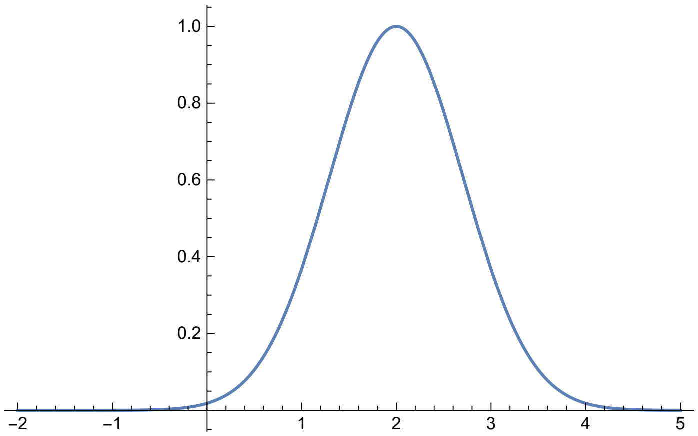
In[∘]:=
```
f[x_] := Exp[- (x - a) ^2];                    (*Not normalized*)
a = 2;
Plot[f[x], {x, -2, 5}, PlotRange → Full, PlotLegends → "Expressions"]
NIntegrate[f[x], {x, -∞, ∞}]
```

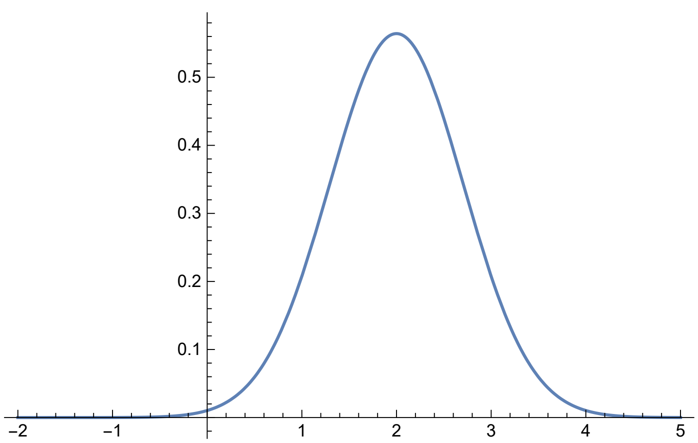
Out[∘]=



Out[∘]= 1.77245

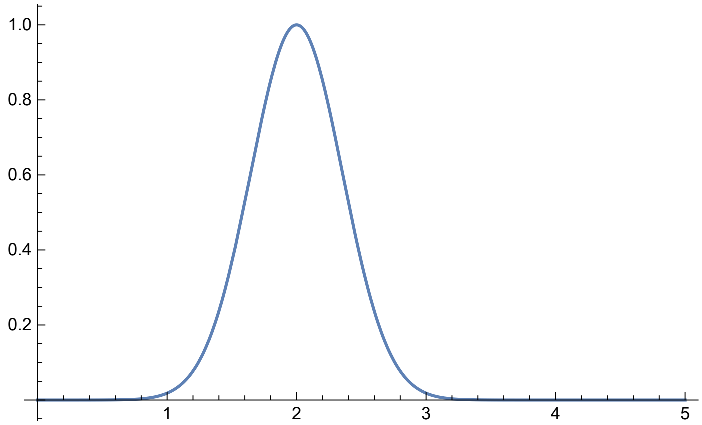
In[∘]:=
```
f[x_] := (1 / Sqrt[π]) * Exp[- (x - a) ^2];          (*Normalized*)
a = 2;
Plot[f[x], {x, -2, 5}, PlotRange → Full, PlotLegends → "Expressions"]
NIntegrate[f[x], {x, -∞, ∞}]
```

Out[∘]=



Out[∘]= 1.

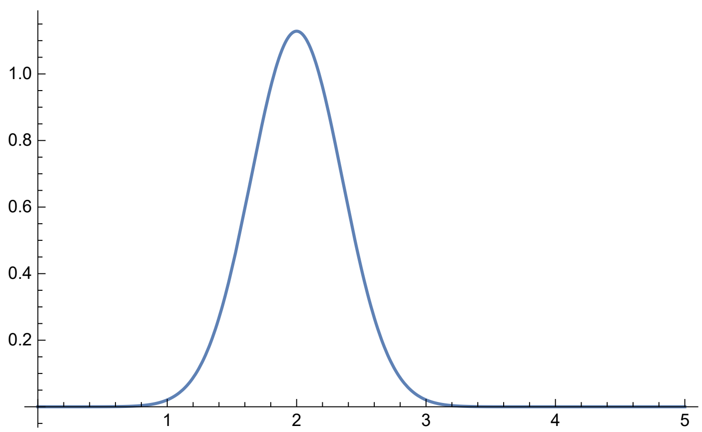
In[ ]:=
```
f[x_, k_] := Exp[-((x - a)^2)/k^2];
(*Not normalized*) (*What do k do in the exponential*)
a = 2;
Plot[f[x, k = 0.5], {x, 0, 5}, PlotRange → Full, PlotLegends → "Expressions"]
NIntegrate[f[x, k = 0.5], {x, -∞, ∞}]
2 * NIntegrate[f[x, k = 0.5], {x, -∞, ∞}]
```

Out[ ]=


Out[ ]= 0.886227

Out[ ]= 1.77245

In[ ]:=
```
f[x_, k_] := (1/(k Sqrt[π])) Exp[-((x - a)^2)/k^2];          (*Normalized*)
a = 2;
Plot[f[x, k = 0.5], {x, 0, 5}, PlotRange → Full, PlotLegends → "Expressions"]
NIntegrate[f[x, k = 0.5], {x, -∞, ∞}]
NIntegrate[f[x, k = 0.05], {x, -∞, ∞}]
```

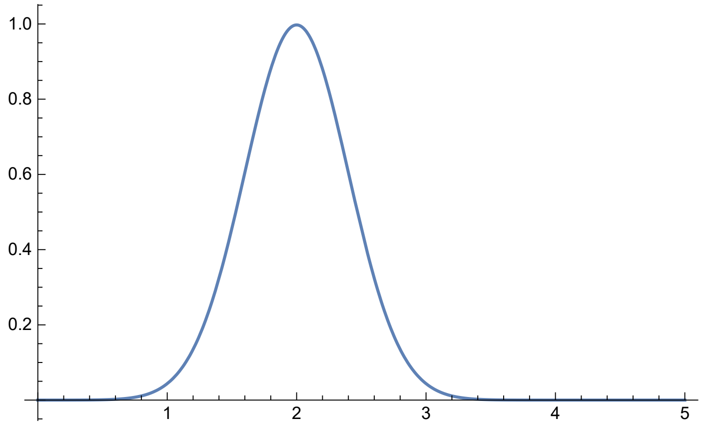
Out[ ]=


Out[ ]= 1.

Out[ ]= 1.

In[◦]:=
```
G[x_] := (1/(σ*Sqrt[2 Pi])) Exp[-((x - a)^2)/(2 σ^2)];
(*What do σ do in the exponenetial*)
a = 2; σ = 0.4;
Plot[G[x], {x, 0, 5}]
```

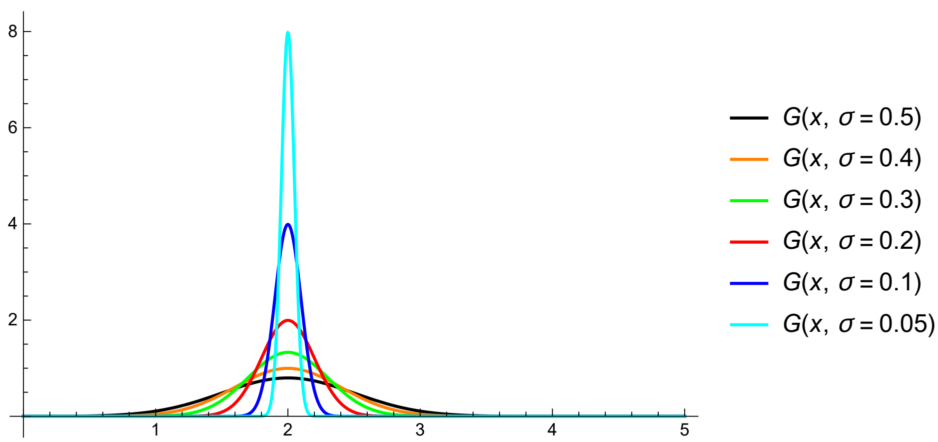
Out[◦]=



In[◦]:=
```
G[x_, σ_] := (1/(σ*Sqrt[2 Pi])) Exp[-((x - a)^2)/(2 σ^2)];
(*What do σ do in the exponenetial*)
a = 2;
Plot[{G[x, σ = 0.5], G[x, σ = 0.4], G[x, σ = 0.3],
  G[x, σ = 0.2], G[x, σ = 0.1], G[x, σ = 0.05]}, {x, 0, 5}, PlotRange → Full,
 PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
NIntegrate[G[x, σ = 0.5], {x, -∞, ∞}]
NIntegrate[G[x, σ = 0.05], {x, -∞, ∞}]
```

General: Exp[−799.918] is too small to represent as a normalized machine number; precision may be lost.

Out[◦]=

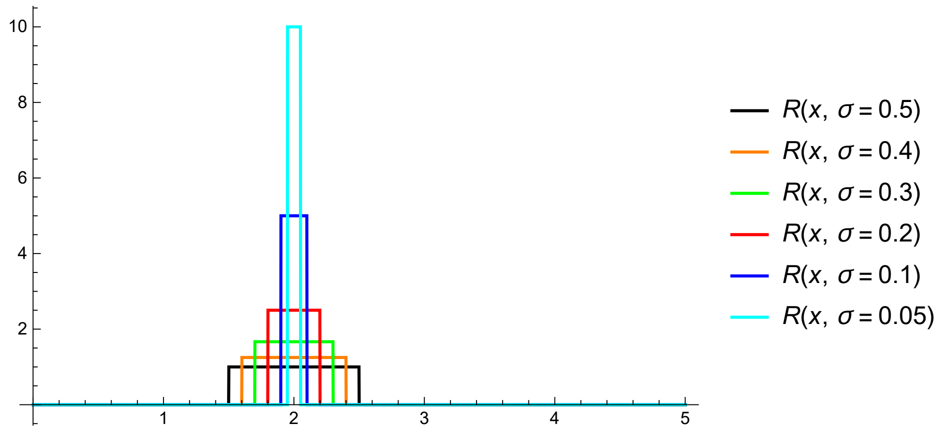


Out[◦]= 1.

Out[◦]= 1.

In[∘]:=
```
R[x_, σ_] := Piecewise[{{1/(2 σ), -σ < x - a < σ}, {0, Modulus[x - a] > σ}}];
(*Rectangular function*)
a = 2;
Plot[{R[x, σ = 0.5], R[x, σ = 0.4], R[x, σ = 0.3],
  R[x, σ = 0.2], R[x, σ = 0.1], R[x, σ = 0.05]}, {x, 0, 5}, PlotRange → Full,
 PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
NIntegrate[R[x, σ = 0.5], {x, -∞, ∞}]
NIntegrate[R[x, σ = 0.05], {x, -∞, ∞}]
```

Out[∘]=

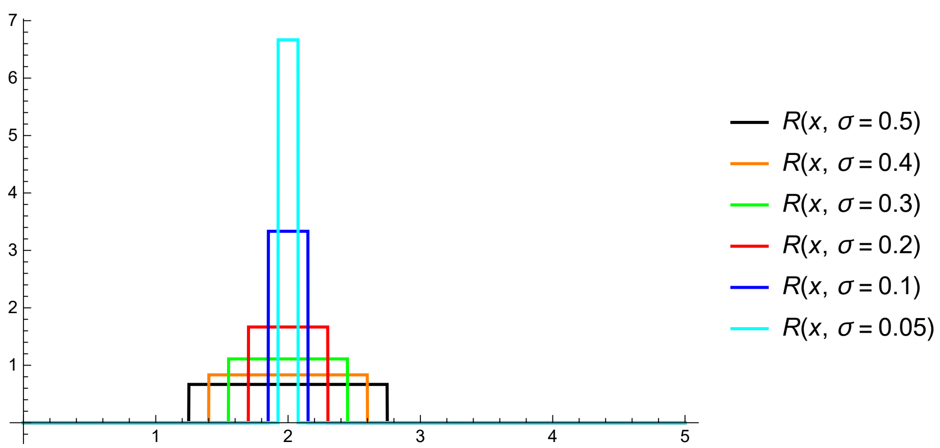


Out[∘]= 1.

Out[∘]= 1.

In[∘]:=
```
R[x_, σ_] := Piecewise[{{1/(3 σ), -3 σ/2 < x - a < 3 σ/2}, {0, Modulus[x - a] > 3 σ/2}}];
a = 2;
Plot[{R[x, σ = 0.5], R[x, σ = 0.4], R[x, σ = 0.3],
  R[x, σ = 0.2], R[x, σ = 0.1], R[x, σ = 0.05]}, {x, 0, 5}, PlotRange → Full,
 PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
NIntegrate[R[x, σ = 0.5], {x, -∞, ∞}]
NIntegrate[R[x, σ = 0.05], {x, -∞, ∞}]
```

Out[∘]=

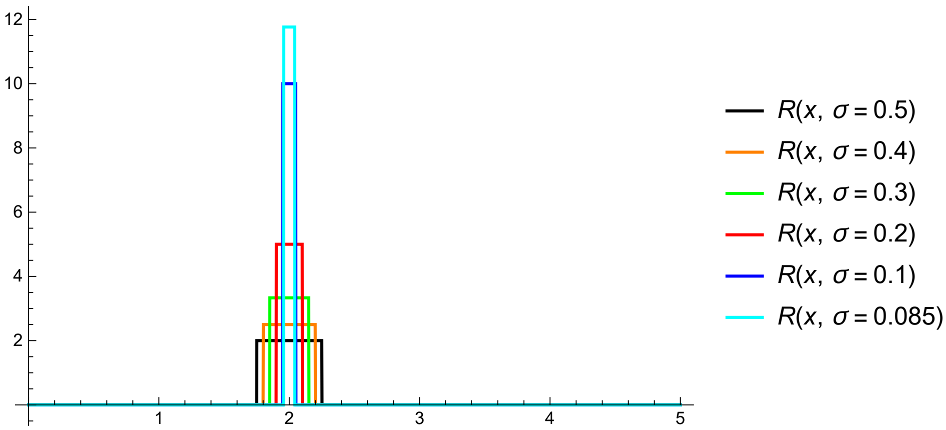


Out[∘]= 1.

Out[∘]= 1.

In[∘]:=
```
R[x_, σ_] := Piecewise[{{1/σ, -σ/2 < x - a < σ/2}, {0, Modulus[x - a] > σ/2}}];
a = 2;
Plot[{R[x, σ = 0.5], R[x, σ = 0.4], R[x, σ = 0.3],
  R[x, σ = 0.2], R[x, σ = 0.1], R[x, σ = 0.085]}, {x, 0, 5}, PlotRange → Full,
 PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
NIntegrate[R[x, σ = 0.5], {x, -∞, ∞}]
NIntegrate[R[x, σ = 0.05], {x, -∞, ∞}]
```

Out[∘]=



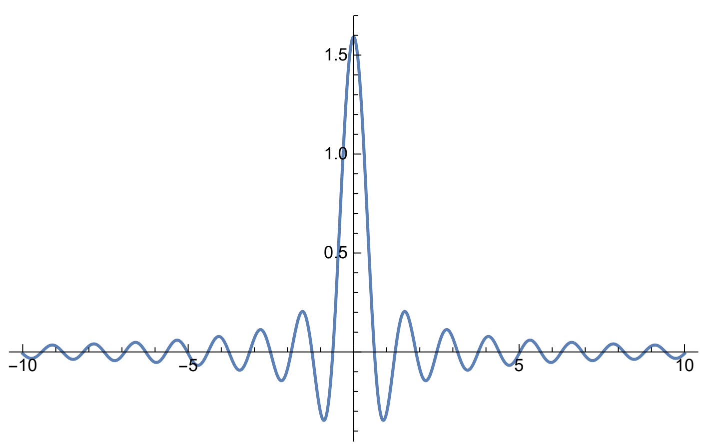
Out[∘]= 1.

Out[∘]= 1.

In[∘]:=
```
s[x_] := Sin[g x] / (π x);
Limit[s[x], x → 0]
g = 5;
Plot[s[x], {x, -10, 10}, PlotRange → Full, PlotLegends → "Expressions"]
```

Out[∘]= $\frac{g}{\pi}$

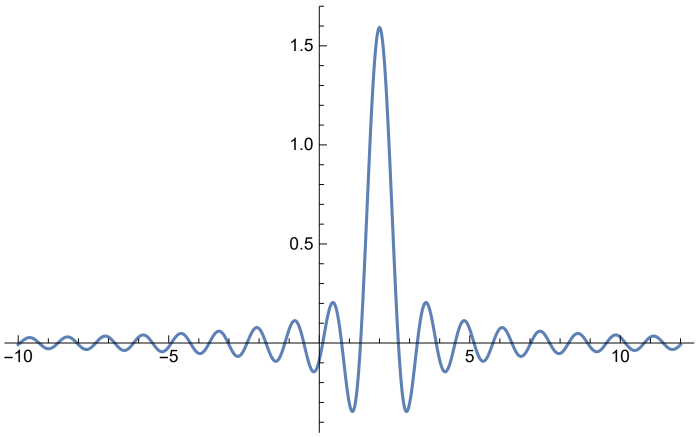
Out[∘]=



In[∘]:=
```
s[x_] := Sin[g (x - a)] / (π (x - a));
Limit[s[x], x → 0]
a = 2; g = 5;
Plot[s[x], {x, -10, 10 + a}, PlotRange → Full, PlotLegends → "Expressions"]
```

Out[∘]= $\frac{\text{Sin}[10]}{2\,\pi}$

Out[∘]=



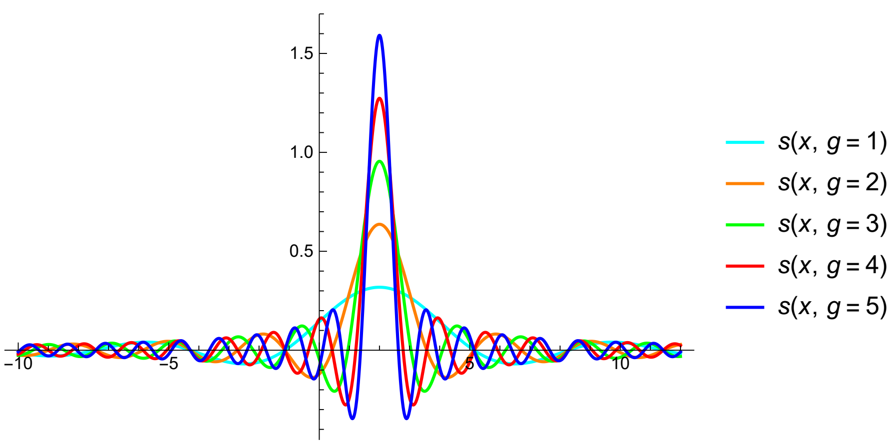
In[∘]:=
```
S[x_, p_] := Sin[p (x - a)] / (π (x - a));
Limit[S[x, p], x → 0]
Limit[S[x, p], p → ∞]
```

Out[∘]= $\frac{\text{Sin}[2\,p]}{2\,\pi}$

Out[∘]= ConditionalExpression[Indeterminate, x ∈ ℝ]

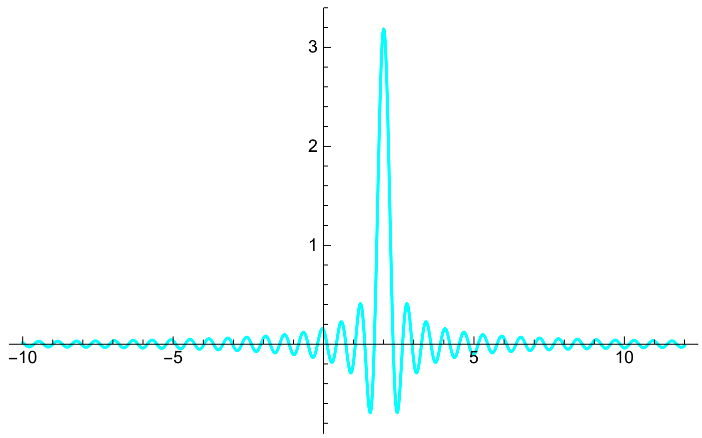
In[∘]:=
```
s[x_, g_] := Sin[g (x - a)] / (π (x - a));
a = 2;
Plot[{s[x, g = 1], s[x, g = 2], s[x, g = 3], s[x, g = 4], s[x, g = 5]}, {x, -10, 10 + a},
 PlotRange → Full, PlotStyle → {Cyan, Orange, Green, Red, Blue}, PlotLegends → "Expressions"]
```

Out[∘]=

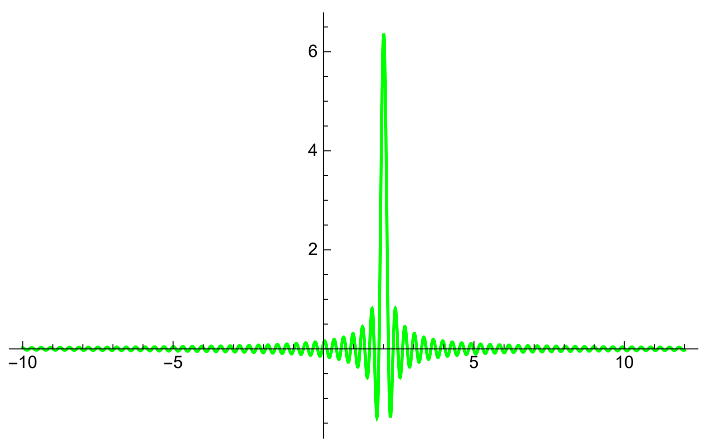


In[ ]:=
```
s[x_, g_] := Sin[g (x - a)] / (π (x - a));
a = 2;
Plot[s[x, g = 10], {x, -10, 10 + a}, PlotRange → Full,
 PlotStyle → Cyan, PlotLegends → "Expressions"]
Plot[s[x, g = 20], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Green]
Plot[s[x, g = 50], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Blue]
Plot[s[x, g = 100], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Orange]
Plot[s[x, g = 1000], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Purple]
Plot[s[x, g = 1000], {x, -10, 10 + a}, PlotRange → Full, PlotStyle → Red]
```

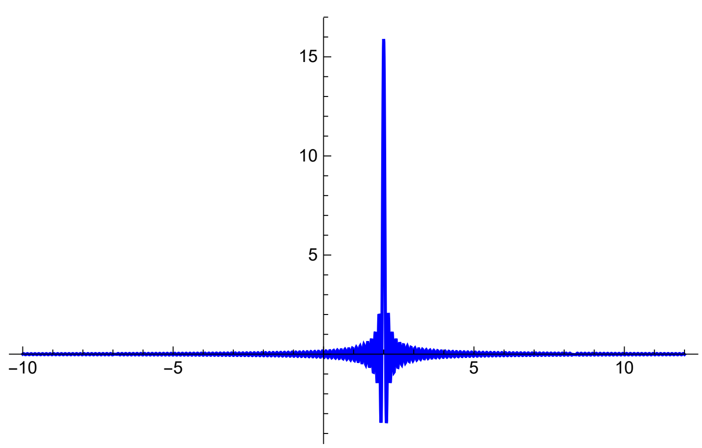
Out[ ]=



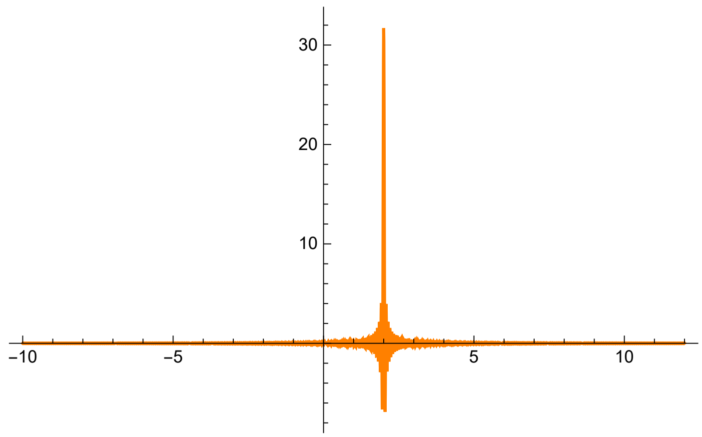
Out[ ]=



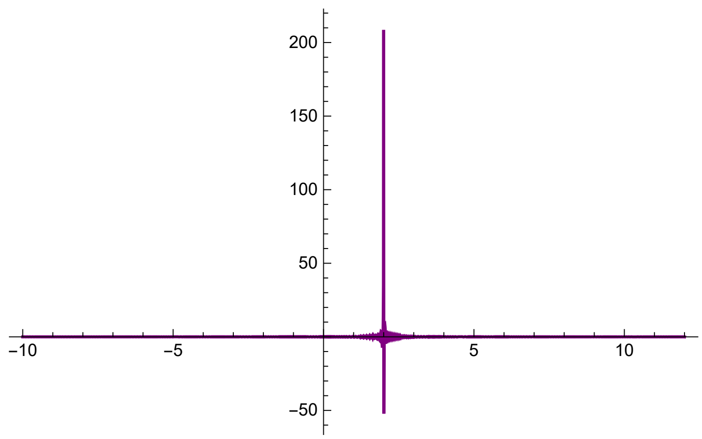
Out[ ]=



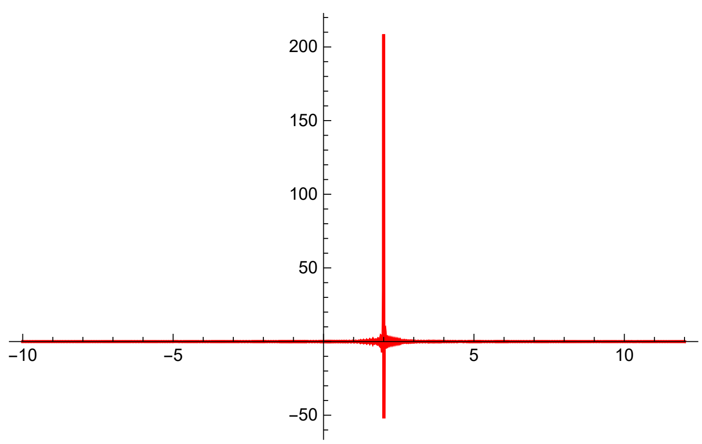
Out[ ]=


30
20
10
–10
–5
5
10

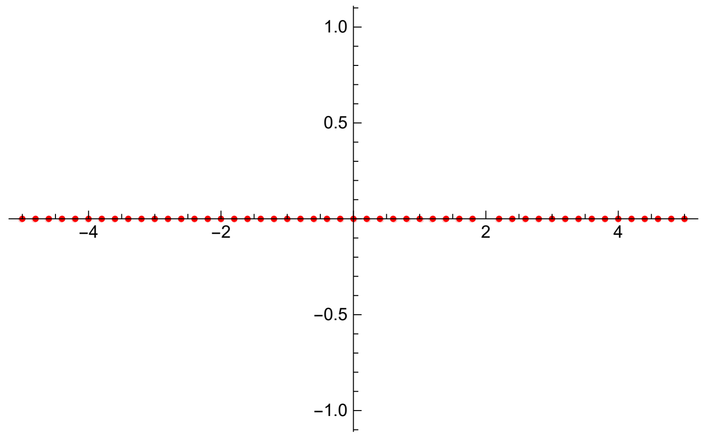
Out[ ]=


200
150
100
50
–10
–5
5
10
–50

Out[ ]=

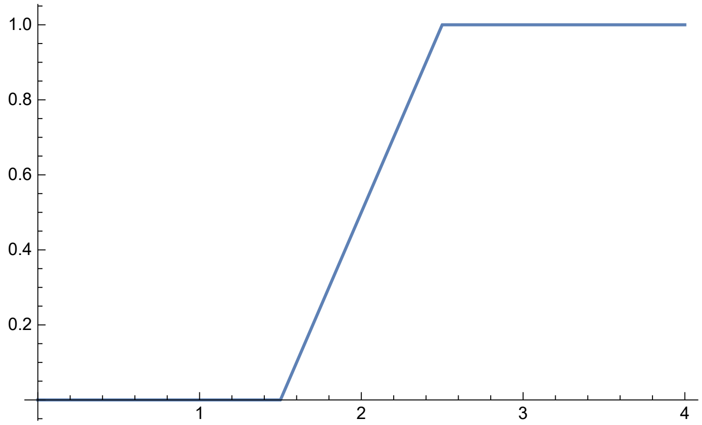

200
150
100
50
–10
–5
5
10
–50

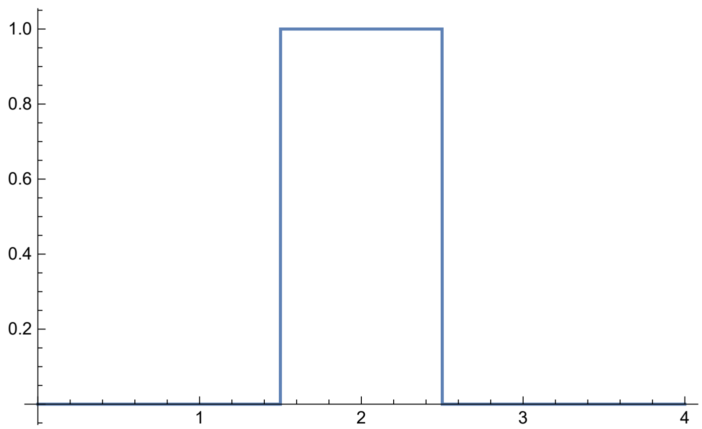
In[ ]:=
```
f[x_] := DiracDelta[x - a];
a = 2;
DiscretePlot[f[x], {x, -5, 5, 0.2}, PlotStyle → {Red, Thick}]
```

Out[ ]=


In[ ]:=
```
Exit
```

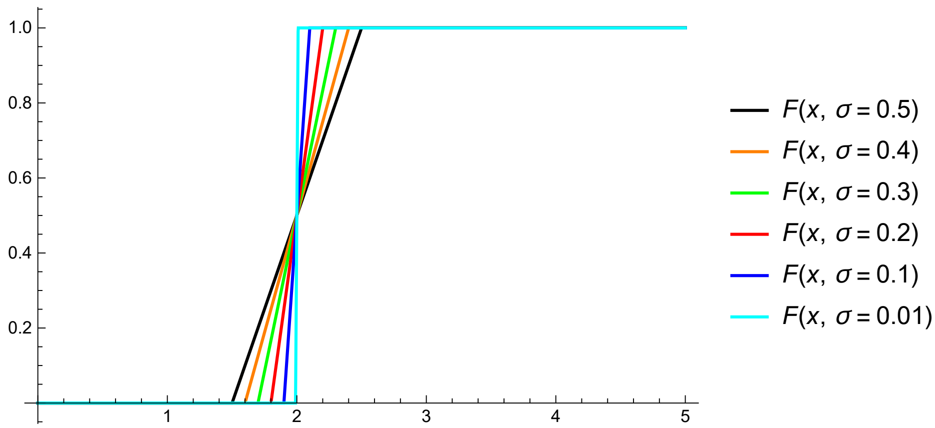
In[∘]:=
```
F[x_, σ_] := Piecewise[{{0, x < a - σ}, {(1/(2 σ)) (x - a + σ), -σ < x - a < σ}, {1, x > a + σ}}];
(*Ramp function*)
der[x_, σ_] =
  D[F[x, σ], x];
(*Derivative of Ramp function is Rectangular function R*)
a = 2;
Plot[F[x, σ = 0.5], {x, 0, 4}, PlotRange → Full]
Plot[der[x, σ = 0.5], {x, 0, 4}, PlotRange → Full]
```

Out[∘]=


Out[∘]=


In[•]:=
```
F[x_, σ_] := Piecewise[{{0, x < a - σ}, {(1/(2 σ)) (x - a + σ), -σ < x - a < σ}, {1, x > a + σ}}];
a = 2;
Plot[{F[x, σ = 0.5], F[x, σ = 0.4], F[x, σ = 0.3],
  F[x, σ = 0.2], F[x, σ = 0.1], F[x, σ = 0.01]}, {x, 0, 5}, PlotRange → Full,
 PlotStyle → {Black, Orange, Green, Red, Blue, Cyan}, PlotLegends → "Expressions"]
```

Out[•]=



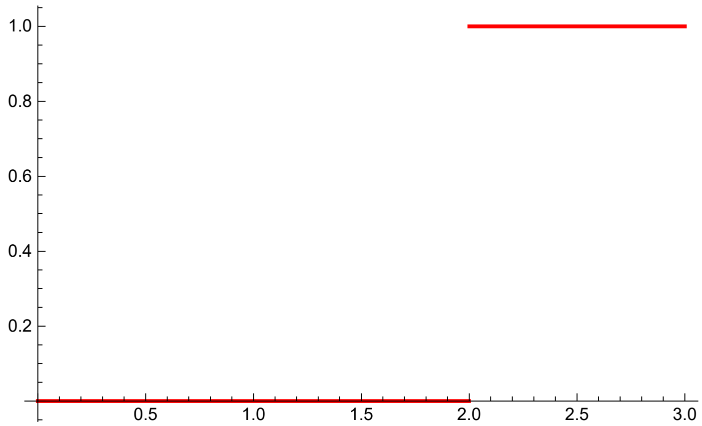
In[•]:=
```
Exit

F[x_, σ_] := Piecewise[{{0, x < a - σ}, {(1/(2 σ)) (x - a + σ), -σ < x - a < σ}, {1, x > a + σ}}];
H[x_, a_] =
  Limit[F[x, σ], σ → 0];
(*For limit σ→0 Ramp function becomes Heaviside unit step function*)
H[x, a]
delta[x_, a_] =
 D[H[x, a], x]
 (*Derivative of dicontinuous Heaviside unit step function is Dirac delta function*)
```

Out[•]=
```
⎧ 1              a < x
⎨ 0              a > x
⎩ Indeterminate  True
```

Out[•]=
```
⎧ 0              a - x < 0 || a - x > 0
⎩ Indeterminate  True
```

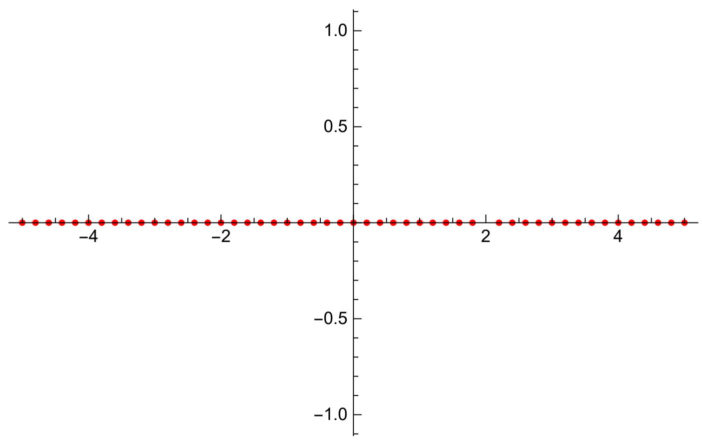
In[◦]:=
```
a = 2;
Plot[H[x, a], {x, 0, 3}, PlotStyle → {Red, Thick},
 PlotLegends → "H(x-a):Heaviside unit step function,\ndiscontinuous function"]
DiscretePlot[delta[x, a], {x, -5, 5, 0.2}, PlotStyle → {Red, Thick},
 PlotLegends → "δ(x-2): Dirac delta function"]
```

Out[◦]=



H(x−a):Heaviside unit step function,
discontinuous function

Out[◦]=



δ(x−2): Dirac delta function