=
```
R[x_, σ_] := Piecewise[{{1/(2 σ), -σ < x - a < σ}, {0, Modulus[x - a] > σ}}];
(*Rectangular function*)
a = 2;
Plot[{R[x, σ = 0.5], R[x, σ = 0.4], R[x, σ = 0.3],
   R[x, σ = 0.2], R[x, σ = 0.1], R[x, σ = 0.05]}, {x, 0, 5}, PlotRange → Full,
 PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
NIntegrate[R[x, σ = 0.5], {x, -∞, ∞}]
NIntegrate[R[x, σ = 0.05], {x, -∞, ∞}]
```

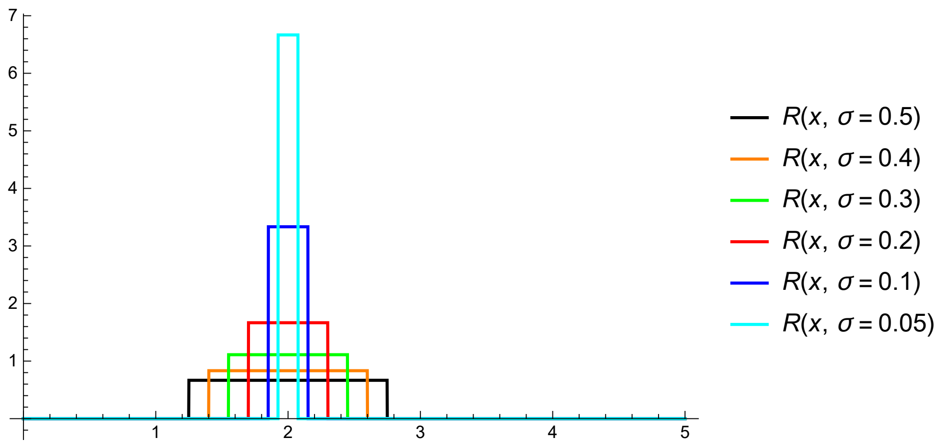
Out[•]=




Out[•]= 1.

Out[•]= 1.

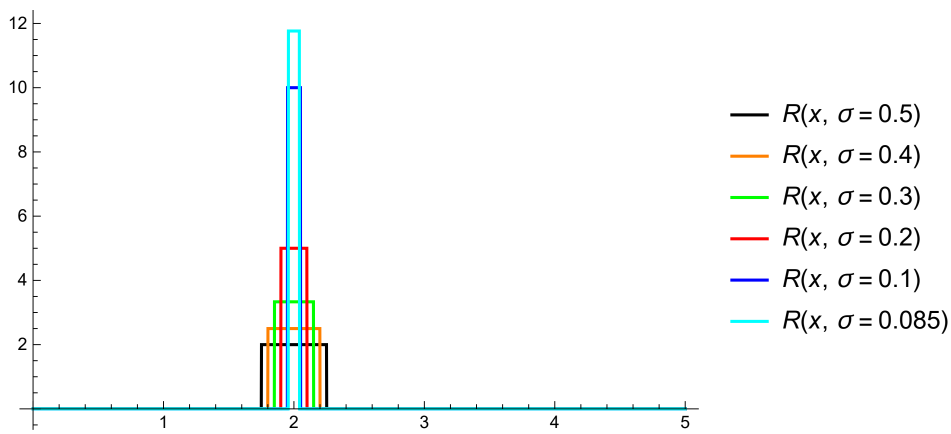
In[◦]:=
```
R[x_, σ_] := Piecewise[{{1/(3 σ), -3 σ/2 < x - a < 3 σ/2}, {0, Modulus[x - a] > 3 σ/2}}];
a = 2;
Plot[{R[x, σ = 0.5], R[x, σ = 0.4], R[x, σ = 0.3],
  R[x, σ = 0.2], R[x, σ = 0.1], R[x, σ = 0.05]}, {x, 0, 5}, PlotRange → Full,
 PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
NIntegrate[R[x, σ = 0.5], {x, -∞, ∞}]
NIntegrate[R[x, σ = 0.05], {x, -∞, ∞}]
```

Out[◦]=



Out[◦]= 1.

Out[◦]= 1.

In[◦]:=
```
R[x_, σ_] := Piecewise[{{1/σ, -σ/2 < x - a < σ/2}, {0, Modulus[x - a] > σ/2}}];
a = 2;
Plot[{R[x, σ = 0.5], R[x, σ = 0.4], R[x, σ = 0.3],
  R[x, σ = 0.2], R[x, σ = 0.1], R[x, σ = 0.085]}, {x, 0, 5}, PlotRange → Full,
 PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
NIntegrate[R[x, σ = 0.5], {x, -∞, ∞}]
NIntegrate[R[x, σ = 0.05], {x, -∞, ∞}]
```

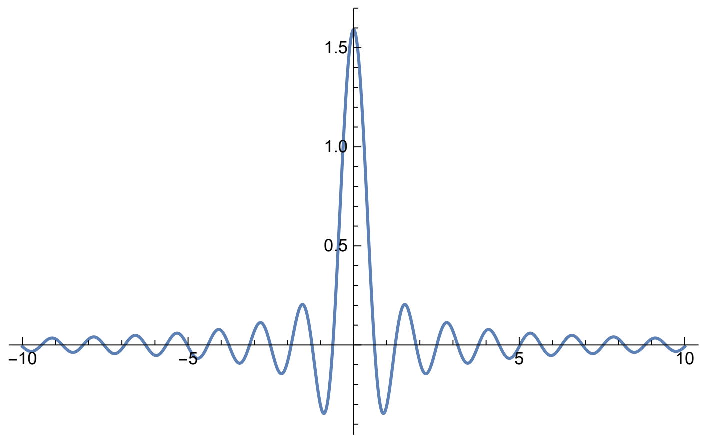
Out[◦]=



Out[◦]= 1.

Out[◦]= 1.

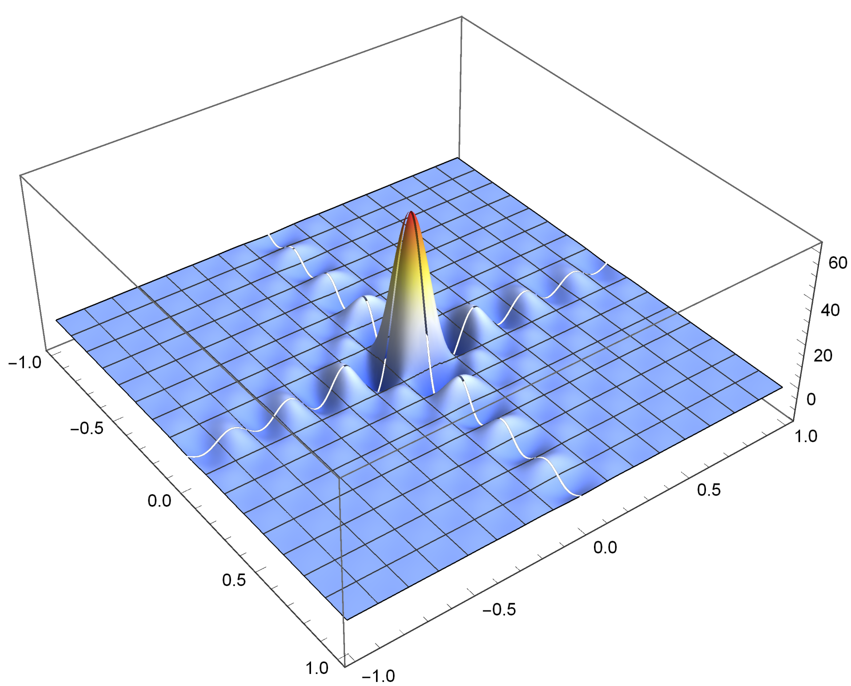
In[∘]:=
```
s[x_] := Sin[g x] / (π x);
Limit[s[x], x → 0]
g = 5;
Plot[s[x], {x, -10, 10}, PlotRange → Full, PlotLegends → "Expressions"]
```

Out[∘]= $\frac{g}{\pi}$

Out[∘]=



In[∘]:=
```
s[x_, y_] := (Sin[g x] / (π x)) (Sin[g y] / (π y));
Limit[s[x], x → 0]
g = 25;
Plot3D[s[x, y], {x, -1, 1}, {y, -1, 1}, PlotPoints → 100,
 PlotRange → Full, ColorFunction → "TemperatureMap"]
```

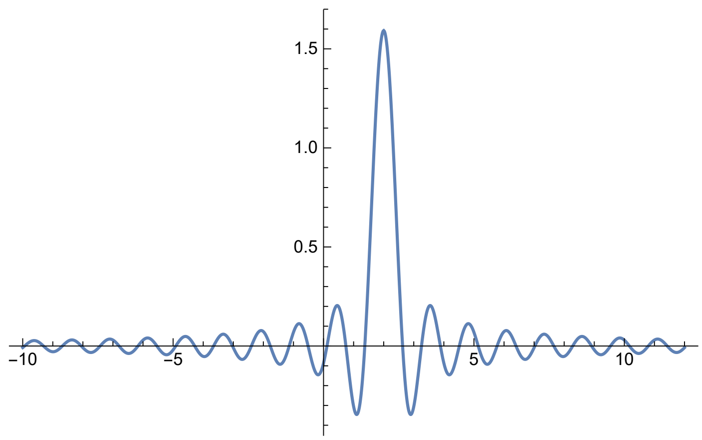
Out[∘]= $\frac{5}{\pi}$

Out[∘]=


In[∘]:=
```
s[x_] := Sin[g (x - a)] / (π (x - a));
Limit[s[x], x → 0]
a = 2; g = 5;
Plot[s[x], {x, -10, 10 + a}, PlotRange → Full, PlotLegends → "Expressions"]
```

Out[∘]= $\frac{\text{Sin}[50]}{2\pi}$

Out[∘]=



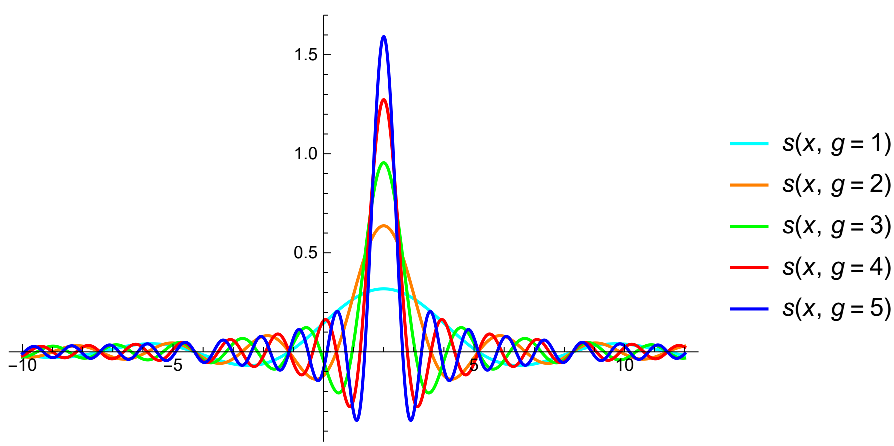

In[∘]:=
```
S[x_, p_] := Sin[p (x - a)] / (π (x - a));
Limit[S[x, p], x → 0]
Limit[S[x, p], p → ∞]
```

Out[∘]= $\frac{\text{Sin}[2\,p]}{2\pi}$

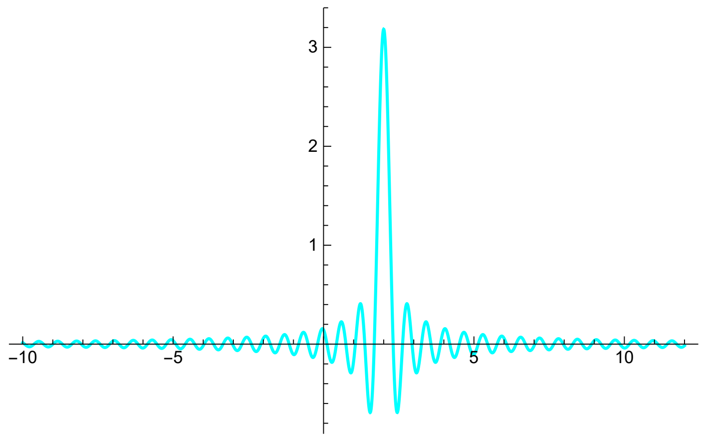
Out[∘]= ConditionalExpression[Indeterminate, x ∈ ℝ]

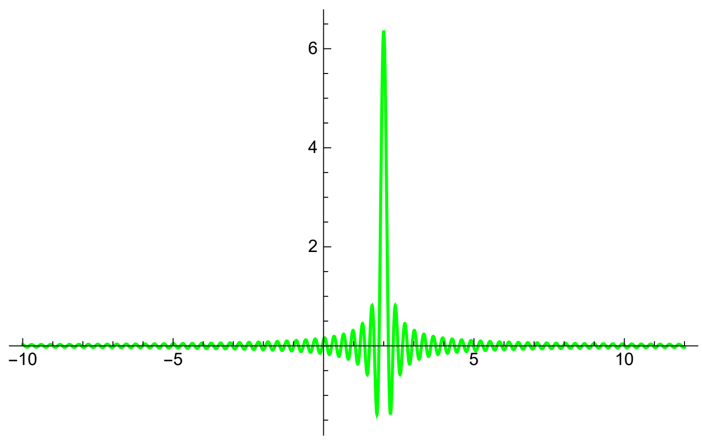
In[∘]:=
```
s[x_, g_] := Sin[g (x - a)] / (π (x - a));
a = 2;
Plot[{s[x, g = 1], s[x, g = 2], s[x, g = 3], s[x, g = 4], s[x, g = 5]}, {x, -10, 10 + a},
 PlotRange → Full, PlotStyle → {Cyan, Orange, Green, Red, Blue}, PlotLegends → "Expressions"]
```

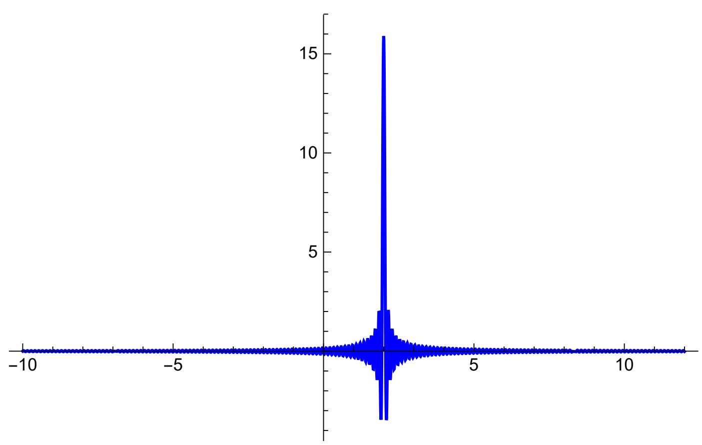
Out[∘]=



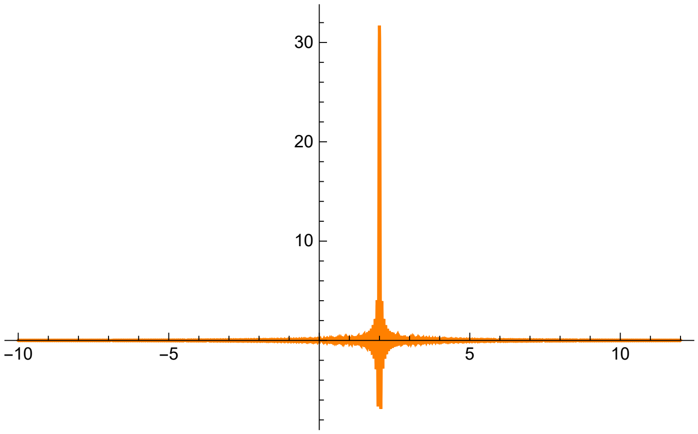
In[ ]:=
```
s[x_, g_] := Sin[g (x - a)] / (π (x - a));
a = 2;
Plot[s[x, g = 10], {x, -10, 10 + a}, PlotRange → Full,
 PlotStyle → Cyan, PlotLegends → "Expressions"]
Plot[s[x, g = 20], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Green]
Plot[s[x, g = 50], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Blue]
Plot[s[x, g = 100], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Orange]
Plot[s[x, g = 1000], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Purple]
Plot[s[x, g = 1000], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Red]
```

Out[ ]=


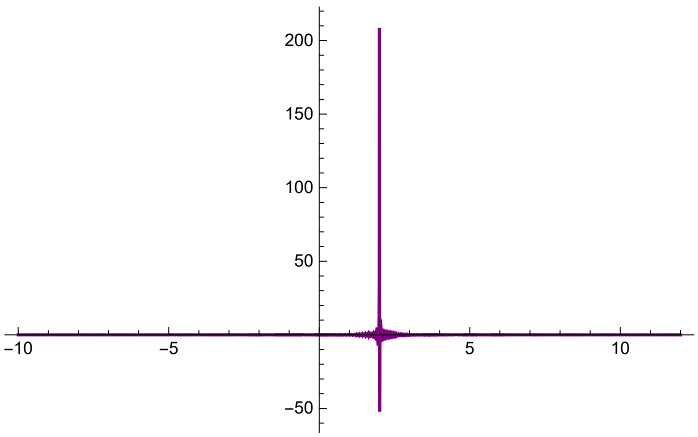

Out[ ]=



Out[ ]=


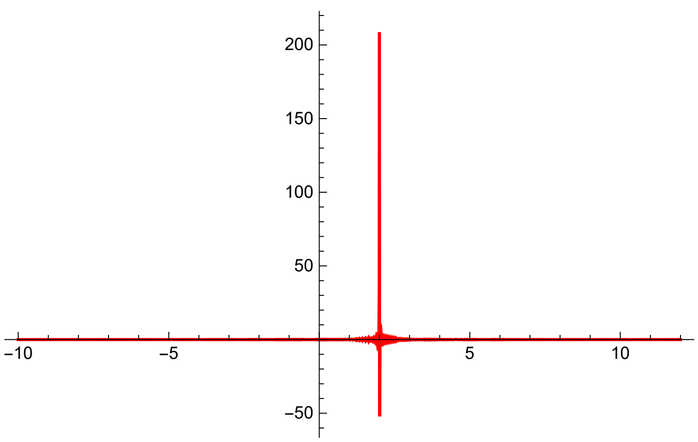
Out[ ]=


30
20
10
–10
–5
5
10

Out[ ]=

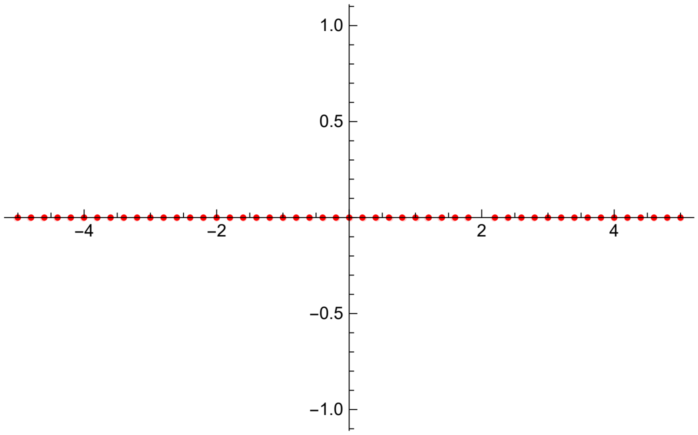

200
150
100
50
–10
–5
5
10
–50

Out[ ]=


200
150
100
50
–10
–5
5
10
–50

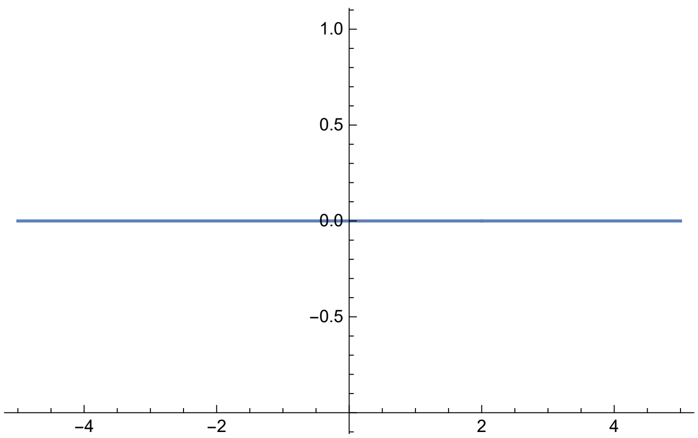
In[ ]:=
```
f[x_] := DiracDelta[x - a];
a = 2;
DiscretePlot[f[x], {x, -5, 5, 0.2}, PlotStyle → {Red, Thick}]
```

Out[ ]=



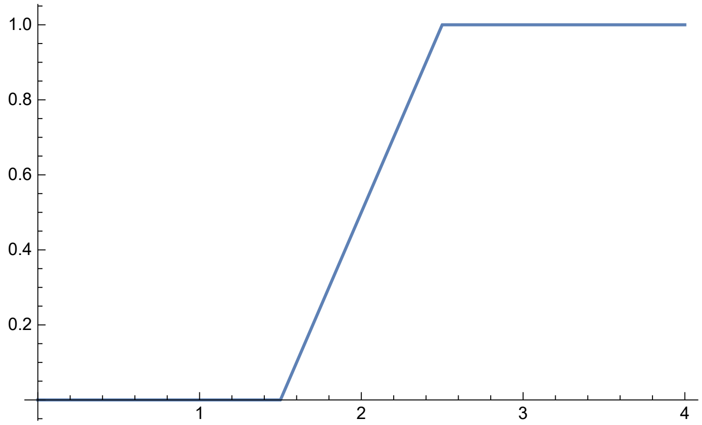

In[ ]:=
```
f[x_] := DiracDelta[x - a];
a = 2;
Plot[f[x], {x, -5, 5}, AxesOrigin → {0, -1}]
```

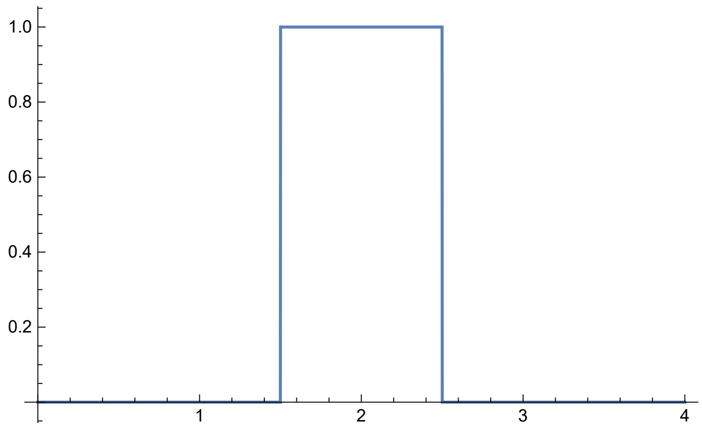
Out[ ]=




In[ ]:=
```
Exit
```

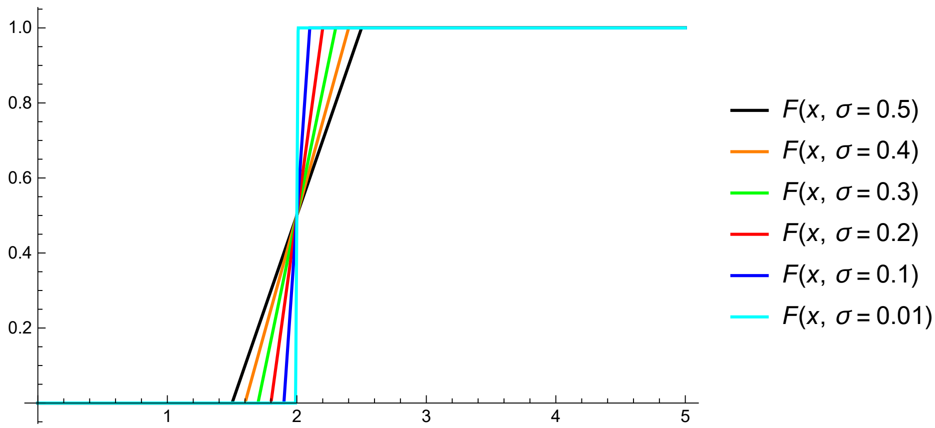
In[∘]:=
```
F[x_, σ_] := Piecewise[{{0, x < a - σ}, {(1/(2 σ)) (x - a + σ), -σ < x - a < σ}, {1, x > a + σ}}];
(*Ramp function*)
der[x_, σ_] =
  D[F[x, σ], x];
(*Derivative of Ramp function is Rectangular function R*)
a = 2;
Plot[F[x, σ = 0.5], {x, 0, 4}, PlotRange → Full]
Plot[der[x, σ = 0.5], {x, 0, 4}, PlotRange → Full]
```

Out[∘]=



Out[∘]=


In[∘]:=
```
F[x_, σ_] := Piecewise[{{0, x < a - σ}, {(1/(2 σ)) (x - a + σ), -σ < x - a < σ}, {1, x > a + σ}}];
a = 2;
Plot[{F[x, σ = 0.5], F[x, σ = 0.4], F[x, σ = 0.3],
  F[x, σ = 0.2], F[x, σ = 0.1], F[x, σ = 0.01]}, {x, 0, 5}, PlotRange → Full,
 PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
```

Out[∘]=




In[∘]:=
```
Exit
```

In[∘]:=
```
F[x_, σ_] := Piecewise[{{0, x < a - σ}, {(1/(2 σ)) (x - a + σ), -σ < x - a < σ}, {1, x > a + σ}}];
H[x_, a_] =
  Limit[F[x, σ], σ → 0];
(*For limit σ→0 Ramp function becomes Heaviside unit step function*)
H[x, a]
delta[x_, a_] =
 D[H[x, a], x];
(*Derivative of dicontinuous Heaviside unit step function is Dirac delta function*)
delta[x, a]
Integrate[delta[x, a = 2], {x, -5, 5}]
```

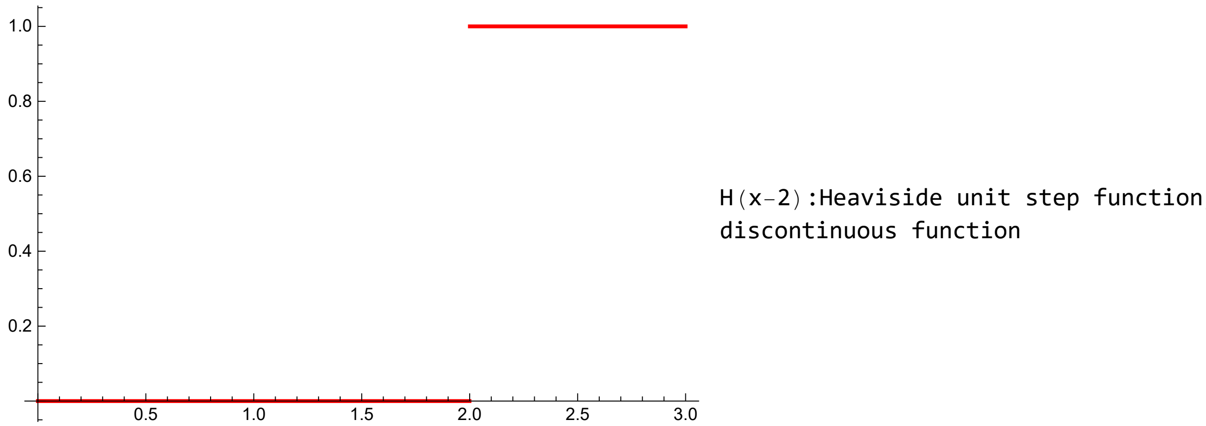
Out[∘]=
```
⎧ 1              a < x
⎨ 0              a > x
⎩ Indeterminate  True
```

Out[∘]=
```
⎧ 0              a - x < 0 || a - x > 0
⎩ Indeterminate  True
```

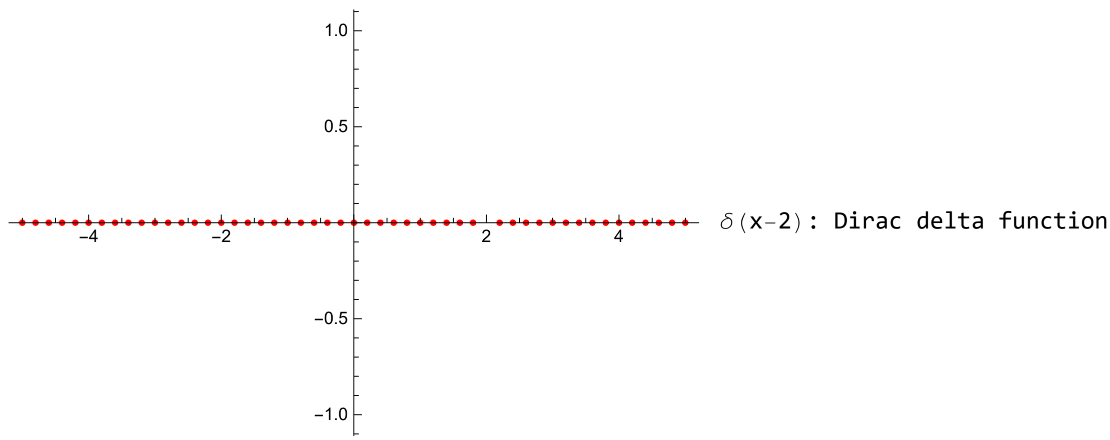
Out[∘]=
```
0
```

```
In[∘]:= a = 2;
Plot[H[x, a], {x, 0, 3}, PlotStyle → {Red, Thick},
 PlotLegends → "H(x-2):Heaviside unit step function,\ndiscontinuous function"]
DiscretePlot[delta[x, a], {x, -5, 5, 0.2}, PlotStyle → {Red, Thick},
 PlotLegends → "δ(x-2): Dirac delta function"]
```

Out[∘]=

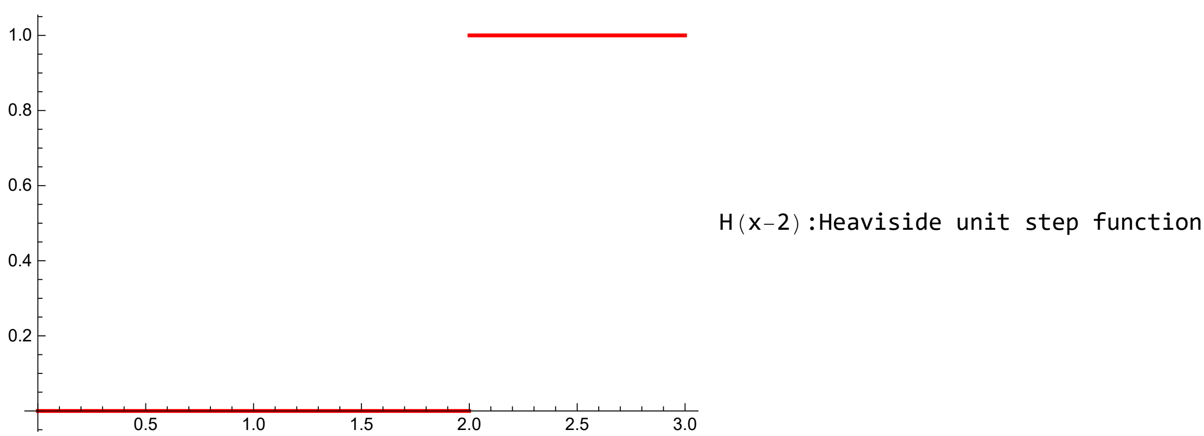



Out[∘]=




```
In[∘]:= Plot[UnitStep[x - 2], {x, 0, 3}, PlotStyle → {Red, Thick},
 PlotLegends → "H(x-2):Heaviside unit step function"]
```

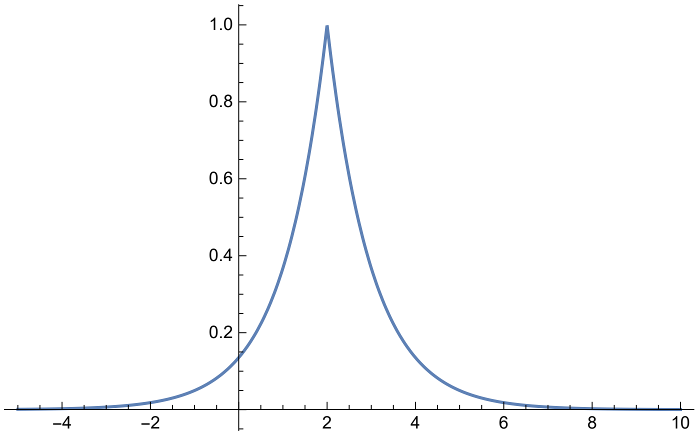
Out[∘]=




```
In[∘]:= Exit
```

In[1]:=
```
f[x_] := Exp[-Abs[x - a]];
a = 2;
Plot[f[x], {x, -5, 10}, PlotRange → Full, PlotLegends → "Expressions"]
```

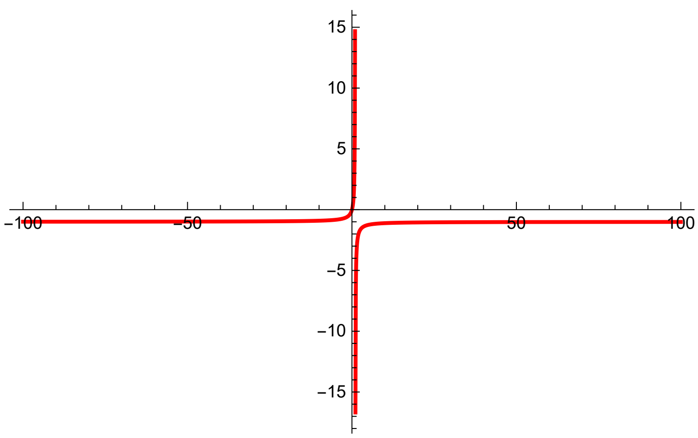
Out[3]=



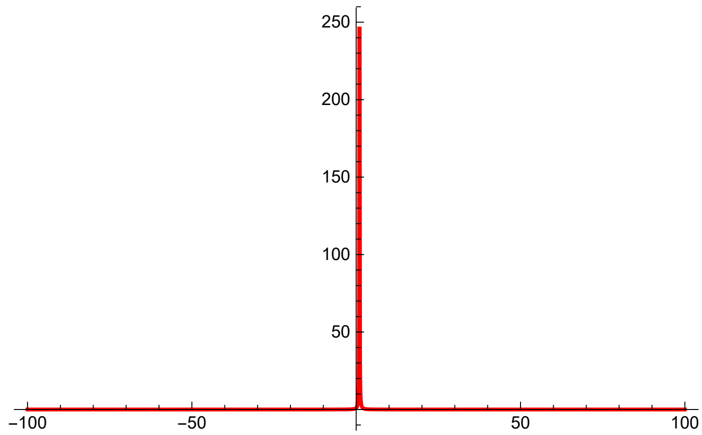

In[4]:=
```
f[z_] := z / (1 - z);
der2[z_] = D[f[z], z]
Plot[f[z], {z, -100, 100}, PlotRange → Full, PlotStyle → {Red, Thick}]
Plot[der2[z], {z, -100, 100}, PlotRange → Full, PlotStyle → {Red, Thick}]
Plot[der2[z], {z, -1, 1}, PlotRange → Full, PlotStyle → {Red, Thick}]
Plot[der2[z], {z, -0.1, 0.1}, PlotRange → Full, PlotStyle → {Red, Thick}]
```

Out[5]= $\frac{1}{1-z}+\frac{z}{(1-z)^2}$

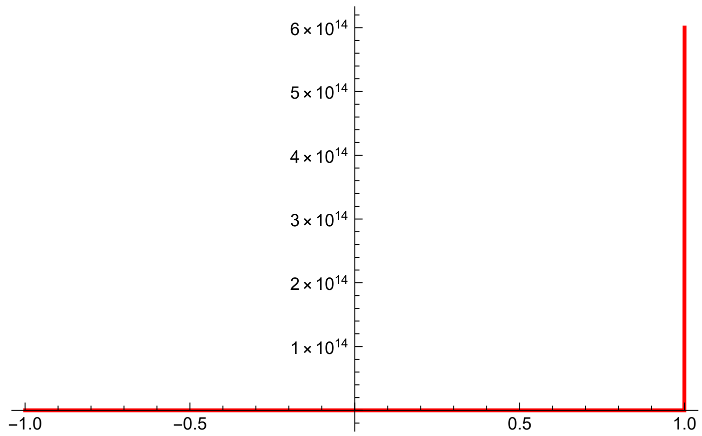
Out[6]=



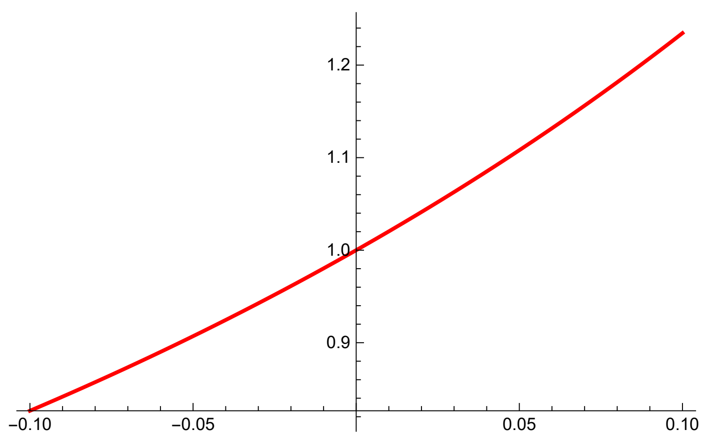
Out[7]=




Out[8]=



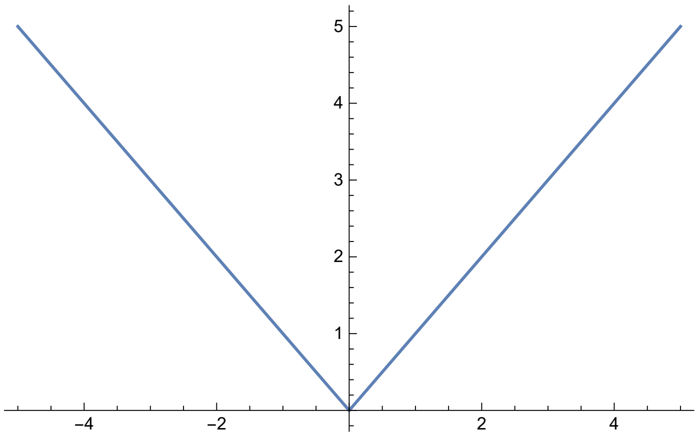

Out[9]=



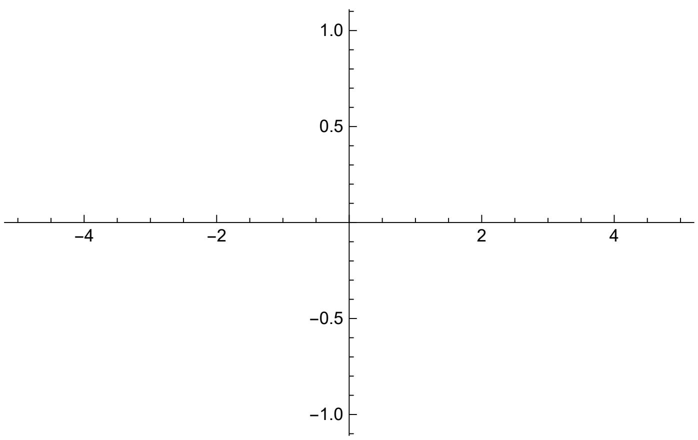
```
In[10]:= h[x_] := Abs[x];
derh[x_] = D[h[x], x]
Plot[h[x], {x, -5, 5}]
Plot[derh[x], {x, -5, 5}, PlotRange → Full, PlotStyle → {Red, Thick}]

Out[11]= Abs′[x]
```

Out[12]=




Out[13]=



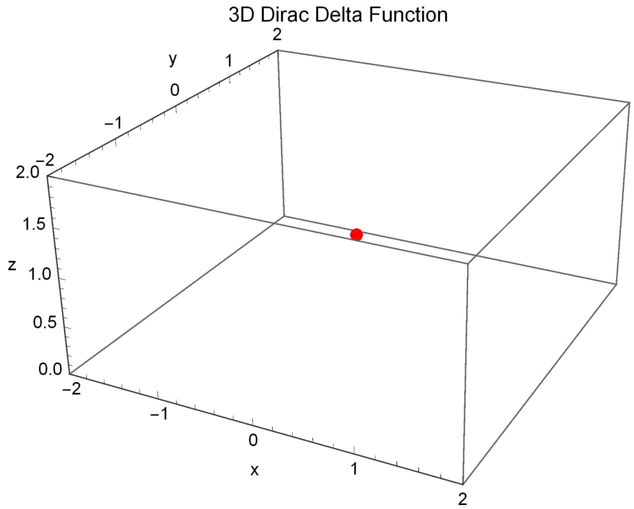

```
In[ ]:= Exit

In[ ]:= f[x_] := x^2;
Integrate[f[x] × DiracDelta[x - 2], {x, -5, 5}]

Out[ ]= 4

In[ ]:= G3[x_, y_, z_] :=
   (1 / ((σ^3) * (2 Pi) ^ (3 / 2))) Exp[- (((x - a) ^2) + ((y - b) ^2) + ((z - x) ^2)) / (2 σ^2)];
a = 2; b = 3; c = 2, σ = 0.1;
ContourPlot3D[G3[x, y, z], {x, 0, 5},
 {y, 0, 5}, {z, 0, 5}, PlotPoints → 100, PlotRange → Full]
```

In[ ]:=
```
Graphics3D[{Red, PointSize[Large], Point[{0, 0, 1}]}, Axes → True,
  AxesLabel → {"x", "y", "z"}, PlotRange → {{-2, 2}, {-2, 2}, {0, 2}},
  PlotLabel → "3D Dirac Delta Function"]
```

Out[ ]=



In[ ]:=
```
Graphics3D[{Blue, PointSize[Large], Point[{-1, -1, 1}], Red,
   PointSize[Large], Point[{1, 1, 1.5}], Green, PointSize[Large], Point[{0, 0, 2}]},
  Axes → True, AxesLabel → {"x", "y", "z"}, PlotRange → {{-2, 2}, {-2, 2}, {0, 3}},
  PlotLabel → "Multiple Dirac Delta Functions in 3D"]
```

Out[ ]=