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Devin Powell, Manhattan Prep Instructor

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Chapter 7

of

Arithmetic

In This Chapter…

Arithmetic

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

Quantity A	**Quantity B**
39 - (25 - 17)	39 - 25 - 17

2.

Quantity A	**Quantity B**
14 - 3(4 - 6)	(4)(-3)(2)(-1)

3.

Quantity A	**Quantity B**
$-5 \times 1 \div 5$	$-6 \times 1 \div 6$

4. 5 - (4 - (3 - (2 - 1))) =

5.

Quantity A	**Quantity B**
$17(6) + 3(6)$	$6(17) + 6(3)$

6.

Quantity A	**Quantity B**
$-2^3/2$	$(-2)^2$

7.

Quantity A	**Quantity B**
$5^3 - 5^2$	5

8.

Quantity A	**Quantity B**
$-10 - (-3)^2$	$-[10 + (-3)^2]$

9. $32/(4 + 6 \times 2) =$

(A) 8/5
(B) 16/5
(C) 2
(D) 20
(E) 28

10.

Quantity A	**Quantity B**
(30,000,000)(2,000,000)	(15,000,000)(4,000,000)

11. What is the sum of the numbers in the grid below?

–2	–1	1	2	3	4
–4	–2	2	4	6	8
–6	–3	3	6	9	12
–8	–4	4	8	12	16
–10	–5	5	10	15	20
–12	–6	6	12	18	24

12. Mitchell plans to work at a day camp over the summer. Each week, he will be paid according to the following schedule: at the end of the first week, he will receive $1. At the end of each subsequent week, he will receive $1, plus an additional amount equal to the sum of all payments he's received in previous weeks. How much money will Mitchell be paid in total during the summer, if he works for the entire duration of the 8-week-long camp?

$

13.

A book with 80,000 words costs $24 and a short story has 1,000 words and costs $1.

Quantity A	**Quantity B**
Price per word of the book	Price per word of the short story

14. A taxi driver makes $50 an hour, but pays $100 a day rent for his taxi and has other costs that amount to $0.50 per mile. If he works three 7-hour days and one 9-hour day and drives a total of 600 miles in one week, what is his profit?

(A) $700
(B) $800
(C) $1,100
(D) $1,200
(E) $1,500

15.

Ticket Prices at the Natural History Museum

	Weekdays	Weekends & Holidays
Child (ages 5–18)	$7	$9
Adult (ages 19–64)	$14	$16
Senior (ages 65+)	$8	$10
*Children under age 5 attend free		

Mr. and Mrs. Gonzales, ages 42 and 39, wish to visit the Natural History Museum with their three children (ages 4, 6, and 10), and Mr. Gonzales's 69-year-old father.

Quantity A	**Quantity B**
The cost of admission for the group on a weekday	The cost of admission for the group on a weekend after applying a coupon offering $10 off the total purchase

16.

On a certain train, children's tickets cost $6 and adult tickets cost $9. Six people are charged between $44 and $50, total, for their tickets.

Quantity A	**Quantity B**
The number of children in the group	The number of adults in the group

17. If twice 4,632 is divided by 100, what is the tenths digit?

$\boxed{}$

18. If 617 is divided by 49, the sum of the tens digit and the tenths digit is equal to

(A) 1
(B) 5
(C) 6
(D) 7
(E) 9

19.

Quantity A	**Quantity B**
The age at death, in years and days, of a person who lived from January 31, 1817 to January 15, 1901	The age at death, in years and days, of a person who lived from January 15, 1904 to January 31, 1988

20. For the month of May, Ali's Coffee Shop is offering a "buy five drinks get one free" special, and Bob's Coffee Shop is offering 20% off all drinks. At both shops, the regular price of a coffee is $2.25.

Quantity A	Quantity B
The total cost of one coffee per day from Ali's for every day of May	The total cost of one coffee per day from Bob's for every day of May

21. In a certain ancient kingdom, the length of a foot was the length of the current king's foot. If the newly crowned monarch had a 12 inch foot as opposed to the 10 inch foot of his predecessor, a formerly 300 foot fence would now have a length of how many feet? (Assume that the length of an inch remains constant.)

(A) 250
(B) 302
(C) 350
(D) 360
(E) 400

22. Mary has six more tapes than Pedro. If Pedro gives two tapes to John and then Pedro buys 5 new tapes, how many more tapes does Mary have than Pedro?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

23. 25 employees donated a total of $450 to charity. If 15 employees donated at least $12 and 9 employees donated at least $19, what is the maximum amount, in dollars, that the last employee could have donated?

$ ☐

24. Maribel must divide 60 candies among herself and her 12 cousins, although there is no requirement that the candies be divided equally. If Maribel is to have more candies than everyone else, what is the least number of candies she could have?

☐

25.

Softies Facial Tissues come 84 to a box for $2.99.
Enviro Facial Tissues come 56 to a box for $1.89.

Quantity A	Quantity B
The positive difference between the per-tissue cost of Softies tissues and the per-tissue cost of Enviro tissues	0.2 cents

26. A tank has a capacity of 200 pints. How many gallons of water would it take to fill the tank to 3/10 of its capacity? (1 gallon = 8 pints) gallons

$\boxed{}$ gallons

27.

(1 kilogram = 2.2 pounds)

Quantity A	Quantity B
The number of kilograms in 44 pounds	The number of pounds in 44 kilograms

28. If the formula for converting degrees Fahrenheit to degrees Celsius is $C = \frac{5}{9}(F - 32)$, what is *F* when *C* = 30?

(A) -10/9
(B) 338/9
(C) 86
(D) 558/5
(E) 112

29.

Quantity A	Quantity B
The number of seconds in 12 hours	The number of minutes in 720 hours

30. Joe's car can travel 36 miles per gallon of fuel. Approximately how many kilometers can the car travel on 10 liters of fuel? (5 miles = approximately 8 kilometers; 1 gallon = approximately 4 liters)

$\boxed{}$ kilometers

31. How many 1-inch square tiles would it take to cover the floor of a closet that has dimensions 5 feet by 4 feet? (1 foot = 12 inches)

(A) 20
(B) 240
(C) 1,440
(D) 2,160
(E) 2,880

32. A pool has sprung a leak and is losing water at a rate of 5 milliliters per second. How many liters of water is this pool losing per hour? (1 liter = 1,000 milliliters)

(A) 3
(B) 6
(C) 12
(D) 18
(E) 24

33.

Child A ate 3/5 of a kilogram of chocolate and Child B ate 300 grams of chocolate. (1 kilogram = 1000 grams)

Quantity A	**Quantity B**
The weight, in grams, of the chocolate that Child A ate	Twice the weight, in grams, of the chocolate that Child B ate

34. It takes $4\frac{2}{3}$ feet of wood to make a frame for a lithograph. If wood is sold at \$5 a yard and only by the yard, and a collector needs to make 4 frames, how much will the wood cost? (1 yard = 3 feet)

(A) \$23.33
(B) \$25
(C) \$33.33
(D) \$35
(E) \$100

35. Out of 5.5 billion bacteria grown for an experiment, 1 in 75 million has a particular mutation. Approximately how many of the bacteria have the mutation?

(A) 7
(B) 73
(C) 733
(D) 7,333
(E) 73,333

36. A particular nation's GDP (Gross Domestic Product) is \$4.5 billion. If the population of the nation is 1.75 million, what is the per capita (per person) GDP, rounded to the nearest dollar?

(A) \$3
(B) \$25
(C) \$257
(D) \$2,571
(E) \$25,714

37. Global GDP (Gross Domestic Product) was \$69.97 trillion in 2011. If the world population for 2011 was best estimated at 6,973,738,433, approximately what is the global GDP per person?

(A) \$10
(B) \$100
(C) \$1,000
(D) \$10,000
(E) \$100,000

38. The distance between Mercury and Earth changes due to the orbits of the planets. When Mercury is at its closest point to Earth, it is 48 million miles away. When Mercury is at its furthest point from Earth, it is 138 million miles away. For a science project, Ruby calculates the maximum and minimum amount of time it would take to travel from Earth to Mercury in a spacecraft traveling 55 miles per hour. Approximately what are the times, in days?

(A) 3,636 and 10,454
(B) 14,545 and 41,818

(C) 36,364 and 104,545
(D) 87,272 and 250,909
(E) 872,727 and 2,509,091

Arithmetic Answers

1. **(A).** First simplify inside the parentheses:

$39 - (25 - 17) =$
$39 - 8 =$
31

You could also distribute the minus sign to get 39 - 25 + 17 if you prefer. Quantity B is equal to -3, so the final answer is (A). If you noticed right away that the minus sign would distribute in Quantity A but not Quantity B, you could have picked (A) without doing any arithmetic.

2. **(B).** This question is simply testing PEMDAS (Parentheses/Exponents, then Multiplication/Division, then Addition/Subtraction), at least in Quantity A. Make sure that you simplify inside the parentheses, and then multiply, before subtracting:

$14 - 3(4 - 6) =$
$14 - 3(-2) =$
$14 + 6 =$
20

Quantity B simply equals (4)(-3)(2)(-1) = 24.

3. **(C).** The quantities are equal. Note that

$-5 \times 1 \div 5 =$
$-5 \div 5 =$
-1

In Quantity B:

$-6 \times 1 \div 6 =$
$-6 \div 6 =$
-1

4. **3.** Make sure to begin with the innermost parentheses:

$5 - (4 - (3 - (2 - 1))) =$
$5 - (4 - (3 - 1)) =$
$5 - (4 - 2) =$
$5 - (2) =$
3

5. **(C).** According to the distributive property, the two quantities are the same. Or:

$17(6) + 3(6) =$
$102 + 18 =$
120

In Quantity B:

$6(17) + 6(3) =$
$102 + 18 =$
120

6. **(B).** In Quantity A, the exponent should be computed before taking the negative of the value — in accordance with PEMDAS.

In Quantity B:

$(-2)^2 =$
$(-2)(-2) =$
4

7. **(A).** Do not make the mistake of thinking that $5^3 - 5^2 = 5^1$. You may not simply subtract the exponents when you are subtracting two terms with the same base! Observe:

$5^3 - 5^2 =$
$125 - 25 =$
100

Obviously, Quantity A is much larger. Alternatively, you could factor out 5^2 (this is an important technique for larger numbers and exponents where pure arithmetic would be impractical):

$5^3 - 5^2 =$
$5^2(5^1 - 1) =$
$5^2(4) =$
100

8. **(C).** In Quantity A:

$-10 - (-3)^2 =$
$-10 - (9) =$
-19

In Quantity B:

$-[10 + (-3)^2]$
$-[10 + (-3)^2] =$
$-[10 + (9)] =$
-19

9. **(C).** Begin inside the parentheses and — in accordance with PEMDAS — simplify 6×2 first:

$32/(4 + 6 \times 2) =$
$32/(4 + 12) =$
$32/(16) =$
2

10. **(C).** The GRE calculator will not be able to handle that many zeroes. You will want to do this calculation by hand. To make things easier, you could cancel as many zeroes as you want, as long as you do the same operation to both sides. For instance, you could divide both sides by 1,000,000,000,000 (just think of this as "1 with twelve zeroes"), to get:

Quantity A	**Quantity B**
(30)(2)	(15)(4)

Or, just use a bit of logic: 30 million times 2 million is 60 million *million*, and 15 million times 4 million is also 60 million *million*. (A "million million" is a trillion, but this doesn't matter as long as you're sure that each Quantity will have the same number of zeroes.)

11. **147.** There are several patterns in the grid, depending on whether you look by row or by column. Within each row, there are positive and negative terms at the beginning that cancel each other. For example, in the first row, you have $-2 + 2 = 0$ and $-1 + 1 = 0$. The only terms in the first row that contribute to the sum are 3 and 4, in the far-right columns. The same is true for the other rows.

Thus, the sum of the grid is equal to the sum of only the two far-right columns. The sum in the first row in those columns is $3 + 4 = 7$; the sum in the next row is $6 + 8 = 14$, etc. The sum in the final row is $18 + 24 = 42$. Simply add $7 + 14 + 21 + 28 + 35 + 42$ in your calculator to get 147.

12. **255.** At the end of the first week, Mitchell receives \$1. At the end of the second week, he gets \$1, plus \$1 for the total he had been paid up to that point, for a total of \$2. At the end of the third week, he gets \$1, plus (\$1 + \$2), or \$3, for the total he had been paid up to that point, so this third week total is \$4. Let's put this in a table:

Week #	Paid this week($)	Cumulative Pay including this week ($)
1	1	1
2	1 + 1 = 2	1 + 2 = 3
3	1 + 3 = 4	3 + 4 = 7
4	1 + 7 = 8	7 + 8 = 15
5	1 + 15 = 16	15 + 16 = 31
6	1 + 31 = 32	31 + 32 = 63
7	1 + 63 = 64	63 + 64 = 127
8	1 + 127 = 128	127 + 128 = 255

13. **(B).** In Quantity A, $24/80{,}000 = 0.0003$, or 0.03 cents per word. In Quantity B, $1/1{,}000 = 0.001$, or 0.1 cents per word. Quantity B is much larger. Note that your calculation was not strictly necessary — it would have been more

efficient to notice that the book costs 24 times the story but has 80 times the words. (Then remember to choose the larger amount!)

14. **(B).** The driver works 3 days × 7 hours per day, plus a 9-hour day, for a total of 30 hours. At $50 an hour, he makes $1,500 but pays $400 in rent and $300 in mileage expenses. $1,500 - $700 = $800.

15. **(C).** You don't actually need to do a lot of tedious arithmetic to answer this problem. Six people will be attending the museum, but the 4-year-old does not require a ticket (kids under 5 are free). Thus, 5 tickets need to be purchased, whether the family attends on a weekday or a weekend.

Notice that all of the weekend tickets are each $2 more expensive. Therefore, buying 5 weekend tickets will cost a total of $10 more. Thus, after the $10-off coupon, the two quantities are the same.

16. **(D).** Even though the range of costs ($44 to $50) is fairly small, there is still more than one possibility. A good way to work this out is to start with the simplest scenario: 3 adults and 3 kids. Their tickets would cost 3(9) + 3(6) = $45. That's in the range, so it's one possibility.

Since kids are cheaper, you don't want to add more kids to the mix (2 kids, 4 adults will give you too small a total), but try switching out 1 kid for 1 adult.

For 4 adults and 2 kids, tickets would cost 4(9) + 2(6) = $48. Thus, Quantity A and Quantity B could be equal, or Quantity B could be larger, so the answer is (D).

17. **6.** Multiply 4,632 by 2 to get 9,264, then divide by 100 to get 92.64. The **tenths** digit is 6. (Do not confuse **tenths** with **tens**. The tens digit is 9.)

18. **(C).** Divide 617 by 49 in your calculator to get 12.5918…. The **tens** digit is 1. The **tenths** digit is 5. The answer is 1 + 5 = 6.

19. **(B).** For Quantity A, subtract 1901 - 1817 to get 84 years. However, this person lived from January 31, 1817 to January 15 (not January 31), 1901, so you must subtract the days from January 16 to January 31, or 16 days.

For Quantity B, subtract 1988 - 1904 to get 84 years. However, the person lived from January 15, 1905 to January 31 (not January 15), 1988, so add 16 days.

The person in Quantity A lived almost 84 years. The person in Quantity B lived 84 years, plus a bit more. Quantity B is larger.

After calculating that both persons lived roughly 84 years you could also notice that all within the same month (though in different years), the person in Quantity B was born earlier in the month and died later in the month than the person in Quantity A, meaning the person in Quantity B lived for more of that month.

20. **(A).** The actual price ($2.25) of the coffee is irrelevant, and no actual calculation is required here. All that's needed to solve this problem is to realize that Bob's is a much better deal.

20% off each coffee is 1/5 off. "Buy five drinks get one free" means that, for everything six drinks one purchases, the last one is free. That’s one in SIX drinks free, or 1/6 off.

So, provided that everything else (the regular price of a coffee at both shops, buying the same number of coffees in the month of May) is the same, one will pay less at Bob’s than at Ali’s. Thus, the total cost at Ali’s is larger. By the way, remember to pick the *larger quantity* (Quantity A), NOT the “better deal”!

21. **(A).** A fence that measured 300 feet under the old king’s regime would be $300 \times 10 = 3{,}000$ inches long. This same 3,000-inch fence, under the new king’s regime, would be $3{,}000/12 = 250$ feet long. (Keep in mind that, if a “foot” gets bigger, fewer such “feet” fit into a fence of fixed length.)

22. **(A).** Let’s make a chart for how many tapes everyone has before they start giving each other tapes or getting new ones. Since Mary has 6 more than Pedro:

Mary	Pedro	John
$P+6$	P	?

Now, Pedro gives 2 to John:

Mary	Pedro	John
$P+6$	$P-2$	$?+2$

Now, Pedro buys 5 new tapes. Note that “$P-2$” plus 5 is $P-2+5=P+3$.

Mary	Pedro	John
$P+6$	$P+3$	$?+2$

You never learned very much about John, so the question only asks you about Mary and Pedro. Mary and Pedro each have P tapes, but Mary has 6 more than that and Pedro has 3 more than that. So, Mary has 3 more tapes than Pedro.

It also would be fairly simple to assign values to Mary and Pedro (for instance, say that Mary has 10 tapes and Pedro has 4) and proceed using a real-number example.

23. **99.** Since you want to maximize the last employee’s contribution, minimize everyone else’s. If 15 employees donated at least \$12 and 9 employees donated at least \$19:

$$15(12) + 9(19) = 180 + 171 = 351$$

So, the minimum that all 24 of these employees could have given is \$351. Therefore, the maximum that the 25th employee could have given is $450 - 351 = 99$, or \$99.

24. **6.** One good way to solve this problem is to first evenly divide the candies, and then give Maribel more (by taking candies away from the others) until the conditions of the problem are met. 60 candies divided by 13 people = 4.615….

So, if Maribel had 5 candies, would she have more than everyone else? Well, if the 12 cousins each had only 4 candies, that’s 48 candies total plus Maribel’s 5 = 53 candies. 7 candies are unaccounted for, meaning that some other cousin or cousins will have to have the same as or more than Maribel.

If Maribel had 6 candies, would she have more than everyone else? Well, if the 12 cousins each had only 5 candies, they would have 60. Since there are only 60 candies total, Maribel could have 6 and the other cousins could have 4 or 5 each. The answer is 6.

Keep in mind that when the question asks for a minimum, you can't just go messing around until you find a case that works — to find the *smallest* case that works, you need to start small and work up from there.

25. **(B).** In the calculator, divide 2.99 by 84 to get a per-tissue cost of 0.03559…, (or 3.559… cents).

Divide 1.89 by 56 to get a per-tissue cost of 0.03375… (or 3.375… cents).

Subtract the smaller number from the larger number to get 0.00184…, or 0.184… cents.

This is less than 0.2 cents. The answer is (B).

26. **7.5 gallons:**

First find out how many pints 3/10 of the capacity is:

$200 \times 3/10 = 600/10 = 60$

Now you need to convert pints to gallons:

$$60 \text{ pints} \times \frac{1 \text{ gallon}}{8 \text{ pints}} = \frac{60}{8} = 7.5 \text{ gallons}$$

27. **(B).** To compare the values, you need to convert the quantity on the left from pounds to kilograms and the quantity on the right from kilograms to pounds:

<u>Quantity A</u>	<u>Quantity B</u>
$44 \text{ pounds} \times \frac{1 \text{ kilogram}}{2.2 \text{ pounds}}$	$44 \text{ pounds} \times \frac{2.2 \text{ pounds}}{1 \text{ kilogram}}$

Before actually multiplying, notice that the quantity on the left is divided by 2.2, while the quantity on the right is multiplied by 2.2. The quantity on the right will be greater.

One could also solve this by noticing that the two quantities involve reverse calculations, with the same number of units (44). Since a kilogram is heavier than a pound, it takes more of the lighter pounds to equal 44 heavier kilograms than it takes of the heavier kilograms to equal 44 of the lighter pounds.

28. **(C).** Start by plugging 30 in for *C* in the equation:

$$30 = \frac{5}{9}(F - 32)$$

Now isolate *C*. Begin by multiplying both sides by 9/5:

$$\begin{aligned} 30 &= \frac{5}{9}(F - 32) \\ \frac{9}{5} \times 30 &= F - 32 \end{aligned}$$

To multiply 30 by 9/5 quickly, reduce before multiplying:

$$\begin{aligned} \frac{9}{5} \times 30 &= F - 32 \\ \frac{9}{1} \times 6 &= F - 32 \\ 54 &= F - 32 \\ 86 &= F \end{aligned}$$

29. **(C).** Before doing either calculation, note that there are 60 seconds in a minute and 60 seconds in an hour. Compare the two calculations:

12 hours × 60 minutes/hour × 60 seconds/minute	720 hours × 60 minutes/hour

Notice that 12 × 60 = 720. That means that both amounts will equal 720 × 12. The two values are equal.

30. **144 kilometers.** Convert miles per gallon to kilometers per liter by multiplying by the conversion ratios such that both the miles and gallons units are canceled out:

$$\frac{36 \text{ miles}}{1 \text{ gallon}} \times \frac{8 \text{ kilometers}}{5 \text{ miles}} = \frac{288 \text{ kilometers}}{5 \text{ gallons}}$$

$$\frac{288 \text{ kilometers}}{5 \text{ gallons}} \times \frac{1 \text{ gallon}}{4 \text{ liters}} = \frac{288 \text{ kilometers}}{20 \text{ liters}} = \frac{14.4 \text{ kilometers}}{1 \text{ liter}}$$

The car has 10 liters of fuel in the tank:

10 liters × 14.4 kilometers/liter = 144 kilometers

31. **(E).** There is a hidden trap in this question. Remember that the dimensions of this room are ft^2, not ft (because 5 feet × 4 feet = 20 square feet). To avoid this trap, you should convert the dimensions to inches first, then multiply.

5 feet × 4 feet = 60 inches × 48 inches

The dimensions of the closet in inches are 60 inches by 48 inches, or 60 × 48 = 2,880 square inches. Each tile is 1 square inch, so it will take 2,880 tiles to cover the floor.

32. **(D).** To answer this question, you need to convert milliliters to liters, and convert seconds to hours. The order in which you make the conversions does not matter. First, convert seconds to hours. There are 60 seconds in 1 minute, and 60 minutes in 1 hour:

$$\frac{5 \text{ milliliters}}{1 \text{ second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{18{,}000 \text{ milliliters}}{1 \text{ hour}}$$

Now convert milliliters to liters:

$$\frac{18{,}000 \text{ milliliters}}{1 \text{ hour}} \times \frac{1 \text{ liter}}{1{,}000 \text{ milliliters}} = \frac{18 \text{ liters}}{1 \text{ hour}}$$

33. **(C).** 3/5 of a kilogram is 600 grams. Twice 300 grams is also 600 grams. The columns are equal.

34. **(D).** For 4 frames, the collector needs $18\frac{2}{3}$ feet or $6\frac{2}{9}$ yards of wood. Since the wood is sold "only by the yard," the collector must buy 7 yards [21 feet] at $5 a yard. The answer is 7(5) = 35.

35. **(B).** One good way to keep track of large numbers (especially those that won't fit in the GRE calculator!) is to use scientific notation (or a loose version thereof — for instance, 5.5 billion in scientific notation is 5.5×10^9, but it would be equally correct for your purposes to write it as 55×10^8).

5.5 billion = 5,500,000,000 = 5.5×10^9
75 million = 75,000,000 = 75×10^6

Since 1 in 75 million of the bacteria have the mutation, divide 5.5 billion by 75 million:

$\frac{5.5\times10^9}{75\times10^6}$, which can also be written as $\frac{5.5}{75} \times \frac{10^9}{10^6}$. Only $\frac{5.5}{75}$. needs to go in the calculator, to yield 0.0733333….

Since $\frac{10^9}{10^6}$ is 10^3, move the decimal three places to the right to get 73.333…, or answer choice (B).

Or, write one number over the other and *cancel out the same number of zeroes from the top and bottom* before trying to use the calculator: $\frac{5{,}500{,}000{,}000}{75{,}000{,}000} = \frac{5{,}500,\cancel{000{,}000}}{75,\cancel{000{,}000}} = \frac{5{,}500}{75} = 73.333\ldots$

36. **(D).** This problem is asking you to divide \$4.5 billion by 1.75 million. When dealing with numbers that have many zeroes, you can avoid mistakes by using scientific notation or by writing out the numbers and canceling zeroes before using the calculator.

4.5 billion = 4,500,000,000 = 4.5×10^9
1.75 million = 1,750,000 = 1.75×10^6

$$\frac{4.5\times10^9}{1.75\times10^6} = 2.57142\ldots\times10^3 = 2{,}571.42\ldots$$

The answer is (D). Alternatively, write one number on top of the other in fully-expanded form, and cancel zeroes before using the calculator:

$$\frac{4{,}500{,}000{,}000}{1{,}750{,}000} = \frac{4{,}500{,}00\cancel{0{,}000}}{1{,}75\cancel{0{,}000}} = \frac{450{,}000}{175} = 2{,}571.42\ldots$$

37. **(D).** This problem is asking you to divide \$69.97 trillion by 6,973,738,433. When dealing with numbers that have many zeroes, you can avoid mistakes by using scientific notation or by writing out the numbers and canceling zeroes before using the calculator.

Before doing that, however, look at the answers — they are very far apart from one another, which gives you license to estimate. GDP is about 70 trillion. Population is about 7 billion. Thus:

$$\frac{70{,}000{,}000{,}000{,}000}{7{,}000{,}000{,}000} = \frac{70{,}000{,}\cancel{000{,}000{,}000}}{7{,}\cancel{000{,}000{,}000}} = 10{,}000$$

38. **(C).** Since Rate × Time = Distance, thus $\frac{\text{Distance}}{\text{Rate}} = \text{Time}$ = Time.

If you don't yet have that formula memorized, a little common sense will tell you that if Mercury and Earth were 110 miles apart, for instance, and you traveled at 55 mph, you would get there in two hours. Thus, the correct operation is division.

48 million miles = 48,000,000, so:

$$\frac{48{,}000{,}000}{55} = 872{,}727.2727\ldots$$

138 million miles = 138,000,000, so:

$$\frac{138{,}000{,}000}{55} = 2{,}509{,}090.909$$

Thus, it would take between 872,727 hours and 2,509,091 hours (rounded to the nearest hour) to travel to Mercury at 55 mph.

To convert to **days**, simply divide each of these numbers by 24 to get 36,364 days and 104,545 days.

Chapter 8
of

Algebra

In This Chapter…

Algebra

Algebra Answers

Algebra

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. If $3x + 2(x + 2) = 2x + 16$, then $x =$

(A) 3
(B) 4
(C) 20/3
(D) 10
(E) 12

2. If $x \neq 0$ and $\dfrac{3x+7}{x} = 10$, what is the value of x?

$\boxed{}$

3. If $4(-3x - 8) = 8(-x + 9)$, what is x^2?

4. If $3x + 7 - 4x + 8 = 2(-2x - 6)$, what is the value of x?

5. If $2x(4 - 6) = -2x + 12$, what is the value of x?

6. If $x \neq 0$ and $\frac{3(6-x)}{2x} = -6$, what is the value of x?

7. If $x \neq -13$ and $\frac{13}{x+13} = 1$, what is the value of x?

8. If $x \neq 2$ and $\frac{10(-3x+4)}{10-5x} = 2$, what is the value of x?

9. If $x \neq 2$ and $\frac{8-2(-4+10x)}{2-x} = 17$, what is the value of x?

10.

-5 is 7 more than $-z$.

Quantity A	**Quantity B**
z	-12

11. If $(x + 3)^2 = 225$, which of the following could be the value of $x - 1$?

(A) 13
(B) 12
(C) -12
(D) -16
(E) -19

12.

$x = 2$

Quantity A	Quantity B
$x^2 - 4x + 3$	1

13.

$$p = 300c^2 - c$$
$$c = 100$$

Quantity A	Quantity B
p	$29{,}000c$

14. If $3(7 - x) = 4(1.5)$, then $x =$

15.

$$1{,}200x + 6{,}000 = 13{,}200$$
$$12y + 60 = 132$$

Quantity A	Quantity B
x	y

16.

$$-(x)^3 = 64$$

Quantity A	Quantity B
x^4	x^5

17. If $3t^3 - 7 = 74$, what is $t^2 - t$?

(A) -3
(B) 3
(C) 6
(D) 9
(E) 18

18. If $3x + 7 - 4x + 8 = 2(-2x - 6)$, what is the value of x?

19. If $y = 4x + 10$ and $y = 7x - 5$, what is the value of y?

20. If $2h - 4k = 0$ and $k = h - 3$, what is the value of $h + k$?

21. If $x - y = 4$ and $2x + y = 5$, what is the value of x?

22. If $x + 2y = 5$ and $x - 4y = -7$, what is the value of x?

23. $4x + y + 3z = 34$
$4x + 3z = 21$

What is the value of y?

24.

Quantity A	**Quantity B**
$(x + 2)(x - 3)$	$x^2 - x - 6$

25.

Quantity A	**Quantity B**
$(2s + 1)(s + 5)$	$2s^2 + 11s + 4$

26.

$xy > 0$

Quantity A	**Quantity B**
$(2x - y)(x + 4y)$	$2x^2 + 8xy - 4y^2$

27.

$$x^2 - 2x = 0$$

Quantity A	**Quantity B**
x	2

28.

Quantity A	**Quantity B**
$d(d^2 - 2d + 1)$	$d(d^2 - 2d) + 1$

29.

Quantity A	**Quantity B**
$xy^2z(x^2z + yz^2 - xy^2)$	$x^3y^2z^2 + xy^3z^3 - x^2y^4z$

30.

$a = 2b = 4c$ and a, b, and c are integers.

Quantity A	**Quantity B**
$a + b$	$a + c$

31.

$k = 2m = 4n$ and k, m, and n are nonnegative integers.

Quantity A	**Quantity B**
km	kn

32.

For the positive integers a, b, c, and d, a is half of b, which is one-third of c. The value of d is triple that of c.

Quantity A	**Quantity B**
$\dfrac{a+b}{c}$	$\dfrac{a+b+c}{d}$

33. If $x^2 - y^2 = 0$ and $xy \neq 0$, which of the following MUST be true?

Indicate <u>all</u> such statements.

☐ $x = y$

☐ $|x| = |y|$

☐ $\dfrac{x^2}{y^2} = 1$

34.

$$3x + 6y = 27$$
$$x + 2y + z = 11$$

Quantity A	**Quantity B**
$z + 5$	$x + 2y - 2$

35. If $(x - y) = \sqrt{12}$ and $(x + y) = \sqrt{3}$, what is the value of $x^2 - y^2$?

(A) 3
(B) 6
(C) 9
(D) 36
(E) It cannot be determined from the information given.

36.

$$a \neq b$$

Quantity A	**Quantity B**
$\dfrac{a-b}{b-a}$	1

37.

$$a = \frac{b}{2}$$

$$c = 3b$$

Quantity A	**Quantity B**
a	c

38. If $xy \neq 0$ and $x \neq -y$, $\dfrac{x^{36} - y^{36}}{(x^{18} + y^{18})(x^9 + y^9)} =$ =

(A) 1
(B) $x^2 - y^2$
(C) $x^9 - y^9$

(D) $x^{18} - y^{18}$

(E) $\dfrac{1}{x^9 - y^9}$

39. If $x \neq -y$, $\dfrac{x^2 + 2xy + y^2}{2(x+y)^2} =$

(A) 1
(B) 1/2
(C) $\dfrac{1}{x+y}$
(D) xy
(E) $2xy$

40. If $ab \neq 0$, $\dfrac{a^8 - b^8}{(a^4 + b^4)(a^2 + b^2)} =$

(A) 1
(B) $a - b$
(C) $(a + b)(a - b)$
(D) $(a^2 + b^2)(a^2 - b^2)$
(E) $\dfrac{a-b}{a+b}$

41.

$$x > y$$
$$xy \neq 0$$

Quantity A	**Quantity B**
$\dfrac{x^2}{y + \frac{1}{y}}$	$\dfrac{y^2}{x + \frac{1}{x}}$

42. If $x + y = -3$ and $x^2 + y^2 = 12$, what is the value of $2xy$?

43. If $x - y = 1/2$ and $x^2 - y^2 = 3$, what is the value of $x + y$?

44. If $x^2 - 2xy = 84$ and $x - y = -10$, what is the value of $|y|$?

45. $(x - 2)^2 + (x - 1)^2 + x^2 + (x + 1)^2 + (x + 2)^2 =$

(A) $5x^2$
(B) $5x^2 + 10$
(C) $x^2 + 10$
(D) $5x^2 + 6x + 10$
(E) $5x^2 - 6x + 10$

46. If $a = (x + y)^2$ and $b = x^2 + y^2$ and $xy > 0$, which of the following must be true?

Indicate <u>all</u> such statements.

☐ $a = b$
☐ $a > b$
☐ a is positive

47. a is directly proportional to b. If $a = 8$ when $b = 2$, what is a when $b = 4$?

(A) 10
(B) 16
(C) 32
(D) 64
(E) 128

48. a is inversely proportional to b. If $a = 16$ when $b = 1$, what is b when $a = 8$?

(A) -2
(B) -1
(C) 2
(D) 4
(E) 8

49. The time it takes to erect a bonfire is inversely proportional to the number of students doing the work. If it takes 20 students 1.5 hours to do the job, how long will it take 35 students to do the job, to the nearest minute?

(A) 51
(B) 52
(C) 53
(D) 54
(E) 55

50.

$$3a + 2b = 20 \text{ and } 2a + 3b = 5$$

Quantity A	**Quantity B**
$a + b$	a

51.

$$m + 2n = 10 \text{ and } m \text{ is 50\% of } n$$

Quantity A	**Quantity B**
m^2	n

52.

For the integers a, b, and c, the sum of a and b is 75% of c.

Quantity A	**Quantity B**
$(3/4)(a + b)$	$(4/3)(c)$

53. If $2a = 4b = 8c = 10$, then $64abc =$

(A) 64,000
(B) 16,000
(C) 8,000
(D) 4,000
(E) 1,000

54. If $4m^2 + 6n^3 - 9 = 16$, what is the value of $2m^2 + 3n^3$?

55. If $a + b = 8$, $b + c = 11$, and $a + c = 5$, what is the value of $a + b + c$?

Algebra Answers

1. **(B).** Distribute the 2 on the left side over the $(x + 2)$, then combine like terms and simplify:

$3x + 2(x + 2) = 2x + 16$
$3x + 2x + 4 = 2x + 16$
$5x + 4 = 2x + 16$
$3x + 4 = 16$
$3x = 12$
$x = 4$

2. **1.** First, multiply both sides by x to get:

$3x + 7 = 10x$
$7 = 7x$
$1 = x$

The answer is 1. By the way, "$x \neq 0$" was in the problem simply because the problem had x on the bottom of a fraction, and dividing by zero is illegal. This is just the problem writer's way of assuring you that the problem, in fact, has an answer. So, you generally don't have to worry about verbiage like "$x \neq 0$."

3. **676.** Distribute, group like terms, and solve for x:

$4(-3x - 8) = 8(-x + 9)$
$-12x - 32 = -8x + 72$
$-32 = 4x + 72$
$-104 = 4x$
$-26 = x$

Then, multiply 26 by 26 in your calculator (or -26 by -26, although the negatives will cancel each other out anyway) to get x^2, which is 676.

4. **-9.** $3x + 7 - 4x + 8 = 2(-2x - 6)$
$-x + 15 = -4x - 12$
$3x + 15 = -12$
$3x = -27$
$x = -9$

5. **-6.** $2x(4 - 6) = -2x + 12$
$2x(-2) = -2x + 12$
$-4x = -2x + 12$
$-2x = 12$
$x = -6$

6. **-2.** $\frac{3(6-x)}{2x}=-6$

$3(6 - x) = -6(2x)$
$18 - 3x = -12x$
$18 = -9x$
$-2 = x$

7. **0.** $\frac{13}{x+13}=1$

$13 = 1(x + 13)$
$13 = x + 13$
$0 = x$

8. **1.** $\frac{10(-3x+4)}{10-5x}=2$

$10(-3x + 4) = 2(10 - 5x)$
$-30x + 40 = 20 - 10x$
$40 = 20 + 20x$
$20 = 20x$
$1 = x$

9. **-6.** $\frac{8-2(-4+10x)}{2-x}=17$

$8 - 2(-4 + 10x) = 17(2 - x)$
$8 + 8 - 20x = 34 - 17x$
$16 - 20x = 34 - 17x$
$16 = 34 + 3x$
$-18 = 3x$
$-6 = x$

10. **(A).** Translate the question stem into an equation and solve for z:

$-5 = -z + 7$
$-12 = -z$
$12 = z$

Because $z = 12 > -12$, Quantity A is greater.

11. **(E).** Begin by square-rooting both sides of the equation, but remember that square-rooting 225 will yield both 15 and -15 as results. (The calculator will not remind you of this! It's your job to keep this in mind.) So:

$x + 3 = 15$
$x = 12$
so, $x - 1 = 11$

OR

$x + 3 = -15$
$x = -18$
so, $x - 1 = -19$

Only -19 appears in the choices.

12. **(B).** To evaluate the expression in Quantity A, replace x with 2.

$x^2 - 4x + 3 =$
$(2)^2 - 4(2) + 3 =$
$4 - 8 + 3 = -1 < 1$

Therefore, Quantity B is greater.

13. **(A).** To find the value of p, first replace c with 100 to find the value for Quantity A:

$p = 300c^2 - c$
$p = 300(100)^2 - 100$
$p = 300(10{,}000) - 100$
$p = 3{,}000{,}000 - 100 = 2{,}999{,}900$

Since $c = 100$, the value for Quantity B is 29,000(100) = 2,900,000.

Thus, Quantity A is greater.

14. **5.** Distribute the 3 on the left side and multiply 4(1.5). Feel free to use the calculator:

$3(7 - x) = 4(1.5)$
$21 - 3x = 6$
$-3x = -15$
$x = 5$

15. **(C).** First, solve for x:

$1{,}200x + 6{,}000 = 13{,}200$
$1{,}200x = 7{,}200$
$x = 6$

Now, solve for y:

$12y + 60 = 132$
$12y = 72$
$y = 6$

The quantities are equal. Alternatively, you could have noticed that dividing both sides of the first equation by 100 would yield an equation identical to the second one, except with x in place of y. Thus, without solving the equations, you could note that the two quantities must be the same.

16. **(A).** First, solve for x:

$$-(x)^3 = 64$$
$$(x)^3 = -64$$

Your calculator will not do a cube root for you. Fortunately, on the GRE, cube roots will tend to be quite small and easy to puzzle out. Ask yourself what number times itself three times equals -64? The answer is $x = -4$.

Since x is negative, Quantity A will be positive (a negative number times itself four times will be positive) and Quantity B will be negative (a negative number times itself five times will be negative). No further calculations are needed to see that Quantity A is greater.

17. **(C).** First, solve for t:

$$3t^3 - 7 = 74$$
$$3t^3 = 81$$
$$t^3 = 27$$
$$t = 3$$

Now, plug $t = 3$ into t^2 - t:

$$(3)^2 - 3 = 9 - 3 = 6$$

18. **-9.** First, combine like terms on each side:

$$3x + 7 - 4x + 8 = 2(-2x - 6)$$
$$-x + 15 = -4x - 12$$
$$3x + 15 = -12$$
$$3x = -27$$
$$x = -9$$

19. **30.** Since each equation is already solved for y, set the right side of each equation equal to the other.

$$4x + 10 = 7x - 5$$
$$10 = 3x - 5$$
$$15 = 3x$$
$$5 = x$$

Substitute 5 for x in the first equation and solve for y.

$$y = 4(5) + 10$$
$$y = 30$$

$x = 5$ and $y = 30$. Be sure to answer for y, not x.

20. **9.** Since the second equation is already solved for k, plug $(h - 3)$ in for k in the first equation:

$2h - 4k = 0$
$2h - 4(h - 3) = 0$
$2h - 4h + 12 = 0$
$-2h = -12$
$h = 6$

Substitute 6 for h in the second equation and solve for k.

$k = (6) - 3$
$k = 3$

$h = 6$ and $k = 3$, so $h + k = 9$.

21. **3.** Notice that the first equation has the term $-y$ while the second equation has the term $+y$. While you could use the substitution method, adding the equations together will make $-y$ and y cancel, so this is the easiest way to solve for x.

$x - y = 4$
$2x + y = 5$
$3x = 9$
$x = 3$

22. **1.** Both equations have the term $+x$, so you can eliminate the variable x by subtracting the second equation from the first:

$x + 2y = 5$
$-(x - 4y = -7)$
$6y = 12$
$y = 2$

Plug this value for y into the first equation to get $x + 2(2) = 5$, or $x = 1$.

Be very careful to change the sign of each term in the second equation when subtracting. For example, $-(-4y) = +4y$ and $-(-7) = +7$.

Alternatively, you could have multiplied the entire second equation by -1 to get $-x + 4y = 7$ and then added this equation to the first. Either way, $x = 1$.

23. **13.** This question contains only two equations, but three variables. To isolate y, both x and z must be eliminated. Notice that the coefficients of x and z are the same in both equations. Subtract the second equation from the first to eliminate x and z.

$4x + y + 3z = 34$
$\underline{-(4x \quad + 3z = 21)}$

$y = 13$

24. **(C).** FOIL the terms in Quantity A:

$$(x + 2)(x - 3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$$

The two quantities are equal.

25. **(A).** FOIL the terms in Quantity A:

$$(2s + 1)(s + 5) = 2s^2 + 10s + s + 5 = 2s^2 + 11s + 5$$

Since $2s^2 + 11s$ appears in both quantities, eliminate it. Because 5 is greater than 4, Quantity A is greater.

26. **(B).** FOIL the terms in Quantity A:

$$(2x - y)(x + 4y) = 2x^2 + 8xy - xy - 4y^2 = 2x^2 + 7xy - 4y^2$$

Since $2x^2$ and $-4y^2$ appear in both quantities, eliminate them. Quantity A is now equal to $7xy$ and Quantity B is now equal to $8xy$. Because $xy > 0$, Quantity B is greater. (Don't assume! If xy were 0, the two quantities would have been equal. If xy were negative, Quantity A would have been greater.)

27. **(D).** Factor $x^2 - 2x = 0$:

$$x^2 - 2x = 0$$
$$x(x - 2) = 0$$
$$x = 0 \text{ OR } (x - 2) = 0$$

$x = 0$ or 2.

Thus, Quantity A could be less than or equal to Quantity B. The answer is (D).

(Note that you CANNOT simply divide both sides of the original equation by x. It is illegal to divide by a variable unless you have evidence that that variable does not equal zero.)

28. **(D).** In Quantity A, multiply d by every term in the parentheses:

$$d(d^2 - 2d + 1) =$$
$$(d \times d^2) - (d \times 2d) + (d \times 1) =$$
$$d^3 - 2d^2 + d$$

In Quantity B, multiply d by the two terms in the parentheses:

$$d(d^2 - 2d) + 1 =$$
$$(d \times d^2) - (d \times 2d) + 1 =$$

$d^3 - 2d^2 + 1$

Because $d^3 - 2d^2$ is common to both quantities, it can be ignored. The comparison is really between d and 1. Without more information about d, there is no way to know which quantity is greater.

29. **(C).** In Quantity A, the term xy^2z on the outside of the parentheses must be multiplied by each of the three terms inside the parentheses. Then simplify the expression as much as possible.

Taking one term at a time, the first is $xy^2z \times x^2z = x^3y^2z^2$, because there are three factors of x, two factors of y, and two factors of z. Similarly, the second term is $xy^2z \times yz^2 = xy^3z^3$ and the third is $xy^2z \times (-xy^2) = -x^2y^4z$. Adding these three terms together gives the distributed form of Quantity A: $x^3y^2z^2 + xy^3z^3 - x^2y^4z$.

This is identical to Quantity B, so no more work is required.

30. **(D).** Since a is common to both quantities, it can be ignored. The comparison is really between b and c. Because $2b = 4c$, it is true that $b = 2c$, so the comparison is really between $2c$ and c. Watch out for negatives. If the variables are positive, Quantity A is greater, but if the variables are negative, Quantity B is greater.

31. **(D).** If the variables are positive, Quantity A is greater. However, all three variables could equal zero, in which case the two quantities are equal. Watch out for the word "nonnegative," which means "positive or zero."

32. **(C).** The following relationships are given: $a = \frac{b}{2}$, $b = \frac{c}{3}$, and $d = 3c$. Pick one variable and put everything in terms of that variable. For instance, variable a:

$b = 2a$
$c = 3b = 3(2a) = 6a$
$d = 3c = 3(6a) = 18a$

Substitute into the quantities and simplify.

Quantity A: $\frac{a+b}{c} = \frac{a+2a}{6a} = \frac{3a}{6a} = \frac{1}{2}$

Quantity B: $\frac{a+b+c}{d} = \frac{a+2a+6a}{18a} = \frac{9a}{18a} = \frac{1}{2}$

The two quantities are equal.

33. **II and III only.** Since $x^2 - y^2 = 0$, add y^2 to both sides to get $x^2 = y^2$. It might look as though $x = y$, but this is not necessarily the case. For example, x could be 2 and y could be -2. Algebraically, when you square root both sides of $x^2 = y^2$, you do NOT get $x = y$, but rather $|x| = |y|$. Thus, statement I is not necessarily true and statement II is true. Statement III is also true and can be easily generated algebraically:

$x^2 - y^2 = 0$
$x^2 = y^2$
$$\frac{x^2}{y^2} = 1$$

34. **(C).** This question may at first look difficult, as there are three variables and only two equations. However, notice that the top equation can be divided by 3, yielding $x + 2y = 9$. This can be plugged into the second equation:

$(x + 2y) + z = 11$
$(9) + z = 11$
$z = 2$

Quantity A is simply 2 + 5 = 7.

For Quantity B, remember that $x + 2y = 9$. Thus, Quantity B is 9 - 2 = 7.

The two quantities are equal.

35. **(B).** The factored form of the Difference of Squares (one of the "special products" you need to memorize for the exam) is comprised of the terms given in this problem.

$x^2 - y^2 = (x + y)(x - y)$

Substitute the values $\sqrt{12}$ and $\sqrt{3}$ in place of $(x - y)$ and $(x + y)$, respectively:

$x^2 - y^2 = \sqrt{12} \times \sqrt{3}$

Combine 12 and 3 under the same root sign and solve:

$x^2 - y^2 = \sqrt{12 \times 3}$
$x^2 - y^2 = \sqrt{36}$
$x^2 - y^2 = 6$

36. **(B).** Plug in any two unequal values for a and b, and Quantity A will always be equal to -1. This is because you can factor a negative out of the top or bottom of the fraction to show that the top and bottom are the same, except for their signs:

$$\frac{a-b}{b-a} = \frac{a-b}{-(a-b)} = -1$$

37. **(D).** To compare a and c, put c in terms of a. Multiply the first equation by 2 to find that $b = 2a$. Substitute into the second equation: $c = 3b = 3(2a) = 6a$. If all three variables are positive, then $6a > a$. If all three variables are negative, then $a > 6a$. Finally, all three variables could equal 0, making the two quantities equal.

38. **(C).** The Difference of Squares (one of the "special products" you need to memorize for the exam) is $x^2 - y^2 = (x + y)(x - y)$. This pattern works for any perfect square minus another perfect square. Thus, $x^{36} - y^{36}$ will factor according to this pattern. Note that $\sqrt{x^{36}} = (x^{36})^{½} = x^{36/2} = x^{18}$, or $x^{36} = (x^{18})^2$. First, factor $x^{36} - y^{36}$ in the numerator, then cancel $x^{18} + y^{18}$ with the $x^{18} + y^{18}$ on the bottom:

$$\frac{x^{36}-y^{36}}{\left(x^{18}+y^{18}\right)\left(x^{9}+y^{9}\right)}=\frac{\cancel{\left(x^{18}+y^{18}\right)}\left(x^{18}-y^{18}\right)}{\cancel{\left(x^{18}+y^{18}\right)}\left(x^{9}+y^{9}\right)}=\frac{\left(x^{18}-y^{18}\right)}{\left(x^{9}+y^{9}\right)}$$

The $x^{18} - y^{18}$ in the numerator will also factor according to this pattern. Then cancel $x^9 + y^9$ with the $x^9 + y^9$ on the bottom:

$$\frac{\left(x^{18}-y^{18}\right)}{\left(x^{9}+y^{9}\right)}=\frac{\cancel{\left(x^{9}+y^{9}\right)}\left(x^{9}-y^{9}\right)}{\cancel{\left(x^{9}+y^{9}\right)}}=x^{9}-y^{9}$$

39. **(B)**. First, you need to know that $x^2 + 2xy + y^2 = (x + y)^2$. This is one of the "special products" you need to memorize for the exam. Factor the top, then cancel:

$$\frac{x^{2}+2xy+y^{2}}{2(x+y)^{2}}=\frac{\cancel{(x+y)^{2}}}{2\cancel{(x+y)^{2}}}=\frac{1}{2}$$

40. **(C).** The Difference of Squares (one of the "special products" you need to memorize for the exam) tells you that $x^2 - y^2 = (x + y)(x - y)$. This pattern works for any perfect square minus another perfect square. Note that $\sqrt{a^8} = (a^8)^{½} = a^{8/2} = a^4$, or $a^8 = (a^4)^2$. Thus, $a^8 - b^8$ will factor according to this pattern:

$$\frac{a^{8}-b^{8}}{\left(a^{4}+b^{4}\right)\left(a^{2}+b^{2}\right)}=\frac{\cancel{\left(a^{4}+b^{4}\right)}\left(a^{4}-b^{4}\right)}{\cancel{\left(a^{4}+b^{4}\right)}\left(a^{2}+b^{2}\right)}=\frac{a^{4}-b^{4}}{a^{2}+b^{2}}$$

Now, factor $a^4 - b^4$ according to the same pattern:

$$\frac{a^{4}-b^{4}}{a^{2}+b^{2}}=\frac{\cancel{\left(a^{2}+b^{2}\right)}\left(a^{2}-b^{2}\right)}{\cancel{a^{2}+b^{2}}}=a^{2}-b^{2}$$

Since $a^2 - b^2$ does not appear in the choices, factor one more time to get $(a + b)(a - b)$, which is choice (C).

41. **(D)**. You could simplify first and then plug in examples, or just plug in examples without simplifying. For instance, if $x = 2$ and $y = 1$:

Quantity A: $$\frac{x^2}{y+\frac{1}{y}} = \frac{2^2}{1+\frac{1}{1}} = \frac{4}{2} = 2$$

Quantity B: $$\frac{y^2}{x+\frac{1}{x}} = \frac{1^2}{2+\frac{1}{2}} = \frac{1}{\frac{5}{2}} = \frac{2}{5}$$

In this case, Quantity A is greater. Then, try negatives. If $x = -1$ and $y = -2$ (remember, x must be greater than y):

Quantity A: $$\frac{x^2}{y+\frac{1}{y}} = \frac{(-1)^2}{-2+\frac{1}{-2}} = \frac{1}{\frac{5}{-2}} = \frac{-2}{5}$$

Quantity B: $$\frac{y^2}{x+\frac{1}{x}} = \frac{(-2)^2}{(-1)+\frac{1}{-1}} = \frac{4}{-2} = -2$$

Quantity A is still greater. However, before assuming that Quantity A is *always* greater, make sure you have tried every category of possibilities for x and y. What if x is positive and y is negative? For instance, $x = 2$ and $y = -2$:

Quantity A: $$\frac{x^2}{y+\frac{1}{y}} = \frac{2^2}{-2+\frac{1}{-2}} = \frac{4}{-\frac{5}{2}} = 4 \times -\frac{2}{5} = -\frac{8}{5}$$

Quantity B: $$\frac{y^2}{x+\frac{1}{x}} = \frac{(-2)^2}{(2)+\frac{1}{2}} = \frac{4}{\frac{5}{2}} = 4 \times \frac{2}{5} = \frac{8}{5}$$

42. **-3.** One of the "special products" you need to memorize for the GRE is $x^2 + 2xy + y^2 = (x + y)^2$. Write this pattern on your paper, plug in the given values, and simplify:

$x^2 + 2xy + y^2 = (x + y)^2$
$(x^2 + y^2) + 2xy = (x + y)^2$
$(12) + 2xy = (-3)^2$
$12 + 2xy = 9$
$2xy = -3$

43. **6.** The Difference of Squares (one of the "special products" you need to memorize for the exam) is $x^2 - y^2 = (x + y)(x - y)$. Write this pattern on your paper and plug in the given values:

$x^2 - y^2 = (x + y)(x - y)$
$3 = (x + y)(1/2)$
$6 = x + y$

44. **4.** One of the "special products" you need to memorize for the exam is $x^2 - 2xy + y^2 = (x - y)^2$. Write this pattern on your paper and plug in the given values:

$x^2 - 2xy + y^2 = (x - y)^2$
$84 + y^2 = (-10)^2$
$84 + y^2 = 100$
$y^2 = 16$
$y = 4$ or -4, so $|y| = 4$.

45. **(B)**. First, multiply out (remember FOIL = First, Outer, Inner, Last) each of the terms in parentheses:

$(x^2 - 2x - 2x + 4) + (x^2 - 1x - 1x + 1) + (x^2) + (x^2 + 1x + 1x + 1) + (x^2 + 2x + 2x + 4)$

Note that some of the terms will cancel each other out (e.g., $-x$ and x, $-2x$ and $2x$):

$(x^2 + 4) + (x^2 + 1) + (x^2) + (x^2 + 1) + (x^2 + 4)$

Finally, combine:

$5x^2 + 10$

46. **II and III only.** Distribute for a: $a = (x + y)^2 = x^2 + 2xy + y^2$. Since $b = x^2 + y^2$, a and b are the same except for the "extra" $2xy$ in a. Since xy is positive, a is greater than b. Statement I is false and statement II is true.

Each term in the sum for a is positive: xy is given as positive, and x^2 and y^2 are definitely positive, as they are squared and not equal to zero. Therefore, $a = x^2 + 2xy + y^2$ is positive. Statement III is true.

47. **(B).** To answer this question, it is important to understand what is meant by the phrase "directly proportional." It means that $a = kb$, where k is a constant. In alternative form: $\frac{a}{b} = k$, where k is a constant. So, because they both equal the constant, $\frac{a_{old}}{b_{old}} = \frac{a_{new}}{b_{new}}$. Plugging in values: $\frac{8}{2} = \frac{a_{new}}{4}$. Cross multiply and solve:

$32 = 2a_{new}$
$a_{new} = 16$

48. **(C)**. To answer this question, it is important to understand what is meant by the phrase "inversely proportional." It

means that $a = \frac{k}{b}$, where k is a constant. In alternative form, $ab = k$, where k is a constant.

So, because the product of a and b is always constant: $(16)(1) = (8)(b)$, or $b = 2$.

49. **(A).** To answer this question, it is important to understand what is meant by the phrase "inversely proportional." It means that $\text{time} = \frac{k}{\text{\# of students}}$, where k is a constant. In alternative form, (time)(# of students) = k, where k is a constant.

So, because the product of (time) and (# of students) is always constant:

(1.5 hours)(20 students) = (t hours)(35 students)

$$t = \frac{(1.5)(20)}{35} = \frac{30}{35} = \frac{6}{7}$$

Remember that t is in hours, so t is $\frac{6}{7} \times 60 = \frac{360}{7} \approx 51.43$ minutes. To the nearest minute, the time is 51 minutes.

50. **(B).** Because variable a is common to both quantities, the real comparison is between $+b$ and 0. Solve the system of equations for b.

Multiply the first equation by 2: $3a + 2b = 20 \longrightarrow 6a + 4b = 40$
Multiply the second equation by 3: $2a + 3b = 5 \longrightarrow 6a + 9b = 15$

Subtract the resulting second equation from the resulting first equation, canceling the a terms:

$-5b = 25$
$b = -5$

Because b is negative, Quantity B is greater.

51. **(C).** Since $m + 2n = 10$ and $m = 0.5n$, substitute $0.5n$ for m to get:

$0.5n + 2n = 10$
$2.5n = 10$
$n = 4$

Substitute $n = 4$ back into either equation to get $m = 2$. Since $2^2 = 4$, the two quantities are equal.

52. **(D).** If a, b, and c are positive, Quantity B is greater. If the variables are negative, Quantity A is greater. For instance, $a = 1$, $b = 2$, and $c = 4$ are valid numbers to test (since $1 + 2$ is 75% of 4). In such a case, Quantity B is

obviously greater. But $a = -1$, $b = -2$, and $c = -4$ are also valid numbers to test, in which case Quantity A is greater.

53. **(E).** First, divide the entire given equation by 2 to simplify: $a = 2b = 4c = 5$

Then, break up the equation into several smaller equations, setting each variable expression equal to 5:

$a = 5$
$2b = 5$ (so $b = 2.5$)
$4c = 5$ (so $c = 1.25$)

Thus, $64abc = 64(5)(2.5)(1.25) = 1{,}000$.

54. **12.5.** You do not need to solve for m and n to answer this question. (Nor is it possible to do so!) Simplify the equation:

$4m^2 + 6n^3 - 9 = 16$
$4m^2 + 6n^3 = 25$

Now divide both sides of the equation by 2:

$2m^2 + 3n^3 = 12.5$

This is exactly the quantity the problem is asking for. No further work is required.

55. **12.** While this problem can be solved by substitution, it is much easier and faster to simply stack and add all three equations. To keep them lined up properly, insert "placeholder terms"—for instance, instead of $a + b = 8$, write $a + b + 0c = 11$ or $1a + 1b + 0c = 11$ (since this equation does not use the variable c, it has "zero c"):

$1a + 1b + 0c = 8$
$0a + 1b + 1c = 11$
$1a + 0b + 1c = 5$
$2a + 2b + 2c = 24$

Divide both sides of the equation by 2 to get $a + b + c = 12$.

Chapter 9
of

Inequalities and Absolute Values

In This Chapter…

Inequalities and Absolute Values

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.
All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

$$|3x - 18| = 9$$

Quantity A	**Quantity B**
x	6

2. If $2z + 4 \geq -18$, which of the following must be true?

(A) $z \leq -11$
(B) $z \leq 11$
(C) $z \geq -11$
(D) $z \geq -7$
(E) $z \geq 7$

3.

$$7y - 3 \leq 4y + 9$$

Quantity A	**Quantity B**
y	4

4.

$$d + \frac{3}{2} < 8$$

Quantity A	**Quantity B**
$2d$	13

5.

$$\frac{4x}{7} \leq 15 + x$$

$$2y - 1.5 > 7$$

Quantity A	**Quantity B**
x	y

6.

$$3|x - 4| = 16$$

Quantity A	**Quantity B**
x	$\frac{28}{3}$

7. If $b \neq 0$ and $\frac{a}{b} > 0$, then which of the following must be true?

- ☐ $a > b$
- ☐ $b > 0$
- ☐ $ab > 0$

8. If $6 < 2x - 4 < 12$, which of the following could be a value of x?

(A) 4
(B) 5
(C) 7
(D) 8
(E) 9

9. If $y < 0$ and $4x > y$, which of the following could be equal to $\frac{x}{y}$?

(A) 0
(B) $\frac{1}{4}$
(C) $\frac{1}{2}$
(D) 1
(E) 4

10.

$$|x + 6| = 3$$
$$|2y| = 6$$

Quantity A	**Quantity B**
The greatest possible value for x	The least possible value for y

11. If $|4y + 2| = 18$, which of the following could be the value of y^2?

Indicate <u>two</u> such values.

☐ 2
☐ 5
☐ 16
☐ 25
☐ 36

12.

$$3(x - 7) \geq 9$$
$$0.25y - 3 \leq 1$$

Quantity A	**Quantity B**
x	y

13. If $|1 - x| = 6$ and $|2y - 6| = 10$, which of the following could be the value of xy?

Indicate <u>all</u> such values.

☐ -40
☐ -14
☐ -10
☐ 56

14. If $2(x - 1)^3 + 3 \leq 19$, then the value of x must be

(A) greater than or equal to 3
(B) less than or equal to 3

(C) greater than or equal to -3
(D) less than or equal to -3
(E) less than -3 or greater than 3

15. If $3P < 51$ and $5P > 75$, what is the value of the integer P?

(A) 15
(B) 16
(C) 24
(D) 25
(E) 26

16. A bicycle wheel has spokes that go from a center point in the hub to equally spaced points on the rim of the wheel. If there are fewer than six spokes, what is the smallest possible angle between any two spokes?

(A) 18 degrees
(B) 30 degrees
(C) 40 degrees
(D) 60 degrees
(E) 72 degrees

17.

$$|-x| \geq 6$$
$xy^2 < 0$ where y is an integer.

Quantity A	**Quantity B**
x	-4

18. If $\frac{|x+4|}{2} > 5$ and $x < 0$, which of the following could be the value of x?

Indicate <u>all</u> such values.

☐ -6
☐ -14
☐ -18

19.

$$|x^3| < 64$$

Quantity A	**Quantity B**
$-x$	$-|x|$

20. If $|0.1x - 3| \geq 1$, then x could be which of the following values?

Indicate <u>all</u> such values.

- ☐ 10
- ☐ 20
- ☐ 30
- ☐ 40
- ☐ 50
- ☐ 60

21. If $|3x + 7| \geq 2x + 12$, then

(A) $x \leq \frac{-19}{5}$
(B) $x \geq \frac{-19}{5}$
(C) $x \geq 5$
(D) $x \leq \frac{-19}{5}$ or $x \geq 5$
(E) $\frac{-19}{5} \leq x \leq 5$

22.

$$|3 + 3x| < -2x$$

Quantity A	**Quantity B**
$\lvert x\rvert$	4

23. If $|y| \leq -4x$ and $|3x - 4| = 2x + 6$, what is the value of x?

(A) -3
(B) -1/3
(C) -2/5
(D) 1/3
(E) 10

24.

x is an integer such that $-x|x| > 4$.

Quantity A	**Quantity B**
x	2

25.

$$|x| < 1 \text{ and } y > 0$$

Quantity A	**Quantity B**
$\lvert x\rvert + y$	xy

26.

x and y are positive numbers such that $x + y + z < 1$ and $xy = 1$

Quantity A	**Quantity B**
z	-1

27.

$|x| > |y|$ and $x + y > 0$

Quantity A	**Quantity B**
y	x

28.

x and y are integers such that $|x|(y) + 9 < 0$ and $|y| \leq 1$.

Quantity A	**Quantity B**
x	-9

29. If $x + y + z = 0$ and $z = 8$, which of the following must be true?

(A) $x < 0$
(B) $y < 0$
(C) $x - y < 0$
(D) $z - y > 0$
(E) $x + y < 0$

30.

$p + |k| > |p| + k$

Quantity A	**Quantity B**
p	k

31.

$|x| + |y| > |x + z|$

Quantity A	**Quantity B**
y	z

32.

$b \neq 0$

$$\frac{|a|}{b} > 1$$

$$a + b < 0$$

Quantity A	**Quantity B**
a	0

33. If $\frac{a}{b} > \frac{c}{d}$, which of the following statements must be true?

Indicate all such statements.

- ☐ $\frac{a}{b} - \frac{c}{d} > 0$
- ☐ $ad < bc$
- ☐ $ad > bc$

34. If $f^2g < 0$, which of the following must be true?

(A) $f < 0$
(B) $g < 0$
(C) $fg < 0$
(D) $fg > 0$
(E) $f^2 < 0$

35. $\sqrt{96} < x\sqrt{6}$ and $\frac{x}{\sqrt{6}} < \sqrt{6}$. If x is an integer, which of the following is the value of x?

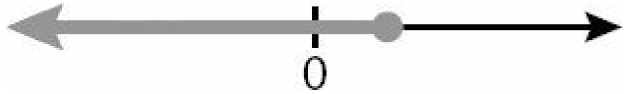
(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

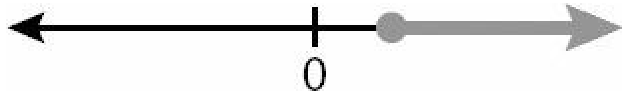
36.

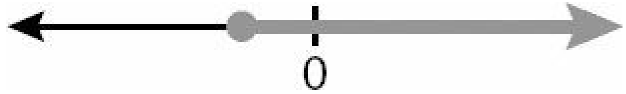
$$|x|y > x|y|$$

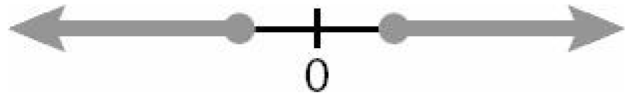
Quantity A	**Quantity B**
$(x + y)^2$	$(x - y)^2$

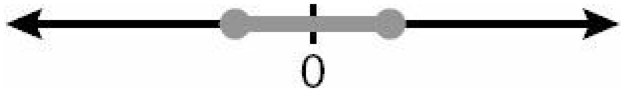
37. Which of the following could be the graph of all values of x that satisfy the inequality $4 - 11x \geq \frac{-2x+3}{2}$?

(A) 

(B) 

(C) 

(D) 

(E) 

38. If $|x^2 - 6| = x$, which of the following could be the value of x?

(A) -2
(B) 0
(C) 1
(D) 3
(E) 5

39.

$$-1 < a < 0 < |a| < b < 1$$

Quantity A	**Quantity B**
$\left(\dfrac{a^2\sqrt{b}}{\sqrt{a}}\right)^2$	$\dfrac{ab^5}{\left(\sqrt{b}\right)^4}$

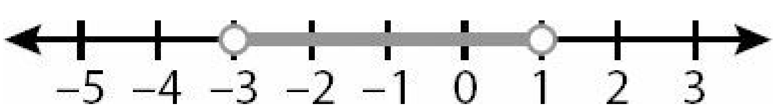
40.

$$x > |y| > z$$

Quantity A	**Quantity B**
$x + y$	$|y| + z$

41. The integers k, l, and m are consecutive even integers between 23 and 33. Which of the following could be the average of k, l, and m?

(A) 24
(B) 25
(C) 25.5
(D) 28
(E) 32

42. 

The number line above represents which of the following inequalities?

(A) $x < 1$
(B) $-6 < 2x < 2$
(C) $-9 < 3x < 6$
(D) $1 < 2x < 3$
(E) $x > -3$

43. For a jambalaya cook-off, there will be x judges sitting in a single row of x chairs. If x is greater than 3 but no more than 6, which of the following could be the number of possible seating arrangements for the judges?

Indicate two such numbers.

☐ 6
☐ 25
☐ 120
☐ 500
☐ 720

44. If $b \neq 0$, which of the following inequalities must be equivalent to $\frac{a}{-3b} < c$?

Indicate all such inequalities.

☐ $\frac{a}{b} > -3c$
☐ $\frac{a}{-3} < bc$
☐ $a > -3bc$

45.

$$a - b > a + b + c$$

Quantity A	**Quantity B**
$2b + c$	$b + c$

46.

$$|x + y| = 10$$
$$x > 0$$
$$z < y - x$$

Quantity A	**Quantity B**
z	10

47.

$$0 < a < \frac{b}{2} < 9$$

Quantity A	Quantity B
9 - *a*	$\frac{b}{2} - a$

48.

For all values of the integer *p* such that $1.9 < |p| < 5.3$, the function $f(p) = p^2$

Quantity A	Quantity B
$f(p)$ for the greatest value of *p*	$f(p)$ for the least value of *p*

49. If $\left|\frac{a}{b}\right|$ and $\left|\frac{x}{y}\right|$ are reciprocals and $\frac{a}{b}\left(\frac{x}{y}\right) < 0$ which of the following must be true?

(A) $ab < 0$
(B) $\frac{a}{b}\left(\frac{x}{y}\right) < -1$
(C) $\frac{a}{b} < 1$
(D) $\frac{a}{b} = -\frac{y}{x}$
(E) $\frac{y}{x} > \frac{a}{b}$

50. If $mn < 0$ and $\frac{k}{m} + \frac{l}{n} < mn$, which of the following must be true?

(A) $km + ln < (mn)^2$
(B) $kn + lm < 1$
(C) $kn + lm > (mn)^2$
(D) $k + l > mn$
(E) $kn > -lm$

51. Which of the following inequalities is equivalent to $|m + 2| < 3$?

(A) $m < 5$
(B) $m < 1$
(C) $-5 < m < 5$

(D) $m > -1$
(E) $-5 < m < 1$

52. If the reciprocal of the negative integer X is greater than the sum of Y and Z, then which of the following must be true?

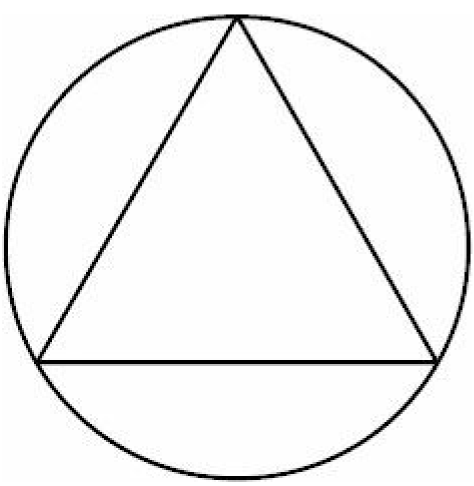
(A) $X > Y + Z$
(B) Y and Z are positive
(C) $1 > X(Y + Z)$
(D) $1 < XY + XZ$
(E) $\frac{1}{X} > Z - Y$

53. If $m + n - 2p < p + n + 4m$, which of the following inequalities must be true?

(A) $5m < 3p$
(B) $p > -m$
(C) $3m > 3p + 2n$
(D) $p > 2$
(E) $n < p$

54. If u and $-3v$ are greater than 0 and $\sqrt{u} < \sqrt{-3v}$, which of the following cannot be true?

(A) $u/3 < -v$
(B) $u/v > -3$
(C) $\sqrt{\frac{u}{-v}} < \sqrt{3}$
(D) $u + 3v > 0$
(E) $u < -3v$



55. In the figure above, an equilateral triangle is inscribed in a circle. If the arc bounded by adjacent corners of the triangle is between 4π and 6π long, which of the following could be the diameter of the circle?

(A) 6.5
(B) 9
(C) 11.9
(D) 15
(E) 23.5

Inequalities and Absolute Values Answers

1. **(D).** Since $3x$ - 18 is inside an absolute value, it could be either positive or negative 9 to have an absolute value of 9. Thus, solve the equation twice, once as though $3x$ - 18 is positive and once as though it is negative.

$$|3x - 18| = 9$$

$(3x - 18) = +9$	or	$(3x - 18) = -9$
$3x = 27$		$3x = 9$
$x = 9$		$x = 3$

$$x = 9 \text{ or } 3$$

Because x could be 9 or 3, x could be greater or less than 6, so the correct answer is (D).

2. **(C).** Solve the inequality algebraically:

$2z + 4 \geq -18$
$2z \geq -22$
$z \geq -11$

3. **(D).** Solve the inequality algebraically:

$7y - 3 \leq 4y + 9$
$3y - 3 \leq 9$
$3y \leq 12$
$y \leq 4$

Because y could be equal to 4 or greater than 4, the correct answer is (D).

4. **(B).** Solve the inequality algebraically:

$d + \frac{3}{2} < 8$
$d < 8 - 1.5$
$d < 6.5$

Quantity A is $2d$, so multiply both sides of the inequality by 2:

$2d < 13$

Quantity B is equal to 13, while $2d$ is less than 13, so the correct answer is (B).

5. **(D).** Solve algebraically for x and y:

$$\frac{4x}{7} \leq 15 + x$$
$$4x \leq 105 + 7x$$
$$-3x \leq 105$$
$$x \geq -35$$

(Remember to flip the inequality sign when dividing by -3!)

$$2y - 1.5 > 7$$
$$2y > 8.5$$
$$y > 4.25$$

Knowing that $x \geq -35$ and $y > 4.25$ is not enough to tell which is greater — the two ranges have a lot of overlap. For instance, x could be -30 and y could be 5 (Quantity B is greater), or x could be 100 and y could be 20 (Quantity A is greater). The correct answer is (D).

6. **(D).** Solve the inequality by first dividing both sides by 3 to isolate the absolute value, then solving for the positive and negative possibilities of (x - 4).

$$3|x-4| = 16$$
$$|x-4| = \frac{16}{3}$$
$$(x-4) = \frac{16}{3} \quad \text{or} \quad (x-4) = -\frac{16}{3}$$
$$x - 4 = \frac{16}{3} \qquad x = -\frac{16}{3} + 4$$
$$x = \frac{16}{3} + 4 \qquad x = -\frac{16}{3} + \frac{12}{3}$$
$$x = \frac{16}{3} + \frac{12}{3} \qquad x = -\frac{4}{3}$$
$$x = \frac{28}{3}$$

x could be $\frac{28}{3}$ or $-\frac{4}{3}$, making the two quantities equal or Quantity B greater, respectively. The correct answer is (D).

7. **III only.** If $\frac{a}{b} > 0$, then both a and b must have the same sign. That is, a and b are either both positive or both negative. Statement I could be true, but is not necessarily true. The relative values of a and b are not indicated by the inequality in the question stem. Statement II could be true, but is not necessarily true. If a were negative, b could be negative. Statement III must be true, as it indicates that a and b have the same sign.

8. **(C).** When manipulating a "three-sided" inequality, you must perform the same operations on all "sides." Therefore, the first step to simplify this inequality would be to add 4 to all sides: $10 < 2x < 16$. Next, divide all sides by 2. The result is $5 < x < 8$. The only answer choice that fits within the parameters of this inequality is 7. The correct answer is (C).

9. **(A).** If y is negative, then dividing both sides of the second inequality by y yields $\frac{4x}{y} < 1$. Remember, you must switch the direction of the inequality sign when multiplying or dividing by a negative (whether that negative is in number or variable form). Next, dividing both sides by 4 changes the inequality to $\frac{x}{y} < \frac{1}{4}$. The only answer choice less than $\frac{1}{4}$ is 0. The correct answer is (A).

10. **(C).** Solve each inequality, remembering that the phrase inside an absolute value can be positive or negative, so solve for each possibility:

$$|x + 6| = 3$$

$(x + 6) = 3$	or	$(x + 6) = -3$
$x = -3$		$x + 6 = -3$
		$x = -9$

$$x = -3 \text{ or } -9$$

$$|2y| = 6$$

$(2y) = 6$	or	$(2y) = -6$
$y = 3$		$y = -3$

$$y = 3 \text{ or } -3$$

The greatest possible value for x is -3. The least possible value for y is -3. The two quantities are equal, and the correct answer is (C).

11. **16, 25.** Solve the inequality, remembering that $4y + 2$ could be positive or negative, so solve for both possibilities:

$$|4y + 2| = 18$$

$(4y + 2) = 18$ or $(4y + 2) = -18$

$4y = 16$ $4y = -20$

$y = 4$ $y = -5$

$y = 4$ or -5

The value of y^2 could be 16 or 25.

12. **(D).** Solve each inequality algebraically:

$3(x - 7) \geq 9$
$x - 7 \geq 3$
$x \geq 10$

$0.25y - 3 \leq 1$
$0.25y \leq 4$
$y \leq 16$

Since the ranges for x and y overlap, either quantity could be greater. For instance, x could be 11 and y could be 15 (y is greater), or x could be 1,000 and y could be -5 (x is greater). The correct answer is (D).

13. **-40, -14, and 56 only.** Solve each absolute value:

$|1 - x| = 6$

$(1 - x) = 6$ or $(1 - x) = -6$

$-x = 5$ $-x = -7$

$x = -5$ $x = 7$

$x = -5$ or 7

$|2y - 6| = 10$

$(2y - 6) = 10$ or $(2y - 6) = -10$

$2y = 16$ $2y = -4$

$y = 8$ $y = -2$

$y = 8$ or -2

Since $x = -5$ or 7 and $y = 8$ or -2, calculate all four possible combinations for xy:

$(-5)(8) = -40$

$(-5)(-2) = 10$
$(7)(8) = 56$
$(7)(-2) = -14$

Select -40, -14, and 56. (Do NOT pick -10, as xy could be 10, but not -10).

14. **(B).** $2(x - 1)^3 + 3 \leq 19$
$2(x - 1)^3 \leq 16$
$(x - 1)^3 \leq 8$

You can take the cube root of both sides of an inequality, because cubing a number, unlike squaring it, does not change its sign.

$x - 1 \leq 2$
$x \leq 3$

This matches the language in answer choice (B).

15. **(B).** Dividing the first inequality by 3 results in $P < 17$. Dividing the second inequality by 5 results in $P > 15$. Therefore, $15 < P < 17$. Because P is an integer, it must be 16.

16. **(E).** In this scenario, if there are n spokes, there are n angles between them. Thus the measure of the angle between spokes is $\frac{360}{n}$. Since $n < 6$, you can rewrite this expression as $\frac{360}{\text{(less than 6)}}$. Dividing by a "less than" produces a "greater than" result. Therefore, $\frac{360}{\text{(less than 6)}}$ = greater than 60. The only answer that is greater than 60 is (E). To verify, note that n can be at most 5, as n is an integer. Because there are 360 degrees in a circle, a wheel with 5 spokes would have $\frac{360}{\text{(less than 6)}}$ degrees between adjacent spokes. The correct answer is (E).

17. **(B).** First, solve the inequality for x, remembering the two cases you must consider when dealing with absolute value: $-x$ is positive and $-x$ is negative.

$|-x| \geq 6$

$+(-x) \geq 6$	or	$-(-x) \geq 6$
$-x \geq 6$		$x \geq 6$
$x \leq -6$		

$x \leq -6$ or $x \geq 6$

Because $xy^2 < 0$, neither x nor y equals zero. A squared term cannot be negative, so y^2 must be positive. For xy^2 to be negative, x must be negative. This rules out the $x \geq 6$ range of solutions for x. Thus, $x \leq -6$ is the only range of valid solutions. Since all values less than or equal to -6 are less than -4, the correct answer is (B).

18. **III only.** Solve the absolute value inequality by first isolating the absolute value:

$$\frac{|x+4|}{2} > 5$$

$|x + 4| > 10$

If $(x + 4)$ is positive or zero, the absolute value bars do nothing and can be removed:

$x + 4 > 10$
$x > 6$

This is not a valid solution range, as the other inequality indicates that x is negative.

Then solve for negative case. Note that $|x + 4| > 10$ when $(x + 4)$ is more positive than 10 or *more negative* than -10.

$(x + 4) < -10$
$x < -14$

Alternatively, using the identity that $|a| = -a$ when a is negative:

$|x + 4| > 10$
$-(x + 4) > 10$ when $(x + 4)$ is negative
$-x - 4 > 10$
$-x > 14$

$x < -14$ (flip the inequality sign when multiplying both sides by -1.)

If $x < -14$, only -18 is a valid answer.

19. **(D).** First, solve the absolute value inequality, using the identity that $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative:

$|x^3| < 64$

$+(x^3) < 64$	or	$-(x^3) < 64$
$x < 4$		$x^3 > -64$ (Flip the inequality sign when multiplying by -1.)
		$x > -4$

$-4 < x < 4$

x could be positive, negative, or zero. If x is positive or zero, the two quantities are equal. If x is negative, Quantity A is greater. The correct answer is (D).

20. **10, 20, 40, 50, 60 only.** Solve the absolute value inequality, using the identity that $|a| = a$ when a is positive or

zero and $|a| = -a$ when a is negative:

$$|0.1x - 3| \geq 1$$

$+(0.1x - 3) \geq 1$ or $-(0.1x - 3) \geq 1$

$+(0.1x - 3) \geq 1$	$-(0.1x - 3) \geq 1$
$0.1x - 3 \geq 1$	$-0.1x + 3 \geq 1$
$0.1x \geq 4$	$-0.1x \geq -2$
$x \geq 40$	$x \leq 20$ (Flip the inequality sign when dividing by -0.1)

Since $x \leq 20$ or $x \geq 40$, x cannot equal 30, but it can be any of the other values from the choices.

Alternatively, plug the choices to test which values "work."

10: $|0.1(10) - 3| = |1 - 3| = |-2| = 2$, which is ≥ 1.
20: $|0.1(20) - 3| = |2 - 3| = |-1| = 1$, which is ≥ 1.
30: $|0.1(30) - 3| = |3 - 3| = |0| = 0$, which is NOT ≥ 1.
40: $|0.1(40) - 3| = |4 - 3| = |1| = 1$, which is ≥ 1.
50: $|0.1(50) - 3| = |5 - 3| = |2| = 2$, which is ≥ 1.
60: $|0.1(60) - 3| = |6 - 3| = |3| = 3$, which is ≥ 1.

21. **(D).** Solve $|3x + 7| \geq 2x + 12$, using the identity that $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative:

$+(3x + 7) \geq 2x + 12$	or	$-(3x + 7) \geq 2x + 12$
$x + 7 \geq 12$		$-3x - 7 \geq 2x + 12$
$x \geq 5$		$-7 \geq 5x + 12$
		$-19 \geq 5x$
		$\frac{-19}{5} \geq x$

$x \leq \frac{-19}{5}$ or $x \geq 5$

22. **(B).** Solve the absolute value inequality, using the identity that $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative:

$$|3 + 3x| < -2x$$

$+(3 + 3x) < -2x$	or	$-(3 + 3x) < -2x$
$3 + 5x < 0$		$-3 - 3x < -2x$
$5x < -3$		$-3 < x$
$x < -\frac{3}{5}$		

$$-3 < x < -\frac{3}{5}$$

Since x is between -3 and $-\frac{3}{5}$, its absolute value is between $\frac{3}{5}$ and 3. Thus, Quantity A is less than Quantity B.

23. **(C).** The inequality is not strictly solvable, as it has two unknowns. However, any absolute value cannot be negative. Putting $0 \leq |y|$ and $|y| \leq -4x$ together, $0 \leq -4x$. Dividing both sides by -4 and flipping the inequality sign, this implies that $0 \geq x$.

Now solve the absolute value equation:

$$|3x - 4| = 2x + 6$$

$+(3x - 4) = 2x + 6$	or	$-(3x - 4) = 2x + 6$
$3x - 4 = 2x + 6$		$-3x + 4 = 2x + 6$
$x - 4 = 6$		$4 = 5x + 6$
$x = 10$		$-2 = 5x$

$$x = 10 \text{ or } -2/5$$

If $x = 10$ or $-2/5$, but $0 \geq x$, then x can only be $-2/5$.

24. **(B).** If $-x|x| \geq 4$, $-x|x|$ is positive. Because $|x|$ is positive by definition, $-x|x|$ is positive only when $-x$ is also positive. This occurs when x is negative. For example, $x = -2$ is one solution allowed by the inequality: $-x|x| = -(-2) \times |-2| = 2 \times 2 = 4$.

So, Quantity A can be -2, -3, -4, -5, -6, etc. The maximum value of Quantity A is less than 2, so Quantity B is greater.

25. **(A).** The inequality $|x| < 1$ allows x to be either a positive or negative fraction (or zero). Interpreting the absolute value sign, it is equivalent to $-1 < x < 1$. As indicated, y is positive.

When x is a negative fraction,

Quantity A: $|x| + y$ = positive fraction + positive = positive

Quantity B: xy = negative fraction × positive = negative

Quantity A is greater in these cases.

When x is zero,

Quantity A: $|x| + y$ = 0 + positive = positive

Quantity B: $xy = 0 \times \text{positive} = 0$

Quantity A is greater in this case.

When x is a positive fraction,

Quantity A: $|x| + y$ = positive fraction + y = greater than y

Quantity B: xy = positive fraction × y = less than y

Quantity A is greater in these cases.

In all cases, Quantity A is greater.

26. **(B).** Solve the inequality for z.

$x + y + z < 1$
$z < 1 - (x + y)$

Based on the facts that x and y are positive and $xy = 1$, either x and y both equal 1 or they are reciprocals (e.g., 2 and $\frac{1}{2}$, 3 and $\frac{1}{3}$, 4 and $\frac{1}{4}$, etc.). Thus, the minimum value of $x + y$ is 2. Plugging into the inequality for z:

$z < 1 - (x + y)$
$z < 1$ - (at least 2)
$z <$ at most -1

Because z cannot equal -1 (z is less than -1) Quantity B is greater.

27. **(B).** In general, there are four cases for the signs of x and y, some of which can be ruled out by the constraints of this question.

x	y	$x + y > 0$
pos	pos	true
pos	neg	true when $\|x\| > \|y\|$
neg	pos	false when $\|x\| > \|y\|$
neg	neg	false

So only the first two cases need to be considered for this question.

If x and y are both positive, $|x| > |y|$ just means that $x > y$.

If x is positive and y is negative, $x > y$ simply because positive > negative.

In both cases, $x > y$. Quantity B is greater.

28. **(D).** If y is an integer and $|y| \leq 1$, then $y = -1$, 0, or 1. The other inequality can be simplified from $|x|(y) + 9 < 0$ to $|x|(y) < -9$. In words, $|x|(y)$ is negative. Because $|x|$ cannot be negative by definition, y must be negative, so only $y = -1$ is possible.

If $y = -1$, then $|x|(y) = |x|(-1) = -|x| < -9$. So, $-|x| = -10, -11, -12, -13$, etc.

Thus, $x = \pm10, \pm11, \pm12, \pm13$, etc. Some of these x values are greater than -9 and some are less than -9.

29. **(E).** If $x + y + z = 0$ and $z = 8$, then $x + y = -8$. It is definitely true that $-8 < 0$, so $x + y < 0$ must be true.

Alternatively, find a counterexample to disprove the other choices.

(A) x could be positive: $x = 5$ and $y = -13$ make $x + y = 5 + (-13) = -8$.

(B) y could be positive: $x = -13$ and $y = 5$ make $x + y = -13 + 5 = -8$.

(C) $x - y$ could be positive: $x = 5$ and $y = -13$ make $x - y = 5 - (-13) = 18$ and $x + y + z = 5 + (-13) + 8 = 0$.

(D) $z - y$ could be positive: $z = 8$ and $y = -13$ make $z - y = 8 - (-13) = 21$ and $x = 5$ would make the sum $x + y + z = 5 + (-13) + 8 = 0$.

(E) $x + y$ CANNOT be positive or zero, as $x + y + z$ would then be at least 8, not equal to 0.

30. **(A).** In general, there are four cases for the signs of p and k, some of which can be ruled out by the constraints of this question.

p	k	$p + \|k\| > \|p\| + k$
pos	pos	Not in this case: For positive numbers, absolute value "does nothing," so both sides are equal to $p + k$.
pos	neg	True for this case: p + (a positive absolute value) is greater than p + (a negative value).
neg	pos	Not in this case: k + (a negative value) is less than k + (a positive absolute value).
neg	neg	Possible in this case: It depends on relative values. Both sides are a positive plus a negative.

Additionally, check whether p or k could be zero.

If $p = 0$, $p + |k| > |p| + k$ is equivalent to $|k| > k$. This is true when k is negative.

If $k = 0$, $p + |k| > |p| + k$ is equivalent to $p > |p|$. This is not true for any p value.

So, there are three possible cases for p and k values. For the second one, use the identity that $|a| = -a$ when a is negative.

p	k	Interpret:
pos	neg	$p = \text{pos} > \text{neg} = k$ $p > k$
neg	neg	$p + \lvert k\rvert > \lvert p\rvert + k$ $p + -(k) > -(p) + k$ $p - k > -p + k$ $2p - k > k$ $2p > 2k$ $p > k$
0	neg	$p = 0 > \text{neg} = k$ $p > k$

In all the cases that are valid according to the constraint inequality, p is greater than k. Quantity A is greater.

31. **(D).** Given only one inequality with three unknowns, solving will not be possible. Instead, test numbers with the goal of proving (D).

For example, $x = 2$, $y = 5$, and $z = 3$.

Check that $|x| + |y| > |x + z|$: $|2| + |5| > |2 + 3|$ is $7 > 5$, which is true.

In this case, $y > z$.

Try to find another example such that $y < z$. Always consider negatives in inequalities and absolute value questions.

Consider another example: $x = 2$, $y = -5$ and $z = 3$.

Check that $|x| + |y| > |x + z|$: $|2| + |-5| > |2 + 3|$ is $7 > 5$, which is true.

In this case, $z > y$.

Either statement could be greater. It cannot be determined from the information given.

32. **(B).** If $\frac{|a|}{b}$ is greater than 1, then it is positive. Because $|a|$ is nonnegative by definition, b would have to be positive. Thus, if you cross multiply, you do not have to flip the sign of the inequality:

$$\frac{|a|}{b} > 1$$

$$|a| > b$$

To summarize, $b > 0$ and $|a| > b$. Putting this together, $|a| > b > 0$.

In order for $a + b$ to be negative, a must be more negative than b is positive. For example, $a = -4$ and $b = 2$ agree with all the constraints so far. Note that a cannot be zero (because $\frac{|a|}{b} = 0$ in this case, not > 1) and a cannot be positive (because $a + b > 0$ in this case, not < 0).

Therefore, $a < 0$. Quantity B is greater.

33. **I only.**

Statement I: TRUE. Subtract $\frac{c}{d}$ from both sides of the inequality $\frac{a}{b} > \frac{c}{d}$, and you will get $\frac{a}{b} - \frac{c}{d} > 0$. It must be true.

Statement II: Maybe. This is only true if b and d have opposite signs, because it is the result of multiplying both sides by bd and flipping the inequality sign, which you would only do when bd is negative.

Statement III: Maybe. This is only true if b and d have the same sign, because it is the result of multiplying both sides by bd without flipping the inequality sign, which is only acceptable when bd is positive.

34. **(B).** Neither f nor g can be zero, or f^2g would be zero. The square of either a positive or negative base is always positive, so f^2 is positive. In order for $f^2g < 0$ to be true, g must be negative. Therefore, the correct answer is (B). Answer choices (A), (B), and (C) are not correct because f could be either positive or negative. Answer choice (E) directly contradicts the truth that f^2 is positive.

35. **(D).** Solve the first inequality:

$$\sqrt{96} < x\sqrt{6}$$

$$\frac{\sqrt{96}}{\sqrt{6}} < x$$

$$\sqrt{16} < x$$

$$4 < x$$

Solve the second inequality:

$$\frac{x}{\sqrt{6}} < \sqrt{6}$$

$$x < \sqrt{6}\sqrt{6}$$

$$x < \sqrt{36}$$

$$x < 6$$

Combining the two inequalities, $4 < x < 6$ so x must be 5. The correct answer is (D).

36. **(B).** In general, there are four cases for the signs of x and y, some of which can be ruled out by the constraint in the question stem. Use the identity that $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative:

x	y	$\|x\|y > x\|y\|$ is equivalent to:	True or False?
pos	pos	$xy > xy$	False: $xy = xy$
pos	neg	$xy > x(-y)$	False: xy is negative, and $-xy$ is positive.
neg	pos	$(-x)y > xy$	True: xy is negative, and $-xy$ is positive.
neg	neg	$(-x)y > x(-y)$	False: $-xy = -xy$

Note that if either x or y equals 0, that case would also fail the constraint.

The only valid case is when x is negative and y is positive.

Quantity A: $(x + y)^2 = x^2 + 2xy + y^2$
Quantity B: $(x - y)^2 = x^2 - 2xy + y^2$

Ignore (or subtract) $x^2 + y^2$ as it is common to both quantities. Thus,

Quantity A: $2xy$ = 2(negative)(positive) = negative
Quantity B: $-2xy$ = -2(negative)(positive) = positive

Quantity B is greater.

37. **(A).** First, solve $4 - 11x \geq \frac{-2x+3}{2}$ for x:

$$4 - 11x \geq \frac{-2x+3}{2}$$

$$8 - 22x \geq -2x + 3$$

$$5 - 22 \geq -2x$$

$$5 \geq 20x$$

$$\frac{5}{20} \geq x$$

$$\frac{1}{4} \geq x$$

Thus, the correct choice should show the black line beginning to the right of zero (in the positive zone), and continuing indefinitely into the negative zone. Even without actual values (other than zero) marked on the graphs, only (A) meets these criteria.

38. **(D).** While you could set x^2 - 6 equal to both x and -x and then solve both equations (there is a positive and negative case because of the absolute value), it is probably easier for most people to plug in the answers:

	x	x^2 - 6	x^2 - 6
(A)	-2	$(-2)^2 - 6 = 4 - 6 = -2$	2
(B)	0	$(0)^2 - 6 = 0 - 6 = -6$	6
(C)	1	$(1)^2 - 6 = 1 - 6 = 5$	5
(D)	**3**	$\mathbf{(3)^2 - 6 = 9 - 6 = 3}$	**3**
(E)	5	$(5)^2 - 6 = 25 - 6 = 19$	19

Note that only $x = 3$ works. While this chart shows the results of trying every choice, note that if you were doing this on your own, you could stop as soon as you got a choice that worked.

39. **(A).** From $-1 < a < 0 < |a| < b < 1$, the following can be determined:

a is a negative fraction,
b is a positive fraction, and
b is more positive than a is negative. (i.e., $|b| > |a|$, or b is farther from 0 on the number line than a is.)

Using exponent rules, simplify the quantities.

Quantity A: $$\left(\frac{a^2\sqrt{b}}{\sqrt{a}}\right)^2 = \frac{\left(a^2\right)^2\left(\sqrt{b}\right)^2}{\left(\sqrt{a}\right)^2} = \frac{a^4 b}{a} = a^3 b$$

Quantity B: $$\frac{ab^5}{\left(\sqrt{b}\right)^4} = \frac{ab^5}{\left(b^{1/2}\right)^4} = \frac{ab^5}{b^{\frac{1}{2}\times 4}} = \frac{ab^5}{b^2} = ab^3$$

Dividing both quantities by b would be acceptable, as b is positive and doing so won't flip the relative sizes of the quantities. It would be nice to cancel a's, too, but it is problematic that a is negative. Dividing both quantities by a^2 would be okay, though, as a^2 is positive.

Divide both quantities by a^2b.

Quantity A: $\frac{a^3b}{a^2b} = a$

Quantity B: $\frac{ab^3}{a^2b} = \frac{b^2}{a}$

Just to make the quantities more similar in form, divide again by b, which is positive.

Quantity A: $\frac{a}{b}$

Quantity B: $\frac{b}{a}$

Both quantities are negative, as a and b have opposite signs. Remember that b is more positive than a is negative. (i.e., $|b| > |a|$, or b is farther from 0 on the number line than a is.) Thus, each fraction can be compared to -1.

Quantity A: $\frac{a}{b}$ is less negative than -1. That is, $-1 < \frac{a}{b}$.

Quantity B: $\frac{b}{a}$ is more negative than -1. That is, $\frac{b}{a} < -1$.

Quantity A is greater.

40. **(D).** Given only a compound inequality with three unknowns, solving will not be possible. Instead, test numbers with the goal of proving (D). Always consider negatives in inequalities and absolute value questions.

For example, $x = 10$, $y = -9$, and $z = 8$.

Check that $x > |y| > z$: $10 > |-9| > 8$, which is true.

In this case, $x + y = 10 + (-9) = 1$ and $|y| + z = 9 + 8 = 17$. Quantity B is greater.

Try to find another example such that Quantity A is greater.

For example, $x = 2$, $y = 1$, and $z = -3$.

Check that $x > |y| > z$: $2 > |1| > -3$, which is true.

In this case, $x + y = 2 + 1 = 3$ and $|y| + z = 1 + (-3) = -2$. Quantity A is greater.

Either statement could be greater. It cannot be determined from the information given.

41. **(D).** The values for k, l, and m, respectively, could be any of the following three sets:

Set 1: 24, 26, and 28
Set 2: 26, 28, and 30
Set 3: 28, 30, and 32

For evenly spaced sets with an odd number of terms, the average is the middle value. Therefore, the average of k, l, and m could be 26, 28, or 30. Only answer choice (D) matches one of these possibilities.

42. **(B).** The number line indicates a range between, but not including, -3 and 1. However, $-3 < x < 1$ is not a given option. However, answer choice (B) gives the inequality $-6 < 2x < 2$. Dividing all three sides of this inequality by 2 yields $-3 < x < 1$.

43. **120 and 720 only.** If x is "greater than 3 but no more than 6," then x is 4, 5, or 6. If there are 4 judges sitting in 4 seats, they can be arranged $4! = 4 \times 3 \times 2 \times 1 = 24$ ways. If there are 5 judges sitting in 5 seats, they can be arranged $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ ways. If there are 6 judges sitting in 6 seats, they can be arranged $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways. Thus, 24, 120, and 720 are all possible answers. Only 120 and 720 appear in the choices.

44. **I only.** For this problem, you need to know that multiplying or dividing an inequality by a negative requires you to flip the inequality sign. Thus, multiplying or dividing an inequality by a variable should *not* be done unless you know *whether* to flip the inequality sign (i.e., whether the variable represents a positive or negative number).

Statement I: TRUE. Multiply both sides of the original inequality by -3 and flip the inequality sign.

Statement II: Maybe. Multiply both sides of the original inequality by b to get Statement II, but only if b is positive. If b is negative, the direction of the inequality sign would have to be changed.

Statement III: Maybe. Multiplying both sides of the original inequality by $-3b$ could lead to Statement III, but because the inequality sign flipped, this is only true if $-3b$ is negative (i.e., if b is positive).

45. **(D).** There are three variables in the original question, but not all of them are relevant. Simplify the original constraint:

$a - b > a + b + c$
$-b > b + 4$
$0 > 2b + c$

So, $2b + c$ is negative.

Next, notice that $b + c$ is common to both quantities, so subtracting it from both will not change their relative values:

Quantity A: $2b + c - (b + c) = 2b + c - b - c = b$
Quantity B: $b + c - (b + c) = 0$

This question is really about the sign of b!

Based on the constraint that $2b + c$ is negative, b could be positive or negative.

If $b = 2$ and $c = -6$, $2b + c = 4 - 6 = -2$, which is negative. In this case, Quantity A is greater.

If $b = -2$ and $c = 1$, $2b + c = -4 + 1 = -3$, which is negative. In this case, Quantity B is greater.

The correct answer is (D).

46. **(B).** From $z < y - x$, the value of z depends on x and y. So, solve for x and y as much as possible. There are two cases for the absolute value equation: $|x + y| = 10$ means that $(x + y) = \pm 10$. Consider these two cases separately

The positive case:

$x + y = 10$, so $y = 10 - x$.
Substitute into $z < y - x$, getting $z < (10 - x) - x$, or $z < 10 - 2x$.
Because x is at least zero, $10 - 2x \leq 10$.
Putting the inequalities together, $z < 10 - 2x \leq 10$.
Thus, $z < 10$.

The negative case:

$x + y = -10$, so $y = -10 - x$.
Substitute into $z < y - x$, getting $z < (-10 - x) - x$, or $z < -10 - 2x$.
Because x is at least zero, $-10 - 2x \leq -10$.
Putting the inequalities together, $z < -10 - 2x \leq -10$.
Thus, $z < -10$.

In both cases, 10 is greater than z. The correct answer is (B).

47. **(A).** The variable a is common to both quantities, and adding it to both quantities to cancel will not change the relative values of the quantities.

Quantity A: $(9 - a) + a = 9$

Quantity B: $\left(\frac{b}{2} - a\right) + a = \frac{b}{2}$

According to the given constraint, $\frac{b}{2} < 9$, so Quantity A is greater. The correct answer is (A).

48. **(C).** If p is an integer such that $1.9 < |p| < 5.3$, p could be 2, 3, 4, or 5, as well as -2, -3, -4, -5. The greatest value

of p is 5, for which the value of $f(p) = 5^2 = 25$. The least value of p is -5, for which the value of $f(p) = (-5)^2 = 25$.

49. **(D).** If $\frac{a}{b}\left(\frac{x}{y}\right) < 0$, then the two fractions have opposite signs. Therefore, by the definition of reciprocals, $\frac{a}{b}$ must be the negative inverse of $\frac{x}{y}$, no matter which one of the fractions is positive. In equation form, this means $\frac{a}{b} = -\frac{y}{x}$, which is choice (D). The other choices are possible but not certain.

50. **(C).** In order to get m and n out of the denominators of the fractions on the left side of the inequality, multiply both sides of the inequality by mn. The result is $kn + lm > (mn)^2$. The direction of the inequality sign changes because mn is negative. This is an exact match with (C), which must be the correct answer.

51. **(E).** When dealing with absolute values, always consider two cases.

The first case is when the expression within the absolute value signs is positive. If $m + 2 > 0$, then $|m + 2| = m + 2$, and therefore $m + 2 < 3$. Subtracting 2 from both sides, this inequality becomes $m < 1$.

The second case is when $m + 2 < 0$, so that $|m + 2| = -(m + 2)$, and therefore $-(m + 2) < 3$. Divide both sides by -1 to get $m + 2 > -3$, remembering to flip the inequality sign. Subtracting 2 from both sides, this inequality becomes $m > -5$.

Combining these two inequalities, the result is $-5 < m < 1$.

52. **(D).** The inequality described in the question is $0 > \frac{1}{X} > Y + Z$. Multiplying both sides of this inequality by X, the result is $0 < 1 < XY + XZ$. Notice that the direction of the inequality sign must change because X is negative.

(A) Maybe true: true only if X equals -1.
(B) Maybe true: either Y or Z or both can be negative.
(C) False: the direction of the inequality sign is opposite the correct direction determined above.
(D) TRUE. It is a proper rephrasing of the original inequality.
(E) Maybe true: it is not a correct rephrasing of the original inequality.

53. **(B).** The given inequality can be simplified as follows:

$m + n - 2p < p + n + 4m$
$m - 2p < p + 4m$
$-3p < 3m$
$p > -m$ (Remember to flip the inequality sign when dividing by -3.)

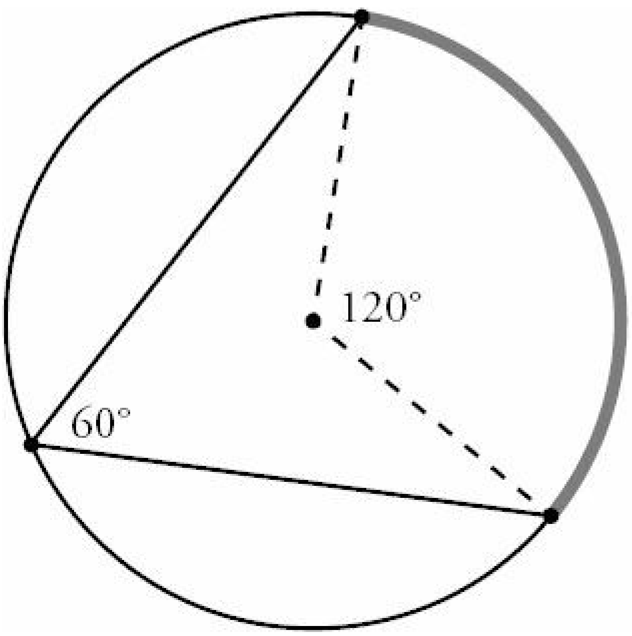
The correct answer is (B).

54. **(D).** When the GRE writes a root sign, the question writers are indicating a nonnegative root only. Therefore both sides of this inequality are positive. Thus, you can square both sides without changing the direction of the inequality

sign. So $u < -3v$. Now evaluate each answer choice:

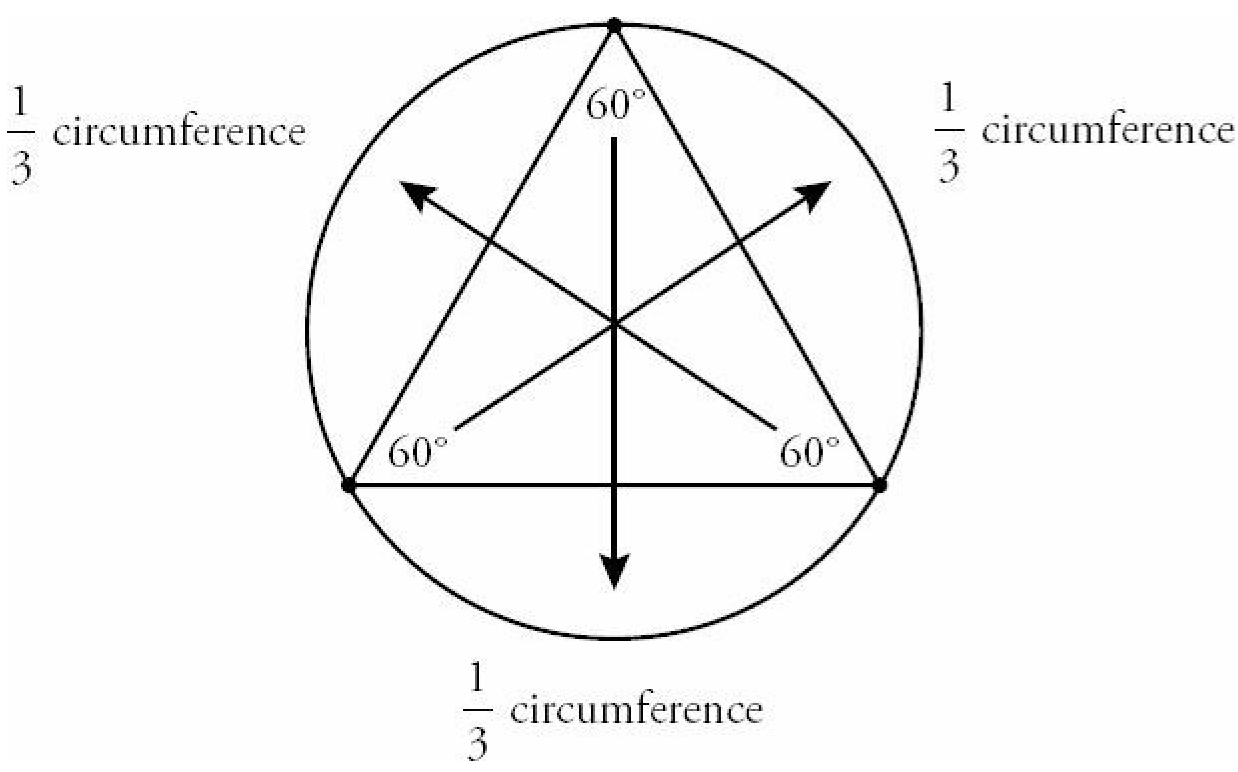
(A) Must be true. Divide both sides of $u < -3v$ by 3.
(B) Must be true. It is given that $-3v > 0$ and therefore, $v < 0$. Then, when dividing both sides of $u < -3v$ by v, you must flip the inequality sign and get $u/v > -3$.
(C) Must be true. This is the result after dividing both sides of the original inequality by $\sqrt{-v}$.
(D) CANNOT be true. Adding $3v$ to both sides of $u < -3v$ results in $u + 3v < 0$, not $u + 3v > 0$.
(E) Must be true. This is the result of squaring both sides of the original inequality.

55. **(D).** Since each of the three arcs corresponds to one of the 60 degree angles of the equilateral triangle, each arc represents 1/3 of the circumference of the circle. The diagram below illustrates this for just one of the three angles in the triangle:



$$\frac{120°}{360°} = \frac{1}{3}$$

The same is true for each of the three angles:



Since each of the three arcs is between 4π and 6π, triple these values to determine that the circumference of the circle is between 12π and 18π. Because circumference equals π times the diameter, the diameter of this circle must be between 12 and 18. Only choice (D) is in this range.

Chapter 10
of

Functions, Formulas, and Sequences

In This Chapter…

Functions, Formulas, and Sequences

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. If $f(x) = x^2 + 1$, what is $f(2) + f(-2)$?

(A) 0
(B) 1
(C) 4
(D) 5
(E) 10

2. If $f(x) = 2x$ and $g(x) = x^3$, what is $f(g(-3))$?

(A) -6
(B) -27
(C) 54
(D) -54
(E) -216

3. If $h(x) = 2x^3 - 3$ and $h(m) = -19$, what is the value of m?

(A) -3

(B) -2
(C) 2
(D) 6,856
(E) 6,862

4. If $f(x) = x - 3$ and $2[f(g)] = 14$, what is the value of $f(4g)$?

5. If $f(a, b) = a^2b^4$, and $f(m, n) = 5$, what is $f(3m, 2n)$?

6. If $f(x) = x^2 - 1$, what is the value of $f(y) + f(-1)$?

(A) $y^2 - 1$
(B) y^2
(C) $y^2 + 1$
(D) $y^2 - 2y$
(E) $y^2 - 2y - 1$

7. If $g(x) = 3x - 3$, what is the value of $g(3) + g\left(-\frac{1}{3}\right)$? ?

8. If $f(x) = \frac{x}{2} - 1$, what is the value of $f(f(10))$?

9. If $h(x) = 5x^2 + x$, then $h(a + b) =$

(A) $5a^2 + 5b^2$
(B) $5a^3 + 5b^3$
(C) $5a^2 + 5b^2 + a + b$
(D) $5a^3 + 10ab + 5b^3$
(E) $5a^2 + 10ab + 5b^2 + a + b$

10. If $\boxed{x} = 2x^2 + 2$, what is $\boxed{4}$?

(A) $\boxed{\boxed{-1}}$
(B) $\boxed{-2}$
(C) $\boxed{\boxed{2}}$
(D) $\boxed{17}$
(E) $\boxed{34}$

11. $\boxed{x}$ is defined as the least integer greater than x for all odd values of x, and the greatest integer less than x for all even values of x. What is $\boxed{-2} - \boxed{5}$?

(A) -12
(B) -9
(C) -8
(D) -7
(E) 3

12. $g(x) = x^2 - 4$ and $g(c) = 12$. If $c < 0$, what is $g(c - 2)$?

13. If $h(x) = 2x - 1$ and $g(x) = x^2 - 3$, what is $h(g(5))$?

14. $h(x) = 2x - 1$ and $g(x) = x^2 - 3$. If $g(m) = 61$, what is $h(m)$ if $m > 0$?

15. If &x& is defined as one-half the square of x, what is the value of $\frac{\&4\&}{\&6\&}$?

16. If $\sim x = |14x|$, which of the following must be true?

Indicate <u>all</u> such answers.

- ☐ ~2 = ~(-2)
- ☐ ~3 + ~4 = ~7
- ☐ The minimum possible value of ~x is zero

17. #x = the square of the number that is 2 less than x. What is the value of #5 - #(-1)?

☐

18. If $g(x) = \dfrac{x^2(4x+9)}{(3x-3)(x+2)}$, which answer represents all values of x for which $g(x)$ is undefined?

(A) 0
(B) $-\dfrac{9}{4}$
(C) -2, 1
(D) -2, 0, 1
(E) $-2, -\dfrac{9}{4}, 1$

19. If $f(x) = \dfrac{\sqrt{x-2}}{x}$ for all integer values of x, for how many values of x is $f(x)$ undefined?

(A) 0
(B) 1
(C) 2
(D) 3
(E) more than 3

20.

$$f(x) = 2x - 3$$
$$f(m) = -11$$

Quantity A	**Quantity B**
The value of m	Half the value of $f(m)$

21. The price of a phone call consists of a standard connection fee, which does not change, plus a per-minute charge. A 10-minute call costs \$2.90 and a 16-minute call costs \$4.40. How much does a 13-minute call cost?

(A) \$3.55
(B) \$3.57
(C) \$3.58
(D) \$3.65

(E) $3.77

22. The first three terms in an arithmetic sequence are 30, 33, and 36. What is the 80th term?

23. The sequence S is defined as $S_n = 2(S_{n-1}) - 4$. If $S_1 = 6$, what is S_5?

(A) -20
(B) 16
(C) 20
(D) 24
(E) 36

24. The sequence S is defined as $S_n = 3(S_{n-1}) + 1$. If $S_1 = -2$, what is S_4?

(A) -39
(B) -41
(C) -43
(D) -45
(E) -47

25. If $S_n = S_{n-1} + S_{n-2} - 3$, then what is S_6 when $S_1 = 5$ and $S_2 = 0$?

(A) -6
(B) -5
(C) -3
(D) -1
(E) 1

26. If $S_n = S_{n-1} + S_{n-2} - 1$, then what is S_4 when $S_0 = -10$ and $S_2 = 0$?

(A) -3
(B) 0
(C) 9
(D) 10
(E) 14

27. If $S_n = S_{n-1} + S_{n-2} + S_{n-3} - 5$, what is S_6 when $S_1 = 4$, $S_2 = 0$, and $S_4 = -4$?

(A) -2
(B) -12
(C) -16
(D) -20
(E) -24

28. The sequence P is defined as $P_n = 10(P_{n-1}) - 2$. If $P_1 = 2$, what is P_4?

29. The sequence S is defined as $S_{n-1} = \frac{1}{4}(S_n)$. If $S_1 = -4$, what is S_4?

(A) -256
(B) -64
(C) -1/16
(D) 1/16
(E) 256

30. In sequence A_n, $A_1 = 45$, and $A_n = A_{n-1} + 2$ for all integers $n > 1$. What is the sum of the first 100 terms in sequence A_n?

(A) 243
(B) 14,400
(C) 14,500
(D) 24,300
(E) 24,545

31. In a certain sequence, the term a_n is given by the formula $a_n = a_{n-1} + 10$. What is the positive difference between the 10th term and the 15th term?

(A) 5
(B) 10
(C) 25
(D) 50
(E) 100

32. In a certain sequence, the term a_n is given by the formula $a_n = 10(a_{n-1})$. How many times greater is a_{10} than a_8?

(A) 1
(B) 3
(C) 10
(D) 30
(E) 100

33. Ashley and Beatrice received the same score on a physical fitness test. The scores for this test, t, are determined by the formula $t = 3ps - 25m$ where s and p are the numbers of sit-ups and push-ups the athlete can do in one minute and m is the number of minutes she takes to run a mile. Ashley did 10 sit-ups and 10 push-ups and ran an 8-minute mile. Beatrice did half as many sit-ups and twice as many push-ups. If both girls received the same overall score, how many minutes did it take Beatrice to run the mile?

(A) 4
(B) 8
(C) 10
(D) 16
(E) 20

34.

The expression $a\{\}b$ is defined as $a\{\}b = (a - b)(a + b)$.

Quantity A **Quantity B**

35. If 5||10 = 5 and 1||(-2) = 1, which of the following could define the expression $a||b$?

(A) $b - a$
(B) $\frac{a^2 - b}{3}$
(C) $-ab/4$
(D) $\frac{b+15}{a}$
(E) $a + b + 4$

36. The maximum height reached by a ball thrown straight up into the air can be determined by the formula $h = -16t^2 + vt + d$, where t is the number of seconds since it was thrown, v is the initial speed of the throw (in feet per second), d is the height (in feet) at which the ball was released, and h is the height of ball t seconds after the throw. Two seconds after a ball is thrown, how high in the air is the ball if it was released at a height of 6 feet and a speed of 80 feet per second?

(A) 96 feet
(B) 100 feet
(C) 102 feet
(D) 134 feet
(E) 230 feet

37. If $a\#b = a^2\sqrt{b} - a$, where $b \geq 0$, what is the value of (-4)#4?

(A) -36
(B) -28
(C) 12
(D) 28
(E) 36

38. The expression $x\$y$ is defined as $\frac{x^2}{y}$, where $y \neq 0$. What is the value of 9\$(6\$2)?

(A) 1/2
(B) 9/4
(C) 9/2
(D) 18
(E) 108

39. Amy deposited \$1,000 into an account that earns 8% annual interest compounded every 6 months. Bob deposited \$1,000 into an account that earns 8% annual interest compounded quarterly. If neither Amy nor Bob makes any additional deposits or withdrawals, in 6 months how much more money will Bob have in his account than Amy?

(A) \$40
(B) \$8
(C) \$4
(D) \$0.40
(E) \$0.04

40. The half-life of an isotope is the amount of time required for 50% of a sample of the isotope to undergo radioactive decay. The half-life of the carbon-14 isotope is 5,730 years. How many years must pass until a sample that starts out with 16,000 carbon-14 isotopes decays into a sample with only 500 carbon-14 isotopes?

(A) 180 years
(B) 1,146 years
(C) 5,730 years
(D) 28,650 years
(E) 183,360 years

41. $f(x) = \frac{2-x}{5}$ and $g(x) = 3x - 2$. If $f(g(x)) = 1$, what is the value of x?

(A) -5/3
(B) -1/3
(C) 2/3
(D) 1
(E) 5/3

42. $f(x) = 2x - 2$ and $g(x) = \frac{x}{2} + 2$. What is the value of $2f(g(2))$?

(A) 0
(B) 4
(C) 6
(D) 8
(E) 12

43. In a particular competition, a skater's score, s, is calculated by the formula $s = \frac{10(3t - 5f)}{m}$ where t is the number of successful triple axels performed, f is the number of times the skater fell, and m is the length, in minutes, of the performance. If Seiko received a score of 6 for a 5 minute performance in which she had twice as many successful triple axels as she had falls, how many times did she fall?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

44. An investor doubles his money every 8 years. At 33 years old, he had 13 million dollars ($13m). How much money will he have when he retires at 65 years old?

(A) $26m
(B) $104m
(C) $130m
(D) $208m
(E) $260m

45. $a\sim b$ is defined as $a - \frac{5-b}{a}$. What is the value of $1\sim((-1)\sim1)$?

(A) -9
(B) -1
(C) 1
(D) 2
(E) 5

46. An archer's score is calculated by the formula $\frac{50b - 10a}{10 + s}$ where b is the number of bull's-eyes hit, a is the total number of arrows shot, and s is the time in seconds it took the archer to shoot. By how many points would an archer who took 10 seconds to shoot 10 arrows and hit all bull's-eyes beat an archer who shot twice as many arrows and hit half as many bull's-eyes in 15 seconds?

(A) 2
(B) 7
(C) 10
(D) 18
(E) 20

47. Each term of a certain sequence is calculated by adding a particular constant to the previous term. The second term of this sequence is 27 and the fiftjh term is 84. What is the 1st term of this sequence?

(A) 20
(B) 15
(C) 13
(D) 12
(E) 8

48. If $a\#b = \frac{1}{2a - 3b}$ and $a@b = 3a - 2b$, what is the value of 1@2 - 3#4 ?

(A) -7/6
(B) -1
(C) -5/6
(D) 2/3
(E) 7/6

49. In a certain sequence, the term a_n is given by the formula $a_n = a_{n-1} + 5$ where $a_1 = 1$. What is the sum of the first 75 terms of this sequence?

(A) 10,150
(B) 11,375
(C) 12,500
(D) 13,950
(E) 15,375

50. In a certain sequence, the term a_n is given by the formula $a_n = 2 \times a_{n-1}$ where $a_1 = 1$. What is the positive difference between the sum of the first 10 terms of the sequence and the sum of the 11th and 12th terms of the same sequence?

(A) 1
(B) 1,024
(C) 1,025

(D) 2,048
(E) 2,049

51.

An operation @ is defined by the equation $a@b = (a - 1)(b - 2)$.
$x@5 = 3@x$

Quantity A	**Quantity B**
The value of x	1

52. The wait time in hours, w, for a certain popular restaurant can be estimated by the formula $w = \frac{n + ks}{10}$ where k is a constant, n is the number of parties waiting ahead of you, and s is the size of your party. If a family of 4 has a wait time of 30 minutes when 2 other parties are ahead of it, how long would a family of 6 expect to wait if there are 8 parties ahead of it?

(A) 45.5 minutes
(B) 1 hour 15 minutes
(C) 1 hour 25 minutes
(D) 1 hour 45 minutes
(E) 2 hours

53. A sequence is defined as $a_n = 5(a_{n-1}) - 3$ where $a_4 = 32$. What is the first term of the sequence, a_1?

(A) 1
(B) 7
(C) 16
(D) 128
(E) 157

54.

A certain sequence is defined by the formula $a_n = a_{n-1} - 7$.
$a_7 = 7$

Quantity A	**Quantity B**
The value of a_1	-35

55. Monthly rent for units in a certain apartment building is determined by the formula $k\left(\frac{5r^2 + 10t}{f + 5}\right)$ where k is a constant, r and t are the number of bedrooms and bathrooms in the unit, respectively, and f is the floor number of the unit. A 2-bedroom, 2-bathroom unit on the first floor is going for \$800/month. How much is the monthly rent on a 3-bedroom unit with 1 bathroom on the 3rd floor?

(A) \$825
(B) \$875
(C) \$900

(D) $925
(E) $1,000

56.

Quantity A	Quantity B
The sum of all the multiples of 3 between 250 and 350	9,990

57. Town A has a population of 160,000 and is growing at a rate of 20% annually. Town B has a population of 80,000 and is growing at a rate of 50% annually.

Quantity A	Quantity B
The number of years until Town B's population is larger than that of Town A	3

58. If $f(x) = x^2$, what is $f(m + n) + f(m - n)$?

(A) $m^2 + n^2$
(B) $m^2 - n^2$
(C) $2m^2 + 2n^2$
(D) $2m^2 - 2n^2$
(E) m^2n^2

59. S_n is a sequence such that $S_n = (-1)^n$, where $n \geq 1$. What is the sum of the first 20 terms in S_n?

60. If //x// means "the least integer greater than or equal to x," what is //-2.5// + //3.6//?

61. If $f(x, y) = x^2y$ and $f(a, b) = 6$, what is $f(2a, 4b)$?

62.

$f(x) = m$ where m is the number of distinct prime factors of x.

Quantity A	Quantity B

$f(30)$	$f(64)$

63. In a particular sequence, S_n is equal to the units digit of 3^n where n is a positive integer. If $S_1 = 3$, how many of the first 75 terms of the sequence are equal to 9?

64. The sequence $a_1, a_2, a_3, \ldots, a_n$ is such that $a_n = 9 + a_{n-1}$ for all $n > 1$. If $a_1 = 11$, what is the value of a_{35}?

65. In sequence Q, the first number is 3, and each subsequent number in the sequence is determined by doubling the previous number and then adding 2. How many times does the digit 8 appear in the units digit of the first 10 terms of the sequence?

66. If $g(2m) = 2g(m)$ and $g(3) = 5.5$, what is $g(6)$?

67. For which of the following functions $f(x)$ is $f(a + b) = f(a) + f(b)$?

(A) $f(x) = x^2$
(B) $f(x) = 5x$
(C) $f(x) = 2x + 1$
(D) $f(x) = \sqrt{x}$
(E) $f(x) = x - 2$

68.

Sam invests a principal of $10,000, which earns annually compounded interest over a period of years.

Quantity A	**Quantity B**
The final value of the investment after 2 years at 8% interest, compounded annually	The final value of the investment after 4 years at 4% interest, compounded annually

69. The number of years it would take for the value of an investment to double, at 26% interest compounded annually, is approximately

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

70. The interest rate, compounded annually, that would bring a principal of $1,200 to a final value of $1,650 in 2 years is closest to

(A) 17%
(B) 18%
(C) 19%
(D) 20%
(E) 21%

71. An investment is made at 12.5% annual simple interest. The number of years it will take for the cumulative value of the interest to equal the original investment is

(A) 4
(B) 5
(C) 6
(D) 7
(E) 8

72. If $f(2a) = 2f(a)$ and $f(6) = 11$, what is $f(24)$?

(A) 22
(B) 24
(C) 44
(D) 66
(E) 88

73. $\frac{1}{2}f(x)=f\left(\frac{1}{2}x\right)$, which is true for all values of $f(x)$?

(A) $f(x) = 2x + 2$
(B) $f(x) = 13x$
(C) $f(x) = x^2$
(D) $f(x) = x - 10$
(E) $f(x) = \sqrt{x-4}$

Functions, Formulas, and Sequences Answers

1. **(E).** The notation "$f(x)$" and "$f(2)$" indicates that you should plug 2 in for x in the given equation:

$f(2) = (2)^2 + 1$
$f(2) = 5$

Likewise, plug -2 in for x:

$f(-2) = (-2)^2 + 1$
$f(-2) = 5$

Now add: $5 + 5 = 10$.

2. **(D).** Start with the innermost function, $g(-3)$. The notation indicates that you should plug -3 in for x in the given equation:

$g(-3) = (-3)^3$
$g(-3) = -27$

Now the problem reads "what is $f(-27)$?" Plug -27 in for x in the $f(x)$ function:

$f(-27) = 2(-27) = -54$

3. **(B).** Be careful with the notation here. The problem indicates that $h(m) = -19$, *not* that $h(-19)$ = something else. Do not plug -19 in for x; rather, plug m in for x and set the answer equal to -19:

$2m^3 - 3 = -19$
$2m^3 = -16$
$m^3 = -8$
$m = -2$

4. **37.** Be careful with the notation here. $2[f(g)] = 14$ represents some number (denoted by variable g) plugged into the function $f(g)$, and then multiplied by 2, to yield the answer 14. If $2[f(g)] = 14$, then divide both sides by 2 to get $f(g) = 7$.

The main function is $f(x) = x - 3$. The notation $f(g)$ indicates that you should plug g in for all instances of x: $f(g) = g - 3$. You also determined that $f(g) = 7$, so set the two right-hand halves of the equations equal to each other: $g - 3 = 7$. The value of g is therefore 10.

The question asks for the value of $f(4g)$. Since $g = 10$, $4g = 40$, and $f(40)$:

$f(40) = 40 - 3 = 37$

5. **720.** Plug m and n into the function in place of a and b. If $f(m, n) = 5$, then:

$$m^2n^4 = 5$$

This cannot be further simplified, so continue to the second part of the problem: plug $3m$ and $2n$ into the function for a and b:

$$f(3m, 2n) = (3m)^2(2n)^4 = 9m^2\ 16n^4 = 144m^2n^4$$

Since $m^2n^4 = 5$, $144m^2n^4 = 144(5) = 720$.

6. **(A).** The question is asking you to plug y into the function, then plug -1 into the function, then add the two answers together.

$$f(y) = y^2 - 1$$
$$f(-1) = (-1)^2 - 1$$
$$f(-1) = 0$$

Thus, $f(y) + f(-1) = y^2 - 1 + 0 = y^2 - 1$, or choice (A).

7. **2.** The question is asking you to plug 3 into the function, then plug $-\frac{1}{3}$ into the function, and then add the two answers together.

$$g(3) = 3(3) - 3$$
$$g(3) = 6$$
$$g\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) - 3$$
$$g\left(-\frac{1}{3}\right) = -1 - 3$$
$$g\left(-\frac{1}{3}\right) = -4$$

Thus, $g(3) + g\left(-\frac{1}{3}\right) = 6 + (-4) = 2.$

8. **1.** When dealing with "nested" functions, tackle the innermost function first.

$$f(10) = \frac{10}{2} - 1 = 4$$

$$f(4) = \frac{4}{2} - 1 = 1$$

Thus, $f(f(10)) = 1$.

9. **(E).** The notation $h(a + b)$ indicates that you need to replace each x with the expression $(a + b)$

$$h(a + b) = 5(a + b)^2 + (a + b)$$
$$h(a + b) = 5(a^2 + 2ab + b^2) + a + b$$
$$h(a + b) = 5a^2 + 10ab + 5b^2 + a + b$$

This is equivalent to answer choice (E).

10. **(A).** The question uses a made-up symbol in place of the traditional notation $f(x)$. The question "If $\ulcorner x = 2x^2 + 2$, what is $\ulcorner 4$?" is asking you to plug 4 into the function.

$$\ulcorner 4 = 2(4)^2 + 2$$
$$\ulcorner 4 = 34$$

Do not fall for trap answer choice (E). The correct answer is 34, which does not appear in the choices in that form.

Trap choice (E) is $\ulcorner 34$, which equals $2(34)^2 + 2$; this is much larger than 34.

You need to solve each answer choice until you find one that equals 34. Choice (A), $\ulcorner\ulcorner -1$, uses the function symbol twice, which requires you to plug -1 into the function, then plug your answer back into the function again:

$$\ulcorner -1 = 2(-1)^2 + 2 = 4$$

$\ulcorner 4 = 2(4)^2 + 2 = 34$ (Note: you do not need to complete this math if you notice that $\ulcorner 4$ must have the same value as the original $\ulcorner 4$ in the question stem.)

Thus, $\ulcorner\ulcorner -1 = 34$ and choice (A) is correct. It is not necessary to try the other answer choices.

11. **(B).** This problem uses a made-up symbol which is then defined verbally, rather than with a formula. $\boxed{x}$ has two different definitions:

If x is odd, $\boxed{x}$ equals the least integer greater than x (for example, if $x = 3$, then the "least integer greater than 3" is equal to 4)

If x is even, $\boxed{x}$ equals the greatest integer less than x (for example, if $x = 6$, the "greatest integer less than x" is equal to 5.

Since -2 is even, $\boxed{-2}$ = the greatest integer less than -2, or -3.

Since 5 is odd, $\boxed{5}$ = the least integer greater than 5, or 6.

Thus, $\boxed{-2} - \boxed{5} = -3 - 6 = -9$.

12. **32.** For the function $g(x) = x^2 - 4$, plugging c in for x gives the answer 12. Thus:

$$c^2 - 4 = 12$$
$$c^2 = 16$$
$$c = 4 \text{ or } -4$$

The problem indicates that $c < 0$, so c must be -4.

The problem then asks for $g(c - 2)$. Since $c = -4$, $c - 2 = -6$. Plug -6 into the function:

$$g(-6) = (-6)^2 - 4$$
$$g(-6) = 36 - 4 = 32$$

13. **43.** The problem introduces two functions and asks for $h(g(5))$. When dealing with "nested" functions, begin with the innermost function.

$$g(5) = 5^2 - 3 = 22$$
$$h(22) = 2(22) - 1 = 43$$

Thus, $h(g(5)) = 43$.

14. **15.** The problem introduces two functions as well as the fact that $g(m) = 61$. First, solve for m:

$$m^2 - 3 = 61$$
$$m^2 = 64$$
$$m = 8 \text{ or } -8$$

The problem indicates that $m > 0$, so m must equal 8. The problem asks for $h(m)$. Since $m = 8$, find $h(8)$:

$$h(8) = 2(8) - 1$$
$$h(8) = 15$$

15. $\frac{4}{9}$ **(or any equivalent fraction).** This function defines a made-up symbol rather than using traditional function notation such as $f(x)$. Since &x& is defined as "one-half the square of x":

$$\&x\& = \frac{1}{2}x^2$$

The problem asks for &4& divided by &6&:

$$\&4\& = \frac{1}{2}(4)^2 = 8$$
$$\&6\& = \frac{1}{2}(6)^2 = 18$$

Therefore, $\frac{\&4\&}{\&6\&} = \frac{8}{18} = \frac{4}{9}$.

16. **I, II, and III.** This function defines a made-up symbol: ~x is equivalent to $|14x|$. The question asks which statements must be true, so test each one.

$$\sim 2 = \sim(-2)$$
$$|14(2)| = |14(-2)|$$
$$|28| = |-28|$$
$$28 = 28$$

This statement must be TRUE.

Similarly test the second statement:

$$\sim 3 + \sim 4 = \sim 7$$
$$|14(3)| + |14(4)| = |14(7)|$$
$$42 + 56 = 98$$
$$98 = 98$$

This statement must be TRUE.

Finally, the third statement is also true. Since ~x is equal to a statement inside an absolute value, this value can never be negative. If $x = 0$, then the value of $|14x|$ is also 0. The minimum possible value for ~x is 0.

17. **0.** This function defines a made-up symbol, rather than using traditional notation such as $f(x)$. First, translate the function:

$$\#x = (x - 2)^2$$
$$\#5 = (5 - 2)^2 = 9 \text{ and } \#(-1) = (-1 - 2)^2 = 9.$$

$\#5 - \#(-1) = 9 - 9 = 0$.

18. **(C).** The term "undefined" refers to the circumstance when the solution is not a real number — for example, when a value would ultimately cause you to divide by 0, that situation is considered "undefined." There aren't many circumstances that result in an undefined answer. Essentially, you can't take the square root of a negative, and you can't divide by 0. There are no square roots in this problem, but it's possible that 0 could end up on the denominator of the fraction. Set each of the terms in the denominator equal to 0:

$3x - 3 = 0$
$3x = 3$
$x = 1$

$x + 2 = 0$
$x = -2$

Thus, if $x = 1$ or $x = -2$, then you'd have to divide by 0, making $g(x)$ undefined. All other values are acceptable.

19. **(E).** The term "undefined" refers to the circumstance when the solution is not a real number — for example, when a value would ultimately cause you to divide by 0, that situation is considered "undefined." There aren't many circumstances that result in an undefined answer. Essentially, you can't take the square root of a negative, and you can't divide by 0.

Since you can't divide by 0 and the bottom of the fraction is simply x, then x cannot be 0. So far, there is 1 "prohibited" value for x.

Since the top of the fraction is a square root and you can't take the square root of a negative, you can conclude that the quantity inside the absolute value, $x - 2$, must be positive or zero:

$x - 2 \geq 0$
$x \geq 2$

Therefore, there are an infinite number of values that x cannot be (1, 0, -1, -2, -3, -4, -5…).

20. **(A).** The problem gives a function, $f(x) = 2x - 3$, and then indicates that, when m is plugged in to the function, the answer is -11. Therefore:

$2m - 3 = -11$
$2m = -8$
$m = -4$

Quantity A is equal to -4. The problem indicates that $f(m) = -11$, so Quantity B is equal to $\frac{11}{2} = -5.5$. Quantity A is larger.

21. **(D).** Since "the price of a phone call consists of a standard connection fee, which does not change, plus a per-minute charge," you can write a formula, using variables for the unknown information. Let c equal the connection fee and r equal the per-minute rate:

$2.90 = c + r(10)$
$4.40 = c + r(16)$

Now, either substitute and solve, or stack and combine the equation. Note that there is one *c* in each equation, so subtracting is likely going to be fastest:

$$\begin{array}{r} 4.40 = c + 16r \\ \underline{-\ (2.90 = c + 10r)} \\ 1.50 = 6r \end{array}$$

$r = 0.25$

The calls cost 25 cents per minute. Note that most people will next plug *r* back into either equation to find *c*, but *c* isn't necessary to solve!

A 10-minute call costs \$2.90. That \$2.90 already includes the basic connection fee (which never changes) as well as the per-minute fee for 10 minutes. The problem asks how much a 13-minute call costs. Add the cost for another 3 minutes (\$0.75) to the cost for a 10-minute call (\$2.90): 2.90 + 0.75 = \$3.65.

In fact, if you notice earlier that both the 10-minute and 16-minute calls include the same connection fee (which never changes), you can use a shortcut to solve. The extra 6 minutes for the 16-minute call cost a total of \$4.40 - \$2.90 = \$1.50. From there, you can calculate the cost per minute ($1.5 \div 6 = 0.25$) or you can notice that 13 minutes is halfway between 10 minutes and 16 minutes, so the cost for a 13-minute call must also be halfway between the cost for a 10-minute call and the cost for a 16-minute call. Add half of \$1.50, or \$0.75, to \$2.90 to get \$3.65.

22. **267.** While the sequence is clear (30, 33, 36, 39, 42, etc.), you don't have time to count to the 80th term. Instead, find a pattern. Each new term in the list adds 3 to the previous term, so determine how many times you need to add 3. (By the way, the term "arithmetic sequence" means a sequence in which the same number is added or subtracted for each new term.)

Start with the first term, 30. To get from the first term to the second term, you start with 30 and add 3 *once.* To get from the first term to the third term, you start with 30 and add 3 *twice.* In other words, for the third term, you add one fewer instance of 3: twice rather than three times. To write this mathematically, say: $30 + 3(n-1)$, where *n* is the number of the term. (Note: you don't need to write that, as long as you understand the pattern.)

To get to the 80th term, then, start with 30 and add 3 exactly 79 times:

$30 + (79 \times 3) = 267$

23. **(E).** The sequence $S_n = 2(S_{n-1}) - 4$ can be read as "to get any term in sequence *S*, double the previous term and subtract 4."

The problem gives S_1 (the first term) and asks for S_5 (the fifth term):

6				
S_1	S_2	S_3	S_4	S_5

To get any term, double the previous term and subtract 4. To get S_2, double S_1 (which is 6) and subtract 4: $S_2 = 2(6) - 4 = 8$. Continue doubling each term and subtracting 4 to get the subsequent term:

6	8	12	20	36
S_1	S_2	S_3	S_4	S_5

24. **(B).** The sequence $S_n = 3(S_{n-1}) + 1$ can be read as "to get any term in sequence S, triple the previous term and add 1."

The problem gives S_1 (the first term) and asks for S_4 (the fourth term):

-2			
S_1	S_2	S_3	S_4

To get any term, triple the previous term and add 1. To get S_2, triple S_1 (which is -2) and add 1. Thus, $S_2 = 3(-2) + 1 = -5$. Continue tripling each term and adding 1 to get the subsequent term:

-2	-5	-14	-41
S_1	S_2	S_3	S_4

25. **(A).** The sequence $S_n = S_{n-1} + S_{n-2} - 3$ can be read as "to get any term in sequence S, add the two previous terms and subtract 3."

The problem gives the first two terms and asks for the sixth term:

5	0				
S_1	S_2	S_3	S_4	S_5	S_6

To get any term, add the two previous terms and subtract 3. So the third term will equal $5 + 0 - 3 = 2$. The fourth term will equal $0 + 2 - 3 = -1$. The fifth term will equal $2 + (-1) - 3 = -2$. The sixth term will equal $-1 + (-2) - 3 = -6$.

5	0	2	-1	-2	-6
S_1	S_2	S_3	S_4	S_5	S_6

26. **(C).** The sequence $S_n = S_{n-1} + S_{n-2} - 1$ can be read as "to get any term in sequence S, add the two previous terms and subtract 1."

The problem gives the zero-th term and the *second* term and asks for the fourth term:

-10		0		
S_0	S_1	S_2	S_3	S_4

Within the sequence S_0 to S_2, the problem gives two values but not the third (S_1). What version of the formula would include those three terms?

$S_2 = S_1 + S_0 - 1$
$0 = S_1 + (-10) - 1$
$0 = S_1 - 11$
$11 = S_1$

−10	11	0		
S_0	S_1	S_2	S_3	S_4

To get each subsequent term, add the two previous terms and subtract 1. $S_3 = 0 + 11 - 1 = 10$. $S_4 = 10 + 0 - 1 = 9$.

−10	11	0	10	9
S_0	S_1	S_2	S_3	S_4

27. **(E).** The sequence $S_n = S_{n-1} + S_{n-2} + S_{n-3} - 5$ can be read as "to get any term in sequence S, add the three previous terms and subtract 5."

The problem gives the first, second, and *fourth* terms and asks for the sixth term:

4	0		−4		
S_1	S_2	S_3	S_4	S_5	S_6

Within the sequence S_1 to S_4, the problem gives three values but not the fourth (S_3). What version of the formula would include those four terms?

$S_4 = S_3 + S_2 + S_1 - 5$
$-4 = S_3 + 4 + 0 - 5$
$-4 = S_3 - 1$
$-3 = S_3$

Fill in the newly-calculated value. To find each subsequent value, continue to add the three previous terms and subtract 5. $S_5 = -4 + (-3) + 0 - 5 = -12$. $S_6 = -12 + (-4) + (-3) - 5 = -24$.

4	0	−3	−4	−12	−24
S_1	S_2	S_3	S_4	S_5	S_6

28. **1,778.** The sequence $P_n = 10(P_{n-1}) - 2$ can be read as "to get any term in sequence P, multiply the previous term by 10 and subtract 2."

The problem gives the first term and asks for the fourth:

$$\frac{2}{P_1} \quad \frac{\quad}{P_2} \quad \frac{\quad}{P_3} \quad \frac{\quad}{P_4}$$

To get P_2, multiply 2 × 10, then subtract 2 to get 18. Continue this procedure to find each subsequent term ("to get any term in sequence P, multiply the previous term by 10 and subtract 2.") $P_3 = 10(18) - 2 = 178$. $P_4 = 10(178) - 2 = 1{,}778$.

$$\frac{2}{P_1} \quad \frac{18}{P_2} \quad \frac{178}{P_3} \quad \frac{1{,}778}{P_4}$$

29. **(A).** The sequence $S_{n-1} = \frac{1}{4}(S_n)$ can be read as "to get any term in sequence S, multiply the term *after* that term by $\frac{1}{4}$." Since this formula is "backwards" (usually, you define later terms with regard to previous terms), you may wish to solve the formula for for S_n:

$$S_{n-1} = \frac{1}{4}(S_n)$$
$$4S_{n-1} = S_n$$
$$S_n = 4S_{n-1}$$

This can be read as "to get any term in sequence S, multiply the previous term by 4."

The problem gives the first term and asks for the fourth:

$$\frac{-4}{S_1} \quad \frac{\quad}{S_2} \quad \frac{\quad}{S_3} \quad \frac{\quad}{S_4}$$

To get S_2, multiply the previous term by 4: (4)(-4) = -16. Continue this procedure to find each subsequent term. $S_3 = (4)(-16) = -64$. $S_4 = (4)(-64) = -256$.

$$\frac{-4}{S_1} \quad \frac{-16}{S_2} \quad \frac{-64}{S_3} \quad \frac{-256}{S_4}$$

30. **(B).** The first term of the sequence is 45, and each subsequent term is determined by adding 2. The problem asks for the sum of the first 100 terms, which cannot be calculated directly in the given time frame; instead, find the pattern. The first few terms of the sequence are 45, 47, 49, 51, ...

What's the pattern? To get to the 2nd term, start with 45 and add 2 once. To get to the 3rd term, start with 45 and add 2 twice. To get to the 100th term, then, start with 45 and add 2 ninety-nine times: 45 + (2)(99) = 243.

Next, find the sum of all odd integers from 45 to 243, inclusive. To sum up any evenly-spaced set, multiply the average by the number of elements in the set. To get the average, average the first and last terms. Since $\frac{45+243}{2} = 144$, the average is 144.

To find the total number of elements in the set, subtract 243 - 45 = 198, then divide by 2 (count only the odd numbers, not the even ones). 198/2 = 99 terms. Now, add 1 (to count both endpoints in a consecutive set, first subtract and then "add 1 before you're done"). The list has 100 terms.

Multiply the average and the number of terms:

$144 \times 100 = 14{,}400$

31. **(D).** This is an arithmetic sequence where the difference between successive terms is always +10. The difference between, for example, a_{10} and a_{11}, is exactly 10, regardless of the actual values of the two terms. The difference between a_{10} and a_{12} is $10 + 10 = 20$ or $10 \times 2 = 20$, because there are two "steps," or terms, to get from a_{10} to a_{12}. Starting from a_{10}, there is a sequence of 5 terms to get to a_{15}. Therefore, the difference between a_{10} and a_{15} is $10 \times 5 = 50$.

32. **(E).** This is a geometric sequence, in which every term is 10 times the term before. The problem does not provide the actual value of any terms in the sequence, but the sequence could be something like 10, 100, 1,000…, or 35, 350, 3,500…. Thus, any term is 100 times as large as the term that comes two before it.

Alternatively, try an algebraic approach. From the formula, $a_9 = 10a_8$ and $a_{10} = 10a_9$. Substitute for a_9 in the second equation to give: $a_{10} = 10(10a_8) = 100a_8$.

33. **(B).** First, calculate Ashley's score: $t_a = 3 \times 10 \times 10 - 25 \times 8 = 300 - 200 = 100$. If Ashley and Beatrice scored the same score and Beatrice did half as many sit-ups (5) and twice as many push-ups (20): $t_b = 3 \times 5 \times 20 - 25m = 100$. Solve for m: $300 - 25m = 100$ and $25m = 200$ so $m = 8$.

Alternatively, you could use logic. Since p and s are multiplied together in the score formula, if you multiply s by $\frac{1}{2}$ and p by 2, the two girls will have the same overall value for $3ps$. Beatrice will need the same mile time, 8 minutes, in order to achieve the same overall score as Ashley.

34. **(A).** This problem defines a function for the made-up symbol {}. To calculate the value of Quantity A, follow the rules of PEMDAS. First calculate the expressions within parentheses:

$7\{\}6 = (7 - 6)(7 + 6) = 1 \times 13 = 13$
$11\{\}11 = (11 - 11)(11 + 11) = 0$

Then substitute these values back into the original expression:

$(13)\{\}(0) = (13 - 0)(13 + 0) = 13 \times 13 = 169$. Quantity A is larger.

35. **(B).** This question provides two examples of the input and output into a made-up function, and asks which answer choice could be that function. In other words, the function in which answer choice gives the answer 5 when you evaluate 5||10, and also gives the answer 1 when you evaluate 1||(-2)?

Only one answer can work for both examples, so first test a = 5 and b = 10 to determine whether the function returns 5. If it doesn't, cross off that choice. If it does, test a = 1 and b = -2. When you find an answer that works for both, you can stop.

(A) Does 10 - 5 = 5? YES. Does (-2) -1 = 1? No. Cross off answer (A).

(B) Does $\frac{5^2-10}{3}=5$? YES. Does $\frac{1^2-(-2)}{3}=1$? YES. This is the correct answer.

It is not necessary to test the remaining answers; the work is shown below for completeness.

(C) Does $\frac{-5\times 10}{4}=5$ NO. Cross off answer (C).

(D) Does $\frac{10+15}{5}=5$? YES. Does $\frac{-2+15}{1}=1$? No. Cross off answer (D).

(E) Does 5 + 10 + 4 = 5? NO. Cross off answer (E).
The answer is (B).

36. **(C).** The question presents a formula with a number of variables and also provides values for all but one of those variables (h, the height of the ball). Solve for h by plugging in the values given for the other variables: t = 2 seconds, v = 80 feet/second, d = 6 feet.

h = -16(2)^2 + (80)(2) + 6 = 102 feet

37. **(E).** This problem defines a function for the made-up symbol #. In this problem a = (-4) and b = 4. Plug the values into the function: $(-4)^2\sqrt{4}-(-4) = 16\times 2+4=36$. Do not forget to keep the parentheses around the -4! Also note that you take only the positive root of 4 (2) because the problem has been presented in the form of a real number underneath the square root sign.

38. **(B).** This problem defines a function for the made-up symbol $. Order of operation rules (PEMDAS) stay the same even when the problem uses made-up symbols. First, calculate the value of the expression in parentheses, 6$2. Plug x = 6 and y = 2 into the function: 6^2/2 =36/2 = 18. Replace 6$2 with 18 in the original expression to give 9$18. Again, plug x = 9 and y = 18 into the function: 9$18 = 9^2/18 = 81/18 = 9/2.

39. **(D).** Both Amy and Bob start with $1,000 and earn 8% interest annually; the difference is in how often this interest is compounded. Amy's interest is compounded twice a year at 4% each time (8% annual interest compounded 2 times a year means that she gets half the interest, or 4%, every six months). Bob's interest is compounded four times a year at 2% (8% divided by 4 times per year) each time. After 6 months, Amy has $1,000 × 1.04 = $1,040.00 (1 interest

payment at 4%) and Bob has $\$1,000 \times (1.02)^2 = \$1,040.40$ (2 interest payments at 2%). The difference is $1,040.40 - $1,040.00 = $0.40.

For Bob's interest, you can also calculate the two separate payments. After three months, Bob will have $1,000 × 1.02 = $1,020.00. After six months, Bob will have $1,020 × 1.02 = $1,040.40.

40. **(D).** After each half-life, the sample is left with half of the isotopes it started with in the previous period. After one half-life, the sample goes from 16,000 isotopes to 8,000. After two half-lives, it goes from 8,000 to 4,000. Continue this pattern to determine the total number of half-lives that have passed: 4,000 becomes 2,000 after 3 half-lives, 2,000 becomes 1,000 after 4 half-lives, 1,000 becomes 500 after 5 half-lives. The sample will have 500 isotopes after 5 half-lives. Thus, multiply 5 times the half-life, or 5 × 5730 = 28,650 years.

Note that the answer choices are very spread apart. Once you have determined that 5 half-lives have passes, you can estimate: 5 × 5000 = 25,000 years; answer (D) is the only possible answer.

41. **(B).** The question is asking you to substitute the expression for $g(x)$ into the function for $f(x)$, and set the answer equal to 1. Since $g(x) = 3x - 2$, substitute the expression $3x - 2$ in for x in the expression for $f(x)$:

$$f(g(x)) = \frac{2 - g(x)}{5} = \frac{2 - (3x - 2)}{5} = \frac{4 - 3x}{5}$$

Since $f(g(x)) = 1$, solve the equation $\frac{4 - 3x}{5} = 1$: :

$$\begin{aligned} 4 - 3x &= 5 \\ -3x &= 1 \\ x &= -\frac{1}{3} \end{aligned}$$

42. **(D).** Start with the innermost portion of $2f(g(2))$:

$g(2) = 2/2 + 2 = 3$

Since $g(2) = 3$, now the expression is $2f(3)$. First evaluate $f(3)$: $2(3) - 2 = 4$.

Therefore $2f(3) = 2 \times 4 = 8$.

43. **(B).** If Seiko had twice as many triple axels as falls then $t = 2f$. Substitute $2f$ in for t, along with the other given information:

$$6 = \frac{10(3(2f) - 5f)}{5}$$

$$6 = \frac{10(6f - 5f)}{5}$$

$$6 = \frac{10f}{5}$$

$$30 = 10f$$

$$f = 3$$

Note that you want to substitute 2*f* in for *t*, and not the reverse, because the problem asks for the value of *f*.

44. **(D).** The investor's amount of money doubles every 8 years. Calculate the amount of money for each 8-year period:

Age	$ (millions)
33	13
33 + 8 = 41	13 × 2 = 26
41 + 8 = 49	26 × 2 = 52
49 + 8 = 57	52 × 2 = 104
57 + 8 = 65	104 × 2 = 208

At age 65, the investor will have $208 million.

45. **(B).** This problem defines a function for the made-up symbol ~. Follow PEMDAS order and start with the inner most parentheses, (-1)~1. In this case *a* = -1 and *b* = 1: $-1-\left(\frac{5-1}{-1}\right)=-1-\left(\frac{4}{-1}\right)=-1+4=3$. (Note: it's important to insert the parentheses! Replace (-1)~1 with 3 and evaluate the next function: 1~3. Now *a* = 1 and *b* = 3:

$$1-\left(\frac{5-3}{1}\right)=1-\left(\frac{2}{1}\right)=1-2=-1$$

46. **(D).** Calculate each of the archer's scores by plugging in the appropriate values for *b*, *a*, and *s*. For the first archer, *b* = *a* = *s* = 10 and the score is $\frac{(50\times 10)-(10\times 10)}{10+10}=\frac{400}{20}=20$. For the second archer, *b* = half of 10 = 5, *a* = twice as many as 10 = 20, and *s* = 15. The score for the second archer is $\frac{(50\times 5)-(10\times 20)}{10+15}=\frac{50}{25}=2$.

The difference in scores is 20 - 2 = 18.

47. **(E).** Let k equal the constant added to a term to get the next term. If the second term = 27, then the third term = 27 + k, the 4th term = 27 + 2k, and the 5th term = 27 + 3k. The 5th term equals 84, so create an equation:

$27 + 3k = 84$
$3k = 57$
$k = 19$

To find the first term, subtract k from the second term. The first term = 27 - 19 = 8.

48. **(C).** This problem defines functions for the made-up symbols # and @. Substitute a = 1 and b = 2 into the function for $a@b$: 3(1) - 2(2) = -1. Substitute a = 3 and b = 4 into the function for $a\#b$: $\frac{1}{2(3)-3(4)} = \frac{1}{-6} = -\frac{1}{6}$. Subtract: $(-1) - \left(-\frac{1}{6}\right) = -1 + \frac{1}{6} = -\frac{5}{6}$.

49. **(D).** This is an arithmetic sequence: each new number is created by adding 5 to the previous number in the sequence. Calculate the first few terms of the sequence: 1, 6, 11, 16, 21, and so on. Arithmetic sequences can be written in this form: $a_n = a_1 + k(n - 1)$, where k is the added constant and n is the number of the desired term. In this case, the function is: $a_n = 1 + 5(n - 1)$. The 75th term of this sequence is $a_{75} = 1 + 5(74) = 371$.

To find the sum of an arithmetic sequence, multiply the average value of the terms by the number of terms. The average of any evenly-spaced set is equal to the midpoint between the first and last terms. The average of the 1st and 75th terms is $\frac{1+371}{2} = 186$. There are 75 terms. Therefore, the sum of the first 75 terms = 186 × 75 = 13,950.

50. **(E).** This is a geometric sequence: each new number is created by multiplying the previous number by 2. Calculate the first few terms of the series to find the pattern: 1, 2, 4, 8, 16, and so on. Geometric sequences can be written in this form: $a_n = r^{n-1}$, where r is the multiplied constant and n is the number of the desired term. In this case, the function is $a_n = 2^{n-1}$.

The question asks for the difference between the sum of the first 10 terms and the sum of the 11th and 12th terms. While there is a clever pattern at play, it is hard to spot. If you don't see the pattern, one way to solve is to use the calculator to add the first ten terms: 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 = 1,023.

The 11th + 12th terms = 1,024 + 2,048 = 3,072

Subtract to get 2,049.

Alternatively, you may see the pattern in the first few terms (1, 2, 4, 8, 16…): Every term is equal to 1 more than the sum of the ones before it. For example, 1 + 2 = 3 and the next term is 4. 1 + 2 + 4 = 7 and the next term is 8. Thus, the sum of the first 10 terms of the sequence is 1 less than the 11th term. The 11th term = 2^{10} = 1,024, so the sum of the first 10 terms = 1,023. and the difference between the 10th and 11th terms equals 1. Add the value of the 12th term or 1 + 2,048 = 2,049.

51. **(B).** This problem defines a function for the made-up symbol @. Use the definition of the new symbol to rewrite the equation $x@5 = 3@x$ without the @ operator.

For $x@5$, $a = x$ and $b = 5$: $x@5 = (x - 1)(5 - 2) = 3x - 3$.
For $3@x$, $a = 3$ and $b = x$: $3@x = (3 - 1)(x - 2) = 2x - 4$.

Equating these two expressions gives us:

$$3x - 3 = 2x - 4$$
$$x = -1$$

Quantity B is larger.

52. **(B).** Start by solving for the constant, k. A family of 4 ($s = 4$) has a wait time of 30 minutes ($w = 0.5$ hours — don't forget that w is in hours!) when 2 parties are ahead of it ($n = 2$). Plug these values into the formula: $0.5 = \frac{2 + 4k}{10}$. Solve for k:

$$5 = 2 + 4k$$
$$3 = 4k$$
$$k = \frac{3}{4}$$

To solve for the wait time of the family of 6 with 8 parties ahead of it, plug these values into the formula along with $k = \frac{3}{4}$: $w = \frac{8 + \left(\frac{3}{4}\right)6}{10} = 1.25$ hours. The answer choices are shown in hours and minutes. 0.25 hours is equal to 0.25×60 minutes = 15 minutes. The answer is 1 hour 15 minutes.

53. **(A).** The given sequence can be read as, "To get any term in a, multiply the previous term by 5 and then subtract 3." The problem indicates that $a_4 = 32$ and asks for the value of a_1.

$$\underset{A_1}{\underline{\qquad}} \quad \underset{A_2}{\underline{\qquad}} \quad \underset{A_3}{\underline{\qquad}} \quad \underset{A_4}{\underline{\;\;32\;\;}}$$

Start with a_4 and find a_3:

$$32 = 5a_3 - 3$$
$$35 = 5a_3$$
$$7 = a_3$$

Use a_3 to find a_2:

$$7 = 5a_2 - 3$$

$10 = 5a_2$
$2 = a_2$

Use a_2 to find a_1:

$2 = 5a_1 - 3$
$5 = 5a_1$
$1 = a_1$

54. **(A).** The sequence $a_n = a_{n-1} - 7$ can be read as "to get any term in sequence a, subtract 7 from the previous term." The problem provides the 7th term; plug the term into the function in order to determine the pattern. Note that Quantity A asks for the value of a_1, so try to find the 6th term:

$7 = a_6 - 7$
$a_6 = 14$

In other words, each previous term will be 7 larger than the subsequent term. Therefore, $a_7 = 7$, $a_6 = 14$, $a_5 = 21$, and so on. The term a_1, then, is larger than the starting point, 7, and must also be larger than the negative value in Quantity B. Quantity A is larger. Note that the value in Quantity B is the result of incorrectly *subtracting* 7 six times, rather than adding it.

55. **(A).** First, solve for the constant k using the price information of the 2-bedroom, 2-bath unit ($m = 800$, $r = t = 2$ and $f = 1$):

$$800 = k\left(\frac{5(2)^2 + 10(2)}{1+5}\right)$$

$$800 = k\left(\frac{20+20}{6}\right)$$

$$800 = k\left(\frac{20}{3}\right)$$

$$800\left(\frac{3}{20}\right) = k$$

$$40(3) = k$$

$$120 = k$$

Next, solve for the rent on the 3-bedroom, 1-bath unit on the 3rd floor ($r = 3$, $t = 1$, and $f = 3$):

$$m = 120\left(\frac{5(3)^2 + 10(1)}{3+5}\right)$$

$$m = 120\left(\frac{45+10}{8}\right)$$

$$m = 120\left(\frac{55}{8}\right)$$

$$m = 15(55)$$

$$m = 825$$

56. **(B).** First, find the smallest multiple of 3 in this range: 250 is not a multiple of 3 (2 + 5 + 0 = 7, which is not a multiple of 3). The smallest multiple of 3 in this range is 252 (2 + 5 + 2 = 9, which is a multiple of 3). Next, find the largest multiple of 3 in this range. 350 is not a multiple of 3 (3 + 5 + 0 = 8); the largest multiple of 3 in this range is 348.

The sum of an evenly-spaced set of numbers equals the average value multiplied by the number of terms. The average value is the midpoint between 252 and 348: (252 + 348) ÷ 2 = 300. To find the number of terms, first subtract 348 - 252 = 96. This figure represents all numbers between 348 and 252, inclusive. To count only the multiples of 3, divide 96 by the 3: 96/3 = 32. Finally, "add 1 before you're done" because you do want to count both end points of the range: 32 + 1 = 33.

The sum is 300 × 33 = 9,900. Since 9,900 is smaller than 9,990, Quantity B is larger.

57. **(A).** Set up a table and calculate the population of each town after every year; use the calculator to calculate Town A's population. If you feel comfortable multiplying by 1.5 yourself, you do not need to use the calculator for Town B. Instead, add 50% each time (e.g., from 80,000, add 50% or 40,000 to get 120,000).

	Town A	**Town B**
Now	160,000	80,000
Year 1	160,000(1.2) = 192,000	80,000 + 40,000 = 120,000
Year 2	192,000(1.2) = 230,400	120,000 + 60,000 = 180,000
Year 3	230,400(1.2) = 276,480	180,000 + 90,000 = 270,000

Note that, after three years, Town A still has more people than Town B. It will take longer than 3 years, then, for Town B to surpass Town A, so Quantity A is larger.

58. **(C).** The problem provides the function $f(x) = x^2$ and asks you to evaluate $f(m + n) + f(m - n)$. Plug into this function twice — first, to insert $m + n$ in place of x, and then to insert $m - n$ in place of x.

$f(m + n) = (m + n)^2 = m^2 + 2mn + n^2$
$f(m - n) = (m - n)^2 = m^2 - 2mn + n^2$

Now add the two:

$$(m^2 + 2mn + n^2) + (m^2 - 2mn + n^2) = 2m^2 + 2n^2$$

59. **0.** Adding 20 individual terms would take quite a long time. Look for a pattern. The first several terms in $S_n = (-1)^n$, where $n \geq 1$:

$$S_1 = (-1)^1 = -1$$
$$S_2 = (-1)^2 = 1$$
$$S_3 = (-1)^3 = -1$$
$$S_4 = (-1)^4 = 1$$

The terms alternate -1, 1, -1, 1, and so on. If the terms are added, every pair of -1 and 1 will add to zero; in other words, for an even number of terms, the sum will be zero. 20 is an even number, so the first 20 terms add to zero.

60. **2.** The "least integer greater than or equal to" language is tricky because of the words "least" and "greater" in the same sentence. First, is the problem asking for a number that is larger or smaller than the starting number? "Greater than or equal to" indicates that the resulting number will be larger than the starting number. Rephrase this as "what is the next largest integer?"

The next largest integer starting from -2.5 is -2. (Remember that, for negative numbers, larger means "closer to 0.")

The next largest integer starting from 3.6 is 4.

$$-2 + 4 = 2$$

61. **96.** The problem provides the function $f(x, y) = x^2y$ and also the fact that when a and b are plugged in for x and y, the answer is 6. In other words:

$$f(x, y) = x^2y$$
$$f(a, b) = a^2b = 6$$

The problem asks for the value of $f(2a,4b)$. First, plug $2a$ in for x and $4b$ in for y:

$$f(2a, 4b) = (2a)^2(4b)$$
$$f(2a, 4b) = 4a^2(4b)$$
$$f(2a, 4b) = 16a^2b$$

The problem already provides the value for the variables: $a^2b = 6$. Therefore, $16a^2b = 16(6) = 96$.

62. **(A).** The problem indicates that $f(x) = m$ where m is the number of distinct (or different) prime factors of x. For example, if $x = 6$, 6 has two distinct prime factors: 2 and 3. Therefore, the corresponding answer (m value) would be 2.

For Quantity A $f(30)$: 30 has 3 distinct prime factors (2, 3, and 5), so $f(30) = 3$.

For Quantity B $f(64)$: 64 is made of the prime factors 2, 2, 2, 2, 2, and 2). This is only one distinct prime factor, so $f(64) = 1$. Quantity A is larger.

63. **19.** It's too much work to write out the first 75 terms of a sequence, so there must be some kind of pattern. Figure it out (the units digit is bolded):

$3^1 = \mathbf{3}$
$3^2 = \mathbf{9}$
$3^3 = 2\mathbf{7}$
$3^4 = 8\mathbf{1}$
$3^5 = 24\mathbf{3}$
$3^6 = 72\mathbf{9}$

The pattern is 3, 9, 7, 1 and repeats every 4 terms. (Note: you can memorize this pattern or recreate it when you need it. For the numbers 0 to 9, there are no more than 4 units digits in the pattern; if you remember this, you only need to test the first 4 terms.)

The 2nd term in the pattern of 4 terms is equal to 9; for each set of 4 terms, then, there will be one number with a units digit of 9. Since 75 divided by 4 = 18 remainder 3, the entire pattern will repeat 18 times, for a total of 18 terms with a units digit of 9, followed by 3 extra terms. The three extra terms (3, 9, 7) include one extra 9. Therefore, a units digit of 9 appears 18 + 1 = 19 times in this sequence.

64. **317.** Each term in the sequence is 9 greater than the previous term. To make this obvious, you may want to write a few terms of the sequence: 11, 20, 29, 38, etc.

a_{35} comes 34 terms after a_1 in the sequence. In other words, a_{35} is $34 \times 9 = 306$ greater than a_1.

Thus, $a_{35} = 11 + 306 = 317$.

65. **9.** After the first term in the sequence, every term has a units digit of 8:

$Q_1 = 3$
$Q_2 = 2(3) + 2 = 8$
$Q_3 = 2(8) + 2 = 18$
$Q_4 = 2(18) + 2 = 38$
$Q_5 = 2(38) + 2 = 78$
…

So 8 will be the units digit nine out of the first ten times.

66. **11.** This question concerns some function for which the full formula is not given. The problem indicates that $g(2m) = 2g(m)$. In other words, this function is such that plugging in $2m$ is the same as plugging in m and then multiplying by 2. Plug $g(3)$ into the equation for $g(m)$:

$g(6) = 2g(3)$
$g(6) = 2(5.5)$
$g(6) = 11$

67. **(B).** The question asks which of the functions in the answer choices is such that performing the function on $a + b$ yields the same answer as performing the function to a and b individually and then adding those answers together.

The correct answer should be such that $f(a + b) = f(a) + f(b)$ is true for any values of a and b. Test some numbers, for example $a = 2$ and $b = 3$.

	$f(a + b) = f(5)$	$f(a) = f(2)$	$f(b) = f(3)$	Does $f(a + b) = f(a) + f(b)$?
(A)	$f(5) = 5^2 = 25$	$f(2) = 2^2 = 4$	$f(3) = 3^2 = 9$	No
(B)	$f(5) = 5(5) = 25$	$f(2) = 5(2) = 10$	$f(3) = 5(3) = 15$	Yes
(C)	$f(5) = 2(5) + 1 = 11$	$f(2) = 2(2) + 1 = 5$	$f(3) = 2(3) + 1 = 7$	No
(D)	$f(5) = \sqrt{5}$	$f(2) = \sqrt{2}$	$f(3) = \sqrt{3}$	No
(E)	$f(5) = 5 - 2 = 3$	$f(2) = 2 - 2 = 0$	$f(3) = 3 - 2 = 1$	No

Alternatively, use logic — for what kinds of operations are performing the operation on two numbers and then adding them together the same as adding the original numbers together and then performing the operation? Multiplication or division would work, but squaring, square-rooting, adding, or subtracting would not. The correct function can contain ONLY multiplication and/or division.

68. **(B).** You can solve this problem by applying the compound interest formula:

$$V = P\left(1 + \frac{r}{100}\right)^t$$

Since the principal P is the same in both cases, you can leave it out and just compare the rest.

Quantity A: $\left(1 + \frac{r}{100}\right)^t = \left(1 + \frac{8}{100}\right)^2 = (1.08)^2 = 1.08 \times 1.08 = 1.1664$

Quantity B: $\left(1 + \frac{r}{100}\right)^t = \left(1 + \frac{4}{100}\right)^4 = (1.04)^4 = 1.04 \times 1.04 \times 1.04 \times 1.04 \approx 1.1699$

Quantity B is larger.

Alternatively, you can use logic. Notice that the *simple* interest in each case would be the same: 2 years of 8% *simple* interest (of an unchanging principal) is equal to 4 years of 4% *simple* interest of the same principal. Now go back to the compounded world. If the simple interest scenarios are the same, then it will always be true that the compounded scenario with *more frequent* compounding will give you a larger principal in the end, because you're earning "interest on the interest" more often.

2 periods of 8% compounded interest	<	4 periods of 4% compounded interest	<	8 periods of 2% compounded interest	<	16 periods of 1% compounded interest

The differences are small but real.

69. **(B).** Start with \$1, and multiply by $\left(1+\frac{26}{100}\right)$ = 1.26 for each year that passes. In order for the amount to double, it would have to reach \$2.

End of Year 1: \$1 × 1.26 = \$1.26
End of Year 2: \$1.26 × 1.26 = \$1.5876
End of Year 3: \$1.5876 × 1.26 = \$2.000376 $\approx$ \$2.00

It takes 3 years for the investment to double in value. If you mistakenly thought in terms of *simple* interest, you might think it would take about 4 years (since 26% is just a tiny bit more than 25% = 1/4). In the compounded case, you're earning "interest on the interest," though, so the investment grows more quickly.

70. **(A).** You can try the answer choices to find the answer. Start with either choice (B) or choice (D). An 18% interest rate corresponds to multiplying by 1.18. Compounding over two years means multiplying by 1.18 twice:

\$1,200 × 1.18 × 1.18 = \$1,670.88.

This is close to \$1,650 but 17% could be closer. Try answer choice (A):

\$1,200 × 1.17 × 1.17 = \$1,642.68

Since this result is in fact closer to \$1,650, the interest rate must be closer to 17% than to 18%.

Alternatively, use the compound interest formula and solve for the missing interest rate:

$$V = P\left(1+\frac{r}{100}\right)^t$$

$$1{,}650 = 1{,}200\left(1+\frac{r}{100}\right)^2$$

$$\frac{1{,}650}{1{,}200} = \left(1+\frac{r}{100}\right)^2$$

$$\sqrt{1.375} = 1+\frac{r}{100}$$

$$1.172 = 1+\frac{r}{100}$$

$$(0.172)100 = r$$

$$r \approx 17$$

71. **(E).** "Simple" interest means that the interest is calculated based on the initial amount every time; the interest earned is not included in future calculations. Each year, the investment pays 12.5%, or 1/8, of the original investment as simple interest. As a result, it will take exactly 8 years for the cumulative interest to add up to the original investment.

Be careful not to apply the compound interest formula here. If the 12.5% interest is in fact compounded annually, it will take only about 6 years for the investment to double in value.

72. **(C).** This question concerns some function for which the full formula is not provided. The problem indicates that $f(2a) = 2f(a)$. In other words, this function is such that plugging in $2a$ is the same as plugging in a and then multiplying by 2. Plug $f(6) = 11$ into the equation $f(2a) = 2f(a)$:

$f(2(6)) = 2(11)$
$f(12) = 22$

Use the same process a second time. If $a = 12$ and $f(12) = 22$:

$f(2(12)) = 2(22)$
$f(24) = 44$

Alternatively, use logic. When you plug in $2a$, you'll get the same answer as when you plug in a and then multiply by 2. Plugging in 24 is the same as plugging in 6 a total of 4 times, and will give you an answer 4 times as big as plugging in 6. Since plugging in 6 yields 11, plugging in 24 yields 44.

73. **(B).** The question is asking "For which function is performing the function on x and THEN multiplying by 1/2 the equivalent of performing the function on 1/2 of x?"

The fastest method is to use logic: since the order of operations says that order does not matter with multiplication and division but DOES matter between multiplication and addition/subtraction, or multiplication and exponents, you need a function that has only multiplication and/or division. Only answer choice (B) qualifies.

Alternatively, try each choice.

	$\frac{1}{2}f(x)$	$f\left(\frac{1}{2}x\right)$	equal?
(A)	$f\left(\frac{1}{2}x\right)$	$\frac{1}{2}(2x+2)=x+1$	No
(B)	$\frac{1}{2}(13x)=\frac{13x}{2}$	$13\left(\frac{1}{2}x\right)=\frac{13x}{2}$	Yes
(C)	$\frac{1}{2}x^2$	$\left(\frac{1}{2}x\right)^2=\frac{1}{4}x^2$	No
(D)	$\frac{1}{2}(x-10)=\frac{x}{2}-5$	$\frac{1}{2}x-10=\frac{x}{2}-10$	No
(E)	$\frac{1}{2}\sqrt{x-4}$	$\sqrt{\frac{1}{2}x-4}$	No

If you aren't sure whether the two terms in choice (E) are equal, try plugging in a real number for x. If $x = 8$, then the left-hand value becomes 1 and the right-hand value becomes the square root of 0. The two values are *not* the same.

Chapter 11

of

Fractions and Decimals

In This Chapter…

Fractions and Decimals

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}=$

2.

Quantity A	Quantity B
$-\frac{3}{4}+\frac{2}{3}$	$-\frac{3}{4}\times\frac{2}{3}$

3. The temperature in Limerick is 3/4 that in Cairo, where the temperature is 8/5 that in Halifax. If the temperature in

Limerick is 60°, what is the temperature in Halifax?

(A) 50°
(B) 55°
(C) 64°
(D) 72°
(E) 75°

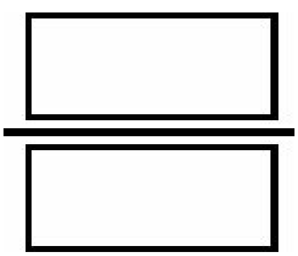
4. At a convention of monsters, 2/5 have no horns, 1/7 have one horn, 1/3 have two horns, and the remaining 26 have three or more horns. How many monsters are attending the convention?

(A) 100
(B) 130
(C) 180
(D) 210
(E) 260

5. One dose of secret formula is made from 1/6 ounce of Substance X and 2/3 ounce of Substance Z. How many doses are in a 30-ounce vial of secret formula?

(A) 20
(B) 24
(C) 30
(D) 36
(E) 45

6. Devora spends 1/4 of her money on a textbook, and then buys a notebook that costs 1/6 the price of the textbook. Assuming she makes no other purchases, what fraction of her money does Devora have left over?



7. 0.003482 =

Indicate <u>all</u> such statements.

☐ -0.003482×10^{-1}
☐ 0.3482×10^{-2}
☐ 34.82×10^{4}
☐ 34.82×10^{-4}
☐ $3,482 \times 10^{-6}$

8. 12.12×10^{-3} =

Indicate <u>all</u> such statements.

☐ -1.21×10^{3}

☐ 0.012
☐ 0.00001212×10^3
☐ 0.01212×10^3

9. 5 is how many fifths of 10?

(A) 2.5
(B) 5
(C) 10
(D) 20
(E) 50

10.

$x > 0$ and $y > 0$

Quantity A	**Quantity B**
$\frac{1}{x}+\frac{1}{y}$	$\frac{xy}{x+y}$

11.

Quantity A	**Quantity B**
$\frac{75}{4^2}\times\frac{3^2}{45}\times\frac{2^4}{45}$	$\frac{3^2}{4^2}\times\frac{2^2}{5^2}\times\frac{10}{3}$

12. 5/12 of all the students are girls and 1/4 of all the students are girls who take Spanish. What fraction of the girls take Spanish?

(A) 5/48
(B) 5/12
(C) 2/5
(D) 3/5
(E) 7/12

13. 1/5 of all the cars on a certain auto lot are red, and 2/3 of all the red cars are convertibles. What fraction of all the cars are NOT red convertibles?

$$\frac{\square}{\square}$$

14. Two identical pies are cut into a total of 16 equal parts. If each part is then split equally among three people, what fraction of a pie does each person receive?

(A) 1/48

(B) 1/24
(C) 1/16
(D) 3/16
(E) 3/8

15. Which of the following are bigger than twice 21/49?

Indicate <u>all</u> such values.

☐ 0.84
☐ 0.857
☐ 0.858
☐ 0.86

16.

$$xy \neq 0$$

Quantity A	**Quantity B**
$2+\frac{1}{xy}$	$\frac{2xy+1}{xy}$

17.

Quantity A	**Quantity B**
$\dfrac{\frac{1}{4}}{\frac{2}{3}-\frac{1-2}{\frac{1}{3}}}$	$\dfrac{\frac{1}{3}}{\frac{1}{4}-\frac{3-4}{\frac{2}{3}}}$

18.

At Store A, 3/4 of the apples are red.
At Store B, which has twice as many apples, 0.375 of them are red.

Quantity A	**Quantity B**
The number of red apples at Store A	The number of red apples at Store B

19.

Dweezil has one third the number of black marbles that Gina has, but he has twice as many white marbles.
Both people have only black marbles and white marbles.

Quantity A	**Quantity B**

The number of marbles Dweezil has	The number of marbles Gina has

20. A pot of soup is divided equally into two bowls. If Manuel eats 1/4 of one of the bowls of soup and 2/5 of the other bowl of soup, how much of the soup did Manuel eat?

$$\frac{\boxed{}}{\boxed{}}$$

21. What is half of $\frac{x^2}{8}$?

(A) $\frac{x}{4}$
(B) $\frac{x^2}{4}$
(C) $\frac{x}{8}$
(D) $\frac{x^2}{16}$
(E) It cannot be determined.

22. $\dfrac{\frac{ab}{c}}{\frac{cd}{a}} =$

(A) ac
(B) bd
(C) $\frac{1}{bd}$
(D) $\frac{a^2b}{c^2d}$
(E) $\frac{ab^2}{cd^2}$

23. $\left(\frac{\sqrt{12}}{5}\right)\left(\frac{\sqrt{60}}{2^4}\right)\left(\frac{\sqrt{45}}{3^2}\right) =$

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) $\frac{1}{2}$

24. $\frac{-1}{2x}-\frac{1}{4y}+\frac{1}{xy}+\frac{1}{8}=$

(A) $\frac{(x-4)(2-y)}{8xy}$

(B) $\frac{(x-2)(y-4)}{8xy}$

(C) $\frac{(x-4)(y-2)}{8xy}$

(D) $\frac{(x+2)(4-y)}{8xy}$

(E) $\frac{(x-2)(4-y)}{8xy}$

25.

x is a digit in the decimal 12.15x9, which, if rounded to the nearest hundredth, would equal 12.16.

<u>Quantity A</u>	**<u>Quantity B</u>**
x	4

26. $\frac{\frac{a}{b}}{\frac{c}{d}+\frac{e}{f}}=$

(A) $\dfrac{acd}{bcf+def}$

(B) $\dfrac{ace}{bdf+bcd}$

(C) $\dfrac{acf}{bde+cdf}$

(D) $\dfrac{ade}{bef+cdf}$

(E) $\dfrac{adf}{bcf+bde}$

27. $\dfrac{(17^2)(22)(38)(41)(91)}{(19)(34)(123)(11)(119)(26)} =$

$$\frac{\boxed{}}{\boxed{}}$$

28. In a decimal number, a bar over one or more consecutive digits means that the pattern of digits under the bar repeats without end. As a fraction, $7.58\overline{3} =$

$$\frac{\boxed{}}{\boxed{}}$$

29.

Quantity A	**Quantity B**
$\left(\dfrac{\sqrt{25}}{\sqrt{10}}\right)\left(\dfrac{\sqrt{8}}{\sqrt{15}}\right)$	$\left(\dfrac{\sqrt{51}}{\sqrt{46}}\right)\left(\dfrac{\sqrt{23}}{\sqrt{34}}\right)$

$$\sqrt{\frac{3}{2}}-\sqrt{\frac{2}{3}}=$$

(A) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{6}}$

(B) $\frac{1}{\sqrt{6}}$

(C) $\frac{\sqrt{3}}{3}$

(D) $\frac{\sqrt{3}}{2}$

(E) $\frac{\sqrt{5}}{\sqrt{6}}$

31. If $abc \neq 0$, then $\frac{ab}{cb}+\frac{a}{c}-\frac{a^2b^3}{abc}=$

(A) $\frac{a-b^2}{c}$

(B) $\frac{a^2-2b^2}{c}$

(C) $\frac{2a^2-b^2}{c}$

(D) $\frac{a\left(2-b^2\right)}{c}$

(E) $\frac{a^2b\left(2-b^2\right)}{c}$

32. If 3/4 of all the cookies have nuts and 1/3 of all the cookies have both nuts and fruit, what fraction of all the cookies have nuts but no fruit?

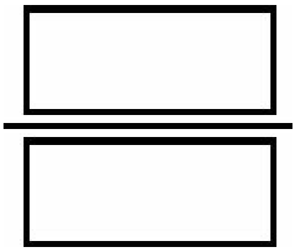
(A) 1/4
(B) 5/12
(C) 1/2
(D) 7/12
(E) 5/6

33. 1/4 of all the juniors and 2/3 of all the seniors are going on a trip. If there are 2/3 as many juniors as seniors, what

fraction of the students are not going on the trip?

(A) 4/9
(B) 1/2
(C) 2/3
(D) 1/3
(E) 5/6

34. 4/5 of the women and 3/4 of the men speak Spanish. If there are 40% as many men as women, what fraction of the group speaks Spanish?



35.

$$abcd \neq 0$$

Quantity A	**Quantity B**
$\frac{a^2b}{cd^2} \times \frac{d^3}{abc}$	$\frac{d^2}{bc} \times \frac{ab^2}{bd}$

36.

Quantity A	**Quantity B**
$\frac{24}{3\sqrt{2}} - \frac{4}{\sqrt{2}}$	$\sqrt{6}$

37.

$$m \neq 0$$

Quantity A	**Quantity B**
$\left(\frac{1}{2} + \frac{1}{m}\right)(m+2)$	$\frac{(m+2)^2}{2m}$

38.

The reciprocal of x's non-integer decimal part equals $x + 1$, and $x > 0$.

Quantity A	**Quantity B**
x	$\sqrt{2}$

39. Which two of the following numbers have a sum between 1 and 2?

Indicate both of the numbers.

☐ $\dfrac{7(2^3)}{3^3-7}$

☐ $\dfrac{2^4}{1+2+3+4}$

☐ $\dfrac{3}{11} \div \dfrac{6}{11}$

☐ $\dfrac{-2^3 3^2}{2^2 5^2}$

☐ $\dfrac{-11^2-11^3}{(30)(44)}$

40. Which three of the following answers, when multiplied by each other, yield a product less than -1?

Indicate all three numbers.

☐ $\dfrac{-15}{17}$

☐ $\dfrac{-18}{19}$

☐ $\dfrac{23}{-22}$

☐ $\dfrac{17}{-16}$

41. The decimal representation of the reciprocal of integer n contains an infinitely repeating pattern of digits, expressed with a bar over the repeating digits. The minimum length of the bar (in digits) is n - 1.

Indicate all of the integers below that could be n.

☐ 3
☐ 5
☐ 7
☐ 9
☐ 11

42. $(3-\frac{1}{3})^2+(3+\frac{1}{3})^2=$

(A) 122/9
(B) 164/9
(C) 36
(D) 164/3
(E) 162

43. If $\dfrac{3}{\frac{m+1}{m}+1}=1$, then *m* must equal

(A) -2
(B) -1
(C) 0
(D) 1
(E) 2

44.

$$rs=\sqrt{3}$$

Quantity A	**Quantity B**
$\dfrac{2r\sqrt{12}}{r^2s\sqrt{72}}$	$\dfrac{14rs^2}{42s}$

45.

Quantity A	**Quantity B**
$\dfrac{\sqrt{10}}{\sqrt{8}}\div\dfrac{\sqrt{9}}{\sqrt{10}}$	$\dfrac{\sqrt{11}}{\sqrt{9}}\div\dfrac{\sqrt{10}}{\sqrt{11}}$

46.

$$\frac{x}{m}>0$$

Quantity A	**Quantity B**
$\dfrac{11m+17x}{11m}$	$\dfrac{17m+11x}{17m}$

47. Which of the following fractions has the greatest value?

(A) $\dfrac{7}{(16^2)(25)}$

(B) $\dfrac{5}{(32)(5^4)}$

(C) $\dfrac{30}{(512)(5^3)}$

(D) $\dfrac{5}{(4^6)(5)}$

(E) $\dfrac{4}{(2^{11})(5^2)}$

48.

$$\frac{m}{p} > \frac{n}{p}$$

Quantity A	**Quantity B**
m	*n*

49. If $2x \neq y$ and $5x \neq 4y$, then

$$\frac{\dfrac{5x-4y}{2x-y}}{\dfrac{3y}{y-2x}+5} =$$

(A) $\dfrac{1}{2}$

(B) $\dfrac{3}{2}$

(C) $\dfrac{5}{2}$

(D) $\dfrac{7}{2}$

(E) $\dfrac{9}{2}$

50. $\frac{39^2}{2^4} \div \frac{13^3}{4^2} =$

(A) $\frac{13}{2}$
(B) $\frac{9}{2}$
(C) $\frac{3}{2}$
(D) $\frac{3}{13}$
(E) $\frac{9}{13}$

51. To the nearest integer, the non-negative fourth root of integer *n* rounds to 3. Inclusive, *n* is between

(A) 0 and 1
(B) 2 and 3
(C) 4 and 9
(D) 10 and 39
(E) 40 and 150

Fractions and Decimals Answers

1. $\frac{71}{20}$ **(or any equivalent fraction).** Add all the fractions by finding a common denominator, which is a multiple of 2, 3, 4, 5, and 6. The smallest number that will work is 60.

$$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}=\frac{30}{60}+\frac{40}{60}+\frac{45}{60}+\frac{48}{60}+\frac{50}{60}=\frac{30+40+45+48+50}{60}=\frac{213}{60}=\frac{71}{20}$$

2. **(A).** In Quantity A, get a common denominator and then add:

$$-\frac{3}{4}+\frac{2}{3}=-\frac{9}{12}+\frac{8}{12}=-\frac{1}{12}$$

In Quantity B, multiply across (common denominators are only needed for addition and subtraction). You can cancel the 3's first if you wish:

$$-\frac{3}{4}\times\frac{2}{3}=-\frac{\cancel{3}}{4}\times\frac{2}{\cancel{3}}=-\frac{2}{4}=-\frac{1}{2}$$

$-\frac{1}{12}$ is larger than $-\frac{1}{2}$. (Be careful with negatives! The closer to 0 a negative is, the larger it is.)

3. **(A).** When you are given two relationships in one sentence, follow the grammar carefully to make sure you produce the right equations. The first sentence of the problem gives you two relationships:

> The temperature in Limerick is 3/4 that in Cairo.
> The temperature in Cairo is 8/5 that in Halifax.

$$L=\frac{3}{4}C$$

$$C=\frac{8}{5}H$$

Replace C with $(8/5)H$ in the first equation:

$$L = \frac{3}{4}\left(\frac{8}{5}H\right)$$
$$L = \frac{24}{20}H$$
$$L = \frac{6}{5}H$$

Now plug in 60 for *L*:

$$60 = \frac{6}{5}H$$
$$\frac{5}{6} \times 60 = H$$
$$50 = H$$

4. **(D).** This is a common GRE setup—you have been given several fractions and one actual number. Once you know what fraction of the whole that number represents, you can solve for the total (call the total *m*). Notice that all of the denominators are primes, so they don't share any factors. Therefore you will have to multiply them all together to find a common denominator. 5 × 7 × 3 = 105:

$$\frac{2}{5} + \frac{1}{7} + \frac{1}{3} = \frac{42}{105} + \frac{15}{105} + \frac{35}{105} = \frac{92}{105}$$

That means that the remaining 26 monsters represent 13/105 of the total monsters at the convention:

$$26 = \frac{13}{105}m$$
$$\frac{105}{13} \times 26 = m$$
$$105 \times 2 = m$$
$$210 = m$$

5. **(D).** To find the number of doses in the vial, you need to divide the total volume of the formula in the vial by the volume of one dose.

One dose = 1/6 + 2/3 = 1/6 + 4/6 = 5/6

Now divide 30 ounces by 5/6:

30 ÷ 5/6 = 30 × 6/5 = 36

6. $\frac{17}{24}$ **(or any equivalent fraction).** The textbook costs 1/4 of Devora's money. The notebook costs 1/6 *of that amount,* or $\frac{1}{6}\left(\frac{1}{4}\right)=\frac{1}{24}$ of Devora's money. Thus, Devora has spent $\frac{1}{4}+\frac{1}{24}=\frac{6}{24}+\frac{1}{24}=\frac{7}{24}$ of her money. Subtract from 1 to get the fraction she has left: $1-\frac{7}{24}=\frac{24}{24}-\frac{7}{24}=\frac{17}{24}$.

Alternatively, pick a value for Devora's money. (Look at the denominators in the problem—4 and 6—and pick a value that both numbers go into.) For instance, say Devora has \$120. She would spend 1/4, or \$30 on the textbook. She would spend 1/6 *of that* amount, or \$5, on the notebook. She would have spent \$35 and have \$85 left, and thus $\frac{85}{120}$ of her money left. Reduce $\frac{85}{120}$ to get $\frac{17}{24}$, or simply enter $\frac{85}{120}$ in the boxes. This will work with any value you pick for Devora's total.

7. **II, IV, and V only.** Note that the first answer is negative, so it cannot be correct. For the second answer, move the decimal 2 places to the left: $0.3482 \times 10^{-2} = 0.003482$ (correct). For the third answer, move the decimal 4 places to the *right* (since the exponent is positive)—this move makes the number much larger and cannot be correct. For the fourth answer, move the decimal 4 places to the left: $34.82 \times 10^{-4} = 0.003482$ (correct). For the fifth answer, move the decimal 6 places to the left: $3{,}482 \times 10^{-6} = 0.003482$ (correct).

8. **III only.** First, simplify $12.12 \times 10^{-3} = 0.01212$. Now, test which answers are equal to this value. The first answer is negative, so it cannot be correct. The second answer is simply 0.012 and is therefore incorrect (the end has been "chopped off," so the number is not the same value). The third answer is $0.00001212 \times 10^3 = 0.01212$ and is correct. The fourth answer is $0.01212 \times 10^3 = 12.12$ and is not correct.

9. **(A).** Translate the words into math. If x means "how many," then "how many fifths" is $\frac{x}{5}$:

$$5=\frac{x}{5}\times 10$$
$$5=\frac{10x}{5}$$
$$25=10x$$
$$\frac{25}{10}=x$$
$$x=2.5$$

10. **(D)**. Add the fractions in Quantity A by making a common denominator (xy):

$$\frac{1}{x}+\frac{1}{y}=\frac{y}{xy}+\frac{x}{xy}=\frac{y+x}{xy}$$

Quantity B is just the reciprocal of Quantity A—one fraction is the "flipped" version of the other. You also know that both quantities are positive. However, without knowing more about x and y, you don't know whether Quantity A or Quantity B is bigger.

11. **(A).** Simplify each quantity by breaking down to primes and canceling factors:

Quantity A: $$\frac{75}{4^2}\times\frac{3^2}{45}\times\frac{2^4}{45}=\frac{3\times5\times5}{\left(2^2\right)^2}\times\frac{3^2}{3\times3\times5}\times\frac{2^4}{3\times3\times5}=\frac{2^4\times3^3\times5^2}{2^4\times3^4\times5^2}=\frac{1}{3}$$

Quantity B: $$\frac{3^2}{4^2}\times\frac{2^2}{5^2}\times\frac{10}{3}=\frac{3^2}{\left(2^2\right)^2}\times\frac{2^2}{5^2}\times\frac{2\times5}{3}=\frac{2^3\times3^2\times5}{2^4\times3\times5^2}=\frac{3}{2\times5}=\frac{3}{10}$$

Since $\frac{1}{3}>\frac{3}{10}$, Quantity A is bigger than Quantity B. You can compare these fractions by making a common denominator, by cross multiplying, or by comparing the decimal equivalents 0.333… and 0.3.

If you notice the same factors on each side in the same position (e.g., 3^2 on top or 4^2 on bottom), then you can save time by canceling those factors simultaneously from both quantities.

12. **(D).** The wording here is very confusing. The problem is *not* asking you to take 1/4 of 5/12. Rather, 1/4 and 5/12 are fractions of the same number (the number of students in the whole class). A good way to avoid this confusion is to plug in a number for the class. Pick 12, as you're asked to take 5/12ths of the class:

Class = 12
Girls = 5
Girls who take Spanish = 3 (1/4 of all the students)

You are asked for the number of girls who take Spanish over the number of girls. Thus, the answer is 3/5.

13. **13/15 (or any equivalent fraction).** If 1/5 of all the cars are red and 2/3 of THOSE are convertibles, then the fraction of all the cars that are red convertibles = (1/5)(2/3) = 2/15. Since you want all of the cars that are NOT red convertibles, subtract 2/15 from 1 to get 13/15.

14. **(B).** If *two* pies are cut into 16 parts, *each* pie is cut into eighths. Thus, 1/8 of a pie is *divided* among three people. "One third of one eighth" = (1/3)(1/8) = 1/24.

15. **III and IV only.** Simply plug 21/49 into the calculator, and then multiply by 2 to get 0.857142… You need all values larger than this number. Obviously, 0.84 is smaller. The next choice, 0.857, might seem attractive; however, it is smaller than 0.857142… You can easily see this by adding zeroes to the end of 0.857 in order to more easily compare:

Choice II: 0.857000

Your number: 0.857142…

The third and fourth choices are, of course, larger than 0.857142…

16. **(C).** Transform Quantity B by splitting the numerator:

$$\frac{2xy+1}{xy}=\frac{2xy}{xy}+\frac{1}{xy}$$

Then cancel the common factor *xy* from top and bottom of the first fraction:

$$\frac{2xy}{xy}+\frac{1}{xy}=2+\frac{1}{xy}$$, which is the same as Quantity A.

Alternatively, you can transform Quantity A by turning 2 into a fraction with the same denominator (xy) as the second term.

$$2+\frac{1}{xy}=\frac{2xy}{xy}+\frac{1}{xy}=\frac{2xy+1}{xy}$$, which is the same as Quantity B.

17. **(B).** Simplify each quantity from the inside out.

Quantity A: $$\frac{\frac{1}{4}}{\frac{2}{3}-\frac{1-2}{\frac{1}{3}}}=\frac{\frac{1}{4}}{\frac{2}{3}-\frac{-1}{\frac{1}{3}}}=\frac{\frac{1}{4}}{\frac{2}{3}-(-3)}=\frac{\frac{1}{4}}{\frac{2}{3}+3}=\frac{\frac{1}{4}}{\frac{2}{3}+\frac{9}{3}}=\frac{\frac{1}{4}}{\frac{11}{3}}=\frac{1}{4}\times\frac{3}{11}=\frac{3}{44}$$

Quantity B: $$\frac{\frac{1}{3}}{\frac{1}{4}-\frac{3-4}{\frac{2}{3}}}=\frac{\frac{1}{3}}{\frac{1}{4}-\frac{-1}{\frac{2}{3}}}=\frac{\frac{1}{3}}{\frac{1}{4}-\left(\frac{-3}{2}\right)}=\frac{\frac{1}{3}}{\frac{1}{4}+\frac{3}{2}}=\frac{\frac{1}{3}}{\frac{1}{4}+\frac{6}{4}}=\frac{\frac{1}{3}}{\frac{7}{4}}=\frac{1}{3}\times\frac{4}{7}=\frac{4}{21}$$

Since Quantity B has a larger numerator *and* a smaller denominator, it is larger than Quantity A. This rule works for any positive fractions. Of course, you can also use the calculator to compute the decimal equivalents.

18. **(C).** Whether you choose fractions or decimals, you want to make 3/4 and 0.375 the same form. Either way, you will see that 3/4 is double 0.375 (which is 3/8). Since Store B has twice as many apples, 3/8 of Store B's apples is the same value as 3/4 of Store A's apples.

Alternatively, pick numbers such that Store B has twice as many apples. If Store A has 4 apples and Store B has 8 apples, then Store A would have (3/4)(4) = 3 red apples and Store B would have (0.375)(8) = 3 red apples. The values will always be the same.

19. **(D).** To demonstrate that there is not enough information to determine who has more marbles, try extreme examples. Dweezil has 1/3 as many black marbles as Gina, but twice as many white marbles:

EXAMPLE 1: LOTS OF BLACK MARBLES

Dweezil:	1,000 black marbles	2 white marbles
Gina:	3,000 black marbles	1 white marble

In this example, Gina has more.

EXAMPLE 2: LOTS OF WHITE MARBLES

Dweezil:	1 black marble	2,000 white marbles
Gina:	3 black marbles	1,000 white marbles

In this example, Dweezil has more.

As always, when trying examples in Quantitative Comparison problems, you *must* try more than one example—with the goal of proving (D).

20. **13/40 (or any equivalent fraction).** Manuel eats 1/4 of one-half of all the soup, and then 2/5 of the other half of all the soup. As math:

$$\frac{1}{4}\left(\frac{1}{2}\right)+\frac{2}{5}\left(\frac{1}{2}\right)=\frac{1}{8}+\frac{1}{5}=\frac{13}{40}$$

Alternatively, pick numbers. Since you'll be dividing this number several times, pick a large number with many factors. For example, say there are 120 ounces of soup. Each bowl would then have 60 ounces. Manuel would then eat 1/4 of one bowl (15 ounces) and 2/5 of the other bowl (24 ounces). In total, he would eat 39 ounces out of 120. While 39/120 would be counted as correct, it is also possible to reduce 39/120 (divide both numerator and denominator by 3) to get 13/40, the answer you reached via the other method above.

21. **(D).** To take half of a number, multiply by $\frac{1}{2}$:

$$\frac{1}{2}\times\frac{x^2}{8}=\frac{x^2}{16}$$

22. **(D).** To divide by a fraction, multiply by its reciprocal:

$$\frac{\frac{ab}{c}}{\frac{cd}{a}} = \frac{ab}{c} \times \frac{a}{cd} = \frac{a^2 b}{c^2 d}$$

23. **(C).** Pull squares out of the square roots and cancel common factors:

$$\left(\frac{\sqrt{12}}{5}\right)\left(\frac{\sqrt{60}}{2^4}\right)\left(\frac{\sqrt{45}}{3^2}\right) = \frac{2\sqrt{3}}{5} \times \frac{2\sqrt{15}}{2^4} \times \frac{3\sqrt{5}}{3^2} = \frac{\sqrt{3}}{5} \times \frac{\sqrt{15}}{2^2} \times \frac{\sqrt{5}}{3}$$

Since $\sqrt{15} = \sqrt{3}\sqrt{5}$, you get

$$\frac{\sqrt{3}}{5} \times \frac{\sqrt{15}}{2^2} \times \frac{\sqrt{5}}{3} = \frac{\sqrt{3}}{5} \times \frac{\sqrt{3}\sqrt{5}}{2^2} \times \frac{\sqrt{5}}{3} = \frac{3 \times 5}{5 \times 2^2 \times 3} = \frac{1}{2^2} = \frac{1}{4}$$

24. **(C).** Combine the four fractions by finding a common denominator ($8xy$, which is also suggested by the answer choices):

$$\frac{-1}{2x} - \frac{1}{4y} + \frac{1}{xy} + \frac{1}{8} = \frac{-1(4y)}{2x(4y)} - \frac{1(2x)}{4y(2x)} + \frac{1(8)}{xy(8)} + \frac{1(xy)}{8(xy)}$$
$$= \frac{-4y}{8xy} - \frac{2x}{8xy} + \frac{8}{8xy} + \frac{xy}{8xy} = \frac{xy - 4y - 2x + 8}{8xy}$$

Now the key is to factor the top expression correctly.

$xy - 4y - 2x + 8 = (x - 4)(y - 2)$

You can always FOIL the expression on the right to make sure it matches the left-hand side.

So, in the end you have $\frac{xy - 4y - 2x + 8}{8xy} = \frac{(x-4)(y-2)}{8xy}$.

25. **(A).** Since the decimal rounds to 12.16, the thousandths digit x must be 5 or greater (6, 7, 8, or 9). All of these possibilities are greater than 4.

26. **(E).** Simplify from the inside out by finding a common denominator (df) for the two fractions “inside”:

$$\frac{\frac{a}{b}}{\frac{c}{d}+\frac{e}{f}}=\frac{\frac{a}{b}}{\frac{cf}{df}+\frac{de}{df}}$$

Next, add those two inside fractions; then flip and multiply:

$$\frac{\frac{a}{b}}{\frac{cf}{df}+\frac{de}{df}}=\frac{\frac{a}{b}}{\frac{cf+de}{df}}=\frac{a}{b}\times\frac{df}{cf+de}=\frac{adf}{bcf+bde}$$

27. $\frac{1}{3}$ **(or any equivalent fraction).** You could just punch the whole numerator and the whole denominator into the calculator and submit each product. If you're very careful, that will work. However, it might be wise to try canceling some common factors out of the fraction, to save time and to avoid errors. It's fine to switch to the calculator whenever the cancellations aren't obvious.

$$\frac{(17^2)\cancel{(22)}(38)(41)(91)}{(19)(34)(123)\cancel{(11)}(119)(26)}=\frac{(17^2)(2)(\cancel{38}\,2)(41)(91)}{\cancel{(19)}(34)(123)(119)(26)}$$
$$=\frac{(17^{\cancel{2}})(2)(2)(41)(91)}{(\cancel{34}\,2)(123)(119)(26)}=\frac{(17)(2)(2)\cancel{(41)}(91)}{(2)(\cancel{123}\,3)(119)(26)}$$
$$=\frac{(17)(2)(2)(\cancel{91}\,7)}{(2)(3)(119)(\cancel{26}\,2)}=\frac{\cancel{(17)}(2)(2)\cancel{(7)}}{(2)(3)(\cancel{17\times 7})(2)}$$
$$=\frac{\cancel{(2)}\cancel{(2)}}{\cancel{(2)}(3)\cancel{(2)}}=\frac{1}{3}$$

28. $\frac{91}{12}$ **(or any equivalent fraction).** First, turn the decimal into a sum of two pieces, to separate the repeating portion.

$$7.58\overline{3}=7.58+0.00\overline{3}$$

Deal with each piece in turn. Like any other terminating decimal, 7.58 can be written as a fraction with a power of 10 in the denominator.

$$7.58=\frac{758}{100}$$

Now, when you look at the repeating portion, you should be reminded that $0.\overline{3} = 0.3333... = \frac{1}{3}$

So $0.00\overline{3}$. is just $\frac{1}{3}$, moved by a couple of decimal places. $0.00\overline{3} = (0.\overline{3})(0.01) = \left(\frac{1}{3}\right)\left(\frac{1}{100}\right) = \frac{1}{300}$

Finally, you can write the original decimal as a sum of fractions, and then combine those fractions.

$$7.58\overline{3} = 7.58 + 0.00\overline{3} = \frac{758}{100} + \frac{1}{300} = \frac{758 \times 3}{300} + \frac{1}{300} = \frac{2,275}{300}$$

You can enter $\frac{2,275}{300}$, unreduced, or you can reduce it to $\frac{91}{12}$ if you want.

29. **(A).** Simplify each quantity by factoring the square roots, then canceling.

Quantity A: $\left(\frac{\sqrt{25}}{\sqrt{10}}\right)\left(\frac{\sqrt{8}}{\sqrt{15}}\right) = \left(\frac{5}{\sqrt{2}\sqrt{5}}\right)\left(\frac{2\sqrt{2}}{\sqrt{3}\sqrt{5}}\right) = \left(\frac{\cancel{5}}{\sqrt{2}\cancel{\sqrt{5}}}\right)\left(\frac{2\sqrt{2}}{\sqrt{3}\cancel{\sqrt{5}}}\right) =$
$\left(\frac{1}{\cancel{\sqrt{2}}}\right)\left(\frac{2\cancel{\sqrt{2}}}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}$

Quantity B: $\left(\frac{\sqrt{51}}{\sqrt{46}}\right)\left(\frac{\sqrt{23}}{\sqrt{34}}\right) = \left(\frac{\sqrt{3}\cancel{\sqrt{17}}}{\sqrt{2}\sqrt{23}}\right)\left(\frac{\sqrt{23}}{\sqrt{2}\cancel{\sqrt{17}}}\right) = \left(\frac{\sqrt{3}}{\sqrt{2}\cancel{\sqrt{23}}}\right)\left(\frac{\cancel{\sqrt{23}}}{\sqrt{2}}\right) = \frac{\sqrt{3}}{2}$

Since $\sqrt{3} < 2$, Quantity B is smaller than 1, whereas Quantity A is greater than 1.

Of course, you can use the calculator here, but the process would be slower and more prone to error.

30. **(B).** The square root of a fraction is the square root of the top over the square root of the bottom.

$$\sqrt{\frac{3}{2}} - \sqrt{\frac{2}{3}} = \frac{\sqrt{3}}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{3}}$$

Then make a common denominator: $\sqrt{3}\sqrt{2} = \sqrt{6}$.

$$\frac{\sqrt{3}}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{3}\sqrt{3}}{\sqrt{3}\sqrt{2}} - \frac{\sqrt{2}\sqrt{2}}{\sqrt{3}\sqrt{2}} = \frac{3}{\sqrt{6}} - \frac{2}{\sqrt{6}} = \frac{1}{\sqrt{6}}$$

31. **(D).** Simplify each fraction first by canceling common terms from top and bottom.

$$\frac{ab}{cb}+\frac{a}{c}-\frac{a^2b^3}{abc}=\frac{a\cancel{b}}{c\cancel{b}}+\frac{a}{c}-\frac{a^{\cancel{2}}b^{\cancel{3}2}}{\cancel{a}\cancel{b}c}=\frac{a}{c}+\frac{a}{c}-\frac{ab^2}{c}$$

Luckily, every fraction now has the same denominator, so you can just add/subtract the numerators.

$$\frac{a}{c}+\frac{a}{c}-\frac{ab^2}{c}=\frac{2a-ab^2}{c}=\frac{a\left(2-b^2\right)}{c}$$

32. **(B).** Since 3/4 of the cookies have nuts and 1/3 of the cookies *also* have fruit, simply subtract 3/4 - 1/3 to get all the cookies with nuts but no fruit.

$$\frac{3}{4}-\frac{1}{3}=\frac{9}{12}-\frac{4}{12}=\frac{5}{12}$$

Alternatively, pick numbers. Since you will be dividing by 4 and 3, pick a number divisible by 4 and 3. If there are 12 cookies, then 9 have nuts and 4 have nuts and fruit, so 5—and thus 5/12 of the total—would have nuts but no fruit.

33. **(B).** If you have to take fractions of different numbers that are *also* related by a fraction, then you should try plugging numbers. Since there are 2/3 as many juniors as seniors, some easy numbers are:

Juniors = 20

Seniors = 30

Juniors going on trip = $\frac{1}{4}$ (20) = 5

Seniors going on trip = $\frac{2}{3}$ (30) = 20

Out of 50 total students, 25 are going on the trip, so 25 are NOT going on the trip. The answer is 25/50 = 1/2.

34. **11/14 (or any equivalent fraction).** If you have to take fractions of different numbers that are *also* related by a fraction or percent, then you should try plugging numbers. Since there are 40% as many men as women, some easy numbers are:

Men: 40

Women: 100

Women who speak Spanish = $\frac{4}{5}$ (100) = 80

Men who speak Spanish = $\frac{3}{4}$ (40) = 30

The group has 140 total people and 110 Spanish speakers. 110/140 = 11/14 (you are not required to reduce, as long as your answer is correct and fits in the box).

35. **(D).** Cancel factors on top and bottom of each product.

Quantity A: $\frac{a^2b}{cd^2} \times \frac{d^3}{abc} = \frac{a^2bd^3}{abc^2d^2} = \frac{ad}{c^2}$

Quantity B: $\frac{d^2}{bc} \times \frac{ab^2}{bd} = \frac{ab^2d^2}{b^2cd} = \frac{ad}{c}$

The two quantities differ in the denominators: A has c^2, while B has c. So you can't tell which quantity is bigger, because sometimes c^2 is greater than c, and other times c^2 is less than c.

36. **(A).** Simplify Quantity A by canceling the common factor of 3 from top and bottom of the first fraction, then subtracting the numerators.

$$\frac{24}{3\sqrt{2}} - \frac{4}{\sqrt{2}} = \frac{8}{\sqrt{2}} - \frac{4}{\sqrt{2}} = \frac{8-4}{\sqrt{2}} = \frac{4}{\sqrt{2}}$$

Now compare Quantity A $\left(\frac{4}{\sqrt{2}}\right)$ with Quantity B $(\sqrt{6})$. Multiply both quantities by $\sqrt{2}$ to eliminate the denominator on the left.

$$\text{Quantity A} = \frac{4}{\sqrt{2}}\sqrt{2} = 4\text{, while Quantity B} = \sqrt{6}\sqrt{2} = \sqrt{12}\,.$$

Finally, square both quantities to get rid of the square-root sign:

Quantity A = 4^2 = 16, while Quantity B = $\left(\sqrt{12}\right)^2$ = 12.

Quantity A is obviously larger.

37. **(D).** Multiply out Quantity A by FOILing:

$$\left(\frac{1}{2}+\frac{1}{m}\right)(m+2) = \frac{1}{2}(m) + \frac{1}{2}(2) + \frac{1}{m}(m) + \frac{1}{m}(2)$$

$$= \frac{m}{2} + 1 + 1 + \frac{2}{m} = \frac{m}{2} + 2 + \frac{2}{m}$$

Make a common denominator ($2m$) to add these terms:

$$\frac{m}{2}+2+\frac{2}{m}=\frac{m(m)}{2(m)}+2\left(\frac{2m}{2m}\right)+\frac{(2)2}{(2)m}=\frac{m^2+4m+4}{2m}$$

Finally, look at Quantity B. Since $(m + 2)^2 = m^2 + 4m + 4$, you know that the quantities are the same.

38. **(C).** You can approach this problem by testing Quantity B ($\sqrt{2}$) as x. Using the calculator, you get $\sqrt{2}$… This decimal number doesn't repeat, but isolate the non-integer decimal part.

$$\sqrt{2}-1\approx 0.41421356...$$

Now take the reciprocal of both sides.

$$\frac{1}{\sqrt{2}-1}\approx\frac{1}{0.41421356...}=2.41421356...$$

The result seems to be 1 more than the original number, $\sqrt{2}$. To prove this outcome exactly, change the right side of the equation to $\sqrt{2}+1$ and rearrange. If the equation is true, you should be able to prove that the two sides are equal.

$$\frac{1}{\sqrt{2}-1}=\sqrt{2}+1?$$

Cross multiply. Notice the difference of squares:

$$1=\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)?$$
$$1=\left(\sqrt{2}\right)^2-1^2=2-1=1$$

Since 1 = 1, the original equation is true, and $x = \sqrt{2}$.

39. **I and V only.** Compute each value. For the simple cases, practice not using the calculator:

☐ $\frac{7(2^3)}{3^3-7}=\frac{7\times 8}{27-7}=\frac{56}{20}=2.8$

☐ $\frac{2^4}{1+2+3+4}=\frac{16}{10}=1.6$

☐ $\frac{3}{11}\div\frac{6}{11}=\frac{3}{11}\times\frac{11}{6}=\frac{3}{6}=\frac{1}{2}=0.5$

☐ $\frac{-2^3 3^2}{2^2 5^2}=\frac{-8\times 9}{10^2}=\frac{-72}{100}=-0.72$

☐ $\frac{-11^2-11^3}{(30)(44)}=\frac{-11^2(1+11)}{(30)(44)}=\frac{-121(12)}{(30)(44)}=\frac{-1,452}{-1,320}=-1.1$

In the last case, you could cancel factors and solve without the calculator, or you could punch in the products on top and bottom as shown, then divide on the calculator.

You've converted each value to decimal form, to make them easy to add. You are looking for two values that add up to a number between 1 and 2. You know that there can only be two such values.

By inspecting the positive numbers, you can see that no two of them add up to a number between 1 and 2. So you need a positive and a negative. The only two possibilities that work are 2.8 and -1.1.

40. **II, III, and IV only.** You want the product of three of the numbers to be less than -1. You can brute-force the calculation by trying all possible products, but you can use the relative size of the numbers to reduce the effort.

Notice that the four answer choices are all very close to -1, but some are greater than -1, and others are less than -1. To get the exact order, you can use the calculator, or you can think about the difference between each fraction and -1:

$\frac{-15}{17}=\frac{-17}{17}+\frac{2}{17}=-1+\frac{2}{17}$

$\frac{-18}{19}=\frac{-19}{19}+\frac{1}{19}=-1+\frac{1}{19}$, which is less than the previous number (since $\frac{2}{17}>\frac{1}{19}$)

$\frac{23}{-22}=\frac{-23}{22}=\frac{-22}{22}-\frac{1}{22}=-1-\frac{1}{22}$

$\frac{17}{-16}=\frac{-17}{16}=\frac{-16}{16}-\frac{1}{16}=-1-\frac{1}{16}$, a greater decrease from -1 than the previous number.

So the order of the original numbers relative to each other and to -1 is this: $\frac{17}{-16} < \frac{23}{-22} < -1 < \frac{-18}{19} < \frac{-15}{17}$

Try multiplying the three lowest numbers first, since they will produce the lowest product. Only *one* product of the three numbers can be less than -1 (or there would be more than one right answer), so the three numbers must be as follows, as you can check on the calculator:

$$\frac{17}{-16} \times \frac{23}{-22} \times \frac{-18}{19} \approx -1.052... < -1$$

41. **III only.** First, eliminate any decimals that *don't* repeat. The reciprocal of 5, which is 15 $\frac{1}{5}$, equals 0.2, which doesn't repeat. Next, use your calculator to compute the repeating decimals that correspond to the other reciprocals.

$\frac{1}{3} = 0.333... = 0.\overline{3}$ The bar only has to be 1 digit long, which does not equal n - 1 (= 3 - 1 = 2).

$\frac{1}{7} = 0.14285714... = 0.\overline{142857}$ The bar is 6 digits long, which equals n - 1 (= 7 - 1 = 6).

$\frac{1}{9} = 0.111... = 0.\overline{1}$ The bar only has to be 1 digit long, which does not equal n - 1 (= 9 - 1 = 8).

$\frac{1}{11} = 0.0909... = 0.\overline{09}$ The bar only has to be 2 digits long, which does not equal n - 1 (= 11 - 1 = 10).

So, 7 is the only possibility.

42. **(B).** First, simplify inside the parentheses. Then, square and add:

$$\left(\frac{8}{3}\right)^2 + \left(\frac{10}{3}\right)^2$$

$$\frac{64}{9} + \frac{100}{9}$$

The answer is 164/9.

43. **(D).** If the left-hand side of the equation is equal to 1, then the numerator and denominator must be equal. Thus, the denominator must also be equal to 3:

$$\frac{m+1}{m}+1=3$$
$$\frac{m+1}{m}=2$$
$$m+1=2m$$
$$1=m$$

Alternatively, you can just plug in each answer choice (into both instances of *m* in the original equation), and stop when you hit a choice that works.

44. **(B).** Cancel common factors in each quantity and substitute in for *rs*.

Quantity A: $\frac{2r\sqrt{12}}{r^2s\sqrt{72}}=\frac{2\sqrt{12}}{rs\sqrt{72}}=\frac{2\sqrt{12}}{\sqrt{3}\sqrt{72}}=\frac{2\sqrt{4}}{\sqrt{72}}=\frac{2\times 2}{\sqrt{36}\sqrt{2}}=\frac{4}{6\sqrt{2}}=\frac{2}{3\sqrt{2}}$

Quantity B: $\frac{14rs^2}{42s}=\frac{14rs}{42}=\frac{14\sqrt{3}}{3\times 14}=\frac{\sqrt{3}}{3}$

At this point, you can use the calculator, or you can compare the two quantities with an "invisible inequality."

$$\frac{2}{3\sqrt{2}} \quad ?? \quad \frac{\sqrt{3}}{3}$$

Since everything is positive, you can cross multiply (be sure to do so *upward*):

$$2\times 3 \quad ?? \quad 3\times\sqrt{2}\sqrt{3}$$

Now square both sides. Since everything is positive, the invisible inequality is unaffected:

$(2 \times 3)^2$?? $3^2 \times 2 \times 3$
36 ?? 54

Since 36 < 54, Quantity B is bigger.

45. **(A).** To divide fractions, multiply by the reciprocal.

Quantity A: $\frac{\sqrt{10}}{\sqrt{8}}\div\frac{\sqrt{9}}{\sqrt{10}}=\frac{\sqrt{10}}{\sqrt{8}}\times\frac{\sqrt{10}}{\sqrt{9}}=\frac{10}{\sqrt{72}}=\frac{10}{6\sqrt{2}}=\frac{5}{3\sqrt{2}}$

Quantity B: $\frac{\sqrt{11}}{\sqrt{9}} \div \frac{\sqrt{10}}{\sqrt{11}} = \frac{\sqrt{11}}{\sqrt{9}} \times \frac{\sqrt{11}}{\sqrt{10}} = \frac{11}{3\sqrt{10}}$

Square both quantities to get rid of the square roots.

Quantity A: $\left(\frac{5}{3\sqrt{2}}\right)^2 = \frac{5^2}{3^2 2} = \frac{25}{18}$

Quantity B: $\left(\frac{11}{3\sqrt{10}}\right)^2 = \frac{11^2}{3^2 10} = \frac{121}{90}$

At this point, use the calculator. Quantity A is approximately 1.389, whereas Quantity B is approximately 1.344.

46. **(A).** Always start by considering the initial given(s). If $x/m > 0$, then you know the two variables have the same sign (and that neither of them are 0). Looking down at the columns, you'll notice that there are common terms in the numerators and denominators of both fractions, so it will probably pay off to separate out the fractions:

Quantity A: $\frac{11m+17x}{11m} = \frac{11m}{11m} + \frac{17x}{11m} = 1 + \frac{17x}{11m}$

Quantity B: $\frac{17m+11x}{17m} = \frac{17m}{17m} + \frac{11x}{17m} = 1 + \frac{11x}{17m}$

Now that you've rephrased your two quantities, put them in an "invisible inequality" and see what you can do (since you don't know which side is greater, use a ? instead of the < or > symbols).

$$1 + \frac{17x}{11m} \ ? \ 1 + \frac{11x}{17m}$$

$$\frac{17x}{11m} \ ? \ \frac{11x}{17m}$$

Now isolate x/m on both sides:

$$\frac{17}{11} \times \frac{x}{m} \ ? \ \frac{11}{17} \times \frac{x}{m}$$

Now, because you know that x/m is positive, you can simply divide both sides by it:

$$\frac{17}{11} \ ? \ \frac{11}{17}$$

You're left with two fractions, one of which is greater than 1, and one of which is less than 1. The answer is (A).

47. **(A).** To determine which fraction is largest, cancel common terms from all five fractions until the remaining values are small enough for the calculator. Note that every choice has at least one 5 on the bottom, so cancel 5^1 from all of the denominators.

Note also that every fraction has a power of 2 on the bottom, so convert 16^2, 32, 512, 4^6, and 2^{11} to powers of 2. Since $16 = 2^4$, $32 = 2^5$, $512 = 2^9$, and $4^6 = (2^2)^6 = 2^{12}$, so you now have:

(A) $\dfrac{7}{(2^4)(5)}$

(B) $\dfrac{5}{(2^5)(5^3)}$

(D) $\dfrac{30}{(2^9)(5^2)}$

(C) $\dfrac{5}{(2^{12})}$

(E) $\dfrac{4}{(2^{11})(5)}$

Since every choice has at least 2^4 on the bottom, cancel 2^4 from all 5 choices:

(A) $\dfrac{7}{5}$

(B) $\dfrac{5}{(2)(5^3)}$

(C) $\dfrac{30}{(2^5)(5^2)}$

(D) $\dfrac{5}{2^8}$

(E) $\dfrac{4}{(2^7)(5)}$

Note that the numerators also have some powers of 2 and 5 that will cancel out with the bottoms of each of the fractions. In choice (C), 30 = (2)(3)(5):

(A) $\dfrac{7}{5}$

(B) $\dfrac{1}{(2)(5^2)}$

(C) $\frac{3}{(2^4)(5)}$
(D) $\frac{5}{2^8}$
(E) $\frac{1}{(2^5)(5)}$

These values are now small enough for the calculator. Note that the GRE calculator does not have an exponent button —to get 2^8, you must multiply 2 by itself 8 times. This is why you should memorize powers of 2 up to 2^{10}, and powers of 3, 4, and 5 up to about the 4th power.

(A) 1.4
(B) 0.02
(C) 0.0125
(D) 0.01953125
(E) 0.00625

The answer is (A).

48. **(D).** Without knowing the signs of any of the variables, you cannot assume that m is larger. While it certainly *could* be (for instance, $m = 4$, $n = 2$, and $p = 1$), if p is negative, the reverse will be true (for instance, $m = 2$, $n = 4$, and $p = -1$).

49. **(A).** Since this expression is complicated, deal with the denominator first. To add 5 to the fraction, make a common denominator (y - $2x$):

$$\frac{3y}{y-2x}+5=\frac{3y}{y-2x}+5\frac{y-2x}{y-2x}=\frac{3y}{y-2x}+\frac{5(y-2x)}{y-2x}$$
$$=\frac{3y}{y-2x}+\frac{5y-10x}{y-2x}=\frac{8y-10x}{y-2x}$$

Now put $\frac{8y-10x}{y-2x}$ back into the original expression. You can flip and multiply:

$$\frac{\frac{5x-4y}{2x-y}}{\frac{3y}{y-2x}+5}=\frac{\frac{5x-4y}{2x-y}}{\frac{8y-10x}{y-2x}}=\left(\frac{5x-4y}{2x-y}\right)\left(\frac{y-2x}{8y-10x}\right)$$

By looking at the answer choices, you can tell that this expression must be reducible to a number. How? Look at y - $2x$ and $2x$ - y. They are actually negatives of each other: $2x - y = -(y - 2x)$. So you can then cancel the whole expression y - $2x$ from top and bottom:

$$\left(\frac{5x-4y}{2x-y}\right)\left(\frac{y-2x}{8y-10x}\right)=\left(\frac{5x-4y}{-(y-2x)}\right)\left(\frac{y-2x}{8y-10x}\right)=\left(\frac{5x-4y}{-1}\right)\left(\frac{1}{8y-10x}\right)$$

You can do the same thing with the remaining terms: $8y - 10x = -2(5x - 4y)$.

$$\left(\frac{5x-4y}{-1}\right)\left(\frac{1}{8y-10x}\right)=\left(\frac{5x-4y}{-1}\right)\left(\frac{1}{-2(5x-4y)}\right)=\left(\frac{1}{-1}\right)\left(\frac{1}{-2}\right)=\frac{1}{2}$$

50. **(E).** To divide fractions, multiply by the reciprocal.

$$\frac{39^2}{2^4}\div\frac{13^3}{4^2}=\frac{39^2}{2^4}\times\frac{4^2}{13^3}$$

Now break down to primes and cancel common factors.

$$\frac{39^2}{2^4}\times\frac{4^2}{13^3}=\frac{(3\times13)^2\times\left(2^2\right)^2}{2^4\times13^3}=\frac{3^2\times13^2\times2^4}{2^4\times13^3}=\frac{3^2}{13}=\frac{9}{13}$$

51. **(E).** You can take the fourth root by taking the square root *twice*. So you should expect the fourth root of an integer greater than 1 to be much smaller than the number itself (for instance, the fourth root of 16 is 2, and the fourth root of 625 is 5).

One way to approach this problem is this: what integer n would give you *exactly* 3 as its fourth root?

$$\sqrt[4]{n}=3$$

Raise each side to the fourth power:

$$n = 3^4 = 81$$

Only the interval in choice (E) contains 81.

Alternatively, take the fourth root (by using the square root button twice) of the values in the answer choices. The fourth root of 40 is 2.5148… which rounds to 3. Likewise, if you take the fourth root of 150, you get 3.4996… which also rounds to 3.

Chapter 12
of

Percents

In This Chapter…

Percents

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

Quantity A	**Quantity B**
50 as a percent of 30	The percent increase from 30 to 80

2. If Ken's salary were 20% higher, it would be 20% less than Lorena's. If Lorena's salary is $60,000, what is Ken's salary?

(A) $36,000
(B) $40,000
(C) $42,500
(D) $42,850
(E) $45,000

3. On a certain morning, Stock X is worth $$x$, where x is positive.

Quantity A	**Quantity B**
The price of Stock X if it decreases in value	The price of Stock X if it decreases in value

by 12%, then increases by 18%.	by 13%, then increases by 19%.

4. Greta's salary is x thousand dollars per year, and she receives a y% raise. Annika's salary is y thousand dollars per year, and she receives an x% raise. x and y are positive integers.

Quantity A	**Quantity B**
The dollar amount of Greta's raise	The dollar amount of Annika's raise

5. Roselba's income exceeds twice Jane's income and both pay the same percentage of income in transportation fees.

Quantity A	**Quantity B**
The amount Jane pays in transportation fees	Half the amount Roselba pays in transportation fees

6. 250% of x is increased by 250% to become 350. What is the value of x?

7. An item's price is discounted by 16%. Subsequently, the discounted price is increased by 16%.

Quantity A	**Quantity B**
The original price	The price after the discount and increase

8. 12 is 5 percent of what number?

9. 7 percent of 9 is what percent of 7?

%

10. What percent of 13 is 20 percent of 195?

%

11. If 14 is added to 56, by what percent does the original number increase?

% increase

12. 25 percent of 30 is 75 percent of what number?

13. What is the percent increase from 50 to 60?

%

14. If x is reduced by 30%, the resulting number is 63. The value of x =

15. 75 reduced by x% is 54. The value of x =

%

16. What is 230% of 15% of 400?

17. 45% of 80 is x% more than 24. The value of x =

%

18. 10 percent of 30 percent of what number is 200 percent of 6?

19. If $y \neq 0$, what percent of y percent of 50 is 40 percent of y?

%

20. If $a \neq 0$, 200 percent of 4 percent of a is what percent of $a/2$?

$\boxed{}$ %

21. If positive integer *m* is increased by 20%, decreased by 25%, and then increased by 60%, the resulting number is what percent of *m*?

$\boxed{}$ %

22.

Quantity A	**Quantity B**
The price of an item after five consecutive 10% discounts are applied	50% of the price of the item

23. Raymond borrowed $450 at 0% interest. If he pays back 0.5% of the total amount every 7 days, beginning exactly 7 days after the loan was disbursed, and has thus far paid back $18, with the most recent payment made today, how many days ago did he borrow the money?

(A) 6
(B) 8
(C) 25
(D) 42
(E) 56

24. An investment loses half its value in the morning, and then increases in value by 50% that afternoon; no other changes occur to the value of the investment. (Assume the investment's original value was a positive number.)

Quantity A	**Quantity B**
The value of the investment before the day's changes	The value of the investment after the day's changes

25. A house valued at $200,000 two years ago lost 40% of its value in the first year and a further 20% of that reduced value during the second year.

Quantity A	**Quantity B**
The current value of the house	$100,000

26. After one year at her job, Sharon received a 50% increase on her $1,000 weekly salary. Bob, who originally made $1,800 a week, took a 20% percent decrease in salary.

Quantity A	**Quantity B**
Sharon's new salary	Bob's new salary

27. 1% of 200% of 360 is what percent of 0.1% of 60?

$\boxed{}$ %

28. If a number is increased by 20%, decreased by 15%, and increased by 7%, the overall percent change is closest to a:

(A) 2% decrease
(B) 2% increase
(C) 9% increase
(D) 12% increase
(E) 14% increase

29. If Mary has half as many cents as Nora has dollars, then Nora has what percent more cents than Mary does? (100 cents = 1 dollar)

(A) 100%
(B) 200%
(C) 1,990%
(D) 19,900%
(E) 20,000%

30. The number that is 50% greater than 60 is what percent less than the number that is 20% less than 150?

(A) 5%
(B) 10%
(C) 15%
(D) 20%
(E) 25%

31. A cockroach population doubles every three days. If there were *c* cockroaches on June 1st, what was the percent increase in the population on July 1st? (June has 30 days.)

(A) 900%
(B) 1,000%
(C) 9,999%
(D) 102,300%
(E) 102,400%

32. A computer that was discounted by 15% sold for $612. What was the price of the computer before the discount?

(A) $108.00
(B) $520.20
(C) $703.80
(D) $720.00
(E) $744.00

33.

In April, the price of fuel increased by 40%.
In May, the price rose by another 30%.

Quantity A	**Quantity B**

The price increase in April	The price increase in May

34. If 35% of the acreage of a national forest was destroyed in a wildfire, and the remainder regenerates at a rate of 10% a year, after how many years, assuming no further losses, will the forest exceed its original acreage?

(A) 10
(B) 8
(C) 5
(D) 4
(E) 3

35. Aloysius spends 50% of his income on rent, utilities, and insurance, and 20% on food. If he spends 30% of the remainder on video games and has no other expenditures, what percent of his income is left after all of the expenditures?

(A) 30%
(B) 21%
(C) 20%
(D) 9%
(E) 0%

36.

An item costs x dollars where x is a positive integer.

Quantity A	**Quantity B**
The price of the item after a 10% discount and then a 7% tax are applied	The price of the item after a 7% tax and then a 10% discount are applied

37.

An item costs x dollars where x is a positive integer.

Quantity A	**Quantity B**
The price of the item after a 10% discount and then a $10-off coupon are applied	The price of the item after a $10-off coupon and then a 10% discount are applied

38. Adalstein bought a bag of 15 magic beans for $60. One-third of the beans cost $2 each and the rest cost $5 each. If there was a hole in the bag and all of the more expensive beans fell out, the lost beans represented approximately what percentage of the money Adalstein paid for all of them?

(A) 7%
(B) 13%
(C) 67%
(D) 83%
(E) 88%

39. Coffee formerly accounted for 5% of a family's food budget, while fresh fruits and vegetables accounted for 20%, and meat and dairy accounted for 30%. The family spent the rest of the food budget on fast food and desserts. If the price of coffee doubled and the family reduced the fruit and vegetable share to meet that expense and spend the same overall, the ratio of their new fresh fruit and vegetable expenditures to their fast food and

dessert expenditures equals which of the following?

(A) 3/20
(B) 1/5
(C) 1/3
(D) 3/8
(E) 3/5

40. J. R. weighed 200 pounds before starting a diet on January 1st. He lost 15% of his original weight. Then he went off the diet and gained 35 pounds by December 31st of the same year. From the beginning to the end of the year, J.R.'s weight changed by what percent?

(A) +5%
(B) +2.5%
(C) 0%
(D) -2.5%
(E) -5%

41. In 1970, Company X had 2000 employees, 15% of whom were women. 10% of these women were executives. In 2012, the company had 12,000 employees, 45% of whom were women. If 40% of those women were executives, what is the percent increase in the number of women executives from 1970 to 2012?

[ ] %

42. 75% of all the boys and 48% of all the girls of Smith High School take civics. If there are 80% as many boys as girls, what percent of all the students take civics?

[ ] %

43. Airline A and Airline B both charge $400 for a certain flight. Airline A then reduces its price by 25%. Airline B reduces its price by 55% but adds $150 in fees. Then, Airline A raises its reduced price by 10%.

Quantity A	**Quantity B**
The final price of the flight at Airline A	The final price of the flight at Airline B

44. Jake used to spend $10 for lunch at a restaurant in Chinatown. The tea served with lunch was free, and Jake left a $2 tip. However, the restaurant raised its lunch price by 20% and began to charge $1 for tea. Jake continued to order the same lunch and tea, and increased his tip so that he was still tipping the same percentage of his total bill.

Quantity A	**Quantity B**
Jake's new lunch expenditure	$15.40

45. Half of a shipment of toy trucks was left at Store W, 25% at Store X, 20% at Store Y, and the remaining 20 at Store Z.

Quantity A	**Quantity B**

The positive difference difference between the trucks left at Stores X and Y 20

46.

p is 75% of q and $p = 2r$.

Quantity A	**Quantity B**
$0.375q$	r

47. In a class of 40 students, exactly 90% had a lower GPA than Tom. For the new term, 60 new students join Tom's class. If Tom's GPA was higher than those of 80% of the new arrivals, what percent of the combined class now has a higher GPA than Tom?

(A) 86%
(B) 85%
(C) 16%
(D) 15%
(E) 14%

48.

$0 < x < 100$

Quantity A	**Quantity B**
x% of 0.5% of 40,000	0.05% of 2,000% of $40x$

Profit Per Student (in Dollars) at Dan's Dojo, 2000-2004

2000	60
2001	80
2002	80
2003	100
2004	162

49. If the percent increase from 2004 to 2005 (not shown) is the same as the percent increase from 2000 to 2001, what is the profit per student for 2005?

50. If x is 0.5% of y, then y is what percent of x?

(A) 199%
(B) 200%
(C) 2,000%
(D) 19,900%

(E) 20,000%

51. Bill pays 20% tax on his gross salary of $5,000 each month and spends 25% of the remaining amount on rent.

Quantity A	Quantity B
Bill's tax	Bill's rent

52. Four people shared a dinner with an $80 bill and tipped the waiter 15 percent of this amount. If each person contributed equally to paying the bill and tip, how much did each person pay?

(A) $20.00
(B) $23.00
(C) $23.75
(D) $24.00
(E) $25.00

53. The price of a certain stock rose by 25 percent and then decreased by y percent. After the decrease, the stock was back to its original price.

Quantity A	Quantity B
y%	25%

54. A chemist is mixing a solution of acetone and water. She currently has 30 ounces mixed, 10 of which are acetone. How many ounces of acetone should she add to her current mixture to attain a 50/50 mixture of acetone and water if no additional water is added?

(A) 2.5
(B) 5
(C) 10
(D) 15
(E) 20

55. By the end of July, a certain baseball team had played 80% of the total games to be played that season and had won 50% of those games. Of the remaining games for the season, the team won 60%.

Quantity A	Quantity B
Percentage of total games won for the season	52%

56.

Quantity A	Quantity B
0.002	0.4 percent of 4 percent of 1.25

57. Jane has a 40-ounce mixture of apple juice and seltzer that is 30% apple juice. If she pours 10 more ounces of apple juice into the mixture, what percent of the mixture will be seltzer?

(A) 33%
(B) 44%
(C) 50%
(D) 56%

(E) 67%

58. Half of the shirts in a closet are white and 30% of the remaining shirts are gray.

Quantity A	Quantity B
The percent of the shirts in the closet that are not white or gray.	20%

59. If 80 percent of the children in a certain room are more than ten years old and 20 percent of these children play an organized sport, what percent of children in the room are over ten but do not play an organized sport?

(A) 16
(B) 20
(C) 40
(D) 60
(E) 64

60. The length and width of a rectangular box are increased by 10% each.

Quantity A	Quantity B
10%	The percent increase in the volume of the box

61. The radius of a circle is doubled.

Quantity A	Quantity B
The percentage that the area of the circle has been increased	400%

62. If 35% of x equals 140, what is 20% of x?

(A) 9.8
(B) 39.2
(C) 80
(D) 320
(E) 400

63. A population of a colony of bacteria increases by 20 percent every 3 minutes. If at 9:00am the colony had a population of 144,000, what was the population of the colony at 8:54am?

(A) 100,000
(B) 112,000
(C) 120,000
(D) 121,000
(E) 136,000

64. Jane scored 15% higher on her second test than she did on her first test. Jane's score on her third test was a 25% decrease from the score on her second test. If Jane got a 69 on her third test, what was the score on her first test?

(A) 69
(B) 70
(C) 75
(D) 80

(E) 92

65. The price of an item is greater than \$90 and less than \$150

Quantity A	Quantity B
The price of the item after a 10% discount and then a \$20 off discount	The price of the item after a \$10 off discount and then a 20% off discount

66. Two classes participate in a contest stacking blocks. Each person in class 1 stacks 80 percent as many blocks as each person in class 2 and there are 25 percent more people in class 1 than class 2.

Quantity A	Quantity B
The percent of the total blocks that class 1 stacks	The percent of the total blocks that class 2 stacks

67. x is y percent of z.

Quantity A	Quantity B
The percent that z is of x.	$\frac{y}{10,000}$

68. The number that is 20 percent less than 300 is what percent greater than 180?

(A) 25
(B) $33^1/_3$
(C) 50
(D) $66^2/_3$
(E) 75

69. A tank that was 40% full of oil is emptied into a 20-gallon bucket. If the oil fills 35% of the bucket's volume, then what is the total capacity of the tank, in gallons?

(A) 8.75
(B) 15
(C) 16
(D) 17.5
(E) 19

70. A full glass of juice is a mixture of 20% grape juice and 80% apple juice. The contents of the glass are poured into a pitcher that is 200 percent larger than the glass. The remainder of the pitcher is filled with 16 ounces of water. What was the original volume of grape juice in the mixture?

(A) 1.6 ounces
(B) 3.2 ounces
(C) 4.8 ounces
(D) 6.4 ounces
(E) 8 ounces

71. If 150 is increased by 60% and then decreased by y percent the result is 192. What is y?

(A) 20
(B) 28
(C) 32
(D) 72
(E) 80

72. A number x is 150% greater than 200. What percent greater is x than 50% of 500?

(A) 0
(B) 20
(C) 50
(D) 100
(E) 200

73. Mixture A weighs 18 grams and is 50% aluminum. Mixture B weighs 32 grams and is 37.5% aluminum. The two mixtures are combined.

Quantity A	Quantity B
The percent of the resultant combination that is not aluminum	58%

74. A stockbroker has made a profit on 80% of his 40 trades this year.

Quantity A	Quantity B
23	The maximum number of consecutive trades that the stockbroker can lose before his profitable trades drop below 50% for the year

75. In 2011, each member of a committee voted for one of two possible candidates for president. Candidate A received 40% of the vote and Candidate B received the remainder. In 2012, the same two candidates ran for president. Candidate A received 3 more votes than the previous year, a 5% increase in his total number of votes. How many votes did Candidate B receive in 2011?

(A) 40
(B) 60
(C) 75
(D) 90
(E) 150

76. A 16 ounce jar of birdseed is 10% sesame. How much sesame must be added to make the jar 20% sesame?

(A) 1 ounce
(B) 1.6 ounce
(C) 2 ounce
(D) 2.4 ounce
(E) 4 ounce

77. a, b, and c are positive.

Quantity A	Quantity B
$(a + b)$% of c	c% of $(a + b)$

78. A certain boat sales lot sells both sailboats and boats that are not sailboats. 25% of the boats are used sailboats. Of

non-sailboats, 3/5 are new. If 33% of all boats are used, approximately what percentage of the sailboats are new?

(A) 31%
(B) 33%
(C) 67%
(D) 68%
(E) 69%

Conference Ticket Advance Discounts	
5-29 days in advance	15%
30-59 days in advance	30%
60-89 days in advance	40%

79. Helen bought a ticket for $252. If she'd bought it 1 day later, she would have paid $306. How many days in advance did she buy her ticket?

(A) 5
(B) 30
(C) 59
(D) 60
(E) 89

Percents Answers

1. **(C).** 50 as a percent of 30 is $\frac{50}{30} \times 100 = 166.\overline{6}\%$. (Note: it's incorrect to calculate "50 percent of 30," which is 15. This asked for 50 *as a percent* of 30, which is equivalent to asking, "What percent of 30 is 50?")

To find the percent increase from 30 to 80, use the percent change formula:

Percent Change = $\frac{Difference}{Original} \times 100$

Percent Change = $\frac{80-30}{30} \times 100 = 166.\overline{6}\%$

The two quantities are the same. Note that if you set up both quantities first, you will be able to see that the two quantities are equal without solving (because both equal $\frac{50}{30} \times 100$).

2. **(B).** The question asks for Ken's salary, so set a variable: call Ken's salary k. The problem indicates that Lorena's salary is 60,000. Now, translate the equation in the first sentence.

"If Ken's salary were 20% higher" can be translated as Ken's salary + 20% of Ken's salary, or $k + 0.2k$. "It would be 20% less than Lorena's" can be translated as Lorena's salary - 20% of Lorena's salary, or 60,000 - (0.2)(60,000). The last part is equivalent to (0.8)(60,000); use whichever form you prefer.

$1.2k = 0.8(60{,}000)$
$1.2k = 48{,}000$
$k = 40{,}000$

Ken's salary is \$40,000.

3. **(A).** The problem never indicates an actual value for the stock, so pick your own value. Let the original price of the stock be \$100. For Quantity A, if the price of the stock decreases by 12%, then the new price is 88% of the original, or \$88. If that price then increases by 18%, then the final price of the stock is 88(1.18) = \$103.84.

For Quantity B, if the \$100 price decreases by 13%, the new price is 87% of \$100, or \$87. If that price then increases by 19%, then the final price of the stock is 87(1.19) = \$103.53.

Quantity A is slightly larger.

4. **(C).** Because the problem never indicates real values, you can pick your own smart numbers. If $x = 100$ and $y = 50$, then:

Greta's salary is \$100,000 and she will receive a 50% raise. Greta's raise, therefore, is \$50,000.

Annika's salary is \$50,000 and she will receive a 100% raise. Annika's raise, therefore, is \$50,000.

The two quantities are the same. This will work for any numbers you choose for x and y, because x percent of $y = y$ percent of x. This fact holds for any two numbers — just as 50% of 100 = 100% of 50, it is also true that 1% of 2,000 = 2,000% of 1, or a% of $b = b$% of a.

5. **(B).** Roselba's income is more than twice as great as Jane's income. If both pay the same percentage of income in transportation fees, that means Roselba must pay *more* than twice as much as Jane in transportation fees. Quantity B is greater.

Alternatively, you can use smart numbers. Call Jane's income 100. Roselba's income, then, is greater than 200. If both pay 10% in transportation fees, then Jane pays \$10 and Roselba pays more than \$20. Half of Roselba's amount equals more than \$10.

6. **40.** To begin, 250% of x is equal to $\frac{250}{100}x$, or $2.5x$

However, when you *increase* a number by 250%, you are NOT simply multiplying it by 2.5. Rather, you are finding 250% (or 2.5 times) that number and then *adding it back to the original.* (For example, 100% OF 15 equals 15, but INCREASING 15 by 100% means doubling it: 15 + 15 = 30.)

Thus, to increase a number by 250%, you are adding 100% to 250% for a total of 350%. Therefore, multiply by 3.5 (NOT 2.5).

$2.5x(3.5) = 350$
$8.75x = 350$
$x = 40$

40 is the final answer. If you are unsure of the logic behind these calculations, check your answer:

250% of 40 equals 2.5(40), or 100.

Next, 100 is INCREASED by 250%, so take 250% of 100, or 250, and add it back to the original 100: 100 + 250 = 350. The problem does indicate that the final answer is 350, so you have just proven that x does indeed equal 40.

7. **(A).** The problem doesn't indicate a specific value anywhere, so you can choose your own smart number. Because this is a percent problem, call the original price \$100. Quantity A is equal to \$100.

Decreasing a value by 16% is the same as taking 100 - 16 = 84% of the number: so (0.84)(100) = \$84. To increase the value by 16%, take 116% of the number, or multiply by 1.16: (1.16)(84) = \$97.44.

Quantity A is larger.

8. **240.** Translate the question as $12 = 0.05x$ and solve on the calculator. $x = 240$. Alternatively, translate the question

as $12 = \frac{5}{100}x$ and solve on paper:

$$12 = \frac{1}{20}x$$

$$(12)(20) = x$$

$$x = 240$$

9. **9**. Always translate the phrase "what percent" as $\frac{x}{100}$. Translate the question as:

$$0.07(9) = \frac{x}{100}(7)$$

$$0.63 = \frac{7x}{100}$$

$$63 = 7x$$

$$9 = x$$

Incidentally, the pattern "x percent of y = y percent of x" always holds true! Here, 7% of 9 = 9% of 7, but it is also true that 2% of 57 = 57% of 2, etc. This works with any two numbers. If you notice this, then you can "fill in the blank" on the answer immediately: "what percent" must be 9 percent.

Finally, notice that the answer is 9 and not 0.09 or 9%. The question asks "what percent," so the percent is already incorporated into the sentence — the "what" by itself represents only the number itself, 9.

10. **300.** Always translate the phrase "what percent" as $\frac{x}{100}$. Translate the question as:

$$\frac{x}{100}(13) = 0.2(195)$$

$$\frac{13x}{100} = 39$$

$$13x = 3{,}900$$

$$x = 300$$

Alternatively, take 20 percent of 195 ($0.2 \times 195 = 39$) and rephrase the question: "What percent of 13 is 39?" Since 39 is three times as big as 13, the answer is 300%.

11. **25%.** In this problem, the change is 14 and the original number is 56. Use the percent change formula:

Percent Change $= \dfrac{Difference}{Original} \times 100$

14/56 = 1/4 or 0.25. Multiply by 100 to get 25%.

Notice that you never need to find the new number (14 + 56 = 70), as you have only been asked about the percent increase, not the new value.

12. **10.** Translate the question as 0.25(30) = 0.75x and solve on the calculator. x = 10.

Alternatively, write the percentages in simplified fraction form and solve on paper:

$$\frac{1}{4}(30) = \frac{3}{4}x$$

$$30 = 3x$$

$$x = 10$$

13. **20% increase.** Use the percent change formula:

Percent Change $= \dfrac{Difference}{Original} \times 100$

The difference is 60 - 50 = 10 and the original is 50. 10/50 = 1/5 or 0.2. Multiply by 100 to get 20%.

14. **90.** Because 30% less than x is the same as 70% of x, you can translate as follows: 0.7x = 63. Use the calculator to get x = 90. Alternatively, solve on paper:

$$\frac{7}{10}x = 63$$

$$x = (63)\left(\frac{10}{7}\right)$$

$$x = (9)(10)$$

$$x = 90$$

15. **28.** Reword the given information as "75 minus x percent of 75 is 54."

$$75 - \frac{75}{100}x = 54$$

$$75 - 54 = \frac{3}{4}x$$

$$21\left(\frac{4}{3}\right) = x$$

$$28 = x$$

16. **138.** Translate into decimals (for the percentages, move the decimal two places to the left) and use the calculator to solve:

$x = 2.3(0.15)(400)$
$x = 138$

Alternatively, translate into fractions and solve on paper:

$$\frac{230}{100} \times \frac{15}{100} \times 400 =$$

$$\frac{23}{10} \times \frac{15}{1} \times 4 =$$

$$\frac{23}{2} \times \frac{3}{1} \times 4 =$$

$$\frac{23}{1} \times \frac{3}{1} \times 2 = 138$$

17. **50.** The left-hand side of the equation is given: 45% of 80 is (0.45)(80) = 36. The problem then becomes: "36 is x% more than 24." From this step, there are two possible approaches.

One approach is to translate the equation and solve:

$$36 = 24 + \frac{x}{100}(24) =$$

$$12 = \frac{24x}{100}$$

$$12\left(\frac{100}{24}\right) = x$$

$$50 = x$$

Alternatively, the increase (36 - 24) is 12, so rephrase the statement as "12 is x% of 24." If you recognize that 12 is half of 24, then you know x must be 50%. You can also translate and solve:

$$12 = \frac{x}{100}(24)$$

$$12\left(\frac{100}{24}\right) = x$$

$$50 = x$$

18. **400.** Translate as decimals and use the calculator to solve, keeping in mind that taking 200% of a number is the same as doubling it, or multiplying by 2:

$0.10(0.30)x = 2(6)$
$0.03x = 12$
$x = 400$

Alternatively, translate as fractions and solve on paper:

$$\left(\frac{1}{10}\right)\left(\frac{3}{10}\right)x = 2(6)$$

$$x = 12\left(\frac{100}{3}\right)$$

$$x = 400$$

19. **80.** The question already contains a variable (y). Use another variable to represent the desired value. Represent "what" with the variable x, and isolate x to solve. Notice that by the end, the y variables cancel out.

$$\left(\frac{x}{100}\right)\left(\frac{y}{100}\right)50 = \left(\frac{40}{100}\right)y$$

At this point, there are *many* options for simplifying, but do simplify before you start multiplying anything. Here is one way to simplify:

$$\left(\frac{x}{100}\right)\left(\frac{y}{2}\right)=\left(\frac{2}{5}\right)y$$

$$x=\frac{2y(100)(2)}{5y}$$

$$x=80$$

20. **16.** 200% of 4% is the same as 2 × 4% (note that 200% equals the plain number 2), or 8%. Rephrase the question as "8% of *a* is what percent of *a*/2?" Without translating to an equation, you can simplify by multiplying both sides of the "equation" by 2 (remember that "is" means "equals"):

8% of *a* is what percent of *a*/2?

16% of *a* is what percent of *a*?

The answer is 16.

Alternatively, translate the words into math; this will take longer, but this method does work if you're not comfortable "thinking through" the math as shown above:

$$\left(\frac{200}{100}\right)\left(\frac{4}{100}\right)a=\left(\frac{x}{100}\right)\left(\frac{a}{2}\right)$$

$$\left(\frac{2}{25}\right)a=\frac{xa}{200}$$

$$\left(\frac{2}{25}\right)a\left(\frac{200}{a}\right)=x$$

$$16=x$$

21. **144.** If *m* is increased by 20%, decreased by 25%, and then increased by 60%, it is being multiplied by 1.2, then 0.75, then 1.6. Since (1.2)(0.75)(1.6) = 1.44, doing these manipulations is the same as increasing by 44%, or taking 144% of a number (this is true regardless of the value of *m*).

Alternatively, pick a real value for *m*. Because this is a percent problem, 100 is a good number to pick. First, 100 is increased by 20%: (100)(1.2) = 120. Next, 120 is decreased by 25%, which is the same as multiplying by 75%: (120)(0.75) = 90. Finally, 90 is increased by 60%: (90)(1.6) = 144. The new number is 144 and the starting number was 100, so the new number is 144/100% of the original number, or 144%.

22. **(A).** Say the item costs \$100. After the first 10% discount, the item costs \$90. After the second, the item costs

$81 (the new discount is only $9, or 10% of 90). After the third discount, the item costs $81 - $8.10 = $72.90. What is the trend here? The cost goes down with each discount, yes, but the discount itself also gets smaller each time; it is only a $10 discount the very first time. The total of the five discounts, then, will be something less than $50.

If the item costs $100 to start, then the value for Quantity B will be $50, or a total discount of $50. This is larger than the total discount described for Quantity A.

Finally, make sure you answer (A) for the higher price — don't accidentally pick (B) for the "better deal"!

23. **(E).** 1% of $450 is $4.50, so 0.5% is $2.25. That's the amount Raymond pays back every week. Because he has paid back $18 in total, divide 18 by 2.25 to determine the total number of payments: $18/$2.25 = 8.

So Raymond has made 8 payments, once every 7 days. The payments themselves spread over only a 7-week period (in the same way that 2 payments spread over only a 1-week period). Raymond waited 1 week to begin repayment, however, so a total of 8 weeks, or 56 days, have passed since he borrowed the money. The answer is (E).

24. **(A).** The investment's value first decreases by 50%, then increases by 50%. If the investment begin at x dollars, $x(0.50)(1.5) = 0.75x$. The investment ends at 75% of its original value. The value after the changes is lower than the value before the changes.

Alternatively, choose a smart number to test. If $x = \$100$, then the investment first decreased to $50, and then increased from $50 to $75. If Quantity A = 100, then Quantity B = 75.

Finally, you could solve this question with logic. The 50% decrease is taken as a percentage of the original number. The 50% increase, however, is taken as a percentage of the new, *smaller* number. The increase, therefore, must be smaller than the decrease.

25. **(B).** To reduce $200,000 by 40%, multiply by 0.6 (reducing by 40% is the same as keeping 60%): 200,000(0.6) = 120,000.

To reduce 120,000 by 20%, multiply by 0.8 (reducing by 20% is the same as keeping 80%): 120,000(0.8) = 96,000. The answer is (B).

26. **(A).** Sharon's 50% increase raised her salary by half, to 1,000(1.5) = $1,500. A 20% decrease from $1,800 reduces Bob's salary to (1,800)(0.8) = $1,440.

27. **12,000%.** Translate the statement into an equation. Since one of the percentages is a variable, fractions are preferable to decimals:

$$\frac{1}{100} \times \frac{200}{100} \times 360 = \frac{x}{100} \times \frac{0.1}{100} \times 60$$

Because 100 appears twice on the bottom of both sides of the equation, multiply each side of the equation by 10,000 (or 100 twice) to cancel the 100's out:

$$\frac{1}{\cancel{100}} \times \frac{200}{\cancel{100}} \times 360 = \frac{x}{\cancel{100}} \times \frac{0.1}{\cancel{100}} \times 60$$

$$200 \times 360 = x(0.1)(60)$$

$$\frac{200 \times 360}{60} = x\left(\frac{1}{10}\right)$$

$$200 \times 6 \times 10 = x$$

$$x = 12{,}000$$

The answer is 12,000%. (The phrase "what percent" translates into math as $\frac{x}{100}$. Additionally, $\frac{12{,}000}{100}$, is the same thing as 12,000%, just as $\frac{50}{100}$ is equal to 50%. While 12,000% may seem quite large, it is correct.)

Alternatively, you can use decimals, though you still have to write "what percent" as a fraction; also, use the calculator to solve:

$$(0.01)(2)(360) = \frac{x}{100}(0.001)(60)$$

$$7.2 = \frac{x}{100}(0.06)$$

$$120 = \frac{x}{100}$$

$$12{,}000 = x$$

28. **(C).** Increasing by 20% is equivalent to multiplying by 1.2, decreasing by 15% is equivalent to multiplying by 1 - 0.15, or 0.85, and increasing by 7% is equivalent to multiplying by 1.07:

$$x(1.2)(0.85)(1.07) = x(1.0914)$$

Overall, these three changes are equivalent to multiplying by 1.0914, or increasing by 9.14%. Choice (C) is closest.

You *cannot* simply add 20 + 7 and subtract 15 to get a 12% increase. These percents cannot be added and subtracted because 20% is a percent of the original number, while 15% is a percent of the new, increased number, and so on.

Alternatively, you can also use a smart number. If the original number is 100, first increase 100 by 20%: 100 + 20 =120. Next, decrease 120 by 15%: 120 - 18 = 102 or 120(0.85) = 102. Finally, increase 102 by 7%. Note: the question indicates that you're trying to find the "closest" answer: a quick glance at the answers shows that they are relatively spread apart. It will be sufficient, then, to approximate the last step: 102 is almost 100 and 7% of 100 is 7.

The increase, then, is 102 + 7 = 109.

Compared to the original figure, 100, you have increased the number by about 9, or approximately 9%.

29. **(D).** You can use smart numbers to solve this problem. If Mary has half as many cents as Nora has dollars, then, as an example, if Nora had \$10, Mary would have 5 cents. Nora's \$10 equals 1,000 cents. To determine what *percent more* cents Nora has, use the percent change formula:

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\text{Percent Change} = \frac{1{,}000 - 5}{5} \times 100 = 19{,}900\%$$

Any example you use where "Mary has half as many cents as Nora has dollars" will yield the same result. Note that you must use the percent change formula — a percent *more* (or percent increase) is not the same as a percent *of* something.

To do the problem algebraically (which is *much* harder than using an example, as above), use M for Mary's cents and N for Nora's cents. Divide N by 100 in order to convert from cents to dollars: $\frac{N}{100}$ and set up an equation to reflect that Mary has half as many cents as Nora has dollars:

$$M = \frac{1}{2}\left(\frac{N}{100}\right)$$

$$M = \frac{N}{200}$$

$$200M = N$$

Therefore, Nora has 200 times as many cents. 200 times AS MANY is 199 times MORE. To convert 199 times MORE to a percent, add two zeros to get 19,900%.

30. **(E).** Rather than trying to write out the whole statement as math, note that "the number that is 50% greater than 60" can be calculated: 1.5(60) = 90. Similarly, "the number that is 20% less than 150" is 0.8(150) = 120. The question can be rephrased as "90 is what percent less than 120?" Use the percent change formula. Since you want a "percent *less,*" the "original" number is 120:

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100 = \frac{30}{120} \times 100 = 25\%$$

31. **(D).** The percent increase is the difference between the amounts divided by the original, converted to a percentage. If the population doubles, mathematically the increase can be written as a power of 2. In the 30-day interval, if the original population is 1, it will double to 2 after three days — so, 21 represents the population after the first increase,

the second increase would then be 2^2 and so on. Since there are ten increases, the final population would be 2^{10} or 1,024. Therefore, the difference, 1024 - 1, is 1023. Use the percent change formula to calculate percent increase:

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100 = \frac{1023}{1} \times 100 = 102{,}300\%$$

Note that the new number *is* 102,400% of the original, but that was not the question asked — the percent *increase* is 102,300%.

32. **(D).** Call the original price x. That price is discounted by 15% to get 612:

$0.85x = \$612$
$x = \$720$

Do not attempt to add 15% of \$612 to \$612. The 15% figure is a percentage of the unknown original number, not of \$612.

33. **(B).** Call the original price x. In April, the total price was $1.4x$. The price *increase* was $1.4x - 1x = 0.4x$.

In May, the price increased an additional 30% over April's price of $1.4x$. Thus, the total price was $(1.3)(1.4)x$, or $1.82x$. The price increase was $1.82x - 1.4x = 0.42x$.

Since $0.42x$ (42% of x) is larger than $0.4x$ (40% of x), Quantity B is larger.

Alternatively, use smart numbers. If the original price is \$100, April's increase would result in a price of \$140 and May's increase would be (1.3)(140) = \$182. Thus, April's increase was \$40 and May's increase was \$42. May's increase will be larger no matter what number you pick as the starting price (it is reasonable in GRE problems to assume that a "price" must be a positive number.)

34. **(C).** Picking numbers is a good strategy here. If the forest had 100 acres:

After the fire:	65 acres
After 1 year:	(65)(1.1) = 71.5 acres
After 2 years:	(71.5)(1.1) = 78.65 acres
After 3 years:	(78.65)(1.1) = 86.515 acres
After 4 years:	(86.515)(1.1) = 95.1665 acres
After 5 years:	(95.1665)(1.1) = 104.68315 acres

Note that each solution is multiplied by 1.1, so you can keep multiplying by 1.1 using the calculator — just be extra careful to keep track of how many times you multiply!

Alternatively, write an inequality in which a is the original acreage and y is the number of years:

$(0.65a)(1.1)^y > a$

Notice that the a values cancel out:

$$(0.65)(1.1)^y > 1$$

The GRE calculator is not equipped to solve this directly (you would need to use a logarithm, a topic not tested on the GRE), so instead plug in the answer choices for y, starting with the smallest value (3), until you find the smallest one that works. 5 is the smallest value that makes the inequality true.

35. **(B).** The 50% spent on rent, utilities, and insurance and the 20% spent on food are both percents of the total, so you can simply add the percents. 50% + 20% = 70%. After these expenditures, Aloysius has 30% left. He then spends 30% *of the remaining 30%* on video games. 30% of 30% = 0.30 × 0.30 = 0.09, or 9%. 30% - 9% = 21% remaining.

Alternatively, use smart numbers. If Aloysius's income is $100, he would spend $50 on rent, utilities, and insurance, and $20 on food, for a total of $70. Of his remaining $30, he would spend 30%, or $9, on video games, leaving $21, or 21% of the original amount.

36. **(C).** This question can be done in one of two ways. If you remember that all multiplications happen simultaneously (in terms of order of operations), you can simply look at this question and see that the order in which the discount and the tax are applied is irrelevant, so the answer must be (C). In other words, if x is the original price, $(0.9)(1.07)x = (1.07)(0.9)x$.

Alternatively, pick a real number and use the calculator. Because the problem deals with percentages, try 100. For Quantity A, first multiply by 0.9 to reflect the 10% discount: (100)(0.9) = $90. Next, multiply by 1.07 to apply the 7% tax: (90)(1.07) = $96.30

For Quantity B, first multiply by 1.07 to get $107 and then by 0.9 to get $96.30.

The answer is (C).

37. **(B).** You may be tempted to pick (C) here right away, but check the work; this problem mixes multiplication and subtraction. Pick a number to test the two quantities; because the problem deals with percentages, 100 is a good number to pick.

For Quantity A, a 10% discount would reduce the $100 price to $90, and $10 off $90 would reduce the price to $80.

For Quantity B, a $10-off coupon would reduce the price to $90, and then 10% off would reduce the price to $81, not $80! The discount is only $9 because you take 10% of 90, not 10% of 100.

Alternatively, you could derive both quantities algebraically:

Quantity A = $0.9x - 10$

Quantity B = $0.9(x - 10) = 0.9x - 9$

The answer is (B). Note that the order of the two discounts mattered here because one change was multiplication (10% off) and one was subtraction ($10 off). If both changes had been the same operation (e.g., both multiplication), the order would not have mattered.

38. **(D).** 1/3 of all the beans is 5 beans. These 5 beans each cost \$2, for a total of \$10. The remaining 10 beans cost \$5 each, for a total of \$50. If all of these more expensive beans are lost, then the lost beans represent $\frac{50}{60}$ of all the money paid. To convert to a percent: $\frac{50}{60} \times 100 = 83.\overline{3}\,\%$.

39. **(C).** Originally, the three figures given total 55%, so 45% was spent on fast food and desserts. If coffee doubled in price and the overall budget did not increase, coffee would then be 10% of the total, and that extra 5% would be taken from the fruit and vegetable expenditures. Thus, fruits and vegetables become 15% of the total while fast food and dessert stays at 45%. The ratio is 15/45, which reduces to 1/3.

40. **(B).** His loss was 15% of 200, or 30 pounds. If he then gained 35 pounds, he finished the year weighing 205.

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\text{Percent Change} = \frac{5}{200} \times 100 = 2.5\%$$

Be sure to select (B) for a 2.5% increase, and not (D) for a 2.5% decrease.

41. **7,100%.** In 1970, Company X had 0.15(2000) = 300 female employees. Of those, 0.10(300) = 30 were female executives.

In 2012, Company X had 0.45(12,000) = 5,400 female employees. Of those, 0.40(5,400) = 2,160 were female executives.

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\text{Percent Change} = \frac{2130}{30} \times 100 = 7{,}100\%$$

42. **60%.** Use smart numbers. There are 80% as many boys as girls, so choose 100 for the girls (100 is a good number to pick for percent problems). The boys, then, must be (100)(0.8) = 80. 75% of all the boys take civics, therefore there are 0.75(80) = 60 boys who take civics. 48% of all the girls take civics, therefore there are 0.48(100) = 48 girls who take civics.

60 + 48 = 108 students take civics and there are 180 total students:

$$\frac{108}{180} \times 100 = 60\%$$

43. **(C).** Airline A reduces its price by 25% to (400)(0.75) = \$300 but then raises that price by 10% to (300)(1.1) = \$330. Airline B reduces its fare to (400)(0.45) = \$180 but adds \$150 in fees, bringing the total price to 180 + 150 = \$330.

44. **(A).** The new lunch price is (10)(1.2) = \$12. With the tea charge, the new lunch bill is 12 + 1 = \$13. Since Jake leaves a 20% tip, the new tip is (0.20)(13) = \$2.60, so his new cost is 13 + 2.6 = \$15.60. This is larger than Quantity B.

45. **(C).** This problem can be solved algebraically, using t as the total. Note: because the equation requires addition, it is easier to use decimals than fractions. (Adding fractions requires finding a common denominator.)

$$0.5t + 0.25t + 0.2t + 20 = t$$
$$0.95t + 20 = t$$
$$20 = 0.05t$$
$$t = 400$$

Thus, the trucks left at Store X = 0.25(400) = 100 and the trucks left at Store Y = 0.20(400) = 80. The difference is 20.

Alternatively, Store W = 50%, Store X = 25%, and Store Y = 20%. Collectively, these three stores received 95% of the trucks, so Store Z receives 5% of the trucks. You know that Store Z receives 20 trucks, so 5% = 20 trucks. The difference between Store X (25%) and Store Y (20%) is also 5%, so that 5% difference is also 20.

46. **(C).** Write an equation from the first part of the given information: $p = 0.75q$. Since $p = 2r$, substitute $2r$ for p in the first equation:

$$2r = 0.75q$$
$$r = 0.375q$$

The two quantities are equal.

Alternatively, use smart numbers. If q is 8, then p is (8)(0.75) = 6. (Note: because you have to multiply q by 0.75, or 3/4, try to pick something divisible by 4 for q, so that p will be an integer.) Therefore, r is 6/2 = 3.

$0.375q = (0.375)(8) = 3$. The value for r is also 3, so the two quantities are equal.

47. **(D).** 90% of 40 students or 0.9(40) = 36 students had a lower GPA than Tom. Of the 60 new students, 80% or 0.80(60) = 48 had a lower GPA than Tom. Thus, 36 + 48 = 84 students in the new, larger class have GPAs lower than Tom.

The new class has 100 people, 84 of whom have lower GPAs than Tom. There are 16 people unaccounted for — don’t forget that Tom is one of them! Since Tom has the lowest GPA of this group of 16 people, there are 15 people above him. Since the class has exactly 100 people, 15/100 = 15%.

48. **(A).** When a percentage contains a variable, use fractions to translate. Quantity A is equal to:

$$\frac{x}{100}\times\frac{0.5}{100}\times\frac{40,000}{1}=x(0.5)(4)=2x$$

Quantity B is equal to:

$$\frac{0.05}{100}\times\frac{2,000}{100}\times\frac{40x}{1}=(0.05)(2)(4x)=0.4x$$

Since x is positive, you can be sure that Quantity A is larger (this is true even if x is a fraction).

Alternatively, use smart numbers. If $x = 50$, then Quantity A equals:

$$\frac{50}{100}\times\frac{0.5}{100}\times\frac{40,000}{1}=(0.5)(0.5)(400)=100$$

Quantity B equals:

$$\frac{0.05}{100}\times\frac{2,000}{100}\times\frac{(40)(50)}{1}=(0.05)(2)(4)(50)=20$$

Quantity A is larger.

49. **216.** The percent increase from 2000 to 2001 is:

$$\text{Percent Change}=\frac{\textit{Difference}}{\textit{Original}}\times 100$$

$$\text{Percent Change}=\frac{20}{60}\times 100=33.\overline{3}\%$$

Now, apply a $33.\overline{3}\%$, or 1/3, increase to 2004's figure. You can't type a repeating decimal into the calculator; instead, multiply 162 by 1/3 to get the amount of increase, and then add to 162 for the new profit per student in 2005: (162) (1/3) + 162 = 216.

50. **(E).** First, write "x is 0.5% of y" as math. Make sure you don't accidentally interpret 0.5% as 50%!

$$x=\frac{0.5}{100}\times y$$

The question asks "y is what percent of x?", so solve for y:

$100x = 0.5y$
$200x = y$

If y is 200 times x, multiply by 100 to convert to a percent:

$$\frac{200x}{1} \times \frac{100}{100} = \frac{20{,}000x}{100}$$

The answer is 20,000%. (For reference, if one number is 2 times as big as the other, it is 200% the size — add two zeros. So, 200 times as big = 20,000%.)

Alternatively, you could use smart numbers. If $y = 100$, then $x = \frac{0.5}{100}(100) = 0.5$. Next, answer the question "100 is what percent of 0.5?" Pick a new variable to translate the "what percent" portion of the sentence:

$$100 = \frac{n}{100} \times 0.5$$
$$10{,}000 = 0.5n$$
$$20{,}000 = n$$

(In translating percents problems to math, always translate "what percent" as a variable over 100.)

51. **(C).** Bill's tax is (0.20)5000 = \$1,000. Thus, his remaining salary is \$4,000. His rent is therefore (0.25)4000 = \$1,000.

52. **(B).** If four people shared the \$80 bill equally, then each person paid for one-quarter of the bill, or \$80/4 = \$20.

The tip is calculated as a percentage of the bill. Because the question asks about the amount that each (one) person paid, you can calculate the 15% tip based solely on one person's portion of the bill (\$20): (0.15)(20) = \$3.

In total, each person paid \$20 + \$3 = \$23.

Alternatively, you could find the total of the bill plus tip and take one-fourth of that for the total contribution of each person. The total of bill and tip is \$80 + (0.15)(80) = \$80 + \$12 = \$92. One-fourth of this is \$92/4 = \$23.

53. **(B).** Use a smart number for the price of the stock; for a percent problem, \$100 is a good choice. The price of the stock after a 25% increase is (1.25) × \$100 = \$125.

Next, find the percent decrease (y) needed to reduce the price back to the original \$100. \$125 - \$25 = \$100, so rephrase the question: 25 is what percent of 125?

$$25 = \frac{x}{100}(125)$$

$$\frac{2{,}500}{125} = x$$

$$x = 20$$

You have to reduce 125 by 20% in order to get back to \$100. Therefore, Quantity A = 20% and is less than Quantity B.

Alternatively, you can use algebra, although algebra is challenging for this problem. Assign the original cost of the stock a variable, such as z. In this case, the price of the stock after a 25% increase would be $1.25z$. The percent decrease, y, is found by multiplying $1.25z$ by $1-\frac{y}{100}$ and setting the quantity equal to the original price, z.

$$z=\left(1-\frac{y}{100}\right)(1.25z)$$

$$\frac{z}{1.25z}=1-\frac{y}{100}$$

$$\frac{1}{1.25}=1-\frac{y}{100}$$

$$0.8=1-\frac{y}{100}$$

$$-0.2=-\frac{y}{100}$$

Multiply all terms by 100 in order to get rid of the fraction:

$80 = 100 - y$
$y = 100 - 80 = 20$

54. **(C).** The chemist now has 10 ounces of acetone in a 30-ounce mixture, so she must have 20 ounces of water. You want to know the amount of acetone you must add in order to make this mixture a 50% solution. No additional water is added, so the solution must finish with 20 ounces of water. Therefore, she also needs a total of 20 ounces of acetone, or 10 more ounces than the mixture currently contains.

Alternatively, you can use algebra. If the chemist adds x ounces of acetone to the mixture, then there will be $10 + x$ ounces of acetone and the total mixture will have $30 + x$ ounces. The goal is to have a mixture that is 50% acetone:

$$50\%=\frac{10+x}{30+x}$$

$$\frac{50}{100}=\frac{10+x}{30+x}$$

$$\frac{1}{2}=\frac{10+x}{30+x}$$

Cross multiply:

$$30 + x = 20 + 2x$$
$$10 = x$$

The answer is (C).

Note that one trap answer is (B), or 5. This answer is not correct because the final number of ounces in the solution is *not* 30; when the chemist adds acetone, the amount of total solution also increases — adding 5 ounces acetone would result in a solution that is 15/(30 + 5) acetone, which is not equivalent to a 50% mixture.

55. **(C).** Choose a smart number for the total number of games; for a percent problem, 100 is a good number to pick. If the total number of games for the season is 100 and the team played 80% of them by July, then the team played (100)(0.8) = 80 games. The team won 50% of these games, or (80)(0.5) = 40 games.

Next, the team won 60% of its *remaining* games. As there were 100 total games and the team has played 80 of them, there are 20 games left to play. Of these, the team won 60%, or (20)(0.6) = 12 games.

Therefore, the team has won a total of 40 + 12 = 52 games out of 100, or 52% of its total games. Quantities A and B are equal.

Alternatively, this problem could be done using weighted averages, where the total percent of games won is equal to the sum of all of the individual percentages multiplied by their weightings. In this case,

$$\text{Total Percentage Won} = (50\%)(80\%) + (60\%)(100\% - 80\%) \times 100$$
$$\text{Total Percentage Won} = [(0.5)(0.8) + (0.6)(0.2)] \times 100$$
$$\text{Total Percentage Won} = [(0.4) + (0.12)] \times 100$$
$$\text{Total Percentage Won} = 0.52 \times 100$$
$$\text{Total Percentage Won} = 52\%$$

56. **(A).** In order to compare, use the calculator to find 0.4 percent of 4 percent of 1.25 (be careful with the decimals!):

$$0.004 \times 0.04 \times 1.25 = 0.0002$$

Or, as fractions:

$$\frac{0.4}{100} \times \frac{4}{100} \times 1.25 = \frac{2}{1{,}000} = 0.0002$$

Quantity A is larger than Quantity B.

57. **(D).** Originally, Jane has a 40-ounce mixture of apple and seltzer that is 30% apple. Since 0.30(40) = 12, 12 ounces were apple and 28 ounces were seltzer.

Jane pours 10 more ounces of apple juice into the mixture, yielding a mixture that is 50 ounces total, still with 28 ounces of seltzer. Now, the percentage of seltzer in the final mixture is $\frac{28}{50} \times 100 = 56\%.$

58. **(A).** Choose a smart number for the total number of shirts in the closest; this is a percent problem, so 100 is a good number to pick. Out of 100 shirts, half, or 50, are white.

30% of the *remaining* shirts are gray. If there are 50 white shirts, there are also 50 remaining shirts and so (0.3)(50) = 15 gray shirts. Therefore, there are 50 + 15 = 65 total shirts that are white or gray, and 100 - 65 = 35 shirts that are neither white nor gray. Since 35 out of 100 shirts are neither white nor gray, exactly 35% of the shirts are neither white nor gray.

Alternatively, you can use algebra, though that is trickier on a problem such as this one. Set a variable, such as x, for the total number of shirts. The number of white shirts is $0.5x$ and the remaining shirts would equal $x - 0.5x = 0.5x$. The number of gray shirts, then, is $(0.5x)(0.3) = 0.15x$. Thus there are $0.5x + 0.15x = 0.65x$ white or gray shirts, and $x - 0.6x = 0.35x$ shirts that are neither white nor gray. $0.35x \div x = 0.35$, or 35%.

59. **(E).** As there are no amounts given in the problem, you can choose a smart number for the total number of children. On percent problems, 100 is a good choice. The problem indicates that 80% of the children, or 80 children total, are more than 10 years old.

20 percent *of these 80 children* play an organized sport. The question asks about the percentage of these children who do NOT play an organized sport. If 20% do, then the remaining 80% of the 80 children do not. (80)(0.8) = 64 children who are over 10 years old and do not play an organized sport.

Alternatively, you can use algebra; set the total number of children in the room equal to x. The problem indicates that 80%, or $0.8x$, of the children are over 10 years old. Of the $0.8x$ children, 20% do play an organized sport, so 80% do not. $(0.8x)(0.8) = 0.64x$. Therefore, 64% of the children are over 10 but do not play an organized sport.

60. **(B).** You can choose smart numbers for the dimensions of the box — for instance, length = 20, width = 10, and height = 1. (For the length and the width, pick values that will still yield an integer when increased by 10%. Since the height doesn't change, pick an easy number such as 1 to keep the overall calculations easy.)

The original volume of the box = length × width × height = 20 × 10 × 1 = 200

After a 10% increase for both the length and the width, the volume becomes 22 × 11 × 1 = 242.

The formula for percent change is:

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\frac{242 - 200}{200} = \frac{42}{200} = \frac{21}{100} = 21\%$$

Quantity B is larger.

Alternatively, you could use algebra. Assign the variables l to the original length, w to the original width, and h to the original height of the box. The volume of the box would then be lwh. A 10% increase in the length and width changes the variables to $1.1l$ and $1.1w$ respectively. The new volume of the box would be $(1.1l)(1.1w)(h) = 1.21lwh$, which constitutes a 21% increase over lwh.

Finally, you could use logic. The formula for volume requires multiplying the length and the width. If just one side is increased by 10%, then the overall volume will increase by 10%. If two sides are increased by 10%, then the overall volume will increase by something larger than 10%.

61. **(B).** You can choose a smart number for the radius of the circle. In this case, because no restrictions are placed on the radius, choose radius = 1 for convenience. The area of a circle is πr^2 and so the area is equal to $\pi 1^2 = \pi$.

The radius of the circle then doubles from 1 to 2. The new area of the circle is $\pi(2)^2 = 4\pi$. Now calculate the percent increase in the area of the circle:

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\text{Percent Change} = \frac{4\pi - \pi}{\pi} \times 100 = \frac{3\pi}{\pi} \times 100 = 300\%$$

Quantity B is larger.

Alternatively, you can use algebra. Assign the variable r, giving an original circle area of πr^2. After the radius is doubled to $2r$, the new area becomes $\pi(2r)^2 = 4\pi r^2$. Again, use the formula for percent increase:

$$\text{Percent Change} = \frac{4\pi r^2 - \pi r^2}{\pi r^2} \times 100 = \frac{3\pi r^2}{\pi r^2} \times 100 = 300\%$$

62. **(C).** Translate the given information into math:

$$\frac{35}{100} x = 140$$

$$x = 140 \times \frac{100}{35}$$

$$x = 400$$

Next, find 20% of x, or $0.20(400) = 80$.

63. **(A).** Every 3 minutes, the population increases by 20% (which is the same as multiplying by 1.2). Beginning at 8:54am, this change would occur at 8:57am and again at 9:00am. Use the variable x to represent the original quantity. Note that the 20% increase occurs twice:

$x(1.2)(1.2) = 144{,}000$
$x = 100{,}000$

Note that you cannot simply reduce 144,000 by 20% twice, because 20% is not a percentage of 144,000 — it is a percentage of the unknown, original number.

Alternatively, you could begin from 144,000 and calculate "backwards":

From 8:57am to 9:00am: $y(1.2) = 144{,}000$, so $y = 144{,}000/1.2 = 120{,}000$.

From 8:54am to 8:57am: $z(1.2) = 120{,}000$, so $z = 120{,}000/1.2 = 100{,}000$.

64. **(D).** Call the first test score x. A 15% increase and then a 25% decrease yields 69. Thus:

$x(1.15)(0.75) = 69$
$x = 80$

Alternatively, begin from the final score, 69, and solve "backwards":

25% decrease from 2nd test to 3rd test: $y(0.75) = 69$, so $y = 92$. 2nd test was 92.

15% increase from 1st test to 2nd test: $z(1.15) = 92$, so $z = 80$. 1st test was 80.

65. **(D).** Reducing a number by a percentage involves multiplication; reducing a number by a fixed amount involves subtraction. The order of operations (PEMDAS) will make a difference.

One possible value for the item is \$100. In this case, the value of Quantity A = (100)(0.9) - 20 = \$70. The value of Quantity B = (100 - 10)(0.80) = \$72. Here, Quantity B is larger.

However, a larger starting value may change the result, because a 20% discount off a larger starting value can result in a much bigger decrease. For a \$140 item, the value of Quantity A = (140)(0.9) - 20 = \$106. The value of Quantity B = (140 - 10)(0.80) = \$104. Here, Quantity A is larger. The answer is (D).

66. **(C).** You can pick smart numbers since there are no amounts specified. Each person in class 1 stacks 80 percent as many boxes as each person in class 2. You can choose 80 and 100, but it's better to pick smaller numbers to make the later math easier.

Class 1 = 8 blocks per person
Class 2 = 10 blocks per person

There are 25% more people in class 1 than class 2. If there are 4 people in class 2, then there are (4)(1.25) = 5 people in class 1.

Blocks Stacked by Class 1 = 8 people × 5 blocks per person = 40 blocks

Blocks Stacked by Class 2 = 10 people × 4 blocks per person = 40 blocks

Since each class stacks 40 blocks, each class stacks 50% of the total blocks. The quantities are equal.

Alternatively, you could use algebra.

Class 1 = $0.8x$ blocks per person
Class 2 = x blocks per person
Class 1 = $1.25y$ people
Class 2 = y people

The total blocks stacked by class 1 = $(0.8x)(1.25y) = xy$. The total blocks stacked by class 2 = xy. Since each class stacks the same number of blocks, each class stacks 50% of the total blocks.

67. **(D).** Because the problem does not specify any real values for the variables, you can test your own numbers. You might be tempted to choose 100 for z, but then x and y will be the same; for example, 60 is 60 percent of 100. It's better to pick three different values for the three variables.

One possible case: 4 is 40% of 10. In this example, $x = 4$, $y = 40$, and $z = 10$. The value of Quantity A is $z/x = 10/4$ =2.5, or 250%. The value of Quantity B is 40/10,000, or 0.4%. In this example, Quantity A is larger.

Will that always be true? Or will a larger or smaller number change the result? In Quantity B, y is divided by a static number: 10,000 never changes. If y is a much larger number, then, perhaps Quantity B will become larger than Quantity A.

Try $y = 10{,}000$. Also change x and z to more manageable numbers. 400 is 10,000% of 4. (Note: 4 is 100% of 4. 40 is 1,000% of 4. 400 is 10,000% of 4.) In this example, $x = 400$, $y = 10{,}000$, and $z = 4$. The value of Quantity A is z/x = 4/400 = 1/100, or 1%. The value of Quantity B is 10,000/10,000 = 1. In this case, the two quantities are equal. Therefore, the answer is (D).

You can also use algebra, though the algebra is challenging for this problem. Translate the given equation, x is y percent of z:

$$x = \frac{y}{100} \times z$$

Translate Quantity A, the percent that z is of x; you can rephrase as "z is what percent of x?" Note that you have to introduce a new variable:

$$z = \frac{p}{100} \times x$$

Solve for p because Quantity A is asking for the unknown percent:

$$\frac{100z}{x} = p$$

Quantity B contains an expression that uses only the variable y, while Quantity A contains 3 variables. Use the given equation to try to write the left-hand side of Quantity A only in terms of y:

Given: $x = \frac{y}{100} \times z$

$$\frac{x}{z} = \frac{y}{100}$$

$$\frac{z}{x} = \frac{100}{y}$$

Substitute for the $\frac{z}{x}$ term in the equation for Quantity A:

$$(100)\left(\frac{100}{y}\right) = p$$

$$\frac{10,000}{y} = p$$

Quantity A equals $\frac{10,000}{y}$ and Quantity B equals $\frac{y}{10,000}$. Without knowing the value of *y*, however, it is impossible to determine that one quantity is larger. Specifically, if *y* is less than 10,000, Quantity A is larger, but if *y* is greater than 10,000, Quantity B is larger. The answer is (D).

68. **(B).** 20% less than 300 is the same as 80% of 300, or 0.80(300) = 240. The question is "240 is what percent greater than 180?"

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\text{Percent Change} = \frac{60}{180} \times 100 = 33.\bar{3}\%$$

69. **(D).** First find the volume of oil in the bucket. The oil fills 35% of the bucket's 20-gallon volume, or (20)(0.35) = 7 gallons of oil

These 7 gallons originally filled 40% of the tank. Call the volume of the tank *T*. *T*(0.4) = 7, so *T* = 17.5 gallons.

70. **(A).** "200% larger" means "*three* times as big as" the original; "200% as large as" would mean twice as big. If the pitcher is three times as big as the glass, then pouring the contents of the glass into the pitcher will make the pitcher 1/3 full. If adding another 16 ounces fills up the pitcher, the 16 ounces must be equal to the remaining 2/3 of the pitcher's capacity. 1/3 of the pitcher's capacity, then, is 16/2, or 8 ounces. The juice mixture totals 8 ounces. 20% of the juice is grape juice, so there are (8)(0.2) = 1.6 ounces of grape juice.

71. **(A).** First, find the value of 150 increased by 60%: (150)(1.6) = 240. 240 is then decreased by y percent to get 192. 240 - 192 = 48, so 240 is decreased by 48 to get 192. Rephrase the question: 48 represents what percentage of 240?

$$48 = \frac{x}{100}(240)$$

$$48\left(\frac{10}{24}\right) = x$$

$$x = 20$$

72. **(D).** "150% greater than 200" means 150% of 200, or 300, *added back to* 200. This is the not the same figure as 150% *of* 200. Thus, 150% greater than 200 is 200 + (200)(1.5) = 500.

50% of 500 = 250. Translate the question as "What percent greater is 500 than 250?" Since 500 is twice 250, it is 100% greater than 250.

Alternatively, use the percent change formula.

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\text{Percent Change} = \frac{500 - 250}{250} \times 100 = 100\%$$

73. **(C).** First, the question asks you to find the percent of the combination that is NOT aluminum. If Mixture A is 50% aluminum, then it is also 50% NOT aluminum. Mixture A weighs 18 grams, so the portion that is NOT aluminum is (18)(0.5) = 9 grams.

If Mixture B is 37.5% (or 3/8) aluminum, then it is 100% - 37.5% = 62.5% (or 5/8) NOT aluminum. Mixture B weighs 32 grams, so the portion that is NOT aluminum is (32)(5/8) = 20.

20 + 9 = 29 grams are NOT aluminum out of the 18 + 32 = 50 total grams. The percentage is 29/50 = 58/100 = 58%.

The two quantities are equal.

74. **(B).** The stockbroker has made a profit on 80% of his 40 trades this year, so (0.80)(40) = 32 of his trades so far have been profitable.

Quantity B asks for the maximum number of additional losses in a row he can have without dropping below 50%. If he stays at 32 profitable trades and 8 non-profitable trades, and all future trades are losses, then he can't go above 32 × 2 = 64 trades without dropping below a 50% success rate. At 64 trades exactly, he would have 32 profitable trades and 32 non-profitable trades, for a "success" percentage of 50% profitable trades. 64 - 40 = 24 trades, so he can have 24 losses in a row without dropping *below* 50%. Quantity B is larger.

Alternatively, you can use algebra. Set the number of additional losing trades (above 40) to x. Then, the number of winning trades will remain constant at 32, the total number of trades will increase to $40 + x$, and the total number of losing trades will be $8 + x$. Quantity B asks for the maximum number of additional losses in a row he can have without dropping below 50% of profitable trades, so set up an inequality. The percentage of profitable trades must be greater than or equal to 50:

$$\frac{\text{Number Profitable}}{\text{Total Number}} \times 100 \geq 50$$

$$\frac{32}{40+x} \times 100 \geq 50$$

$$\frac{32}{40+x} \geq \frac{1}{2}$$

Cross multiply (note: you know the variable represents a positive number, so you don't need to do anything to the inequality sign):

$64 \times 40 + x$

$24 \times x$

Therefore, the stockbroker can lose money on 24 trades in a row and still have 50% of trades be profitable, so Quantity B is 24. Quantity B is larger.

75. **(D).** Candidate A had a 5% increase in votes between 2011 and 2012; a percent increase is calculated based upon the "original" number, which in this case was the number of votes for Candidate A in 2011. This 5% increase was equivalent to 3 total votes. You can use algebra to solve; let x equal the number of votes for Candidate A in 2011.

$$\frac{5}{100} = \frac{3}{x}$$

$$\frac{1}{20} = \frac{3}{x}$$

$$x = 60$$

Alternatively, if 5% = 3, then 50% = 30 (multiply both sides by 10) and 100% = 60 (multiply both sides by 2).

Therefore, Candidate A received 60 votes in 2011. The problem also indicates that Candidate A received 40% of the total vote in 2011. You can solve for the total votes and subtract to find the number of votes for Candidate B: Let T equal the total number of votes in 2011.

$60 = (0.4)T$

$150 = T$

$T - A = 150 = 60 = 90$ votes for Candidate B in 2011.

Alternatively, you could set up a proportion to solve. The advantage of this method: you won't solve for the total number of votes, so you won't get distracted by trap wrong answer (E). 60 votes represent 40% of the total, and you want to solve for 60% of the total:

$$\frac{60 \text{ (A votes)}}{40 \text{ (\% of total votes)}} = \frac{x \text{ (B votes)}}{60 \text{ (\% of total votes)}}$$

$$\frac{3}{2} = \frac{x}{60}$$

$$180 = 2x$$

$$x = 90$$

76. **(C).** A 16-ounce jar that is 10% sesame has 1.6 ounces of sesame. From there, you might infer that all you need to do is add 1.6 ounces again, and the mixture will be 20% sesame. However, this is incorrect — adding 1.6 ounces of sesame will also add 1.6 ounces to the total amount of seed in the jar. 3.2 ounces sesame/17.6 ounces total = 18.18% which is NOT equal to 20%.

Instead, write an equation expressing the ratio of sesame to the total mixture, where x is the amount of sesame to add; this equals the desired 20% (or 1/5) figure:

$$\frac{1.6 + x}{16 + x} = \frac{1}{5}$$

Cross multiply:

$5(1.6 + x) = 16 + x$
$8 + 5x = 16 + x$
$4x = 8$
$x = 2$

77. **(C).** It is always the case that, for two positive quantities, M% of N = N% of M. In this case, $(a + b)$ makes the problem appear more complicated, but the principle still applies. Algebraically:

Quantity A	**Quantity B**
$\frac{(a+b)}{100} \times c$	$\frac{c}{100} \times (a+b)$

Both quantities can be simplified to equal $\frac{c(a+b)}{100}$.

78. **(E).** This problem is best solved with a double-set matrix. Since all figures are given in percents or fractions (no real numbers of boats), you can use any number you want for the total; 100 is the easiest choice. Since 25% of 100 = 25 boats are used sailboats and 33% of 100 = 33 boats are used, you can infer that 33 - 25 = 8 boats are used non-sailboats:

	sailboat	**non-sailboat**	**Total**
new			
used	25	8	33
Total			100

You are told that *of non-sailboats,* 3/5 (or 60%) are new. Since you don't know the number of non-sailboats:

	sailboat	**non-sailboat**	**Total**
new		$0.6x$	
used	25	8	33
Total		x	100

Since, in the non-sailboat column, new + used = total:

$0.6x + 8 = x$
$8 = 0.4x$
$20 = x$

This is enough to fill in the rest of the chart:

	sailboat	**non-sailboat**	**Total**
new	55	12	67
used	25	8	33
Total	80	20	100

You can now see that 55/80 of the sailboats are new. This is 68.75%. Rounded to the nearest percent, the answer is 69%.

79. **(B).** Helen bought a ticket for $252; if she had bought it 1 day later, she would have paid $54 more. There are three possibilities that represent the dividing lines between the given discount levels:

Possibility 1: She bought the ticket 60 days in advance for a 40% discount (if she'd bought it 1 day later, or 59 days in advance, she would have received a 30% discount instead).

Possibility 2: She bought the ticket 30 days in advance for a 30% discount (if she'd bought it 1 day later, or 29 days in advance, she would have received a 15% discount instead).

Possibility 3: She bought the ticket 5 days in advance for a 15% discount (if she'd bought it 1 day later, or 4 days in advance, she would not have received any kind of discount).

This question is harder than it looks, because you cannot just calculate a percent change between \$252 and \$306. The discounts are *percentages of the full-price ticket*, and you don't know that number. Call it x.

Do note that the only three possible answers are 5, 30, and 60 (answers (A), (B), and (D), respectively). 59 days ahead and 89 days ahead do not represent days for which the next day (58 and 88 days ahead, respectively) results in a change in the discount.

Possibility 1 (60 days in advance): \$252 would represent a 40% discount from the original price, so the original price would be $252 = 0.6x$, and x would be \$420.

If the full ticket price is \$420, then buying the ticket 1 day later would result in a 30% discount instead, or (420)(0.7) = \$294. The problem indicates, however, that Helen would have paid \$306, so Possibility 1 is not correct.

Possibility 2 (30 days in advance): \$252 would represent a 30% discount from the original price, so the original price would be $252 = 0.7x$ and x would be \$360.

If the full ticket price is \$360, then buying the ticket 1 day later would result in a 15% discount instead, or (360)(0.85) = \$306. This matches the figure given in the problem, so Possibility 2 is correct; you do not need to test Possibility 3. Helen bought the ticket 30 days in advance.

Chapter 13
of

Divisibility and Primes

In This Chapter…

Divisibility and Primes

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. For how many positive integer values of x is $\frac{65}{x}$ an integer?

$\boxed{}$

2. If x is a number such that $0 < x \leq 20$, for how many values of x is $\frac{20}{x}$ an integer?

(A) 4
(B) 6
(C) 8
(D) 10
(E) More than 10

3.

Quantity A | **Quantity B**

The number of even factors of 27 | The number of even factors of 81

4.

Quantity A | **Quantity B**

The number of distinct factors of 10 | The number of distinct prime factors of 210

5.

Quantity A | **Quantity B**

The least common multiple of 22 and 6 | The greatest common factor of 66 and 99

6. The number of students who attend a school could be divided among 10, 12, or 16 buses, such that each bus transports an equal number of students. What is the minimum number of students that could attend the school?

(A) 120
(B) 160
(C) 240
(D) 320
(E) 480

7.

Quantity A | **Quantity B**

The number of distinct prime factors of 27 | The number of distinct prime factors of 18

8.

Quantity A | **Quantity B**

The number of distinct prime factors of 31 | The number of distinct prime factors of 32

9. How many factors greater than 1 do 120, 210, and 270 have in common?

(A) 1
(B) 3
(C) 6
(D) 7
(E) 30

10. Company H distributed $4,000 and 180 pencils evenly among its employees, with each employee getting an equal integer number of dollars and an equal integer number of pencils. What is the greatest number of employees that could work for Company H?

(A) 9
(B) 10
(C) 20
(D) 40
(E) 180

11. n is divisible by 14 and 3. Which of the following statements must be true?

Indicate all such statements.

- ☐ 12 is a factor of n
- ☐ 21 is a factor of n
- ☐ n is a multiple of 42

12. Positive integers a and b each have exactly four factors. If a is a one-digit number and $b = a + 9$, what is the value of a?

13. Ramon wants to cut a rectangular board into identical square pieces. If the board is 18 inches by 30 inches, what is the least number of square pieces he can cut without wasting any of the board?

(A) 4
(B) 6
(C) 9
(D) 12
(E) 15

14. If n is the product of 2, 3, and a two-digit prime number, how many of its factors are greater than 6?

15.

m is a positive integer that has a factor of 8.

Quantity A	**Quantity B**
The remainder when m is divided by 6	The remainder when m is divided by 12

16. When the positive integer x is divided by 6, the remainder is 4. Each of the following could also be an integer EXCEPT

(A) $\frac{x}{2}$

(B) $\frac{x}{3}$

(C) $\frac{x}{7}$

(D) $\frac{x}{11}$

(E) $\frac{x}{17}$

17. If $x^y = 64$ and x and y are positive integers, which of the following could be the value of $x + y$?

Indicate all such values.

- ☐ 2
- ☐ 6
- ☐ 7
- ☐ 8
- ☐ 10
- ☐ 12

18. If k is a multiple of 24 but not a multiple of 16, which of the following cannot be an integer?

(A) $\frac{k}{8}$

(B) $\frac{k}{9}$

(C) $\frac{k}{32}$

(D) $\frac{k}{36}$

(E) $\frac{k}{81}$

19. If $a = 16b$ and b is a prime number greater than 2, how many positive distinct factors does a have?

20. If a and c are positive integers and $4a + 3 = b$ and $4c + 1 = d$, which of the following could be the value of $b + d$?

(A) 46
(B) 58
(C) 68
(D) 74
(E) 82

21. Each factor of 210 is inscribed on its own plastic ball, and all of the balls are placed in a jar. If a ball is randomly selected from the jar, what is the probability that the ball is inscribed with a multiple of 42?

(A) $\frac{1}{16}$

(B) $\frac{5}{42}$

(C) $\frac{1}{8}$

(D) $\frac{3}{16}$

(E) $\frac{1}{4}$

22. At the Canterbury Dog Fair, 1/4 of the poodles are also show dogs and 1/7 of the show dogs are poodles. What is the least possible number of dogs at the fair?

23. A "prime power" is an integer that has only one prime factor. For example, $5 = 5$, $25 = 5 \times 5$, and $27 = 3 \times 3 \times 3$ are all prime powers, while $6 = 2 \times 3$ and $12 = 2 \times 2 \times 3$ are not. Which of the following numbers is not a prime power?

(A) 49
(B) 81
(C) 100
(D) 121
(E) 243

24. If a and b are integers such that $a > b > 1$, which of the following cannot be a multiple of either a or b?

(A) $a - 1$

(B) $b + 1$
(C) $b - 1$
(D) $a + b$
(E) ab

25. 616 divided by 6 yields remainder p, and 525 divided by 11 yields remainder q. What is $p + q$?

26. If x is divisible by 18 and y is divisible by 12, which of the following statements must be true?

Indicate <u>all</u> such statements.

☐ $x + y$ is divisible by 6
☐ xy is divisible by 48
☐ x/y is divisible by 6

27. If p is divisible by 7 and q is divisible by 6, pq must have at least how many factors greater than 1?

(A) 1
(B) 3
(C) 6
(D) 7
(E) 8

28. If r is divisible by 10 and s is divisible by 9, rs must have at least how many factors?

(A) 2
(B) 4
(C) 12
(D) 14
(E) 16

29. If t is divisible by 12, what is the least possible integer value of a for which $\frac{t^2}{2^a}$ might not be an integer?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

30. If a, b, and c are multiples of 3 such that $a > b > c > 0$, which of the following values must be divisible by 3?

Indicate <u>all</u> such values.

☐ $a + b + c$
☐ $a - b + c$
☐ $abc/9$

31. New cars leave a car factory in a repeating pattern of red, blue, black, and gray cars. If the first car to exit the factory was red, what color is the 463rd car to exit the factory?

(A) red
(B) blue
(C) black
(D) gray
(E) It cannot be determined from the information given.

32. Jason deposits money at a bank on a Tuesday and returns to the bank 100 days later to withdraw the money. On what day of the week did Jason withdraw the money from the bank?

(A) Monday
(B) Tuesday
(C) Wednesday
(D) Thursday
(E) Friday

33. x and h are both positive integers. When x is divided by 7, the quotient is h with a remainder of 3. Which of the following could be the value of x?

(A) 7
(B) 21
(C) 50
(D) 52
(E) 57

34. a, b, c, and d are all positive integers. If $\frac{ab}{c+d} = 3.7$, which of the following statements must be true?

Indicate <u>all</u> such statements.

☐ ab is divisible by 5.
☐ $c + d$ is divisible by 5.
☐ If c is even, then d must be even.

35. When x is divided by 10, the quotient is y with a remainder of 4. If x and y are both positive integers, what is the remainder when x is divided by 5?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

36. What is the remainder when $13^{17} + 17^{13}$ is divided by 10?

☐

37. a, b, c, and d are positive integers. If $\frac{a}{b}$ has a remainder of 9 and $\frac{c}{d}$ has a remainder of 10, what is the minimum possible value for bd?

38. If n is an integer and n^3 is divisible by 24, what is the largest number that must be a factor of n?

(A) 1
(B) 2
(C) 6
(D) 8
(E) 12

39.

10! is divisible by 3^x5^y, where x and y are positive integers.

Quantity A	**Quantity B**
The greatest possible value for x	Twice the greatest possible value for y

40.

Quantity A	**Quantity B**
The number of distinct prime factors of 100,000	The number of distinct prime factors of 99,000

41. For which two of the following values is the product a multiple of 27?

Indicate two such values.

☐ 1
☐ 7
☐ 20
☐ 28
☐ 63
☐ 217
☐ 600
☐ 700

42. Which of the following values times 12 is not a multiple of 64?

Indicate all such values.

☐ 6^6
☐ 12^2
☐ 18^3
☐ 30^3
☐ 222

43. If $3^x(5^2)$ is divided by $3^5(5^3)$, the quotient terminates with one decimal digit. If $x > 0$, which of the following statements must be true?

(A) x is even
(B) x is odd
(C) $x < 5$
(D) $x \geq 5$
(E) $x = 5$

44. $\underline{abc}$ is a three-digit number in which a is the hundreds digit, b is the tens digit, and c is the units digit. Let &($\underline{abc}$)& = $(2^a)(3^b)(5^c)$. For example, &(203)& = $(2^2)(3^0)(5^3) = 500$. For how many three-digit numbers $\underline{abc}$ will the function &($\underline{abc}$)& yield a prime number?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 9

Divisibility and Primes Answers

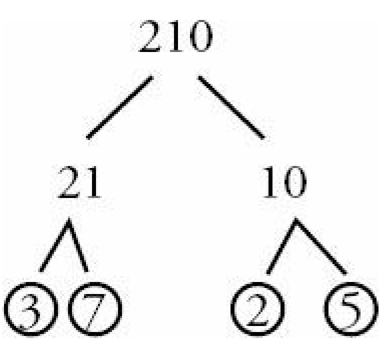
1. **4.** If x is a positive integer such that $\frac{65}{x}$ is also an integer, then x must be a factor of 65. The factors of 65 are 1, 5, 13, and 65. Thus, there are 4 positive integer values of x such that $\frac{65}{x}$ is an integer.

2. **(E).** Notice that the problem did NOT say that x had to be an integer. Therefore, the factors of 20 will work (1, 2, 4, 5, 10, 20), but so will 0.5, 0.1, 0.25, 2.5, etc. It is possible to divide 20 into fractional parts—for instance, something 20 inches long could be divided evenly into quarter inches (there would be 80 of them, as $\frac{20}{0.25} = 80$). There are an infinite number of x values that would work (it is possible to divide 20 into thousandths, millionths, etc.), so the answer is (E). It is very important on the GRE to notice whether there is an integer constraint on a variable or not! Any answer like "More than 10" should be a clue that this problem may be less straightforward than it seems.

3. **(C).** When counting factors, it helps to list them in pairs so you don't miss any. The factors of 27 are: 1 & 27, 3 & 9. The factors of 81 are: 1 & 81, 3 & 27, 9 & 9. Neither number has any even factors, so Quantity A and Quantity B are each 0 and therefore equal.

4. **(C).** The *factors* of 10 are 1 & 10, and 2 & 5. Since there are 4 factors, Quantity A is 4.

The *prime factors* of 210 are 2, 3, 5, and 7.



210 has 4 prime factors, so Quantity B is 4. Thus, the two quantities are equal.

5. **(A).** The least common multiple of 22 and 6 is 66. One way to find the least common multiple is to list the larger number's multiples (it is more efficient to begin with the larger number) until you reach a multiple that the other number goes into. The multiples of 22 are 22, 44, 66, 88, etc. The smallest of these that 6 goes into is 66.

The greatest common factor of 66 and 99 is 33. One way to find the greatest common factor is to list all the factors of one of the numbers, and then pick the greatest one that also goes into the other number. For instance, the factors of 66 are 1 & 66, 2 & 33, 3 & 22, and 6 & 11. The greatest of these that also goes into 99 is 33. Thus, Quantity A is greater.

6. **(C).** The number of students must be divisible by 10, 12, and 16. So the question is really asking, "What is the least common multiple of 10, 12, and 16?" Since all of the answer choices end in 0, each is divisible by 10. Just use your calculator to test which choices are also divisible by 12 and 16. Because you are looking for the minimum, start by checking the smallest choices. Since $\frac{120}{16}$ and $\frac{160}{12}$ are not integers, the smallest choice that works is 240.

7. **(B).** *Distinct* means *different from each other*. To find *distinct prime factors*, make a prime tree, and then disregard any repeated prime factors. The integer 27 breaks down into $3 \times 3 \times 3$. Thus, 27 has only 1 *distinct* prime factor. The integer 18 breaks down into $2 \times 3 \times 3$. Thus, 18 has 2 *distinct* prime factors.

8. **(C).** *Distinct* means *different from each other*. To find *distinct prime factors*, you would generally make a prime tree, and then disregard any repeated prime factors. However, 31 *is* prime, so 31 is the only prime factor of 31 and Quantity A is 1.

Any correct prime tree you make for 32 will result in five 2's, so 32 equals 2^5. Since this is the same prime factor repeated five times, 32 has only one *distinct* prime factor. Quantity B is 1, so the quantities are equal.

9. **(D).** Pick one of the numbers and list all of its factors on your paper. The factors of 120 are: 1 & 120, 2 & 60, 3 & 40, 4 & 30, 5 & 24, 6 & 20, 8 & 15, 10 & 12. Since the problem specifically asks for factors "greater than 1," eliminate 1 now. Now cross off any factors that do NOT go into 210:

~~120~~, 2 & ~~60~~, 3 & ~~40~~, ~~4~~ & 30, 5 & ~~24~~, 6 & ~~20~~, ~~8~~ & 15, 10 & ~~12~~

Now cross off any factors remaining that do NOT go into 270. Interestingly, all of the remaining factors (2, 3, 5, 6, 10, 15, 30) *do* go into 270. This is 7 shared factors.

10. **(C).** In order to distribute \$4,000 and 180 pencils evenly, the number of employees must be a factor of each of these two numbers. Because you are looking for the greatest number of employees possible, start by checking the greatest choices.

(E) \$4000 could not be evenly distributed among 180 employees (although 180 pencils could).
(D) \$4,000 could be evenly divided among 40 people, but 180 pencils could not.
(C) is the greatest choice that works—\$4,000 and 180 pencils could each be evenly distributed among 20 people.

Alternatively, find the greatest common factor (GCF) of the two numbers. Factor: $4{,}000 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^5 \times 5^3$ and $180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$. These numbers have $2 \times 2 \times 5$ in common, so 20 is the GCF. The correct answer is (C).

11. **II and III only.** Since n is divisible by 14 and 3, n contains the prime factors of both 14 and 3, which are 2, 7, and 3. Thus, any numbers that can be constructed using only these prime factors (no additional factors) are factors of n. Since $12 = 2 \times 2 \times 3$, you CANNOT make 12 by multiplying the prime factors of n (you would need one more 2). However, you CAN construct 21 by multiplying two of the known prime factors of n ($7 \times 3 = 21$), so the second statement is true. Finally, n must be at least 42 ($= 2 \times 7 \times 3$, the *least common multiple* of 14 and 3), so n is definitely a multiple of 42. That is, n can only be 42, 84, 126, etc…

12. **6.** Start by considering integer a, which is the most constrained variable. It is a positive one-digit number (between 1 and 9, inclusive) and it has four factors. Prime numbers have exactly two factors: themselves and one, so you only need to look at non-prime one-digit positive integers. That's a short enough list:

1 has just one factor!
4 has 3 factors: 1, 2, and 4
6 has 4 factors: 1, 2, 3, and 6
8 has 4 factors: 1, 2, 4, and 8
9 has 3 factors: 1, 3, and 9

So the two possibilities for a are 6 and 8. Now apply the two constraints for b. It is 9 greater than a, and it has exactly four factors. Check the possibilities:

If $a = 6$, then $b = 15$, which has 4 factors: 1, 3, 5, and 15.
If $a = 8$, then $b = 17$, which is prime, so it has only has 2 factors: 1 and 17.
Only $b = 15$ works, so a must be 6.

13. **(E).** Cutting a rectangular board into square pieces means that Ramon needs to cut pieces that are equal in length and width. "Without wasting any of the board" means that he needs to choose a side length that divides evenly into both 18 and 30. "The least number of square pieces" means that he needs to choose the largest possible squares. With these three stipulations, choose the largest integer that divides evenly into 18 and 30, or the greatest common factor, which is 6. This would give Ramon 3 pieces going one way and 5 pieces going the other. He would cut $3 \times 5 = 15$ squares of dimension 6" × 6". Note that this solution ignored squares with non-integer side length for the sake of convenience, a potentially dangerous thing to do. (After all, identical squares of 1.5" by 1.5" could be cut without wasting any of the board.) However, to cut fewer squares that are larger than 6" × 6", Ramon could only cut 2 squares of 9" or 1 square of 18" from the 18" dimension of the rectangle, neither of which would evenly divide the 30" dimension of the rectangle. The computed answer is correct.

14. **4.** Because this is a numeric entry question, you can infer that the answer will be the same regardless of which two-digit prime you pick. So for the sake of simplicity, pick the smallest and most familiar two-digit prime: 11.

If n is the product of 2, 3, and 11, n equals 66 and its factors are:

Small	Large
1	66
2	33
3	22
6	11

There are four factors greater than 6: 11, 22, 33, and 66.

Notice that because the other given prime factors of n (2 and 3) multiply to get exactly 6, you can only produce a factor greater than 6 by multiplying by the third factor, the "two-digit prime number." The right-hand column represents that third factor multiplied by all of the other factors: 11×6, 11×3, 11×2, and 11×1. If you replace 11 with any other two-digit prime, you will get the same result. (If you're not sure, try it!)

15. **(D).** Test values for m with the goal of proving (D). Because m has a factor of 8, m could equal 8, 16, 24, 32, 40, etc. If m is 24, both quantities are equal to 0. But if m is 32, Quantity A is 2 and Quantity B is 8.

16. **(B).** When dealing with remainder questions on the GRE, the best thing to do is test a few real numbers.

Multiples of 6 are 0, 6, 12, 18, 24, 30, 36, etc.

Numbers with a remainder of 4 when divided by 6 are those 4 greater than the multiples of 6:

x could be 4, 10, 16, 22, 28, 34, 40, etc.

You could keep listing numbers, but this is probably enough to establish a pattern.

(A) $x/2 \longrightarrow$ ALL of the listed x values are divisible by 2. Eliminate (A).
(B) $x/3 \longrightarrow$ NONE of the listed x values are divisible by 3, but continue checking.
(C) $x/7 \longrightarrow$ 28 is divisible by 7.
(D) $x/11 \longrightarrow$ 22 is divisible by 11.
(E) $x/17 \longrightarrow$ 34 is divisible by 17.

The question is "Each of the following could also be an integer EXCEPT." Since four of the choices could be integers, (B) must be the answer.

17. **III, IV, and IV only.** If $x^y = 64$ and x and y are positive integers, perhaps the most obvious possibility is that $x = 8$ and $y = 2$. However, "<u>all</u> such values" implies that other solutions are possible. One shortcut is noting that only an even base, when raised to a power, could equal 64. So you only have to worry about even possibilities for x. Here are all the possibilities:

$2^6 = 64 \longrightarrow x + y = 8$
$4^3 = 64 \longrightarrow x + y = 7$
$8^2 = 64 \longrightarrow x + y = 10$
$64^1 = 64 \longrightarrow x + y = 65$

The only possible values of $x + y$ listed among the choices are 7, 8, and 10.

18. **(C).** If k is a multiple of 24, it contains the prime factors of 24: 2, 2, 2, and 3. (It could also contain other prime factors, but you can only be sure of the prime factors contained in 24.)

If k were a multiple of 16, it would contain the prime factors of 16: 2, 2, 2, and 2.

Thus, if k is a multiple of 24 but NOT of 16, k must contain 2, 2, and 2, but NOT a fourth 2 (otherwise, it would be a multiple of 16).

Thus: k definitely has 2, 2, 2, and 3. It could have any other prime factors (including more 3's) EXCEPT for more 2's.

An answer choice in which the denominator contains more than three 2's would guarantee a non-integer result. Only choice (C) works. Since k has fewer 2's than 32, $k/32$ can never be an integer.

Alternatively, list multiples of 24 for k: 24, 48, 72, 96, 120, 144, 168, etc.

Then, eliminate multiples of 16 from this list: 24, ~~48~~, 72, ~~96~~, 120, ~~144~~, 168, etc.

A pattern emerges: k = (an odd integer) × 24.

(A) $k/8$ can be an integer, for example when $k = 24$.
(B) $k/9$ can be an integer, for example when $k = 72$.
(C) $k/32$ is correct by process of elimination.
(D) $k/36$ can be an integer, for example when $k = 72$.
(E) $k/81$ can be an integer, for example when $k = 81 \times 24$.

19. **10.** Because this is a numeric entry question, there can be only one correct answer. So, plugging in any prime number greater than 2 for b must yield the same result. Try $b = 3$.

If $a = 16b$ and $b = 3$, then a is 48. The factors (NOT prime factors) of 48 are: 1 & 48, 2 & 24, 3 & 16, 4 & 12, and 6 & 8. There are 10 distinct factors.

20. **(C).** The two equations are already solved for b and d, and the question is about the value of $b + d$. So, stack the equations and add:

$$\begin{array}{l} 4a + 3 = b \\ \underline{4c + 1 = d} \\ 4a + 4c + 4 = b + d \end{array}$$

Because a and c are integers, $4a + 4c + 4$ is the sum of three multiples of 4, which is a multiple of 4 itself. Therefore, the other side of the equation, $b + d$, must also equal a multiple of 4.

You could also factor out the 4:

$$\begin{array}{l} 4a + 4c + 4 \\ 4(a + c + 1) \end{array}$$

Since a and c are integers, $a + c + 1$ is an integer, so $4(a + c + 1)$ is definitely a multiple of 4, and $b + d$ is also a multiple of 4. Only choice (C) is a multiple of 4.

21. **(C).** The factors of 210 are as follows:

1 & 210
2 & 105
3 & 70
5 & 42
6 & 35
7 & 30

10 & 21
14 & 15

Out of the list of 16 factors, there are two multiples of 42 (42 and 210).

Thus, the answer is 2/16 or 1/8.

22. **10.** If 1/4 of the poodles are also show dogs, the number of poodles must be divisible by 4. (The number of dogs is necessarily an integer.) Since the least possible number is the goal, try an example with 4 poodles.

If 1/7 of the show dogs are poodles, the number of show dogs must be divisible by 7. Since the least possible number is the goal, try an example with 7 show dogs.

So far there are:

4 poodles, 1 of which is a show dog
7 show dogs, 1 of which is a poodle

Note that the one poodle that is also a show dog *is the same dog* as the one show dog that is also a poodle! To get the total number of dogs, only count that dog *once,* not twice. In total:

3 poodles (non-show dogs)
1 dog that is both poodle and show dog
6 show dogs (non-poodles)

This equals 10 dogs in total. This example met all the constraints of the question while using minimum values at each step, so this is the least possible number of dogs at the fair.

23. **(C).** Break down each of the numbers into its prime factors.

(A) $49 = 7 \times 7$
(B) $81 = 3 \times 3 \times 3 \times 3$
(C) $100 = 2 \times 2 \times 5 \times 5$
(D) $121 = 11 \times 11$
(E) $243 = 3 \times 3 \times 3 \times 3 \times 3$

Since 100 has both 2 and 5 as prime factors, it is not a prime power. The correct answer is (C).

24. **(C).** Since a positive multiple must be equal to or larger than the number it is a multiple of, answer choice (C) cannot be a multiple of a or b, as it is smaller than both integers a and b.

You can also try testing numbers such that a is larger than b.

(A) If $a = 3$ and $b = 2$, $a - 1 = 2$, which is a multiple of b.
(B) If $a = 3$ and $b = 2$, $b + 1 = 3$, which is a multiple of a.
(C) Is the correct answer by process of elimination.
(D) If $a = 4$ and $b = 2$, $a + b = 6$, which is a multiple of b.
(E) If $a = 3$ and $b = 2$, $ab = 6$, which is a multiple of both a and b.

25. **12.** Remember, remainders are always whole numbers, so dividing 616 by 6 in your calculator won't quite give you what you need. Rather, find the largest number less than 616 that 6 *does* go into (not 615, not 614, not 613 …). That number is 612. Since 616 - 612 = 4, the remainder p is equal to 4.

Alternatively, you could divide 616 by 6 in your calculator to get 102.66…. Since 6 goes into 616 precisely 102 whole times, multiply 6 × 102 to get 612, then subtract from 616 to get remainder 4.

This second method might be best for finding q. Divide 525 by 11 to get 47.7272…. Since 47 × 11 = 517, the remainder is 525 - 517 = 8.

Therefore, $p + q = 4 + 8 = 12$.

26. **I only.** To solve this problem with examples, make a short list of possibilities for each of x and y:

$x = 18, 36, 54\ldots$
$y = 12, 24, 36\ldots$

Now try to *disprove* the statements by trying several combinations of x and y above. In Statement I, $x + y$ could be 18 + 12 = 30, 54 + 12 = 66, 36 + 24 = 60, or many other combinations. Interestingly, all those combinations are multiples of 6. This makes sense, as x and y individually are multiples of 6, so their sum is too. Statement I is true.

To test statement II, xy could be 18(12) = 216, which is NOT divisible by 48. Eliminate statement II.

As for statement III, x/y could be 18/12, which is not even an integer (and therefore not divisible by 6), so III is not necessarily true.

27. **(D).** This problem is most easily solved with an example. If $p = 7$ and $q = 6$, then $pg = 42$, which has the factors 1 & 42, 2 & 21, 3 & 14, and 6 & 7. That's 8 factors, but read carefully! The question asks how many factors *greater than 1*, so the answer is 7. Note that choosing the smallest possible examples ($p = 7$ and $q = 6$) was the right move here, since the question asks "at least how many factors…?" If testing $p = 70$ and $q = 36$, many, many more factors would have resulted. The question asks for the minimum.

28. **(C).** This problem is most easily solved with an example. If $r = 10$ and $s = 9$, then $rs = 90$. The factors of 90 are 1 & 90, 2 & 45, 3 & 30, 5 & 18, 6 & 15, and 9 & 10. Count to get a minimum of 12 factors.

29. **(D).** If t is divisible by 12, then t^2 must be divisible by 144 or 2 × 2 × 2 × 2 × 3 × 3. Therefore, t^2 can be divided evenly by 2 at least four times, so a must be at least 5 before $\frac{t^2}{2^a}$ might not be an integer.

Alternatively, test values. If $t = 12$, $\frac{t^2}{2^a} = \frac{144}{2^a}$. Plug in the choices as possible a values, starting with the smallest choice and working up.

(A) Since $144/2^2 = 36$, eliminate.
(B) Since $144/2^3 = 18$, eliminate.

(C) Since $144/2^4 = 9$, eliminate.

(D) $144/2^5 = 4.5$. The first choice for which $\frac{t^2}{2^a}$ might not be an integer is (D).

30. **I, II, and III.** Since *a*, *b*, and *c* are all multiples of 3, $a = 3x$, $b = 3y$, $c = 3z$, where $x > y > z > 0$ and all are integers. Substitute these new expressions into the statements.

Statement I: $a + b + c = 3x + 3y + 3z = 3(x + y + z)$. Since $(x + y + z)$ is an integer, this number must be divisible by 3.

Statement II: $a - b + c = 3x - 3y + 3z = 3(x - y + z)$. Since $(x + y + z)$ is an integer, this number must be divisible by 3.

Statement III: $abc/9 = (3x3y3z)/9 = (27xyz)/9 = 3xyz$. Since xyz is an integer, this number must be divisible by 3.

31. **(C).** Pattern problems on the GRE often include a very large series of items that would be impossible (or at least unwise) to write out on paper. Instead, this problem requires you to recognize and exploit the pattern. In this case, after every 4th car, the color pattern repeats. By dividing 463 by 4, you find that there will be 115 cycles through the 4 colors of cars—red, blue, black, gray—for a total of 460 cars to exit the factory. The key to solving these problems is the remainder. Because there are $463 - 460 = 3$ cars remaining, the first such car will be red, the second will be blue, and the third will be black.

32. **(D).** This is a pattern problem. An efficient method is to recognize that the 7th day after the initial deposit would be Tuesday, as would the 14th day, the 21st day, etc. Divide 100 by 7 to get 14 full weeks comprising 98 days, plus 2 days left over. For the two leftover days, think about when they would fall. The first day after the deposit would be a Wednesday, as would the first day after waiting 98 days. The second day after the deposit would be a Thursday, and so would the 100th day.

33. **(D).** Division problems can be interpreted as follows: dividend = divisor × quotient + remainder. This problem is dividing *x* by 7, or distributing *x* items equally to 7 groups. After the items are distributed among the 7 groups, there are 3 things left over, the remainder. This means that the value of *x* must be some number that is 3 larger than a multiple of 7, such as 3, 10, 17, 24, etc. The only answer choice that is 3 larger than a multiple of 7 is 52.

34. **II and III only.** Start by rearranging the equation: $\frac{ab}{c+d} = \frac{37}{10}$ is equivalent to $10ab = 37(c + d)$. Remember that all four variables are positive integers. Because 37 and 10 have no shared factors, $(c + d)$ must be a multiple of 10 and ab must be a multiple of 37, in order to make the equation balance.

I. Could be true. All the requirements are met if $ab = (5)(37)$ and $(c + d) = 50$, so ab could be divisible by 5. But all the requirements are met if $ab = 37$ and $(c + d) = 10$, in which case ab is not divisible by 5.

II. MUST be true. Because $(c + d)$ must be divisible by 10, it must be divisible by both 5 and 2.

III. MUST be true. Because $(c + d)$ must be divisible by 10, it must be divisible by both 5 and 2. Thus, $(c + d)$ must be even, so if *c* were even, *d* would have to be even, too.

35. **(E).** This is a bit of a trick question—any number that yields remainder 4 when divided by 10 will also yield remainder 4 when divided by 5. This is because the remainder 4 is less than both divisors, and all multiples of 10 are

also multiples of 5. For example, 14 yields remainder 4 when divided either by 10 or by 5. This also works for 24, 34, 44, 54, etc.

36. **0.** The remainder when dividing an integer by 10 always equals the units digit. You can also ignore all but the units digits, so the question can be rephrased as: *What is the units digit of* $3^{17} + 7^{13}$*?*

The pattern for the units digits of 3 is [3, 9, 7, 1]. Every fourth term is the same. The 17th power is 1 past the end of the repeat: 17 - 16 = 1. Thus, 3^{17} must end in 3.

The pattern for the units digits of 7 is [7, 9, 3, 1]. Every fourth term is the same. The 13th power is 1 past the end of the repeat: 13 - 12 = 1. Thus, 7^{13} must end in 7. The sum of these units digits is 3 + 7 = 10. Thus, the units digit is 0.

37. **110.** When dividing, the remainder is always less than the divisor. If you divided a by b to get a remainder of 9, then b must have been greater than 9. Similarly, d must be greater than 10. Since b and d are integers, the smallest they could be is 10 and 11, respectively.

Thus, the minimum that bd could be is $10 \times 11 = 110$.

As an example, try $a = 19$, $b = 10$, $c = 21$, and $d = 11$ (generate a by adding remainder 9 to the value of b, and generate c by adding remainder 10 to the value of d.)

It is not possible to generate an example in which *any* of the four numbers are smaller. The least possible value of bd is 110.

38. **(C).** Start by considering the relationship between n and n^3. Because n is an integer, for every prime factor n has, n^3 must have three of them. Thus, n^3 must have prime numbers in multiples of 3. If n^3 has one prime factor of 3, it must actually have two more, because n^3's prime factors can only come in triples.

The question says that n^3 is divisible by 24, so n^3's prime factors must include at least three 2's and a 3. But since n^3 is a cube, it must contain at least three 3's. Therefore n must contain at least one 2 and one 3, or $2 \times 3 = 6$.

39. **(C).** First, expand 10! as $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$.

(Do NOT multiply all of those numbers together to get 3,628,800—it's true that 3,628,800 is the value of 10!, but analysis of the prime factors of 10! is easier in the current form.)

Note that 10! is divisible by 3^x5^y, and the quantities concern the greatest possible values of x and y, which is equivalent to asking, "What is the maximum number of times you can divide 3 and 5, respectively, out of 10! while still getting an integer answer?"

In the product $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$, only the multiples of 3 have 3 in their prime factors, and only the multiples of 5 have 5 in their prime factors. Here are all the primes contained in $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ and therefore in 10!:

$10 = 5 \times 2$

$9 = 3 \times 3$
$8 = 2 \times 2 \times 2$
$7 = 7$
$6 = 2 \times 3$
$5 = 5$
$4 = 2 \times 2$
$3 = 3$
$2 = 2$
$1 =$ no primes

There are four 3's and two 5's total. The maximum values are $x = 4$ and $y = 2$. Therefore, Quantity A and Quantity B are each 4, so the correct answer is (C).

40. **(B).** Since only the number of *distinct* prime factors matter, not what they are or how many times they are present, you can tell on sight that Quantity A has only 2 distinct prime factors, because 100,000 is a power of 10. (Any prime tree for 10, 100, or 1,000, etc. will contain only the prime factors 2 and 5, occurring in pairs.)

In Quantity B, 99,000 breaks down as $99 \times 1{,}000$. Since 1,000 also contains 2's and 5's, and 99 contains even more factors (specifically 3, 3, and 11), Quantity B has more distinct prime factors. It is not necessary to make prime trees for each number.

41. **V and VII only.** For two numbers to have a product that is a multiple of 27, the two numbers need to have at least three 3's among their combined prime factors, since 27 = 33. Only 63 and 600 are multiples of 3, so the other choices could be eliminated very quickly if you see that. There's no need to actually multiply the numbers together. Since 63 is $3 \times 3 \times 7$ and 600 is 3×200, their product will have the three 3's required for a multiple of 27.

42. **III, IV, and V only.** Because $64 = 2^6$, multiples of 64 would have at least six 2's among their prime factors.

Since 12 (which is $2 \times 2 \times 3$) has two 2's already, a number that could be multiplied by 12 to generate a multiple of 64 would need to have, at minimum, the *other* four 2's needed to generate a multiple of 64.

Since you want the choices that don't multiply with 12 to generate a multiple of 64, select only the choices that have *four or fewer 2's* within their prime factors.

6^6	$= (2 \times 3)^6$	six 2's	INCORRECT
12^2	$= (2^2 \times 3)^2$	four 2's	INCORRECT
18^3	$= (2 \times 3^2)^3$	three 2's	CORRECT
30^3	$= (2 \times 3 \times 5)^3$	three 2's	CORRECT
222	$= (2 \times 3 \times 37)$	one 2	CORRECT

43. **(D).** When a non-multiple of 3 is divided by 3, the quotient does not terminate (for instance, $1/3 = 0.333\ldots$).

Since $3^x(5^2)/3^5(5^3)$ does NOT repeat forever, x must be large enough to cancel out the 3^5 in the denominator. Thus, x must be at least 5. Note that the question asks what MUST be true. Choice (D) must be true. Choice (E), $x = 5$, represents one value that would work, but this choice does not *have* to be true.

44. **(B).** Since a prime number has only two factors, 1 and itself, $(2^a)(3^b)(5^c)$ cannot be prime unless the digits a, b and c are such that two of the digits are 0 and the third is 1. For instance, $(2^0)(3^1)(5^0) = (1)(3)(1) = 3$ is prime. Thus, the only three values of $\underline{abc}$ that would result in a prime number &($\underline{abc}$)& are 100, 010, and 001. However, only one of those three numbers (100) is a three digit number.

Chapter 14
of

Exponents and Roots

In This Chapter…

Exponents and Roots

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

Quantity A	**Quantity B**
25^7	5^{15}

2.

$216 = 2^x3^y$
x and y are integers.

Quantity A	**Quantity B**
x	y

3.

Quantity A	**Quantity B**

$\sqrt{18}\sqrt{2}$	$\sqrt{6}$

4.

Quantity A	**Quantity B**
$\sqrt{3}+\sqrt{6}$	$\sqrt{9}$

5.

Quantity A	**Quantity B**
$\sqrt{7{,}777{,}777{,}777}$	88,000

6. If $5{,}000 = 2^x5^y$ and x and y are integers, what is $x + y$?

7. If $3^2 9^2 = 3^x$, what is x?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

8.

80 is divisible by 2^x.

Quantity A	**Quantity B**
x	3

9.

Quantity A	**Quantity B**
$(81)^2(900)^3$	270^6

10. If $17\sqrt[3]{m} = 34$, what is $6\sqrt[3]{m}$?

11. $\dfrac{1}{\dfrac{1}{\dfrac{1}{5^{-2}}}}$ is equivalent to:

(A) 1/25
(B) 1/5
(C) 1
(D) 5
(E) 25

12. If $77{,}742y^{11} = 4x^2$, what is $\dfrac{77{,}742y^{11}}{8x^2}$?

$$\frac{\square}{\square}$$

13. $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{4}}}} =$

(A) $\sqrt{2}$
(B) 2
(C) $2\sqrt{2}$
(D) 4
(E) $4\sqrt{2}$

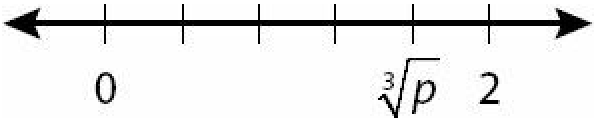
14.

Quantity A	**Quantity B**
$\dfrac{200}{\sqrt{200}}$	$\sqrt{200}$

15. For what positive integer is the square of the integer divided by the cube root of the integer equal to nine times the integer?

(A) 4
(B) 8
(C) 16
(D) 27
(E) 125

16.



If the hash marks above are equally spaced, what is the value of *p*?

(A) 3/2
(B) 8/5
(C) 24/15
(D) 512/125
(E) 625/256

17. What is the greatest prime factor of $2^{99} - 2^{96}$?

18. If $2^k - 2^{k+1} + 2^{k-1} = 2^k\,m$ what is *m*?

(A) -1
(B) -1/2
(C) 1/2
(D) 1
(E) 2

19.

Quantity A	**Quantity B**
$\frac{2}{9}(81)^{50}$	$\frac{(3^2)(9)^{99}}{2}$

20. If $5^{k+1} = 2{,}000$, what is $5^k + 1$?

(A) 399
(B) 401
(C) 1,996
(D) 2,000
(E) 2,001

21. If $3^{11} = 9^x$, what is the value of *x*?

22. If $x^7 = 2.5$, what is x^{14}?

23. If $\sqrt[5]{x^6} = x^{\frac{a}{b}}$, then the value of a/b =

24. $$\frac{20^{-5}5^{10}8^6}{10^8 25^{-2}} = ?$$

(A) 1
(B) 4
(C) 5
(D) 6
(E) 10

25. If $\frac{5^7}{5^{-4}} = 5^a$ and $\frac{2^{-3}}{2^{-2}} = 2^b$ and $3^8(3) = 3^c$, what is the value of $a + b + c$?

26. If 12^x is odd and x is an integer, what is the value of x^{12}?

27. $$\frac{200^{\frac{5}{2}}}{\sqrt{200}} = ?$$

(A) 4
(B) 40
(C) 400
(D) 4,000
(E) 40,000

28.

$$\frac{(10^3)(0.027)}{(900)(10^{-2})} = (3)(10^m)$$

Quantity A	Quantity B
The value of m	3

29. $\frac{1}{3}(10^6 - 10^4) = ?$

(A) $33.\overline{3}$
(B) $3,333.\overline{3}$
(C) 33,000
(D) 330,000
(E) 333,333

30. Simplify: $\frac{2^2 + 2^2 + 2^3 + 2^4}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$

(A) 2
(B) 4
(C) 8
(D) 16
(E) 32

31. $\frac{2^{-4}3^{-20}}{4^{-1}9^{-6}} =$

(A) $2^2 3^8$
(B) $2^1 3^{12}$
(C) $\frac{1}{2^2 3^8}$
(D) $\frac{1}{2^1 3^{12}}$
(E) $\frac{1}{2^2 3^{12}}$

32. If $\frac{0.000027 \times 10^x}{900 \times 10^{-4}} = 0.03 \times 10^{11}$, what is the value of x?

(A) 13
(B) 14
(C) 15
(D) 16
(E) 17

33. $(\sqrt[2]{x})(\sqrt[3]{x}) =$

(A) $\sqrt[5]{x}$

(B) $\sqrt[6]{x}$

(C) $\sqrt[3]{x^2}$

(D) $\sqrt[5]{x^6}$

(E) $\sqrt[6]{x^5}$

34. $\left(\sqrt[3]{x^2}\right)\left(\sqrt[4]{x^5}\right) =$

(A) $\sqrt[7]{x^{10}}$

(B) $\sqrt[12]{x^{10}}$

(C) $\sqrt[12]{x^7}$

(D) $\sqrt[9]{x^{23}}$

(E) $\sqrt[12]{x^{23}}$

35.

$n = 0.00025 \times 10^4$ and $m = 0.005 \times 10^2$

Quantity A	**Quantity B**
$\frac{n}{m}$	0.5

36. $\frac{40^{50} - 40^{48}}{2^{96}} \times 10^{-45} =$

(A) 20
(B) $10^3(1{,}599)$
(C) $10^2(1{,}601)$
(D) 200^6
(E) 200^{53}

37. Which of the following is equal to $x^{\frac{3}{2}}$?

(A) $x^2\sqrt{x}$

(B) $x\sqrt{x}$

(C) $\sqrt[3]{x^2}$

(D) $\sqrt[3]{x}$

(E) $(x^3)^2$

38. $\sqrt{(360)(240)(3)(2)} =$

(A) 180
(B) 360
(C) 720
(D) 1,440
(E) 3,600

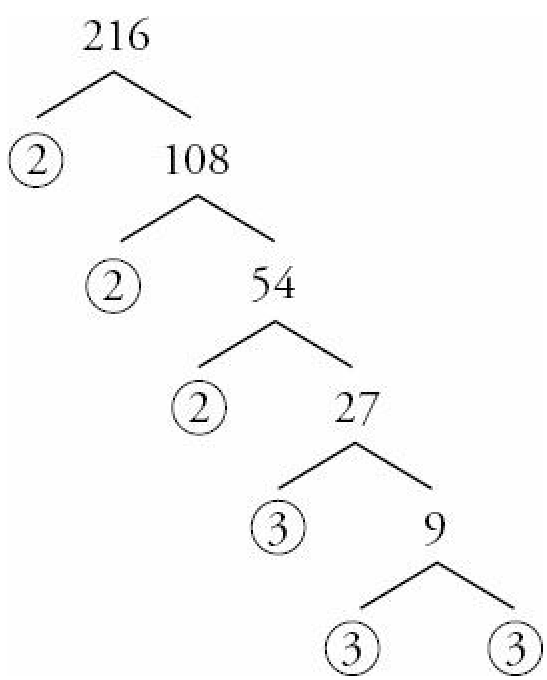
39. If $125^{14}48^{8}$ is written out as an integer, how many consecutive zeroes will that integer have at the end?

(A) 22
(B) 32
(C) 42
(D) 50
(E) 112

Exponents and Roots Answers

1. **(B).** In problems asking you to compare or combine exponents with different bases, convert to the same base when possible. Since $25 = 5^2$, Quantity A is equal to $(5^2)^7$. When raising a power to a power, multiply the exponents. Quantity A is equal to 5^{14}, so Quantity B is larger.

2. **(C).** Make a prime tree for 216:



$216 = 2^3 3^3$, so $x = 3$ and $y = 3$.

3. **(A).** In Quantity A, $\sqrt{18}\sqrt{2} = \sqrt{36} = 6$. Since 6 is greater than $\sqrt{6}$, Quantity A is larger.

4. **(A).** You may NOT add $\sqrt{3}$ and $\sqrt{6}$ to get $\sqrt{9}$, but you can simply put each value in your calculator. $\sqrt{3}$ = 1.732… and $\sqrt{6}$ = 2.449…, and their sum is about 4.18. Since Quantity B is $\sqrt{9}$ = 3, Quantity A is larger.

5. **(A).** You have a calculator with a square root button, but 7,777,777,777 is too large for the calculator.

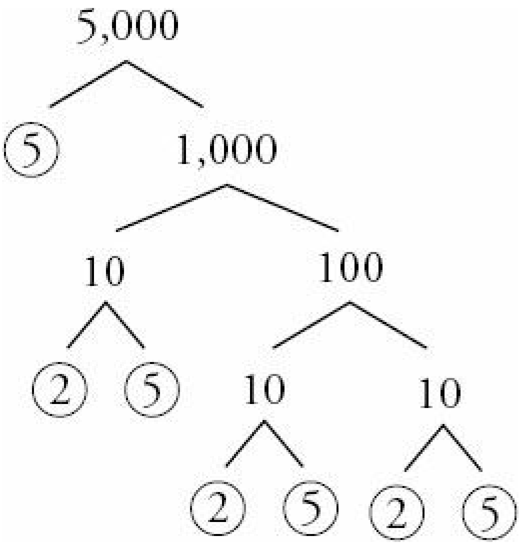
You could also square both quantities, getting 7,777,777,777 in Quantity A and $88{,}000^2$ in Quantity B. However, $88{,}000^2$ is also too large for the calculator. Here’s a quick workaround. Square 88 in your calculator (there is no “squared” button on the GRE calculator — you have to type 88×88):

$88^2 = 7{,}744$

Therefore $88{,}000^2 = 7{,}744 \times 10^6 = 7{,}744{,}000{,}000$

Thus, Quantity A is larger.

6. **7.** Make a prime tree for 5,000:



You can see that $5{,}000 = 2^3 5^4$, therefore $x = 3$ and $y = 4$, and the answer is $3 + 4 = 7$.

7. **(E).** In problems asking you to compare or combine exponents with different bases, convert to the same base when possible. Since $9 = 3^2$:

$$3^2(3^2)^2 = 3^x$$

Multiply exponents when raising a power to a power:

$$3^2 3^4 = 3^x$$

Add exponents when multiplying with the same base:

$$3^6 = 3^x$$

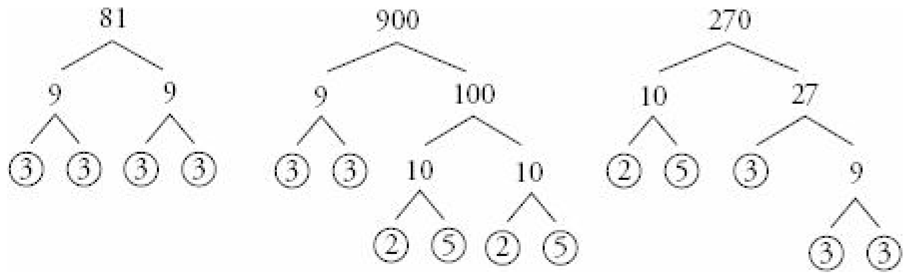
Therefore, $x = 6$.

8. **(D).** Make a prime tree for 80, or simply divide 80 by 2 in your calculator as many times as you can before you get a non-integer answer: 2 goes into 80 exactly 4 times.

That doesn't mean x *is 4, however!* The problem did NOT say "80 is *equal to* 2^x". Rather, it said "divisible by."

80 is divisible by 2^4, and therefore also by 2^3, 2^2, 2^1, and even 2^0 (anything to the 0th power = 1). Thus, x could be 0, 1, 2, 3, or 4, and could therefore be smaller than, equal to, or greater than 3.

9. **(B).** Break down 81, 900, and 270 into their prime factors:



$81 = 3^4$, $900 = 2^2 3^2 5^2$, and $270 = 2^1 3^3 5^1$. Therefore:

Quantity A = $(3^4)^2(2^2 3^2 5^2)^3 = (3^8)(2^6 3^6 5^6) = 2^6 3^{14} 5^6$

Quantity B = $(2^1 3^3 5^1)^6 = 2^6 3^{18} 5^6$

Since Quantity A and Quantity B both have 2^6 and 5^6, focus on 3^{14} vs. 3^{18}. Quantity B is larger.

10. **12.** This question looks much more complicated than it really is — note that you are not asked for *m* itself, but rather for $\sqrt[3]{m}$. Just think of $\sqrt[3]{m}$ as a very fancy variable that you don't have to break down:

$$17\sqrt[3]{m} = 34$$

$$\sqrt[3]{m} = \frac{34}{17}$$

$$\sqrt[3]{m} = 2$$

Therefore, $6\sqrt[3]{m}$.

11. **(E).** One quick trick to simplifying efficiently here is knowing that a negative exponent in the denominator turns into a positive exponent in the numerator. In other words, the lowermost portion of the fraction, $\frac{1}{5^{-2}}$, is simply equal to 5^2.

$$\frac{1}{\frac{1}{\frac{1}{5^{-2}}}} = \frac{1}{\frac{1}{5^2}}$$

Dividing by $\frac{1}{5^{-2}}$ is the same as multiplying by the reciprocal, which will leave 5^2 in the numerator:

$$\frac{1}{\frac{1}{5^2}} = 1 \times \frac{5^2}{1} = 5^2$$

The answer is simply 5^2, or 25.

12. **1/2.** This question looks much more complicated than it really is! Since $77{,}742y^{11} = 4x^2$, simply substitute $4x^2$ for $7{,}742y^{11}$ in the numerator:

$$\frac{4x^2}{8x^2} = \frac{1}{2}$$

13. **(B).** To begin solving, start at the "inner core" — that is, the physically smallest root sign:

$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{4}}}} =$$
$$\sqrt{2+\sqrt{2+\sqrt{2+2}}} =$$
$$\sqrt{2+\sqrt{2+2}} =$$
$$\sqrt{2+2} = 2$$

14. **(C).** Quantity A has a root sign on the bottom of a fraction. When you see this, rationalize the denominator by multiplying the fraction by the denominator over the denominator (so that the number you are multiplying by is equal to 1, so you are not changing the value of the fraction):

$$\frac{200}{\sqrt{200}}\left(\frac{\sqrt{200}}{\sqrt{200}}\right) = \frac{200\sqrt{200}}{200} = \sqrt{200}$$

(Please note that $\sqrt{200} \times \sqrt{200}$ is simply 200. There is no need to multiply out to get $\sqrt{40{,}000} = \sqrt{200} \times \sqrt{200} = 200$. Instead, simply think of the root signs canceling out.) The two quantities are the same.

15. **(D).** To solve this question, you need to write the information from the question as an equation. Call "the square of the integer" x^2, "the cube root of the integer" $\sqrt[3]{x}$, and "nine times the integer" $9x$:

$$\frac{x^2}{\sqrt[3]{x}} = 9x$$

There are a few ways to proceed from here, but it might be most helpful to convert $\sqrt[3]{x}$ into its other form, $x^{\frac{1}{3}}$, and then subtract the exponents on the left side of the equation (always subtract exponents, of course, when dividing with the same base):

$$\frac{x^2}{x^{\frac{1}{3}}} = 9x$$

$$x^{2-\frac{1}{3}} = 9x$$

$$x^{\frac{5}{3}} = 9x$$

A good next move would be to raise both sides to the 3rd power:

$$\left(x^{\frac{5}{3}}\right)^3 = (9x)^3$$
$$x^5 = 9^3x^3$$

Now simply divide both sides by x^3:

$$x^2 = 9^3$$

Since 9 is really just 3^2:

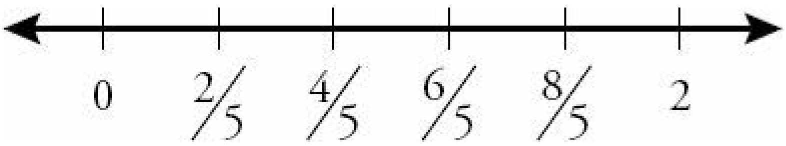
$$x^2 = (3^2)^3$$
$$x^2 = 3^6$$
$$\sqrt{x^2} = \sqrt{3^6}$$
$$x = 3^3$$
$$x = 27$$

Alternatively, you could try the answers. For instance, for choice (E):

$$\frac{27^2}{\sqrt[3]{27}} = 9(27)$$
$$\frac{27^2}{3} = 9(27)$$
$$27^2 = 9(27)(3)$$
$$27 = 9(3)$$

Thus, choice (E) is correct. While this backsolving approach works, it may be a bit slower than the algebra approach.

16. **(D).** To determine the distance between hash marks, divide 2 (the distance from 0 to 2) by 5 (the number of segments the number line has been divided into). The result is 2/5. Therefore:



Note that 2 is equal to 10/5, so you can see that the number line is labeled correctly.

Since $\sqrt[3]{p}$ marks the same hash mark on the number line as 8/5:

$$\sqrt[3]{p} = \frac{8}{5}$$

$$p = \left(\frac{8}{5}\right)^3$$

$$p = \frac{512}{125}$$

The answer is (D). Watch out for trap answer choice (B), which represents $\sqrt[3]{p}$, not *p*.

17. **7**. You cannot subtract $2^{99} - 2^{96}$ to get 2^3! You cannot directly combine, even with the same base, when adding or subtracting. (As it turns out, the difference between 2^{99} and 2^{96} is much, much larger than 2^3.) Instead, factor out the largest number 2^{99} and 2^{96} have in common:

$$2^{99} - 2^{96} = 2^{96}\,(2^3 - 1) = 2^{96}\,(7)$$

Since $2^{99} - 2^{96}$ is equal to $2^{96}\,7^1$, its greatest prime factor is 7.

18. **(B).** First, break down 2^{k+1} as $2^k 2^1$ and 2^{k-1} as $2^k 2^{-1}$:

$$2^k - 2^k 2^1 + 2^k 2^{-1} = 2^k m$$

Factor out 2^k from the left, then cancel 2^k from both sides:

$$2^k(1 - 2^1 + 2^{-1}) = 2^k m$$

$$1 - 2^1 + 2^{-1} = m$$

$$1 - 2 + \frac{1}{2} = m$$

$$-\frac{1}{2} = m$$

19. **(B).** A good way to begin comparing these quantities is to look for similarities — specifically, 81^{50} and 9^{99} can each be broken down to powers of 3, as $81 = 3^4$ and $9 = 3^2$:

Quantity A: $\frac{2}{9}(3^4)^{50} = \frac{2}{9}(3^{200})$

Quantity B: $\frac{(3^2)(3^2)^{99}}{2} = \frac{(3^2)(3^{198})}{2} = \frac{3^{200}}{2}$ or $\frac{1}{2}(3^{200})$

Since 3^{200} is the same on both sides, ignore it (or eliminate it by dividing both quantities by 3^{200}). Since 1/2 is greater than 2/9, Quantity B is larger.

20. **(B).** The key to solving this problem is realizing that you can split 5^{k+1} into 5^k5^1. (Exponents are added when multiplying with the same base, so the process can also be reversed; thus, any expression with the form x^{a+b} can be split into x^ax^b.)

$5^{k+1} = 2{,}000$

$5^k5^1 = 2{,}000$

Now divide both sides by 5:

$5^k = 400$

So, $5^k + 1 = 401$.

Notice that you can't solve for k itself — k is not an integer, since 400 is not a "normal" power of 5. But you don't need to solve for k. You just need 5^k.

21. **5.5**. Begin by converting 9 to a power of 3:

$3^{11} = (3^2)^x$
$3^{11} = 3^{2x}$

Thus, $11 = 2x$ and $x = 5.5$.

22. **6.25**. It is not necessary to find x to solve this problem. Simply square both sides:

$x^7 = 2.5$
$(x^7)^2 = (2.5)^2$
$x^{14} = 6.25$

23. **6/5.** Just as a square root is the same as a 1/2 exponent, so too is a fifth root the same as a 1/5 exponent. Thus:

$$\sqrt[5]{x^6} = (x^6)^{\frac{1}{5}} = x^{\frac{6}{5}}$$

Since $x^{\frac{6}{5}} = x^{\frac{a}{b}}$, $a/b = 6/5$.

24. **(C).** Since $20^{-5} = \frac{1}{20^5}$ and $25^{-2} = \frac{1}{25^2}$, one quick shortcut is to convert any term with a negative exponent to one with a positive exponent by moving it from the numerator to the denominator or vice-versa:

$$\frac{20^{-5}5^{10}8^6}{10^8 25^{-2}} = \frac{5^{10}8^6 25^2}{20^5 10^8}$$

Then, convert the non-prime terms to primes:

$$\frac{5^{10}8^6 25^2}{20^5 10^8} = \frac{5^{10}(2^3)^6(5^2)^2}{(2^2 5^1)^5(2^1 5^1)^8} = \frac{5^{10}2^{18}5^4}{2^{10}5^5 2^8 5^8} = \frac{2^{18}5^{14}}{2^{18}5^{13}} = 5$$

25. **19.** To solve this problem, you need to know that when dividing with the same base, you subtract the exponents, and when multiplying with the same base, you add the exponents. Thus:

$$\frac{5^7}{5^{-4}} = 5^{7-(-4)} = 5^{11}\text{, so } a = 11.$$

$$\frac{2^{-3}}{2^{-2}} = 2^{-3-(-2)} = 2^{-1}\text{, so } b = -1.$$

$$3^8(3) = 3^8(3^1) = 3^9\text{, so } c = 9.$$

Therefore, $a + b + c = 11 + (-1) = 9 = 19$.

26. **0.** This is a bit of a trick question. 12^x is odd? How strange! 12^1 is 12, 12^2 is 144 … it soon becomes easy to see that every "normal" power of 12 is going to be even. (An even number such as 12 multiplied by itself any number of times will yield an even answer.) These normal powers are 12 raised to a positive integer. What about negative integer exponents? They are all fractions of this form: $\frac{1}{12^{\text{positive integer}}}$.

The only way for 12^x to be odd is for x to equal 0. Any nonzero number to the 0^{th} power = 1. Since $x = 0$ and the question asks for x^{12}, the answer is 0.

27. **(E).** To solve this problem, you need to know that a square root is the same as a 1/2 exponent:

$$\frac{200^{\frac{5}{2}}}{\sqrt{200}} = \frac{200^{\frac{5}{2}}}{200^{\frac{1}{2}}} = 200^{\frac{5}{2}-\frac{1}{2}} = 200^{\frac{4}{2}} = 200^2 = 40{,}000$$

28. **(B).** Since $(10^3)(0.027)$ is simply 27 and $(900)(10^{-2})$ is simply 9:

$$\frac{27}{9} = (3)(10^m)$$

$$3 = 3(10^m)$$

$$1 = 10^m$$

You might be a little confused at this point as to how 10^m can equal 1. However, you can still answer the question correctly. If m were 3, as in Quantity B, 10^m would equal 1,000. However, 10^m actually equals 1. So m must be smaller than 3.

As it turns out, the only way 10^m can equal 1 is if $m = 0$. Any nonzero number to the 0th power is equal to 1.

29. **(D).** You CANNOT simply subtract $10^6 = 10^4$ to get 10^2. This is because you cannot do any operation directly to the exponents when subtracting with the same base. Rather, you must factor out the largest power of 10 each term has in common:

$$\frac{1}{3}\left[10^4(10^2-1)\right] = \frac{1}{3}\left[10^4(99)\right] = \frac{1}{3}[990{,}000] = 330{,}000$$

30. **(D).** You could factor 2^2 out of the top, but the numbers are small enough you might as well just say that the numerator is 4 + 4 + 8 + 16 = 32.

FOIL the denominator:

$$(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$$
$$\sqrt{25}+\sqrt{3}\sqrt{5}-\sqrt{3}\sqrt{5}-\sqrt{9}$$
$$\sqrt{25}-\sqrt{9}$$

5 - 3 = 2

32/2 = 16 is your final answer.

31. **(C).** Since $2^{-4} = \frac{1}{2^4}$, $3^{-20} = \frac{1}{3^{20}}$, etc., one quick shortcut here is to note that $\frac{2^{-4}3^{-20}}{4^{-1}9^{-6}} = \frac{4^1 9^6}{2^4 3^{20}}$, and

solve from there:

$$\frac{4^1 9^6}{2^4 3^{20}} = \frac{(2^2)^1 (3^2)^6}{2^4 3^{20}} = \frac{2^2 3^{12}}{2^4 3^{20}} = \frac{1}{2^2 3^8}$$

32. **(A).** One good approach is to convert 0.000027, 900, and 0.03 to powers of 10:

$$\frac{27 \times 10^{-6} \times 10^x}{9 \times 10^2 \times 10^{-4}} = 3 \times 10^{-2} \times 10^{11}$$

Now combine the exponents on the terms with base 10:

$$\frac{27 \times 10^{-6+x}}{9 \times 10^{-2}} = 3 \times 10^9$$

Since 27/9 = 3, cancel the 3 from both sides, then combine powers of 10:

$$\frac{10^{-6+x}}{10^{-2}} = 10^9$$

$$10^{-6+x-(-2)} = 10^9$$

$$10^{-4+x} = 10^9$$

You can now see that $-4 + x = 9$, so $x = 13$.

33. **(E).** A good first step is to convert to fractional exponents. Since a square root is the same as a 1/2 exponent and a cube root is the same as a 1/3 exponent:

$$x^{\frac{1}{2}} x^{\frac{1}{3}} = x^{\frac{1}{2}+\frac{1}{3}} = x^{\left(\frac{3}{6}+\frac{2}{6}\right)} = x^{\frac{5}{6}} = \sqrt[6]{x^5}$$

34. **(E).** A good first step is to convert to fractional exponents. Since a cube root is the same as a 1/3 exponent and a 4th root is the same as a 1/4 exponent:

$$\left(x^2\right)^{\frac{1}{3}} \left(x^5\right)^{\frac{1}{4}} = \left(x^{\frac{2}{3}}\right)\left(x^{\frac{5}{4}}\right) = x^{\left(\frac{2}{3}+\frac{5}{4}\right)} = x^{\left(\frac{8}{12}+\frac{15}{12}\right)} = x^{\frac{23}{12}} = \sqrt[12]{x^{23}}$$

35. **(A).** To simplify 0.00025×10^4, simply move the decimal in 0.00025 four places to the right to get 2.5. To simplify 0.005×10^2, move the decimal in 0.005 two places to the right to get 0.5. Thus, $n = 2.5$, $m = 0.5$, and n/m $= 2.5/0.5 = 5$.

36. **(B).** Since you cannot directly combine exponential terms with the same base when you are adding and subtracting, you will need to factor out the top of the fraction:

$$\frac{40^{50} - 40^{48}}{2^{96}} \times 10^{-45}$$

$$\frac{40^{48}(40^2 - 1)}{2^{96}} \times 10^{-45}$$

$$\frac{40^{48}(1599)}{2^{96}} \times 10^{-45}$$

Now, your goal should be to eliminate the fraction entirely by getting the denominator to cancel out. One good way to do this is to break up the 40, so as to isolate some 2's that will allow 2^{96} to be canceled out. (Note that it would also be possible to break 40 into 8 and 5, but in this particular problem, it seems wise to leave the 10 intact so it can ultimately combine with the 10^{-45}).

$$\frac{(4 \times 10)^{48}(1599)}{2^{96}} \times 10^{-45}$$

$$\frac{4^{48}10^{48}(1599)}{2^{96}} \times 10^{-45}$$

$$\frac{(2^2)^{48}10^{48}(1599)}{2^{96}} \times 10^{-45}$$

$$\frac{2^{96}10^{48}(1599)}{2^{96}} \times 10^{-45}$$

2^{96} cancels! Combine 10^{48} and 10^{-45} for the final answer.

1048(1,599) × 10^{-45}
103(1,599)

37. **(B)**. Since a one-half exponent is the same as a square root, $x^{\frac{3}{2}}$ could also be written as $\sqrt{x^3}$. This, however, does not appear in the choices. Note, however, that $\sqrt{x^3}$ can be simplified a bit:

$$\sqrt{x^2 \times x}$$

$$\sqrt{x^2} \times \sqrt{x}$$

$$x\sqrt{x}$$

This matches choice (B). Alternatively, convert the answer choices. For instance, in incorrect choice (A), $x^2\sqrt{x} = x^2 x^{\frac{1}{2}} = x^{\frac{5}{2}}$. Since this is not equal to $x^{\frac{3}{2}}$, eliminate (A). Correct choice (B) can be converted as such:

$$x\sqrt{x} = x^1 x^{\frac{1}{2}} = x^{\frac{3}{2}}.$$

38. **(C).** The GRE calculator has a square root button, but it won't work on numbers as large as 360 × 240 × 3 × 2. Thus, you want to break this number down enough to pull out some perfect squares. You can already see that 360 is just 36 × 10. Now break it down a bit more:

$$\sqrt{(36)(10)(24)(10)(3)(2)}$$

Note that the two 10's inside can make 100, which is a perfect square:

$$\sqrt{(36)(100)(24)(3)(2)}$$

Multiply (24)(3)(2) to see if you get a perfect square. You do! It's 144:

$$\sqrt{(36)(100)(144)}$$

Since the operation inside the root sign is multiplication, it is allowable to break up the root sign into three separate root signs, as such:

$$\sqrt{36}\sqrt{100}\sqrt{144}$$

The answer is 6 × 10 × 12 = 720.

39. **(B).** Exponents questions are usually about primes, because you always want to create common bases, and the easiest common bases are primes. In order to answer this question, you have to understand what creates zeroes at the end of a number.

10 = 5 × 2
40 = 8 × 5 × 2
100 = 10 × 10 = 2 × 5 × 2 × 5
1,000 = 10 × 10 × 10 = 2 × 5 × 2 × 5 × 2 × 5

What you'll notice is that zeroes are created by 10's, each of which is created by one 2 and one 5. So to answer this question, you simply need to work out how many pairs of 2's and 5's are in the expression:

$$125^{14}48^{8} = (5^3)^{14} \times (2^4 \times 3)^8 = 5^{42} \times 2^{32} \times 3^8$$

Even though there are 42 powers of 5, there are only 32 powers of 2, so you can only make 32 pairs of one 5 and one 2. The answer is (B).

Chapter 15
of

Number Properties

In This Chapter…

Number Properties

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

On a number line, the distance from A to B is 4 and the distance from B to C is 5.

Quantity A	**Quantity B**
The distance from A to C	9

2.

$a, b, c,$ and d are consecutive integers such that $a < b < c < d$

Quantity A	**Quantity B**
The average of $a, b, c,$ and d	The average of b and c

3. $w, x, y,$ and z are consecutive odd integers such that $w < x < y < z$. Which of the following statements must be true?

Indicate <u>all</u> such statements.

☐ $wxyz$ is odd
☐ $w + x + y + z$ is odd
☐ $w + z = x + y$

4.

Quantity A	**Quantity B**
The sum of all the odd integers from 1 to 100, inclusive	The sum of all the even integers from 1 to 100, inclusive

5. If $a + b + c + d + e$ is odd, and $a, b, c, d,$ and e are integers, which of the following could be the number of integers among $a, b, c, d,$ and e that are even?

Indicate <u>all</u> such numbers.

☐ 0
☐ 1
☐ 2
☐ 3
☐ 4
☐ 5

6.

Quantity A	**Quantity B**
The least odd number greater than or equal to 5!	The greatest even number less than or equal to 6!

7. If set S consists of all positive integers that are multiples of both 2 and 7, how many numbers in set S are between 140 and 240, inclusive?

☐

8.

$$ab > 0$$
$$bc < 0$$

Quantity A	**Quantity B**
ac	0

9.

$$mn < 0$$
$$mp > 0$$

Quantity A	Quantity B
np	0

10.

$$abc < 0$$
$$b^2c > 0$$

Quantity A	Quantity B
ab	0

11.

a, b, and c are integers such that $a < b < c$

Quantity A	Quantity B
$\frac{a+b+c}{3}$	b

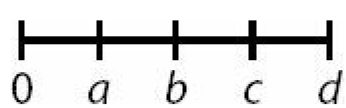
12. If $x^2 = y^2$, which of the following must be true?

- ☐ $x = y$
- ☐ $x^2 - y^2 = 0$
- ☐ $|x| - |y| = 0$

13. If p and k are even, and q is odd, which of the following cannot be even?

(A) pk
(B) pq
(C) $\frac{k}{p}$
(D) $\frac{qp}{k}$
(E) $\frac{q}{p}$

14.



Quantity A	Quantity B
$a \times c$	$b \times d$

15. If $a > b > c > d$ and $a = 2$, which of the following must be negative?

(A) ab
(B) ac
(C) ad
(D) bd
(E) None of the above.

16. If $y^2 = 4$ and $x^2y = 18$, $x + y$ could equal which of the following values?

Indicate two such values.

- ☐ -5
- ☐ -1
- ☐ 1
- ☐ 5
- ☐ 6

17.

Quantity A	**Quantity B**
The remainder when 10^{11} is divided by 2	The remainder when 3^{13} is divided by 3

18.

q is odd

Quantity A	**Quantity B**
$(-1)^q$	$(-1)^{q+1}$

19.

n is a positive integer

Quantity A	**Quantity B**
$(-1)^{4n} \times (-1)^{202}$	$(3)^3 \times (-5)^5$

20. If n is the smallest of three consecutive positive integers, which of the following must be true?

(A) n is divisible by 3
(B) n is even
(C) n is odd
(D) $(n)(n + 2)$ is even
(E) $n(n + 1)(n + 2)$ is divisible by 3

21. If x, y, and z are integers, $y + z = 13$, and $xz = 9$, which of the following must be true?

(A) x is even

(B) $x = 3$
(C) y is odd
(D) $y > 3$
(E) $z < x$

22.

$$abc > 0,$$
$$a < b,$$
$$\text{and } a^2(c) < 0$$

Quantity A	**Quantity B**
ab	$b(ac)^2$

23. On a number line, A is 6 units from B and B is 2 units from C. What is the distance between A and C?

(A) 4
(B) 8
(C) 12
(D) 4 or 8
(E) 4, 8, or 12

24. The average of 11 integers is 35. What is the sum of all the integers?

25. What is the sum of all the integers from 1 to 80, inclusive?

(A) 3,200
(B) 3,210
(C) 3,230
(D) 3,240
(E) 3,450

26. The average of a set of 18 consecutive integers is 22.5. What is the smallest integer in the set?

27. p is the sum of all the integers from 1 to 150, inclusive. q is the sum of all the integers from 1 to 148, inclusive. What is the value of $p - q$?

28. If m is the product of all the integers from 2 to 11, inclusive, and n is the product of all the integers from 4 to 11,

inclusive, what is the value of $\frac{n}{m}$?

Give your answer as a fraction.

29. If $\sqrt{x}$ is an integer and $xy^2 = 36$, how many values are possible for the integer y?

(A) 2
(B) 3
(C) 4
(D) 6
(E) 8

30.

a, b, and c are positive even integers such that $8 > a > b > c$

Quantity A	**Quantity B**
The range of a, b, and c	The average of a, b, and c

31. If x is a non-zero integer and $0 < y < 1$, which of the following must be greater than 1?

(A) x
(B) $\frac{x}{y}$
(C) xy^2
(D) x^2y
(E) $\frac{x^2}{y}$

32.

a, b, and c are consecutive integers such that $a < b < c < 4$

Quantity A	**Quantity B**
The range of a, b, and c	The average of a, b, and c

33.

$x = 2y = 4z$ and x, y, and z are integers

Quantity A	**Quantity B**
The average of x and $2y$	$4z + x - 2y$

34.

$\sqrt{xy}$ is a prime number, xy is even, and $x > 4y > 0$

Quantity A	**Quantity B**
y	1

35.

$abcd$ is even and positive, and abc is odd and positive

Quantity A	**Quantity B**
1	d

36.

$b - a < 0$ and $a + 2c < 0$

Quantity A	**Quantity B**
b	$-2c$

37.

In set N consisting of n integers, the average equals the median.

Quantity A	**Quantity B**
The remainder when n is divided by 2	The remainder when $n - 1$ is divided by 2

38.

x is even, $\sqrt{x}$ is a prime number, and $x + y = 11$

Quantity A	Quantity B
x	y

39.

The product of integers f, g, and h is even and the product of integers f and g is odd

Quantity A	**Quantity B**

The remainder when f is divided by 2 | The remainder when h is divided by 2

40.

x, y, and z are integers
$xyz \geq 0$
$yz < 0$
$y < 0$

Quantity A | **Quantity B**

x | z

41.

$\sqrt{y} = 3$
$x^2 = 16$
$y - x > 10$

Quantity A | **Quantity B**

x | xy

42. If $\frac{17}{2^{10}5^{13}}$ is expressed as a terminating decimal, how many zeroes are located to the right of the decimal point before the first non-zero digit?

(A) 10
(B) 12
(C) 13
(D) 15
(E) 17

43. If x is odd, all of the following must be odd EXCEPT:

(A) $x^2 + 4x + 6$
(B) $x^3 + 5x + 3$
(C) $x^4 + 6x + 7$
(D) $x^5 + 7x + 1$
(E) $x^6 + 8x + 4$

44.

$x^2 > 25$ and $x + y < 0$

Quantity A | **Quantity B**

x | y

45.

The positive integer a is divisible by 2, and $0 < ab < 1$

Quantity A	**Quantity B**
b	$\frac{1}{2}$

46.

p and w are single-digit prime numbers
$p + w < 6$
p^2 is odd

Quantity A	**Quantity B**
3	w

47. If $\frac{x}{23}$ is a positive integer with a factor of 6, which of the following statements must be true?

Indicate <u>two</u> such statements.

- ☐ $x > 23$
- ☐ x has at least 3 prime factors
- ☐ x is odd
- ☐ x is prime
- ☐ x is not divisible by 5

48.

$x^2 > y^2$ and $x > -|y|$

Quantity A	**Quantity B**
x	y

49.

The sum of four consecutive integers is -2

Quantity A	**Quantity B**
The smallest of the four integers	-2

50. If g is an integer and x is a prime number, which of the following must be an integer?

Indicate <u>all</u> such expressions.

☐ $\frac{g^2x+5gx}{x}$

☐ $g^2-x^2\left(\frac{1}{3}\right)$

☐ $6\left(\frac{g}{2}\right)-100\left(\frac{g}{2}\right)^2=$

51. If $k=\frac{19!}{16!}$, which of the following is the smallest choice that does not have a prime factor in common with k?

(A) 19
(B) 34
(C) 77
(D) 115
(E) 133

52. If $4^6 25^5 = 10^x + k$, and x is an integer, what is the minimum positive value k could be?

(A) 0
(B) 30,000
(C) 30,000,000
(D) 10,000,000,000
(E) 30,000,000,000

53. Jose is making a necklace with beads in a repeating pattern of blue, red, green, orange, purple. If the 1st bead is blue, what color will the 234th bead be?

(A) blue
(B) red
(C) green
(D) orange
(E) purple

54. What is the units digit of 7^{94}?

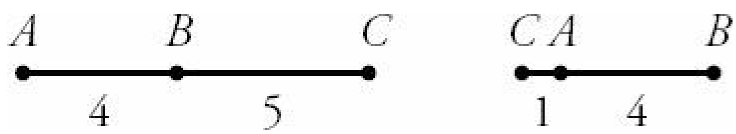
$\boxed{}$

55. What is the units digit of the sum $3^{47} + 5^{43} + 2^{12}$?

$\boxed{}$

Number Properties Answers

1. **(D).** Whenever a question looks this straightforward ($4 + 5 = 9$, so the quantities initially appear equal), be suspicious. Draw the number line described. If the points A, B, and C are in alphabetical order from left to right, then the distance from A to C will be 9. However, alphabetical order is not required. If the points are in the order C, A, and B from left to right, then the distance from A to C is $5 - 4 = 1$.



2. **(C).** When integers are consecutive (or simply evenly spaced), the average equals the median. Since the median of this list is the average of the two middle numbers, Quantity A and Quantity B both equal the average of b and c. Alternatively, try this with real numbers. If the set is 2, 3, 4, 5, both quantities equal 3.5. No matter what consecutive integers you choose, the two quantities are equal.

3. **I and III only.** This question tests your knowledge of the properties of odd numbers as well as of consecutives.

Statement I is TRUE, as multiplying only odd integers together (and no evens) will always yield an odd answer.

However, when adding, the rule is "an odd number of odds makes an odd." Summing an even number of odds produces an even, so Statement II is FALSE.

Statement III is TRUE. Since $w, x, y,$ and z are consecutive odd integers, you could define them all in terms of w:

$w = w$
$x = w + 2$
$y = w + 4$
$z = w + 6$

Thus, $w + z = w + (w + 6) = 2w + 6$
And $x + y = (w + 2) + (w + 4) = 2w + 6$

Therefore, $w + z = x + y$. Alternatively, try real numbers, such as 1, 3, 5, and 7. It is true that $1 + 7 = 3 + 5$. This would hold true for any set of four consecutive, ordered odd numbers you select.

4. **(B).** No math is required to solve this problem. Note that the numbers from 1 to 100 include 50 even integers and 50 odd integers. The first few odds are 1, 3, 5, etc. The first few evens are 2, 4, 6, etc. Every even is 1 greater than its counterpart (2 is 1 greater than 1, 4 is 1 greater than 3, 6 is 1 greater than 5, etc.) Not only is Quantity B greater, it's greater by precisely 50.

5. **I, III, and V only.** When adding integers, the rule is "an odd number of odds makes an odd" (you can ignore the evens for purposes of evaluating only whether the sum is even or odd — just count how many odds are being added). If 5 integers sum to an odd, the possibilities are:

1 odd, 4 evens
3 odds, 2 evens
5 odds, 0 evens

Make sure to answer for the number of *evens*. The answers are 0, 2, and 4.

6. **(B).** When multiplying integers, just one even will make the product even. Thus all factorials greater than 1! (which is just 1) are even.

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$. Since 120 is even, the "least odd number greater than or equal to 5!" is 121.

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$. Since 720 is even, the "greatest even number less than or equal to 6!" is 720.

Quantity B is greater.

7. **8.** A positive integer that is a multiple of both 2 and 7 is just a multiple of 14. Since 140 is a multiple of 14, start listing there and count the terms in the range: 140, 154, 168, 182, 196, 210, 224, 238.

Alternatively, note that 140 is the 10th multiple of 14, and $240/14 \approx 17.143$ (use the calculator). Therefore, the 10th through the 17th multiples of 14, inclusive, are in this range. The number of terms is $17 - 10 + 1 = 8$ ("add one before you are done" for an inclusive list).

8. **(B).** If $ab > 0$, then a and b have the same sign. If $bc < 0$, then b and c have opposite signs. Therefore, a and c must have opposite signs. Therefore, ac is negative, and Quantity B is greater.

If you find the logic difficult (a and b are same sign, b and c are opposite signs, therefore a and c are opposite signs), you could make a quick chart of the possibilities using plus and minus signs:

a	b	c	
+	+	−	← first possibility, a and c have different signs
−	−	+	← second possibility, a and c have different signs

9. **(B).** If $mn < 0$, then m and n have opposite signs. If $mp > 0$, then m and p have the same sign. Therefore, n and p must have opposite signs. Therefore, np is negative, and Quantity B is greater.

Alternatively, you could make a quick chart of the possibilities using plus and minus signs:

m	n	p	
+	−	+	← first possibility, n and p have different signs
−	+	−	← second possibility, n and p have different signs

10. **(B).** If abc is negative, then either exactly 1 or all 3 of the values a, b, and c are negative:

−	−	−	← first possibility, all negative
−	+	+	← second possibility, 1 negative, 2 positives (order can vary)

If b^2c is positive, then c must be positive, since b^2 cannot be negative. If c is positive, eliminate the first possibility since all 3 variables cannot be negative. Thus, only one of a, b, and c are negative, but the one negative cannot be c. Either a or b is negative, and the other is positive. It doesn't matter which one of a or b is negative — that's enough to know that ab is negative and Quantity B is greater.

11. **(D).** Note that $\frac{a+b+c}{3}$ is just another way to express "the average of a, b, and c." The average of a, b, and c would equal b if the numbers were evenly spaced (such as 1, 2, 3, or 5, 7, 9), but that is not specified. For instance, the integers could be 1, 2, 57 and still satisfy the $a < b < c$ constraint. In that case, the average is 20, which is greater than $b = 2$.

The correct answer is (D).

12. **II and III only.** When you take the square root of $x^2 = y^2$, you do NOT get $x = y$. Actually, you get $|x| = |y|$. After all, if $x^2 = y^2$, the variables could represent 2 and -2, 5 and 5, -1 and -1, etc. The information about the signs of x and y is lost when the numbers are squared; thus, taking the square root results in absolute values, which allow both sign possibilities for x and y. Thus, statement I is not necessarily true.

From $x^2 = y^2$, simply subtract y^2 from both sides to yield statement II. If you can algebraically generate a statement from the original equation $x^2 = y^2$, that statement must be true.

To generate statement III, take the square root of both sides of $x^2 = y^2$ to get $|x| = |y|$, then subtract $|y|$ from both sides.

13. **(E).** "Cannot be even" means it must be either odd or a non-integer.

(A) *must be* even, as an even times any integer equals an even.
(B) *must be* even, as an even times any integer equals an even.
(C) *can be* even (for instance, if $k = 8$ and $p = 4$).
(D) *can be* even (for instance, if $p = 16$, $k = 2$, and $q = 1$).
(E) *cannot be* even, as an odd divided by an even is *never* an integer. CORRECT.

14. **(B).** The exact values of a, b, c, and d are unknown, as is whether they are evenly spaced (do NOT assume that they are, just because the figure looks that way). However, it is known that all of the variables are positive such that $0 < a < b < c < d$.

Because $a < b$ and $c < d$ and all the variables are positive, $a \times c < b \times d$. In words, the product of the two smaller numbers is less than the product of the two greater numbers. Quantity B is greater.

You could also try this with real numbers. You could try $a = 1$, $b = 2$, $c = 3$, and $d = 4$, or you could mix up the spacing, as in $a = 0.5$, $b = 7$, $c = 11$, $d = 45$. For any scenario that matches the conditions of the problem, Quantity B is greater.

15. **(E).** Don't assume the variables are integers or that they are equally spaced. It is possible that b, c, and d are all positive (integers or fractions), so it is not true that the products in choices (A) through (D) must be negative.

16. **-1, 5.** From the first equation it seems that y could equal either 2 or -2, but if $x^2y = 18$, then y must equal only 2 (otherwise, x^2y would be negative). Still, the squared x indicates that x can equal 3 or -3. So the possibilities for $x + y$ are:

$3 + 2 = 5$
$(-3) + 2 = -1$

17. **(C).** It is not necessary to calculate 10^{11} or 3^{13}. Because 10 is an even number, so is 10^{11}, and 0 is the remainder when any even is divided by 2. Similarly, 3^{13} is a multiple of 3 (it has 3 among its prime factors), and 0 is the remainder when any multiple of 3 is divided by 3.

18. **(B).** The negative base -1 to any odd power is -1, and the negative base -1 to any even power is 1. Since q is odd, Quantity A = -1 and Quantity B = 1.

19. **(A).** Before doing any calculations on a problem with negative bases raised to integer exponents, check to see whether one quantity is positive and one quantity is negative, in which case no further calculation is necessary. Note that a negative base to an even exponent is positive, while a negative base to an odd exponent is negative.

Since n is an integer, $4n$ is even. Thus, in Quantity A, $(-1)^{4n}$ and $(-1)^{202}$ are both positive, and Quantity A is positive. In Quantity B, $(3)^3$ is positive but $(-5)^5$ is negative, and thus Quantity B is negative. Since a positive is by definition greater than a negative, Quantity A is greater.

20. **(E).** For three consecutive integers, the only possibilities are [odd, even, odd] or [even, odd, even]. Since n could be odd or even, eliminate (B) and (C). Choice (D) is true if n is even, but not if n is odd, so eliminate (D). In any set of three consecutive integers, one of the integers must be divisible by 3, but not necessarily n. Eliminate (A).

For the same reason, (E) must be true, as $n(n + 1)(n + 2)$ can be thought of as "the product of any three consecutive integers." Since one of these integers must be divisible by 3, the product of those three numbers must also be divisible by 3.

21. **(D).** If $xz = 9$ and x and z must both be integers, then they are 1 and 9 (or -1 and -9) or 3 and 3 (or -3 and -3). Therefore, they are both odd. More generally, the product of two integers will only be odd if the component integers themselves are both odd. Because z is odd, and $y + z$ equals 13 (an odd), y must be even.

Eliminate (A): x is NOT even.

Eliminate (B): x could be 3 but doesn't have to be.

Eliminate (C): y is NOT odd.

Eliminate (E): z does not have to be less than x (for instance, they could both be 3).

At this point, only (D) remains, so it must be the answer. To prove it, consider the constraint that limits the value of y: $y + z = 13$. Since z could be -1, 1, -3, 3, -9, or 9, the maximum possible value for z is 9, so y must be at least 4. All values that are at least 4 are also greater than 3, so (D) must be true.

22. **(B).** Since $a^2(c) < 0$ and it is not possible for a^2 to be negative, c must be negative.

If $abc > 0$ and c is negative, ab must be negative also, implying that a and b have opposite signs. Because $a < b$, a must be negative and b positive.

Thus, Quantity A is negative and Quantity B is pos × (neg × neg)^2 = pos × pos, which is positive.

23. **(D).** There is no rule that *A*, *B*, and *C* must occur in alphabetical order. If the points are ordered *A*, *B*, *C*, the distance from *A* to *C* is 6 + 2 = 8. But if in the order *A*, *C*, *B*, the distance from *A* to *C* is 6 – 2 = 4. Any other orders allowed by the relative distances (*C*, *B*, *A*, for instance, or *B*, *C*, *A*) will also yield either 4 or 8.

24. **385.** To find the sum of a set of numbers, given the average and number of terms, use the average formula. Average $= \frac{\text{Sum}}{\text{Number of Terms}}$, so Sum = Average × Number of Terms = 35 × 11 = 385.

25. **(D).** To find the sum of a set of evenly-spaced numbers, multiply the median (which is also the average) by the number of terms in the set. The median of the numbers from 1 to 80 inclusive is 40.5 (the first 40 numbers are 1 through 40, and the second 40 numbers are 41 through 80, so the middle is 40.5). You can also use the formula $\frac{\text{First} + \text{Last}}{2}$ to calculate the median of an evenly-spaced set: $\frac{1+80}{2}$. Multiply 40.5 times 80 to get the answer:

26. **14.** In an evenly-spaced set, the average is also the median. Thus, 22.5 is the median of the set. There are an even number of terms in the set, so the median is halfway between the two middle numbers. Thus, the 9th number is 22 and the 10th number is 23. Count backwards to get the answer, or write out the set:

First 9 integers (counting down): 22, 21, 20, 19, 18, 17, 16, 15, 14
Last 9 integers (counting up): 23, 24, 25, 26, 27, 28, 29, 30, 31

Alternatively, the 1st number is 8 "steps" down from the 9th number, so the smallest integer in the set is 22 - 8 = 14.

27. **299.** *p* is a large number, but it consists entirely of *q* + 149 + 150. Thus, *p* - *q* is what's left of *p* once the common terms are subtracted: 149 + 150 = 299.

28. $\mathbf{\frac{1}{6}}$. There is a trick to this problem — all of the integers in the product *n* will be canceled out by the same integers appearing in the product *m*:

$$\frac{n}{m} = \frac{\cancel{4\times5\times6\times7\times8\times9\times10\times11}}{2\times3\times\cancel{4\times5\times6\times7\times8\times9\times10\times11}} = \frac{1}{2\times3} = \frac{1}{6}$$

29. **(E).** If $\sqrt{x}$ is an integer, then *x* must be a perfect square. If *x* is a perfect square and $xy^2 = 36$, then *x* could actually equal *any* of the perfect square factors of 36, which are 1, 4, 9, or 36. (Only consider positive factors, because in order to have a valid square root, *x* must be positive.) Thus, y^2 could equal 36, 9, 4, or 1, respectively.

Of course, y^2 is positive, but y itself could be positive or negative. Thus $y = \pm 6, \pm 3, \pm 2$, or ± 1, for a total of 8 possible values.

30. **(C).** Integers a, b, and c must be 6, 4, and 2, respectively, as they are positive even integers less than 8 and ordered according to the given inequality. The range of a, b, and c is 6 - 2 = 4. The average of a, b, and c is $\frac{6+4+2}{3} = \frac{12}{3} = 4$. The two quantities are equal.

31. **(E).** Find the choices that do not have to be greater than 1. Most obviously, x could be negative, which eliminates (A), (B), and (C). For choice (D), if $x^2 = 1$, that times the positive fraction y would be less than 1. In choice (E), x^2 must be positive and at least 1, so dividing by the positive fraction y increases the value.

32. **(D).** If the variables were also constrained to be positive, they would have to be 1, 2, and 3, making the quantities both equal to 2. However, the variables could be negative, for example, $a = -10$, $b = -9$, $c = -8$. The range of a, b, and c will always be 2 because the integers are consecutive, but the average can vary depending on the specific values.

33. **(C).** Since $x = 2y$, the average of x and $2y$ is $\frac{x+2y}{2} = \frac{x+x}{2} = x$. Similarly, $4z + x - 2y$ is $x + x - x = x$.

Alternatively, pick values, such as $x = 4$, $y = 2$, and $z = 1$.

Quantity A is the average of 4 and 2(2), which is equal to 4.

Quantity B is 4(1) + 4 - 2(2) = 4.

The two quantities are equal for any set of values that conform to $x = 2y = 4z$, even negative test cases.

34. **(B).** If $\sqrt{xy}$ is a prime number, $\sqrt{xy}$ could be 2, 3, 5, 7, 11, 13, etc. Square these possibilities to get a list of possibilities for xy: 4, 9, 25, 49, 121, 169, etc. However, xy is even, so xy must equal 4.

Finally, $x > 4y > 0$, which implies that both x and y are positive. Solve $xy = 4$ for x, then substitute to eliminate the variable x and solve for y.

If $xy = 4$, then $x = \frac{4}{y}$.

If $x > 4y$, then $\frac{4}{y} > 4y$.

Because y is positive, you can multiply both sides of the inequality by y and you don't have to flip the sign of the inequality: $4 > 4y^2$

Finally, divide both sides of the inequality by 4: $1 > y^2$

Thus, y is a positive fraction less than 1 (you already know $y > 0$). Quantity B is greater.

35. **(D).** Since abc is odd and $abcd$ is even, d has to be even — if it is an integer. For instance, it could be the case that $a = 1$, $b = 1$, $c = 1$, and $d = 2$. In this case, d would be greater than 1. However, d could be a fraction — for example, $abcd$ could equal $3 \times 3 \times 3 \times \frac{2}{3} = 18$. In this case, d would be $\frac{2}{3}$, which is less than 1.

36. **(B).** Solve both inequalities for a. Add a to both sides of $b - a < 0$ to get $b < a$. From $a + 2c < 0$, subtract $2c$ on both sides to get $a < -2c$. Put these together: $b < a < -2c$. Thus, b must be less than $-2c$. Quantity B is greater.

37. **(D).** "The remainder when … divided by 2" is a fancy way of asking whether a integer is odd or even (evens yield remainder 0 when divided by 2; odds yield remainder 1 when divided by 2).

If n is even, Quantity A is 0 and Quantity B is 1.

If n is odd, Quantity A is 1 and Quantity B is 0.

In this problem, n is the number of numbers in the set. So, could a set in which the average equals the median have either an even or an odd number of numbers? Absolutely. In fact, it is true of *any* evenly spaced set (and of some other sets) that the average equals the median. For instance:

1, 2, 3, 4	average and median are both 2.5	$n = 4$	$\longrightarrow$	Quantity B is greater
1, 2, 3	average and median are both 2	$n = 3$	$\longrightarrow$	Quantity A is greater

The correct answer is (D).

38. **(B).** If $\sqrt{x}$ is a prime number, $x = (\sqrt{x})(\sqrt{x})$ is the square of a prime number. Squaring a number does not change whether it is odd or even (the square of an odd number is odd and the square of an even number is even). Since x is even, it must be the square of the only even prime number. Thus, $\sqrt{x} = 2$ and $x = 4$. Since $x + y = 11$, $y = 9$ and Quantity B is greater.

39. **(A).** If fg is odd and both f and g are integers, both f and g are odd. The remainder when odd f is divided by 2 is 1. Since fgh is even and f and g are odd, integer h must be even. Thus, when h is divided by 2, the remainder is 0. Quantity A is greater.

40. **(B).** If $yz < 0$ and $y < 0$, z must be positive and y negative. If $xyz \geq 0$ and $yz < 0$, then x must be negative or 0. Quantity A is at most 0, while Quantity B is positive.

41. **(A).** If $\sqrt{y} = 3$, $y = 9$. If $x^2 = 16$, then $x = 4$ or -4. Since $y - x > 10$ and $y = 9$:

$9 - x > 10$
$-x > 1$
$x < -1$

This rules out the $x = 4$ possibility. Thus, x equals -4 and $xy = (-4)(9) = -36$. Since -4 is greater than -36, Quantity A is greater.

42. **(A).** Decimal placement can be determined by how many times a number is multiplied or divided by 10. Multiplying moves the decimal to the right, and dividing moves the decimal to the left. Look for powers of 10 in the given fraction, remembering that $10 = 2 \times 5$.

$$\frac{17}{2^{10}5^{13}} = \frac{17}{2^{10}5^{10}5^{3}} = \frac{17}{(2\times5)^{10}5^{3}} = \frac{17}{10^{10}5^{3}} = \frac{17/125}{10^{10}} = \frac{0.136}{10^{10}}$$

There are no zeros to the right of the decimal point before the first non-zero digit in 0.136. However, dividing by 10^{10} will move the decimal to the left 10 places, resulting in 10 zeros between the decimal and the "136" part of the number.

43. **(C).** For even and odd questions, you can either think it out logically, or plug in a number. Since one choice requires raising the number to the 6th power, pick something small! Plug in $x = 1$:

(A) $x^2 + 4x + 6 = 1 + 4 + 6 = 11$
(B) $x^3 + 5x + 3 = 1 + 5 + 3 = 9$
(C) $x^4 + 6x + 7 = 1 + 6 + 7 = 14$
(D) $x^5 + 7x + 1 = 1 + 7 + 1 = 9$
(E) $x^6 + 8x + 4 = 1 + 8 + 4 = 13$

For the logic approach, remember that an odd number raised to an integer power is always odd, an odd number multiplied by an odd number is always odd, and an odd number multiplied by an even number is always even:

(A) $x^2 + 4x + 6$ = odd + even + even = odd
(B) $x^3 + 5x + 3$ = odd + odd + odd = odd
(C) $x^4 + 6x + 7$ = odd + even + odd = even
(D) $x^5 + 7x + 1$ = odd + odd + odd = odd
(E) $x^6 + 8x + 4$ = odd + even + even = odd

44. **(D).** If $x^2 > 25$, then $x > 5$ OR $x < -5$. For instance, x could be 6 or -6.

If $x = 6$:

$6 + y < 0$
$y < -6$

x is greater than y.

If $x = -6$:

$-6 + y < 0$
$y < 6$

y could be less than x (e.g., $y = -7$) or greater than x (e.g., $y = 4$).

45. **(B).** If the positive integer a is divisible by 2, it is a positive even integer. Thus, the minimum value for a is 2. Thus, since $ab < 1$, b must be less than $\frac{1}{2}$.

46. **(A).** If the sum of two primes is less than 6, either the numbers are 2 and 3 (the two smallest unique primes), or both numbers are 2 (just because the variables are different letters doesn't mean that p cannot equal w). Both numbers cannot equal 3, though, or $p + w$ would be too great. If p^2 is odd, p is odd, and therefore $p = 3$, so w can only be 2.

47. **I and II only.** If $\frac{x}{23}$ is divisible by 6, x must have factors of 2 and 3, as well as 23. Another way to write this is $\frac{x/23}{6}$ = positive integer, so $\frac{x}{23}$ = 6 × positive integer, and finally $x = 2 \times 3 \times 23 \times$ positive integer.

I. TRUE. x must be greater than 23, as it is 23 × positive values.
II. TRUE. x has at least 3 prime factors, namely, 2, 3, and 23.
III. False. Because x has a factor of 2, it is not odd.
IV. False. Because it has more than one prime factor, x is definitely *not* prime.
V. Maybe. While x must have factors of 2, 3, and 23, it could also have other prime factors. For instance, x could be 2(3)(23), or it could be a very large number with more factors, such as $2^2 3^2 23^4 5^{11} 13^2$. Thus, x may or may not be divisible by 5.

48. **(A).** If $x^2 > y^2$, x must have a greater absolute value than y. For instance:

	x	y
Example 1	3	2
Example 2	-3	2
Example 3	3	-2
Example 4	-3	-2

If $x > -|y|$ must also be true, which of the examples continue to be valid?

	x	y	$x > -\lvert y\rvert$?	
Example 1	3	2	$3 > -\lvert 2\rvert$	TRUE
Example 2	-3	2	$-3 > -\lvert 2\rvert$	FALSE
Example 3	3	-2	$3 > -\lvert -2\rvert$	TRUE
Example 4	-3	-2	$-3 > -\lvert -2\rvert$	FALSE

Only Example 1 and Example 3 remain.

x	y

Example 1	3	2
Example 3	3	-2

Thus, either x and y are both positive and x has a larger absolute value (Quantity A is greater) or x is positive and y is negative (Quantity A is greater). In either case, Quantity A is greater.

49. **(C).** Write an equation: $x + (x + 1) + (x + 2) + (x + 3)= -2$.

$4x + 6 = -2$
$4x = -8$
$x = -2$

Thus, the integers are -2, -1, 0, and 1. The smallest of the four integers equals -2.

50. **I and III only.** In Statement I, x can be factored out and eliminated: $\frac{g^2x+5gx}{x}=\frac{x(g^2+5g)}{x} = g^2 + 5g$. Since g is an integer, so is $g^2 + 5g$.

In Statement II, g^2 is certainly an integer, but $x^2\left(\frac{1}{3}\right)$ is only an integer if $x = 3$ (since 3 is the only prime number divisible by 3), so statement II does not have to be an integer.

When Statement III is simplified, $6\left(\frac{g}{2}\right)-100\left(\frac{g}{2}\right)^2=3g-\frac{100g^2}{4}= 3g - 25g^2$ results; since g is an integer, $3g - 25g^2$ is also an integer, albeit a negative one.

51. **(C).** $\frac{19!}{16!}=\frac{19\times18\times17\times\cancel{16\times15\times14\times13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}}{\cancel{16\times15\times14\times13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}}$ $= 19 \times 18 \times 17$. No need to multiply these out, because the question is only about common prime factors. If $k = 19 \times 18 \times 17$, k's prime factors are 19, 2, 3, 3, and 17 (19 and 17 are prime, and 18 breaks down into prime factors 2, 3, and 3).

Choices (A) is wrong because 19 has a prime factor in common with k (that factor is 19 itself). Choice (B) is wrong because 34 has a prime factor of 17, as does k. Choice (C) is correct. 77 has prime factors 7 and 11 and does not have a prime factor in common with k. Note that 115 also does not share any prime factors with k, but the questions asks for the *smallest* choice that works.

52. **(E).** Since $4^6 25^5$ is too big for the calculator, try another strategy: note that $4 \times 25 = 100$, and the other side of the equation involves a power of 10. Separate out "pairs" of 4 and 25 on the left:

$4^6 25^5 = 10^x + k$
$4^1(4^5 25^5) = 10^x + k$
$4^1(4 \times 25)^5 = 10^x + k$

$4^1(100)^5 \quad = 10^x + k$

Thus, the left side of the equation is $4(100^5)$, or $4(10^{10})$, or 40,000,000,000. Thus:

40,000,000,000 = [a power of 10] + k

To minimize k while keeping it positive, maximize the power of 10 while keeping it less than $4^6 25^5$. The greatest power of 10 that is less than 40,000,000,000 is 10,000,000,000, or 10^{10}. Thus:

40,000,000,000 = 10,000,000,000 + k
30,000,000,000 = k

53. **(D).** Since the pattern has 5 elements, find the remainder when 234 is divided by 5, which is just the units digit in this case. Alternatively, 5 goes evenly into 230, and since 234 - 230 = 4, the remainder is 4. The 4th color in the pattern, orange, is the answer.

54. **9.** The units digits of 7 to positive integers create a repeating pattern (this works for digits other than 7 also). By multiplying 7 by itself repeatedly in the calculator, you can generate the pattern:

$7^1 = \mathbf{7}$
$7^2 = 4\mathbf{9}$
$7^3 = 34\mathbf{3}$
$7^4 = 2{,}40\mathbf{1}$
$7^5 = 16{,}80\mathbf{7}$
$7^6 = 117{,}64\mathbf{9}$
$7^7 = 823{,}54\mathbf{3}$
$7^8 = 5{,}764{,}80\mathbf{1}$
Pattern: 7, 9, 3, 1

The pattern for the units digits of 7 to a power is 7, 9, 3, 1. (You should know that none of the patterns ever have more than 4 elements before repeating, so you don't actually have to multiply out 8 or more times, as shown above.)

To find the 94th item in a pattern of 4 repeating items, find the remainder when 94 is divided by 4. 94 divided by 4 in the calculator is 23.5. Ignore the decimal and multiply 4 × 23 to find the largest number (less than 94) that 4 *does* go into: it's 92. 94 divided by 4 gives remainder 2 (since 94 – 92 = 2).

Thus, to get to the units digit of 7^{94}, the pattern (7, 9, 3, 1) repeats 23 times, and then continues 2 more elements into the pattern. The second element in the pattern is 9.

55. **8.** For any digit, that digit to increasing positive integer powers will end in units digits that create a repeating pattern:

$3^1 = \mathbf{3}$
$3^2 = \mathbf{9}$
$3^3 = 2\mathbf{7}$

$3^4 = 8\mathbf{1}$
$3^5 = 24\mathbf{3}$
$3^6 = 72\mathbf{9}$
$3^7 = 2{,}18\mathbf{7}$
$3^8 = 6{,}56\mathbf{1}$
Pattern: 3, 9, 7, 1

3^{47} will end in the digit that is the 47th item in the pattern 3, 9, 7, 1. Since it's a pattern of 4 elements, find the largest multiple of 4 that is still less than 47: it's 44. Since 47 - 44 = 3, the remainder when 47 is divided by 4 is 3. Thus, 3^{47} will end in the units digit that is the 3rd item in the pattern. Thus, 3^{47} ends in 7.

$5^1 = \mathbf{5}$
$5^2 = 2\mathbf{5}$
$5^3 = 12\mathbf{5}$

…this one's a "freebie," as 5 to any power ends in 5 and 5^{43} must end in 5.

$2^1 = \mathbf{2}$
$2^2 = \mathbf{4}$
$2^3 = \mathbf{8}$
$2^4 = 1\mathbf{6}$
$2^5 = 3\mathbf{2}$
$2^6 = 6\mathbf{4}$
$2^7 = 12\mathbf{8}$
$2^8 = 25\mathbf{6}$
$2^9 = 51\mathbf{2}$
$2^{10} = 1{,}02\mathbf{4}$
Pattern: 2, 4, 8, 6

2^{12} will end in the digit that is the 12th item in the pattern 2, 4, 8, 6. Since 4 goes into 12 evenly, 2^{12} will end in the last item in the pattern, which is 6. (You also could just keep going with the pattern above: 2^{11} is 2,048, and 2^{12} = 4,096, which ends in 6). Thus, 2^{12} ends in 6.

Thus, 3^{47}, 5^{43}, and 2^{12} are very large numbers that end in 7, 5, and 6, respectively. Imagine that you were adding those very large numbers by hand (the xxxx just indicates the unknown and unimportant parts of these numbers):

```
  xxxxxxx7
  xxxxxxx5
+ xxxxxxx6
```

To begin adding these numbers, you would add 7 + 5 + 6 = 18. You would put 8 in the units place of your answer, and carry the 1:

```
         1
  xxxxxxx7
```

```
   xxxxxxx5
+ xxxxxxx6
        8
```

The units digit of the answer is 8.

Chapter 16
of

Word Problems

In This Chapter…

Word Problems

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. If a taxi charges $8.00 for the first mile, and $1.00 for each additional quarter mile, how much does the taxi charge for a 4.5 mile ride?

(A) $16.00
(B) $18.00
(C) $22.00
(D) $24.00
(E) $26.00

2. If Nash had 12 grandchildren and three times as many granddaughters as grandsons, how many granddaughters did he have?

(A) 3
(B) 4
(C) 6
(D) 8
(E) 9

3. If Deepak pays 30% of his income in taxes and his take-home pay after taxes is $2,800 per month, how much does Deepak make per month?

$

4. Movie theater X charges \$6 per ticket, and each movie showing costs the theatre \$1,750. How many people need to see a movie so that the theater makes \$1 of profit per customer?

(A) 300
(B) 325
(C) 350
(D) 375
(E) 400

5. Arnaldo earns \$11 for each ticket that he sells, and a bonus of \$2 per ticket for each ticket he sells over 100. If Arnaldo was paid \$2,400, how many tickets did he sell?

(A) 120
(B) 160
(C) 180
(D) 200
(E) 250

6. Attendees at a charity dinner each gave at least \$85 to the charity. If \$6,450 was collected, what is the maximum number of people who could have attended?

(A) 73
(B) 74
(C) 75
(D) 76
(E) 77

7. Eva meditates for 20 minutes at a time, with a 5-minute break in between sessions. If she begins meditating at 10:10, what time will it be when she completes her third session?

(A) 11:20
(B) 11:25
(C) 11:50
(D) 11:55
(E) 12:25

8. A washing machine takes 35 minutes to wash one load of laundry, and in between washing different loads of laundry it takes Derek 2 minutes to unload and another 4 minutes to reload the machine. If the washing machine begins washing one load of laundry at 12:30pm, how many loads of laundry can Derek wash and unload before 6:35pm?

(A) 8
(B) 9
(C) 10
(D) 14
(E) 15

9.

Kendra is more than 5 years old.

Quantity A	Quantity B
Five years less than twice Kendra's age	Twice what Kendra's age was five years ago

10.

Six years ago, Billy was twice as old as Allie.

Quantity A	Quantity B
The difference between their ages today	2 years

11. Every day the drama club has a car wash, the club pays a fixed amount for supplies. If the club charges $12 for a car wash and earned a total profit of $190 in one day by giving 20 car washes, how much did the club pay for supplies, assuming that there are no other expenses?

12. A store owner pays her assistant $22 per hour for every hour the store is open. If all other expenses for the store work out to $160 per day, and the store is open for 8 hours on Monday and sells $720 worth of merchandise on that day, what is the store's profit for the day?

(A) 384
(B) 396
(C) 530
(D) 538
(E) 560

13. Regular gas costs $3.00 a gallon and is consumed at a rate of 25 miles per gallon. Premium costs $4.00 a gallon and is consumed at a rate of 30 miles per gallon. How much more will it cost to use premium rather than regular for a 300-mile trip?

(A) $1
(B) $4
(C) $5
(D) $36
(E) $40

14. A toy retailer buys toys from the toy company and marks the price up 25% to sell to customers. If the retailer has a sale of 80% off retail price, what is its percent loss in terms of the original amount it paid for the toys?

(A) 25%
(B) 30%
(C) 40%
(D) 75%
(E) 80%

15. Mr. Choudury's fourth-grade class consists of 20 students: 12 boys and 8 girls. If the boys weigh an average of 80 pounds each, and the girls weigh an average of 70 pounds each, what is the average weight in pounds of all 20 students?

(A) 71
(B) 74
(C) 75
(D) 76
(E) 79

16. It costs a certain bicycle factory $11,000 to operate for one month, plus $300 for each bicycle produced during the month. Each of the bicycles sells for a retail price of $700. What is the minimum number of bicycles that the factory must sell in one month to make a profit?

(A) 26
(B) 27
(C) 28
(D) 29
(E) 30

17. The yoga company Yoga for Life offers 45-minute classes at $12 per class. If the number of minutes Randolf spent doing yoga this month was 132 greater than the number of dollars he paid, how many classes did he attend?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 8

18. When Mark fills his car with regular gasoline, he gets 20 miles/gallon. When he fills his car with premium gasoline, he gets 25 miles/gallon. If the price of regular gasoline is $4.00 per gallon and the price of premium gas is $6.25 per gallon, then the cost efficiency of regular gasoline, in miles/dollar, is what percent greater than the cost efficiency of premium gasoline?

(A) 4%
(B) 10%
(C) 20%
(D) 25%
(E) 50%

19. An online merchant sells wine for $20 a bottle or $220 for a case of 12. Either way, his cost for the wine is $10 per bottle. Shipping costs him $5 for a bottle and $40 for a case. If, in a month, he sells 12 cases and 60 bottles and has no other revenue or expenses, his profit is equal to which of the following?

(A) $780
(B) $1,020
(C) $2,160
(D) $2,640
(E) $3,840

20. If everyone who contributed to a charity drive paid at least $14 and $237 was collected, what is the maximum number of people who could have contributed?

(A) 13
(B) 14
(C) 15
(D) 16
(E) 17

21.

In a certain barter system, one sack of rice can be traded for two and a half chickens or a third of a medallion.

Quantity A	**Quantity B**
The value of a chicken in sacks of rice	The value of a medallion in sacks of rice

22. A hunting lodge has enough fuel to keep 20 rooms heated for fourteen days. If the lodge decides to save fuel by turning off the heat in 5 unoccupied rooms, and each room requires the same amount of fuel to heat it, how many extra FULL days will the fuel supply last?

(A) 3
(B) 4
(C) 5
(D) 18
(E) 19

23. Last year, a magazine charged a $50 subscription fee. This year, the price will be increased by $10. If the magazine could lose 4 subscribers this year and still collect the same revenue as it did last year, how many subscribers did the magazine have last year?

(A) 20
(B) 21
(C) 22
(D) 23
(E) 24

24. Store Y sells two brands of socks, one at $3 a pair and the other at $4 a pair. If Janine must spend exactly $29 on socks and there is no sales tax, how many pairs of socks can she buy?

Indicate all such values:

☐ 6
☐ 7
☐ 8
☐ 9
☐ 10

Word Problems Answers

1. **(C).** Break the trip into two parts: the first mile and the final 3.5 miles. The first mile costs \$8, and the final 3.5 miles cost \$1 per 1/4 mile, or \$4 per mile. $8 + 3.5(4) = 8 + 14 = 22$.

2. **(E).** Rather than assigning separate variables to the granddaughters and grandsons, define them both in terms of the same unknown multiplier, based on the ratio given:

Number of granddaughters = $3m$
Number of grandsons = m

Note that you are solving for $3m$, not simply for m!

$3m + m = 12$
$4m = 12$
$m = 3$
$3m = 9$

SHORTCUT: Another method depends on the same underlying logic, but forgoes the algebra. Suppose that Nash had exactly one grandson and three granddaughters. That would add to four grandchildren altogether. Triple the number of grandsons and granddaughters to triple the number of grandchildren.

Further, note that only (D) and (E) are greater than half of 12, and that 8 isn't a multiple of 3.

3. **4,000.** If Deepak pays 30% in taxes, his take-home pay after taxes is 70%. Since this amount is equal to \$2,800:

$0.70x = 2{,}800$
$x = 4{,}000$

4. **(C).** This problem requires you to know that profit equals revenue minus cost. You should memorize the formula Profit = Revenue - Cost (or Profit = Revenue - Expenses), but you could just think about it logically — of course a business has to pay its expenses out of the money it makes: the rest is profit.

The question tells you that the cost is \$1,750. If the theater charges \$6 per ticket, the revenue will be equal to 6 times the number of customers. Let c be the number of customers. The revenue is $6c$. Lastly, the question asks for the number of customers so that the profit is \$1 per customer. That means that the profit will be \$1 times the number of customers, or c. Plug these values into the equation Profit = Revenue - Cost:

$c = 6c - 1750$
$-5c = -1750$
$c = 350$

5. **(D).** Let x = the total number of tickets sold. Therefore, $(x - 100)$ = the number of tickets Arnaldo sold beyond the first 100

$11x + 2(x - 100) = 2{,}400$
$11x + 2x - 200 = 2{,}400$
$13x = 2{,}600$
$x = 200$

6. **(C).** Divide \$6,450 by \$85 to get 75.88…. But don't just round up! You are told that each person gave at least \$85. If 76 people attended and each gave the minimum of \$85, then \$6,460 would have been collected. Since only \$6,450 was collected, that 76th person could not have attended. You need to round down to 75. (This means at least one person gave more than the minimum.)

7. **(A).** Simply list Eva's meditation sessions and breaks:

10:10 - 10:30	session 1
10:30 - 10:35	break
10:35 - 10:55	session 2
10:55 - 11:00	break
11:00 - 11:20	session 3

Note that you are asked for the time when she completes her third session, so do not add a third break!

A quicker way to do this problem would be to add 20(3) + 5(2) to get 70 minutes. 70 minutes after 10:10 is 11:20.

8. **(B).** You *could* simply list Derek's activities:

12:30 - 1:05	load 1
1:05 - 1:11	unload/reload
1:11 - 1:46	load 2
1:46 - 1:52	unload/reload
1:52 - 2:27	load 3
Etc.	

However, completing this rather tedious list all the way up to 6:35pm is not a good expenditure of time on the GRE. A better approach would be to determine how many minutes are available for Derek to do laundry. From 12:30 to 6:35 is 6 hours and 5 minutes, or 365 minutes. (Many students make silly mistakes here by calculating as though there were 100 rather than 60 minutes in an hour!)

It takes 41 minutes to do one load of laundry and then switch to the next one (34 + 4 + 2 minutes).

Divide 365 minutes by 41 to get 8.9…. So, Derek can definitely do 8 total loads of laundry plus switching time.

What about that extra 0.9…? You need to figure out whether Derek can fit in one more laundry load. Importantly, for this last load he needs only 2 extra minutes to unload, since he will not be reloading the machine.

Multiply 8 (the total number of loads Derek can definitely do) by 41 minutes to get 328 minutes. Subtract 328 from the 365 available minutes to get 37 minutes. Amazingly, that is *exactly* how much time it takes Derek to do one load of laundry (35 minutes) and then unload it (2 minutes). So, Derek can wash and unload 9 total loads of laundry.

9. **(A).** This is an algebraic translation, meaning you need to translate the text into algebra. Use k to represent Kendra's

age, and you know $k > 5$ (they only tell you this because Quantity B requires you to consider Kendra's age five years ago, and if she were younger than five years old, that would create an impossible negative age!).

Quantity A becomes $2k - 5$, while Quantity B becomes $2(k - 5)$ or $2k - 10$. From here, the easiest thing is to manipulate the columns as if they were a giant inequality:

$$2k - 5 > 2k - 10$$

(The direction of the inequality sign in the center doesn't matter.)

The $2k$ on either side cancels, and you're left with -5 on the left and -10 on the right. -5 is bigger than -10, so Quantity A is bigger.

You could also solve this question by plugging in values, but there's no need, as the algebra is nice and simple. The answer is (A).

10. **(D).** This is an algebraic translation question, so you should start by translating the given information into algebra. Start by creating variables, A for Allie and B for Billy. Notice that the question is asking about six years ago, so you need to subtract six from both Billy and Allie's ages:

$$B - 6 = 2(A - 6)$$
$$B - 6 = 2A - 12$$
$$B = 2A - 6$$

That's as far as you can take the algebra. From here, Quantity A asks you for $B - A$ (we know that Billy is older from the given information). You can't solve for that with the equation you've been given, so you should quickly look at some values:

$$B = 2A - 6$$

Notice that you can't pick whatever values you want here. You need both Billy and Allie to be older than 6, or else the issue of six years ago won't make a lot of sense (you don't want negative ages!). Start out with Allie at 7 years old. That would make Billy 8 years old.

$$8 - 7 = 1$$

In this case, Quantity B is bigger, so the answer has to be (B) or (D).

Because you've minimized A and B, you should try something big next. If A were 100, B would be 194.

$$B - A = 194 - 100 = 94$$

In this case, Quantity A is bigger, so the answer is (D).

11. **50.** Since Profit = Revenue - Expenses, and \$12 for a car wash multiplied by 20 car washes = \$240:

$$190 = 240 - E$$

Subtract 240 from each side, then multiply each side by -1 to flip the signs:

$190 = 240 - E$
$-50 = -E$
$50 = E$

12. **(A).** Since Profit = Revenue - Expenses, and you are told that revenue = $720:

$P = 720 - E$

Expenses are equal to $22 per hour times 8 hours, plus a fixed $160, or 22(8) + 160 = $336. Thus:

$P = 720 - 336$
$P = 384$

13. **(B).** 12 gallons of regular are needed to go 300 miles (300 divided by 25 miles per gallon), costing $36 (12 gallons × $3 per gallon). 10 gallons of premium would be needed to go 300 miles (300 divided by 30 miles per gallon), costing $40 (10 gallons × $4 per gallon). You need the difference, so $40 - $36 = $4.

14. **(D).** For problems like this that ask for percents and use no real numbers, it is almost always easy and possible to use 100 as a starting number. Suppose the toy store buy the toys for $100, and marks them up to $125. An 80% sale will drop the price down to $25. The percent loss is the amount it loses per toy expressed as a percentage of the original price per toy:

$(100 - 25)/100 \times 100 = 75\%$

15. **(D).** The most straightforward approach is to determine the total weight of all 20 students, and divide that total by 20.

12 boys × 80 pounds per boy = 960 pounds
8 girls × 70 pounds per girl = 560 pounds
Total = 1,520 pounds

1520/20 = 76

SHORTCUT: Pretty much all GRE multiple-choice weighted average problems have the same five answers:

Much closer to the lesser value
A little closer to the lesser value
The unweighted average of the two values
A little closer to the greater value
Much closer to the greater value

Any of these five choices *could* be correct, but the correct answer is usually "a little closer to the lesser value" or "a little closer to the greater value." In this case, because there are a few more boys than girls, the weighted average will be a little closer to the boys' average than to the girls'.

16. **(C).** The question asks how many bicycles the factory must sell to make a profit. One way of phrasing that is to say

the profit must be greater than 0. Since Profit = Revenue - Cost, you can rewrite the equation to say:

Revenue - Cost > 0

Let b equal the number of bicycles sold. Each bike sells for \$700, so the total revenue is $700b$. The cost is equal to \$11,000 plus \$300 for every bicycle sold.

$(700b) - (11{,}000 + 300b) > 0$

Isolate b on one side of the inequality:

$700b - 11{,}000 + 300b > 0$
$400b - 11{,}000 > 0$
$400b > 11{,}000$
$b > 27.5$

If b must be greater than 27.5, then the factory needs to sell at least 28 bicycles to make a profit.

17. **(B).** The normal way to do this problem would be to assign variables and set up equations, using X to represent the number of classes Randolf took, $12X$ to represent the amount he paid, and $45X$ to represent the number of minutes he spent.

A quicker way might be to notice that with every class Randolf takes, the difference between the number of minutes he spends and the amount he pays increases by 33. If Randolf takes 1 class, then the number of minutes he spends is 33 greater than the number of dollars he pays. If he takes 2 classes, the number of minutes is 66 greater than the number of dollars, and so on. $132 = 4 \times 33$, so Randolf must have taken 4 classes.

18. **(D).** In order to solve this problem, you first need to convert each of the gasoline types into miles/dollar. To do this, you take the miles/gallon and divide it by dollars/gallon. Thus, for regular gasoline, if it gets 20 miles/gallon and each gallon costs \$4.00, then the miles/dollar is $\frac{20}{4} = 5$. Similarly, the miles/dollar of premium gasoline is $\frac{25}{6.25} = 4$.

To express this as a percentage, use the equation:

$$\text{Percent Increase} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\text{Thus, } \frac{5-4}{4} \times 100 = 25\%.$$

19. **(B).** Profit is equal to Revenue - Expenses. First, calculate revenue:

12 cases sold for \$220 each = \$2,640
60 bottles sold for \$20 each = \$1,200

TOTAL REVENUE = $3,840

Now, calculate expenses. How many total bottles of wine were sold? 12 cases × 12 bottles, plus 60 individual bottles = 204 bottles. Note that the bottles sold individually versus those sold in cases have the same inventory cost ($10), but different shipping costs. Thus:

204 bottles at $10 each = $2,040
Shipping on bottles = 60 × $5 = $300
Shipping on cases = 12 × $40 = $480
TOTAL EXPENSES = $2,820

Profit = Revenue - Expenses
Profit = $3,840 - $2,820
Profit = $1,020

20. **(D).** This is a maximization question. To solve maximization questions, you often have to minimize something else. In order to get the most people involved here, you need the donation per person to be as small as possible. In this case, everyone could pay exactly $14:

237/14 = 16.92

You can't round up to 17, because it is not possible that 17 people donated $14 each (you would end up with something bigger than $237). The answer is (D), or 16.

21. **(B).** If one sack of rice is worth one-third of a medallion, buying the whole medallion would require three sacks of rice. Quantity B is equal to 3. The math is a bit tougher in Quantity A, but no calculation is really required — if a sack of rice gets you 2.5 chickens, a single chicken is worth less than a sack of rice. Quantity A is less than 1.

22. **(D).** The lodge has 20(14) = 280 "fuel-days" of fuel. (A "fuel-day" is enough fuel for 1 room for 1 day.) If the lodge only needs to heat 15 rooms instead of 20, divide 280 by 15 to get 18.666…. You are asked for FULL days, so round down to get 18.

23. **(E).** Assign the variable s for subscribers.

Last year:
$50 per subscription
s subscribers

This year:
$60 per subscription
s - 4 subscribers

You are told that the magazine "could" lose 4 subscribers and that the magazine would then collect the same revenue as last year — don't let the "could" throw you off. You are being asked to calculate using this hypothetical situation:

$50s = 60(s - 4)$
$50s = 60s - 240$
$-10s = -240$
$s = 24$

24. **III and IV only.** Let's say that Janine buys x of the first pair of socks and y of the second. This means that she spends $3x$ on the first and $4y$ on the second. You can rephrase the question as follows: if $3x + 4y = 29$, what could be the value of $x + y$? (Keep in mind that x and y must be integers.)

Consider all multiples of 4 that are less than 29, and find which of those multiples, when subtracted from 29, will leave you with a multiple of 3. Here are all the multiples of 4 under 29: 4, 8, 12, 16, 20, 24, 28. Only 8 and 20 can leave you with a multiple of 3 when subtracted from 29. If Janine spends \$8 on the second brand of socks, she buys 2 pairs. She then has \$21 to spend on the first brand of socks, meaning that she can buy 7 pairs of the first brand. If you add up the pairs of socks, she buys $2 + 7 = 9$ pairs. If she spends \$20 on the second brand of socks, which gets her 5 pairs, then she has \$9 left to spend on the first brand, at \$3 per pair, which lets her buy 3 pairs. In this scenario she can buy $3 + 5 = 8$ pairs. The correct answers are 8 and 9 pairs.

Chapter 17
of

Two-Variable Word Problems

In This Chapter…

Two-Variable Word Problems

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. There are five more computers in the office than employees. If there are 10 employees in the office, what is the ratio of computers to employees in the office?

(A) 2 : 3
(B) 2 : 5
(C) 3 : 2
(D) 3 : 5
(E) 5 : 2

2. Two parking lots can hold a total of 115 cars. The Green lot can hold 35 fewer cars than the Red lot. How many cars can the Red lot hold?

(A) 35
(B) 40
(C) 70
(D) 75
(E) 80

3. Three friends sit down to eat 14 slices of pizza. If two of the friends eat the same number of slices, and the third eats two more slices than each of the other two, how many slices are eaten by the third friend?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

4.

In 8 years, Polly's age (which is currently *p*) will be twice Quan's age (which is currently *q*).

Quantity A	**Quantity B**
$p - 8$	$2q$

5. Pens cost 70 cents each, and pencils cost 40 cents each. If Iris spent $5.20 on 10 pens and pencils, how many pencils did she purchase? ($1 = 100 cents)

(A) 4
(B) 6
(C) 8
(D) 10
(E) 13

6. Jack downloaded ten songs and two books for $48, Jill downloaded fifteen songs and one book for $44. How much did Jack spend on books, if all songs are the same price and all books are the same price?

(A) $14
(B) $20
(C) $28
(D) $29
(E) $30

7. Marisa has $40 more than Ben, and Ben has one-third as much money as Marisa. How many dollars does Ben have?

$ $\boxed{}$

8. Norman is 12 years older than Michael. In 6 years, he will be twice as old as Michael. How old is Michael now?

(A) 3
(B) 6
(C) 12
(D) 18
(E) 24

9. 3 people split a $100 stereo but pay different amounts. If A pays $5 less than B, C pays more than $35, and all 3 people pay integer amounts, what is the most A could pay?

(A) 29
(B) 29.5
(C) 30
(D) 33

(E) 34

10. Krunchy Kustard sells only two kinds of doughnuts, glazed and cream-filled. A glazed doughnut has 200 calories, and a cream-filled doughnut has 360 calories. If Michael ate 5 doughnuts totaling 1,640 calories, how many were glazed?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

11. Olympic lifting consists of two disciplines, the Snatch, and the Clean and Jerk. Halil's best Snatch and best Clean and Jerk sum to 295 kilograms. If his best Clean and Jerk was 25 kilograms heavier than his best Snatch, what was the weight of his best Clean and Jerk?

(A) 135
(B) 142.5
(C) 145
(D) 147.5
(E) 160

12. The "aspect ratio" of a computer monitor is the ratio of the monitor's width to its height. If a particular monitor has an aspect ratio of 16 : 9, and a perimeter of 100 inches, how many inches wide is the monitor?

(A) 18
(B) 25
(C) 32
(D) 36
(E) 64

13. Cindy bought 48 containers of soda, all either 12-ounce cans or 20-ounce bottles. If the number of ounces she purchased in cans was equal to the number of ounces she purchased in bottles, how many bottles of soda did Cindy buy?

(A) 18
(B) 21
(C) 24
(D) 27
(E) 30

14. Red chips all have the same value as one another, blue chips all have the same value as one another, and yellow chips also all have the same value as one another. If the value of a red chip plus a blue chip is 4.25, the value of a blue chip plus a yellow chip is 2.75, and the value of a red chip plus a blue chip plus a yellow chip is 4.5, what is the value of a red chip plus a yellow chip?

(A) 0.25
(B) 2
(C) 2.25
(D) 2.75
(E) 3

15. Two runners' race times add to 170 seconds and one of the race times is ten seconds less than twice the other. What is the faster race time, in seconds?

(A) 40
(B) 50
(C) 60
(D) 70
(E) 110

16. Beth is twelve years younger than Alan. In 20 years, Beth will be 80% of Alan’s age. How old is Beth now?

$\boxed{}$ years old

17. Rey is 12 years younger than Sebastian. Five years ago, Rey was half Sebastian’s age. How old will Sebastian be next year?

(A) 15
(B) 20
(C) 25
(D) 30
(E) 35

18. During a sale, the local outlet of the Chasm sold three times as many jeans as chinos. If they made twice as much profit for a pair of chinos as for a pair of jeans, and sold no other items, what percent of their profits during the sale came from chinos?

(A) $16\frac{2}{3}\%$
(B) 20%
(C) 40%
(D) 60%
(E) $83\frac{1}{3}\%$

19. Marisol is twice as old as Vikram. Eight years ago, Marisol was 6 years younger than three times Vikram’s age at that time. How old will Marisol be in 5 years?

$\boxed{}$

20. Mark is twice as old as Vicky. Four years ago, Mark was 6 years younger than three times Vicky’s age at that time. How old will Mark be in 2 years?

$\boxed{}$

21. The length of a rectangle is two more than twice the width, and the area of the rectangle is 40. What is the rectangle’s perimeter?

$\boxed{}$

22. Marcy bought one pair of jeans at 70% off and one blouse at 40% off. If she paid $12 more for the blouse than for the jeans, and she spent a total of $84, what was the original price of the jeans?

(A) 76
(B) 96
(C) 100
(D) 120
(E) 124

23. Cranwell Golf Course offers two different pricing packages for golf lessons. Under the "Sapphire" pricing plan, lessons can be bought for a flat rate of $80 per hour. Under the "Diamond" pricing plan, for an initial fee of $495, lessons can be bought for a rate of $15 per hour. If Jeanie buys the "Diamond" pricing plan, how many golf lessons does she need to take in order to have spent exactly 40% less than she would have under the "Sapphire" plan?

(A) 10
(B) 12
(C) 15
(D) 18
(E) 20

24. Wall-to-wall carpeting is installed in a certain hallway. The carpeting costs $4.25 per square foot. If the perimeter of the hallway (in feet) is equal to 44% of the area of the hallway (in square feet) and the hallway is 50 feet long, how much did it cost to install the carpeting?

(A) $182.50
(B) $212.50
(C) $505.25
(D) $1,062.50
(E) $1,100.00

25. Jamal gets three monthly credit card statements over the course of three months. If his average monthly statement over these three months is $44 more than the median amount, and the sum of the largest and the smallest statement is $412, what is the total amount that Jamal spent over these three months?

(A) $456
(B) $552
(C) $600
(D) $824
(E) $1,000

26. A certain kennel houses only collies, labs, and golden retrievers. If the ratio of collies to labs is 5 : 9, there are 66 golden retrievers, and 12 more golden retrievers than labs, what percent of the total number of dogs in the kennel are collies?

(A) 5%
(B) 9%
(C) 12%
(D) 20%
(E) 25%

27. If Mason is now twice as old as Gunther was 10 years ago, and G is Gunther's current age in years, which of the following represents the sum of Mason and Gunther's ages 4 years from now?

(A) $\frac{3G}{2}+3$
(B) $3G + 28$
(C) $3G - 12$
(D) $8 - G$
(E) $14-\frac{3G}{2}$

28. A baker makes a combination of chocolate chip cookies and peanut butter cookies for a school bake sale. His recipes only allow him to make chocolate chip cookies in batches of 7, and peanut butter cookies in batches of 6. If he makes exactly 95 cookies for the bake sale, what is the minimum number of chocolate chip cookies that he makes?

(A) 7
(B) 14
(C) 21
(D) 28
(E) 35

29. Janie has 5 fewer candies than Mark. If Janie gives Mark 5 candies, Mark will then have 4 times as many candies as Janie. How many candies does Janie have?

(A) 5
(B) 10
(C) 15
(D) 20
(E) 25

30. If Standard Jeans cost $60 and Designer Jeans cost 150% more, and 29 total pairs of jeans are sold for a total of $3,540, how many pairs were Designer Jeans?

(A) 2
(B) 9
(C) 18
(D) 20
(E) 23

31. Lou has three daughters: Wen, Mildred, and Tyla. Three years ago, when Lou was twice as old as Tyla, he was thirty years older than Mildred. Now, he is forty-seven years older than Wen. In four years, Wen will be half as old as Tyla. What is Lou’s, Wen’s, Mildred’s and Tyla’s combined age?

(A) 138
(B) 144
(C) 154
(D) 166
(E) 181

32. A farmer has exactly 1,000 square feet of farmland, on which he can grow both soy and corn. Every square foot can produce either one pound of soy or three pounds of corn. If soy can be sold on the market for $12/pound for the first hundred pounds and then $6 per pound after that, and corn can be sold on the market for $10/pound, what is the number of square feet of farmland that the farmer should devote to soy to make a profit of exactly $13,080?

(A) 30
(B) 100
(C) 270
(D) 600
(E) 730

33. Dwayne planted 70 acres with two types of field beans, navy beans, and pinto beans. Each acre of navy beans yielded 27 bushels, and each acre of pinto beans yielded 36 bushels. If Dwayne grew twice as many bushels of pinto beans as navy beans, how many acres of pinto beans did he plant?

(A) 28
(B) 30
(C) 35
(D) 40
(E) 42

Two-Variable Word Problems Answers

1. **(C).** Let c = number of computers. Let e = number of employees

There are 5 more computers than employees. You can translate that into an equation:

$c = e + 5$
If $e = 10$, then $c = (10) + 5$
$c = 15$

The ratio of computers to employees is 15 : 10, which can be reduced to 3 : 2.

2. **(D).** Let g = the number of cars that the Green lot can hold. Let r = the number of cars that the Red lot can hold

The first two sentences can be translated into two equations:

$g + r = 115$
$g = r - 35$

You want to solve for r, so you should substitute (r - 35) for g in the first equation:

$(r - 35) + r = 115$
$2r - 35 = 115$
$2r = 150$
$r = 75$

3. **(D).** Let P = the number of slices of pizza eaten by each of the two friends who eat the same amount. Let T = the number of slices of pizza eaten by the third friend.

$T = P + 2$
$P + P + T = 14$

Substitute (P + 2) for T in the second equation:

$P + P + (P + 2) = 14$
$3P + 2 = 14$
$3P = 12$
$P = 4$

You can use the value of P to solve for T:

$T = P + 2 = 4 + 2 = 6$

4. **(C).** This is an algebraic translation question, so you should start by translating the given information into equations.

Remember to add eight to both Polly and Quan's ages, because they will *both* be eight years older in eight years!

$p + 8 = 2(q + 8)$
$p + 8 = 2q + 16$
$p = 2q + 8$

Looking at the two columns, you can see that it would be helpful to manipulate the equation one last time:

$p - 8 = 2q$

You can see from that equation that the two columns are equal. The answer is (C).

5. **(B).** Many questions that involve two unknowns (e.g., the number of pens and the number of pencils) can be translated either as one equation involving one variable, or two equations involving two variables each.

It's a bit more work to *translate* using just one variable than using two, but generally much less work to *solve* one equation with one variable than to solve a system of two equations with two variables.

With One Variable

First, assign one variable to the pencils, and then define the pens in terms of the pencils:

Number of pencils = x
Number of pens = $10 - x$

Since this problem describes a real-life situation, it is not too difficult to write the formula:

(Cost per pen × number of pens) + (cost per pencil × number of pencils) = total cost

Plugging in the numbers from the problem (70 cents per pen and 40 cents per pencil):

$70(10 - x) + 40x = 520$
$700 - 70x + 40x = 520$
$700 - 30x = 520$
$180 = 30x$
$x = 6$

With Two Variables

First, assign one variable to the pencils, and another variable to the pens.

Number of pencils = x
Number of pens = y

$x + y = 10$
$70y + 40x = 520$

Next, isolate y (from the first, simpler equation) so you can substitute:

$y = 10 - x$

Substitute $10 - x$ for y:

$$70(10 - x) + 40x = 520$$
$$700 - 70x + 40x = 520$$
$$700 - 30x = 520$$
$$180 = 30x$$
$$x = 6$$

SHORTCUT: This question allows backsolving, choosing an answer choice to see whether it works. Generally, you'll want to start with answer choice (C); if it turns out to be too large, you can eliminate it and the two answer choices larger than it; if it turns out to be too little, you can eliminate it and the two answer choices less than it. (C) is not always the most efficient answer choice to test first, though. Here, for instance, (D) and (E) are implausibly large, so start with (B), the middle value of the remaining answers.

But proceed without that insight. Assume that Iris bought 8 pencils, and therefore 2 pens. $8 \times 40 + 2 \times 70 = 320 + 140 = 460$. That's 60 cents too little, so Iris must have bought fewer pencils and more pens. Try 6 pencils and 4 pens. $6 \times 40 + 4 \times 70 = 240 + 280 = 520$. (You might also have noticed that every time Iris swaps a pencil for a pen, she spends an extra 30 cents.)

6. **(C).** The equations are $10s + 2b = 48$ and $15s + b = 44$. The easiest next move would be to solve the second equation for b:

$$b = 44 - 15s$$

Substitute that into the first equation:

$$10s + 2(44 - 15s) = 48$$
$$10s + 88 - 30s = 48$$
$$-20s + 88 = 48$$
$$-20s = -40$$
$$s = 2$$

Plug $s = 2$ back into whichever equation you prefer to get that $b = 14$, and thus TWO books cost \$28.

7. **20.** Write both facts from the problem as simple equations:

$$M = B + 40$$
$$B = \frac{1}{3}M$$

Since you want the dollar amount for Ben, substitute for the other variable, M. Since M equals $B + 40$, write $B + 40$ in parentheses in place of M in the second equation:

$$B = \frac{1}{3}(B + 40)$$

Now distribute:

$$B = \frac{B}{3} + \frac{40}{3}$$

Subtract $\frac{B}{3}$ from both sides (note that $\frac{B}{3}$ is the same as $\frac{1}{3}B$, so subtracting $\frac{1}{3}B$ from B will give you $\frac{2}{3}B$, or $\frac{2B}{3}$.

$$\frac{2B}{3} = \frac{40}{3}$$
$$2B = 40$$
$$B = 20$$

The final answer is \$20.

Alternatively, you could reason that if Ben has one-third what Marisa does, then he's *missing* two-thirds of her amount. Since that two-thirds turns out to equal \$40, then two-thirds of Marisa's money is \$40. Thus, her total amount is \$60. Divide by 3 to get Ben's amount, \$20.

8. **(B).** Let N = Norman's age now $(N + 6)$ = Norman's age in 6 years.
Let M = Michael's age now $(M + 6)$ = Michael's age in 6 years.

Translate the first two sentences into equations. Note that the second equation deals with Norman and Michael's ages in 6 years:

$$N = M + 12$$
$$(N + 6) = 2(M + 6)$$

You want to solve for N, so substitute $(M + 12)$ for N in the second equation:

$$(M + 12) + 6 = 2(M + 6)$$
$$M + 18 = 2M + 12$$
$$M + 6 = 2M$$
$$6 = M$$

9. **(A).** This is a maximization question (what is the *most* A could pay). In order to solve maximization questions, you often have to minimize the other terms. In this case, you need to minimize B and C in order to maximize A.

The minimum possible C is \$36, leaving \$64 for A and B to pay. Since B pays \$5 more than A:

$$B = A + 5$$
$$A + (A + 5) = 64$$
$$2A = 59$$
$$A = 29.5$$

Of course, this isn't possible, because the three people have to pay integer values. So you need to move C up to \$37, leaving \$63 for A and B to pay.

$$A + (A + 5) = 63$$
$$2A = 58$$
$$A = 29$$

The correct answer is (A), or \$29.

10. **(A).** Many questions that involve two unknowns (e.g., the number of glazed doughnuts and the number of cream-filled doughnuts) can be translated either as one equation involving one variable, or two equations involving two variables each.

It's a bit more work to *translate* using just one variable than using two, but generally much less work to *solve* one equation with one variable than to solve a system of two equations with two variables.

With One Variable

Michael ate 5 doughnuts.

Number of glazed = x
Number of cream-filled = $5 - x$

(number of glazed × calories per glazed) + (number of cream-filled × calories per cream-filled) = total calories

$$200x + 360(5 - x) = 1{,}640$$
$$200x + 1{,}800 - 360x = 1{,}640$$
$$-160x = -160$$
$$x = 1$$

With Two Variables

Number of glazed = x
Number of cream-filled = y

$x + y = 5$
$200x + 360y$ Isolate y in first equation to allow substitution.

$y = 5 - x$ Substitute $5 - x$ for y in second equation.

$$200x + 360(5 - x) = 1{,}640$$
$$200x + 1{,}800 - 360x = 1{,}640$$
$$-160x = 160$$
$$x = 1$$

Alternative Method

This question allows backsolving, choosing an answer choice to see whether it works. Generally, you'll want to start with answer choice (C); if it turns out to be too great, you can eliminate it and the two answer choices less than it; if it turns out to be too little, you can eliminate it and the two answer choices less than it.

Assume that Michael ate 3 glazed and therefore 2 cream-filled doughnuts. $3(200) + 2(360) = 600 + 720 = 1{,}320$. That's 320 calories too few. You might notice that every time Michael swaps a glazed for a cream-filled, he consumes another 160 calories. Since $2(160) = 320$, he needs to swap 2 of his 3 glazed for cream-filled. If you don't notice that, just try (B) 2 glazed and 3 cream-filled, and see that yields only 1,480 calories.

11. **(E).** Many questions that involve two unknowns (e.g., the weight of the Snatch and the weight of the Clean and Jerk) can be translated either as one equation involving one variable, or two equations involving two variables each.

It's a bit more work to *translate* using just one variable than using two, but generally much less work to *solve* one equation with one variable than to solve a system of two equations with two variables.

With One Variable

Halil's best Snatch and best Clean and Jerk sum to 295 kilograms.

Weight of best Clean and Jerk = x
Weight of best Snatch = $295 - x$

His best Clean and Jerk was 25 kilograms heavier than his best Snatch.

$$x = (295 - x) + 25$$
$$x = 320 - x$$
$$2x = 320$$
$$x = 160$$

With Two Variables

Weight of best Clean and Jerk = x
Weight of best Snatch = y

$x + y = 295$ Isolate y to allow substitution.

$y = 295 - x$ Substitute $295 - x$ for y.

$$x = (295 - x) + 25$$
$$x = 320 - x$$
$$2x = 320$$
$$x = 160$$

SHORTCUT: You could also subtract the difference (25) from the sum (295), then divide the result by 2. This will give you the weight of the *lighter* lift. Or, you might notice simply that the Clean and Jerk must be more than half of the total of 295. Half of 295 is 147.5, and only one answer is greater than that.

12. **(C).** Rather than assigning separate variables to the width and height, define them both in terms of the same unknown multiplier, based on the ratio given:

Width = $16m$
Height = $9m$

Note that, since you want the width, YOU ARE SOLVING FOR $16m$, NOT SIMPLY FOR m!

The perimeter of a rectangle = 2(length + width), or in this case 2(width + height)

$100 = 2 \times (16m + 9m)$
$100 = 50m$
$m = 2$
$16m = 32$

SHORTCUT: Another method depends on the same underlying logic, but forgoes the algebra.

Suppose the dimensions were simply 16 inches and 9 inches. This would yield a perimeter of 50 inches. Double the width and height to double the perimeter.

13. **(A).** Many questions that involve two unknowns (e.g., the number of bottles and the number of cans) can be translated either as one equation involving one variable, or two equations involving two variables each.

With One Variable

Cindy bought 48 containers of soda
Number of bottles = x
Number of cans = $48 - x$

The number of ounces she purchased in cans was equal to the number of ounces she purchased in bottles
(ounces/can)(number of cans) = (ounces/bottle)(number of bottles)

$12(48 - x) = 20x$
$576 - 12x = 20x$
$576 = 32x$
$x = 18$

With Two Variables

Number of bottles = x
Number of cans = y
Cindy bought 48 containers of soda

$x + y = 48$

the number of ounces she purchased in cans was equal to the number of ounces she purchased in bottles

(ounces/can)(number of cans) = (ounces/bottle)(number of bottle)

$12y = 20x$ Isolate y in first equation to allow substitution.

$y = 48 - x$ Substitute into second equation.

$$12(48 - x) = 20x$$
$$576 - 12x = 20x$$
$$576 = 32x$$
$$x = 18$$

Alternative Method

This question allows backsolving, choosing an answer choice to see whether it works. Generally, you'll want to start with answer choice (C); if it turns out to be too large, you can eliminate it and the two answer choices larger than it; if it turns out to be too little, you can eliminate it and the two answer choices less than it. There are exceptions, though. Here, for instance, you notice that (C), (D) and (E) are implausibly large. There must be fewer bottles than cans, if the total volumes of the (large) bottles is to equal the total volume of the (small) cans, which means that the answer must be either (A) or (B).

Start with (B). Assume that Cindy bought 21 bottles, and therefore 27 cans. $21 \times 20 = 420$, $27 \times 12 = 324$. These numbers aren't equal. Cindy must have bought fewer than 21 bottles, and more than 27 cans, and only (A) matches that.

SHORTCUT: Because (ounces/can)(number of cans) = (ounces/bottle)(number of bottles), the ratio of (number of bottles) to (number of cans) will be the inverse of the ratio of (ounces per bottle) to (ounces per can). So the ratio of bottles to cans is 12 : 20, or 3 : 5. This means that only 3/8 of all the containers are bottles, 3/(5 + 3). 3/8 of 48 is 18. This shortcut requires very little math, but a great deal of mathematical sophistication. It comes in handy, but don't worry if you didn't spot it. There are many valid ways to solve problems like this one.

14. **(B).** First, write all the statements as equations:

$$r + b = 4.25$$
$$b + y = 2.75$$
$$r + b + y = 4.5$$

Also write "$r + y$?" on your paper, as a reminder of what you are looking for.

If $r + b = 4.25$, then $r + b + y = 4.5$ could be rewritten as:

$$4.25 + y = 4.5$$
$$y = 0.25$$

Since $b + y = 2.75$ and $y = 0.25$:

$$b + 0.25 = 2.75$$
$$b = 2.5$$

Since $r + b = 4.25$ and $b = 2.5$:

$$r + 2.5 = 4.25$$

$r = 1.75$

Therefore, $r + y = 1.75 + 0.25 = 2$.

15. **(C).** Call the race times x and y. Since you are told a sum:

$x + y = 170$

One of the race times is ten seconds less than twice the other

$x = 2y - 10$

Since the second equation is already solved for x, plug $(2y - 10)$ in for x in the first equation:

$2y - 10 + y = 170$
$3y - 10 = 170$
$3y = 180$
$y = 60$

If $y = 60$ and the times add to 170, then $x = 110$.

Note that you are asked for the *faster* race time — that means the smaller number! The answer is 60.

16. **28.** Since Beth is twelve years younger than Alan:

$B = A - 12$

To translate, *In 20 years, Beth will be 80% of Alan's age*, make sure that Beth becomes $B + 20$ and Alan becomes $A + 20$ (if you will be doing other operations to these values, put parentheses around them to make sure the rules of PEMDAS are not violated):

$B + 20 = 0.8(A + 20)$
$B + 20 = 0.8A + 16$
$B + 4 = 0.8A$

Since the first equation is already solved for B, plug $(A - 12)$ into the simplified version of the second equation in place of B.

$B + 4 = 0.8A$
$(A - 12) + 4 = 0.8A$
$A - 8 = 0.8A$
$0.2A - 8 = 0$
$0.2A = 8$
$A = 40$

Alan is 40. Since $B = A - 12$, Beth is 28. Since you are answering for Beth, the answer is 28.

17. **(D).** Many questions that involve two unknowns (e.g., Rey's age and Sebastian's age) can be translated either as one

equation involving one variable, or two equations involving two variables each.

It's a bit more work to *translate* using just one variable than using two, but generally much less work to *solve* one equation with one variable than to solve a system of two equations with two variables.

With One Variable

Rey is 12 years younger than Sebastian

s = Sebastian's age NOW
$s - 12$ = Rey's age NOW

In problems that involve values that change over time (ages, prices, number of employees, etc.), make sure to fix the variable to some particular time.

How old will Sebastian be next year?

Solve for $s + 1$

Five years ago, Rey was half Sebastian's age

Five years ago, Rey was $(s - 12) - 5$, and Sebastian was $(s - 5)$, so…

$(s - 12) - 5 = (s - 5)/2$
$s - 17 = (s - 5)/2$

Multiply both sides by 2 so you no longer have to deal with a fraction:

$2(s - 17) = s - 5$
$2s - 34 = s - 5$
$2s = s + 29$
$s = 29$
$s + 1 = 30$

With Two Variables

r = Rey's age NOW
s = Sebastian's age NOW

How old will Sebastian be next year?

Solve for $s + 1$

Rey is 12 years younger than Sebastian

$r = s - 12$

Five years ago, Rey was half Sebastian's age

$r - 5 = (s - 5)/2$

Substitute $s - 12$ for r.

$(s - 12) - 5 = (s - 5)/2$
$s - 17 = (s - 5)/2$

Multiply both sides by 2 to clear the fraction.

$2s - 34 = s - 5$
$2s = s + 29$
$s = 29$
$s + 1 = 30$

18. **(C).** If all the values given in a problem and its answers are *percents*, *ratios*, or *fractions of* some unknown, then the problem will probably be easiest to solve by stipulating values for the unknowns. In this problem, the two ratios given are 3 : 1 (jeans sold : chinos sold) and 2 : 1 (profits per pair of chinos : profits per pair of jeans). The easiest number to stipulate are:

3 pairs of jeans sold
1 pair of chinos sold
$2 profit/pair of chinos
$1 profit/pair of jeans

This yields $2 profit from the chinos, out of a total $5 in profit. 2/5 = 40%

Notice that this relatively simple problem is surprisingly complicated with variables. Even if you're clever enough to use just two variables rather than four, it's still a bit of work.

c = number of chinos sold
$3c$ = number of jeans sold
p = profit per jeans sold
$2p$ = profit per chinos sold

solve for (profit for chinos)/[(profit for chinos) + (profit for jeans)]

$(c \times 2p)/(c \times 2p + 3cp)$
$2pc/(5pc) = 2/5 = 40\%$

If you use four variables, you'll almost certainly make a translation error.

19. **49.** Write each sentence as its own equation:

Equation 1: $M = 2V$
Equation 2: $(M - 8) = 3(V - 8) - 6$

Next, simplify Equation 2 and substitute. Since Equation 1 is already solved for M, the easiest way to substitute is to plug in $2V$ in place of M in Equation 2.

$$\begin{aligned} 2V - 8 &= 3V - 24 - 6 \\ 2V - 8 &= 3V - 30 \\ 2V + 22 &= 3V \\ 22 &= V \end{aligned}$$

Thus, $M = 44$, and Marisol in 5 years, or $M + 5$, is equal to 49.

20. **30.** Translate the first two sentences in the problem into two separate equations:

Equation 1: $M = 2V$
Equation 2: $(M - 4) = 3(V - 4) - 6$

Next, simplify Equation 2 and substitute. Since Equation 1 is already solved for M, the easiest way to substitute is to plug in $2V$ in place of M in Equation 2.

$$\begin{aligned} 2V - 4 &= 3V - 12 - 6 \\ 2V - 4 &= 3V - 18 \\ 2V + 14 &= 3V \\ 14 &= V \end{aligned}$$

Thus, $V = 14$, $M = 28$, and Mark in 2 years, or $M + 2$, is equal to 30.

21. **28.** Convert this word problem into two equations with two variables. "The length is two more than twice the width" can be written as:

$$L = 2W + 2$$

Since the area is 40 and area is length × width:

$$LW = 40$$

Since the first equation is already solved for L, plug $(2W + 2)$ in for L into the second equation:

$$\begin{aligned} (2W + 2)W &= 40 \\ 2W^2 + 2W &= 40 \end{aligned}$$

Since you now have a quadratic on your hands (you have both a W^2 and a W term), get all terms on one side to set them to zero:

$$2W^2 + 2W - 40 = 0$$

Simplify as much as possible — in this case, divide the entire equation by 2 — before trying to factor:

$$\begin{aligned} W^2 + W - 20 &= 0 \\ (W - 4)(W + 5) &= 0 \\ W &= 4 \text{ or } -5 \end{aligned}$$

Since a width cannot be negative, the width = 4. Since $LW = 40$, the length must be 10.

Perimeter = $2L + 2W$
Perimeter = $2(10) + 2(4)$
Perimeter = 28

Note that it might have been possible for you to puzzle out that the sides were 4 and 10 just by trying values. However, if you did this, you got lucky — no one said that the values even had to be integers! The ability to translate into equations and solve is very important for the GRE.

22. **(D).** To solve this problem, establish the following variables:

J = original jean price
B = original blouse price

Then establish a system of equations, keeping in mind that "70% off" is the same as 100% - 70% = 30%, or 0.3, of the original price:

$0.3J + 12 = 0.6B$
$0.3J + 0.6B = 84$

Now simply use whatever strategy you're most comfortable with to solve a system of equations — for example, aligning the equations and then subtracting them:

$$\begin{array}{l} 0.3J + 12 = 0.6B \\ 0.3J - 84 = -0.6B \\ \hline 0 + 96 = 1.2B \end{array}$$

$B = 96/1.2$
$B = 80$

And you can plug the price of the blouse back into the original equation to get the price of the jeans:

$0.3J + 12 = 0.6B$
$0.3J + 12 = 48$
$0.3J = 36$
$J = 120$

Alternatively, you could first figure out the price of the discounted jeans with this equation:

$x + (x + 12) = 84$
$2x + 12 = 84$
$2x = 72$
$x = 36$

Then plug that discounted price into the equation *discounted price = original price × (100% - percent discount):*

$36 = 0.3P$

$360 = 3P$
$120 = P$

23. **(C).** Start by assigning variables:

D = price under the Diamond plan
S = price under the Sapphire plan
x = the number of lessons Jeanie takes

…then establish equations:

$D = 495 + 15x$
$S = 80x$
$0.6S = D$

…then solve by substitution:

$0.6S = 495 + 15x$
$0.6(80x) = 495 + 15x$
$48x = 495 + 15x$
$33x = 495$
$x = 15$

24. **(D).** The equation for the perimeter of a space is $2W + 2L = P$, where W is width and L is length.

The equation for the area is $A = W \times L$. Thus,

$0.44(W \times L) = 2W + 2L$
$0.44(50W) = 2W + 2(50)$
$22W = 2W + 100$
$20W = 100$
$W = 5$

If $W = 5$ and $L = 50$, then the total square footage of the room is 250, and the total cost is:

$4.25 × 250 = $1,062.50

25. **(B).** Call the smallest statement S, the middle statement M, and the largest statement L.

From your knowledge of medians, you should know that M is the same as the median, since there are only three values. The equation for averages is:

$$\frac{\text{Sum of \#'s}}{\text{Number of \#'s}} = \text{Average}$$

Incorporate the equation for averages into the following equation:

$$\frac{S+M+L}{3} = M + 44$$

$S + M + L = 3M + 132$

$S + L = 2M + 132$

While you don't know the actual quantities of *S* and *L*, you know their sum:

$412 = 2M + 132$

$280 = 2M$

$140 = M$

Finally, add *M* to the sum of *S* and *L*:

$140 + 412 = 552$

26. **(D).** Start by assigning variables:

C = Number of collies
L = Number of labs
G = Number of golden retrievers

…and work from what you know.

$G = 66$

$L = 66 - 12$

$L = 54$

Ratios work like fractions, and you can set them up accordingly.

$$\frac{5}{9} = \frac{C}{54}$$

Cross multiplying and simplifying, you get:

$C = 30$

Now take the number of collies and express it as a percentage of the total number of dogs:

Total # of Dogs = 30 + 54 + 66 = 150

$$\frac{30}{150} \times 100 = 20\%$$

27. **(C).** The sum of Mason and Gunther's ages 4 years from now requires adding 4 to BOTH ages.

First equation (what you are looking for):

$(M + 4) + (G + 4) = ?$
$M + G + 8 = ?$

Since Mason is twice as old as Gunther was 10 years ago, put $(G - 10)$ in parentheses and build the second equation from there (the parentheses are crucial).

Second equation:

$M = 2(G - 10)$
$M = 2G - 20$

Note that the answer choices ask for the sum of the ages 4 years from now, in terms of G, so substitute for M (the variable you substitute for is the one that drops out).

Substituting from the second equation into the first:

$(2G - 20) + G + 8 = ?$
$3G - 12 = ?$

This matches choice (C).

Alternatively, you could write the second equation, $M = 2(G - 10)$, and then come up with two values that "work" in this equation for M and G. The easiest way to do this is to make up G, which will then tell you M. For instance, set $G = 12$ (use any number you want, as long as it's over 10, since the problem strongly implies that Gunther has been alive for more than 10 years).

$M = 2(12 - 10)$
$M = 4$

If Gunther is 12, then Mason is 4. In four years, they will be 16 and 8, respectively. Add these together to get 24.

Now, plug $G = 12$ into each answer choice to see which yields the correct answer (for this example), 24. Only choice (C) works.

28. **(E).** The equation for the situation described is $7x + 6y = 95$, where x stands for the number of batches of chocolate chip cookies and y stands for the number of batches of peanut butter cookies.

It looks as though this equation is not solvable, because you have two variables and only one equation. However, since the baker can only make whole batches, x and y must be integers, which really limits the possibilities.

Furthermore, you want the *minimum* number of chocolate chip cookies the baker could have made. So, try 1 for x and see if you get an integer for y (use your calculator when needed!):

$7(1) + 6y = 95$
$6y = 88$
$y = 14.6\ldots$

Since this did not result in an integer number of batches of peanut butter cookies, this situation doesn't work. Try 2, 3, 4, etc. for x. (Don't try values out of order — remember, there might be more than one x value that works, but you need to be sure that you have the smallest one!)

You will see that the smallest value that works for x is 5:

$7(5) + 6y = 95$
$6y = 60$
$y = 10$

Remember that you need the minimum number of chocolate chip *cookies*, not *batches of cookies*. Since the minimum number of batches is 5 and there are 7 cookies per batch, the minimum number of chocolate chip cookies is 35.

29. **(B).** First, translate the problem into two equations, writing "Janie after she gave Mark 5 candies" as (J - 5) and "Mark after receiving 5 more candies" as (M + 5).

$J = M - 5$
$4(J - 5) = M + 5$

Since $J = M - 5$, plug $M - 5$ in for J in the second equation:

$4(M - 5 - 5) = M + 5$
$4(M - 10) = M + 5$
$4M - 40 = M + 5$
$4M = M + 45$
$3M = 45$
$M = 15$

If $M = 15$, then, since Janie has 5 fewer candies, $J = 10$.

Alternatively, you could backsolve from the answers. Start with choice (C). If Janie had 15 candies, Mark would have 20. If Janie gave Mark 5, she would have 10 and he would have 25. Since 25 is NOT 4 times 10, this answer is not correct. Since you want Mark to have more and Janie to have less, you might intuit that you should try a smaller answer. Try choice (B). If Janie had 10 candies, Mark would have 15. If Janie then gave Mark 5, she would have 5 and he would have 20. Since 20 is 4 times more than 5, this is the answer.

30. **(D).** This is an algebra question with two unknowns: the number of Standard jeans sold and the number of Designer jeans sold.

First, you're told that 29 pairs of jeans are sold altogether: $s + d = 29$

You also know that the cost of Standard Jeans is \$60 and the cost of Designer Jeans is \$150 (Be careful! The question doesn't say Designer Jeans cost 150% *as much* as Standard Jeans, but 150% *more*. If one thing costs 100% more than another thing, it's twice as much, so something that costs 150% more is 2.5 times as much). You've been given the total cost of the jeans, so you can write a second equation:

$60s + 150d = 3{,}540$

Now, look at your two equations:

$$s + d = 29$$
$$60s + 150d = 3{,}540$$

The easiest way to solve from here is to multiply the top equation by 60, then combine the two through elimination.

$$\begin{array}{l} 60s + 150d = 3{,}540 \\ \underline{-\ 60s + 60d = 1{,}740} \\ 90d = 1{,}800 \end{array}$$

Therefore, $d = 20$.

31. **(A).** The key to this tricky-sounding problem is setting up variables correctly and ensuring that you subtract or add appropriately for these variables when representing their ages at different points in time.

L = Lou's age now
W = Wen's age now
M = Mildred's age now
T = Tyla's age now

Represent the second sentence of the problem:

Equation 1: $(L - 3) = 2(T - 3)$
Equation 2: $(L - 3) = (M - 3) + 30$

Next:

Equation 3: $L = W + 47$
Equation 4: $(W + 4) = (T + 4)/2$

In order to solve this problem effectively, look for ways that you can get two of the equations to have the same two variables in them. If you have two equations with only two variables, you can solve for both of those variables. Equation 4 has a W and a T; the only other equation with a T is Equation 1. If you substitute the L in Equation 1 with the W from Equation 3, you will have two equations with just W's and T's.

Equation 1: $(L - 3) = 2(T - 3)$

$$(W + 47) - 3 = 2(T - 3)$$
$$W + 44 = 2T - 6$$
$$W + 50 = 2T$$

Equation 4: $(W + 4) = (T + 4)/2$

$$2W + 8 = T + 4$$
$$2W + 4 = T$$

Now combine the equations to solve for W.

$W + 50 = 2(2W + 4)$
$W + 50 = 4W + 8$
$W + 42 = 4W$
$42 = 3W$
$14 = W$

Now that you know Wen's age, you can solve for the rest.

Equation 3: $L = W + 47$

$L = 14 + 47$
$L = 61$

Equation 1: $(L - 3) = 2(T - 3)$

$(61 - 3) = 2(T - 3)$
$58 = 2T - 6$
$64 = 2T$
$32 = T$

Equation 2: $(L - 3) = (M - 3) + 30$

$61 - 3) = (M - 3) + 30$
$58 = M + 27$
$31 = M$

Now that you know that $L = 61$, $W = 14$, $M = 31$, and $T = 32$, add them together to find the answer.

$61 + 14 + 31 + 32 = 138$

32. **(C).** First, assign variables:

S = number of square feet devoted to soy
C = number of square feet devoted to corn

…and set up the equation:

$[(12)100 + (6)(S - 100)] + [(10)(3)(C)] = 13{,}080$

This means that the farmer gets \$12 per pound for the first 100 pounds, or (12)100, and then \$6 per pound for each pound after 100, or (6)(S - 100). He also gets \$10 per pound for corn, but you have to also account for the fact that he can grow 3 pounds of corn per acre, not just one. Hence, (10)(3)(C).

Simplify:

$[1{,}200 + 6S - 600] + [30C] = 13{,}080$
$600 + 6S + 30C = 13{,}080$

$6S + 30C = 12{,}480$
$S + 5C = 2{,}080$

You also know that:

$S + C = 1{,}000$
$S = 1{,}000 - C$

Combining, you get:

$(1{,}000 - C) + 5C = 2{,}080$
$1{,}000 + 4C = 2{,}080$
$4C = 1{,}080$
$C = 270$
$S = 1{,}000 - 270$
$S = 730$

33. **(E).** This question is difficult to translate. Begin by finding two things that are equal, and build an equation around that equality. *Dwayne grew twice as many bushels of pinto beans as navy beans*:

2(bushels of navy beans) = (bushels of pinto beans)

Break that down further:

bushels of navy beans = acres of navy beans × bushels per acre of navy beans
bushels of pinto beans = acres of pinto beans × bushels per acre of pinto beans

So:

2(acres of navy beans × bushels per acre of navy beans) =
(acres of pinto beans × bushels per acre of pinto beans)

Each acre of navy beans yielded 27 bushels, and each acre of pinto beans yielded 36 bushels

2 × 27 × (acres of navy beans) = 36 × (acres of pinto beans)

You can finish your translation with one variable or with two.

One Variable

Number of acres planted with pinto beans = p
Number of acres planted with navy beans = $70 - p$

$2 \times 27(70 - p) = 36p$
$54(70 - p) = 36p$
$3{,}780 - 54p = 36p$
$3{,}780 = 90p$
$p = 42$

Two Variables

Number of acres planted with pinto beans = p
Number of acres planted with navy beans = n

$$2 \times 27n = 36p$$
$$n + p = 70$$

Isolate n to allow substitution:

$$n = 70 - p$$

Substitute:

$$2 \times 27(70 - p) = 36p$$
$$54(70 - p) = 36p$$
$$3{,}780 - 54p = 36p$$
$$3{,}780 = 90p$$
$$p = 42$$

SHORTCUT: Notice that the ratio of (bushels of pinto beans produced) to (bushels of navy beans produced) is 2 : 1, greater than the ratio of (bushels of pinto beans produced per acre) to (bushels of navy beans produced per acre), 36 : 27. This means that a greater number of acres must have been planted with pinto beans than with navy beans, so the answer must be more than half of the 70 total, so either (D) or (E). You could simply backsolve the easier answer choice, (D).

40 acres of pinto beans × 36 bushels per acre = 1,440 bushels
30 acres of navy beans × 27 bushels per acre = 810 bushels

1440 isn't quite twice 810, so 40 isn't quite enough. The answer must be (E).

Chapter 18
of

Rates and Work

In This Chapter…

Rates and Work

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. Roger took 2 hours to walk from his home to a store 3 miles away, and then returned home along the same path. If Roger's average rate for the round trip was 2 miles per hour, at what rate, in miles per hour, did Roger return home?

(A) $\frac{10}{3}$
(B) 3
(C) $\frac{5}{2}$
(D) 2
(E) 1

2. Running on a 10-mile loop in the same direction, Sue ran at a constant rate of 8 miles per hour and Rob ran at a constant rate of 6 miles per hour. If they began running at the same point on the loop, how many hours later did Sue complete exactly 1 more lap than Rob?

(A) 3
(B) 4

(C) 5
(D) 6
(E) 7

3. Svetlana ran the first 5 kilometers of a 10-kilometer race at a constant rate of 12 kilometers per hour. If she completed the entire 10-kilometer race in 55 minutes, at what constant rate did she run the last 5 kilometers of the race, in kilometers per hour?

(A) 15
(B) 12
(C) 11
(D) 10
(E) 8

4. A standard machine fills paint cans at a rate of 1 gallon every 4 minutes. A deluxe machine fills gallons of paint at twice the rate of a standard machine. How many hours will it take a standard machine and a deluxe machine, working together, to fill 135 gallons of paint?

(A) 1
(B) 1.5
(C) 2
(D) 2.5
(E) 3

5. Wendy builds a birdhouse in 15 hours and Michael builds an identical birdhouse in 10 hours. How many hours will it take Wendy and Michael, working together at their respective constant rates, to build a birdhouse? (Assume that they can work on the same birdhouse without changing each other's work rate.)

(A) 5
(B) 6
(C) 7
(D) 8
(E) 9

6. Machine A, which produces 15 golf clubs per hour, fills a production lot in 6 hours. Machine B fills the same production lot in 1.5 hours. How many golf clubs does Machine B produce per hour?

$\boxed{}$ golf clubs per hour

7. Davis drove from Amityville to Beteltown at 50 miles per hour, and returned by the same route at 60 miles per hour.

Quantity A	**Quantity B**
Davis' average speed for the round trip, in miles per hour	55

8. If a turtle traveled $\frac{1}{30}$ of a mile in 5 minutes, what was its speed in miles per hour?

(A) 0.02
(B) $0.1\overline{6}$

(C) 0.4
(D) 0.6
(E) 2.5

9. Akilah traveled at a rate of x miles per hour for $2x$ hours

Quantity A	Quantity B
The number of miles Akilah traveled	$3x$

10. Claudette travels the first $\frac{2}{3}$ of a 60-mile trip at 20 miles per hour (mph) and the remainder of the trip at 30 mph. How many minutes later would she have arrived if she had completed the entire trip at 20 mph?

minutes

11. Rajesh traveled from home to school at 30 miles per hour, then returned home at 40 miles per hour to retrieve a forgotten item, and finally returned back to school at 60 miles per hour, all along the same route. What was his average speed for the entire trip, in miles per hour?

(A) 32
(B) 36
(C) 40
(D) 45
(E) 47

12. Jack traveled the first 75% of an 80-mile trip at 45 miles per hour and the remainder at 30 miles per hour. What was Jack's average speed for the entire 80-mile trip, in miles per hour?

(A) 37.5
(B) 38.25
(C) 40
(D) 41.25
(E) 42.5

13. Lamont traveled 80 miles in 2.5 hours, at a constant rate. He then decreased his speed by 25% and traveled 120 additional miles at the new constant rate. How many hours did the entire journey take?

(A) 6.25
(B) 7.5
(C) 8.75
(D) 10
(E) 11.25

14. Twelve workers pack boxes at a constant rate of 60 boxes in 9 minutes. How many minutes would it take 27 workers to pack 180 boxes, if all workers work at the same constant rate?

(A) 12
(B) 13
(C) 14
(D) 15
(E) 16

15. Four editors can proofread 4 documents in 4 hours. How many editors would be required to proofread 80 documents in 2 hours, if all editors proofread all documents at the same constant rate?

(A) 120
(B) 130
(C) 140
(D) 150
(E) 160

16. To service a single device in 12 seconds, 700 nanorobots are required, with all nanorobots working at the same constant rate. How many hours would it take for a single nanorobot to service 12 devices?

(A) 7/3
(B) 28
(C) 108
(D) 1,008
(E) 1,680

17. Working at a constant rate, Sarita answered x verbal test questions in 3 hours. Separately, she solved math problems at a constant rate of y math problems every 30 minutes.

Quantity A	**Quantity B**
The number of verbal test questions Sarita answered in 1 hour	The number of math problems Sarita solved in 1 hour

18. If 45 people built $\frac{1}{2}$ of a pyramid in 288 days, how many days did it take 65 people to build the next $\frac{1}{6}$ the pyramid, rounded to the nearest integer, assuming each person works at the same constant rate?

days

19. A machine purifies 100 cubic feet (ft^3) of water in 4 minutes. How many minutes will it take the machine to purify the contents of a 15 foot × 15 foot × 10 foot tank that is $\frac{1}{2}$ of full of water?

(A) 20
(B) 30
(C) 45
(D) 60
(E) 75

20. If a baker made 60 pies in the first 5 hours of his workday, by how many pies per hour did he increase his rate in the last 3 hours of the workday in order to complete 150 pies in the entire 8-hour period?

(A) 12
(B) 14
(C) 16
(D) 18
(E) 20

21. A stockbroker worked 10 hours a day on Monday, Wednesday, and Friday, 11 hours a day on Tuesday and Thursday, and 8 hours on Saturday. She earned \$600 each weekday and \$300 on Saturday.

Quantity A	Quantity B
The stockbroker's average earnings, in dollars per hour, over the 6-day period.	50

22. Two coal carts, A and B, started simultaneously from opposite ends of a 400-yard track. Cart A traveled at a constant rate of 40 feet per second; Cart B traveled at a constant rate of 56 feet per second. After how many seconds of travel did the two carts collide? (1 yard = 3 feet)

(A) 75
(B) 48
(C) $23\frac{1}{3}$
(D) $12\frac{1}{2}$
(E) $4\frac{1}{6}$

23. Nine identical machines, each working at the same constant rate, can stitch 27 jerseys in 4 minutes. How many minutes would it take 4 such machines to stitch 60 jerseys?

(A) 8
(B) 12
(C) 16
(D) 18
(E) 20

24. Brenda walked a 12-mile scenic loop in 3 hours. If she then reduced her walking speed by half, how many hours would it take Brenda to walk the same scenic loop two more times?

(A) 6
(B) 8
(C) 12
(D) 18
(E) 24

25. A gang of criminals hijacked a train heading due south. At exactly the same time, a police car located 50 miles north of the train started driving south toward the train on an adjacent roadway parallel to the train track. If the train traveled at a constant rate of 50 miles per hour, and the police car traveled at a constant rate of 80 miles per hour, how long after the hijacking did the police car catch up with the train?

(A) 1 hour
(B) 1 hour and 20 minutes
(C) 1 hour and 40 minutes
(D) 2 hours
(E) 2 hours and 20 minutes

26. Each working at a constant rate, Rachel assembles a brochure every 10 minutes and Terry assembles a brochure every 8 minutes.

Quantity A	**Quantity B**
The number of minutes it will take Rachel and Terry, working together, to assemble 9 brochures	40

27. With 4 identical servers working at a constant rate, a new Internet search provider processes 9,600 search requests per hour. If the search provider adds 2 more identical servers, and server work rate never varies, the search provider can process 216,000 search requests in how many hours?

(A) 15
(B) 16
(C) 18
(D) 20
(E) 24

28. A pipe siphons ink from an 800-liter drum at a rate of r liters per minute. If two such pipes were used, the drum could be emptied 100 minutes faster than when one pipe is used.

Quantity A	**Quantity B**
r	5

29. If Sabrina can assemble a tank in 8 hours, and Janis can assemble a tank in 13 hours, then Sabrina and Janis working together at their constant respective rates can assemble a tank in approximately how many hours?

(A) 21
(B) 18
(C) 7
(D) 5
(E) 2

30. Etienne began to eat 20 cookies at exactly the same time Jacques began making more cookies, one at a time, at a constant rate of 16 cookies per hour. If Etienne ate 20 cookies per hour, after how many hours were there no cookies?

hours

31. Phil collects virtual gold in an online computer game, and then sells the virtual gold for real dollars. After playing 10 hours a day for 6 days, he collected 540,000 gold pieces. If he immediately sold this virtual gold at a rate of $1 per 1,000 gold pieces, what were his average earnings per hour, in real dollars?

(A) 5
(B) 6
(C) 7
(D) 8
(E) 9

32. After completing a speed training, Alyosha translates Russian literature into English at a rate of 10 more than twice as many words per hour as he was able to translate before the training. If he was previously able to translate 10 words per minute, how many words can he now translate in an hour?

(A) 30

(B) 70
(C) 610
(D) 1,210
(E) 1,800

33. Jenny takes 3 hours to sand a picnic table; Laila can do the same job in $\frac{1}{2}$ hour. Working together at their respective constant rates, Jenny and Laila can sand a picnic table in how many hours?

(A) 1/6
(B) 2/9
(C) 1/3
(D) 3/7
(E) 5/6

34.

One worker strings 2 violins in 3 minutes. All workers string violins at the same constant rate.

Quantity A	**Quantity B**
The number of minutes required for 12 workers to string 720 violins	The number of violins that 5 workers can string in 24 minutes

35. Riders board the Jelly Coaster in groups of 4 every 15 seconds. If there are 200 people in front of Kurt in line, in approximately how many minutes will Kurt board the Jelly Coaster?

(A) 5
(B) 8
(C) 10
(D) 13
(E) 20

36. Machines A and B both shrink-wrap CDs continuously, each working at a constant rate, but Machine B works 50% faster than Machine A. If Machine B shrink-wraps 48,000 more CDs in a 24-hour period than Machine A does, what is Machine A's shrink-wrapping rate in CDs per hour?

(A) 4,000
(B) 6,000
(C) 8,000
(D) 12,000
(E) 16,000

37. A team of 8 chefs produce 3,200 tarts in 5 days. All chefs produce tarts at the same constant rate.

Quantity A	**Quantity B**
The number of chefs needed to produce 3,600 tarts in 3 days	The number of days that 4 chefs need to produce 4,800 tarts

38. Working together at their respective constant rates, robot A and robot B polish 88 pounds of gemstones in 6 minutes. If robot A's rate of polishing is 3/5 that of robot B, how many minutes would it take robot A alone to

polish 165 pounds of gemstones?

(A) 15.75
(B) 18
(C) 18.75
(D) 27.5
(E) 30

39. Car A started driving north from point X, traveling at a constant rate of 40 miles per hour. One hour later, car B started driving north from point X at a constant rate of 30 miles per hour. Neither car changed direction of travel. If each car started with 8 gallons of fuel, which is consumed at a rate of 30 miles per gallon, how many miles apart were the two cars when car A ran out of fuel?

(A) 30
(B) 60
(C) 90
(D) 120
(E) 150

40. A population of bacteria doubled at a constant rate, increasing from 50 to 3,200 bacteria in exactly two days.

Quantity A	**Quantity B**
Twice the population of bacteria after 16 more hours	The population of bacteria after 32 more hours

41. One robot, working independently at a constant rate, can assemble a doghouse in 12 minutes. What is the maximum number of complete doghouses that can be assembled by 10 such identical robots, each working on separate doghouses at the same rate for $2\frac{1}{2}$ hours?

(A) 20
(B) 25
(C) 120
(D) 125
(E) 150

42. A semiconductor company predicts that it will be able to double the density of transistors on its circuits (measured in transistors per square mm) every 18 months. If this prediction holds true, and the company's circuits currently have a density of 5 million transistors per square mm, what will be the density of transistors on the company's circuits, measured in millions of transistors per square mm, exactly 30 years from now?

(A) 5×2^{18}
(B) 5×2^{20}
(C) 5×2^{26}
(D) 5×2^{36}
(E) 5×2^{45}

43. Working continuously 24 hours a day, a factory bottles Soda Q at a rate of 500 liters per second and Soda V at a rate of 300 liters per second. If twice as many bottles of Soda V as of Soda Q are filled at the factory each day, what is the ratio of the volume of a bottle of Soda Q to a bottle of Soda V?

(A) 3/10
(B) 5/6
(C) 6/5
(D) 8/3
(E) 10/3

44. Working alone at their respective constant rates, Audrey can complete a certain job in 4 hours, while Ferris can do the same job in 3 hours. Audrey and Ferris worked together on the job and completed it in 2 hours, but while Audrey worked this entire time, Ferris worked for some of the time and took 3 breaks of equal length. How many minutes long was each of Ferris's breaks?

(A) 5
(B) 10
(C) 15
(D) 20
(E) 25

45. A turtle climbed to the top of a plateau at a rate of 4 miles an hour, crossed the plateau at a rate of x miles per hour, and descended the other side of the plateau at a rate of x^2 miles per hour. If each portion of the journey was equal in distance, what was the turtle's average speed for the entire trip, in terms of x?

(A) $\dfrac{2x}{x+2}$

(B) $\dfrac{(x+2)^2}{3}$

(C) $(x+2)^2$

(D) $\dfrac{4x^2}{(x+2)^2}$

(E) $\dfrac{12x^2}{(x+2)^2}$

Rates and Work Answers

1. **(B).** The average rate at which Roger travels is the total distance traveled divided by the total time spent traveling. In this case, Roger traveled 3 miles to and 3 miles back from a store, covering a total of 6 miles. The average rate for the whole trip is given as 2 miles per hour. Solve for the total time, using the variable *t*:

$$\text{average rate} = \frac{\text{total distance}}{\text{total time}}$$
$$2 = \frac{6}{t}$$
$$2t = 6$$
$$t = 3$$

The total time that Roger spent traveling was 3 hours. Since he took 2 hours to walk to the store, he only took 3 - 2 = 1 hour returning from the store. Roger traveled the 3 miles back in 1 hour, so he traveled at a rate of 3 miles per hour on the return trip.

2. **(C).** If Sue completed exactly one more lap than Rob, she ran 10 more miles than Rob. If Rob ran *d* miles, then Sue ran *d* + 10 miles. Rob and Sue began running at the same time, so they ran for the same amount of time. Let *t* represent the time they spent running. Fill out a chart for Rob and Sue:

	D (miles)	=	R (miles/hour)	×	T (hours)
Rob	d	=	6	×	t
Sue	$d + 10$	=	8	×	t

There are two equations:

$d = 6t$ $\qquad$ $d + 10 = 8t$

Substitute $6t$ for d in the second equation, and then solve for t:

$6t + 10 = 8t$
$10 = 2t$
$5 = t$

3. **(D).** To calculate Svetlana's speed during the second half of the race, first calculate how long it took her to run the first half of the race. Svetlana ran the first 5 kilometers at a constant rate of 12 kilometers per hour. These values can be used in the $D = RT$ formula.

D (km)	=	R (km/hr)	×	T (hr)
5	=	12	×	t

Svetlana's time for the first part of the race is 5/12 hours, or 25 minutes.

She completed the entire 10-kilometer race in 55 minutes, so she ran the last 5 kilometers in 55 - 25 = 30 minutes, or 0.5 hours.

D (km)	=	R (km/hr)	×	T (hr)
5	=	r	×	0.5

$5 = 0.5r$
$10 = r$

Svetlana ran the second half of the race at a speed of 10 kilometers per hour.

4. **(E).** The question asks for the amount of time in hours, so re-express the work rates in gallons per hour, not gallons per minute. First, calculate the rate of the standard machine:

$$\frac{1 \text{ gallon}}{4 \text{ minutes}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{60 \text{ gallons}}{4 \text{ hours}} = 15 \text{ gallons/hour}$$

Since the deluxe machine's rate is twice the standard machine's rate, the deluxe machine can fill 15 × 2 = 30 gallons of paint per hour. Together, the machines can fill 15 + 30 = 45 gallons of paint per hour. Now apply $W = R \times T$:

$135 = 45 \times T$
$3 = T$

5. **(B).** Use two separate lines in a $W = RT$ chart, one for Wendy and one for Michael, to calculate their respective rates. Building 1 birdhouse equals doing 1 unit of work:

	W (birdhouses)	=	R (birdhouses/hour)	×	T (hours)
Wendy	1	=	R_W	×	15
Michael	1	=	R_M	×	10

Thus, Wendy's rate is 1/15 birdhouses per hour, and Michael's rate is 1/10 birdhouses per hour. Since Wendy and Michael are working together, add their rates:

	W (birdhouses)	=	R (birdhouses/hour)	×	T (hours)
Wendy + Michael	1	=	$\frac{1}{15}+\frac{1}{10}$	×	t

Now solve for t by first combining the fractions:

$$1=\left(\frac{1}{15}+\frac{1}{10}\right)t$$
$$1=\left(\frac{2}{30}+\frac{3}{30}\right)t$$
$$1=\left(\frac{5}{30}\right)t$$
$$\frac{30}{5}=t$$
$$6=t$$

6. **60 golf clubs per hour.** First, calculate the size of a production lot. Machine A works at a rate of 15 golf clubs per hour and completes a production lot in 6 hrs. Plug this information into the $W = RT$ formula.

W (clubs)	=	R (clubs/hour)	×	T (hours)
w	=	15	×	6

w = (15 clubs per hour)(6 hours) = 90 clubs

Therefore, a production lot consists of 90 golf clubs. Since Machine B can complete the lot in 1.5 hours, use the $W = RT$ chart a second time to calculate the rate for Machine B.

W (clubs)	=	R (clubs/hour)	×	T (hours)
90	=	r	×	1.5

Make the calculation easier by converting 1.5 hours to 3/2 hours.

$$90 = \frac{3}{2}r$$

$$\frac{2}{3} \times 90 = r$$

$$2 \times 30 = r$$

$$60 = r$$

7. **(B).** Never take an average speed by simply averaging the two speeds (50 mph and 60 mph). You must use the formula Average Speed = Total Distance/Total Time. Fortunately, for Quantitative Comparisons, you can often sidestep actual calculations.

Davis' average speed can be thought of as an average of the speed he was traveling at every single moment during his journey — for instance, say Davis wrote down the speed he was going during every second he was driving, then he averaged all the seconds. Since Davis spent more *time* going 50 mph than going 60 mph, the average speed will be closer to 50 than 60, and Quantity B is larger. If the distances are the same, average speed is always weighted towards the *slower* speed.

If you want to actually do the math, pick a convenient number for the distance between Amityville and Beteltown — for instance, 300 miles (divisible by both 50 and 60). If the distance is 300 miles, it took Davis 6 hours to drive there at 50 mph, and 5 hours to drive back at 60 mph. Using Average Speed = Total Distance/Total Time (and a total distance of 600 miles, for both parts of the journey):

Average Speed = 600 miles/11 hours
Average Speed = 54.54…

You will get the same result with any value you choose for the distance. Thus, Quantity B is greater.

8. **(C).** The turtle traveled 1/30th of a mile in 5 minutes, which is 1/12 of an hour. Using the $D = RT$ formula, solve for r:

D (mile)	=	R (miles/hour)	×	T (hours)
$\frac{1}{30}$	=	r	×	$\frac{1}{12}$

$$\frac{1}{30} = \frac{1}{12}r$$

$$\frac{12}{30} = r$$

$$0.4 = r$$

9. **(D)** Use $D = RT$:

Distance = $x(2x)$
Distance = $2x^2$

Which is greater, $2x^2$ or $3x$? If $x = 1$, $3x$ is greater. But if $x = 2$, $2x^2$ is greater.

Without information about the value of x, the relationship cannot be determined.

10. **20 minutes.** First, figure out how long it took Claudette to travel 60 miles under the actual conditions. The first leg of the trip was 2/3 of 60 miles, or 40 miles. To travel 40 miles at a rate of 20 miles per hour, Claudette spent 40/20 = 2 hours = 120 minutes. The second leg of the trip was the remaining 60 - 40 = 20 miles. To travel that distance at a rate of 30 miles per hour, Claudette spent 20/30 = 2/3 hour = 40 minutes. In total, Claudette traveled for 120 + 40 = 160 minutes.

Now consider the hypothetical trip. If Claudette had traveled the whole distance of 60 miles at 20 miles per hour, the trip would have taken 60/20 = 3 hours = 180 minutes.

Finally, compare the two trips. The real trip took 160 minutes, so the hypothetical trip would have taken 180 - 160 = 20 minutes longer.

11. **(C).** Do not simply average the three speeds. You will always get the wrong answer that way. To compute the average speed for a trip, figure out the total distance and divide by the total time.

Pick a convenient distance from home to school, one that is divisible by 30, 40, and 60—say 120 miles (tough for Rajesh, but easier for you).

The first part of the journey (from home to school) takes 120/30 = 4 hours. The second part of the journey takes 120/40 = 3 hours. The third part of the journey takes 120/60 = 2 hours.

The total distance Rajesh travels is 120 + 120 + 120 = 360 miles. The total time is 4 + 3 + 2 = 9 hours. Finally, his average speed for the entire trip was 360/9 = 40 miles per hour.

12. **(C).** To find the average speed, divide the total distance by the total time. You have to figure out the time for each part of the journey separately. First, figure out the miles traveled for each part of the journey.

First part: 75% of 80 miles = $\left(\frac{3}{4}\right)(80) = 60$ miles

Second part: 80 - 60 = 20 miles.

Now use $D = RT$ for each part of the journey. The two rates are 45 miles per hour and 30 miles per hour.

First part: $60 = 45t$, which gives $t = 4/3$ for this part
Second part: $20 = 30t$, which gives $t = 2/3$ hour for the second part

So the total time is 4/3 + 2/3 = 6/3 = 2 hours. The total distance is 80 miles, so the average speed was 80/2 = 40 miles per hour.

13. **(B).** First, determine Lamont's rate for the first part of the journey. Since he traveled 80 miles in 2.5 hours, his rate was $\frac{80}{2.5}$ miles per hour. This fraction can be reduced: $\frac{80}{2.5} = \frac{800}{25} = \frac{8 \times 100}{25} = 8 \times 4 = 32$ miles per hour. Now, for the second part of the journey, Lamont decreased his speed by 25%. In other words, his new speed was 100% - 25% = 75% of the original speed.

New speed = $75\% \times 32 = \left(\frac{3}{4}\right)(32) = 24$ miles per hour.

He traveled 120 miles at 24 miles per hour, so the time for the second part of the journey was 120/24 = 5 hours.

Finally, the entire journey took 2.5 + 5 = 7.5 hours.

14. **(A).** To solve a Rates & Work problem with multiple workers, modify the standard formula *Work* = *Rate* × *Time* to this:

Work = *Individual Rate* × *Number of Workers* × *Time*

Use the first sentence to solve for an individual worker's rate. Plug in the fact that 12 workers pack boxes at a constant rate of 60 boxes in 9 minutes:

Work = *Individual Rate* × *Number of Workers* × *Time*

60 = (R)(12)(9 minutes)

R = 5/9 boxes per minute

In other words, each worker can pack 5/9 of a box per minute. Plug that rate back into the formula, but use the details from the second sentence in the problem:

Work = *Individual Rate* × *Number of Workers* × *Time*
180 = (5/9)(27)(T)
180 = 15T
12 = T

15. **(E).** To solve a Rates & Work problem with multiple workers, modify the standard formula *Work* = *Rate* × *Time* to this:

Work = *Individual Rate* × *Number of Workers* × *Time*

Solve for an individual worker's rate, using the fact that 4 editors can proofread 4 documents in 4 hours:

Work = *Individual Rate* × *Number of Workers* × *Time*
4 documents = (R)(4)(4 hours)
R = 1/4 document per hour

So each editor can proofread 1/4 of a document per hour.

Note that it is NOT correct to infer that if 4 editors can proofread 4 documents in 4 hours, then 1 editor can proofread 1 document in 1 hour. (After all, if 4 editors can proofread 4 documents in 4 hours, then each editor proofreads one of the documents over the whole 4 hours, not in 1 hour.)

Plug the 1/4 rate back into the formula, but using the details from the second sentence in the problem (using E for the unknown number of editors):

$80 = (1/4)(E)(2)$
$E = 160$

160 editors are required. Alternatively, you could reason that, since it takes an editor 4 hours to proofread one document, you could get 80 documents proofread by 80 editors in that same period of time (4 hours). To get the job done in half the time, you need twice as many editors, or 160 of them.

16. **(B).** To solve a Rates & Work problem with multiple workers, modify the standard formula *Work* = *Rate* × *Time* to this:

Work = *Individual Rate* × *Number of Workers* × *Time*

Solve for an individual nanorobot's rate, using the fact that 700 nanorobots can service 1 device in 12 seconds. Notice that the "work" here is 1 device:

Work = *Individual Rate* × *Number of Workers* × *Time*
1 device = (R)(700)(12 seconds)
R = 1/8,400 devices per second

That is, each nanorobot can service $\frac{1}{8,400}$ of a device in 1 second. Plug that rate back into the formula, but using the details from the second sentence in the problem:

Work = *Individual Rate* × *Number of Workers* × *Time*
$12 = (1/8,400)(1)(T)$
$T = 100,800$

The answer is 100,800 seconds. Divide by 60 to convert this time to 1,680 minutes; divide by 60 again to get 28 hours.

17. **(D).** Since Sarita answered x verbal test questions in 3 hours, she answered $x/3$ verbal test questions in 1 hour. So Quantity A is $x/3$.

Thirty minutes is 1/2 an hour. If Sarita can do y math problems in ½ an hour, then she can do $2y$ math problems in an hour. So Quantity B is $2y$.

Without more information about x and y, it cannot be determined whether $x/3$ or $2y$ is a greater number. The correct answer is (D).

18. **66 days.** To solve a Rates & Work problem with multiple workers, modify the standard formula *Work* = *Rate* × *Time* to this:

Work = *Individual Rate* × *Number of Workers* × *Time*

Solve for an individual worker's rate, plugging in the fact that 45 people built ½ of a pyramid in 288 days:

Work = *Individual Rate* × *Number of Workers* × *Time*
1/2 pyramid = (R)(45)(288 days)
R = 1/25,920 pyramid per day (don't be shy about using the calculator here)

That is, each person can build $\frac{1}{25{,}920}$ of a pyramid in 1 day. Now put this rate into the work equation, together with the remaining details in the problem:

Work = *Individual Rate* × *Number of Workers* × *Time*
1/6 = (1/25,920)(65)(T)
$T \approx$ 66.46 days

Rounded to the nearest integer, the answer is 66 days.

19. **(B).** Since the question asks for time, you need: the rate and the amount of work. To find the rate, divide the total work the purifier can do by the time required. 100 cubic feet ÷ 4 minutes = 25 cubic feet per minute.

Now find the amount of work required. The tank has a total volume of (15)(15)(10) = 2,250 cubic feet, but is only 1/2 full of water. Thus, the volume of water to be purified is 1,125 cubic feet.

Plug these numbers back into the $W = RT$ formula and solve.

$W = RT$
(1,125) = (25)T
T = 1,125 ÷ 25 = 45 minutes

20. **(D).** First identify what you are looking for. To find the amount by which the baker's rate of pie-making increased, so you need both his rate for the first 5 hours and his rate in the last 3 hours. The difference is the ultimate answer:

Rate for last 3 hours - Rate for first 5 hours = Increase

The rate for the first 5 hours was 60 pies ÷ 5 hours = 12 pies per hour.

In the last 3 hours, the baker made 150 - 60 = 90 pies. The rate in the last 3 hours of the workday was thus 90 pies ÷ 3 hours = 30 pies per hour.

Now find the difference between the two rates of work:

30 pies per hour - 12 pies per hour = 18 pies per hour

21. **(A).** To find average earnings per hour, divide the total earnings by the total time (in hours). So compute the total earnings and the total time.

5 week days × \$600 = \$3,000
1 Saturday × \$300 = \$300
Total earnings = \$3,300

3 days × 10 hours = 30
2 days × 11 hours = 22
1 day × 8 hours = 8
Total hours = 60

The broker's average earnings per hour = \$3,300 ÷ 60 = \$55 per hour. Since 55 is greater than 50, Quantity A is greater.

22. **(D).** This is a classic combined rates problem. Since the carts are moving directly toward each other, add their rates together. Remember that when two objects are moving in opposite directions — either toward each other or away from each other — add their rates to find how fast the gap is closing (or opening up).

To avoid any unit conversion trap (answer choice (E)), do the yards-to-feet conversion up front: 400 yards × 3 ft/yard = 1,200 feet. The combined rate of the two carts is 96 ft/sec. Therefore the time it takes for them to meet is 1,200 feet ÷ 96 feet/second = $12\frac{1}{2}$ seconds.

	Distance	=	Rate	×	Time
Cart A	$40t$	=	40 ft/sec	×	t
Cart B	$56t$	=	56 ft/sec	×	t
Combined Rate	1200 feet	=	96 ft/sec	×	$12\frac{1}{2}$ sec

23. **(E).** To solve a Rates & Work problem with multiple workers, modify the standard formula *Work* = *Rate* × *Time* to this:

Work = *Individual Rate* × *Number of Workers* × *Time*

Solve for an individual machine's rate, using the fact that 9 machines can stitch 27 jerseys in 4 minutes.

Work = *Individual Rate* × *Number of Workers* × *Time*
27 jerseys = (R)(9)(4 minutes)
R = 3/4 jersey per minute

That is, each machine can stitch 3/4 of a jersey in 1 minute. Plug that rate back into the formula, but using the details from the second sentence in the problem:

Work = *Individual Rate* × *Number of Workers* × *Time*
60 = (3/4)(4)(T)

$T = 20$

24. **(C).** In this problem, you must compare an actual scenario with a hypothetical one. Start by figuring out the rate (speed) for Brenda's actual walk. Since she walked 12 miles in 3 hours, she walked at a rate of 12/3 = 4 miles per hour.

Now, in the hypothetical situation, she would walk the loop twice, for a total distance of 12 × 2 = 24 miles. Her hypothetical speed would be 1/2 of 4 miles per hour, or 2 miles per hour.

Walking 24 miles at a rate of 2 miles per hour would take Brenda 24/2 = 12 hours.

Alternatively, you might note that both of the changes—doubling the distance and halving the rate—have the same effect: Each change makes the trip take twice as long as it would have before. So the time required for this hypothetical situation is multiplied by four: 3 × 2 × 2 = 12 hours.

25. **(C).** In this "chase" problem, the two vehicles are moving in the same direction, with one chasing the other. To determine how long it will take the rear vehicle to catch up, *subtract* the rates to find out how quickly the rear vehicle is gaining on the one in front.

The police car gains on the train at a rate of 80 - 50 = 30 miles per hour. Since the police car needs to close a gap of 50 miles, plug into $D = RT$ to find the time:

$50 = 30t$
$5/3 = t$

The time it takes to catch up is 5/3 hours, or 1 hour and 40 minutes.

26. **(C).** "Cheat" off the easy quantity. In 40 minutes (from Quantity B), Rachel would assemble 40/10 = 4 brochures and Terry would assemble 40/8 = 5 brochures, for a total of 4 + 5 = 9 brochures. Thus, Quantity A is also 40, and the two quantities are equal.

27. **(A).** If the search provider adds 2 identical servers to the original 4, there are now 6 servers. Because 6/4 = 1.5, the rate at which all 6 servers work is 1.5 times the rate at which 4 servers work:

9,600 searches per hour × 1.5 = 14,400 searches per hour

Now apply this rate to the given amount of work (216,000 searches), using the $W = RT$ formula.

$216{,}000 = (14{,}400)T$
$216{,}000 \div 14{,}400 = 15$ hours

28. **(B).** Start by filling in a $W = RT$ chart with the pieces you know. Remember, two identical pipes would siphon ink at double the rate of a single pipe.

	Work (liters)	=	Rate (liters/min)	×	Time (min)
1 pipe	800	=	r	×	**800/*r***

2 pipes	800	=	$2r$	×	**800/(2*r*)**

One pipe does the work in 800/*r* minutes, and two pipes do the work in 800/2*r* minutes.

The questions states that "If two such pipes were used, the drum could be emptied 100 minutes faster than when one pipe is used. Express this as a skeleton equation:

Time for 2 pipes + 100 = Time for 1 pipe

(You can check that you've written this correctly by noting that the time for 2 pipes should be shorter, so it is necessary to add 100 to that side to make it equal the longer time for 1 pipe.)

Now plug in expressions for the times and solve for *r*:

$$\frac{800}{2r} + 100 = \frac{800}{r}$$
$$800 + 200r = 1600$$
$$200r = 800$$
$$r = 4$$

1 pipe siphons 4 liters of ink per minute. Thus, Quantity A equals 4, which is less than Quantity B.

29. **(D).** Since Sabrina and Janis are working together, add their rates. Sabrina completes 1 tank in 8 hours, so she works at a rate of $\frac{1}{8}$ tank per hour. Likewise, Janis works at a rate of $\frac{1}{13}$ tank per hour. Now, add these fractions:

$$\frac{1}{8}+\frac{1}{13}=\frac{13}{104}+\frac{8}{104}=\frac{21}{104}$$ tanks per hour, when working together

Now plug this combined rate into the $W = RT$ formula to find the time. You might also notice that since the work is equal to 1, the time will just be the reciprocal of the rate.

Sabrina & Janis:	1 tank	=	21/104 tank/hr	×	**104/21 minutes**

At this point, you do not need to do long division or break out the calculator! Just approximate: 104/21 is about 100/20 = 5. The answer is (D).

You might also use some intuition to work the answer choices and avoid setting up this problem at all! You can immediately eliminate (A) and (B), since these times exceed either worker's individual time. Also, since Sabrina is the faster worker, Janis's contribution will be less than Sabrina's. The two together won't work twice as fast as Sabrina, but they will work *more* than twice as fast as Janis. Therefore, the total time should be more than half of Sabrina's individual time, and less than half of Janis's individual time. $4 < t < 6.5$, which leaves (D) as the only possible answer.

30. **5 hours.** Etienne and Jacques were working at cross-purposes (although perhaps Etienne didn't mind), so *subtract* their rates. Usually you add work rates, but this situation is just like a car chase: when one car (or person) gains on another, you subtract rates.

20 cookies/hour - 16 cookies/hour = 4 cookies/hour, so the quantity of cookies decreased by 4 per hour. Since Etienne began with a pile of 20 cookies, it took him 20/4 = 5 hours to eat all the cookies.

Note that Etienne ate a lot more than 20 cookies. In 5 hours he ate 100 cookies—the initial 20, plus the 80 that Jacques made in the 5 hours.

31. **(E).** To solve for average earnings, fill in this formula:

Total earnings/Total hours = Average earnings per hour

Since the gold-dollar exchange rate is \$1 per 1,000 gold pieces: Phil's real dollar earnings for the 6 days were 540,000/1,000 = \$540. His total time worked was 10 hours/day × 6 days = 60 hours. Therefore, his average hourly earnings were \$540/60 = \$9/hour.

32. **(D).** To find the new rate in words per hour, start by setting up an equation to find this value:

New words/hr = 10 + 2(Old words/hr)

The old rate was given in words per minute, so convert to words per hours:

10 words/min × 60 min/hr = 600 words/hr.

Now plug into the equation:

New words/hr = 10 + 2(600) = 1,210

Note that you would not want to start by working with the rate per minute. If you did so, you'd get 10 + 2(10) = 30 words/minute, then 30 × 60 = 1,800 words/hr. You would get this inflated number because you added an additional 10 words per minute instead of per hour. This is another reason to perform your conversions right away!

33. **(D).** Since the two women are working together, add their rates. To find their individual rates, divide work by time. Never divide time by work! (Also, be careful when dividing the work by 1/2. The rate is the reciprocal of 1/2, or 2 tables/hour.)

Once you find Jenny and Laila's combined rate, divide the work required (1 table) by this rate. 1 table ÷ 7/3 table/ hour = 3/7 hour. s

	Work (tables)	=	Rate (table/hour)	×	Rate (table/hour)
Jenny	1	=	**1/3**	×	3
Laila	1	=	**2**	×	1/2
Jenny & Laila	1	=	**1/3 + 2 = 7/3**	×	3/7

34. **(A).** First, figure out the individual rate for 1 worker: 2 violins/3 minutes = 2/3 violin per minute. (Always divide work by time to get a rate.) Now apply $W = RT$ separately to Quantity A and Quantity B.

Quantity A:

$R = 12 \times$ the individual rate $= 12 \times 2/3 = 8$ violins per minute.
$W = 720$ violins

Solve for T in $W = RT$:

$720 = 8T$
$90 = T$

Quantity B:

$R = 5 \times$ the individual rate $= 5 \times 2/3 = 10/3$ violins per minute.
$T = 24$ minutes

Solve for W in $W = RT$:

$W = (10/3)(24)$
$W = 80$

Since 90 > 80, Quantity A is greater.

35. **(D).** To find Kurt's wait time, determine how long it will take for 200 people to board the Jelly Coaster. The problem states that 4 people board every 15 seconds. Since there are four 15-second periods in one minute, this rate converts to 16 people/minute. To find the time, divide the "work" (the people) by this rate.

200 people ÷ 16 people/minute = 200/16 = 12.5 minutes. The question asks for an approximation, and this is now close enough to answer (D). In theory there may be an additional 15 seconds while Kurt's group is boarding (the problem doesn't really say), but Kurt's total wait time would still be approximately 13 minutes.

36. **(A).** Machine B is 50% faster than A, so relate their rates with the equation $b = 1.5a$. Put the data into a chart to compare work each machine does in 24 hours.

	Work (CDs)	=	Rate (CDs/hour)	×	Time (hour)
Machine A	$24a$	=	a	×	24
Machine B	$24b$	=	b	×	24

Machine B's work in a 24-hour period exceeds Machine A's work by 48,000 CDs. That is to say:

$36a - 24a = 48{,}000$
$12a = 48{,}000$
$a = 4{,}000$

Machine A shrink-wraps 4,000 CDs per hour.

Another way to solve this problem is to notice that since B is 50% faster than A, the quantity by which its work exceeds A's in an hour will equal 50%, or half, of A's hourly rate. Since B shrink-wraps 48,000/24 = 2,000 more CDs per hour than A, Machine A wraps 2,000 × 2 = 4,000 CDs per hour.

37. **(C).** To solve a Rates & Work problem with multiple workers, modify the standard formula *Work* = *Rate* × *Time* to this:

Work = *Individual Rate* × *Number of Workers* × *Time*

Solve for an individual chef's rate, using the fact that 8 chefs produce 3,200 tarts in 5 days.

Work = *Individual Rate* × *Number of Workers* × *Time*
3,200 tarts = (R)(8)(5 days)
R = 80 tarts per day

That is, each chef can produce 80 tarts per day. Plug that rate back into the formula for each of the quantities.

Quantity A
Work = *Individual Rate* × *Number of Workers* × *Time*
3,600 = (80)(*Number of Workers*)(3)
Number of Workers = 15

Quantity B
Work = *Individual Rate* × *Number of Workers* × *Time*
4,800 = (80)(4)(Time)
Time = 15 days

The number of chefs in Quantity A equals the number of days in Quantity B.

38. **(E).** When rate problems involve multiple situations, it can help to set up an initial "skeleton" $W = RT$ chart for the solution. That way, you can easily determine what data is needed, and fill in that data as you find it. Since you want to know how long Robot A will take alone, the chart will look like this:

	Work (pounds)	=	Rate (pounds/min)	×	Rate (min)
Robot A	165	=	A's rate	×	t

You know the work and you want to know the time, so you just need A's rate. Call the rates a and b. Now set up another chart representing what you know about the two robots working together.

	Work (pounds)	=	Rate (pounds/min)	×	Rate (min)
Robot A	$6a$	=	a	×	6
Robot B	$6b$	=	b	×	6

A & B together	$6(a + b) = 88$	=	$a + b$	×	6

Now that you know that $6(a + b) = 88$, just apply the other piece of information you know: robot A's rate is 3/5 of B's rate. This can be written as $a = (3/5)b$. Since you are looking for a, substitute for b:

$a = (3/5)b$
$(5/3)a = b$

$6(a+(5/3)a) = 88$
$6(8/3)a = 88$
$(48/3)(a) = 88$
$a = 88(3/48)$
$a = 88(1/16) = 88/16 = 11/2$

So A's rate is 11/2 pounds per minute. Now just plug into the original chart.

	Work (pounds)	=	Rate (pounds/min)	×	Rate (min)
Robot A	165	=	11/2	×	**30**

The time Robot A takes to polish 165 pounds of gems is $165/(11/2) = 330/11 = 30$ minutes.

39. **(C).** No distances are given in this problem, so you need to determine how far the two cars end up traveling before finding the distance between them. Since the cars go in the same direction, the skeleton equation is as follows:

Car A's distance - Car B's distance = distance between cars
(all distances refer to the time when car A ran out of fuel).

Since the limiting factor in this case is A's fuel supply, you must calculate how far the car is able to drive before running out of fuel. This in itself is a rate problem of sorts:

30 miles per gallon × 8 gallons = 240 miles

So Car A will end up 240 miles north of its starting point, which happens 240/40 = 6 hours after it started. What about Car B? It started an hour later and thus traveled (30 miles per hour)(6 hours - 1 hour) = 180 miles by that time.

Therefore the two cars were 240 - 150 = 90 miles apart when car A ran out of fuel.

40. **(B).** First, build a chart to see how many doubling periods occured to grow the population from 50 to 3,200. Most problems of this sort rely on converting any growth rate to a doubling period.

50
100
200

400
800
1,600
3,200

The population doubled 6 times in 2 days. 48 hours/6 doubling periods = 8 hours per doubling period. Therefore the population doubles every 8 hours.

Quantity A:
After 16 more hours, the population has gone through 2 doubling periods, so it has quadrupled (that is, it has increased by a factor of 4) from the final level of 3,200. Since Quantity A is actually twice that population, the quantity is 8 times 3,200.

Quantity B:
After 32 more hours, the population has gone through 4 doubling periods, so it has gone up by a factor of $2 \times 2 \times 2 \times 2 = 2^4 = 16$. Quantity B is 16 times 3,200.

Quantity B is greater. Notice that you don't need to actually figure out 8 × 3,200 or 16 × 3,200.

41. **(C).** (D) is a trap. This issue is relatively rare, but it's worthwhile to be able to recognize it if you see it. Note that in this case, each robot is *independently* assembling complete doghouses. Since the question asks for the number of *completed* doghouses after $2\frac{1}{2}$ hours, you need to remove any *incomplete* doghouses from the calculations.

Since one robot completes a doghouse in 12 minutes, the individual hourly rate is 60/12 = 5 doghouses per hour. Therefore, each robot produces 5 × 2.5 = 12.5 doghouses in $2\frac{1}{2}$ hours. (You could also simply divide the 150 total minutes by 12 minutes per doghouse to get the same result.)

However, since you are interested only in *completed* doghouses, and the robots are working independently, drop the decimal. Each robot completes only 12 doghouses in the time period, for a total of 12 × 10 = 120 doghouses.

42. **(B).** The question gives you the initial value and the rate of doubling, so you can set up the solution in a straightforward manner. The hardest part is cutting through all the language to find what you need. The initial value is stated outright: 5 million transistors/square mm. To find the number of doubling periods, first convert from years to months:

$$\frac{30 \text{ years} \times 12 \text{ months per year}}{18 \text{ months per doubling period}} = \frac{360}{18} = 20 \text{ doubling periods}$$

So there is an initial density of 5 million, doubled 20 times. Thankfully, you don't have to calculate that number! Since the answer choices are in exponential form, set things up that way. Also, note that the question asks for the density in millions of transistors/square mm, so you need to use 5, rather than 5,000,000, in the calculation. 5 doubled 20 times = 5×2^{20}

43. **(E) 10/3.** If twice as many bottles of Soda V as of Soda Q are filled at the factory each day, then twice as many bottles of Soda V as of Soda Q are filled at the factory each second.

Use smart numbers for the number of bottles filled each second. Since twice as many bottles of Soda V are produced, so the output in one second could be 100 bottles of V and 50 bottles of Soda Q. Using these numbers, the volume of the Q bottles is 500 liters / 50 bottles = 10 liters/bottle and the volume of the V bottles is 300 liters/100 bottles = 3 liters/bottle. The ratio of the volume of a bottle of Q to a bottle of V is 10 liters/3 liters = 10/3.

44. **(B).** To determine how long Ferris' breaks were, you need to know the difference between the amount of work the two *should* have completed in two hours, and the amount they actually *did* complete (that is, one full job).

Audrey and Ferris are working together, so first find each worker's individual rate, and then add them together to get the combined rate.

	Work (jobs)	=	Rate (jobs/hour)	×	Time (hours)
Audrey	1	=	**1/4**	×	4
Ferris	1	=	**1/3**	×	3
Audrey & Ferris	**7/6**	=	**1/4 + 1/3 = 3/12 + 4/12 = 7/12**	×	2

Combining the two workers' rates, together they complete 7/12 job per hour, so they should have completed 14/12 = 7/6 job in two hours. Therefore, Ferris' breaks cost them 7/6 - 1 = 1/6 job worth of productivity.

How long was Ferris on break? The amount of time it would have taken him to do 1/6 of the job.

Ferris' breaks	1/6 job	=	1/3 job/hour	×	**1/2 hour**

At the rate of 1/3 job/hour, Ferris must have spent 1/2 hour on break to miss 1/6 job. Therefore, each of his 3 breaks was 30 minutes ÷ 3 = 10 minutes long. The answer is (B).

Alternatively, use smart numbers to eliminate the fractions. Define the job as, say, making 12 toys. Now you can say that Audrey makes 3 toys/hour and Ferris makes 4 toys/hour. The work is all the same as above, but when you get to the point of figuring out Ferris' missed time, the work becomes much easier. Together, the two should produce 14 toys, but they produced only 12. Thus, Ferris' slacking costs them 2 toys. Since his rate is 4 toys/hour, he must have missed 1/2 hour of work (by taking 3 breaks of 10 minutes each).

45. **(E).** To find the turtle's average speed for the trip, divide total distance by total time.

Since the distance has not been defined, call each equal leg of the trip *D*. Therefore, the turtle's total distance is 3*D*. (Note that the *D* must cancel out before you are done.) Find the turtle's total time by calculating the time for each leg of the journey. In each case, the time is equal to *D*/rate.

	Distance	=	Rate	×	Time
Up	D miles	=	4 mi/hr	×	**D/4 hr**
Across	D miles	=	x mi/hr	×	**D/x hr**
Down	D miles	=	x^2 mi/hr	×	**D/x^2 hr**

Now find the total time by adding up the separate legs, using a common denominator:

$$\frac{D}{4}+\frac{D}{x}+\frac{D}{x^2}=$$

$$\frac{Dx^2}{4x^2}+\frac{4Dx}{4x^2}+\frac{4D}{4x^2}=$$

$$\frac{Dx^2+4Dx+4D}{4x^2}=$$

$$\frac{D\left(x^2+4x+4\right)}{4x^2}$$

All that's left to do is plug this total time, along with the total distance of 3D, into the formula for average rate (= Total distance/Total time):

$$\frac{3D}{\frac{D\left(x^2+4x+4\right)}{4x^2}}=$$

$$\frac{3D\times 4x^2}{D\left(x^2+4x+4\right)}=$$

$$\frac{12x^2}{x^2+4x+4}$$

Notice that the D's cancel out, as predicted. To match this to answer choice (E), you need to recognize the denominator as a special product: $(x + 2)^2$.

Alternatively, you can solve this problem with smart numbers. Make $x = 5$. Since $x^2 = 25$, you want a distance for each leg that is divisible by 4, 5, and 25. A distance of 100 will work nicely. Now find the time for each leg of the trip.

	Distance (miles)	=	Rate (miles/hour)	×	Time (hours)
Up	100	=	4	×	**25**
Across	100	=	$x = 5$	×	**20**
Down	100	=	$x^2 = 25$	×	**4**
Entire Trip	300	=			**49 hr**

The turtle's average rate is 300/49 mi/hr. There is no need to simplify, as you just need to plug in for x in each answer choice and see whether the result matches. When you plug $x = 5$ into the answer choices, only (E) produces 300/49.

Chapter 19

of

Variables-in-the-Choices Problems

In This Chapter…

Variables-in-the-Choices Problems

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. If Josephine reads b books per week and each book has, on average, 100,000 words, which best approximates the number of words Josephine reads per day?

(A) $100{,}000b$
(B) $\frac{100{,}000b}{7}$
(C) $\frac{700{,}000}{b}$
(D) $\frac{7b}{100{,}000}$
(E) $\frac{100{,}000b}{(7)(24)}$

2. A rectangle's width w is twice its length. Which of the following expresses the rectangle's area in terms of w?

(A) w

(B) $2w^2$
(C) $3w^2$
(D) $\frac{w^2}{2}$
(E) $\frac{w^2}{4}$

3. A clothing store bought a container of 100 shirts for $\$x$. If the store sold all of the shirts at the same price for a total of \$50, what is the store's profit per shirt, in dollars, in terms of x?

(A) $50 - \frac{x}{100}$
(B) $50 - x$
(C) $5 - x$
(D) $0.5 - x$
(E) $0.5 - \frac{x}{100}$

4. There are two trees in the front yard of a school. The trees have a combined height of 60 feet, and the taller tree is x times the height of the shorter tree. How tall is the shorter tree, in terms of x?

(A) $\frac{60}{1+x}$
(B) $\frac{60}{x}$
(C) $\frac{30}{x}$
(D) $60 - 2x$
(E) $30 - 5x$

5. Louise is three times as old as Mary. Mary is twice as old as Natalie. If Louise is L years old, what is the average age of the three women, in terms of L?

(A) $L/3$
(B) $L/2$
(C) $2L/3$
(D) $L/4$
(E) $L/6$

6. Toshi is four times as old as Kosuke. In x years Toshi will be three times as old as Kosuke. How old is Kosuke, in terms of x?

(A) $2x$
(B) $3x$
(C) $4x$
(D) $8x$
(E) $12x$

7. A shirt that costs k dollars is increased by 30%, then by an additional 50%. What is the new price of the shirt in

dollars, in terms of k?

(A) $0.2k$
(B) $0.35k$
(C) $1.15k$
(D) $1.8k$
(E) $1.95k$

8. Carlos runs a lap around the track in x seconds. His second lap is five seconds slower than the first lap, but the third lap is two seconds faster than the first lap. What is Carlos's average race time, in *minutes*, in terms of x?

(A) $x - 1$
(B) $x + 1$
(C) $\frac{x-1}{60}$
(D) $\frac{x+1}{60}$
(E) $\frac{x+3}{60}$

9. Andrew sells vintage clothing at a flea market at which he pays \$150 per day to rent a table plus \$10 per hour to his assistant. He sells an average of \$78 worth of clothes per hour. Assuming no other costs, which of the functions below best represents profit per day P in terms of hours h that the flea market table is open for business?

(A) $P(h) = 238 - 10h$
(B) $P(h) = 72 - 10h$
(C) $P(h) = 68h - 150$
(D) $P(h) = 78h - 160$
(E) $P(h) = -160h + 78$

10. If a, b, c, and d are consecutive integers and $a < b < c < d$, what is the average of a, b, c, and d in terms of d?

(A) $d - \frac{5}{2}$
(B) $d - 2$
(C) $d - \frac{3}{2}$
(D) $d + \frac{3}{2}$
(E) $\frac{4d-6}{7}$

11. A cheese that costs c cents per ounce costs how many dollars per pound? (16 ounces = 1 pound and 100 cents = 1 dollar)

(A) $4c/25$
(B) $25c/4$
(C) $25/4c$
(D) $c/1,600$

(E) $1{,}600c$

12. A bag of snack mix contains 3 ounces of pretzels, 1 ounce of chocolate chips, 2 ounces of mixed nuts, and x ounces of dried fruit by weight. What percent of the mix is dried fruit, by weight?

(A) $\frac{x}{600}$

(B) $\frac{100}{6x}$

(C) $\frac{100x}{6}$

(D) $\frac{100x}{6+x}$

(E) $\frac{x}{100(6+x)}$

13. At her current job, Mary gets a 1.5% raise twice per year. Which of the following choices represents Mary's current income y years after starting the job at a starting salary of s?

(A) $s(1.5)^{2y}$
(B) $s(0.015)^{2y}$
(C) $s(1.015)^{2y}$
(D) $s(1.5)^{y/2}$
(E) $s(1.015)^{y/2}$

14. Phone Plan A charges \$1.25 for the first minute and \$0.15 for every minute thereafter. Phone Plan B charges a \$0.90 connection fee and \$0.20 per minute. Which of the following equations could be used to find the length, in minutes, of a phone call that costs the same under either plan?

(A) $1.25 + 0.15x = 0.90x + 0.20$
(B) $1.25 + 0.15x = 0.90 + 0.20x$
(C) $1.25 + 0.15(x - 1) = 0.90 + 0.20x$
(D) $1.25 + 0.15(x - 1) = 0.90 + 0.20(x - 1)$
(E) $1.25 + 0.15x + 0.90x + 0.20 = x$

15. If powdered drink mix costs c cents per ounce and p pounds of it are purchased by a supplier who intends to resell it, what will be the total revenue, in dollars, in terms of c and p if all of the drink mix is sold at a price per ounce equivalent to three times what the supplier paid? (16 ounces = 1 pound and 100 cents = 1 dollar)

(A) $48cp$

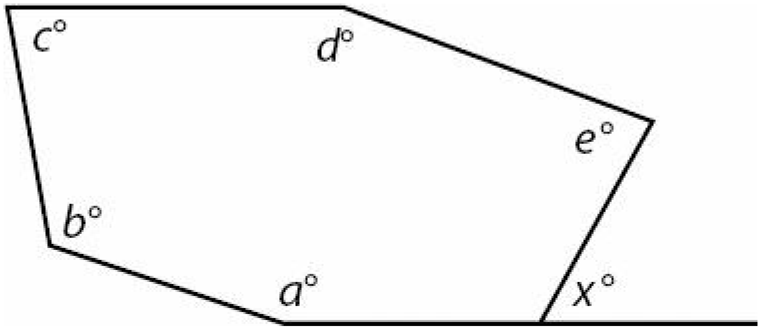
(B) $\frac{32cp}{100}$

(C) $\frac{100(32)}{cp}$

(D) $\frac{12cp}{25}$

(E) $\dfrac{25cp}{12}$

16.



16. If $d = 2c$ and $e = \dfrac{1}{2}a$, what is x in terms of a, b, and c?

(A) $\dfrac{3}{2}a + b + 3c - 540$
(B) $\dfrac{3}{2}a + b + 3c$
(C) $720 - \dfrac{3}{2}a - b - 3c$
(D) $720 - \dfrac{1}{2}a - b - 2c$
(E) $540 - \dfrac{1}{2}a - b - \dfrac{3}{2}c$

17. a, b, and c are 3 consecutive odd integers such that $a < b < c$. If a is halved to become m, b is doubled to become n, c is tripled to become p, and $k = mnp$, which of the following is equal to k in terms of a?

(A) $3a^3 + 18a^2 + 24a$
(B) $3a^3 + 9a^2 + 6a$
(C) $\dfrac{11}{2}a + 16$
(D) $6a^2 + 36a + 24$
(E) $a^3 + 6a^2 + 4a$

18. If m pencils cost the same as n pens, and each pencil costs 20 cents, what is the cost, in dollars, of 10 pens, if each pen costs the same amount (100 cents = 1 dollar)?

(A) $\dfrac{200n}{m}$
(B) $\dfrac{2n}{100m}$

(C) $\frac{2m}{n}$

(D) $\frac{2n}{m}$

(E) $200mn$

19. Randi sells forklifts at a dealership where she makes a base salary of $2,000 per month, plus a commission equal to 5% of the selling price of the first 10 forklifts she sells that month, and 10% of the value of the selling price of any forklifts after that. If all forklifts have the same sale price, *s,* which of the choices below represents Randi's monthly pay, *P*, as a function of number of forklifts sold, *f*, in months in which she sells more than 10 forklifts? (Assume Randi's pay is made up entirely of base salary and commission, and no deductions are taken from this pay.)

(A) $P = 2{,}000 + 0.05sf + 0.10sf$
(B) $P = 2{,}000 + 0.05sf + 0.10s(f - 10)$
(C) $P = 2{,}000 + 0.05s + 0.10s(f - 10)$
(D) $P = 2{,}000 + 0.5s + 0.10sf - 10$
(E) $P = 2{,}000 + 0.5s + 0.10s(f - 10)$

20. If the width of a rectangle is w, the length is l, the perimeter is p, and $w = 2l$, what is the area in terms of p?

(A) $\frac{p^2}{18}$

(B) $\frac{p^2}{36}$

(C) $\frac{p}{9}$

(D) $\frac{p^2}{9}$

(E) $\frac{p}{6}$

Variables-in-the-Choices Problems Answers

1. **(B).** Since Josephine reads b books per week and each book has an average of 100,000 words, she reads $100{,}000b$ words per week. However, the question asks for words per *day,* so divide this quantity by 7.

Alternatively, you could try picking numbers. If $b = 2$, for instance, then Josephine would read 2 books per week and thus 200,000 words per week. Divide by 7 to get 28,571.42… words per day. Plug 2 into each answer choice in place of b, and pick the answer that gives you 28,571.42… Only (B) works.

2. **(D).** Since width is twice length, write $W = 2L$. However, you want your answer in terms of w, so solve for L:

$L = 1/2\ W$ or $\frac{w}{2}$

Since area is $L \times W$ and $L = \frac{w}{2}$:

$$A = \frac{w}{2} \times W$$

$A = \frac{w^2}{2}$, or choice (D).

Alternatively, pick values. If width were 4, length would be 2. The area would therefore be $4 \times 2 = 8$. Plug in 4 for w to see which answer choice yields 8. Only (D) works.

3. **(E).** This problem requires you to know that profit equals revenue minus cost. You could memorize the formula Profit = Revenue - Cost (or Profit = Revenue - Expenses), or you could just think about it logically — of course a business has to pay its expenses out of the money it makes: the rest is profit.

The revenue for all 100 shirts was \$50, and the cost to purchase all 100 shirts was \$$x$. Therefore:

Total profit = $50 - x$

The question does not ask for the total profit, but for the profit per shirt. The store sold 100 shirts, so divide the total profit by 100 to get the profit per shirt:

Profit per shirt = $\frac{50 - x}{100}$

None of the answer choices match this number, so you need to simplify the fraction. Split the numerator into two separate fractions:

$$\frac{50-x}{100}=\frac{50}{100}-\frac{x}{100}=0.5-\frac{x}{100}$$

4. **(A).** Let s = the height of the shorter tree. Let t = the height of the taller tree.

If the combined height of the tress is 60 feet, then:

$s + t = 60$

You also know that the height of the taller tree is x times the height of the shorter tree:

$t = xs$

You need to solve for the height of the shorter tree, so substitute (xs) for t in the first equation:

$s + (xs) = 60$

You need to isolate s, so factor s out of the left side of the equation:

$$s(1+x)=60$$

$$s=\frac{60}{1+x}$$

5. **(B).** First, you need to express all three women's ages in terms of L. If Louise is three times as old as Mary, then Mary's age is $L/3$.

You also know that Mary is twice as old as Natalie. If Mary's age is $L/3$, then Natalie's age is 1/2 of that, or $L/6$.

Now you can plug those values into the average formula. The average of the three ages is:

$$\text{average}=\frac{L+\frac{L}{3}+\frac{L}{6}}{3}$$

To get rid of the fractions in the numerator, multiply the entire fractions by 6/6:

$$\frac{6}{6}\times\left(\frac{L+\frac{L}{3}+\frac{L}{6}}{3}\right)=\frac{6L+2L+L}{18}=\frac{9L}{18}=\frac{L}{2}$$

6. **(A).** Let T = Toshi's age $(T+x)$ = Toshi's age in x years

Let K = Kosuke's age $(K+x)$ = Kosuke's age in x years

If you know that Toshi is four times as old as Kosuke, then you know that:

$T = 4K$

To translate the second sentence correctly, remember that you need to use $(T + x)$ and $(K + x)$ to represent their ages:

$(T + x) = 3(K + x)$

You need to solve for Kosuke's age in terms of x, so replace T with $(4K)$ in the second equation:

$(4K) + x = 3K + 3x$
$K + x = 3x$
$K = 2x$

7. **(E).** If the cost of the shirt is increased 30%, then the new price of the shirt is 130% of the original price. If the original price was k, then the new price is $1.3k$.

Remember that it is this new price that is increased by 50%. You need to multiply $1.3k$ by 1.5 (150%) to get the final price of the shirt:

$1.3k \times 1.5 = 1.95k$

8. **(D).** Carlos's race times can be expressed as x, $x + 5$, and $x - 2$. (Remember, SLOWER race times are LARGER numbers, so "five seconds slower" means *plus* 5, not *minus* 5!) Average the race times:

$$\frac{x+(x+5)+(x-2)}{3} = \frac{3x+3}{3} = x+1$$

His average time is $x + 1$ *seconds*. But you need *minutes*. Since there are 60 seconds in a minute, divide by 60 to get $\frac{x+1}{60}$, or choice (D).

Alternatively, pick values. If x were 60 seconds, for example, Carlos's race times would be 60, 65, and 58. His average time would be 61 seconds, or 1 minute and 1 second, or $1\frac{1}{60}$ minutes, or $\frac{61}{60}$ minutes. Plug in $x = 60$ to see which value yields $\frac{61}{60}$. Only (D) works.

9. **(C).** For every hour Andrew's business is open, he sells \$78 worth of clothes but pays \$10 to his assistant. Thus, he is making \$68 an hour after paying the assistant. He also must pay \$150 for the whole day.

So, the formula for his daily profit, using Revenue - Expenses = Profit and h for hours he is open:

$68h - 150$

Written as a function of profit in terms of hours, this is $P(h) = 68h - 150$, or choice (C).

Be careful that you are reading the answer choices as *functions*. P is not a variable that is being multiplied by h! P is the *name* of the function, and h is the variable on which the output of the function depends.

Note that (D) is a very good trap — this formula represents what the profit would be if Andrew only had to pay the assistant \$10 *total*. However, he pays the assistant \$10 *per hour*.

Alternatively, you could pick numbers. If Andrew were open for an 8-hour day (here, you are testing out $h = 8$), he would make \$68 an hour (\$78 of sales minus \$10 to the assistant), or \$544 total. Subtract the \$150 rental fee to get \$394.

Then, plug 8 into the answer choices in place of h to see which answer yields 394. Only (C) works.

10. **(C).** Since *a, b, c,* and *d* are consecutive and *d* is largest, you can express c as $d - 1$, b as $d - 2$, and a as $d - 3$. Therefore, the average is:

$$\frac{(d-3)+(d-2)+(d-1)+d}{4} = \frac{4d-6}{4} = d - \frac{6}{4} \text{ or } d - \frac{3}{2},$$ which matches choice (C).

Alternatively, plug in numbers. Say *a, b, c,* and *d* are simply 1, 2, 3, and 4 (generally, you want to avoid picking the numbers 0 and 1, lest *several* of the choices appear to be correct and you have to start over, but since only *d* appears in the choices, it's no problem that *a* is 1 in this example).

Thus, the average would be 2.5. Plug in 4 for *d* to see which choice yields an answer of 2.5. Only (C) works.

11. **(A).** If a cheese costs *c* cents per ounce, it costs $16c$ cents per pound. To convert from cents to dollars, divide by 100:

$$\frac{16c}{100} = \frac{4c}{25},$$ or choice (A).

Alternatively, pick numbers. If $c = 50$, a cheese that costs 50 cents per ounce would cost 800 cents, or \$8, per pound. Plug in $c = 50$ and select the answer that gives the answer 8. Only (A) works.

12. **(D).** To figure out what *fraction* of the mix is fruit, put the amount of fruit over the total amount of the mix: $\frac{x}{6+x}$. To convert a fraction to a percent, simply multiply by 100: $\frac{x}{6+x}(100) = \frac{100x}{6+x}$, or answer choice (D).

Alternatively, pick smart numbers. For instance, say $x = 4$. In that case, the total amount of mix would be 10 ounces, 4 of which would be dried fruit. Since 4/10 = 40%, the answer to the question for your example would be 40%. Now, plug $x = 4$ into each answer choice to see which yields 40%. Only choice (D) works: $\frac{100(4)}{6+(4)} = \frac{400}{10} = 40$. This will work for any number you choose for x, provided that you correctly calculate what percent of the mix would be dried fruit in your particular example.

13. **(C).** To increase a number by 1.5%, first convert 1.5% to a decimal by dividing by 100 to get 0.015.

Do NOT multiply the original number by 0.015 — this approach would be very inefficient, because multiplying by 0.015 would give you only the increase, not the new amount (you would then have to add the increase back to the original amount, a process so time-wasting and inefficient that it would not likely appear in a formula in the answer choices).

Instead, multiply by 1.015. Multiplying by 1 keeps the original number the same; multiplying by 1.015 gets you the original number plus 1.5% more.

Finally, if you want to multiply by 1.015 twice per year, you will need to do it $2y$ times. This $2y$ goes in the exponent spot, to give you $s(1.015)^{2y}$, or choice (C).

14. **(C).** A formula to find the cost of a call under Plan A, using x as the number of minutes:

Cost = $1.25 + 0.15(x - 1)$

Note that you need to use $x - 1$ because the caller does *not* pay $0.15 for every single minute — the first minute was already paid for by the $1.25 charge.

A formula to find the cost of a call under Plan B, using x as the number of minutes:

Cost = $0.90 + 0.20x$

Note that here you do *not* use $x - 1$ because the connection fee does not "buy" the first minute — you still have to pay $0.20 for every minute.

To find the length of a call that would cost the same under either plan, set the two formulas equal to one another:

$$1.25 + 0.15(x - 1) = 0.90 + 0.20x$$

This is choice (C). Note that you are not required to solve this equation, but you might be required to solve a similar equation in a different problem on this topic:

$$1.25 + 0.15x - 0.15 = 0.90 + 0.20x$$
$$1.1 + 0.15x = 0.90 + 0.20x$$
$$0.20 = 0.05x$$
$$20 = 5x$$
$$4 = x$$

A 4-minute call would cost the same under either plan. To test this, calculate the cost of a 4-minute call under both plans: it's $1.70 either way.

15. **(D).** The mix costs c cents per ounce. Since you want your final answer in dollars, convert right now:

c cents per ounce = $\frac{c}{100}$ dollars per ounce

The supplier then purchases p pounds of mix. You cannot simply multiply p by $\frac{c}{100}$, because p is in pounds and $\frac{c}{100}$ is in dollars per OUNCE. You must convert again. Since there are 16 ounces in a pound, it makes sense that a pound would cost 16 times more than an ounce:

$\frac{c}{100}$ dollars per ounce = $\frac{16c}{100}$ dollars per pound

Reduce to get $\frac{4c}{25}$ dollars per pound.

Multiply by p, the number of pounds, to get what the supplier paid: $\frac{4cp}{25}$ dollars.

Now, the supplier is going to sell the mix for three times what he or she paid. (Don't worry that the problem says three times the "price per ounce" — whether you measure in ounces or pounds, this stuff just got three times more expensive.)

$\frac{4cp}{25} \times 3 = \frac{12cp}{25}$, or answer choice (D).

Note: Make sure you were calculating for revenue, not profit! You were not asked to subtract expenses (what the supplier paid) from the money he or she will be making from selling the mix.

An alternative solution is to plug in smart numbers. An easy number to pick when working with cents is 50 (or 25 — whatever is easy to think about and convert to dollars). Write a value on your paper along with what the value means in words:

$c = 50$ mix costs 50¢ per ounce

Now, common sense (and the fact that 16 ounces = 1 pound) will easily allow you to convert:

50¢ per ounce = $8.00 per pound

The supplier bought p pounds. Pick any number you want. For example:

$p = 2$ bought 2 pounds, so spent \$16

Notice that no one asked you for this \$16 figure, but when calculating with smart numbers, it's best to write down obvious next steps in the reasoning process.

Finally, the supplier is going to sell the mix for three times what he or she paid, so the supplier will sell for \$48.

Plug in $c = 50$ and $p = 2$ to see which answer choice generates 48. Only (D) works.

16. **(A).** Since the figure has six sides, use the formula $(n - 2)(180)$, where n is the number of sides, to figure out that the sum of the angles inside the figure = $(6 - 2)(180) = 720$.

The angle supplementary to x can be labeled on your paper as $180 - x$ (since two angles that make up a straight line must sum to 180). Thus:

$$a + b + c + d + e + 180 - x = 720$$
$$a + b + c + d + e - x = 540$$

You are asked to solve for x. Since x is being subtracted from the left side, it would be easiest to add x to both sides, and get everything else on the opposite side.

$$a + b + c + d + e - x = 540$$
$$a + b + c + d + e = 540 + x$$
$$a + b + c + d + e - 540 = x$$

Since $d = 2c$ and $e = \frac{1}{2}a$ and the answers are in terms of a, b, and c, you need to make the d and e drop out of $a + b + c + d + e - 540 = x$.

Fortunately $d = 2c$ and $e = \frac{1}{2}a$ are already solved for d and e, the variables you want to drop out. Substitute:

$$a + b + c + 2c + \frac{1}{2}a - 540 = x$$
$$\frac{3}{2}a + b + 3c - 540 = x$$

This is a match with answer choice (A).

Alternatively, pick numbers. To do this, use the formula $(n - 2)(180)$, where n is the number of sides, to figure out that the sum of the angles inside the figure = $(6 - 2)(180) = 720$. Then, pick values for a, b, c, d, and e, so that $d = 2c$ and $e = \frac{1}{2}a$.

$a = 100$

$b = 110$
$c = 120$
$d = 240$ (this is twice the value picked for c)
$e = 50$ (this is 1/2 the value picked for a)

Subtract all of these values from 720 to get that the unlabeled angle, for this example, is equal to 100. This makes x equal to 180 - 100 = 80.

Now plug $a = 100$, $b = 110$, and $c = 120$ into the answers to see which formula yields a value of 80. (A) is the correct answer.

17. **(A).** One algebraic solution involves defining all three terms in terms of a. Since the terms are consecutive odd integers, they are 2 apart from each other, as such:

a
$b = a + 2$
$c = a + 4$

Then, a is halved to become m, b is doubled to become n, and c is tripled to become p, so:

$$\frac{1}{2}a = m$$

$2b = n$
$2(a + 2) = n$
$2a + 4 = n$

$3c = p$
$3(a + 4) = p$
$3a + 12 = p$

Since $k = mnp$, multiply the values for m, n, and p:

$$k = \left(\frac{1}{2}a\right)(2a+4)(3a+12)$$
$$k = \left(\frac{1}{2}a\right)(6a^2 + 24a + 12a + 48)$$
$$k = \left(\frac{1}{2}a\right)(6a^2 + 36a + 48)$$
$$k = 3a^3 + 18a^2 + 24a$$

This is a match with answer choice (A).

A "smart numbers" solution would be to pick three consecutive odd integers for a, b, and c. When picking numbers for a Variables in the Choices problem, avoid picking 0, 1, or any of the numbers in the problem (this can sometimes cause more than one answer to appear to be correct, thus necessitating starting over with another set of numbers). So:

$a = 3$
$b = 5$
$c = 71$

Then, a is halved to become m, b is doubled to become n, and c is tripled to become p, so:

$1.5 = m$
$10 = n$
$21 = p$

Since $k = mnp$, multiply the values for m, n, and p:

$k = (1.5)(10)(21)$
$k = 315$

Now, plug $a = 3$ (the value originally selected) into the answer choices to see which choice equals 315. Only (A) works.

Because the correct answer is simply a mathematical way of writing the situation described in the problem, this will work for any value you pick for a, provided that a, b, and c are consecutive odd integers and you calculate k correctly.

18. **(C).** The phrase "m pencils cost the same as n pens" can be written as an equation, using x for the cost per pencil and y for the cost per pen:

$mx = ny$

Keep in mind here that m stands for the NUMBER of pencils and n for the NUMBER of pens (not the cost). Now, since pencils cost 20 cents or \$0.2 (the answer needs to be in dollars, so convert to dollars now), substitute in for x:

$0.2m = ny$

Solve for y to get the cost of 1 pen:

$$y = \frac{0.2m}{n}$$

Since y is the cost of 1 pen and $y = \frac{0.2m}{n}$, multiply by 10 to get the cost of 10 pens:

$$10y = 10\left(\frac{0.2m}{n}\right)$$

$$10y = \frac{2m}{n}$$

Thus, the answer is $\frac{2m}{n}$, or (C).

Alternatively, plug in smart numbers. Since pencils cost 20 cents, maybe pens cost 40 cents (you can arbitrarily pick this number). You are told "m pencils cost the same as n pens"—pick a number for one of these variables, and then determine what the other variable would be for the example you've chosen. For instance, if $m = 10$, then 10 pencils would cost \$2.00. Since 5 pens can be bought for \$2.00, n would be 5. Now, answer the final question as a number: the cost of 10 pens in this example is \$4.00, so the final answer is 4. Plug in $m = 10$ and $n = 5$ to all of the answer choices to see which yields an answer of 4. Only (C) works. For any working system you choose in which "m pencils cost the same as n pens," choice (C) will work.

19. **(E).** One way to do this problem is simply to construct a formula. Randi's pay is equal to \$2,000 plus commission:

$$P = 2000 + \ldots$$

You are only asked to construct a formula for months during which she sells more than 10 forklifts, so you know that she will definitely be receiving 5% commission on 10 forklifts that each cost s. Since the revenue from the forklifts would then be $10s$, Randi's commission would be $0.05(10s)$, or $0.5s$.

$$P = 2{,}000 + 0.5s + \ldots$$

Now, you must add the commission for the forklifts she sells above the first 10. Since these first 10 forklifts are already accounted for, you can denote the forklifts at this commission level by writing $f - 10$. Since each forklift still costs s, the revenue from these forklifts would be $s(f - 10)$. Since Randi receives 10% of this as commission, the amount she receives would be $0.10s(f - 10)$.

$$P = 2{,}000 + 0.5s + 0.10s(f - 10)$$

It is possible to simplify further by distributing $0.10s(f - 10)$, but before doing more work, check the answers — you have an exact match already, answer choice (E).

Alternatively, plug in numbers. Say forklifts cost \$100 (so, $s = 100$). Randi makes \$5 each for the first ten she sells, so \$50 total. Then she makes \$10 each for any additional forklifts. Pick a value for f (make sure the value is more than 10, since the question asks for a formula for months in which Randi sells more than 10 forklifts). So, in a month in which she sells, for example, 13 forklifts (so, $f = 13$), she would make \$2,000 + \$50 + 3(\$10) = \$2,080.

In this example:

$$s = 100$$
$$f = 13$$

Plug in these values for s and f to see which choice yields \$2,080. Only choice (E) works:

$$P = 2{,}000 + 0.5(100) + 0.10(100)(13 - 10)$$
$$P = 2{,}000 + 50 + 10(3)$$
$$P = 2{,}080$$

20. **(A).** This question can be solved either with smart numbers or algebra. First, consider plugging in smart numbers.

Set $l = 2$, so $w = 4$. The perimeter will be $2l + 2w = 2(2) + 2(4) = 12$. The answer is the area, which is $wl = (2)(4) = 8$ based on these numbers. Now plug $p = 12$ into the choices to see which choice equals 8:

(A) 144/18 = 8
(B) 144/36 = 4
(C) 12/9 = 4/3
(D) 144/9 = 16
(E) 12/6 = 2

The correct answer is (A).

Though smart numbers are easier and faster here, you could also solve with algebra. If $w = 2l$:

$a = l \times w = l \times 2\,l = 2l^2$
$p = 2l + 2w = 2l + 4l = 6l$

Solve the second equation for l:

$l = p/6$

And plug back into the first equation:

$$2\left(\frac{p}{6}\right)^2 = 2\left(\frac{p^2}{36}\right) = \frac{2p^2}{36} = \frac{p^2}{18}$$

This method is much more difficult than plugging in numbers, but you can still get to the correct answer, (A).

Chapter 20
of

Ratios

In This Chapter…

Ratios

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

The ratio of men to women in a senior citizen garden club is 5 to 4.

Quantity A	**Quantity B**
The smallest possible number of members in the garden club	20

2.

In a certain children's class, there is a 2 to 3 ratio of boys to girls. The ratio of students from the north side of town to students from the south side of town is 4 to 3, and no student is from anywhere else.

Quantity A	**Quantity B**
The smallest possible number of students in the class	35

3. A pantry holds x cans of beans, twice as many cans of soup, and half as many cans of tomato paste as there are cans of beans. If there are no other cans in the pantry, which of the following could be the total number of cans in the pantry?

Indicate <u>two</u> such numbers.

☐ 6
☐ 7
☐ 36
☐ 45
☐ 63

4. If there are 20 birds and 6 dogs in a park, which of the following represents the ratio of dogs to birds in the park?

(A) 3 : 13
(B) 3 : 10
(C) 10 : 3
(D) 13 : 3
(E) 1 : 26

5. Of the 24 children in a classroom, 12 are boys. Which of the following is the ratio of boys to girls in the classroom?

(A) 1 : 1
(B) 1 : 2
(C) 2 : 1
(D) 1 : 3
(E) 3 : 1

6. If there are 24 white marbles and 36 blue marbles in a bag, what is the ratio of blue to white marbles?

Give your answer as a fraction.

7. If there are 7 whole bananas, 14 whole strawberries, and no other fruit in a basket, what is the ratio of strawberries to the total pieces of fruit in the basket?

Give your answer as a fraction.

8. The ratio of cheese to sauce for a single pizza is 1 cup to 1/2 cup. If Bob used 15 cups of sauce to make a number of pizzas, how many cups of cheese did he use on those pizzas?

cups

9. Laura established a new flower garden, planting 4 tulip plants to every 1 rose plant, and no other plants. If she planted a total of 50 plants in the garden, how many of those plants were tulips?

tulip plants

10. The ratio of oranges to peaches to strawberries in a basket containing no other kinds of fruit is 2 : 3 : 4. If there are 8 oranges in the basket, a total of how many pieces of fruit are in the basket?

(A) 16
(B) 32
(C) 36
(D) 48
(E) 72

11. A certain automotive dealer sells only cars and trucks, and the ratio of cars to trucks on the lot is 1 to 3. If there are currently 51 trucks for sale, how many cars does the dealer have for sale?

(A) 17
(B) 34
(C) 68
(D) 153
(E) 204

12. Last season, Arjun's tennis record was 3 matches won for every 2 he lost. If he played 30 matches last season, how many did he win?

(A) 10
(B) 12
(C) 18
(D) 20
(E) 50

13. A steel manufacturer combines 98 ounces of iron with 2 ounces of carbon to make one sheet of steel. How many ounces of iron were used to manufacture 1/2 of a sheet of steel?

(A) 1
(B) 49
(C) 50
(D) 198
(E) 200

14. Maria uses a recipe for 36 cupcakes that requires 8 cups of flour, 12 cups of milk, and 4 cups of sugar. How many cups of milk would Maria require for a batch of 9 cupcakes?

(A) 2
(B) 3

(C) 4
(D) 6
(E) 8

15. In a certain orchestra, each musician plays only one instrument and the ratio of musicians who play either the violin or the viola to musicians who play neither instrument is 5 to 9. If 7 members of the orchestra play the viola and four times as many play the violin, how many play neither?

(A) 14
(B) 28
(C) 35
(D) 63
(E) 72

16. The ratio of 0.4 to 5 equals which of the following ratios?

(A) 4 to 55
(B) 5 to 4
(C) 2 to 25
(D) 4 to 5
(E) 4 to 45

17. At an animal shelter, the ratio of cats to dogs is 4 to 7. If there are 27 more dogs than cats, how many cats are at the shelter?

(A) 12
(B) 16
(C) 24
(D) 28
(E) 36

18. On a wildlife preserve, the ratio of giraffes to zebras is 37 : 43. If there are 300 more zebras than giraffes, how many giraffes are on the wildlife preserve?

(A) 1,550
(B) 1,850
(C) 2,150
(D) 2,450
(E) 2,750

19. On a youth soccer team, the ratio of boys to girls is 6 to 7. If there are 2 more girls than boys on the team, how many boys are on the team?

(A) 12
(B) 18
(C) 24
(D) 30
(E) 36

20. At a certain company, the ratio of male to female employees is 3 to 4. If there are 5 more female employees than male employees, how many male employees does the company have?

(A) 12
(B) 15

(C) 18
(D) 21
(E) 24

21. On Monday, a class has 8 girls and 20 boys. On Tuesday, a certain number of girls joined the class just as twice that number of boys left the class, changing the ratio of girls to boys to 7 to 4. How many boys left the class on Tuesday?

(A) 5
(B) 6
(C) 11
(D) 12
(E) 18

22. If a dak is a unit of length and 14 daks = 1 jin, how many squares with a side length of 2 daks can fit in a square with a side length of 2 jin?

(A) 14
(B) 28
(C) 49
(D) 144
(E) 196

23.

In a group of adults, the ratio of women to men is 5 to 6, while the ratio of left-handed people to right-handed people is 7 to 9. Everyone is either left- or right-handed; no one is both.

Quantity A	**Quantity B**
The number of women in the group	The number of left-handed people in the group

24.

Party Cranberry is 3 parts cranberry juice and 1 part seltzer. Fancy Lemonade is 1 part lemon juice and 2 parts seltzer. One glass of Party Cranberry is mixed with an equally sized glass of Fancy Lemonade.

Quantity A	**Quantity B**
The fraction of the resulting mix that is cranberry juice	The fraction of the resulting mix that is seltzer

25.

The ratio of 16 to g is equal to the ratio of g to 49.

Quantity A	**Quantity B**
g	28

26. In a parking lot, 1/3 of the vehicles are black and 1/5 of the remainder are white. How many vehicles could be parked on the lot?

(A) 8
(B) 12
(C) 20
(D) 30
(E) 35

27. Three friends divided a bag of chocolates so that David received a fifth the number of chocolates that Fouad did, and Stina received 80 percent of the total number of chocolates. What is the ratio of the number of chocolates Stina received to the number that David received?

(A) 4 : 3
(B) 8 : 5
(C) 8 : 1
(D) 24 : 1
(E) 80 : 1

28. A new sport is played with teams made up of 2 forwards, 3 guards, and 1 goalie. There are 23 players available to play forward, 21 other players available to play guard, and 9 other players available to play goalie. If the maximum possible number of complete teams are formed, how many of the available players will <u>not</u> be on a team?

(A) 7
(B) 9
(C) 11
(D) 13
(E) 15

29. Oil, vinegar, and water are mixed in a 3 to 2 to 1 ratio to make salad dressing. If Larry has 8 cups of oil, 7 cups of vinegar, and access to any amount of water, what is the maximum number of cups of salad dressing he can make with the ingredients he has available, if fractional cup measurements are possible?

(A) 12
(B) 13
(C) 14
(D) 15
(E) 16

30. With y dollars, 5 oranges can be bought. If all oranges cost the same, how many dollars do 25 oranges cost, in terms of y?

(A) $y/5$
(B) y
(C) $y + 5$
(D) $5y$
(E) $25y$

31. A certain drawer contains only black and white socks. If the ratio of black socks to white socks is 3 : 4 and there are 15 black socks in the drawer, how many socks total are in the drawer?

(A) 15
(B) 20
(C) 30
(D) 35
(E) 45

32.

A tree grows taller at a constant rate. The ratio of its growth in feet to the time spent growing in years is 4 : x.

Quantity A	**Quantity B**
The number of feet the tree grows taller in 10 years	40

33. Ten robots painted 3 identical houses in 5 hours, working at a constant rate. How many hours would it take 20 identical robots to paint 12 such houses, working at the same constant rate?

(A) 2.5
(B) 5
(C) 10
(D) 15
(E) 20

34. Dick takes twice as long as Jane to run any given distance. Starting at the same moment, Dick and Jane run towards each other from opposite ends of the schoolyard, a total distance of x, at their respective constant rates until they meet.

Quantity A	**Quantity B**
The fraction of the total distance x that is covered by Jane	$\frac{2}{3}x$

35. One robot can pack a box in 15 minutes. Working together at the same constant rate, how many boxes can 16 robots pack in 1 hour?

(A) 4
(B) 16
(C) 24
(D) 64
(E) 256

36. A woman spent $\frac{5}{8}$ of her weekly salary on rent, and $\frac{1}{3}$ of the remainder on food, leaving \$40 available for other expenses. What is the woman's weekly salary?

(A) \$160
(B) \$192
(C) \$216
(D) \$240
(E) \$256

37. A mixture contains nothing but water and acetone in a ratio of 1 : 2. After 200 mL of water is added to the mixture, the ratio of water to acetone is 2 : 3.

Quantity A	**Quantity B**
The original volume of the mixture	1,800 mL

38. In a certain rectangle, the ratio of length to width of a rectangle is 3 : 2 and the area is 150 square centimeters. What is the perimeter of the rectangle, in centimeters?

(A) 10
(B) 15
(C) 25
(D) 40
(E) 50

39. At a certain college, the ratio of students to professors is 8 : 1 and the ratio of students to administrators is 5 : 2. No person is in more than one category (for instance, there are no administrators who are also students).

Quantity A	**Quantity B**
The fractional ratio of professors to administrators	$\frac{5}{8}$

40. Mary prepared x pounds of pasta for the y people expected to attend a banquet. If only z of these y people actually attend, such that $z < y$, how many pounds of pasta will be left over if Mary serves the originally intended portion to each of the guests in attendance?

(A) $\frac{x(y-z)}{y}$
(B) $\frac{zx}{y}$
(C) $\frac{x-z}{y}$
(D) $\frac{x(z-y)}{y}$
(E) $\frac{x(y+z)}{y}$

41. Sara purchased a number of wrenches and hammers from a hardware store, such that the ratio of wrenches to hammers purchased was 5 : 4 and she purchased 10 more wrenches than hammers.

Quantity A	**Quantity B**
The number of hammers Sara purchased from the hardware store	50

42. A family drove from home to a vacation destination 100 miles away, driving the first half of the distance at a constant speed of 50 miles per hour and the second half of the distance at a constant speed of 20 miles per hour. Returning home by the same route, they traveled at a constant speed of 30 miles per hour for the whole trip.

Quantity A	**Quantity B**
The number of hours it took to drive from home to the vacation destination	The number of hours it took to drive from the vacation destination back home

43. A hose is filling a large bucket with water at a constant rate of 3 gallons per minute. The bucket is losing water through a leak at a constant rate of 1 gallon per minute. If the bucket can hold a total volume of 8 gallons, how many minutes are required to fill the bucket to capacity, starting from empty?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

44. If Dan can make 10 widgets every 15 seconds, how many widgets can Dan make in 1 hour, working at this constant rate?

(A) 40
(B) 240
(C) 600
(D) 2,400
(E) 4,000

45. In a certain country, 8 rubels are worth 1 schilling, and 5 schillings are worth 1 lemuw. In this country, 6 lemuws are equivalent in value to how many rubels?

(A) 20/3
(B) 30
(C) 40
(D) 48
(E) 240

46. The ratio of Kim's time to paint a house to Jane's time to paint a house is 3 : 5. If Kim and Jane work together at their respective constant rates, they can paint a house in 10 hours.

Quantity A	**Quantity B**
The number of hours it takes Kim to paint the house alone	16

47. Team A and Team B are raising money for a charity event. The ratio of money collected by Team A to money collected by team B is 5 : 6. The ratio of the number of students on Team A to the number of students on Team B is 2 : 3. What is the ratio of money collected per student on team A to money collected per student on team B?

(A) 4 : 5
(B) 5 : 4
(C) 5 : 6
(D) 5 : 9
(E) 9 : 5

48. Ketchup, soy sauce, and mayonnaise are mixed together in a ratio of 3 : 2 : 5 to make Mr. Anderson's special sauce. If Mr. Anderson prepared 25 ounces of special sauce for his upcoming barbeque, how many ounces of soy sauce did he use?

(A) 2.5
(B) 5
(C) 7.5
(D) 10

(E) 12.5

49. Saul ran from point A to point B and then back again by the same route in 63 minutes. It took Saul $\frac{3}{4}$ as much time to run from point A to point B as it took him to run from point B to point A.

Quantity A	**Quantity B**
The number of minutes Saul's point A to point B run took	30

50. Jarod needs 2/3 of an ounce of vinegar for every 2 cups of sushi rice that he prepares. To prepare 7 cups of sushi rice in the same proportion, how many ounces of vinegar does Jarod need?

(A) 3/2
(B) 4/3
(C) 7/3
(D) 7/2
(E) 14/3

51 Joe drove from Springfield to Shelbyville at *x* miles per hour. He then drove from Shelbyville to Bakersfield at (1.5)*x* miles per hour. If the distance between Springfield and Shelbyville is twice the distance between Shelbyville and Bakersfield, what was Joe's average speed for the entire trip?

(A) $\frac{9}{8}x$
(B) $\frac{6}{5}x$
(C) $\frac{5}{4}x$
(D) $\frac{7}{4}x$
(E) $\frac{9}{4}x$

52. The total cost of 3 bananas, 2 apples, and 1 mango is \$3.50. The total cost of 3 bananas, 2 apples, and 1 papaya is \$4.20. The ratio of the cost of a mango to the cost of a papaya is 3 : 5.

Quantity A	**Quantity B**
The cost of a papaya	\$2.00

53. In a certain town, $\frac{2}{5}$ of the total population is employed. Among the unemployed population, the ratio of males to females is 5 : 7. If there are 40,000 employed people in the town, how many females are unemployed?

(A) 16,000
(B) 25,000
(C) 35,000

(D) 65,000
(E) 75,000

54.

Quantity A	Quantity B
The ratio of $2\frac{11}{12}$ to $1\frac{3}{4}$	$\frac{5}{3}$

55. On a certain map of the United States, 3/5 of an inch represents a distance of 400 miles. If Oklahoma City and Detroit are separated on the map by approximately 3/2 of an inch, what is the approximate distance between them in miles?

(A) 240
(B) 360
(C) 600
(D) 800
(E) 1,000

56. A machine can manufacture 20 cans per hour, and exactly 10 such cans fit into every box. Maria packs cans in boxes at a constant rate of 3 boxes per hour. If the machine ran for 2 hours and was then turned off before Maria started packing the cans in boxes, how many minutes would it take Maria to pack all the cans that the machine had made?

(A) 40
(B) 45
(C) 80
(D) 160
(E) 800

57. At Company X, a person's pay is increased by the same dollar amount every year that he or she works for Company X. Since Joe started with Company X, he has seen 10 such raises, and the ratio of his pay now to his pay when he started is 5 : 2. What is the ratio of Joe's yearly pay increase to his starting pay?

(A) 1/4
(B) 3/20
(C) 5/3
(D) 4/1
(E) 20/3

58. If Beth has 1/4 more money than Ari, and each person has an integer number of dollars, which of the following could be the combined value of Beth and Ari's money?

Indicate all such values.

☐ $12
☐ $54
☐ $72
☐ $200

59. If salesperson A sold 35% more motorcycles than salesperson B, which of the following could be the total

number of motorcycles sold by both salespeople?

Indicate <u>all</u> such total numbers of motorcycles.

☐ 47
☐ 70
☐ 135
☐ 235

60. A zoo has twice as many zebras as lions and four times as many monkeys as zebras. Which of the following could be the total number of zebras, lions, and monkeys at the zoo?

Indicate <u>all</u> such totals.

☐ 14
☐ 22
☐ 28
☐ 55
☐ 121

61. In Nation Z, 10 terble coins equal 1 galok. In Nation Y, 6 barbar coins equal 1 murb. If a galok is worth 40% more than a murb, what is the ratio of the value of 1 terble coin to the value of 1 barbar coin?

(A) $\frac{3}{5}$
(B) $\frac{11}{13}$
(C) $\frac{3}{7}$
(D) $\frac{21}{23}$
(E) $\frac{21}{25}$

62. Autolot has a 2 : 1 ratio of blue cars to red cars and a 6 : 1 ratio of red cars to orange cars on the lot. What could be the total number of blue, red and orange cars on the lot?

(A) 38
(B) 39
(C) 40
(D) 41
(E) 42

63. Originally, 70% of the clients at Bob's Dating Bistro were male. After z of the female clients left, the service still had 74 clients. Which of the following could be the value of z?

Indicate <u>all</u> such values.

- ☐ 4
- ☐ 6
- ☐ 12
- ☐ 16
- ☐ 18

64. Beaker B has a volume of b, which is twice the volume c of Beaker C. The volume of Beaker C is one third the volume g of Beaker G.

Quantity A	**Quantity B**
$\frac{b+c}{g}$	1

Ratios Answers

1. **(B).** The ratio of men to women is 5 to 4. Since both 5 and 4 are whole numbers, they could actually *be* the number of men and women, respectively.

5 and 4 are also the *lowest* possible numbers of men and women, because reducing the ratio of 5 to 4 any further is impossible without making one part a non-integer (e.g., 2.5 to 2) or both parts negative, and the numbers of men and women must be positive.

So the smallest number of people who could be in the garden club is 5 + 4 = 9, which is less than 20. Quantity B is greater.

2. **(C).** When a ratio includes things you can count (such as people), each group must be a positive multiple of the numbers in the ratio. For instance, the ratio of boys to girls is 2 to 3. So the number of boys is $2n$ and the number of girls is $3n$, where n is a positive integer. The total number of students is $2n + 3n = 5n$, which is a positive multiple of 5.

Using the second ratio, write the number of students from the north side as $4m$ and the number of students from the south side as $3m$, where m is a positive integer. The total number of students is $4m + 3m = 7m$, which is a positive multiple of 7.

The smallest number that is a positive multiple of both 5 and 7 is 35. The two quantities are equal.

3. **II and V only.** Write the number of each type of can.

Cans of beans = x Cans of soup = $2x$ Cans of tomato paste = $0.5x$

Since each number of cans must be an integer, you know that $0.5x = n$, where n is some positive integer. This means $x = 2n$. (In other words, x is even.)

The total number of cans is $x + 2x + 0.5x = 3.5x$. Now substitute in $2n$:

Total = $3.5x = 3.5(2n) = 7n$. So the total number of cans is a multiple of 7.

Of the answer choices, only 7 and 63 are multiples of 7.

4. **(B)** If there are 6 dogs and 20 birds in the park, the ratio of dogs to birds is 6 : 20, which reduces to 3 : 10.

5. **(A).** If 12 of 24 children in the classroom are boys, then the remaining 24 - 12 = 12 children are girls. Therefore the ratio of boys to girls in the classroom is 12 : 12. Reduce this ratio by dividing both sides by the common factor of 12. The reduced ratio is $\frac{12}{12} : \frac{12}{12}$ or 1 : 1.

Be sure to answer the right question on a quick problem such as this one. The correct ratio of boys to all children is 1 : 2, which is also the correct ratio of girls to all children; however, the question asks for the ratio of boys to girls, which is 1 : 1.

6. $\frac{3}{2}$ **or any equivalent fraction.** If there are 36 blue marbles and 24 white marbles, the ratio of blue to white marbles is 36 : 24. Write this ratio as a fraction and cancel the common factor of 12 from top and bottom. $\frac{36}{24} = \frac{3\times12}{2\times12} = \frac{3}{2}$. The the original ratio of $\frac{36}{24}$ would also be counted as correct if entered as-is.

7. $\frac{2}{3}$ **or any equivalent fraction.** If there are 7 bananas and 14 strawberries, then there are 7 + 14 = 21 total pieces of fruit. The ratio of strawberries to the total is therefore 14 : 21. Write this ratio as a fraction and cancel the common factor of 7 from top and bottom. $\frac{14}{21} = \frac{2\times7}{3\times7} = \frac{2}{3}$. The original ratio of $\frac{14}{21}$ would also be counted as correct if entered as-is.

8. **30 cups.** To solve for the actual amount of cheese Bob used, work with the Part : Part ratio. The ratio of cheese to sauce is $1 : \frac{1}{2}$, or $1x : \frac{1}{2}x$ (putting in an unknown multiplier). When the actual amount of sauce is $\frac{1}{2}x$, the actual amount of cheese is 1x, or simply x. Bob used 15 cups of sauce. Solve for x:

$$\frac{1}{2}x = 15$$
$$x = 15(2)$$
$$x = 30$$

Since x also indicates the actual amount of cheese, Bob used 30 cups of cheese. Don't go so fast on this sort of problem that you make a silly mistake, such as reversing the ratio and getting $\frac{1}{2}$ of 15, or 7.5 cups, as the incorrect answer.

9. **40 tulip plants.** To solve for the number of tulips, work with the Part : Part : Whole ratio. The ratio of tulips to roses is 4 : 1, so the Tulip : Rose : Total relationship is 4 : 1 : 5. This ratio can be written as 4x : 1x : 5x, with x as the unknown integer multiplier. There are 50 total plants in the garden, so set 5x equal to 50 and solve for x:

$$5x = 50$$
$$x = 10$$

Now plug this value into the expression for the actual number of tulips: 4x = 4(10) = 40. Laura planted 40 tulip plants in the garden.

10. **(C).** Work with the Part : Whole ratio to solve for the total number of pieces of fruit. The ratio of oranges to

peaches to strawberries is 2 : 3 : 4, so the Part : Whole relationship would include the total of 2 + 3 + 4 = 9, or 2 : 3 : 4 : 9 (three parts and a whole). This four-way ratio can be written as $2x : 3x : 4x : 9x$, with x as the unknown multiplier. There are 8 oranges in the basket, so set the oranges part ($2x$) equal to 8 and solve for x.

$$2x = 8$$
$$x = 4$$

Now plug this value into the expression for the whole: $9x = 9(4) = 36$. There are 36 pieces of fruit total.

11. **(A).** Focus on the given Part : Part ratio. The ratio of cars to trucks is 1 : 3, or $x : 3x$ with x as the unknown multiplier. Since there are 51 trucks for sale, set $3x$ equal to 51 and solve for x.

$$3x = 51$$
$$x = 17$$

Since x also represents the number of cars, the dealer has 17 cars for sale.

12. **(C).** Work with the Part : Part : Whole ratio to solve for the number of wins. The ratio of matches won to matches lost is 3 to 2, so the Wins : Losses : Total ratio is 3 : 2 : 5. This ratio can be written as $3x : 2x : 5x$, with x as the unknown multiplier. Arjun played 30 matches in all, so set $5x$ equal to 30 and solve.

$$5x = 30$$
$$x = 6$$

Now plug this value into the expression for the number of wins: $3x = 3(6) = 18$. Arjun won 18 matches.

13. **(B).** Iron and carbon combine to make steel in a specific given ratio. The ratio of iron (ounces) to carbon (ounces) to steel (sheets) is 98 : 2 : 1. Because there are different units (ounces and sheets), the Part numbers do not add to the Whole number as they typically do, but don't be concerned.

This ratio can be written as $98x : 2x : x$, with x as the unknown multiplier, which is also the number of sheets. To make 1/2 a sheet of steel, set x equal to 1/2.

Now plug this value into the expression for the number of iron ounces: $98x = (98)(1/2) = 49$. To make 1/2 a sheet of steel, 49 ounces of iron are required.

14. **(B).** As a ratio, Flour : Milk : Sugar : Cupcakes is equal to 8 : 12 : 4 : 36, where the first 3 numbers are in cups. Because there are different units (cups and cupcakes), the Part numbers do not add to the Whole number, but don't be concerned.

This ratio can be written as $8x : 12x : 4x : 36x$, with x as the unknown multiplier. To make 9 cupcakes, set $36x$ equal to 9 and solve for x.

$$36x = 9$$
$$x = 1/4$$

In words, for a batch of 9 cupcakes, Maria would make 1/4 of the original recipe.

Now plug this value into the expression for cups of milk: $12x = (12)(1/4) = 3$. Maria would need 3 cups of milk.

15. **(D).** Since 7 members of the orchestra play the viola and four times as many play the violin, $(7)(4) = 28$ people must play the violin. All together, $7 + 28 = 35$ musicians in the orchestra play either the viola or the violin.

The ratio of *either* to *neither* is 5 : 9, or $5x : 9x$ using the unknown multiplier. Since 35 people play either instrument, set $5x$ equal to 35 and solve for x.

$$5x = 35$$
$$x = 7$$

Now plug this value into the expression for *neither*: $9x = 9(7) = 63$. There are 63 people in the orchestra who play neither instrument.

16. **(C).** You can rewrite ratios as fractions and then multiply or divide top and bottom by the same number, keeping the ratio (or fraction) the same.

First, multiply top and bottom by 10, to remove the decimal. $\frac{0.4}{5} = \frac{0.4 \times 10}{5 \times 10} = \frac{4}{50}$

Next, cancel the common factor of 2. $\frac{4}{50} = \frac{2 \times 2}{25 \times 2} = \frac{2}{25}$

Finally, the fraction $\frac{2}{25}$ is the same as the ratio of 2 to 25, which is therefore equivalent to the original ratio of 0.4 to 5.

17. **(E).** The ratio of cats to dogs is 4 : 7. Because the number of cats or dogs in this question can only be positive integers, introduce the unknown multiplier x: the number of cats is $4x$, and the number of dogs is $7x$, where x is a positive integer.

Now translate the second sentence of the problem into algebra. "There are 27 more dogs than cats" becomes Dogs - Cats = 27, or $7x - 4x = 27$. Solve for x:

$$7x - 4x = 27$$
$$3x = 27$$
$$x = 9$$

Finally, substitute into the expression for the number of cats: $4x = 4(9) = 36$. There are 36 cats.

18. **(B).** The ratio of giraffes to zebras is 37 : 43. Introduce the unknown multiplier x: the number of giraffes is $37x$, and the number of zebras is $43x$, where x is a positive integer.

Now translate the second sentence of the problem into algebra. “There are 300 more zebras than giraffes” becomes Zebras - Giraffes = 300, or $43x - 37x = 300$. Solve for x:

$$43x - 37x = 300$$
$$6x = 300$$
$$x = 50$$

Finally, substitute into the expression for the number of giraffes: $37x = 37(50) = 1{,}850$. There are 1,850 giraffes.

In a pinch, here’s a shortcut: the right answer must be a multiple of 37, because the giraffe number in the ratio is 37, and you need a positive whole number of giraffes. If you test the answer choices, only 1,850 is divisible by 37. This shortcut doesn’t always work this well, of course!

19. **(A).** The ratio of boys to girls is 6 : 7. If you introduce the unknown multiplier x, the number of boys is $6x$, and the number of girls is $7x$, where x is a positive integer.

Now translate the second sentence of the problem into algebra. “There are 2 more girls than boys” becomes Girls - Boys = 2, or $7x - 6x = 2$. Solve for x:

$$7x - 6x = 2$$
$$x = 2$$

Finally, substitute into the expression for the number of boys: $6x = 6(2) = 12$. There are 12 boys on the youth soccer team.

20. **(B).** The ratio of male to female employees is 3 : 4. If you introduce the unknown multiplier x, the number of males is $3x$, and the number of females is $4x$, where x is a positive integer.

Now translate the second sentence of the problem into algebra. “There are 5 more female employees than male employees” becomes Females – Males = 5, or $4x – 3x = 5$. Solve for x:

$$4x – 3x = 5$$
$$x = 5$$

Finally, substitute into the expression for the number of male employees: $3x = 3(5) = 15$. There are 15 male employees.

21. **(D).** Call the number of girls who joined the class x, so the new number of girls in the class was $8 + x$. Twice as many boys left the class, so the number of boys who left the class is $2x$, and the new number of boys in the class was $20 – 2x$.

The resulting ratio of boys to girls was 7 to 4. Since there is already a variable in the problem, don’t use an unknown multiplier. Rather, set up a proportion and solve for x.

$$\frac{\text{Girls}}{\text{Boys}} = \frac{8+x}{20-2x} = \frac{7}{4}$$

$4(8+x) = 7(20-2x)$
$32+4x = 140-14x$
$18x = 108$
$x = 6$

Finally, the question asks for the number of boys who left the class. This is $2x = 2(6) = 12$ boys.

Check: There were 8 girls in the class, then 6 joined for a total of 14 girls. There were 20 boys in the class until 12 left the class, leaving 8 boys in the class. The resulting ratio of girls to boys was $\frac{14}{8} = \frac{7\times 2}{4\times 2} = \frac{7}{4}$, as given.

22. **(E).** Since 14 daks = 1 jin, a length measured in daks is 14 times the same length measured in jins. In other words, the ratio of the length in daks to the length in jins is 14 to 1.

Write this relationship as a fraction: $\frac{14 \text{ daks}}{1 \text{ jin}}$. You can also write $\frac{1 \text{ jin}}{14 \text{ daks}}$. You can convert a measurement from one unit to the other by multiplying by one of these unit conversion factors.

Side of big square: $(2 \text{ jins})\left(\frac{14 \text{ daks}}{1 \text{ jin}}\right) = 28$. Since the small square has a side length of 2 daks, the number of small sides that will fit along a big side is $28 \div 2$, or 14.

However, 14 is not the right answer. 14 is the number of small squares that will fit along *one wall* of the big square, in one row. There will be 14 rows, so in all there will be $(14)(14) = 196$ small squares that fit inside the big square.

23. **(A).** Write two different Part : Part : Whole relationships. In each relationship, the two parts sum to the whole.

Women : Men : Total = 5 : 6 : 11, so Women : Total = 5 : 11.

Left-handed : Right-handed : Total = 7 : 9 : 16, so Left-handed : Total = 7 : 16.

In other words, women account for $\frac{5}{11} = 45.\overline{45}\%$ of the group, left-handed people for $\frac{7}{16} = 43.75\%$ of the group.

Since the total number of people is the same (it's the same group, whether divided by gender or handedness), the percents can be compared directly. There must be more women than left-handed people in the group. Quantity A is greater.

24. **(B).** Be careful — don't just add the "parts" from the different glasses, because the parts will generally not be the same size! Start by writing Part : Part : Whole relationships for each glass. In each relationship, the whole is the sum of the parts.

For Party Cranberry, *Cranberry* : *Seltzer* : *Whole* = 3 : 1 : 4

For Fancy Lemonade, *Lemon* : *Seltzer* : *Whole* = 1 : 2 : 3

Since the two glasses that are mixed are the same size, you can choose a smart number to represent the volume of a glass. This number should be a multiple of both 4 and 3, according to the ratios above, so it is convenient to say that a glass is 12 ounces. Multiply the Party Cranberry ratio by 3 and the Fancy Lemonade ratio by 4, in both cases to get 12 total ounces.

For Party Cranberry, *Cranberry* : *Seltzer* : *Whole* = 9 : 3 : 12

For Fancy Lemonade, *Lemon* : *Seltzer* : *Whole* = 4 : 8 : 12

Finally, when the two glasses are mixed, the resulting total is 24 ounces, of which 9 ounces are cranberry juice but 3 + 8 = 11 ounces are seltzer. There is more seltzer in the resulting mix, so its fraction of the mix is greater than cranberry juice's fraction of the mix. Quantity B is greater.

25. **(D).** Write the ratios as fractions and set them equal to each other.

$$\frac{16}{g}=\frac{g}{49}$$

Cross multiply to get $16 \times 49 = g^2$.

Remember that when you "unsquare" an equation, you must account for the negative possibility. The value of g could be *either* $4 \times 7 = 28$ *or negative* 28. Nothing in the problem indicates that g must be positive. Since Quantity A might equal Quantity B *or* be less than Quantity B, the relationship cannot be determined from the information given.

26. **(D) 30.** Since vehicles must be counted with whole numbers and 1/3 of the cars are black, the total number of cars must be divisible by 3. Otherwise, 1/3 of the total would not be a whole number. The answer must be (B) or (D).

The remainder of the cars is 1 – 1/3 = 2/3 of the total. Of these, 1/5 are white, so 1/5 of 2/3, or $\frac{2}{15}$ of the total number of vehicles are white. Again, because the white cars must be countable with whole numbers, $\frac{2}{15}$ of the total must be an integer. You can write the equation using fractions:

$$\left(\frac{2}{15}\right)(\text{Total})=\text{Integer}$$

To get an integer outcome, the total must be divisible by 15. Of the answer choices, only (D) is divisible by 15.

27. **(D).** The ratio of David's chocolates to Fouad's chocolates is 1 : 5, which you can represent as x and $5x$, respectively, using an unknown multiplier. Together, David and Fouad received $x + 5x = 6x$ chocolates.

Since Stina received 80% of the total, David and Fouad received 20% of the total, or 1/5 of the total. Thus, the total is 5 times what David and Fouad received, or $5(6x) = 30x$. Stina received 80% of this total, or $24x$. You could get to Stina's number directly — since she gets 4/5 of the total and the others get 1/5, she gets 4 times as many chocolates as David and Fouad together, or $4(6x) = 24x$.

Finally, the ratio of Stina's chocolates to David's is $24x$ to x, or 24 : 1.

28. **(C).** To figure out the "limiting factor," take the number of players available for each position and figure out how many teams could be formed in each case, if there were more than enough players in all the other positions.

Forwards: 23 players available ÷ 2 forwards needed per team = 11.5 teams (if there could be partial teams) = 11 complete teams, rounding down.

Guards: 21 players available ÷ 3 guards needed per team = 7 complete teams.

Goalies: 9 players available ÷ 1 goalie needed per team = 9 complete teams.

The guards are the limiting factor, because the fewest complete teams can be formed with them. Only 7 complete teams can be formed, using all of the available guards and some of the other players. A total of 7 × 2 = 14 forwards are required, leaving 23 - 14 = 9 unused forwards. Likewise, 7 × 1 = 7 goalies are required, leaving 9 - 7 = 2 unused goalies. In all, there are 9 + 2 = 11 unused players, who will not be on a team.

29. **(E).** Since the ratio of ingredients is 3 : 2 : 1 in the recipe, imagine that Larry works in cups. Then a recipe makes 3 + 2 + 1 = 6 cups of dressing. To figure out the "limiting factor," take each available amount of ingredient and figure out how many times he could make the recipe, permitting fractions, if he had more than enough of the other ingredients.

Oil: 8 cups available ÷ 3 cups needed per recipe = 8/3 recipes (in other words, $2\frac{2}{3}$ times the recipe). There is no need to round down, because fractional cups of ingredients are allowed.

Vinegar: 7 cups available ÷ 2 cups needed per recipe = 7/2 recipes (in other words, $3\frac{1}{2}$ times through the recipe).

Water availability is not limited, so ignore it.

Oil is the limiting factor, because Larry can make the fewest recipes with it. Thus, he can only make 8/3 recipes. To find the total cups of salad dressing, multiply this fraction by the total number of cups that a recipe makes:

$$\frac{8}{3} \text{ recipe} \times 6 \text{ cups per recipe} = 16 \text{ cups}$$

30. **(D).** Create a unit conversion factor, using the given ratio of oranges to dollars. The conversion factor will look

like either $\frac{5 \text{ oranges}}{y \text{ dollars}}$ or $\frac{y \text{ dollars}}{5 \text{ oranges}}$. Which one you use depends on how you want to convert the units.

You are given 25 oranges and asked how many dollars, in terms of *y*, these oranges will cost. Since you are starting with oranges and want to get to dollars, choose the conversion unit that cancels oranges and leaves dollars on top: $\frac{y \text{ dollars}}{5 \text{ oranges}}$. Then multiply:

$$(25 \text{ oranges})\left(\frac{y \text{ dollars}}{5 \text{ oranges}}\right) = \frac{25y}{5} \text{ dollars} = 5y \text{ dollars}$$

Intuitively, a total of 25 oranges is the same as 5 sets of 5 oranges each. Each set costs *y* dollars. Therefore, the total cost for 5 sets of oranges is $5 \times y = 5y$.

31. **(D).** The ratio of black socks to white socks is 3 : 4, so you can represent the number of black socks as $3x$ and the number of white socks as $4x$, with x as the unknown multiplier.

You are told that there are 15 black socks, so set that equal to the expression for black socks and solve for x:

$$3x = 15$$
$$x = 5$$

The total number of socks is $3x + 4x = 7x$, so there are $7(5) = 35$ socks total in the drawer.

32. **(D).** Translate the second sentence from a ratio to a rate. The growth of the tree, divided by the time it spends growing, is $4/x$. In other words, the constant rate of growth is $4/x$ feet per year.

Now apply the formula Distance = Rate × Time.

$$\text{Distance} = \left(\frac{4 \text{ feet}}{x \text{ years}}\right)(10 \text{ years}) = \frac{40}{x} \text{ feet}$$

Without knowing x, it cannot be determined whether Quantity A is greater or less than Quantity B. For example, if the tree grows 4 feet every year ($x = 1$), the two quantities are equal. If the tree grows 4 feet every 10 years ($x = 10$), then Quantity B is greater than Quantity A (which would be 4).

33. **(C).** Work problems can be solved in various ways; this explanation takes an approach involving ratios. Start with what you know—10 robots can paint 3 houses in 5 hours. Twice as many robots can paint twice as many houses in the same amount of time. In other words, the ratio 10 robots : 3 houses = 20 robots : 6 houses. That is, the 20 robots in question can paint 6 houses in 5 hours.

But 12 houses need painting, not 6. That’s twice as many houses to paint. Since the rate at which the robots work is constant, a given set of robots will take twice as long to paint twice as many houses. So, to paint 12 houses, the 20 robots need 2 × 5 hours = 10 hours.

34. **(C).** Rate problems can be solved in various ways; this explanation takes an approach involving ratios. If Dick takes twice as long as Jane to run any distance, then he must run half as fast. So the ratio of Dick's speed to Jane's speed is 1 : 2. Because *Rate* × *Time* = *Distance*, this also means that for a fixed period of time, the ratio of the distances they run is also 1 : 2. That is, Dick only covers half the distance Jane covers in the same amount of time. They start running at the same moment and stop running when they meet, so they do run for the same amount of time.

Now use a Part : Part : Whole relationship. Since together, Dick and Jane run the whole distance, Dick's distance : Jane's distance : Total distance ratio must be 1 : 2 : 3. Dick runs $\frac{1}{3}x$, Jane runs $\frac{2}{3}x$, and the total distance is $\frac{3}{3}x$. Thus, Quantity A equals Quantity B.

35. **(D).** First find how many boxes 1 robot can pack in 1 hour. Since it is given that 1 robot can pack a box in 15 minutes, multiply by the conversion ratio of 60 minutes to 1 hour:

$$\left(\frac{1\text{ box}}{15\text{ minutes}}\right)\left(\frac{60\text{ minutes}}{1\text{ hour}}\right)=\frac{4\text{ boxes}}{1\text{ hour}}$$

This result should make intuitive sense. Each hour contains four 15-minute segments. A robot can build 1 box in each of these 4 segments, so 4 boxes per hour are packed by one robot.

The rate is the same for all robots, so if 1 robot can build 4 boxes in 1 hour, then 16 robots can build 16 times as many boxes in the same amount of time. 16 × 4 boxes per hour = 64 boxes in one hour.

36. **(A).** The total amount of money left over after paying rent and buying food is $40. From this number, you can find her total weekly salary by determining what fraction this is of her total salary.

Since the woman spent 5/8 of her salary on rent, she had 1 - 5/8 = 3/8 of her salary remaining. Of the remainder, she spent 1/3 on food and had 2/3 left over. So, 2/3 of 3/8 of her total weekly salary was left over for other expenses.

$$\left(\frac{2}{3}\right)\left(\frac{3}{8}\right)=\frac{2}{8}=\frac{1}{4}$$

One quarter of her salary was the $40 left over. If *T* is her total weekly salary, then

$$\left(\frac{1}{4}\right)T=\$40$$

$$T=\$160$$

37. **(C).** Use the unknown multiplier, *x*, for the first ratio given. Since the ratio of water to acetone is 1 : 2, make the amount of water *x* and the amount of acetone 2*x*. After 200 mL of water are added to the mixture, there will be *x* + 200 mL of water. Now write the new ratio:

$$\frac{\text{Water}}{\text{Acetone}} = \frac{x + 200}{2x} = \frac{2}{3}$$

Cross multiply and solve for x:

$3x + 600 = 4x$
$x = 600$

Finally, compute the original volume of the mixture. The original volume of water is $x = 600$ mL, while the original volume of acetone is $2x = 2(600) = 1{,}200$ mL. Therefore, the total original volume is $600 + 1200 = 1800$ mL.

38. **(E).** Rewrite the given ratio using the unknown multiplier x, so that the length of the rectangle is $3x$, while the width is $2x$. Now express the area of the rectangle in these terms, set it equal to 150 square centimeters, then solve for x:

Area = (Length)(Width)
$150 = (3x)(2x)$
$150 = 6x^2$
$25 = x^2$
$x = 5$ cm

In this case, you don't need to worry about the negative possibility for the square root, since lengths cannot be less than zero. The length is $3x = 15$ centimeters, while the width is $2x = 10$ centimeters.

Finally, the perimeter of a rectangle is twice the length, plus twice the width:

Perimeter = 2 × length + 2 × width
Perimeter = 2 × 10 cm + 2 × 15 cm
Perimeter = 20 cm + 30 cm
Perimeter = 50 cm

39. **(B).** One way to approach this problem is to pick a smart number for the number of students, which shows up in both ratios. In the first ratio, students are represented by 8, so you want the smart number of students to be a multiple of 8. Likewise, in the second ratio, students are represented by 5, so you want the smart number of students to be a multiple of 5 as well. So pick 40 for the number of students.

From here, solve for the number of professors.

$$\frac{\text{Students}}{\text{Professors}} = \frac{40}{\text{Professors}} = \frac{8}{1}$$

40 = 8 × Professors
5 = Professors

Likewise, solve for the number of administrators.

$$\frac{\text{Students}}{\text{Administrators}} = \frac{40}{\text{Administrators}} = \frac{5}{2}$$

$40 \times 2 = 5 \times \text{Administrators}$
$16 = \text{Administrators}$

Therefore, the ratio of professors to administrators is 5 : 16. In fractional ratio form, this is $\frac{5}{16}$. Comparing the two quantities, both have the same numerator, but the denominator of Quantity A is greater, making it the smaller value. In fact, Quantity B is exactly twice as great as Quantity A.

40. **(A).** To determine the number of pounds of leftover pasta, find the amount of pasta actually served to the guests who actually attended, and subtract from the total amount of pasta originally prepared. Originally, Mary prepared x pounds of pasta for y people. Create a ratio of pasta to people: each person was supposed to receive $\frac{x}{y}$ pounds of pasta. Since z people attended, with the same $\frac{x}{y}$ portion for each, you can write this equation:

Total Pasta Served = (z guests)($\frac{x}{y}$ pounds per guest) = $\frac{zx}{y}$ pounds of pasta served

Finally, compute the excess. Extra Pasta = Original Amount Prepared – Amount Served $= x - \frac{zx}{y}$

No answer choice matches this expression, so use a common denominator to combine terms and then reduce:

$$\text{Extra Pasta} = \frac{xy}{y} - \frac{zx}{y} = \frac{xy - zx}{y} = \frac{x(y-z)}{y}$$

41. **(B).** Let x be the number of hammers that Sara purchased from the store. Since there were 10 more wrenches than hammers, she received $x + 10$ wrenches. The ratio of wrenches to hammers is 5 : 4, so you can write a proportion:

$$\frac{\text{Wrenches}}{\text{Hammers}} = \frac{x+10}{x} = \frac{5}{4}$$

Cross multiply and solve for x:

$(4)(x + 10) = 5x$
$4x + 40 = 5x$
$x = 40$

Quantity A is 40, which is less than 50.

Quantity B is greater.

42. **(A).** To make the comparison, you must determine the time it takes for the family to drive both to their vacation destination and back home. Recall the formula for rates problems:

Distance = Rate × Time. This can be rearranged to = $\frac{\text{Distance}}{\text{Rate}}$.

The drive is 100 miles each way, so half the distance is 50 miles. The first half of the drive to

the vacation destination was covered at 50 miles per hour:

$$\frac{\text{Distance}}{\text{Rate}} = \frac{50 \text{ miles}}{50 \text{ miles per hour}} = 1 \text{ hour}$$

The second half of the drive was covered at 20 miles per hour:

$$\frac{\text{Distance}}{\text{Rate}} = \frac{50 \text{ miles}}{20 \text{ miles per hour}} = 2.5 \text{ hours}$$

Thus, the total time for the drive to the vacation destination is 1 hour + 2.5 hours = 3.5 hours. Quantity A is 3.5.

The time for the drive home is $\frac{\text{Distance}}{\text{Rate}} = \frac{100 \text{ miles}}{30 \text{ miles per hour}} = 3.333...$ hours. Quantity B is $3.\overline{3}$.

Thus, Quantity A is greater.

43. **(C).** In order to solve this rates problem, remember the formula for work: Work = Rate × Time.

The total work is 8 gallons (a fully filled bucket). The hose is working against the leak, so the effective total rate will be the difference of the two rates:

Effective rate of filling = 3 gallons per minute in – 1 gallon per minute out = 2 gallons per minute

Therefore, by Work = Rate × Time:

$$\frac{8 \text{ gallons}}{2 \text{ gallons per minute}} = \text{Time to fill}$$

Time to fill = 4 minutes

44. **(D).** To solve this problem, convert Dan's rate from widgets per second to widgets per hour. Conceptually, there are two steps: first convert seconds to minutes, then convert minutes to hours. The fast way to do this two-step conversion is to multiply the rate by the right conversion factors, which express identities (such as 60 minutes = 1

hour) in the form of ratios: $\frac{60 \text{ minutes}}{1 \text{ hour}}$ or $\frac{1 \text{ hour}}{60 \text{ minutes}}$. If you make sure that the units cancel correctly, then you can always be sure under pressure whether to multiply or divide by 60.

Here is the conversion, done all in one line:

$$\left(\frac{10 \text{ widgets}}{15 \text{ seconds}}\right)\left(\frac{60 \text{ seconds}}{1 \text{ minute}}\right)\left(\frac{60 \text{ minutes}}{1 \text{ hour}}\right) = \frac{2,400 \text{ widgets}}{1 \text{ hour}}$$

Notice that seconds and minutes both cancel on the left. In 1 hour, Dan can make 2,400 widgets.

45. **(E).** You want to convert an amount of money in "lemuws" to "rubels." Conceptually, there are two steps: first convert lemuws to schillings, then convert schillings to rubels. The fast way to do this two-step conversion is to multiply the money by the right conversion factors, which express identities (such as 8 rubels = 1 schilling) in the form of $\frac{8 \text{ rubels}}{1 \text{ schilling}}$ or $\frac{1 \text{ schilling}}{8 \text{ rubels}}$. If you make sure that the units cancel correctly, then you can always be sure under pressure whether to multiply or divide by 8.

Here is the conversion, done all in one line:

$$(6 \text{ lemuws})\left(\frac{5 \text{ schillings}}{1 \text{ lemuw}}\right)\left(\frac{8 \text{ rubels}}{1 \text{ schilling}}\right) = 240 \text{ rubels}$$

Both lemuws and schillings cancel on the left, leaving rubels. 6 lemuws are worth 240 rubels.

46. **(C).** To compute the time it takes Kim to paint the house alone, use an unknown multiplier. Since the ratio of Kim's time to Jane's time is 3 : 5, Kim's time can be written as $3x$, while Jane's time is $5x$. The work in both cases is 1 house. Since Rate = $\frac{\text{Work}}{\text{Time}}$, you can write Kim's rate as $\frac{1}{3x}$ and Jane's rate as $\frac{1}{5x}$.

To find the speed at which the two people work together, add their rates. This combined rate must equal the rate at which they paint 1 house. Since it takes them 10 hours working together to paint a house, the combined rate is $\frac{1}{10}$.

Kim's rate + Jane's rate = Combined rate

$$\frac{1}{3x} + \frac{1}{5x} = \frac{1}{10}$$

Solve for x by finding a common denominator on the left ($15x$) and adding:

$$\frac{5}{15x}+\frac{3}{15x}=\frac{1}{10}$$

$$\frac{8}{15x}=\frac{1}{10}$$

$$80=15x$$

$$16=3x$$

$$\frac{16}{3}=x$$

Finally, Kim's time is not *x* but $3x=(3)\left(\frac{16}{3}\right)$. The two quantities are equal.

47. **(B).** To solve this ratios problem, choose smart numbers for the money collected for each team and the number of students on each team. Choose multiples of the ratios given, such as the following:

Money collected by Team A = $10
Money collected by Team B = $12
Number of students in Team A = 2
Number of students in Team B = 3

Then compute the money per student:

Money per student in Team A = $10 / 2 = $5 per student
Money per student in Team B = $12 / 3 = $4 per student

Thus, the ratio of money per student in Team A to money per student in Team B is 5 : 4.

Alternatively, you could solve this problem algebraically by creating unknown multipliers that must eventually cancel, but this method is more work.

As a shortcut, you could express each ratio as a fraction, then divide the fractions:

$$\frac{\text{Ratio of money collected}}{\text{Ratio of students}}=\frac{5/6}{2/3}=\frac{5}{6}\times\frac{3}{2}=\frac{5}{4}$$, which is 5:4.

48. **(B).** If ketchup, soy sauce, and mayonnaise are mixed together in a ratio of 3 : 2 : 5, then there are 2 parts of soy sauce for every 3 + 2 + 5 = 10 parts total. In other words, soy sauce comprises $\frac{2}{10}$ of the total mixture. Thus, if the

total mixture volume is 25 ounces, $\frac{2}{10}\times 25=5$ ounces of that is soy sauce.

49. **(B).** Begin by computing the time spent running from point A to point B. Define the variable *x* as the time Saul spent running from B to A. The trip from A to B took $\frac{3}{4}$ as much time, so the time spent from A to B is $\left(\frac{3}{4}\right)x$. The total time was 63 minutes:

$$63=\left(\frac{3}{4}\right)x+x$$

$$63=\left(\frac{7}{4}\right)x$$

$$63\left(\frac{4}{7}\right)=x$$

$$9\times 4=x$$

$$x=36\text{ minutes}$$

The time for the trip from A to B was $\left(\frac{3}{4}\right)x=\left(\frac{3}{4}\right)(36)=27$ minutes

Notice that you never need the typical rate equation (Distance = Rate × Time), as the Part : Part : ratio given for running times and the Whole trip time were enough to solve for time directly.

Quantity B is greater.

50. **(C).** To find how much vinegar Jarod needs, think about how many multiples of his original recipe Jarod wants to make. The original recipe makes 2 cups of sushi rice, so 7 cups of rice is $\frac{7}{2}$ times his original recipe.

Since Jarod is scaling proportionally, to make $\frac{7}{2}$ times the usual amount of rice, he must also use $\frac{7}{2}$ times as much vinegar. Therefore, Joe must use:

$$\left(\frac{7}{2}\right)\left(\frac{2}{3}\text{ ounces}\right)=\frac{7}{3}$$

Alternatively, you can start with 7 cups of rice and multiply by the recipe's ratio of vinegar to rice, cancelling cups of rice and producing ounces of vinegar:

$$(7 \text{ cups of rice})\left(\frac{\frac{2}{3} \text{ ounces of vinegar}}{2 \text{ cups of rice}}\right) = \frac{7}{3} \text{ ounces of vinegar}$$

51. **(A).** You are looking for the average rate for the entire trip. Use Distance = Rate × Time, rearranging to get $\frac{\text{Total Distance}}{\text{Total Time}}$.

Define the distance from Shelbyville to Bakersfield as y. This way, the distance from Springfield to Shelbyville, which is twice as far, is $2y$. (Notice that y will have to vanish by the end of the problem.)

Now compute the time from Springfield to Shelbyville, given that the rate of travel is x miles per hour.

$$\text{Time from Springfield to Shelbyville} = \frac{\text{Distance}}{\text{Rate}} = \frac{2y}{x}$$

Similarly, compute the time from Shelbyville to Bakersfield:

$$\text{Time from Shelbyville to Bakersfield} = \frac{\text{Distance}}{\text{Rate}} = \frac{y}{1.5x}$$

Thus, the total travel time is the sum of the two times: Total Time = $\frac{2y}{x} + \frac{y}{1.5x}$

Using a common denominator ($3x$), add the two fractions: Total Time = $\frac{6y}{3x} + \frac{2y}{3x} = \frac{8y}{3x}$

Compute the total distance, which equals $2y + y = 3y$.

Now you can figure out the average rate:

$$\text{Average Rate} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{3y}{8y/3x} = \frac{9xy}{8y} = \frac{9}{8}x$$

Alternatively, since the variable x appears in the answer choices, you could pick numbers. However, because of the complexity of this problem, it is difficult to pick numbers that both fit the constraints and don't yield messy fractions.

52. **(B).** Solve for the cost of a papaya by translating the information given into mathematical statements. The first sentence tells you that 3 bananas, 2 apples, and 1 mango cost \$3.50. Letting B represent the cost of a banana, A the cost of an apple, and M the cost of a mango, write

$3B + 2A + M = \$3.50$

Similarly, for the second sentence write

$3B + 2A + P = \$4.20$ (where P is the cost of a papaya)

The problem asks for the cost of a papaya and provides the ratio of the costs of a mango and papaya. To use this information, you must remove bananas and apples from the list of unknowns. Here's how: try elimination. Specifically, subtract the first equation from the second:

$$\begin{array}{l} 3B + 2A + P = \$4.20 \\ -\ (3B + 2A + M = \$3.50) \\ \hline P - M = \$0.70 \end{array}$$

Now, since the ratio of the cost of a mango to a papaya is 3 : 5, write a proportion:

$\frac{M}{P} = \frac{3}{5}$, which becomes $M = \frac{3}{5}P$ if you isolate M. Now substitute back into the equation above, to eliminate M and solve for P:

$$P - M = \$0.70$$

$$P - \frac{3}{5}P = \$0.70$$

$$\frac{5}{5}P - \frac{3}{5}P = \$0.70$$

$$\frac{2}{5}P = \$0.70$$

$$P = \frac{5}{2}(\$0.70) = \$1.75$$

Quantity B is greater.

53. **(C).** To solve for the number of unemployed females, first compute the total number of people who are unemployed. You need to represent the total number of people in the town. Call this number x. Since 2/5 of the town is employed, a total of 40,000 people, write the ratio

$$\frac{\text{Employed}}{\text{Total population}} = \frac{40{,}000}{x} = \frac{2}{5}$$

Cross multiply and solve for x:

$5(40,000) = 2x$
$200,000 = 2x$
$100,000 = x$

40,000 people in the town are employed, so 100,000 - 40,000 = 60,000 people are unemployed.

Finally, the ratio of unemployed males to females is 5 : 7. In other words, out of every 5 + 7 = 12 unemployed people, there are 7 unemployed females. Therefore, the fraction of unemployed females in the total unemployed population is 7 out of 12, or 7/12. Setting y as the number of unemployed females, write

$$\frac{\text{Unemployed females}}{\text{Total unemployed}} = \frac{y}{60,000} = \frac{7}{12}$$

Solve for y: $y = \dfrac{7 \times 60,000}{12} = 7 \times 5,000 = 35,000$ unemployed females.

54. **(C).** Reduce the ratio in Quantity A to lowest form. To do so, convert both numbers to improper fractions first:

$$2\frac{11}{12} = 2 + \frac{11}{12} = \frac{2 \times 12}{12} + \frac{11}{12} = \frac{2 \times 12 + 11}{12} = \frac{24 + 11}{12} = \frac{35}{12}$$

$$1\frac{3}{4} = 1 + \frac{3}{4} = \frac{4}{4} + \frac{3}{4} = \frac{4 + 3}{4} = \frac{7}{4}$$

Now compute the ratio by dividing the two numbers and simplifying:

$$\text{Ratio} = \frac{2\frac{11}{12}}{1\frac{3}{4}} = \frac{35/12}{7/4} = \frac{35}{12} \times \frac{4}{7} = \frac{5}{3}$$

The two quantities are equal.

55. **(E).** According to the problem, 3/5 of an inch on the map is equivalent to 400 miles of actual distance. So you can set up a ratio of these two measurements to use as a conversion factor: $\dfrac{3/5 \text{ inch}}{400 \text{ miles}}$ or $\dfrac{400 \text{ miles}}{3/5 \text{ inch}}$. Which one you use depends on which way you're converting: from miles to inches, or vice versa.

You are told that Oklahoma City is separated from Detroit by approximately 3/2 inches on the map, and you are asked how many real miles, approximately, lie between the two cities. Since you want to go from inches to miles, multiply the given measurement (3/2 inches) by the conversion factor that will cancel out inches and give you miles:

$$\left(\frac{3}{2}\text{ inches}\right)\left(\frac{400\text{ miles}}{\frac{3}{5}\text{ inch}}\right)=\left(\frac{3}{2}\right)(400)\left(\frac{5}{3}\right)\text{ miles}=1{,}000\text{ miles}$$

56. **(C).** First, figure out how many boxes worth of cans the machine produced in the 2 hours that it was on. The first step is to find the number of cans produced in 2 hours. Use the formula Work = Rate × Time. 20 cans per hour is the rate, and 2 hours is the time:

Work = (20 cans per hour) × (2 hours) = 40 cans

Now, since there are 10 cans per box, compute the number of boxes:

Number of boxes = 40 cans × $\left(\frac{1\text{ box}}{10\text{ cans}}\right)$ = 4 boxes

So Maria must pack 4 whole boxes to accommodate all the cans that the machine had made.

One more time, use the formula Work = Rate × Time. Maria's rate is 3 boxes per hour, while the total work as 4 boxes. Rearrange and plug in:

$$\text{Time}=\frac{\text{Work}}{\text{Rate}}=\frac{4\text{ boxes}}{3\text{ boxes per hour}}=\frac{4}{3}\text{ hours}$$

Finally, convert from hours to minutes as the question demands.

$$\text{Time}=\frac{4}{3}\text{ hours}\times\left(\frac{60\text{ minutes}}{1\text{ hour}}\right)=80\text{ minutes}$$

57. **(B).** First, assign variables to unknown quantities. Let y be Joe's starting salary with Company X and z be the amount of money that Joe's pay is increased each year. Joe has seen 10 raises of z dollars each, so Joe has had a total raise since he started of $10z$ dollars.

Joe's pay now is his starting pay plus the ten raises, or $y + 10z$.

Also, the ratio of his pay now to his pay when he started is 5 : 2. Write this relationship as a proportion:

$$\frac{y+10z}{y}=\frac{5}{2}$$

Cross multiply and simplify:

$2(y + 10z) = 5y$
$2y + 20z = 5y$

$3y = 20z$

Finally, the problem asks for the ratio of Joe's yearly increase, z, to his starting pay, y. Rearrange the equation to put $\frac{z}{y}$ by itself:

$$\frac{z}{y} = \frac{3}{20}$$

58. **II and III only.** If Beth has 1/4 more money than Ari, their money is in a ratio of 5 : 4 (because 5 is 1/4 more than 4). Another way to see this result is with algebra:

$$B = A + \frac{1}{4}A = \frac{5}{4}A \text{, so } \frac{B}{A} = \frac{5}{4}.$$

As a result, for every \$9 total, Beth has \$5 and Ari has \$4. To keep both Ari and Beth in integer dollar values, the answer needs to be a multiple of 9. Among the answer choices, only 54 and 72 are multiples of 9.

59. **I and IV only.** Since salesperson A sold 35% more motorcycles than salesperson B, their sales are in a ratio of 135 : 100. You can reduce this ratio to 27 : 20 by cancelling a common factor of 5.

As a result, for every 47 motorcycles sold, salesperson A sold 27 and salesperson B sold 20. The number of motorcycles sold must be integer multiples of these numbers (because you can't sell partial motorcycles — not legally anyway), so the total needs to be a multiple of 47. Among the answer choices, only 47 and 235 are multiples of 47.

60. **II, IV, and V only.** First, figure out which animal there are fewest of. "Twice as many zebras as lions" means Zebras > Lions, and "four times as many monkeys as zebras" means Monkeys > Zebras. So lions are found at the zoo in smallest numbers. To make the calculation straightforward, pick 1 lion as a smart number to start with. Since there are twice as many zebras, there are 2 zebras. Finally, there are four times as many monkeys as zebras, so there are 4 × 2 = 8 monkeys. Putting all that together:

Lions : Zebras : Monkeys = 1 : 2 : 8

So, for every 11 animals (1 + 2 + 8), there are 1 lion, 2 zebras, and 8 monkeys. To preserve integer numbers of lions, zebras, and monkeys, the total number of animals could only be a multiple of 11. Among the answer choices, only 22, 55, and 121 fit the bill.

61. **(E).** To tackle this question, rewrite all these ridiculously named currencies in terms of just one currency, ideally a real currency. Use whatever real currency you like, but here's an example with dollars.

Say that 1 murb is worth \$1.

A galok is worth 40% more than a murb, or 40% more than \$1. A galok is worth \$1.40.

10 terble coins equal 1 galok, so 10 terble coins are worth a total of \$1.40. Each terble coin is worth \$0.14 or 14

cents.

6 barbar coins equal 1 murb, so 6 barbar coins equal \$1. Each barbar coin is worth $\$\frac{1}{6}$ or $\frac{100}{6}$ cents.

The ratio of the value of 1 terble coin to the value of 1 barbar coin:

$$\frac{1\text{ terble}}{1\text{ barbar}} = \frac{14\text{ cents}}{100/6\text{ cents}} = 14 \times \frac{6}{100} = \frac{21}{25}$$

62. **(A).** Manipulate the given ratios to create one ratio that includes all three colors. You might use a table:

R	B	O
1	2	
6		1

The problem here is the red car: that column contains both a 1 and a 6. In order to fix this issue, create a common term. Multiply the entire first ratio (the first row) by 6:

R	B	O
6	12	
6		1

Now that the same number is in both rows of the red column, you can combine the two rows into a single ratio:

R : B : O = 6 : 12 : 1

For every 19 cars (6 + 12 + 1), there are 6 red cars, 12 blue cars, and 1 orange car. To maintain whole numbers of cars in each color, the correct answer has to be a multiple of 19. Only 38 is a multiple of 19.

63. **II and IV only.** Since 70% of the clients were originally male, the other 30% were female. So the ratio of men to women was 7 : 3. In other words, Bob's Bistro had $7x$ male and $3x$ female clients, for a total of $10x$ clients. Notice that x must be an integer. Thus, the original number of clients MUST have been a multiple of 10.

After z women leave, the total number of people is equal to $10x - z$, which is given as 74. Since $10x$ is a multiple of 10, it ends in 0. What has to be true about z, so that when you subtract it from a number ending in 0, you get 74? The restriction is that the units digit has to be 6. For instance, $10x$ could be 80, and $z = 6$ to make $10x - z = 74$. Or $10x$ could be 90, in which case z would be 16.

Answer choices 6 and 16 work, as proven above. No other choices have 6 as their units digit.

64. **(C).** The ratio of b to c is 2 : 1, while the ratio of g to c is 3 : 1. Since the variable common to both ratios (c) has

the same number (1) in both ratios, you can just combine to make a three-part ratio:

$$b : c : g = 2 : 1 : 3$$

Put in an unknown multiplier: $b = 2x$, while $c = x$ and $g = 3x$.

Thus, $\frac{b+c}{g} = \frac{2x+x}{3x} = \frac{3x}{3x} = 1$ The two quantities are equal.

Chapter 21
of

Averages, Weighted Averages, Median, and Mode

In This Chapter…

Averages, Weighted Averages, Median, and Mode

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. Husain and Dino have an average of \$20 each. Dino wins a cash prize, which raises their average to \$80. Assuming no other changes occurred, how many dollars did Dino win?

\$ $\boxed{}$

2.

Janani is 6 centimeters taller than Preeti, who is 10 centimeters taller than Rey.

Quantity A	**Quantity B**
The average height of the three people	The median height of the three people

3. The average of Joelle's five quiz scores is 88. What score does Joelle need to get on a sixth quiz to raise her average for all six quizzes to 90?

(A) 88

(B) 94
(C) 98
(D) 100
(E) 102

4.

The average of x and y is 55. The average of y and z is 75.

Quantity A	**Quantity B**
$z - x$	40

5. In Clarice's class, each test weights her overall grade average three times as much as each quiz does. If Clarice scored 88 and 94 on two quizzes, respectively, and she scored 90 on the only test, what is her current overall grade average?

6. What is the average of x, $x - 6$, and $x + 12$?

(A) x
(B) $x + 2$
(C) $x + 9$
(D) $3x + 6$
(E) It cannot be determined from the information given.

7. The average of four numbers is 12. If the set of numbers includes 9, 11, and 12, what is the fourth number?

(A) 12
(B) 14
(C) 16
(D) 20
(E) 24

8.

For a set of 30 integers, the average is 30 and none of the integers are greater than 60.

Quantity A	**Quantity B**
The range of the set	30

9. If x is negative, what is the median of the list 20, x, 7, 11, 3?

(A) 3
(B) 7
(C) 9
(D) 11
(E) 15.5

10. If the average of n and 11 is equal to $2n$, then what is the average of n and $\frac{13}{3}$?

(A) 4
(B) 8
(C) 11
(D) 14
(E) 19

11.

Quantity A	Quantity B
The average of x - 3, x, x + 3, x + 4, and x + 11	The median of x - 3, x, x + 3, x + 4, and x + 11

12. John buys 5 books with an average price of \$12. If John then buys another book with a price of \$18, what is the average price of the 6 books?

(A) \$12.50
(B) \$13
(C) \$13.50
(D) \$14
(E) \$15

13. Every week, Renee is paid 40 dollars per hour for the first 40 hours she works, and 80 dollars per hour for each hour she works after the first 40 hours. How many hours would Renee have to work in one week to earn an average of 60 dollars per hour that week?

(A) 60
(B) 65
(C) 70
(D) 75
(E) 80

14.

At a certain school, the 118 juniors have an average final exam score of 88 and the 100 seniors have an average final exam score of 92.

Quantity A	Quantity B
The average final exam score for all of the juniors and seniors combined.	90

15. Last year a car dealership sold 640 cars over the entire year. This year, the dealership has sold an average of 32 cars per month for the first four months. What is the average number of cars sold per month over the entire 16-month period?

(A) 43
(B) 44
(C) 48
(D) 51
(E) 64

16.

Quantity A	**Quantity B**
The average (arithmetic mean) of x, y, and z	The average (arithmetic mean) of $0.5x$, $0.5y$, and $0.5z$

17. Balpreet's quiz scores in English are 80, 82, 79 and 84. Her quiz scores in History are 90 and 71. What is the sum of the scores she would need to get on her next English quiz and her next History quiz to raise each class' quiz score average to 85?

(A) 109
(B) 192
(C) 194
(D) 198
(E) 218

18. Aaron's first three quiz scores were 75, 84, and 82. If his score on the fourth quiz reduced his average quiz score to 74, what was his score on the fourth quiz?

19. Paco's practice test scores are 650, 700, 630 and 640. What score on the 5th test would result in an average score of 660 for all 5 tests?

20. A quiz is scored from 0 to 110. JaeHa has 5 quiz scores: 90, 95, 88, 84, 92. What does the average on her next 2 quizzes need to be in order to bring her average for all 7 quizzes up to 95?

21.

The integer ages of the three children in the Chen family range from 2 to 13, and no two children are the same age.

Quantity A	**Quantity B**
The average age of all three children in the Chen family	10

22.

Four people have an average age of 18, and none of the people are older than 30.

Quantity B

Quantity A	
The range of the four people’s ages	25

23.

Set A consists of 5 numbers, which have an average value of 43. Set B consists of 5 numbers.

Quantity A	**Quantity B**
The value of x if the average of x and the 5 numbers in Set A is 46	The average of Set B if the average of the 10 numbers in Sets A and B combined is 52

24. The average of 7 numbers is 12. The average of the 4 smallest numbers in this set is 8, while the average of the 4 greatest numbers in this set is 20. How much greater is the sum of the 3 greatest numbers than the sum of the 3 smallest numbers?

(A) 4
(B) 14
(C) 28
(D) 48
(E) 52

25. If the average of a, b, c, 5, and 6 is 6, what is the average of a, b, c, and 13?

(A) 8
(B) 8.5
(C) 9
(D) 9.5
(E) It cannot be determined from the information given.

26. The average (arithmetic mean) of 8 numbers is 42. One of the numbers is removed from the set, and the resulting average (arithmetic mean) of the remaining numbers is 40. What number was removed from the set?

(A) 26
(B) 28
(C) 50
(D) 54
(E) 56

27. The average of 13 numbers is 70. If the average of 10 of these numbers is 90, what is the average of the other 3 numbers?

(A) -130
(B) $\frac{10}{3}$
(C) 30
(D) 90
(E) 290

28. Town A has 6,000 citizens and an average (arithmetic mean) of 2 radios per citizen. Town B has 10,000 citizens and an average (arithmetic mean) of 4 radios per citizen. What is the average number of radios per citizen in both towns combined?

Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

29.

Joe's quiz scores are 80, 82, 78, 77, and 83. Dave's quiz scores are 60, 90, and 80.

Quantity A	**Quantity B**
The score Joe would need on his next quiz to increase his overall average score to 82.	The score Dave would need on his next quiz to increase his overall average score to 82.

30. Fiber X Cereal is 55% fiber. Fiber Max Cereal is 70% fiber. Sheldon combines an amount of the two cereals in a single bowl of mixed cereal that is 65% fiber. If the bowl contains 12 ounces of cereal, how much of the cereal, in ounces, is Fiber X?

(A) 3
(B) 4
(C) 6
(D) 8
(E) 9

31. The average population in Town X was recorded as 22,455 during the years 2000–2010, inclusive. However, an error was later uncovered: the figure for 2009 was erroneously recorded as 22,478, but should have been correctly recorded as 22,500. What is the average population in Town X during the years 2000–2010, inclusive, once the error is corrected?

$\boxed{}$

Liquor Brand	**Percent Alcohol**
Delicate Flower	4%
Fine and Dandy	6%
Monster Smash	12%

32. Based on the chart above, which of the following statements must be true?

Indicate <u>all</u> such statements.

☐ A cocktail made with one part Fine and Dandy and two parts Monster Smash (and no other ingredients) will be more than 9% alcohol.
☐ A cocktail made with 11.5 grams each of all three liquors (and no other ingredients) will be more than

7% alcohol.

☐ A cocktail made with one part Delicate Flower, two parts Fine and Dandy, and one part alcohol-free mixer (and no other ingredients) will be more than 7% alcohol.

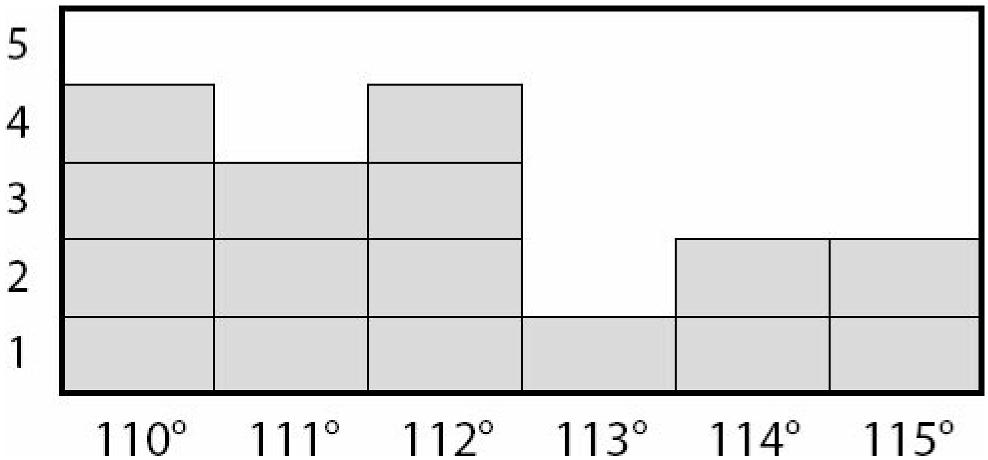
33.

$$S_n = 3n + 3$$

Sequence S is defined for all integers n such that $0 < n < 10{,}000$

Quantity A	**Quantity B**
The median of sequence S	The mean of sequence S

34. The bar graph below displays the number of temperature readings at each value from a sample, measured in degrees Fahrenheit. What was the average temperature reading?



☐ degrees Fahrenheit

35. In a certain dance troupe, there are 55 women and 33 men. If all of the women are 62 inches tall and all of the men are 70 inches tall, what is the average height of the dancers in the troupe?

☐ inches

36. Set A: 1, 3, 5, 7, 9
Set B: 6, 8, 10, 12, 14

For the sets of numbers above, which of the following statements are true?

Indicate <u>all</u> such statements.

☐ The mean of Set B is greater than the mean of Set A.
☐ The median of Set B is greater than the median of Set A.
☐ The standard deviation of Set B is greater than the standard deviation of Set A.
☐ The range of Set B is greater than the range of Set A.

37. Three people have \$32, \$72, and \$98, respectively. If they pool their money then redistribute it among them, what is the maximum value for the median amount of money?

(A) \$72
(B) \$85
(C) \$98
(D) \$101
(E) \$202

38.

Weekly Revenue Per Product Category at Office Supply Store *X*

Product Category	**Weekly Revenue in Dollars**
Pens	164
Pencils	111
Legal Pads	199
Erasers	38
Average of Categories above	128

According to the chart above, the average revenue per week per product category is \$128. However, there is an error in the chart; the revenue for Pens is actually \$176, not \$164. What is the new, correct average revenue per week per product category be, in dollars?

(A) 130
(B) 131
(C) 132
(D) 164
(E) 176

39.

A set of 7 integers has a range of 2, an average of 3, and a mode of 3.

Quantity A	**Quantity B**
The third number in the set when the numbers are arranged in ascending order	The fifth number in the set when the numbers are arranged in ascending order

40.

Set *S* consists of the first 500 positive, even multiples of 7.

Quantity A	**Quantity B**
The average of the set	The median of the set

41.

The average of $3x$, x, and y is equal to $2x$

Quantity A	**Quantity B**
$2x$	y

42. The average age of the buildings on a certain city block is greater than 40 years old. If four of the buildings were built two years ago and none of the buildings are more than 80 years old, which of the following could be the number of buildings on the block?

Indicate all such numbers.

☐ 4
☐ 6
☐ 8
☐ 11
☐ 40

43. Four students contributed to a charity drive, and the average amounts contributed by each student was \$20. If no student gave more than \$25, what is the minimum amount that any student could have contributed?

$ ☐

44.

The average of 7 distinct integers is 12, and the least of these integers is -15.

Quantity A	**Quantity B**
The greatest that any of the integers could be	84

45.

Set N consists of the first 9 positive multiples of 3

Quantity A	**Quantity B**
The average of the first and last terms in the set	The average of the third and seventh terms in the set

46. The average of 15 consecutive integers is 88. What is the greatest of these integers?

☐

47.

The average of 5 integers is 10 and the range of the 5 integers is 10.

Quantity A	Quantity B
The median of the 5 integers	10

48.

3 numbers have a range of 2 and a median of 4.4

Quantity A	Quantity B
The greatest of the numbers	5.4

Averages, Weighted Averages, Median, and Mode Answers

1. **$120.** If the two people had an average of $20 each, they held a sum of 2($20) = $40. After Dino wins a cash prize, the new sum is 40 + *p* and the new average is 80. Plug into the average formula:

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$

$$80 = \frac{40 + p}{2}$$

$160 = 40 + p$

$120 = p$

Dino won $120.

2. **(B).** Pick numbers that agree with the given height constraints. Rey is the shortest person, and if Rey is 100 cm tall, Preeti is 110 cm tall, and Janani is 116 cm tall. The average height is $\frac{100 + 110 + 116}{3} = 108.67$ (rounded to nearest 0.01). The median height is the middle height, which is 110. Quantity B is greater.

Alternatively, note that Preeti's height is the median. Preeti's height is closer to Janani's than to Rey's. Since the average of Janani's and Rey's heights would be midway between those heights, and Preeti's height is higher than that middle, the median is greater than the average.

3. **(D).** There are two ways to solve this question. The first involves using the average formula:

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$

If the average of Joelle's 5 quiz scores is 88, plug these numbers in and solve for the sum of the scores:

$$88 = \frac{\text{Sum}}{5}$$

$88 \times 5 = \text{Sum}$

$440 = \text{Sum}$

Use the average formula again to solve for the sixth quiz score, *x*, keeping in mind that the new average is 90 and the new number of quizzes is 6:

$$90 = \frac{440 + x}{6}$$

$90 \times 6 = 440 + x$
$540 = 440 + x$
$x = 100$

The other way to solve the question is by using the concept of residuals, or differences from the average. If 5 scores of 88 need to be brought up to the new average of 90, there are 5(2) = 10 points needed in the sum. So the sixth quiz score should be high enough to "give away" those 10 points to the sum, while still retaining 90 points for itself. That number is 100.

4. **(C).** Since the average of x and y is 55, $\frac{x+y}{2} = 55$, and $y + z = 150$.

Since the average of y and z is 75, $\frac{y+z}{2} = 75$, and $y + z = 150$.

Stack the two equations and subtract to cancel the y's and get $z - x$ directly:

$z + y = 150$
$\underline{-(x + y = 110)}$
$z - x = 40$

5. **90.4.** To account for the fact that tests weight the grade three times as much as quizzes, include each test score as if it were three identical quizzes. So, 2 quizzes and 1 test = 2 + 3 = 5 quizzes.

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$
Average = (88 + 94 + 90 + 90 + 90)/5
Average = 452/5 = 90.4

6. **(B).** The average formula is just as easily applied to algebraic expressions as arithmetic ones:

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$
$$\text{Average} = \frac{(x)+(x-6)+(x+12)}{3}$$
$$\text{Average} = \frac{3x+6}{3} = x+2$$

The correct answer is (B).

7. **(C).** There are two ways to solve this question. The first involves using the average formula, plugging in 12 for the average and 4 for the number of terms.

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$

$$12 = \frac{\text{Sum}}{4}$$

$$48 = \text{Sum}$$

The sum of the three known terms is 9 + 11 + 12 = 32. If the total of all four numbers must be 48, then the missing fourth number is 48 - 32 = 16.

The other way to solve uses the concept of residuals, or differences from the average. The average of 12 is the "balance point" of the four numbers. The 9 is -3 from this balance point on the number line, 11 is -1 from this balance point, and 12 is on this balance point, because it's equal to the average. Thus, the three known terms are weighted -3 + (-1) = -4 from the average, so the fourth terms needs to be 12 + 4 = 16 in order for the set to balance.

8. **(D).** There are infinite possibilities for a set of 30 integers less than or equal to 60, with an average of 30. For instance:

Example 1: The set consists of fifteen 0's and fifteen 60's
In this example, the range is 60 and the average is 30.

Example 2: The set consists of fifteen 14's and fifteen 16's.
In this example, the range is 2 and the average is 30.

The range could be greater or less than 30.

9. **(B).** The easiest way to start thinking about a question like this is to plug in a value and see what happens. If $x = -1$, the list looks like this when ordered from least to greatest:

-1, 3, 7, 11, 20

The median is 7. Because any negative x you pick will be the least term in the list, the order of the list won't change, so the median will always be 7.

10. **(A).** This question can be quickly solved with the average formula:

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$

$$2n = \frac{n+11}{2}$$

$$4n = n + 11$$

$$3n = 11$$

$$n = \frac{11}{3}$$

Since $n = 11/3$, the average of n and $\frac{13}{3}$ is:

$$\frac{\frac{11}{3}+\frac{13}{3}}{2}=\frac{\frac{24}{3}}{2}=\frac{8}{2}=4$$

Or just notice that the midpoint between $\frac{11}{3}$ and $\frac{13}{3}$ is $\frac{12}{3}$, just as 12 is the midpoint between 11 and 13. The average is $\frac{12}{3}=4$.

11. **(C).** To find the median of the numbers, notice that they are already in order from least to greatest: $x - 3$, x, $x + 3$, $x + 4$, $x + 11$

The median is the middle, or third, term: $x + 3$.

Now find the average of the numbers:

$$\frac{(x-3)+(x)+(x+3)+(x+4)+(x+11)}{5}=\frac{5x+15}{5}=x+3$$

The median and the mean are both $(x + 3)$.

12. **(B).** First, calculate the cost of the first 5 books.

Sum = (Average cost)(Number of books) = (\$12)(5) = \$60

Total cost of all 6 books = \$60 + \$18 = \$78
Total number of books = 6

Average = \$78/6 = \$13 per book.

13. **(E).** Let h = number of hours Renee would have to work. The average rate Renee gets paid is equal to the total wages earned divided by the total number of hours worked. Rene earns \$40 per hour for the first 40 hours, so she makes 40×40 = \$1,600 in the first 40 hours. She also earns \$80 for every hour after 40 hours, for additional pay of \$80(h - 40). The total number of hours worked is h.

$$\frac{1{,}600+80(h-40)}{h}=60$$

$1{,}600 + 80h - 3{,}200 = 60h$

Now isolate h:

$80h - 1{,}600 = 60h$
$-1{,}600 = -20h$
$80 = h$

You could also notice that 60 is exactly halfway between 40 and 80. Therefore, Renee needs to work an equal number of hours at $40 per hour and $80 per hour. If she works 40 hours at $40 per hour, she also needs to work 40 hours at $80 per hour.

14. **(B).** This is a weighted average problem. Because the number of juniors is greater than the number of seniors, the overall average will be closer to the juniors' average than the seniors' average. Since 90 is halfway between 88 and 92, and the weighted average will be closer to 88, Quantity B is larger.

It is not necessary to do the math because this is a Quantitative Comparison question with a very convenient number as Quantity B. However, you can actually calculate the overall average by summing up all 218 scores and dividing by the number of people: $\frac{118(88)+100(92)}{118+100} = 89.83...$

15. **(C).** The dealership sold 640 cars last year. This year, the dealership has sold 32 cars per month for the first 4 months of this year, which is a total of 4(32) = 128 cars. Now, if you're thinking that there's a difference between selling an average of 32 cars per month and selling exactly 32 cars per month, you're right. However, it won't make a difference to the total sum of cars sold over the whole period, which is all that is needed to calculate the average.

Over the entire 16-month period, the dealership sold 640 + 128 = 768. Now use the average formula to calculate the answer:

$$\text{Average number of cars sold per month} = \frac{\text{Sum } (= \text{\# of cars sold})}{\text{Number of Terms } (= \text{\# of months})}$$

$$\text{Average number of cars sold per month} = \frac{768 \text{ total cars}}{16 \text{ months}} = 48 \text{ cars/month}$$

16. **(D).** The average of x, y, and z is $\frac{x+y+z}{3}$. Calculated similarly, the average of $0.5x$, $0.5y$, and $0.5z$ is exactly half that. If the sum of the variables is positive, Quantity A is greater. However, if the sum of the variables is negative, Quantity B is greater. If the sum of the variables is zero, the two quantities are equal.

17. **(C).** For Balpreet to raise her English average to 85:

$$\frac{80+82+79+84+x}{5} = 85$$

$325 + x = 425$
$x = 100 +$

For Balpreet to raise her history average to 85:

$$\frac{90+71+y}{3} = 85$$

$161 + y = 255$

$y = 94$

$x + y = 100 + 94 = 194$

The correct answer is (C).

18. **55.** To find Aaron's fourth quiz score:

$$\frac{75+84+82+x}{4} = 74$$

$241 + x = 296$

$x = 55$

19. **680.** To find the score Paco would need on his 5th test:

$$\frac{650+700+630+640+x}{5} = 660$$

$2{,}620 + x = 3{,}300$

$x = 680$

20. **108.** To find the score JaeHa would need to average on her next 2 quizzes to bring her total average up to a 95:

$$\frac{90+95+88+84+92+x+y}{7} = 95$$

$449 + x + y = 665$

$x + y = 216$

Note that it is not necessary to determine x and y individually. Since the two new quiz scores sum to 216, their average is $\frac{216}{2} = 108$.

21. **(B).** Since there are only three children and the range is from 2 to 13, one child must be 2, one must be 13, and the other child's age must fall somewhere in the middle.

If the average of the children's ages were 10, as in Quantity B, the sum of the three ages would be 30. Subtract 2 and 13 from 30 to get that the third child would need to be 15. This is not possible, because the middle child cannot be older than the 13-year-old. Since this age is too great, the true average age must be less, and Quantity B is greater.

Alternatively, since no two children are the same age, the middle child has a maximum age of 12. If that child were 12, the average age would be $\frac{2+12+13}{3} = \frac{27}{3} = 9$. Thus, the true average age must be 9 or less, and Quantity B is greater.

22. **(D).** If 4 people have an average age of 18, then the sum of their ages is $4 \times 18 = 72$. Since the question is about range, try to minimize and maximize the range. Minimizing the range is easy — if everyone were exactly 18, the average age would be 18 and the range would be 0. So clearly, the range can be smaller than 25.

To maximize the range, make the oldest person the maximum age of 30, and see whether the youngest person could be just 1 year old while still obeying the other rules of the problem: the sum of the ages is 72 and, of course, no one can be a negative age.

One such set: 1, 20, 21, 30

This is just one example that would work. In this case, the range is 30 - 1 = 29, which is greater than 25.

The correct answer is (D).

23. **(C).** If the average of the 5 numbers in Set A is 43, the sum of Set A is (5)(43) = 215.

For Quantity A, use the Average Formula. Sum all 6 numbers, and divide by 6:

$$\frac{\text{sum of the 5 numbers in Set A} + x}{6} = 46$$

$$\frac{215 + x}{6} = 46$$

$$215 + x = 276$$

$$x = 61$$

For Quantity B, use the Average Formula again:

$$\frac{\text{sum of Set A} + \text{sum of Set B}}{10} = 52$$

$$215 + \text{sum of Set B} = 520$$

$$\text{sum of Set B} = 305$$

The average of the 5 numbers in Set B is thus $\frac{305}{5} = 61$.

Alternatively, you could note that each set of 5 numbers has the same "weight" in the average of all 10 numbers. The average of Set A is 43, which is 52 - 43 = 9 below the average of all 10 numbers. The average of Set B must be 9 above the average of all 10 numbers: 52 + 9 = 61.

24. **(D).** Using the average formula, $\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$, build three separate equations:

All 7 numbers:

$$12 = \frac{\text{Sum of all 7 numbers}}{7}$$

Sum of all 7 numbers = 84

The 4 smallest numbers:

$$8 = \frac{\text{Sum of the 4 smallest numbers}}{4}$$

Sum of the 4 smallest numbers = 32

The 4 greatest numbers:

$$20 = \frac{\text{Sum of the 4 greatest numbers}}{4}$$

Sum of the 4 greatest numbers = 80

There are only 7 numbers, yet information is given about the 4 smallest and the 4 greatest, which is a total of 8 numbers! The middle number has been counted twice—it is included in both the 4 greatest and the 4 smallest.

The sum of all 7 numbers is 84, but the sum of the 4 greatest and 4 smallest is 80 + 32 = 112. The difference can only be attributed to the double counting of the middle number in the set of 7: 112 - 84 = 28.

The middle number is 28, so subtract it from the sum of the 4 smallest numbers to get the sum of the 3 smallest numbers: 32 - 28 = 4.

Now subtract the middle number from the sum of the 4 greatest numbers to get the sum of the 3 greatest numbers: 80 - 28 = 52.

The difference between the sum of the 3 greatest numbers and the sum of the 3 smallest numbers is 52 - 4 = 48.

25. **(A).** Since $\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$:

$$6 = \frac{a+b+c+5+6}{5}$$

$30 = a + b + c + 11$
$19 = a + b + c$

It is not necessary, or possible, to determine the values of a, b, and c individually. The second average includes all three variables, so the values will be summed again anyway.

$$\text{Average} = \frac{a+b+c+13}{4}$$

$$\text{Average} = \frac{19+13}{4}$$

$$\text{Average} = \frac{32}{4} = 8$$

26. **(E).** There are two ways to solve this question. The first involves using the average formula: Average = $\frac{\text{Sum}}{\text{Number of Terms}}$ or Sum = Average × Number of Terms.

If the average of 8 numbers is 42, the sum of all 8 numbers = 42 × 8 = 336.

After removing one number, the new average of the remaining 7 numbers is 40. So, the sum of the remaining 7 numbers = 40 × 7 = 280.

The number that was removed accounts for the difference in these sums, so the number that was removed is 336 - 280 = 56.

The other way to solve the question is by using the concept of residuals, or differences from the average. A number was removed, and it caused the average of the remaining 7 numbers to drop by 2 (from 42 to 40). That requires a 7 × 2 = 14 point drop in the sum. The removed number must be 14 more than the preexisting average. That would be 42 + 14 = 56.

27. **(B).** $\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$ or Sum = Average × Number of Terms.

The average of 13 numbers is 70, so:

Sum of all 13 terms = 70 × 13 = 910

The average of 10 of these numbers is 90, so:

Sum of 10 of these numbers = 90 × 10 = 900

Subtract to find the sum of "the other 3 numbers": 910 - 900 = 10

Average of the other 3 numbers $\frac{\text{Sum}}{\text{Number of Terms}} = \frac{10}{3}$.

28. $\frac{13}{4}$ **(or any equivalent fraction).** To find this weighted average, you must find the sum of all the radios in Towns A and B, and divide by the total number of people in both towns:

$$\text{Average} = \frac{6{,}000(2) + 10{,}000(4)}{16{,}000}$$

Cancel three zeros from each term:

$$\text{Average} = \frac{6(2) + 10(4)}{16}$$

$$\text{Average} = \frac{52}{16}$$

This reduces to $\frac{13}{4}$, although you are not required to reduce.

29. **(B).** Average = $\frac{\text{Sum}}{\text{Number of Terms}}$. Set the unknown final test score as x for Joe and y for Dave.

Quantity A:

$82 = \frac{80 + 82 + 78 + 77 + 83 + x}{6}$
$492 = 400 + x$
$x = 92$

Quantity B:

$82 = \frac{60 + 90 + 80 + y}{4}$
$328 = 230 + y$
$y = 98$

30. **(B).** Use the weighted average formula to get the ratio of Fiber X to Fiber Max:

$\frac{0.55x + 0.70m}{x + m} = 0.65$, where x is the amount of Fiber X and m is the amount of Fiber Max.

This is not that different from the regular average formula—on the top, there is the total amount of fiber (55% of Fiber X and 70% of Fiber Max), which is divided by the total amount of cereal ($x + m$) to get the average. Simplify by multiplying both sides by ($x + m$):

$0.55x + 0.70m = 0.65(x + m)$
$0.55x + 0.70m = 0.65x + 0.65m$

If you wish, you can multiply both sides of the equation by 100 to eliminate all the decimals:

$55x + 70m = 65x + 65m$
$55x + 5m = 65x$
$5m = 10x$

$\frac{m}{x} = \frac{10}{5}$ or $\frac{2}{1}$

Since m and x are in a 2 to 1 ratio, 2/3 of the total is m and 1/3 of the total is x. Since the total is 12 ounces, Fiber *X* accounts for $\frac{1}{3}(12) = 4$ ounces of the mixed cereal.

One shortcut to this procedure is to note that the weighted average (65%) is 10% away from Fiber *X*'s percent and 5% away from Fiber Max's percent. Since 10 is twice as much as 5, the ratio of the two cereals is 2 to 1. However, it is a 2 to 1 ratio of Fiber Max to Fiber *X*, not the reverse! Whichever number is closer to the weighted average (in this case, 70% is closer to 65%) gets the larger of the ratio parts. Since the ratio is 2 to 1 (Fiber Max to Fiber *X*), again, 1/3 of the cereal is Fiber *X* and $\frac{1}{3}(12) = 4$.

31. **22,457.** There is a simple shortcut for a change to an average. The figure for 2009 was recorded as 22,478, but actually should have been recorded as 22,500. Thus, 22 people in that year were not counted. Thus, the sum should have been 22 higher when the average was originally calculated.

2000–2010, inclusive, is 11 years (subtract low from high and then add 1 to count an inclusive list of consecutive numbers). When taking an average, you divide the sum by the number of things being averaged (in this case, 11). So the shortcut is to take the change to the sum and "spread it out" over all of the values being averaged by dividing the change by the number of things being averaged.

Divide 22 by 11 to get 2. The average should have been 2 higher. Thus, the correct average for the 11 year period is 22,457.

Alternatively, the traditional method: 22,455 × 11 years = 247,005, the sum of all 11 years' recorded populations. Add the 22 uncounted people: the corrected sum would be 247,027. Divide by 11 to get the real average: 22,457. (Note that while the traditional method is faster to explain, the shortcut is faster to actually execute!)

32. **I and II only.** This is a weighted average problem. Consider the statements individually:

I. TRUE. A cocktail made with one part Fine and Dandy and two parts Monster Smash will have a percent alcohol equal to the average of 6%, 12%, and 12%, or $\frac{6\% + 12\% + 12\%}{3} = \frac{30\%}{3} = 10\%$. (Count the Monster Smash twice, since the cocktail contains twice as much of it.)

II. TRUE. The 11.5 grams is irrelevant—all that matters is that equal amounts of each liquor were used, so there is no need to weight the average. Take an ordinary average of 4, 6, and 12 percent. The average is 7.333…%.

III. FALSE. To find the percent alcohol for a cocktail made with one part Delicate Flower (4% alcohol), two parts Fine and Dandy (6% alcohol) and one part mixer (0% alcohol), average 4, 6, 6, and 0. (Count the 6 twice since two parts Fine and Dandy went into the cocktail versus one part of each of the other components.) The average of 4, 6, 6, and 0 is 4. If you got this wrong, you may have ignored the 0% alcohol mixer. You cannot ignore the effect of a zero in an average—a zero can often lower an average considerably. Alternatively, note that none of the components of this cocktail have more than 6% alcohol. The resulting drink cannot have a greater concentration of alcohol than any of its components.

33. **(C).** Sequence S is an evenly spaced set, which can be seen by plugging in a few n values:

$S_1 = 3(1) + 3 = 6$
$S_2 = 3(2) + 3 = 9$
$S_3 = 3(3) + 3 = 12\ldots$

Terms increase by three every time n increases by 1; this meets the definition of an evenly spaced set. For ANY evenly spaced set, the median equals the mean.

34. **112.** This is a weighted average problem. You CANNOT simply average 110, 111, 112, 113, 114, and 115. You must take into account how many times each number appears. The chart is really another way of writing:

110, 110, 110, 110
111, 111, 111
112, 112, 112, 112
113
114, 114
115, 115

In other words, the average temperature reading is really an average of 16 numbers. The easiest way to do this is:

$$\frac{4(110)+3(111)+4(112)+1(113)+2(114)+2(115)}{16}$$

Use your calculator—the correct answer is 112.

35. **65.** This is a weighted average problem. You CANNOT simply average 62 and 70. You must take into account how many times each number appears (55 and 33 times, respectively). You are actually averaging 88 numbers:

$$\frac{55(62)+33(70)}{88}=$$

Use your calculator. The correct answer is 65.

36. **I and II only.** In both sets, the numbers are evenly spaced. Moreover, both sets are evenly spaced by the same amount (adjacent terms increase by 2) and have the same number of terms (5 numbers in each set). The difference is that each term in Set B is 5 greater than the corresponding term in Set A (i.e., 6 - 1 = 5, 8 - 3 = 5, etc.)

In evenly spaced sets, the mean = median. Also, if an evenly spaced set has an odd number of numbers, the mean and median both equal the middle number. (When a set has an even number of numbers, the mean and median both equal the average of the 2 middle numbers).

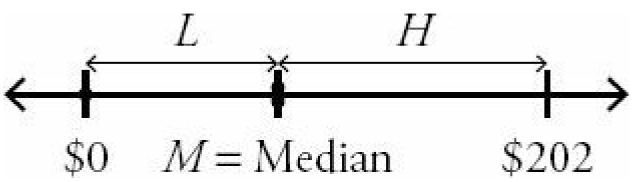
So, Set A has mean and median of 5 and Set B has mean/median = 10. Statement I and Statement II are true.

Since Sets A and B are equally spaced and have the same number of elements, their standard deviations are equal (that is, Set A is exactly as spread out from its own mean as Set B is from its own mean), so Statement III is false.

Since 9 - 1 = 8 and 14 - 6 = 8, the ranges are equal and Statement IV is false.

37. **(D).** The pool of money is \$32 + \$72 + \$98 = \$202. After the redistribution, each person will have an amount between \$0 and \$202, inclusive. Call the amounts L, M, and H (low, median, high). To maximize M, minimize L and H.

The minimum value for H is, in fact, M. The "highest" of the three values can actually be equal to the median (if H were lower than M, the term order and therefore which number is the median would change, but if $H = M$, M can still be the median).



Minimum L = \$0
Minimum $H = M$

Maximum M = Total pool of money - Minimum L - Minimum H
M = \$202 - \$0 - M
$2M$ = \$202
M = \$101

The correct answer is (D).

38. **(B).** The chart provides the average and the number of product categories. If the incorrectly-calculated average was \$128 for the 4 categories, then the sum was 4 × 128 = \$512. Since the revenue for Pens was actually \$176, not \$164, the sum should have been \$12 higher. Thus, the correct sum is \$524. Divide by 4 to get \$131, the answer.

Alternatively, notice that the \$128 average given in the question stem actually does a lot of work for you. If \$164 jumps up to \$176, that's an increase of \$12. Distributed over the four categories, it will bring the overall average up by \$3, from \$128 to \$131.

39. **(C).** A set of 7 integers with a range of 2 and an average of 3 could consist of only these possibilities:

Example 1: 2, 2, 2, 3, 4, 4, 4

Example 2: 2, 2, 3, 3, 3, 4, 4

Example 3: 2, 3, 3, 3, 3, 3, 4

However, the mode of the set must be 3 for this set. The mode is the most common number in the set. Example 1 has two modes: 2 and 4, so Example 1 is invalid for this question. Only Examples 2 and 3 remain.

In both valid examples, the third and fifth numbers in each set are 3. The two quantities are equal.

40. **(C).** The set begins 14, 28, 42, etc. However, the specific numbers—and even the number of elements—in the set are irrelevant, since the average equals the median for any evenly spaced set. Consecutive multiples (in this case, of 14) are evenly spaced.

41. **(C).** Write "the average of $3x$, x, and y is equal to $2x$" as an equation:

$$\frac{3x + x + y}{3} = 2x$$
$$4x + y = 6x$$
$$y = 2x$$

The two quantities are equal.

42. **8, 11, 40.** Because Average = $\frac{\text{Sum}}{\text{Number of Terms}}$, this question about averages depends both on x, the total number of buildings on the block, and on the sum of the building ages. The 4 buildings that are 2 years old have a total age of 4(2), and the $(x - 4)$ other buildings have a total age of $(x - 4)$(no more than 80).

$$\text{Average age} = \frac{4(2) + (x - 4)(\text{no more than } 80)}{x}$$

Having many 80 year old buildings on the block would raise the average much closer to 80. (For instance, if there were a million 80-year-old buildings and four 2-year-old buildings, the average would be very close to 80 years old.) So, there is some minimum number of older buildings that could raise the average above 40.

Ignore the "greater than" 40 years old constraint on the average building age for a moment. What is the minimum x needed to be to make the average age exactly 40 when the age of the other buildings is maximized at 80?

$$40x = 8 + (x - 4)(80)$$
$$40x = 8 + 80x - 320$$
$$-40x = -312$$
$$x = \frac{312}{40} = 7.8$$

Because there can't be a partial building and the age of the buildings can't be greater than 80, x must be at least 8 to bring the average age up over 40. (You would need even more buildings to bring the average above 40 if those older buildings were only between 50 and 70 years old, for example.)

Alternatively, test the answer choices. Try the first choice, 4 buildings. Since 4 of the buildings on the block are only 2 years old, this choice can't work—the average age of the buildings would be 2.

Try the second choice. With 6 total buildings, there would be four 2-year-old buildings, plus two others. To maximize the average age, maximize the ages of the two other buildings by making them both 80 years old.

$$\frac{4(2)+2(80)}{6} = 28$$

Since the average is less than 40 years old, this choice is not correct.

Try the third choice. With 8 total buildings, there would be the four 2-year-old buildings, plus four others. To maximize the average age, maximize the ages of the four other buildings by making them each 80 years old:

$$\frac{4(2)+4(80)}{8} = 41$$

Since the average age is greater than 40 years old, this choice is correct. Since the other, greater choices allow the possibility of even more 80-year-old buildings, increasing the average age further, those choices are also correct.

43. **\$5.** The average of four values is \$20. Thus, the sum of the four values is \$80. To determine the minimum contribution one student could have given, maximize the contributions of the other three students. If the three other students each gave the maximum of \$25, the fourth student would only have to give \$5 to make the sum equal to \$80.

44. **(A).** If the average of 7 integers is 12, then their sum must be $7 \times 12 = 84$. To maximize the largest of the numbers, minimize the others.

The smallest number is -15. The integers are distinct (that is, different from each other), so the minimum values for the smallest 6 integers are -15, -14, -13, -12, -11, and -10. To find the maximum value for the 7th integer, sum -15, -14, -13, -12, -11, -10, and x, while setting that sum equal to 84:

$$-15 + (-14) + (-13) + (-12) + (-11) + (-10) + x = 84$$
$$-75 + x = 84$$
$$x = 159$$

Quantity A is greater.

45. **(C).** In an evenly spaced set, the middle number is also the average. Numbers equally spaced on opposite sides of the middle will also average to the average of the whole set. Thus, the answer is (C). Here is the set written out, with the median underlined:

3 6 9 12 <u>15</u> 18 21 24 27

In an evenly spaced set, the median is equal to the average. Thus, 15 is the average. It is also the case that 12 and 18 average to 15. So do 9 and 21. So do 6 and 24. And so do 3 and 27.

Both quantities are equal to 15.

46. **95.** In any evenly spaced set, the average equals the median. Thus, 88 is the middle number in the set. Since the set has 15 elements, the 8th element is the middle one.

Lowest 7 integers:	81	82	83	84	85	86	87
Middle integers:	88						
Greatest 7 integers:	89	90	91	92	93	94	95

The largest integer in the list is 95. If you were confident about the process, you could skip listing the integers. Instead you could reason that to go from 8th integer to the 15th integer, you must add 7: $88 + 7 = 95$.

47. **(D).** If the average of 5 integers is 10, their sum must be 50. The range is given as 10. Try to make two examples where this is true, but where the medians are as different as possible.

Example 1: 5, 10, 10, 10, 15

In this case, Quantity A is equal to Quantity B.

Is there a list such that the average is still 10 and the range is still 10, but the median is something else? Try adjusting the three middle numbers while keeping 5 as the least integer and 15 as the greatest integer. To adjust the numbers without disturbing the average, anything you subtract from one number should be added to another number, so the sum stays constant.

Example 2: 5, 6, 12, 12, 15

Here, 4 was subtracted from the second term, and then 2 was added to each of the third and fourth terms. The average and range still each equal 10, but now the median is 12. The answer is (D).

48. **(D).** If the set has an odd number of terms, then the median is the middle number. So, the middle number is 4.4. The set has a range of 2. The other two numbers could be 2 apart and also equally distributed around 4.4:

Example 1: 3.4, 4.4, 5.4

Here, the two quantities are equal.

Or, the two other numbers could be 2 apart but both a bit higher, or both a bit lower.

Example 2: 4.3, 4.4, 6.3

Example 3: 2.5, 4.4, 4.5

Thus, Quantity A could be equal to, less than, or greater than Quantity B. The correct answer is (D).

Chapter 22
of

Standard Deviation and Normal Distribution

In This Chapter…

Standard Deviation and Normal Distribution

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. Set *S* {5, 10, 15}

If the number 15 were removed from Set *S* and replaced with the number 1,000, which of the following would change?

Indicate all such statements.

☐ The mean
☐ The median
☐ The standard deviation

2.

Set *W*: -9, -3, 3, 9
Set *X*: 2, 4, 6, 8
Set *Y*: 100, 101, 102, 103
Set *Z*: 7, 7, 7, 7

Which of the following choices lists the four sets above in order from smallest standard deviation to greatest standard deviation?

(A) W, X, Y, Z
(B) W, Y, X, Z
(C) W, X, Z, Y
(D) Z, Y, X, W
(E) Z, X, Y, W

3.

Set N is a set of x distinct positive integers where $x > 2$.

Quantity A	Quantity B
The standard deviation of Set N	The standard deviation of Set N if every number in the set is multiplied by -3

4. Set S is a set of distinct positive integers. The standard deviation of Set S must increase if which of the following were to occur?

Indicate all such statements.

☐ Each number in the set is multiplied by 1/2.
☐ The smallest number is increased to become equal to the median.
☐ The smallest number is increased to become larger than the current largest number.
☐ The largest number is doubled.

5. The 75th percentile on a test corresponded to a score of 700, while the 25th percentile corresponded to a score of 450.

Quantity A	Quantity B
800	A 95th percentile score

6. Set X consists of 9 total terms, but only two different terms. Six of the terms are each equal to twice the value of each of the remaining 3. Which of the following would provide sufficient additional information to determine the average of the set?

Indicate all such statements.

☐ The smaller number is positive and is 3 less than the larger number.
☐ The standard deviation of the set is equal to $2\sqrt{3}$.
☐ The biggest term in the set is 6.

7. Set $S = \{2, 5, 7, 11, 16, 24, 28, 50, 52, 101, 120, 130\}$

What is the average of the first quartile ("Q1") and the third quartile ("Q3") of set S?

(A) 9

(B) 26
(C) 42.75
(D) 76.5
(E) 85.5

8. The test scores at Millbrook High School are normally distributed, and the 60th percentile is equal to a score of 70.

Quantity A	**Quantity B**
The 30th percentile score	35

9. The lengths of a certain population of earthworms are normally distributed with a mean length of 30 centimeters and a standard deviation of 3 centimeters. One of the worms is picked at random.

Quantity A	**Quantity B**
The probability that the worm is between 24 and 30 centimeters, inclusive	The probability that the worm is between 27 and 33 centimeters, inclusive

10. The hourly wage paid to working adults in Maplewood is normally distributed around a mean of \$18 per hour with a standard deviation of \$3.50.

Quantity A	**Quantity B**
The percent of working adults in Maplewood who are paid between \$18 and \$25 per hour, inclusive	40%

11. Home values among the 8,000 homeowners of Town X are normally distributed, with a standard deviation of \$11,000 and a mean of \$90,000.

Quantity A	**Quantity B**
The number of homeowners in Town X whose home value is above \$112,000	300

12. Exam grades among the students in Ms. Harshman's class are normally distributed, and the 50th percentile is equal to a score of 77.

Quantity A	**Quantity B**
The number of students who scored less than 80 on the exam	The number of students who scored greater than 74 on the exam

13. The length of bolts made in factory *Z* is normally distributed, with a mean length of 0.1630 meters and a standard deviation of 0.0084 meters. The probability that a randomly selected bolt is between 0.1546 meters and 0.1756 meters long is between

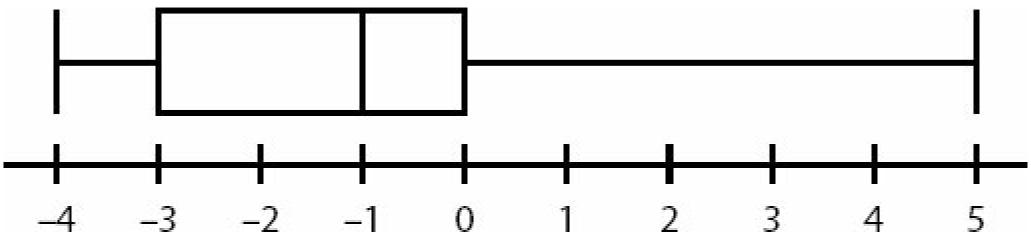
(A) 54% and 61%
(B) 61% and 68%
(C) 68% and 75%
(D) 75% and 82%
(E) 82% and 89%

14. Birth weight of babies born at City Hospital is normally distributed. A baby 2 standard deviations above the mean birth weight weighs 10.8 pounds, and a baby 1 standard deviation below the mean birth weight weighs 5.85

pounds.

Quantity A	**Quantity B**
Twice the weight of a baby 2 standard deviations below the mean	The weight of a baby 1 standard deviation above the mean

15. Which of the following sets of data applies to this graph?



(A) -4, -4, -2, 0, 0, 5
(B) -4, 1, 1, 3, 4, 4
(C) -4, -4, -3, 1, 5
(D) -5, 3, 4, 5
(E) -4, -4, -2, -2, 0, 0, 0, 5

16. If a set of data consists of only the first ten positive multiples of 5, what is the interquartile range of the set?

(A) 15
(B) 25
(C) 27.5
(D) 40
(E) 45

17. On a given math test out of 100 points, the vast majority of the 149 students in a class scored either a perfect score or a zero, with only one student scoring within 5 points of the mean. Which of the following logically follows about Set *T*, made up of the scores on the test?

Indicate all such statements.

☐ Set *T* will not be normally distributed.
☐ The range of Set *T* would be significantly smaller if the scores had been more evenly distributed.
☐ The mean of Set *T* will not equal the median.

18. If Set *X* is a normally distributed set of numbers with a mean of 4 and a standard deviation of 4, approximately what is the probability that a number chosen at random from the set will be negative?

(A) 1/10
(B) 1/6
(C) 1/4
(D) 1/3
(E) 1/2

19. Jane scored in the 68th percentile on a test, and John scored in the 32nd percentile.

Quantity A	**Quantity B**

The proportion of the class that received a score less than John's score | The proportion of the class that scored as high as or higher than Jane

20. If a set of data has a mean of 4.2 and a standard deviation of 7.1, what is the range of values that lie within 2 standard deviations of the mean?

(A) -2.9 to 11.3
(B) -2.9 to 12.6
(C) -10 to 12.6
(D) -10 to 18.4
(E) 4.2 to 18.4

21. If octiles divide up a set of data into 8 ordered groups, each with the same number of terms, what is the median of the sixth octile group of the set of data composed of the integers from 25 to 48, inclusive?

22. In a class with 20 students, a test was administered, scored only in whole numbers from 0 to 10. At least one student got every possible score, and the average was 7.

Quantity A

4

Quantity B

The lowest score that two students could have received

23.

Frequency	6	5	5
Result	4	6	8

Quantity A

The mode of this data set

Quantity B

5

24. A test is scored out of 100 and the scores are divided into five quintile groups. Students are not told their scores, but only their quintile group.

Quantity A

The scores of two students in the bottom quintile group, chosen at random and added together

Quantity B

The score of a student in the top quintile group, chosen at random

25. In a set of 10 million numbers, one percentile would represent what percent of the total number of terms?

(A) 1,000,000
(B) 100,000
(C) 10,000
(D) 100

(E) 1

26. What is the range of the set of numbers comprised entirely of $\{1, 6, x, 17, 20, y\}$ if all terms in the set are positive integers and $xy = 18$?

(A) 16
(B) 17
(C) 18
(D) 19
(E) Cannot be determined from the information given.

27. On a particular test whose scores are distributed normally, the 2nd percentile is 1720, while the 84th percentile is 1990. What score, rounded to the nearest 10, most closely corresponds to the 16th percentile?

(A) 1,750
(B) 1,770
(C) 1,790
(D) 1,810
(E) 1,830

28. A data set contains at least two different integers.

Quantity A	**Quantity B**
The range of the data set	The interquartile range of the data set

29. In a normally distributed set of data, one standard deviation above the mean is 77 and the standard deviation is 10. What is the mean of the data?

$\boxed{}$

30. Some rock samples are weighed, and their weights are determined to be normally distributed. One standard deviation below the mean is 250 grams and one standard deviation above the mean is 420 grams.

Quantity A	**Quantity B**
335 grams	The median weight, in grams

31. In a normally distributed set of data, the mean is 12 and the standard deviation is less than 3.

Quantity A	**Quantity B**
Number of data points in the set located between 9 and 15	60% of the total number of data points

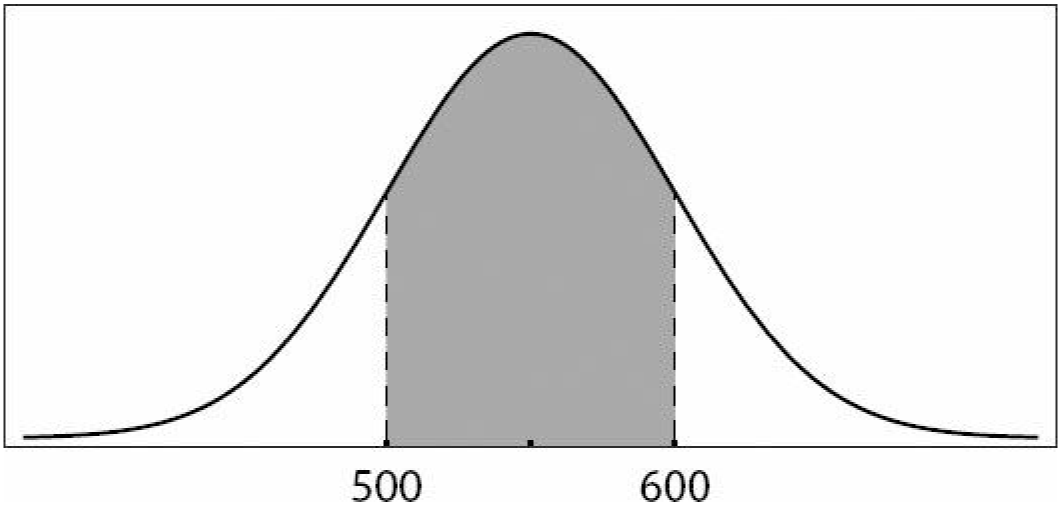
32.

Quantity A	**Quantity B**
The standard deviation of the set 10, 20, 30	The standard deviation of the set 10, 20, 20, 20, 20, 20, 30

33.

Quantity A	**Quantity B**
The standard deviation of a set of numbers with a range of 8	The standard deviation of a set of numbers consisting of 3 consecutive multiples of 3

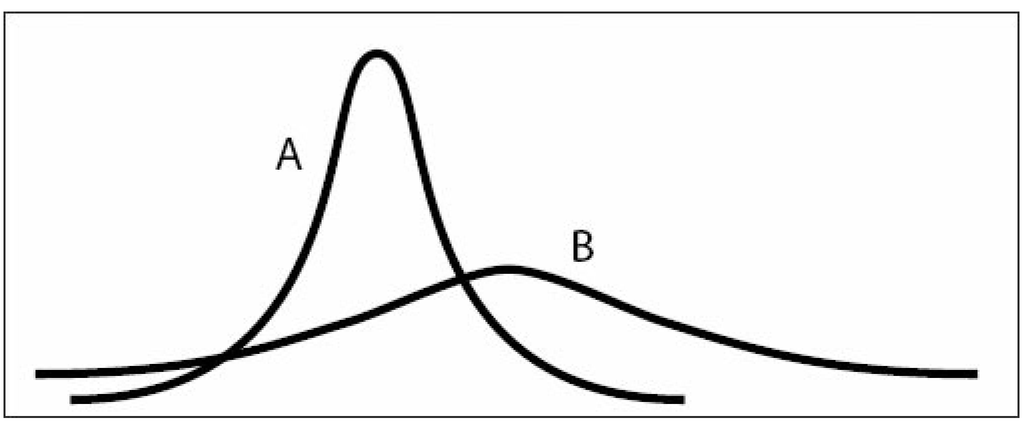
34.



The graph represents the normally distributed scores on a test. The shaded area represents approximately 68% of the scores.

Quantity A	**Quantity B**
The mean	550

35.



A and B are graphical representations of normally distributed random variables *X* and *Y*, respectively, with relative positions, shapes, and sizes as shown. Which of the following must be true?

Indicate <u>all</u> such statements.

- ☐ *Y* has a larger standard deviation than *X*.
- ☐ The probability that *Y* falls within 2 standard deviations of its mean is larger than the probability that *X* falls within 2 standard deviations of its mean.

☐ Y has a larger mean than X.

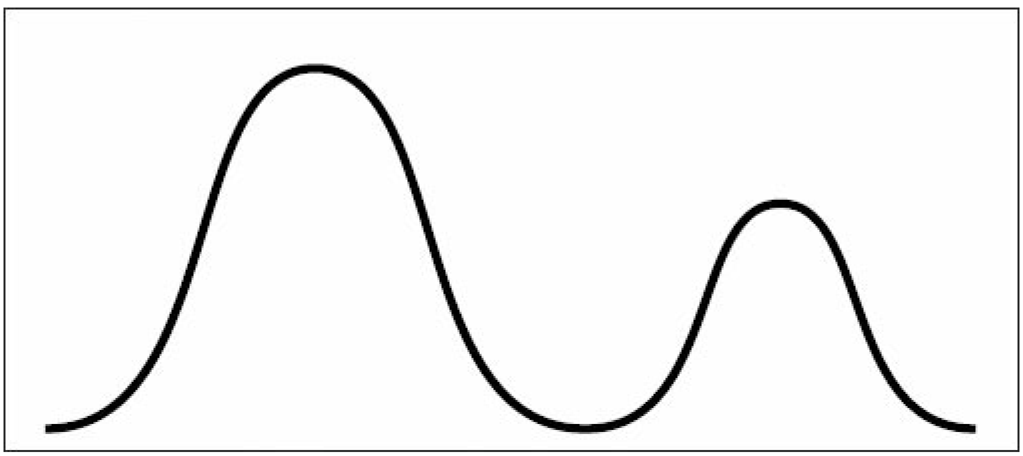
36. 300 test results are integers ranging from 15 to 75, inclusive. Dominick's result is clearly in the 80th percentile of those results, not the 79th or the 81st.

Quantity A	**Quantity B**
Number of other test results in the same percentile as Dominick's	Maximum number of other test-takers with the same result as Dominick

37. The outcome of a standardized test is an integer between 151 and 200, inclusive. The percentiles of 400 test scores are calculated, and the scores are divided into corresponding percentile groups.

Quantity A	**Quantity B**
Minimum number of integers between 151 and 200, inclusive, that include more than one percentile group	Minimum number of percentile groups that correspond to a score of 200

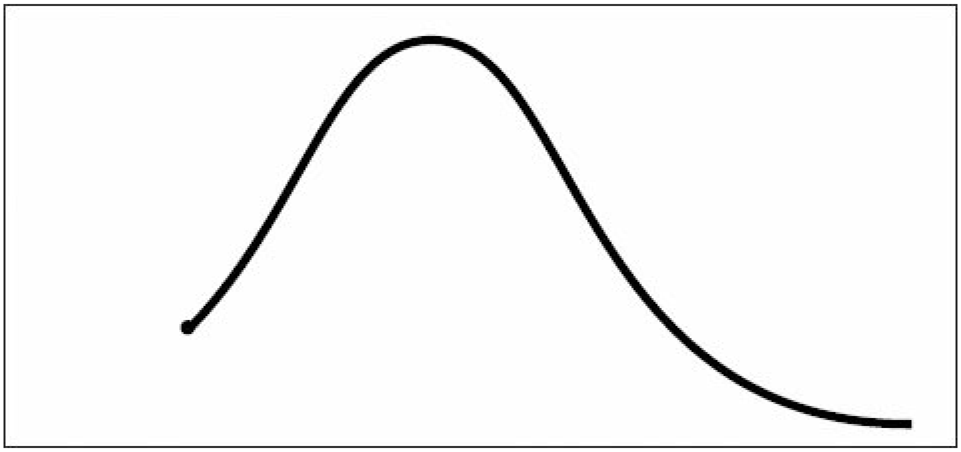
38.



Which of the following would the data pattern shown best describe?

(A) The number of grams of sugar in a selection of drinks is normally distributed.
(B) A number of male high school principals and a larger number of female high school principals have normally distributed salaries, distributed around the same mean.
(C) A number of students have normally distributed heights, and a smaller number of taller, adult teachers also have normally distributed heights.
(D) The salary distribution for biologists skews to the left of the median.
(E) The maximum-weight bench presses for a number of male athletes are normally distributed, and the maximum-weight bench presses for a smaller number of female athletes are also normally distributed, although around a smaller mean.

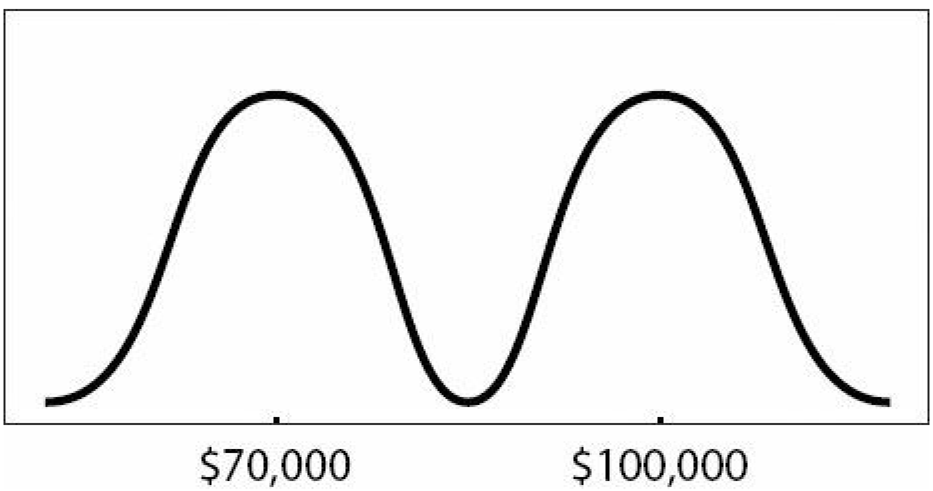
39.



Which of the following would the data pattern shown best describe?

(A) The weights of raccoons in a population are normally distributed.
(B) Salaries in a certain field appear normally distributed, except that salaries cannot dip below the limits of a minimum-wage law.
(C) The fraction of people at a certain age in a certain population is inversely proportional to age.
(D) a set of consecutive integers
(E) a set with a standard deviation of zero

40.

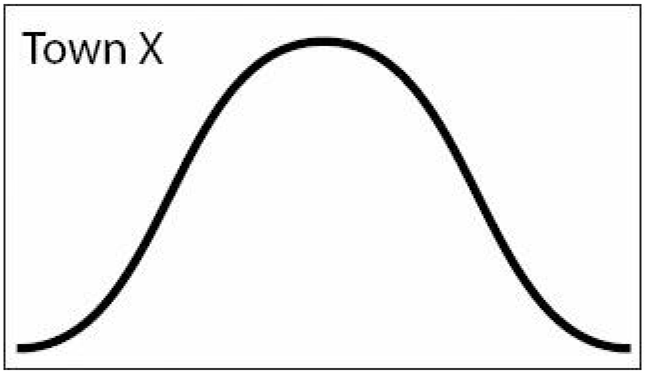


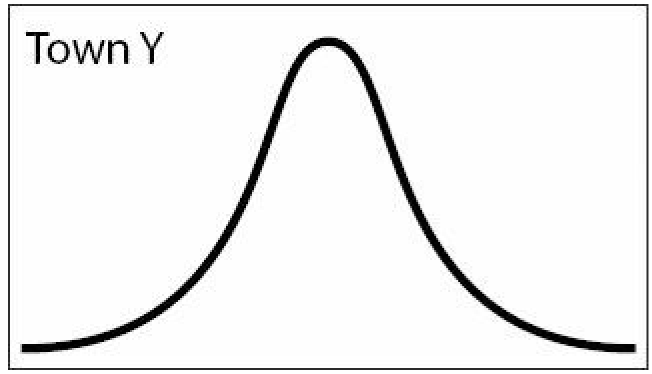
A number of scientists' salaries were reported; physicists' salaries clustered around a mean of $100,000, and biologists' clustered around a mean of $70,000. Which of the following could be true, according to the graph above?

Indicate all such statements.

☐ Some biologists earn more than some physicists.
☐ Both biologists' and physicists' salaries are normally distributed.
☐ The range of salaries is greater than $150,000

41.





The graph on the left represents the number of family members per family in Town X, while the graph on the right represents the number of family members per family in Town Y. The median family size for Town X is equal to the median family size for Town Y. The horizontal and vertical dimensions of the boxes above are identical and correspond to the same measurements. Which of the following must be true?

Indicate all such statements.

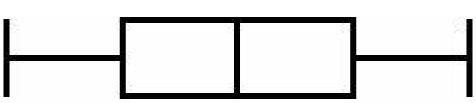
☐ The range of family sizes measured as the number of family members is larger in Town X than in Town Y.
☐ Families in Town Y are more likely to have sizes within 1 family member of the mean than are families in Town X.
☐ The data for Town X has a larger standard deviation than the data for Town Y.

42.

The box-and-whisker plot shown could be a representation of which of the following?

(A) a data set with a range of 100, symmetrically distributed around its median
(B) a data set with a range of 10 and an interquartile range of 6
(C) a data set in which the median of the upper half of the data is closer to the lowest value in the set than to the highest value
(D) a set of consecutive integers
(E) a normal distribution

43.



The box-and-whisker plot shown could be a representation of which of the following sets?

(A) -2, 0, 2, 4
(B) 3, 3, 3, 3, 3, 3
(C) 1, 25, 100
(D) 2, 4, 8, 16, 32
(E) 1, 13, 14, 17

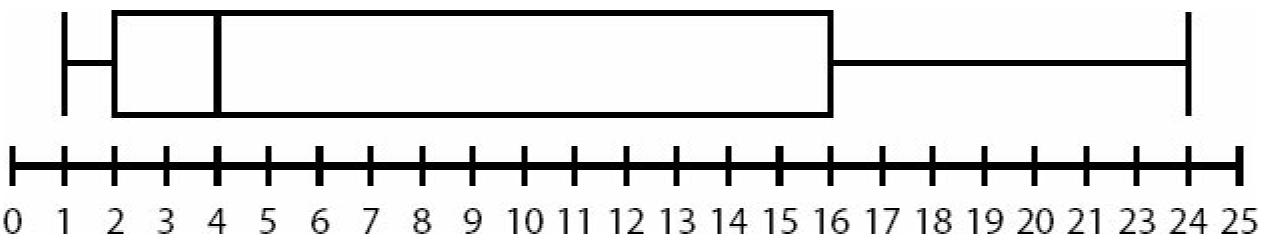
44.

Which of the following must be true about the data described by the box-and-whisker plot above?

Indicate all such statements.

- ☐ The median of the whole set is closer to the median of the lower half of the data than it is to the median of the upper half of the data.
- ☐ The data is normally distributed.
- ☐ The set has a standard deviation greater than zero.

45.



The box-and-whisker plot above represents a data set with:

(A) a mean of 4 and a range of 14
(B) a mean of 4 and a range of 23
(C) a median of 4 and a range of 14
(D) a median of 4 and a range of 23
(E) a median of 4 and a range of 24

46. The earthworms in Sample A have an average length of 2.4 inches, and the earthworms in Sample B have an average length of 3.8 inches. The average length of the earthworms in both samples is 3.0 inches. Which of the following must be true?

Indicate all such statements.

- ☐ There are more earthworms in Sample A than in Sample B.
- ☐ The median length of the earthworms is 3.2 inches.
- ☐ The range of lengths of the earthworms is 1.4.

Standard Deviation and Normal Distribution Answers

1. **I and III only.** The word *mean* is a synonym for the average. Because an average is calculated by taking the sum of the numbers in the set and dividing by the number of numbers in the set, changing *any* one number in a set (without adjusting the others) will change the sum and, therefore, the average. The median is the middle number in a set, so making the biggest number even bigger won't change that (the middle number is still 10). Standard deviation is a measure of how *spread out* the numbers in a set are — the more spread out the numbers, the larger the standard deviation — so making the biggest number *really far away* from the others would greatly increase the standard deviation.

2. **(D).** Standard deviation is a measure of how "spread out" the numbers in a set are — in other words, how far are the individual data points from the average of all of the data points? The GRE will not ask you to calculate standard deviation — in problems like this one, you will be able to eyeball which sets are more spread out and which are less spread out.

Since Set Z's members are identical, the standard deviation is zero. Zero is the smallest possible standard deviation for any set, so it must be the smallest here. You can eliminate answer choices (A), (B), and (C). Set Y's members have a *spread* of 1 between each number, Set X's members are two away from each other, and W's members are six away from each other, so Set Y has the next-smallest standard deviation (note that this is enough to eliminate answer choice (E) and choose answer choice (D)). The correct answer is (D) Z, Y, X, W.

3. **(B).** "Set N is a set of x distinct positive integers where $x > 2$" just means that the members of the set are all positive integers different from each other, and that there are at least 3 of them. You don't know anything about the standard deviation of the set other than that it is not zero. (Because the numbers are different from each other, they are at least a little spread out, which means the standard deviation must be greater than zero. The only way to have a standard deviation of zero is to have a set of identical numbers).

In Quantity B, multiplying each of the distinct integers by -3 would definitely spread out the numbers and thus increase the standard deviation. For instance, if the set had been 1, 2, 3, it would now be -3, -6, -9. The negatives are irrelevant — multiplying any set of *different* integers by 3 will spread them out more.

Thus, whatever the standard deviation is for the set in Quantity A, Quantity B must represent a larger standard deviation because the numbers in that set are more spread out.

4. **IV only.** In a set of distinct (different) integers, if each number is multiplied by 1/2, the numbers will get closer together (for instance, 2, 4, 6 would become 1, 2, 3), so the standard deviation would *decrease.*

If the smallest number in a set became equal to the median, then two numbers in the set would now be the same. The set would become *less* spread out, not more.

If the smallest number were increased to become larger than the current largest number, the standard deviation *could* increase, but wouldn't have to. For instance, if the set were 1, 2, 3, and the 1 were increased to become 100, the standard deviation would increase. But if the set were 1, 100, 101, and the 1 were increased to become 102, the set would get closer together.

Finally, if the largest number were doubled, the standard deviation would have to increase. For instance, if the set were 1, 2, 3 and the largest number doubled to 6, then the set would become more spread out. Because only the largest number is changing, and because the largest number becomes even larger when doubled, the numbers in the set will always become more spread out, thus increasing the standard deviation.

5. **(D).** Scoring scales on a test are not necessarily linear, so while it may look like you can line up the difference in percentiles with the difference in score, you cannot make any predictions about *other* percentiles. For all you know, 750 could be the 95th percentile score — or 963 could be. All you know is that 25% of the scores are $\leq$ 450, while 50% of the scores are > 450 and $\leq$ 700, and 25% of the scores are > 700.

6. **I and III only.** For the first statement, if the smaller number is positive and 3 less than the larger number AND one term is twice the other, the two terms have to be 3 and 6. (If you couldn't work out the numbers, you could write this as two equations: $S + 3 = L$ and $L = 2S$.) If you know the numbers, you can calculate the average.

The second statement is NOT sufficient. The standard deviation tells you how far all the terms are from the mean — but knowing *how spread out* the numbers are doesn't tell you what they're spread out *from.* For instance, the sets [3, 3, 3, 6, 6, 6, 6, 6, 6] and [-3, -3, -3, -6, -6, -6, -6, -6, -6] have the same standard deviation and meet the other constraints of the problem — each set consists of 9 terms, but only two different numbers, and 6 of the terms are each twice the value of each of the remaining 3. However, the two sets would have different averages (without calculating, you can easily see that one average would be positive and one would be negative).

The third statement works — if the biggest term in the set is 6, the smallest would, of course, be 3, allowing you to calculate the average.

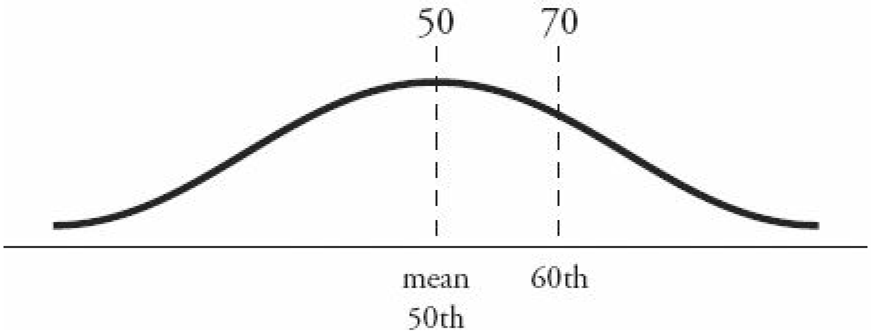
7. **(C).** A quartile is defined as the median of half of a set of data. The first quartile (or Q1) of a set of data is the median of the lower half of the data. For the first half, {2, 5, 7, 11, 16, 24}, the median is (7 + 11)/2 = 9 = Q1.

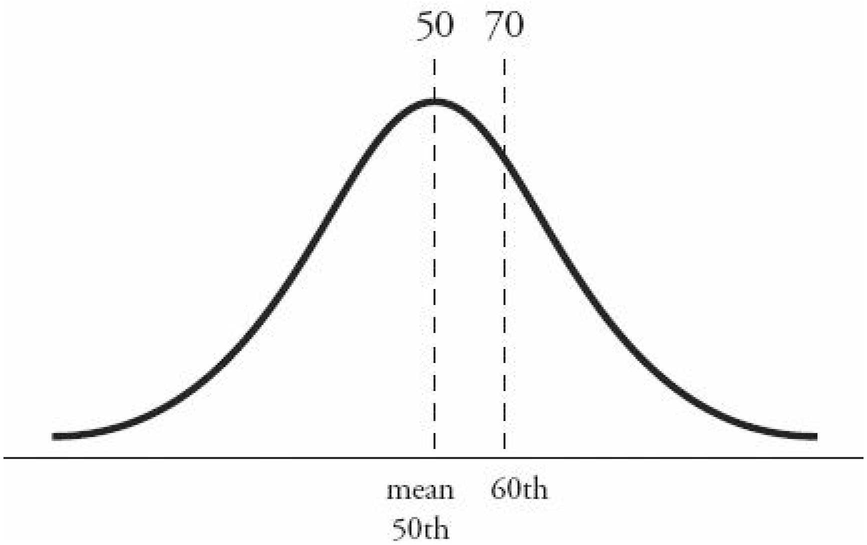
The third quartile (or Q3) of a set of data is the median of the upper half of the data. For the second half, {28, 50, 52, 101, 120, 130}, the median is (52 + 101)/2 = 76.5 = Q3.

Now find the average of Q1 and Q3 = (9 + 76.5) /2 = 42.75.

8. **(D).** The test scores are distributed normally — that is, in a specific hump-shaped pattern. That pattern is determined by two numbers: the mean (where the peak of the hump is) and the standard deviation (a measure of the width of the hump). To know the exact shape of the distribution, you need to know *both* numbers.

It's given that the 60th percentile is equal to a score of 70. The 60th percentile corresponds to a specific point on the right side of the normal distribution's hump—the point where 60% of the area under the curve is to the left and 40% is to the right. However, you don't know the width of the curve. It could be low and flat (imagine a lower mean, such as 50, and a high standard deviation, which allows the 60th percentile score of 70 to fall far from the mean), or it could be high and narrow (with a mean of, say, 68 and a tiny standard deviation). So a score of 35 (Quantity B) could correspond to *any* percentile below 60th, in fact—either less than or greater than the 30th percentile score, which is Quantity A.





9. **(B).** Normal distributions are always centered on and symmetrical around the mean, so the chance that the worm's length will be within a certain 6-centimeter range (or any specific range) is highest when that range is centered on the mean, which in this case is 30 centimeters.

More specifically, Quantity A equals the area between -2 standard deviations and the mean of the distribution. In a normal distribution, roughly 34 + 34 + 14 + 14 = 96% of the sample will fall within 2 standard deviations above or below the mean. If you limit yourself only to the 2 standard deviations below the mean, then half of that, or 96%/2 = 48%, will fall in this range. In contrast, Quantity B equals the area between -1 standard deviation and +1 standard deviation. In a normal distribution, roughly 34 + 34 = 68% of the sample will fall within 1 standard deviation above or below the mean. 68% is larger than 48%, so Quantity B is larger than Quantity A.

But you don't need these exact figures to answer this question! Picture any bell curve — the area under the "hump" (that is, centered around the middle) is bigger! Thus, it has more members of the set (in this case, worms) in it.

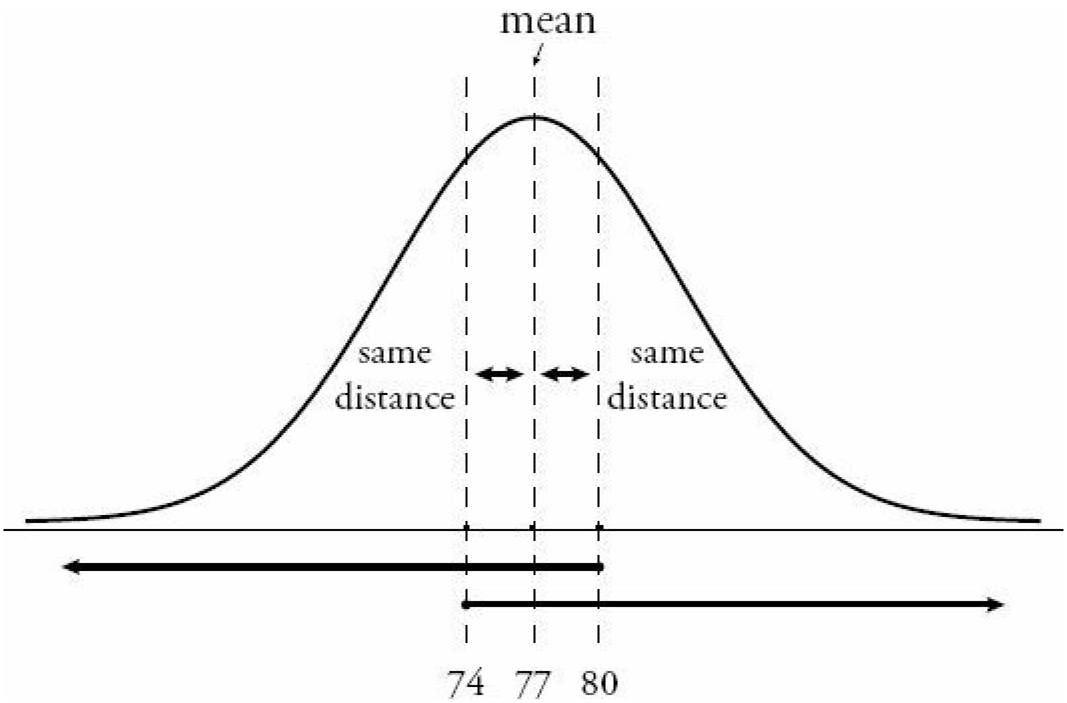
10. **(A).** The area under a normal distribution between the mean (the center) and +1 standard deviation represents approximately 34% of the total area (this is just a fact to memorize for the GRE), while the area between +1 and +2 standard deviations represents approximately 14%. The sum of those areas is 34 + 14 = 48%, so the percent of wages that fall between the mean ($18) and +2 standard deviations ($18 + $3.50 + $3.50 = $25) is 48%. Thus Quantity A is larger than Quantity B.

11. **(B).** How many standard deviations above $90,000 is $112,000? The difference between the two numbers is $22,000, which is 2 times the standard deviation of $11,000. So Quantity A is really the number of home values greater than 2 standard deviations above the mean.

In any normal distribution, roughly 2% will fall more than 2 standard deviations above the mean (this is something to memorize). The value of Quantity A is roughly (8,000)(0.02) = 160, which is definitely smaller than Quantity B

(300).

12. **(C).** The normal distribution is symmetrical around the mean. For any symmetrical distribution, the mean equals the median (also known as the 50th percentile). Even though you don't know the standard deviation, the number of students who scored *less* than 3 points *above* the mean (77 + 3 = 80) must be the same as the number of students who scored *greater* than 3 points *below* the mean (77 - 3 = 74). As long as the boundary scores (80 and 74) are placed symmetrically around the mean, you will have equal proportions. Draw this if it is at all confusing:



Notice that the two conditions overlap and are perfectly symmetrical. Each number consists of a short segment between it and the 50th percentile mark, as well as half of the students (either above or below the 50th percentile mark). That is, the "less than 80" category consists of the segment between 80 and 77, as well as all students below the 50th percentile mark (below 77). The "greater than 74" category consists of the segment between 74 and 77, as well as all students above the 50th percentile mark (above 77).

13. **(D).** First, make the numbers easier to use. You can multiply every number by the same constant, or move the decimal the same number of places for each number. If you move the decimal four places, the mean becomes 1630, the standard deviation becomes 84, and the two other numbers become 1,546 and 1,756.

Next, "normalize" the boundaries you care about. That is, take 1,546 meters (the lower boundary) and 1,756 meters (the upper boundary) and convert each of them to a number of standard deviations away from the mean. To do so, subtract the mean, then divide by the standard deviation.

Lower boundary: 1546 - 1630 = -84

-84 ÷ 84 = -1

So the lower boundary is -1 standard deviation (that is, 1 standard deviation less than the mean).

Upper boundary: 1756 - 1630 = 126

126 ÷ 84 = 1.5

So the upper boundary is 1.5 standard deviations above the mean.

You need to find the probability that a random variable distributed according to the standard normal distribution falls between -1 and 1.5.

Use the approximate areas under the normal curve. Approximately 34 + 34 = 68% will fall within 1 standard deviation above or below the mean, so 68% accounts for the -1 to 1 portion of the standard deviation. What about the portion from 1 to 1.5?

Approximately 14% will fall between 1 and 2 standard deviations above the mean. You are not expected to know the exact area between 1 and 1.5; however, since a normal distribution has its hump around 0, *more* than half of the area between 1 and 2 must fall closer to 0 (between 1 and 1.5). So the area under the normal curve between 1 and 1.5 must be greater than half of the area, or greater than 7%, but less than the full area, 14%.

Put it all together. The area under the normal curve between -1 and 1.5 is approximately 68% + (something between 7% and 14%). The lower estimate is 68% + 7% = 75%, and the upper estimate is 68% + 14% = 82%.

14. **(B).** First, compute the standard deviation from the information given. 10.8 pounds is 3 standard deviations more than 5.85 pounds, since 10.8 is 2 standard deviations *more* than the mean, and 5.85 is 1 standard deviation *less* than the mean.

> 10.8 - 5.85 = 4.95 pounds = 3 standard deviations.
> 4.95 ÷ 3 = 1.65 pounds = 1 standard deviation.

Now compute the mean. Since 5.85 pounds is 1 standard deviation below the mean, 5.85 plus 1 standard deviation equals the mean:

> 5.85 + 1.65 = 7.5 pounds = mean.

Quantity A: *Twice the weight of a baby 2 standard deviations below the mean.* Note that you already know 5.85 is one standard deviation below the mean; subtract another standard deviation (1.65) in order to reach 2 standard deviations below the mean, then multiply by 2.

> = 2 (5.85 - 1.65) = 2 (4.2) = 8.4 pounds.

Quantity B: *The weight of a baby 1 standard deviation above the mean.* Add 1 standard deviation to the mean.

> = 7.5 + 1.65 = 9.15 pounds.

Quantity B is larger than Quantity A.

15. **(E).** The smallest number in the set is -4, so you can eliminate (D). The largest number in the set is 5, so you can eliminate (B).

The median is -1; now check the medians of the remaining answer choices. The median of (A) is between 0 and -2, which is -1; (A) could be the right answer. The median of (C) is -3, which is wrong. The median of (E) is between -2 and 0, which is -1; (E) could also be the right answer.

Q1 is -3; check Q1 for both (A) and (E). The median of the smaller three numbers (-4, -4, -2) for (A) is -4, which is wrong; you want Q1 to be -3. (E) is the only answer choice left and you can pick it without checking if you're confident in your previous work. Here's the actual proof: the median of the smaller four numbers (-4, -4, -2, -2) is -3.

16. **(B).** The interquartile range of a set of data is the distance between Q1 (quartile marker 1, the median of the first half of the set) and Q3 (quartile marker 3, the median of the second half of the set).

The first ten positive multiples of 5 are: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50. Q1 is the median of the first 5 terms, or 15. Q3 is the median of the last 5 terms, or 40.

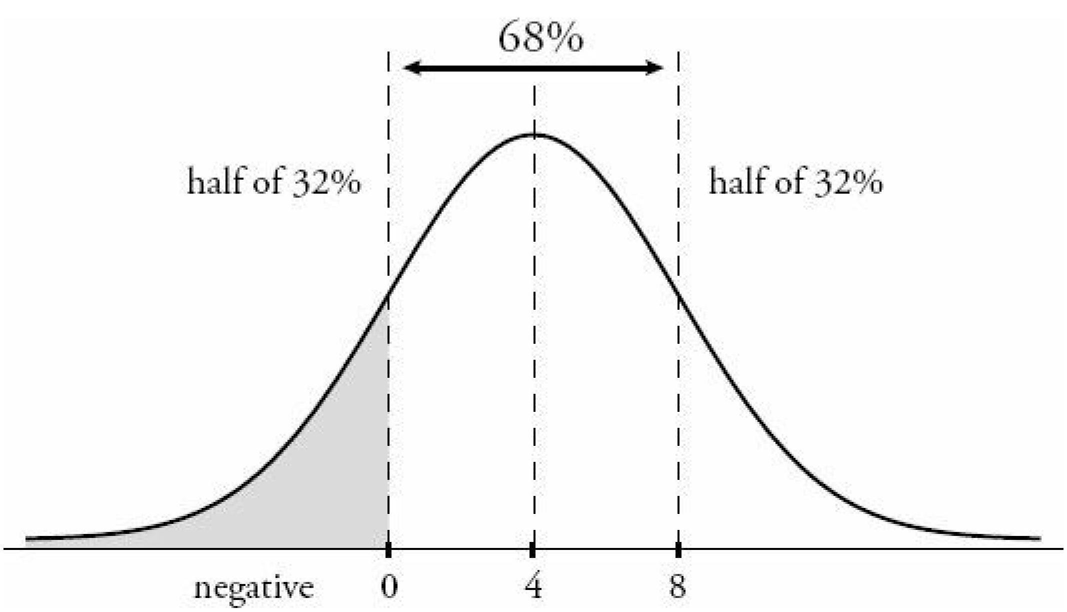
Take the difference between Q3 and Q1: 40 - 15 = 25.

17. **I only.** The definition of a normally distributed set is that about two-thirds of the data falls within one standard deviation of the mean. If only one person scored close to the mean (and most people were at the top or bottom of the curve), that set of data is not normally distributed, so the first statement is true.

The second statement is false—the range of the data would not necessarily change if the set were more evenly distributed. For instance, as long as one person still had a zero and one person still had a score of 100, the other scores could fall anywhere without changing the range.

The third statement is also false. The mean of Set *T* might or might not be equal to the median. For instance, the one student within five points of the mean could have a score actually equal to the mean; of the remaining 148 students, half could have scores of 0 and half could have scores of 100. In this case, the mean would equal the median. However, the same scenario with *unequal* numbers of students scoring 0 and 100 would result in the mean *not* equaling the median.

18. **(B).** In a normally distributed set of data, roughly 34 + 34 = 68% of the data fall within one standard deviation of the mean. This means about 100 - 68 = 32% of the data fall outside of one standard deviation, with HALF of that 32% falling one standard deviation to the RIGHT of the mean, and HALF of that 32% falling to the left. 68%



A negative number would be more than one standard deviation to the left of the mean (4 - 4 = 0), meaning that 1/2 of

32% of the data will fall in that range, or 16%. The closest answer is 1/6.

19. **(C).** Percentiles define the proportion of a group that scores below a particular benchmark. Since John scored in the 32nd percentile, by definition, 32 percent of the class scored worse than John. Quantity A is equal to 32%.

Jane is in the 68th percentile, so 68% of the class scored worse than she did. Since 100 - 68 = 32, 32% of the class scored as high as or higher than Jane. Quantity B is also equal to 32%.

20. **(D).** To find the range of one standard deviation, add and subtract the standard deviation from the mean. For two standard deviations, take twice the value of the standard deviation ($7.1 \times 2 = 14.2$) and both add and subtract it from the mean.

$4.2 - 14.2 = -10$
$4.2 + 14.2 = 18.4$

21. **41.** The set of data from 25 to 48 has 48 - 25 + 1 = 24 terms, so each octile group would be made up of 24/8 = 3 terms. It's easier to find the 6th octile group by working backwards from the 8th one:

8th octile group: 48, 47, 46   7th octile group: 45, 44, 43   6th octile group: 42, 41, 40

The median of the 6th octile group is 41.

22. **(B).** Since the average is 7, you can use the average formula to find the sum of the scores in the class.

Average = Sum ÷ # of terms
7 = Sum ÷ 20
Sum = 140

Now you already know that at least one student got every possible score. There are eleven possible scores: 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10. This is an evenly spaced set, so you can calculate the sum by multiplying the average of the set by the number of terms in the set. The average is (10 + 0)/2 = 5 and the number of terms is 11, so the sum of the set is $5 \times 11 = 55$. If you subtract from the sum you found earlier, the remaining 9 students had to score 140 - 55 = 85 points.

Quantity B is asking you to find the lowest score that two students could have received. If 9 students scored a total of 85 points, and any one student could not score more than 10 points, then what is the lowest possible score that any one student could have received? In order to minimize that number, you need to maximize the numbers for the other students. If 8 students scored 10 points each, for a total of 80 points, then the 9th student must have scored at least 5. Quantity B must be larger than Quantity A. Notice that the average score of 7 forces you to make a lot of scores 10's to balance out the very low scores of 0, 1, 2, etc. that you must have in the class (at least one of each).

23. **(B).** Frequency refers to the number of times a particular result occurred. In other words, if the set represented by this table were written out, it would look like this: 4,4,4,4,4,4, 6,6,6,6,6, 8,8,8,8,8. The mode is the most commonly occurring result. The mode of this data is 4 because 4 appears more often than any other number. Thus, Quantity B is larger.

24. **(D).** Quintiles ("fifths" of the data) define relative scores, not absolute scores. Imagine two possible score distributions:

Example 1: The class's scores are 1, 2, 3, 4, 5 (20% of the class scored each of these). In this case, adding up two lowest quintile students would be $1 + 1 = 2$, which is less than 5, the score of a top quintile student.

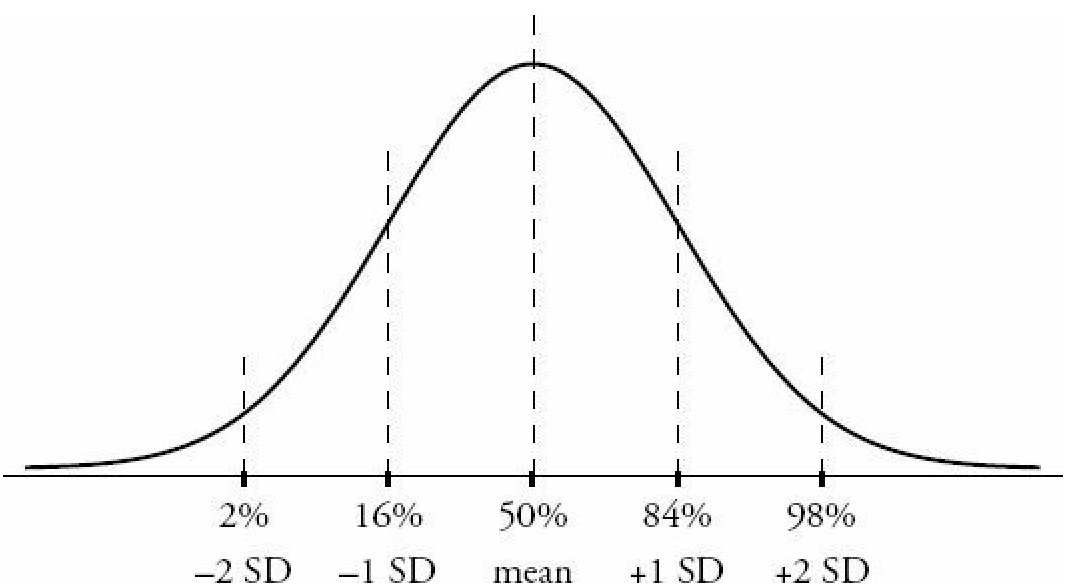
Example 2: The class's scores are 10, 11, 12, 13, 14 (20% of the class scored each of these — still not so sharp!). In this case, adding up two lowest quintile students would be $10 + 10 = 20$, which is greater than 14, the score of a top quintile student.

Thus, you cannot determine which quantity is larger.

25. **(E).** A percentile ALWAYS represents one *percent* of a set of data. If the question had asked how many TERMS one percentile represented, that would be a different question (with a different answer).

26. **(D).** You don't know what x or y are, but since they are both positive integers, they can only be 1 and 18, 2 and 9, or 3 and 6 (because they have to multiply to 18). So the smallest number in the set is 1 and the largest is 20. Since $20 - 1 = 19$, the range is 19.

27. **(D).** The diagram below shows the standard distribution curve for any normally distributed variable. The percent figures correspond roughly to the standard percentiles both 1 and 2 standard deviations (SD) away from the mean.



The 2nd percentile is 1720, roughly corresponding to 2 standard deviations below the mean. Therefore, the mean - 2 standard deviations = 1720.

Likewise, the 84th percentile is 1990. 84% of a normally distributed set of data falls below the mean + 1 standard deviation, so the mean + 1 standard deviation = 1990.

Call the mean M and the standard deviation S. You can now solve for these variables:

$$M - 2S = 1720$$
$$M + S = 1990$$

Subtract the first equation from the second equation:

$3S = 270$
$S = 90$

You are looking for the 16th percentile, which is the mean - 1 standard deviation or $M - S$. (It's a fact to memorize that approximately 2% of normally distributed data falls below $M - 2S$, and approximately 14% of normally distributed data falls between $M - 2S$ and $M - S$.)

You already know that $M - 2S = 1720$, so add another S to get $M - S$.

$(M - 2S) + S = 1720 + 90 = 1810$

Notice that the percentiles are *not* linearly spaced. The normal distribution is hump-shaped, so percentiles will be bunched up around the hump and spread out farther away.

28. **(D).** In most data sets, the range is larger than the interquartile range because the interquartile range ignores the smallest and largest data points. That's actually the purpose of interquartile range — to get a good picture of where *most* of the data is (think of the "big hump" on a bell curve). For instance:

Example Set A: 1, 2, 3, 4, 5, 6, 7, 100

Here, the range is 100 - 1 = 99.

The interquartile range is Q3 - Q1, or the median of the upper half of the data minus the median of the lower half of the data: 6.5 - 2.5 = 4.

In this example, the range is much larger. However, consider this set:

Example Set B: 4, 4, 4, 4, 5, 5, 5, 5

In this set, the range is 5 - 4 = 1. The interquartile range is also 5 - 4 = 1. While the interquartile range can never be *smaller* than the range, they can certainly be equal.

29. **67.** Since the standard deviation is 10 and 1 standard deviation above the mean is 77, simply subtract 77 - 10 = 67 to get the mean.

30. **(C).** Since one standard deviation below the mean is 250 and one standard deviation above the mean is 420, the mean/median must be halfway in between. Since 420 - 250 = 170 and half of 170 is 85, simply add 85 to 250 (or subtract it from 420) to get the mean/median of 335. (Note that in a normal distribution, the mean is equal to the median, so the two terms can be used interchangeably.)

31. **(A).** If the standard deviation were 3, then one standard deviation below the mean would be 9 and one standard deviation above the mean would be 15, so about 2/3 (more precisely 68%) of the data would be between 9 and 15 (in a normal distribution, it is always the case that about 2/3 of the data is within one standard deviation of the mean).

Since the *actual* standard deviation is *less than* 3, about 2/3 of the data is found within an *even smaller range* than 9 to 15. For instance, the standard deviation could be 1, and then about 2/3 of the data would be between 11 and 13. Or the standard deviation could be 2.5, and then about 2/3 of all the data would be found between 9.5 and 14.5.

Since 2/3 of the data is found within an *even smaller range* than 9 to 15, the range from 9 to 15 contains *more than* 2/3 of the data, so it definitely contains more than 60% of all the data points.

Don't be confused by the use of "number of data points." While you don't know the actual total number of data points, you can definitively conclude that Quantity A is equal to a larger percentage of that total than is Quantity B.

32. **(A).** Standard deviation is a measure of the data's spread from the mean. While the two sets have the same range (30 - 10 = 20), they do NOT have the same spread. The four extra terms in Quantity B are identical to the mean, meaning that, on average, the data in Quantity B is closer to the mean than the data in Quantity A. Thus, Quantity A is more spread out, on average, and has the larger standard deviation. You do not have to compute the actual standard deviations to find the answer here.

33. **(D).** Standard deviation depends on the difference between each number in a set and the average (arithmetic mean) of the set.

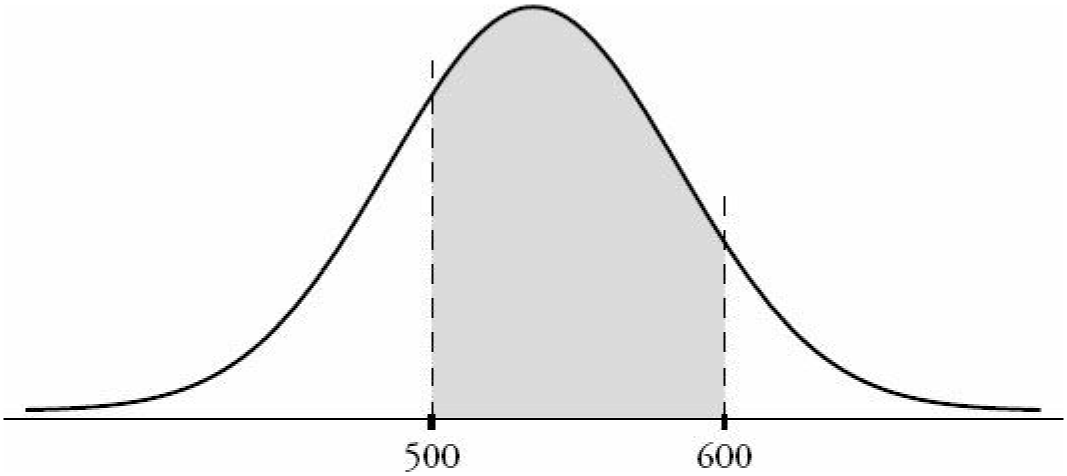
Quantity B is known, since the average of 3 consecutive multiples of 3 (such as 3, 6, 9) is the middle number (6), and the differences between each of the numbers in the set and that average are therefore known (-3, 0, and 3). While you don't need to calculate the actual standard deviation for the GRE, it certainly can be done: standard deviation is the square root of the average of the squares of these differences, or $\sqrt{\frac{(-3)^2+(0)^2+(3)^2}{3}}=\sqrt{\frac{18}{3}}=\sqrt{6}$, which is between 2 and 3.

Quantity A is *not* known, and in fact it can be greater or less than Quantity B. Again, you don't have to calculate standard deviation; you should just realize that the numbers in Quantity A could be more or less spread out than the numbers in Quantity B. Therefore, the answer is (D).

If you're interested in the calculations: the maximum value of Quantity A is 4, which you can get from a set of 2 numbers that differ by 8 (say, 1 and 9). The average of this set is 5; the difference of each number from this average is -4 and 4, and so the standard deviation works out to be $\sqrt{\frac{(-4)^2+(4)^2}{2}}=\sqrt{\frac{32}{2}}=\sqrt{16}=4$. Even without calculating, you should recognized that the set {1, 9} is more spread out than the set {3, 6, 9}.

On the other hand, the standard deviation of a set with range of 8 can be as close to zero as you want to make it. How? Start with 1 and 9 again in your set, but now add a whole bunch of 5's (right at the average). The differences of those 5's from the average of the set (which is still 5) are all 0. In essence, you're averaging in a lot of 0's to the standard deviation, swamping the effect of the two original deviations from 5 (the original numbers 1 and 9). You can make the whole set be super-close to the mean, 5, reducing the standard deviation to minuscule levels—well below that of the set {3, 6, 9}.

34. **(D).** While the shaded area may appear to be evenly located on either side of the mean, it isn't necessarily. For example, the 68% could be more lopsided, like so:



This area could still represent 68% of the scores, even if it's not one standard deviation to either side of the mean. For you to know that the median = 500, the problem would need to state explicitly that 500 and 600 each represent one standard deviation from the mean (or at least that 500 and 600 are equally far from the mean).

The fact that 68% of the data is located between 500 and 600 is definitely intended to trick you into assuming that 500 and 600 are -1 and +1 standard deviation from the mean, but you cannot assume this. While it is always true that, in a normal distribution, about 68% (some people memorize the approximation as two-thirds) of the data is within one standard deviation of the mean, the reverse is not true: you cannot assume that any chunk of data that is about 68% of the whole is therefore within one standard deviation of the mean.

35. **I and III only**.

I. True. Standard deviation describes how much a set of data diverges from the mean. Curve B is more widely spread than curve A, and thus *Y* has a larger standard deviation than *X*.

II. Not True. The probability that *any* normally distributed variable falls within 2 standard deviations of its mean is the same, approximately 0.14 + 0.34 + 0.34 + 0.14 = 0.96, or 96%. This is a value you should memorize for the GRE.

III. True. The mean of a normal curve is the point along the horizontal axis below the "peak" of the curve. The highest point of curve B is clearly to the right of the highest point of curve A, so the mean of Y is larger than the mean of X. Notice that the mean has nothing to do with the *height* of the normal curve, which only corresponds to how tightly the variable is gathered around the mean (i.e., how small the standard deviation is).

36. **(C).** Since the number of test results is divisible by 100, the percentiles cleanly divide the total into 100 percentile groups of 300 ÷ 100 = 3 results each. That is, there are 3 results in each percentile. So 2 other results are in the same percentile as Dominick's. Quantity A is 2.

Dominick's result is clearly in the 80th percentile, not the 79th or the 81st. So it must be possible to distinguish the 80th percentile (that group of 3) from the "next-door" percentiles. Say Dominick got a 58. How many *other* people could have gotten a 58? Maybe no one, maybe one, maybe two — but if *three* other people got a 58, then you'd have a total of four people with the same result. In that case, it would be impossible to assign Dominick's result *definitively* to the 80th percentile and not the neighboring percentiles. So the maximum number of other test-takers with the same result as Dominick is 2. Quantity B is also 2.

37. **(A).** 400 test scores are distributed among 50 possible outcomes (integers between 151 and 200, inclusive, which number 200 - 151 + 1 = 50 integers). There is an average of 400 ÷ 50 = 8 scores per integer outcome, and there are

400 ÷ 100 = 4 scores in each percentile. So, if all the scores were completely evenly distributed with exactly 8 scores per integer, there would be 2 percentile groups per integer outcome (0th and 1st percentiles at 151, 2nd and 3rd percentiles at 152, etc.). In that case, all 50 integers from 151 to 200 would correspond to more than one percentile group.

How can you reduce the number of integers corresponding to more than one percentile group? Bunch up the scores. Imagine that everyone gets a 157. Then that integer is the only one that corresponds to more than one percentile group (it corresponds to all 100 groups, in fact). However, you can't reduce further this way. You're left with 1 integer, so the minimum number of integers corresponding to more than one percentile group is 1, which is Quantity A.

As for Quantity B, though, a particular integer may have *no* percentile groups corresponding to it. In the previous example, if everyone gets a 157, then no one gets a 158, or a 200 for that matter. So the minimum number of percentile groups corresponding to a score of 200 (or to any other particular score) is 0, which is Quantity B.

38. **(C).** A two-humped shape could come from two overlapping normal distributions with different averages. Since the hump on the right is smaller, the distribution with a higher average should contain less data. Of the possible answer choices, only (C) describes such a scenario.

39. **(B).** The data appears essentially hump-shaped, but the left tail is cut off. This means that there is no data below a certain cutoff. Only (B) corresponds to this kind of situation.

40. **I, II, and III.**

I. Could Be True. Although biologists' salaries cluster around a lower number than physicists' salaries do, you cannot claim that *every* biologist's salary is lower than *every* physicist's salary. Some biologists' salaries can be high, and some physicists' salaries can be low.

II. Could Be True. Normal distributions are consistent with the hump shapes you see in the graph. You cannot *prove* that they're normal, but you cannot claim they're definitely not — they certainly *could* be normal.

III. Could Be True. From real-world normal distributions of an unknown amount of data, there's no way to tell the maximum or minimum values of the data. So the range certainly could be more than $150,000.

41. **II and III only.**

I. Not Necessarily True. Range is calculated this way: *largest value* - *smallest value*. From the graphs as shown (assuming that they do not continue "off screen" left and right), you would conclude that the two distributions have the *same* range, because the distributions are above zero on both the far left and the far right. (In the real world, you might assume that the graphs continue off screen, leading to even less confidence about the range of each distribution.)

II. True. The graph on the right (Town Y) has a smaller standard deviation (it is less spread out around its mean). So families in Town Y are more likely to be within 1 family member of the mean than families in Town X are.

III. True. The graph on the left is more spread out, so it has a larger standard deviation.

42. **(C).** The plot is not symmetrical, so you can eliminate (A), (D), and (E), which would all be symmetrically distributed around the median. You can also eliminate (B), which claims a range of 10 (the distance from whisker to

whisker) and an interquartile range of 6 (the width of the box). Since the distance between the whiskers is actually much more than twice the width of the box, (B) cannot be the answer. (C) fits: the median of the upper half of the data (the right edge of the box) is closer to the lowest value in the set (the left whisker) than to the highest value (the right whisker).

43. **(A).** The plot is symmetrical, so you can eliminate any non-symmetrical data sets (such as (C), (D), and (E)). In (B), all the data points are the same, so there would be no width to the box-and-whisker plot. (A) is the only remaining possibility: the data is evenly spaced, leading to equal widths for each segment of the plot, as shown.

44. **III only.**

I. Not True. The median of the whole set is the line in the middle of the box. As shown, it is closer to the *right* side of the box (the median of the upper half of the data) than to the *left* side of the box (the median of the lower half of the data) — the opposite of what this statement claims.

II. Not True. This non-symmetrical plot could never represent a symmetrical distribution such as the normal distribution. In fact, a *true* normal distribution cannot be represented by a box-and-whisker plot at all, because such a distribution stretches infinitely to the right and to the left, in theory.

III. True. Any set represented by a box-and-whisker plot has a standard deviation greater than zero, because the plot displays some spread in the data. The only set that has a zero standard deviation is a set containing identical data points with zero spread between them, such as $\{3, 3, 3, 3\}$.

45. **(D).** The line inside the box always represents the median. If the plot is symmetrical, the median equals the mean, but because this plot is not symmetrical, there's no way that the mean is 4. So you can eliminate (A) and (B).

The range is the distance between the whiskers. That distance is $24 - 1 = 23$.

46. **I only.** Since the overall average length of all the earthworms is closer to the average length of earthworms in Sample A than to the average for Sample B, there are more earthworms in Sample A.

However, you cannot know anything about the median or the range of the data set without the individual values. For instance, the lengths of all the worms in Sample A could be exactly 2.4, or they could be spread out quite a bit from 2.4. Similarly, the worms in Sample B could measure exactly 3.0, or they could have a variety of different lengths that average to 3.0. Thus, the median and range could vary quite a bit.

Chapter 23
of

Probability, Combinatorics, and Overlapping Sets

In This Chapter…

Probability, Combinatorics, and Overlapping Sets

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. A number is randomly chosen from a list of 10 consecutive positive integers. What is the probability that the number is greater than the mean?

(A) 3/10
(B) 2/5
(C) 1/2
(D) 7/10
(E) 4/5

2. A number is randomly chosen from the first 100 positive integers. What is the probability that it is a multiple of 3?

(A) 32/100
(B) 33/100
(C) 1/3
(D) 34/100
(E) 2/3

3. A restaurant menu has several options for tacos. There are 3 types of shells, 4 types of meat, 3 types of cheese, and 5 types of salsa. How many distinct tacos can be ordered assuming that any order contains exactly one of each of the above choices?

4. A history exam features 5 questions. 3 of the questions are multiple-choice with four options each. The other two questions are true or false. If Caroline selects one answer for every question, how many different ways can she answer the exam?

5. A certain company places a six-symbol code on each of their products. The first two symbols are one of the letters A–E and the last four symbols are digits. If repeats are allowed on both letters and numbers, how many such codes are possible?

6. The probability is 1/2 that a coin will turn up heads on any given toss and the probability is 1/6 that a number cube with faces numbered 1 to 6 will turn up any particular number. What is the probability of turning up a heads and a 6?

(A) 1/36
(B) 1/12
(C) 1/6
(D) 1/4
(E) 2/3

7. An integer is randomly chosen from 2 to 20 inclusive. What is the probability that the number is prime?

8. Five students in a classroom are lining up one behind the other for recess. How many different lines are possible?

(A) 5
(B) 10
(C) 24
(D) 25
(E) 120

9. An Italian restaurant boasts 320 distinct pasta dishes. Each dish contains exactly one pasta, one meat, and one sauce. If there are 8 pastas and 4 meats available, how many sauces are there to choose from?

10. A 10-student class is to choose a president, vice president, and secretary from the group. Assuming that no person can occupy more than one post, in how many ways can this be accomplished?

11.

Quantity A	Quantity B
The number of 4-digit positive integers where all 4 digits are less than 5	625

12. BurgerTown offers many options for customizing a burger. There are 3 types of meats and 7 condiments: lettuce, tomatoes, pickles, onions, ketchup, mustard, and special sauce. A burger must include meat, but may include as many or as few condiments as the customer wants. How many different burgers are possible?

(A) 8!
(B) (3)(7!)
(C) (3)(8!)
(D) $(8)(2^7)$
(E) $(3)(2^7)$

13. The probability of rain is 1/6 for any given day next week. What is the chance it rains on both Monday and Tuesday?

(A) 1/36
(B) 1/12
(C) 1/6
(D) 1/3
(E) 2/3

14. How many five-digit numbers can be formed using the digits 5, 6, 7, 8, 9, 0 if no digits can be repeated?

(A) 64
(B) 120
(C) 240
(D) 600
(E) 720

15. A bag contains 3 red, 2 blue, and 7 white marbles. If a marble is randomly chosen from the bag, what is the probability that it is NOT blue?

16. A man has 3 different suits, 4 different shirts, 2 different pairs of socks, and 5 different pairs of shoes. In how

many ways can the man dress himself if he must wear 1 suit, 1 shirt, 1 pair of socks, and 1 pair of shoes?

17. A state issues automobile license plates using two letters selected from a 26-letter alphabet, as well as four numerals selected from the digits 0 through 9, inclusive. Repeats are permitted. For example, one license plate combination could be GF3352.

Quantity A	**Quantity B**
The number of possible unique license plate combinations	6,000,000

18. A small nation issues license plates that consist of just one number (selected from the digits 0 through 9, inclusive) and four letters, selected from a 20-letter alphabet. Repeats are permitted. However, there is one four-letter combination that is not allowed to appear on license plates. How many allowable license plate combinations exist?

(A) 1,599,990
(B) 1,599,999
(C) 1,600,000
(D) 4,569,759
(E) 4,569,760

19. A bag contains 6 black chips numbered 1–6 respectively and 6 white chips numbered 1–6 respectively. If Pavel reaches into the bag of 12 chips and removes 2 chips, one after the other, without replacing them, what is the probability that he will pick black chip #3 and then white chip #3?

20. Tarik has a pile of 6 green chips numbered 1–6 respectively and another pile of 6 blue chips numbered 1–6 respectively. Tarik will randomly pick 1 chip from the green pile and 1 chip from the blue pile.

Quantity A	**Quantity B**
The probability that both chips selected by Tarik will display a number less than 4	1/2

21. A bag contains 6 red chips numbered 1–6 respectively and 6 blue chips numbered 1–6 respectively. If 2 chips are to be picked sequentially from the bag of 12 chips, without replacement, what is the probability of picking a red chip and then a blue chip with the same number?

22. In a school of 150 students, 75 take Latin, 110 take Spanish, and 11 take neither.

Quantity A	**Quantity B**
The number of students who take only Latin	46

23. How many 10-digit numbers can be formed using only the digits 2 and 5?

(A) 2^{10}
(B) (22)(5!)
(C) (5!)(5!)
(D) 10!/2
(E) 10!

24. A 6-sided cube has sides numbered 1 through 6. If the cube is rolled twice, what is the probability that the sum of the two rolls is equal to 8?

(A) 1/9
(B) 1/8
(C) 5/36
(D) 1/6
(E) 7/36

25. A coin with heads on one side and tails on the other has a 1/2 probability of landing on heads. If the coin is flipped 5 times, how many distinct outcomes are possible if the last flip must be heads? Outcomes are distinct if they do not contain exactly the same results in exactly the same order.

26. In a class of 25 students, every student takes either Spanish, Latin, or French, or two of the three, but no students take all three languages. 9 take Spanish, 7 take Latin and 5 take exactly two languages.

Quantity A	**Quantity B**
The number of students who take French	14

27. Bob has a 24-sided die with an integer between 1 and 24 on each face. Every number is featured exactly once. When he rolls, what is the probability that the number showing is a factor of 24?

28. A baby has x total toys. If 9 of the toys are stuffed animals, 7 of the toys were given to the baby by its grandmother, 5 of the toys are stuffed animals given to the baby by its grandmother, and 6 of the toys are neither stuffed animals nor given to the baby by its grandmother, what is the value of x?

29. How many integers between 2,000 and 3,999 have a ones digit that is a prime number?

30. How many integers between 2,000 and 6,999 are even and have a digit that is a prime number in the tens place?

31. A group of 12 people who have never met are in a classroom. How many handshakes are exchanged if each pair shakes hands exactly once?

(A) 12
(B) 22
(C) 66
(D) 132
(E) 244

32. A classroom has 12 girls and 20 boys. One quarter of the girls in the class have blue eyes. If a child is selected at random from the class, what is the probability that he/she is a girl who does not have blue eyes?

(A) 3/32
(B) 9/32
(C) 3/8
(D) 23/32
(E) 29/32

33. A coin with heads on one side and tails on the other has a 1/2 probability of landing on heads. If the coin is flipped three times, what is the probability of flipping 2 tails and 1 head, in any order?

(A) 1/8
(B) 1/3
(C) 3/8
(D) 5/8

(E) 2/3

34. A 6-sided cube has sides numbered 1 through 6. If the cube is rolled twice, what is the probability that at least one of the rolls will result in a number higher than 4?

(A) 2/9
(B) 1/3
(C) 4/9
(D) 5/9
(E) 2/3

35. Tiles are labeled with the integers from 1 to 100 inclusive; no numbers are repeated. If Alma chooses one tile at random, replaces it in the group, and chooses another tile at random, what is the probability that the product of the two integer values on the tiles is odd?

(A) 1/8
(B) 1/4
(C) 1/3
(D) 1/2
(E) 3/4

36. If the word "WOW" can be rearranged in exactly 3 ways (WOW, OWW, WWO), in how many ways can the word "MISSISSIPPI" be rearranged?

37. If a, b, and c are integers randomly chosen from the set of prime numbers greater than 2 and less than 30, what is the probability that $ab + c$ is equal to 23?

38.

The probability of rain is 1/2 on any given day next week.

Quantity A	Quantity B
The probability that it rains on AT LEAST one out of the 7 days next week	127/128

39. Two fair dice with sides numbered 1 to 6 are tossed. What is the probability that the sum of the exposed faces on the dice is a prime number?

40. Jack has a cube with 6 sides numbered 1 through 6. He rolls the cube repeatedly until the first time that the sum of all of his rolls is even, at which time he stops. (Note: it is possible to roll the cube just once.) What is the probability that Jack will need to roll the cube more than 2 times in order to get an even sum?

(A) 1/8
(B) 1/4
(C) 3/8
(D) 1/2
(E) 3/4

41. Jan and 5 other children are in a classroom. The principal of the school walks in and chooses two children at random. What is the probability that Jan is chosen?

(A) 4/5
(B) 1/3
(C) 2/5
(D) 7/15
(E) 1/2

42. The probability that Gary will eat eggs for breakfast on any given day is 3/7. The probability that Gary will eat cereal for breakfast on any given day is 4/7. Gary never has both eggs and cereal for breakfast on the same day.

Quantity A	**Quantity B**
Probability that Gary eats eggs or cereal for breakfast on a particular day	1

43. The probability that Maria will eat breakfast on any given day is 0.5. The probability that Maria will wear a sweater on any given day is 0.3. The two probabilities are independent of each other.

Quantity A	**Quantity B**
The probability that Maria eats breakfast or wears a sweater	0.8

44. The probability of rain in Greg's town on Tuesday is 0.3. The probability that Greg's teacher will give him a pop quiz on Tuesday is 0.2. The events occur independently of each other.

Quantity A	**Quantity B**
The probability that either or both events occur	The probability that neither event occurs

45. The probability of event *X* occurring is the same as the probability of event *Y* occurring. The events occur independently of each other.

Quantity A	**Quantity B**
The probability that both events occur	The probability that neither event occurs.

46. A certain city has a 1/3 chance of rain occurring on any given day. In any given 3-day period, what is the probability that the city experiences rain?

(A) 1/3
(B) 8/27
(C) 2/3
(D) 19/27

(E) 1

47. Five students, Adnan, Beth, Carol, Dan, and Edmund are to be arranged in a line. How many such arrangements are possible if Beth is not allowed to stand next to Dan?

(A) 24
(B) 48
(C) 72
(D) 96
(E) 120

48. A polygon has 12 edges. How many different diagonals does it have? (A diagonal is a line drawn from one vertex to any other vertex inside the given shape. This line cannot touch or cross any of the edges of the shape. For example, a triangle has zero diagonals and a rectangle has two.)

(A) 54
(B) 66
(C) 108
(D) 132
(E) 144

49. A student council is to be chosen from a class of 12 students consisting of a president, a vice president, and 3 committee members. How many such councils are possible?

(A) $\frac{12!}{7!5!}$
(B) $\frac{12!}{7!3!}$
(C) $\frac{12!}{5!3!}$
(D) $\frac{12!}{7!}$
(E) 12!

50.

Quantity A	Quantity B
The number of possible pairings of 2 colors that can be selected from 5 possible options	The number of possible pairings of 8 colors that can be selected from 9 possible options

51.

Quantity A	Quantity B
The number of possible 4-person teams that can be selected from 6 people	The number of possible 2-person teams that can be selected from 6 people

52.

Quantity A	Quantity B

The number of ways 1st, 2nd, and 3rd place prizes could be awarded to 3 out of 6 contestants	The number of ways 1st, 2nd, 3rd, 4th, and 5th place prizes could be awarded to 5 contestants

53. An inventory of coins contains 100 different coins.

Quantity A	**Quantity B**
The number of possible collections of 56 coins that can be selected where the order of the coins does not matter	The number of possible collections of 44 coins that can be selected where the order of the coins does not matter

54. An office supply store carries an inventory of 1,345 different products, all of which it categorizes as "business use," "personal use," or both. 740 products are categorized as "business use" ONLY and 520 products are categorized as both "business use" and "personal use."

Quantity A	**Quantity B**
The number of products characterized as "personal use"	600

55. How many distinct 4-letter "words" can be made from the name "CHRISTYNA"? (A "word" is any arrangement of 4 letters regardless of whether it can be found in a dictionary.)

(A) 9
(B) 24
(C) 36
(D) 504
(E) 3,024

56. Seiko has a 6-sided number cube with sides labeled 1 through 6. If she rolls the cube twice, what is the probability that the product of the two rolls is less than 36?

(A) 1/6
(B) 1/3
(C) 2/3
(D) 5/6
(E) 35/36

57. There is an 80% chance David will eat a healthy breakfast and a 25% chance that it will rain. If these events are independent, what is the probability that David will eat a healthy breakfast OR that it will rain?

(A) 20%
(B) 80%
(C) 85%
(D) 95%
(E) 105%

58. The probability of rain is 1/2 for every day next week. What is the chance that it rains on at least one day during the workweek (Monday through Friday)?

(A) 1/2
(B) 31/32
(C) 63/64
(D) 127/128

(E) 5/2

59. Eight women and two men are available to serve on a committee. If three people are picked, what is the probability that the committee includes at least one man?

(A) 1/32
(B) 1/4
(C) 2/5
(D) 7/15
(E) 8/15

60. At Lexington High School, everyone takes at least one language — Spanish, French, or Latin — but no one takes all three languages. If 100 students take Spanish, 80 take French, 40 take Latin, and 22 take exactly two languages, how many students are there?

(A) 198
(B) 220
(C) 242
(D) 264
(E) 286

61.

Of 60 birds found in a certain location, 20 are songbirds and 23 are migratory. (It is possible for a songbird to be migratory, or not.)

Quantity A	**Quantity B**
The number of the 60 birds that are neither migratory nor songbirds	16

Probability, Combinatorics, and Overlapping Sets Answers

1. **(C).** In a list of 10 consecutive integers, the mean will be the average of the 5th and 6th numbers. Therefore, the 6th through 10th integers (five total integers) will be larger than the mean. Since probability is determined by the number of desired items divided by the total number of choices, the probability that the number chosen is higher than the mean is 5/10 = 1/2.

Another approach to this problem is to create a set of 10 consecutive integers; the easiest such list contains the numbers {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. The mean is one-half the sum of the first element plus the last element, or $\frac{1+10}{2}=5.5$. Therefore, there are 5 numbers higher than the mean in the list: 6, 7, 8, 9 and 10. Again, the probability of choosing a number higher than the mean is 5/10 = 1/2.

2. **(B).** The first 100 positive integers comprise the set of numbers containing the integers 1 to 100. Of these numbers, the only ones that are divisible by 3 are {3, 6, 9, …, 96, 99}, which adds up to exactly 33 numbers. This can be determined in several ways. You could simply count the multiples of 3, but that's a bit slow. Alternatively, you can compute 99/3 = 33 and realize that there are 33 multiples of 3 up to and including 99. The number 100 is not divisible by 3, so the correct answer is 33/100.

Alternatively, you can use the "add one and you're done trick," subtracting the first multiple of 3 from the last multiple of 3, dividing by 3 and then adding 1: $\frac{(99-3)}{3}+1=33$. Then, since probability is determined by the number of desired options divided by the total number of options, the probability that the number chosen is a multiple of 3 is 33/100.

3. **180.** This problem tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. Therefore, the total number of tacos is (3)(4)(3)(5) = 180 tacos.

4. **256.** This question tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. The five separate test questions give you five independent choices. For the three multiple choice questions there are four options each, whereas for the two true/false questions there are two options each. Multiplying the independent choices yields (4)(4)(4)(2)(2) = 256 different ways to answer the exam.

5. **250,000.** This question tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. There are 5 choices for each of the letters at the beginning of the code (A, B, C, D, or E) and 10 choices for each of the 4 digits at the end of the code (0, 1, 2, 3, 4, 5, 6, 7, 8, or 9). Therefore, there are (5)(5)(10)(10)(10)(10) = 250,000 possible codes.

6. **(B).** The probability of independent events A AND B occurring is equal to the product of the probability of event A and the probability of event B. In this case, the probability of the coin turning up heads is 1/2 and the product of rolling a 6 is 1/6. Therefore, the probability of heads AND a 6 is equal to (1/2)(1/6) = 1/12. Alternatively, you could list all the possible outcomes: H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6. There are 12 total outcomes and only 1 with heads and a 6. Therefore, the desired outcome divided by the total number of outcomes is equal to 1/12.

7. **8/19.** Among the integers 2 through 20 inclusive there are 8 primes: 2, 3, 5, 7, 11, 13, 17, and 19. From 2 to 20 inclusive there are exactly 20 - 2 + 1 = 19 integers; remember to "add one before you're done" to include both endpoints. Alternatively, there are 20 integers from 1 to 20 inclusive, so there must be 19 integers from 2 to 20 inclusive. Since probability is defined as the number of desired items divided by the total number of choices, the probability that the number chosen is prime is 8/19.

8. **(E).** This problem describes a specific arrangement of people, so this is an ordering problem. The total number of ways to arrange n items in order is n! (the exclamation point means "factorial"). Therefore, since there are 5 students to be arranged in a line, the total number of possible orderings is 5! = (5)(4)(3)(2)(1) = 120.

Alternatively, ask, "How many choices do I have for each place in the line?" Consider the first place in line. Since no students have been chosen yet, there are 5 total options for the first place in line. Similarly, for the second place there are 4 choices, because one student has already been chosen to occupy the first spot. Applying the same logic, there are 3 choices for the third place, 2 for the fourth, and 1 for the fifth. Using the fundamental counting principle, there are a total of (5)(4)(3)(2)(1) = 120 different lines.

9. **10.** This problem tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. Let the number of sauces be represented by the variable S. The total number of possible pasta dishes can be represented by each separate choice multiplied together: (8)(4)(S), or 32S. The problem also indicates that the total number of pasta dishes must be equal to 320. Therefore, 32S = 320 and S = 10.

10. **720**. One possible approach is to ask, "How many choices do I have for each of the class positions?" Begin by considering the president of the class. Since no one has been chosen yet, there are 10 students from whom to choose. Then, for the vice president there are 9 options because now one student has already been chosen as president. Similarly, there are 8 choices for the secretary. Using the fundamental counting principle, the total number of possible selections is (10)(9)(8) = 720.

Alternatively, you could use factorials. In this case order matters because you are choosing people for specific positions. This problem is synonymous to asking, "How many different ways can you line up 3 students as first, second, and third from a class of 10?" The number of ways to arrange the entire class in line is 10!. However, the problem is only concerned with the first 3 students in line, so exclude rearrangements of the last 7. The way in which these "non-chosen" 7 students can be ordered is 7!. Thus the total number of arrangements for 3 students from a class of 10 is $\frac{10!}{7!} = (10)(9)(8) = 720$.

11. **(B).** This is a combinatorics problem — to calculate Quantity A, make four "slots" (since the numbers are all four-digit numbers), and determine how many possibilities there are for each slot:

____ ____ ____ ____

Since all 4 digits must be less than 5, the possible options are limited to 0, 1, 2, 3 and 4. However, a 4-digit number cannot begin with 0 (the smallest four-digit number is 1,000). So, the first slot has only 4 possibilities, not 5:

$\underline{\ 4\ }\quad\underline{\ 5\ }\quad\underline{\ 5\ }\quad\underline{\ 5\ }$

(4)(5)(5)(5) = 500, which is less than 625. The answer is (B).

Note that the value in Quantity B comes from multiplying $5 \times 5 \times 5 \times 5$. Neglecting to note that 4-digit numbers cannot begin with 0 results in incorrect choice (C).

12. **(E).** This problem tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. The key to this problem is realizing how many choices there are for each option. For the meat, there are obviously 3 choices. For each of the condiments there are exactly 2 choices: yes or no. The only real choice regarding each condiment is whether to include it at all. As there are 7 condiments, the total number of choices is $(3)(2)(2)(2)(2)(2)(2)(2) = (3)(2^7)$.

Note: the condiment options cannot be counted as 8! (0 through 7 = 8 options) because, in this case, the order in which the options are chosen does not matter; a burger with lettuce and pickles is the same as a burger with pickles and lettuce.

13. **(A).** For probability questions, always begin by separating out the probabilities of each individual event. Then, if you need all the events to happen (an "AND question"), multiply the probabilities together. If you only need one of the multiple events to happen (an "OR question"), add the probabilities together.

In this case, there are two events: rain on Monday and rain on Tuesday. The question asks for the probability that it will rain on Monday AND on Tuesday, so multiply the individual probabilities together:

$$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

14. **(D).** This problem relies on the fundamental counting principle, which says that the total number of ways for something to happen is the product of the number of options for each individual choice. The problem asks how many five-digit numbers can be created from the digits 5, 6, 7, 8, 9, and 0. For the first digit, there are only five options (5, 6, 7, 8, and 9) because a five-digit number must start with a non-zero integer. For the second digit, there are 5 choices again, because now zero can be used but one of the other numbers has already been used, and numbers cannot be repeated. For the third number, there are 4 choices, for the fourth there are 3 choices, and for the fifth number there are 2 choices. Thus, the total number of choices is $(5)(5)(4)(3)(2) = 600$.

Alternatively, you can use the same logic and realize there are 5 choices for the first digit. (Separate out the first step because you have to remove the zero from consideration.) The remaining five digits all have an equal chance of being chosen, so choose four out of the remaining five digits to complete the number. The number of ways in which this second step can be accomplished is $(5!)/(1!) = (5)(4)(3)(2)$. Thus, the total number of choices is again equal to $(5)(5)(4)(3)(2) = 600$.

15. **5/6.** In the bag of marbles, there are 3 red marbles and 7 white marbles, for a total of 10 marbles that are NOT blue. There are a total of $3 + 7 + 2 = 12$ marbles in the bag. Since probability is defined as the number of desired items divided by the total number of choices, the probability that the marble chosen is not blue is $10/12 = 5/6$.

16. **120.** This problem utilizes the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. Since the man must choose one suit, one shirt, one pair of socks, and one pair of shoes, the total number of outfits is the number of suits times the number of shirts times the number of socks times the number of shoes: $(3)(4)(2)(5) = 120$.

17. **(A).** This is a combinatorics problem. The license plates have 2 letters and 4 numbers, so make six "slots" and

determine how many possibilities there are for each slot. There are 26 letters in the alphabet and 10 digits to pick from, so:

$$\underline{26} \quad \underline{26} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10}$$

Multiply 26 × 26 on your calculator to get 676. Add four zeroes for the four 10’s to get 6,760,000. Quantity A is larger.

18. **(A).** First, calculate the total number of combinations, ignoring the one illegal “word.” Since there are 10 possibilities for the first slot and 20 possibilities for the other 4 slots:

$$\underline{10} \quad \underline{20} \quad \underline{20} \quad \underline{20} \quad \underline{20}$$

Multiply to get 1,600,000.

Now, consider all the license plates that contain the forbidden word. Say, for example, that the forbidden word is GURG. That means that the plates 0GURG, 1GURG, 2GURG, 3GURG, etc. are forbidden, for a total of 10 forbidden plates. You can also express this mathematically as:

$$\underline{10} \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{1}$$

Multiply to get 10 forbidden plates. Subtract: 1,600,000 - 10 = 1,599,990.

Alternatively, after finding the 1,600,000 figure, glance at the answer choices: the correct answer can’t be (C), (D), or (E). Answer choice (B) subtracts just 1 plate, but more than 1 plate could have the forbidden “word.” Therefore, (A) must be the correct answer.

19. $\mathbf{\frac{1}{121}}$ The probability of picking black chip #3 is $\frac{1}{12}$. Once Pavel has removed the first chip, only 11 chips remain, so the probability of picking white chip #3 is $\frac{1}{11}$. Multiply $\frac{1}{12} \times \frac{1}{11} = \frac{1}{121}$.

20. **(B).** In this problem, Tarik is NOT picking 1 chip out of all 12. Rather, he is picking 1 chip out of 6 green ones, and then picking another chip out of 6 blue ones. There are 3 green chips with numbers less than 4, so Tarik has a $\frac{3}{6}$ chance of selecting a green chip showing a number less than 4. Likewise, Tarik has a $\frac{3}{6}$ chance of selecting a blue chip showing a number less than 4. Therefore, Quantity A is equal to $\frac{3}{6} \times \frac{3}{6} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Quantity B is larger.

21. $\mathbf{\frac{1}{22}}$. The trap answer in this problem is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. This is NOT the answer to the question being asked—

rather, this is the answer to the question, "What is the probability of picking a red chip and then a blue chip that both have #3?" (or any other specific number). This is a more specific question than the one actually asked. In the question, asked, there are six possible ways to fulfill the requirements of the problem, not one, because the problem does not specify whether the number should be 1, 2, 3, 4, 5, or 6.

Thus, ANY of the 6 red chips is acceptable for the first pick. However, on the second pick, only the blue chip with the same number as the red one that was just picked is acceptable (you need whatever chip is the "match" for the first one picked). Thus:

$$\frac{6}{12} \times \frac{1}{11} = \frac{1}{2} \times \frac{1}{11} = \frac{1}{22}$$

22. **(B).** Use the overlapping sets formula for two groups: Total = Group 1 + Group 2 - Both + Neither. (Adding the two groups—in this case Latin and Spanish—double-counts the students who take both classes, so the formula subtracts the "both" students.)

$150 = 75 + 110 - B + 11$
$150 = 196 - B$
$46 = B$

Careful! This is not the value of Quantity A. Since 46 students take both Latin and Spanish, subtract 46 from the total who take Latin to find those who take only Latin:

$75 - 46 = 29$

Thus, Quantity A is equal to 29 and Quantity B is larger.

23. **(A).** This problem relies on the fundamental counting principle, which says that the total number of ways for something to happen is the product of the number of options for each individual choice. For any digit of the 10-digit number there are exactly two options, a 2 or a 5. Thus, since there are two choices for each digit and it is a 10-digit number, there are $(2)(2)(2)(2)(2)(2)(2)(2)(2)(2) = 2^{10}$ total choices.

24. **(C).** The probability of any event equals the number of ways to get the desired outcome divided by the total number of ways for the event to happen. Starting with the denominator, use the fundamental counting principle to compute the total number of ways to roll a cube twice. There are 6 possibilities (1, 2, 3, 4, 5, or 6) for the first roll and 6 for the second, giving a total of (6)(6) = 36 possibilities for the two rolls. For the numerator, determine the number of possible combinations that will add to 8. For example, you might roll a 2 the first time and a 6 the second time. The full set of options is (2,6), (3,5), (4,4), (5,3), and (6,2). Thus there are 5 possible combinations that sum to 8, yielding a probability of 5/36.

25. **16.** This problem utilizes the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. For the first flip there are 2 options: heads or tails. Similarly, for the second flip there 1 2 options, for the third there are 2 options, for the fourth there are 2 options, and for the fifth there is only one option because the problem restricts this final flip to heads. Therefore, the total number of outcomes is (2)(2)(2)(2)(1) = 16. A good rephrasing of this question is, "How many different outcomes are there if the coin is flipped 4 times?" The fifth flip, having been restricted to heads, is irrelevant. Therefore, the total number of ways to flip the coin five times with heads for the fifth flip is equal to the total number of ways to flip the coin four times; either way, the answer is 16.

26. **(C).** The problem specifies that no one takes all three languages. In addition, a total of 5 people take 2 languages. Thus, 5 people have been double-counted. Because you know the total number of people who have been double-counted (5) and triple-counted (0), you can use the standard overlapping sets formula:

Total = Spanish + French + Latin - (Two of the Three) - 2(All Three)

25 = 9 + French + 7 - 5 - 2(0)
25 = 11 + French
14 = French

The two quantities are equal.

27. $\frac{1}{3}$ Probability equals the number of desired outcomes divided by the total number of possible outcomes. Among the integers 1 through 24 there are 4 factor pairs of 24: (1, 24), (2, 12), (3, 8), and (4, 6), for a total of 8 factors. The total number of possible outcomes when rolling the die once is 24. The probability that the number chosen is a factor of 24 is 8/24 = 1/3.

28. **17.** Use the overlapping sets formula for two groups: Total = Group 1 + Group 2 - Both + Neither. Here, the groups are "stuffed animal" and "given by the baby's grandmother." The problem indicates that the "both" category is equal to 5, and that the "neither" number is 6. The total is x.

Total = Group 1 + Group 2 - Both + Neither

$x = 9 + 7 - 5 + 6$
$x = 17$

29. **800.** This is a combinatorics problem. Make four "slots" (since the numbers are all four-digit numbers), and determine how many possibilities there are for each slot:

___ ___ ___ ___

Since the number must begin with 2 or 3, there are 2 possibilities for the first slot. Because the ones digit must be prime and there are only 4 prime 1-digit numbers (2, 3, 5, and 7), there are 4 possibilities for the last slot.

$\underline{\ 2\ }\quad \underline{\quad}\quad \underline{\quad}\quad \underline{\ 4\ }$

The other slots have no restrictions, so put 10, since there are 10 digits from 0–9:

$\underline{\ 2\ }\quad \underline{\ 10\ }\quad \underline{\ 10\ }\quad \underline{\ 4\ }$

Multiply to get 800.

Alternatively, figure out the pattern and add up the number of qualifying 4-digit integers. In the first ten numbers, 2000–2009, there are exactly 4 numbers that have a prime units digit: 2002, 2003, 2005, and 2007. The pattern then repeats in the next group of ten numbers, 2010–2020, and so on. In any group of ten numbers, then, four qualify. In the first one hundred numbers, 2000–2099, there are ten groups of ten, or $10 \times 4 = 40$ numbers that have a prime units digit. In the first one thousand numbers, 2000–2999, there are ten groups of one hundred, or $100 \times 4 = 400$ numbers that have a prime units digit. There are a total of two groups of one thousand numbers (2000–2999 and 3000–3999), so there are a total of $400 \times 2 = 800$ numbers that have a prime units digit.

30. **1,000.** This is a combinatorics problem. Make four "slots" (since the numbers are all four-digit numbers), and determine how many possibilities there are for each slot:

$$\underline{\quad\quad}\quad\underline{\quad\quad}\quad\underline{\quad\quad}\quad\underline{\quad\quad}$$

Since the number must begin with 2, 3, 4, 5, or 6, there are 5 possibilities for the first slot. Because the digit in the tens place must be prime and there are only 4 prime 1-digit numbers (2, 3, 5, and 7), there are 4 options for the tens place.

$$\underline{\quad 5\quad}\quad\underline{\quad\quad}\quad\underline{\quad 4\quad}\quad\underline{\quad\quad}$$

Since the number must be even, the final digit must be 0, 2, 4, 6, or 8—thus, there are 5 possibilities for the units digit. (Note that saying that a 4-digit number is even is simply saying that it ends in an even digit—for instance, 3,792 is even.) No restrictions were given for the hundreds slot, so there are 10 options, since there are 10 digits from 0–9.

$$\underline{\quad 5\quad}\quad\underline{\quad 10\quad}\quad\underline{\quad 4\quad}\quad\underline{\quad 5\quad}$$

Multiply to get 1,000.

Alternatively, figure out the pattern and add up the number of qualifying 4-digit integers. In the first one hundred numbers, 2,000–2,099, there are exactly four sets of ten numbers that have a prime tens digit: 202-, 203-, 205-, and 207-. Within each of those four sets, there are five possibilities for numbers that also end in an even digit. For example, the possibilities for the 202- set would be 2,020, 2,022, 2,024, 2,026, and 2,028. Out of the first one hundred numbers, then, there are $5 \times 4 = 20$ possibilities that have both a prime tens digit and end in an even digit.

From 2,000 to 6,999, there are 50 groups of 100 numbers ($6{,}999 - 2{,}000 + 1 = 5{,}000$, and $5{,}000 \div 100 = 50$). Therefore, there are $20 \times 50 = 1{,}000$ numbers.

31. **(C).** Multiple approaches are possible here. One way is to imagine the scenario and count up the number of handshakes: you are the first person. How many hands do you need to shake? There are 11 other people in the room, so you need to shake hands 11 times. Now, move to the second person: how many hands must he shake? He has already shaken your hand, leaving him 10 others with whom to shake hands. The third person will need to shake hands with 9 others, and so on. Therefore, there are a total of $11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$ handshakes. The fastest way to find the sum of a group of consecutive numbers is to take the average of the first and last terms and multiply it by the number of terms. The average is $(11 + 1)/2 = 6$ and there are $11 - 1 + 1 = 11$ terms (find the difference between the terms and "add one before you're done"). The sum is $6 \times 11 = 66$.

Alternatively, rephrase the question as "How many different ways can any 2 people be chosen from a group of 12?" (This works because the problem ultimately asks you to "choose" each distinct pair of 2 people one time.) The key here is to realize that handshakes are independent of order, i.e., it doesn't matter if A shakes hands with B or if B shakes hands with A; it's the same outcome. Thus, you only care about how many pairs you can make. Any time you recognize a group of order-independent items being selected from a larger set, you can apply the formula $\frac{\text{total!}}{\text{in!out!}}$

to arrive at the total number of combinations. Thus: $\frac{12!}{2!10!} = \frac{12 \cdot 11}{2} = 66$.

32. **(B).** The probability of any outcome is equal to the number of desired outcomes divided by the total number of outcomes. There are 12 girls and 20 boys in the classroom. If one-quarter of the girls have blue eyes, then there are (12)(1/4) = 3 girls with blue eyes. Therefore, there are 12 - 3 = 9 girls who do NOT have blue eyes. The total number of ways in which you could choose a child is simply the total number of children in the class, namely 12 + 20 = 32. Therefore, the probability of choosing a girl who does not have blue eyes equals the number of girls without blue eyes divided by the total number of children, which is 9/32.

33. **(C).** There are only 2 possible outcomes for each flip and only 3 flips total. The most straightforward approach is to list all of the possible outcomes: {HHH, HHT, HTH, HTT, TTT, TTH THH, THT}. Of these 8 possibilities, 3 of the outcomes have one head and two tails, so the probability of this event is 3/8.

Alternatively, you can count the total number of ways of getting 1 head without listing all the possibilities. If the coin is flipped 3 times and you want only 1 head, then there are 3 possible positions for the single head: on the first flip alone, on the second flip alone, or on the third flip alone. Since there are 2 possible outcomes for each flip, heads or tails, there are (2)(2)(2) = 8 total outcomes. Again, the probability is 3/8.

Finally, you can compute the probability directly. The probability of flipping heads is 1/2 and the probability of flipping tails is also 1/2. The probability of getting heads in the first position alone, or HTT, is (1/2)(1/2)(1/2) = 1/8, where you multiply because you have heads AND tails AND tails. This represents the probability of heads in position 1, but heads could also be in position 2 alone or in position 3 alone. Since there are 3 possible positions for the heads, multiply by 3 to get the total probability (3)(1/8) = 3/8.

34. **(D).** Because this problem is asking for an "at least" solution, you can use the 1 - x shortcut. The probability that at least one roll results in a number higher than 4 is equal to 1 minus the probability that both of the rolls result in numbers 4 or lower. For one roll, there are 6 possible outcomes (1 through 6) and 4 ways in which the outcome can be 4 or lower, so the probability is 4/6 = 2/3. Thus, the probability that both rolls results in numbers 4 or lower is (2/3) (2/3) = 4/9. This is the result that you do NOT want; subtract this from 1 to get the probability that you do want. The probability that at least one of the rolls results in a number higher than 4 is 1 - (4/9) = 5/9.

Alternatively, write out the possibilities. The total number of possibilities for two rolls is (6)(6) = 36. Here are the ways in which at least one number higher than 4 can be rolled:

51, 52, 53, 54, 55, 56
61, 62, 63, 64, 65, 66
15, 25, 35, 45 (note: 55 and 65 have already been counted above)
16, 26, 36, 46 (note: 56 and 66 have already been counted above)

There are 20 elements (be careful not to double-count any options). The probability of at least one roll resulting in a number higher than 4 is 20/36 = 5/9.

35. **(B).** You need to use both probability and number properties concepts in order to answer this question. First, in order for two integers to produce an odd integer, the two starting integers must be odd. An odd times an odd equals an odd. An even times an odd, by contrast, produces an even, as does an even times an even.

Within the set of tiles, there are 50 even numbers (2, 4, 6, …, 100) and 50 odd numbers (1, 3, 5, …, 99). One randomly-chosen tile will have a 50/100 = 1/2 probability of being even, and a 1/2 probability of being odd. The probability of choosing an odd tile first is (1/2) and the probability of choosing an odd tile second is also (1/2), so the probability of "first odd AND second odd" is (1/2)(1/2) = 1/4.

Alternatively, you can recognize that there are only four options for odd/even pairs if two tiles are chosen: OO, OE, EO, EE. The only one of these combinations that yields an odd product is OO. Since all of these combinations are equally likely, and since OO is exactly one out of the four possibilities, the probability of choosing OO is 1/4.

36. **34,650.** This is a combinatorics problem, and the WOW example is intended to make it clear to you that any W is considered identical to any other W — switching one W with another would NOT result in a different combination, just as switching one S with another in MISSISSIPPI would not result in a different combination.

Therefore, solve this problem using the classic combinatorics formula for accounting for subgroups among which order does not matter:

<u>Total Number of Items!</u>
First Group! Second Group! Etc…

Because MISSISSIPPI has 11 letters, including one M, four S's, four I's, and two P's:

$$\frac{11!}{1!4!4!2!}$$

Now expand the factorials and cancel; use the calculator for the last step of the calculation:

$$\frac{11\times10\times9\times8\times7\times6\times5\times\cancel{4!}}{\cancel{1!4!}(4\times3\times2\times1)(2\times1)}=\frac{11\times10\times9\times\cancel{8}\times7\times\cancel{6}\times5}{(\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1})(\cancel{2}\times\cancel{1})}=11\times10\times9\times7\times5=34{,}650$$

37. **0.** Because *a, b,* and *c* are prime numbers greater than 2, they must all be odd. So, $ab + c$ = (ODD)(ODD) + ODD. ODD times ODD yields another odd number, so the calculation simplifies to ODD + ODD, which will always yield an even answer.

Thus, it is impossible for $ab + c$ to be odd. The probability is zero. (Note: if you were ever asked to type a nonzero probability into a box, you would need to express it as a decimal between 0 and 1.)

38. **(C).** Since Quantity A is an "at least" problem, you can use the 1 - x shortcut. Rather than calculate the probability of rain on exactly 1 day next week, and then the probability of rain on exactly 2 days next week, and so on (after which you would still have to add all of the probabilities together!), instead calculate the probability of no rain at all on any day, and then subtract that number from 1. That will give the combined probabilities for any scenarios that include rain on at least 1 day.

Probability of NO rain for any of the 7 days = $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{128}$

Subtract this probability from 1:

$$1-\frac{1}{128}=\frac{128}{128}-\frac{1}{128}=\frac{127}{128}$$

Quantities A and B are equal.

39. **5/12.** First think about the prime numbers less than 12, the maximum sum of the numbers on the dice. These primes are 2, 3, 5, 7, 11.

The probability of rolling 2, 3, 5, 7, or 11 = the number of ways to roll any of these sums, divided by the total number of possible rolls. The total number of possible dice rolls is 6 × 6 = 36.

Sum of 2 can happen 1 way: 1 + 1
Sum of 3 can happen 2 ways: 1 + 2 or 2 + 1
Sum of 5 can happen 4 ways: 1 + 4, 2 + 3, 3 + 2, 4 + 1
Sum of 7 can happen 6 ways: 1 + 6, 2 + 5, 3 + 4, 4 + 3, 5 + 2, 6 + 1
Sum of 11 can happen 2 ways: 5 + 6, 6 + 5

That's a total of 1 + 2 + 4 + 6 + 2 = 15 ways to roll a prime sum.

Thus, the probability is 15/36 = 5/12.

40. **(B).** Jack will only continue to roll the cube if the sum of the individual rolls is odd. For the first roll, this will only occur if the number itself is odd; if Jack does not stop after the first roll, then, he must have rolled an odd number for the first roll. For the second roll, in order for the sum of the first and second to be odd, Jack must now roll an even (because odd + even = odd). You can rephrase the question: "What is the probability that Jack will roll an odd first and an even second?" The probability of event A AND event B equals the probability of A times the probability of B. Since the probability of odd = 1/2 and the probability of even = 1/2, the probability of the first number being odd AND the second number being even is (1/2)(1/2) = 1/4.

41. **(B).** The probability of any event equals the number of ways to get the desired outcome divided by the total number of outcomes.

Start with the denominator, which is the total number of ways that the principal can choose two children from the classroom. Use the fundamental counting principle. There are 6 possible options for the first choice and 5 for the second, giving (6)(5) = 30 possibilities. However, this double-counts some cases; for example, choosing Jan and then Robert is the same as choosing Robert and then Jan. Divide the total number of pairs by 2: $\frac{6\cdot 5}{2}=15$. Alternatively, use the formula for a set where the order doesn't matter: $\frac{\text{total!}}{\text{in! out!}}$. In this case: $\frac{6!}{2!4!}=\frac{6\times 5\times 4!}{(2)(4!)}=\frac{6\times 5}{2}=15$.

Now compute the numerator, which is the number of pairs that include Jan. Since the pair only includes two children and one is already decided (Jan), there are exactly 5 options for the other child. Thus, there are 5 total pairs that include Jan: Jan with each of the other students.

The probability of choosing a pair with Jan is 5/15 = 1/3.

As a final alternative, you may simply list all the pairs of students and count how many of them include Jan. Label the students in the class as J, 1, 2, 3, 4, and 5, where J is Jan. Then all the pairs can be listed as (J1), (J2), (J3), (J4), (J5), (12), (13), (14), (15), (23), (24), (25), (34), (35), and (45). (Be careful not to include repeats.) There are 15 total elements in this list and 5 that include Jan, yielding a probability of 5/15 = 1/3.

42. **(C).** The probability that Gary will eat eggs is 3/7. The probability that he will eat cereal is 4/7. In order to calculate the probability of eating one OR the other, add: 3/7 + 4/7 = 7/7 = 1. Quantities A and B are the same.

43. **(B).** The problem indicates that the events occur independently of each other. Therefore, in calculating Quantity A, the first step is to calculate the "or" situation, but you cannot stop there. When you add 0.5 + 0.3 = 0.8, you double-count the occurrences when both events occur. Next, you have to subtract out the probability of both events occurring in order to get rid of the "double counted" occurrences.

Notice that this is a Quantitative Comparison. At this point, you could conclude that, because the 0.8 figure includes at least one "both" occurrence, the real figure for Quantity A must be smaller than 0.8. Therefore, Quantity B must be larger.

To do the actual math, find the probability of both events occurring (breakfast AND sweater): (0.5)(0.3) = 0.15. Subtract the "AND" occurrences from the total "or" probability: 0.8 - 0.15 = 0.65

Quantity B is larger.

44. **(B).** The problem indicates that the events occur independently of each other. Therefore, in calculating Quantity A, do not simply add both events, even though it is an "or" situation. Adding 0.3 + 0.2 = 0.5 is incorrect because the probability that both events occur is counted twice. (Only add probabilities in an "or" situation when the probabilities are mutually exclusive.)

While Quantity A's value should include the probability that both events occur, make sure to count this probability only once, not twice. Since the probability that both events occur is 0.3(0.2) = 0.06, you must subtract this value from the "or" probability.

Quantity A: Add the two probabilities (rain OR pop quiz) and subtract BOTH scenarios (rain AND pop quiz):

$$0.3 + 0.2 - (0.3)(0.2) = 0.44$$

Quantity B: Multiply the probability that rain does NOT occur (0.7) and the probability that the pop quiz does NOT occur (0.8).

$$0.7(0.8) = 0.56$$

You could also note that, collectively, Quantities A and B include every possibility and are mutually exclusive of one another (Quantity A includes "rain and no quiz," "quiz and no rain," and "both rain and quiz," and Quantity B includes "no rain and no quiz"). Therefore, the values of Quantities A and B must add to 1. Calculating the value of either Quantity A or Quantity B would automatically tell you the value for the other quantity.

If you do this, calculate Quantity B first (because it's the easier of the two quantities to calculate) and then subtract

Quantity B from 1 in order to get Quantity A's value. That is, 1 - 0.56 = 0.44.

45. **(D).** The probabilities of both events are the same *as each other,* but that doesn't indicate anything about the value of those probabilities. For instance, if both probabilities are equal to 0.99, then the probability of both events occurring (Quantity A) is MUCH higher than the probability of neither occurring (Quantity B).

But what if the probability of each event is more like 0.000001 ("1 in a million")? Then the chance of neither occurring (Quantity B) would be much higher than the chance of both occurring (Quantity A).

In the case that the probabilities are each equal to 0.5, then — and only then — would the two quantities be equal.

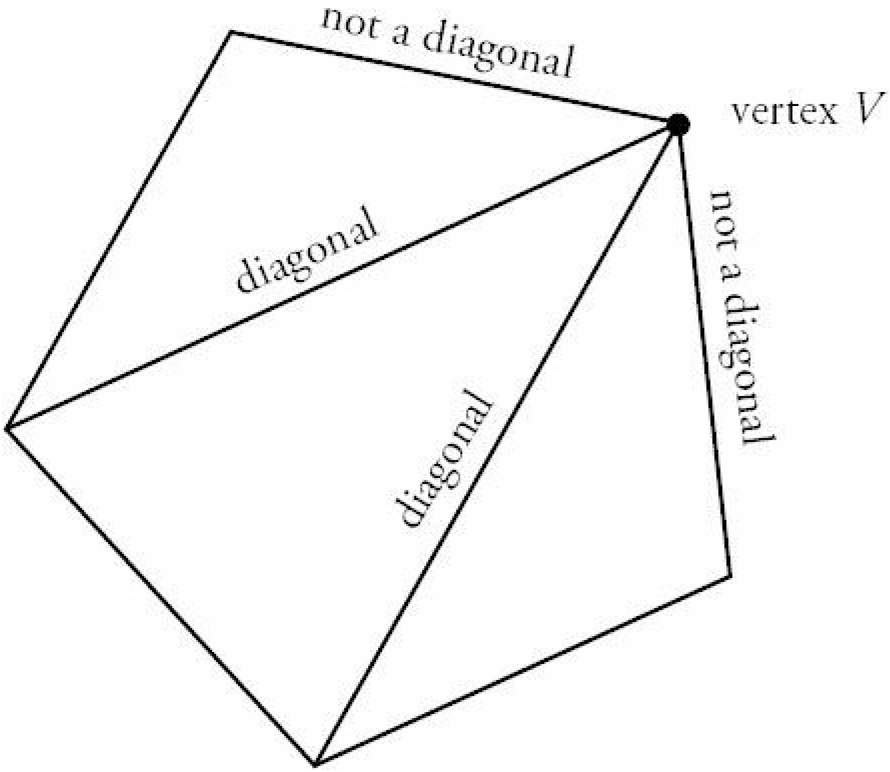
46. **(D).** In essence, the question is asking "What is the probability that one or more days are rainy days?" since any single rainy day would mean the city experiences rain. In this case, employ the 1 - x shortcut, where the probability of rain on one or more days is equal to 1 minus the probability of no rain on any day. Since the probability of rain is 1/3 on any given day, the probability of no rain on any given day is 1 - 1/3 = 2/3. Therefore, the probability of no rain on three consecutive days is (2/3)(2/3)(2/3) = 8/27. Finally, subtract from 1 to find the probability that it rains on one or more days: P(1 or more days) = 1 - P(no rain) = 1 - 8/27 = 19/27.

47. **(C).** The number of ways in which the students can be arranged with Beth and Dan separated is equal to the total number of ways in which the students can be arranged minus the number of ways they can be arranged with Beth and Dan together. The total number of ways to arrange 5 students in a line is 5! = 120. To compute the number of ways to arrange the 5 students such that Beth and Dan are together, group Beth and Dan as "one" person, since they must be lined up together. Then the problem becomes one of lining up 4 students, which gives 4! possibilities. However, remember that there are actually two options for the Beth and Dan arrangement: Beth first and then Dan or Dan first and then Beth. Therefore, there are (4!)(2) = (4)(3)(2)(1)(2) = 48 total ways in which the students can be lined up with Dan and Beth together. Finally, there are 120 - 48 = 72 arrangements where Beth will be separated from Dan.

Alternatively, compute the number of ways to arrange the students directly by considering individual cases. In this case, investigate how many ways there are to arrange the students if Beth occupies each spot in line and sum them to find the total. If Beth is standing in the first spot in line, then there are 3 options for the second spot (since Dan cannot occupy this position), 3 options for the next spot, 2 options for the next spot, and finally 1 option for the last spot. This yields (3)(3)(2)(1) = 18 total possibilities if Beth is first. If Beth is second, then there are 3 options for the first person (Dan cannot be this person), 2 options for the third person (Dan cannot be this person either), 2 options for the fourth person, and 1 for the fifth. This yields (3)(2)(2)(1) = 12 possibilities. In fact, if Beth is third or fourth in line, you arrive at the same situation as when Beth is second. Thus there will be 12 possible arrangements whether Beth is 2nd, 3rd, or 4th in line, yielding 36 total arrangements for these 3 cases. Using similar logic, the situation in which Beth is last in line is exactly equal to the situation where she is first in line. Thus, there are (18)(2) = 36 possibilities where Beth is first or last. In total, this yields 36 + 36 = 72 possible outcomes when considering all of the possible placements for Beth.

48. **(A).** A diagonal of a polygon is an internal line segment connecting any two unique vertices; this line segment does not lie along an edge of the given shape. Consider a polygon with 12 vertices. Construct a diagonal by choosing any two vertices and connecting them with a line. Remember that this is order independent; the line is the same regardless of which is the starting vertex. Therefore, this is analogous to choosing any 2 elements from a set of 12, and can be written as $\frac{12!}{10!2!}=\frac{12\times11\times10!}{10!(2)(1)}=\frac{12\times11}{2}=6\times11=66$. However, this method includes the vertices connected to their adjacent vertices, which form edges instead of diagonals. In order to account for this, subtract the number of edges on the polygon from the above number: 66 - 12 = 54.

Alternatively, if you choose a random vertex of the 12-sided shape, then there are 12 - 1 = 11 lines that can be drawn to other vertices since no line can be drawn from the vertex to itself. However, the lines from this vertex to the two adjacent vertices will lie along the edges of the polygon and therefore cannot be included as diagonals (see the figure of a pentagon below for an example).



Thus, there are 12 - 1 - 2 = 9 diagonals for any given vertex. Since there are 12 vertices, you might think that the total number of diagonals is equal to (12)(9) = 108. However, using this scheme you have counted each diagonal twice, using each side of the diagonal once as the starting point. Therefore, there are half this many different diagonals: 108/2 = 54.

49. **(B).** Consider each independent choice and then use the fundamental counting principle to calculate the total number of possibilities. Start by choosing the president, for which there are 12 total options. Next, for the vice president, there are 11 options. Finally, choose the committee of 3 from the remaining 10 students using the formula for a set where the order doesn't matter: $\frac{\text{total!}}{\text{in! out!}}$. The number of possible arrangements for the 3 committee members is $\frac{10!}{7!3!}$. Finally, using the fundamental counting principle, the total number of ways to choose a president, vice president, and committee of 3 is $\frac{12 \times 11 \times 10!}{7!3!}$ which can be rewritten as $\frac{12!}{7!3!}$.

50. **(A).** This is a classic combinatorics problem in which *order doesn't matter*—that is, the pairing "blue, green" is the same as "green, blue." A color is either "in" or "out." Use the standard "order doesn't matter" formula:

$$\frac{\text{Everything!}}{\text{Picked! NotPicked!}}$$

For Quantity A:

$$\frac{5!}{2!3!}=\frac{5\times4\times3!}{(2)(1)3!}=\frac{5\times4}{2}=10$$

For Quantity B:

$$\frac{9!}{8!1!}=\frac{9\times8!}{8!(1)}=9$$

Note that while the formula is necessary for Quantity A, you could reason your way to the value for Quantity B: every combination that selects 8 out of 9 colors will leave out exactly 1 color. Since there are 9 colors, there are 9 combinations.

51. **(C).** This is a classic combinatorics problem in which *order doesn't matter* — that is, picking Joe and Jane is the same as picking Jane and Joe. A person is either on the team or not. Use the standard "order doesn't matter" formula:

$$\frac{\text{Everything!}}{\text{Picked! NotPicked!}}$$

For Quantity A:

$$\frac{6!}{2!4!}=\frac{6\times5\times4!}{(2)(1)4!}=\frac{6\times5}{2}=15$$

For Quantity B:

$$\frac{6!}{4!2!}=\frac{6\times5\times4!}{4!(2)(1)}=\frac{6\times5}{2}=15$$

The quantities are equal. Note the first line of each Quantity: from that stage, you can already tell that the values will be the same.

This will always work—when order doesn't matter, the number of ways to pick 4 and leave out 2 is the same as the number of ways to pick 2 and leave out 4. Either way, it's one group of 4 and one group of 2. What actually happens to those groups (getting picked, not getting picked, getting a prize, losing a contest, etc.) is irrelevant to the ultimate solution.

52. **(C).** In this problem, order matters; if Jane comes in 1st place and Rohit comes in 2nd, there is a different outcome than when Rohit places 1st and Jane places 2nd. Use the fundamental counting principle to solve. To determine Quantity A make three slots (one for each prize). Six people are available to win 1st, and then five people could win 2nd, and four people could win 3rd:

$$\underline{\ 6\ }\quad \underline{\ 5\ }\quad \underline{\ 4\ }$$

Multiply: (6)(5)(4) = 120.

For Quantity B, make 5 slots, one for each prize. Five people can win 1st prize, then 4 people for 2nd prize, and so on:

$$\underline{\ 5\ }\quad \underline{\ 4\ }\quad \underline{\ 3\ }\quad \underline{\ 2\ }\quad \underline{\ 1\ }$$

Multiply (5)(4)(3)(2)(1) = 120. Quantities A and B are equal.

53. **(C).** This is a classic combinatorics problem in which *order doesn't matter* — in fact, the problem tells you that explicitly. Use the standard "order doesn't matter" formula:

$$\frac{\text{Everything!}}{\text{Picked! NotPicked!}}$$

For Quantity A:

$$\frac{100!}{56!44!}$$

Because the numbers are so large, there must be a way to solve the problem without actually simplifying (even with a calculator, this is unreasonable under GRE time limits). Try Quantity B and compare:

$$\frac{100!}{44!56!}$$

The quantities are equal. Note that this will always work — when order doesn't matter, the number of ways to pick 56 and leave out 44 is the same as the number of ways to pick 44 and leave out 56. Either way, it's one group of 56 and one group of 44. What actually happens to those groups (being part of a collection, being left out of the collection, etc.) is irrelevant to the ultimate solution.

54. **(A).** Use the overlapping sets formula for two groups: Total = Group 1 + Group 2 - Both + Neither. But first, add 740 ("business use ONLY") + 520 ("business use" and "personal use") to get 1260, the total number of products categorized as "business use."

Also note that the problem indicates that *all* of the products fall into one or both of the two categories, so "neither" in this formula is equal to zero.

Total = Business + Personal - Both + Neither
1,345 = 1,260 + P - 520 + 0
1,345 = 740 + P
605 = P

Quantity A is larger. Note that the question asked for the number of products characterized as "personal use" (which includes products in the "both" group). If the problem had asked for the number of products characterized as "personal use" ONLY, you would have had to subtract the "both" group to get 605 - 520 = 85. In this problem, however, Quantity A equals 605.

55. **(E).** Because all of the letters of the name "Christyna" are unique, there are 9 distinct choices of letters to form the 4-letter "word." In addition, order does matter: the "word" CHRI is different from the "word" CRHI. Use the fundamental counting principle to solve.

Begin by considering the first choice, for which there are 9 total options. Similarly, for the second choice there are 8 options, because one letter has already been chosen. Employing the same logic, there are 7 choices for the third letter and 6 choices for the fourth letter. Using the fundamental counting principle, (9)(8)(7)(6) = 3024 words.

Alternatively, try factorials. The total number of ways to arrange all 9 letters is 9! However, the problem is only concerned with "words" using four of these letters, meaning you must exclude rearrangements of the other 5. The number of ways in which you can order the 5 "non-chosen" letters is 5!. Thus the total number of "words" with 4 letters that can be made from the name "Christyna" is $\frac{9!}{5!}$ = (9)(8)(7)(6) = 3,024 "words."

56. **(E).** Use the 1 - *x* shortcut for this problem. How do you know to use this shortcut? In this case, you're being asked to solve for the probability that the product of the values from the two rolls will be less than 36. This will be time consuming to calculate directly because there are many different combinations that would produce a product less than 36 — (1)(6), (4)(5), (6)(3), and so on — the vast majority of combinations, actually! In fact, there's only one case when the product of the two rolls will *not* be less than 36: (6)(6). It is much easier to solve for this one value and then apply the 1 - *x* shortcut. In other words:

(The probability of rolling less than 36) + (The probability of rolling 36) = 1 $\longrightarrow$
(The probability of rolling less than 36) = 1 - (The probability of rolling 36)

This is an "AND question," because you need to get a 6 on the first roll *and* on the second roll. Multiply the two probabilities together:

$$\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$$

Subtract from 1:

$$1-\frac{1}{36}=\frac{35}{36}$$

57. **(C).** For probability questions, begin by separating out the probabilities of each individual event. Then, if all of the events must happen (an "AND question"), multiply the probabilities together. If only one of the multiple events needs to happen (an "OR question"), add. This question is an OR question, because it asks for the probability that David will eat a healthy breakfast *or* that it will rain.

At first glance, this may seem strange, because if you add the two probabilities together, you'll get something bigger than 100%, which is NEVER possible: $0.8 + 0.25 = 1.05$. This figure double-counts the cases where David eats a healthy breakfast AND it rains. Subtract out these cases in order to find the desired value.

In order to calculate the probability that David will eat a healthy breakfast AND that it will rain, multiply the individual probabilities together:

$$0.8 \times 0.25 = 0.2$$

Finally, subtract to find the probability that David will eat a healthy breakfast OR that it will rain:

$$1.05 - 0.2 = 0.85, \text{ or } 85\%$$

58. **(B).** Because this question uses "at least" language, use the $1 - x$ shortcut. In this case, the only outcome you do *not* want is rain on zero days next week. It will be much faster to solve for that probability and subtract from 1 in order to find the probability of all of the other outcomes (that there will be rain on one or more days next week).

(The probability of rain on at least one day) + (The probability of no rain) = 1

How do you find the probability of no rain? You want no rain on Monday AND Tuesday AND Wednesday AND Thursday AND Friday. Multiply together the individual probabilities of no rain on each of the five days.

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32}$$

Subtract this from 1 to get the desired answer:

$$1 - \frac{1}{32} = \frac{31}{32}$$

59. **(E).** Because this is an "at least" question, use the $1 - x$ shortcut.

(The probability of picking at least one man) + (The probability of picking no men) = 1

The probability of picking no men is an AND setup: woman AND woman AND woman.

For the first choice, there are 8 women out of 10 people: $8/10 = 4/5$
For the second choice, there are $7/9$ (because one woman has already been chosen)
For the third choice, there are $6/8 = 3/4$

Multiply the three probabilities together to find the probability that the committee will be comprised of woman AND woman AND woman:

$$\frac{4}{5} \times \frac{7}{9} \times \frac{3}{4} = \frac{1}{5} \times \frac{7}{3} \times \frac{1}{1} = \frac{7}{15}$$

To determine the probability of picking at least one man, subtract this result from 1:

> 1 - 7/15 = 8/15

60. **(A).** This overlapping sets question can be solved with the following equation:

> Total # of People = Group 1 + Group 2 + Group 3 - (# of people in 2 groups) - (2)(# of people in all 3 groups) + (# of people in no groups)

The problem indicates that everyone takes at least one language, so the number of people in no groups is zero. The problem also indicates that nobody takes all three languages, so that value is also zero.

> Total # of Students = 100 + 80 + 40 - 22 - (2)(0) + 0 = 198.

61. **(A).** It is not possible to solve for a single value for Quantity A, but it is possible to tell that Quantity A is greater than 17. Since 20 birds are songbirds and 23 are migratory, the total of these groups is 43, which is less than 60. It is possible for the overlap (the number of migratory songbirds) to be as little as 0, which would result in 20 songbirds, 23 non-songbird migratory birds, and 60 - 20 - 23 = 17 birds that are neither songbirds nor migratory.

It is also possible that there could be as many as 20 birds that overlap the two categories. (Find this figure by taking the number of birds in the smaller group; in this case, there are 20 songbirds). In the case that there are 20 migratory songbirds, there would also be 3 migratory birds that are not songbirds, in which case there would be 60 - 20 - 3 = 37 birds that are neither migratory nor songbirds.

Thus, the number of birds that are neither migratory nor songbirds is at least 17 and at most 37. No matter where in the range that number may be, it is greater than Quantity B, which is only 16.

Chapter 24
of

Data Interpretation

In This Chapter…

Data Interpretation

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as xy-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

Problem Set A

9th Grade Students at Millbrook High School

	Boys	Girls
Enrolled in Spanish	12	13
Not Enrolled in Spanish	19	16

1. Approximately what percent of the 9th grade girls at Millbrook High School are enrolled in Spanish?

(A) 21%
(B) 37%
(C) 45%
(D) 50%
(E) 57%

2. What fraction of the students in 9th grade at Millbrook High School are boys who are enrolled in Spanish?

(A) 1/5
(B) 19/60
(C) 5/12
(D) 12/31
(E) 12/25

3. What is the ratio of 9th grade girls not enrolled in Spanish to all 9th grade students at Millbrook Middle School?

(A) 1 : 16
(B) 13 : 60
(C) 4 : 15
(D) 19 : 60
(E) 16 : 29

4. If x percent more 9th grade students at Millbrook High School are not enrolled in Spanish than are enrolled in Spanish, what is x?

(A) 20
(B) 25
(C) 30
(D) 40
(E) 50

5. If 2 of the 9th grade boys at Millbrook High school who are not enrolled in Spanish decided to enroll in Spanish, and then 8 new girls and 7 new boys enrolled in the 9th grade at Millbrook Middle School and also in Spanish, what percent of 9th grade students at Millbrook would then be taking Spanish?

(A) 52%
(B) 53%
(C) 54%
(D) 55%
(E) 56%

Problem Set B

Number of Hours Worked Per Week per Employee at Marshville Toy Company

# of employees	Hours worked per week
4	15
9	25
15	35
27	40
5	50

6. What is the median number of hours worked per week per employee at Marshville Toy Company?

(A) 25
(B) 30
(C) 35
(D) 37.5
(E) 40

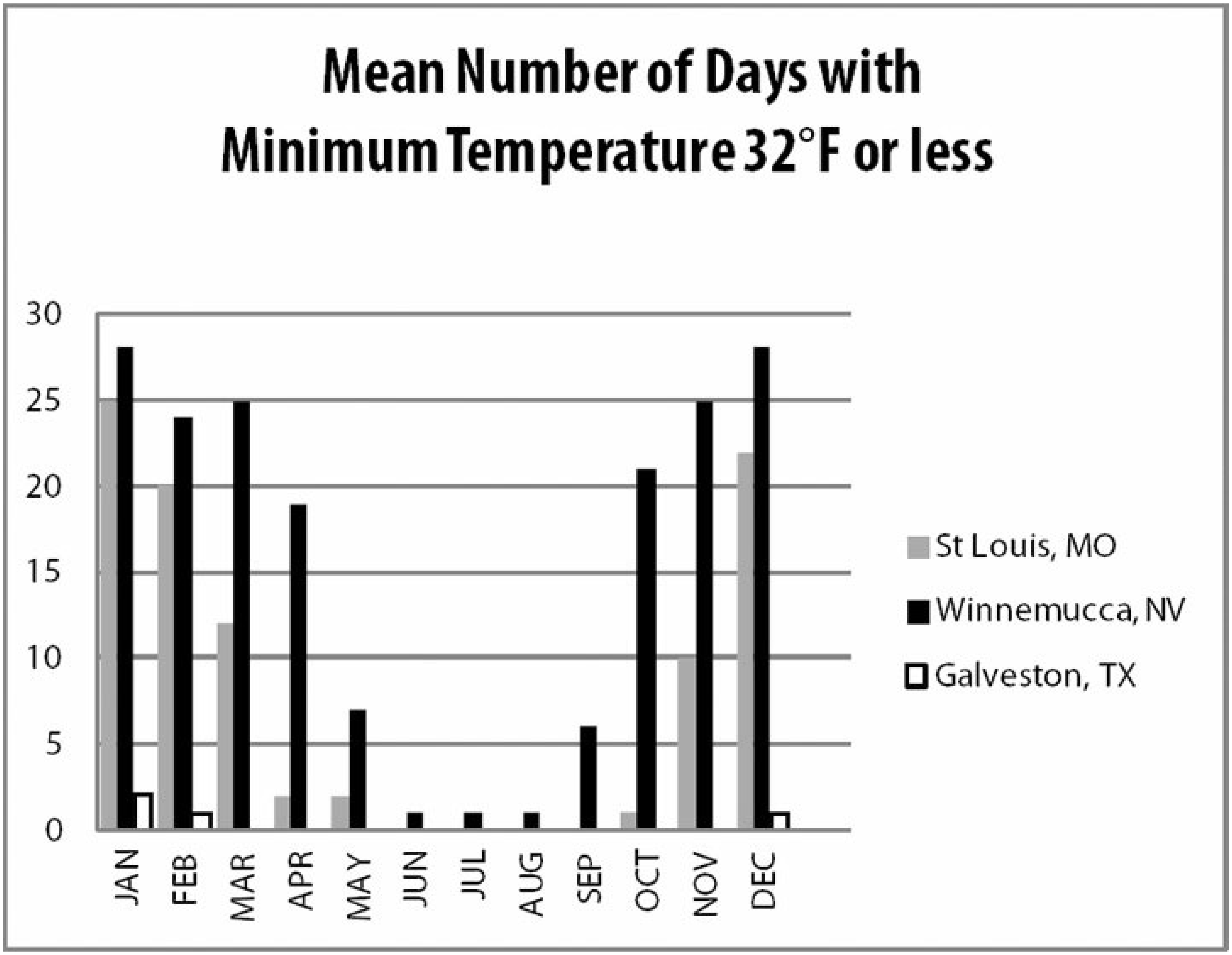
7. What is the average number of hours worked per week per employee at Marshville Toy Company?

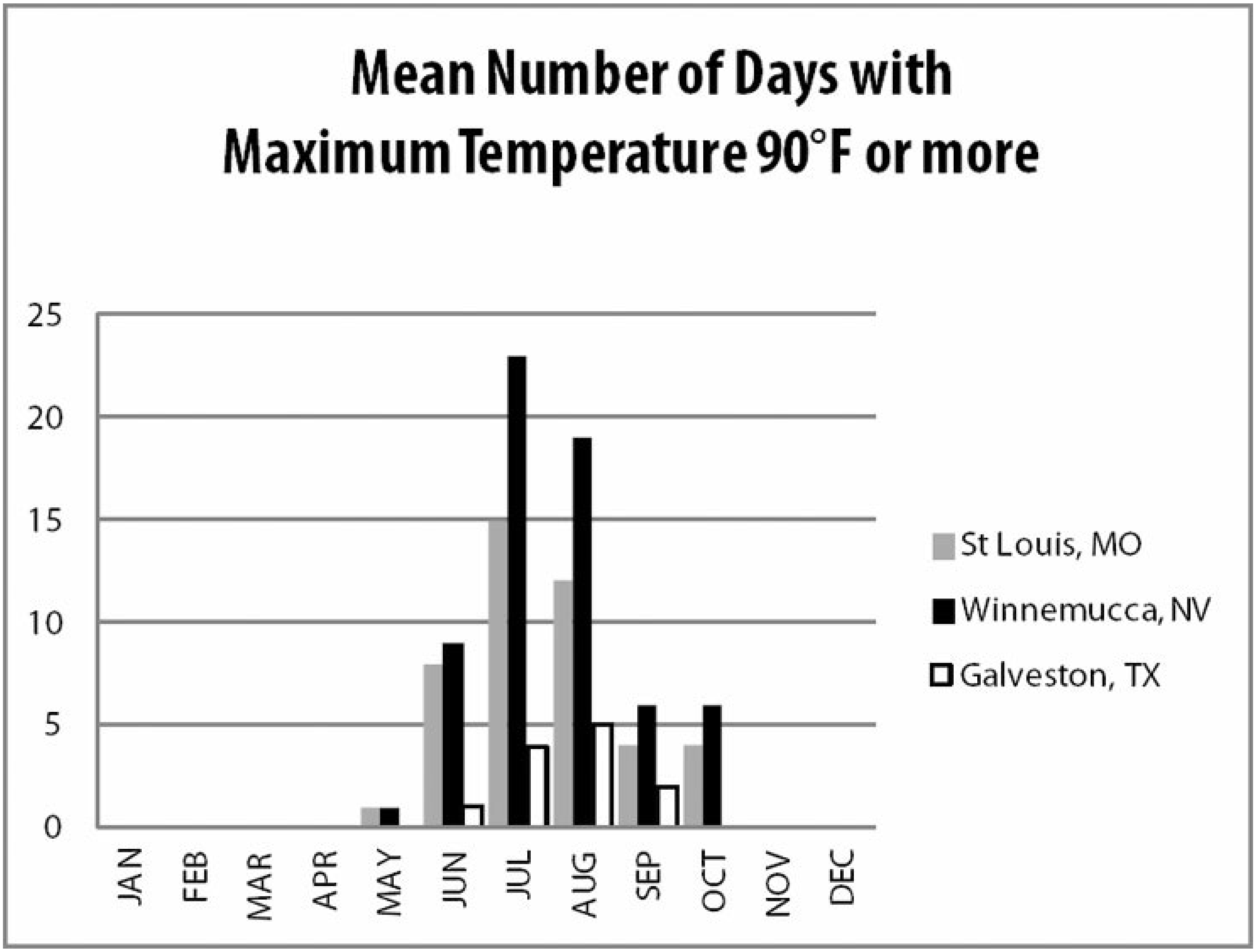
(A) 32
(B) 33
(C) 35
(D) $35\frac{2}{3}$
(E) $36\frac{1}{3}$

8. What is the positive difference between the mode and the range of the number of hours worked per week per employee at Marshville Toy Company?

(A) 0
(B) 4
(C) 5
(D) 8
(E) 26

Problem Set C





9. In how many months of the year were there more than 20 days with temperatures 32°F or less in Winnemucca?

(A) 2
(B) 3
(C) 4
(D) 6
(E) 7

10. On how many days in the entire year did the temperature in Galveston rise to at least 90°F or fall to, or below, 32°F?

(A) 11
(B) 16
(C) 28
(D) 42
(E) 59

11. Approximately what percent of the days with maximum temperature of 90°F or more in St. Louis occurred in July?

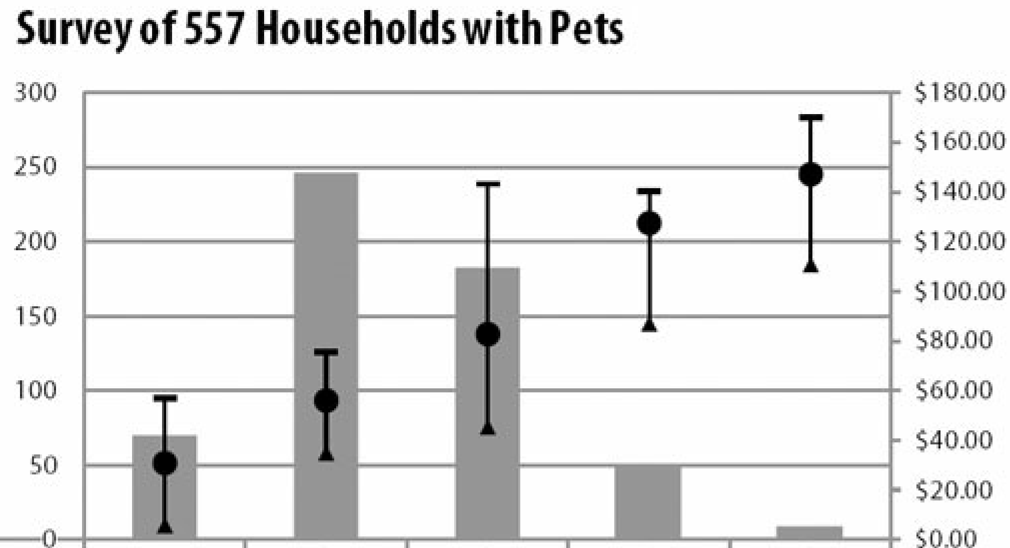
(A) 6%
(B) 15%
(C) 17%
(D) 34%

(E) 44%

12. The number of freezing January days in Winnemucca was approximately what percent more than the number of freezing January days in St. Louis? (A “freezing” day is one in which the minimum temperature is 32°F or less.)

(A) 3%
(B) 6%
(C) 12%
(D) 24%
(E) 28%

Problem Set D



Number of pets	1	2	3	4	5
Number of households with this number of pets	70	246	183	49	9
Maximum Monthly Spending on Pet Supplies	$57.32	$75.98	$143.57	$140.48	$170.23
Minimum Monthly Spending on Pet Supplies	$6.34	$35.47	$45.84	$87.46	$111.20
Average Monthly Spending on Pet Supplies	$31.25	$56.42	$83.11	$127.74	$147.38

13. Approximately what percent of the surveyed households have more than three pets?

(A) 10%
(B) 20%
(C) 30%
(D) 40%
(E) 50%

14. What is the median number of pets owned by the households in the survey?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

15. Grouping households by number of pets, what is the range of monthly spending on pet supplies for the group with the largest range?

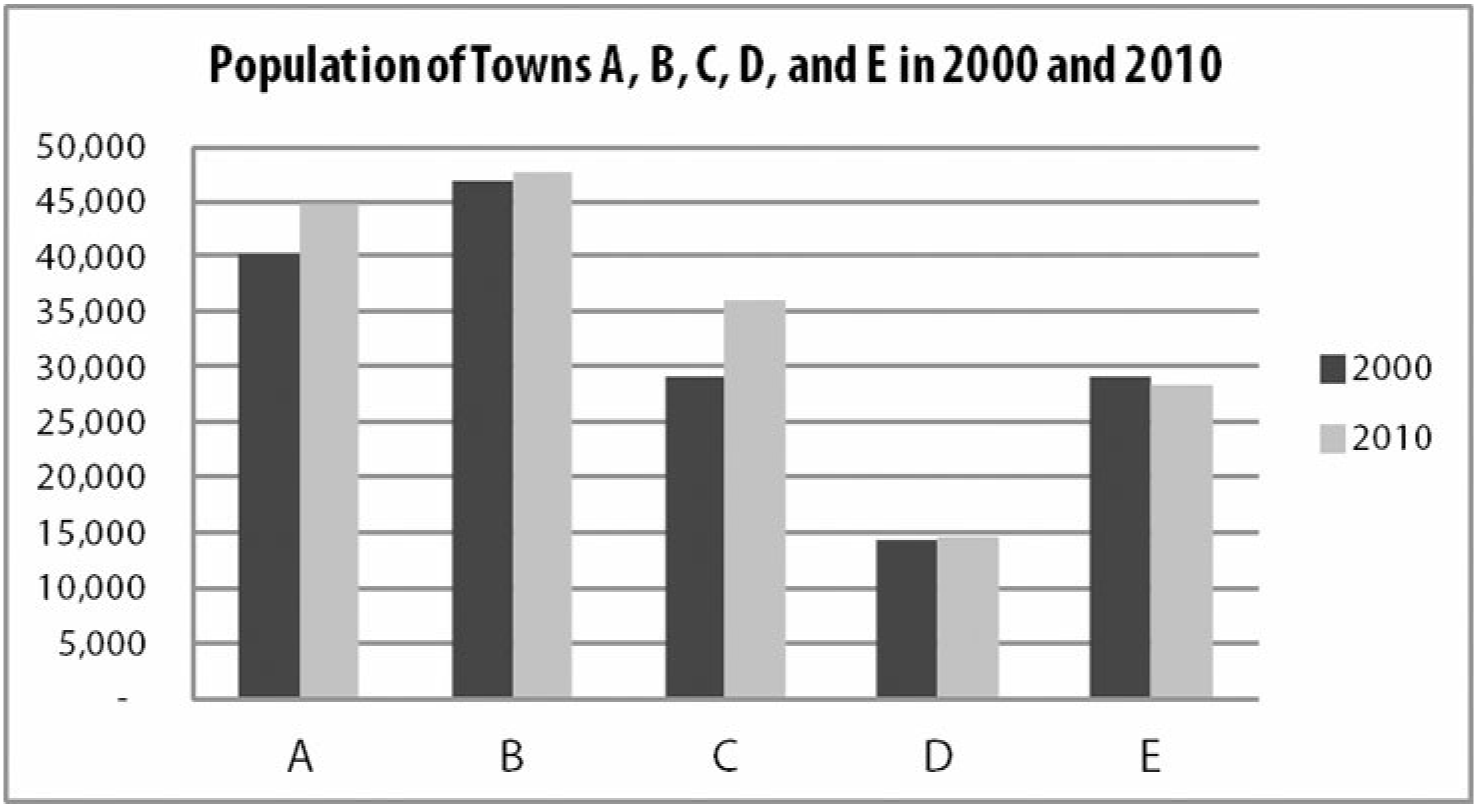
(A) $69.03
(B) $97.73
(C) $116.13
(D) $138.98

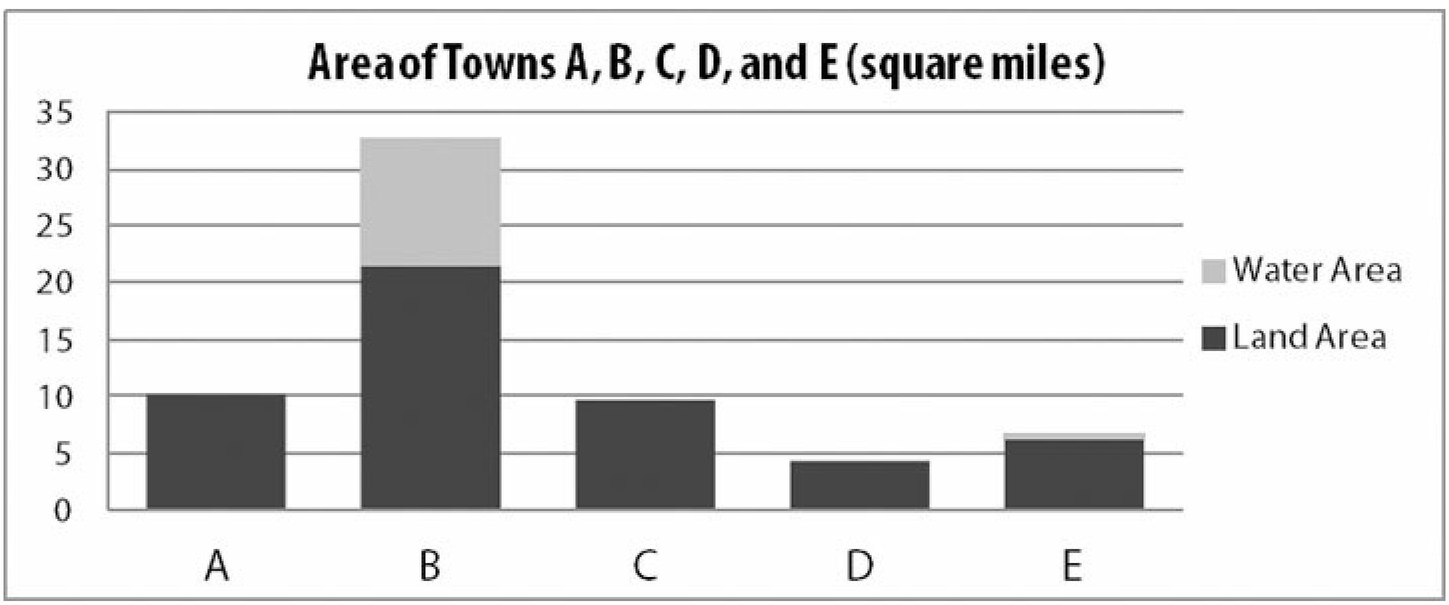
(E) $170.23

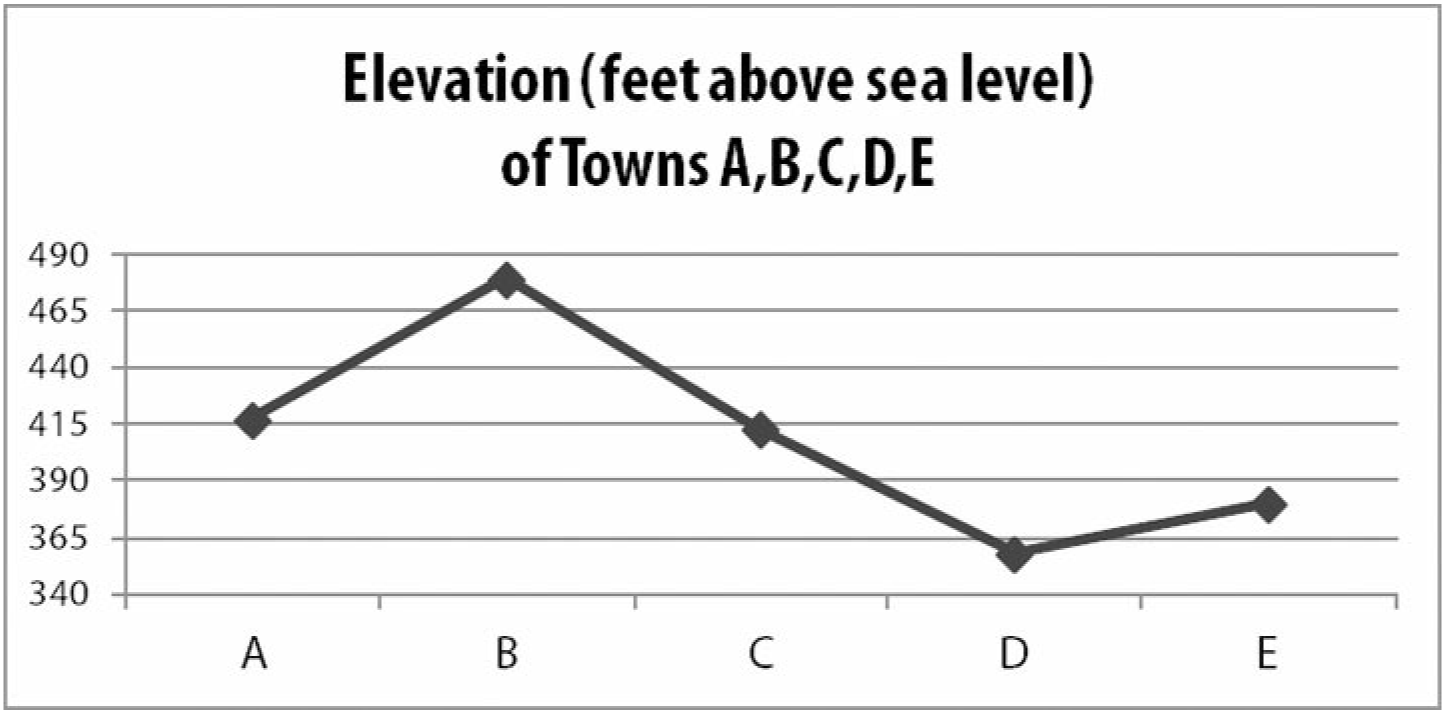
16. Households with how many pets have the greatest average monthly spending per pet?

(A) 1 pet
(B) 2 pets
(C) 3 pets
(D) 4 pets
(E) 5 pets

Problem Set E







17. In what town did the population increase by the greatest percent between 2000 and 2010?

(A) Town A
(B) Town B
(C) Town C
(D) Town D
(E) Town E

18. The ratio of the population of one town to the population of another remained most unchanged between 2000 and 2010 for which two towns?

Indicate two such towns.

☐ Town A
☐ Town B
☐ Town C
☐ Town D
☐ Town E

19. The water area of town B is most nearly equal the total land area of which two towns?

Indicate two such towns.

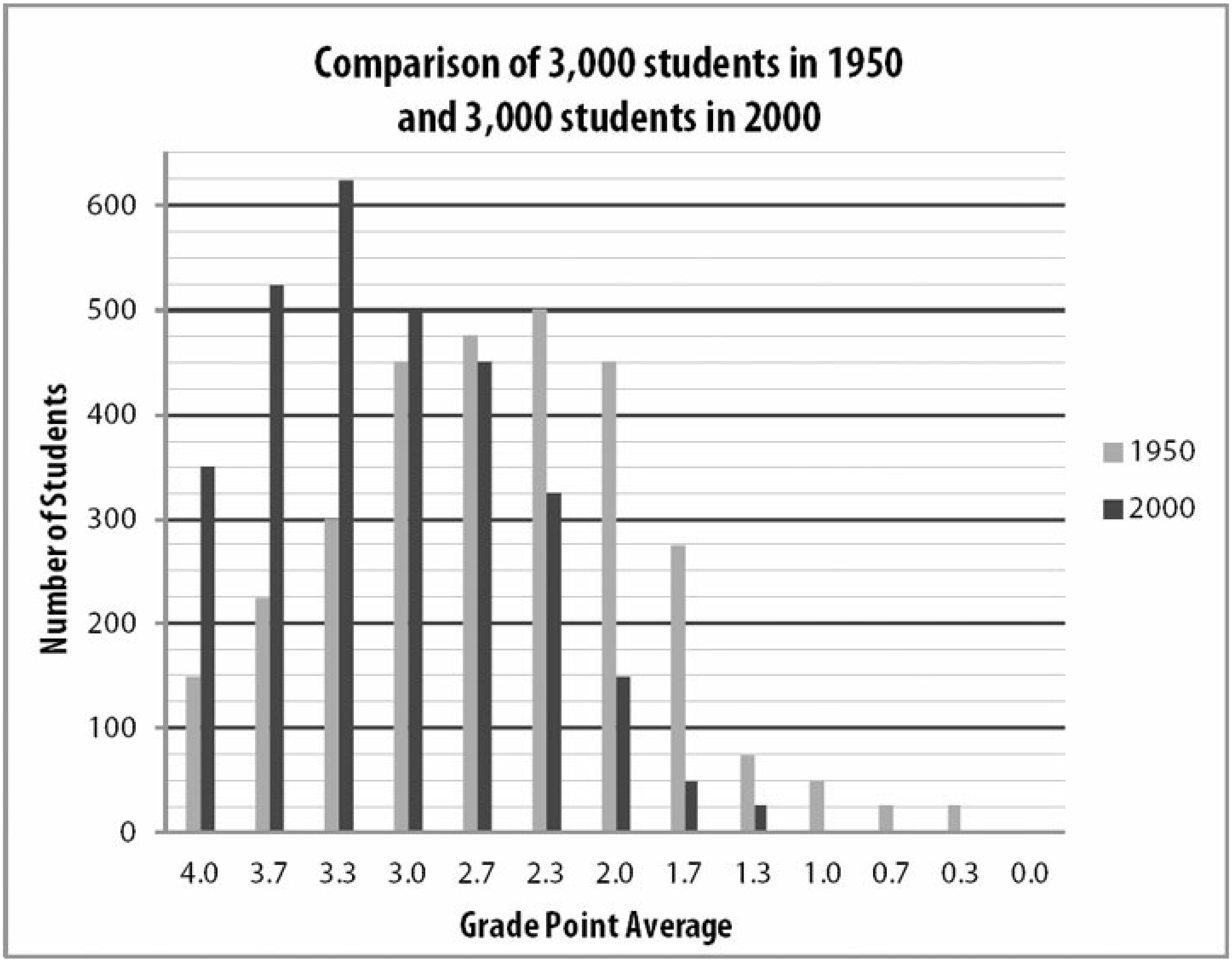
☐ Town A
☐ Town B
☐ Town C
☐ Town D
☐ Town E

20. Which two towns have the most nearly equal elevation in feet above sea level?

Indicate two such towns.

- [ ] Town A
- [ ] Town B
- [ ] Town C
- [ ] Town D
- [ ] Town E

Problem Set F



21. What was the mode for grade point average among the 3,000 students in 2000?

(A) 3.7
(B) 3.3
(C) 3.0
(D) 2.7
(E) 2.3

22. What was the median grade point average among the 3,000 students in 1950?

(A) 3.7
(B) 3.3
(C) 3.0
(D) 2.7
(E) 2.3

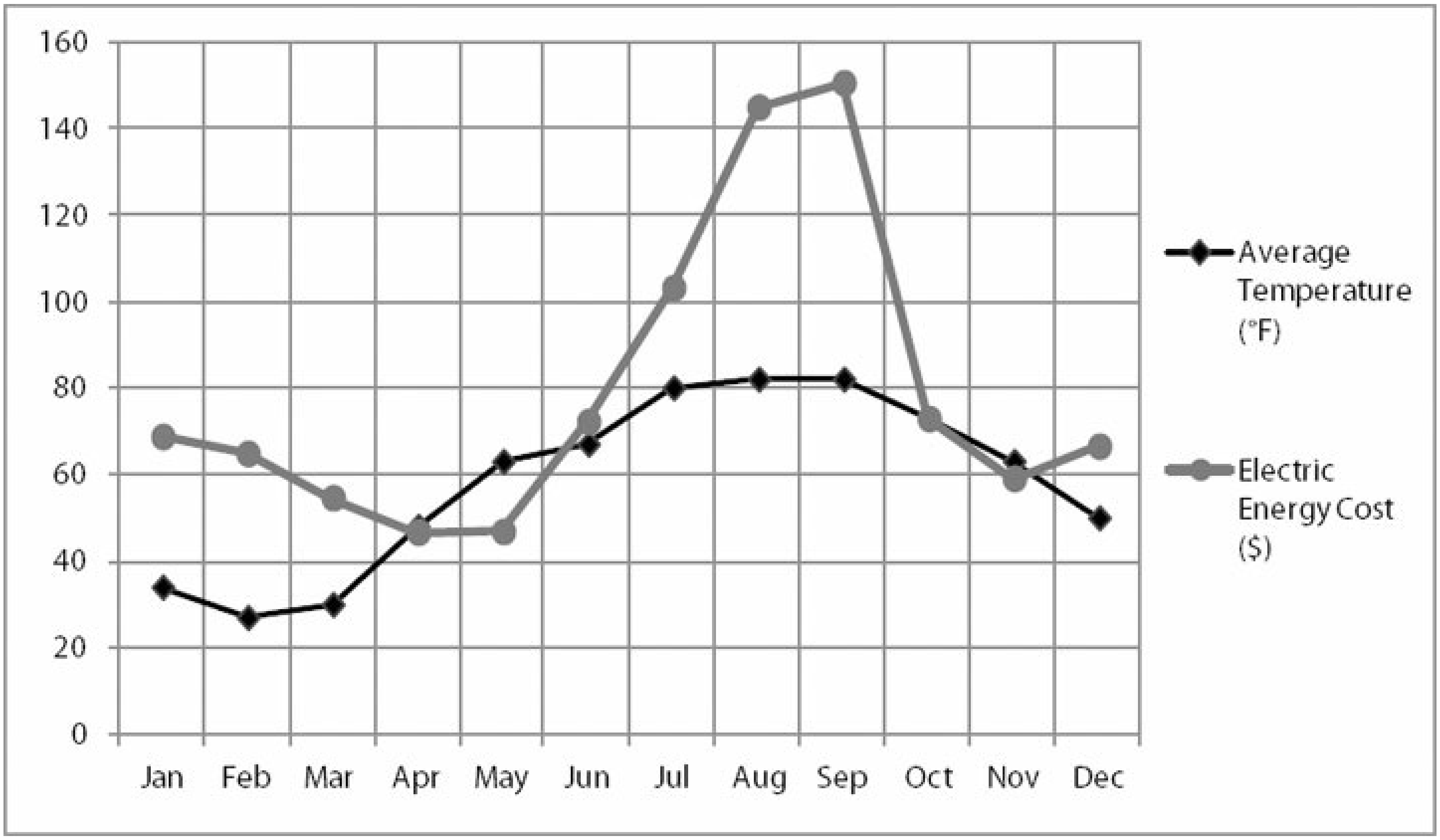
23. Approximately what percent of the students in 2000 earned at least a 3.0 grade point average?

(A) 25%
(B) 50%
(C) 67%
(D) 80%
(E) 97.5%

24. Approximately what percent of the students in 1950 earned a grade point average less than 3.0?

(A) 33%
(B) 37.5%
(C) 50%
(D) 62.5%
(E) 75%

Problem Set G



25. Electric energy cost increased most between which two consecutive months?

Indicate <u>two</u> such months.

- ☐ January
- ☐ February
- ☐ March
- ☐ April
- ☐ May
- ☐ June
- ☐ July
- ☐ August
- ☐ September
- ☐ October
- ☐ November
- ☐ December

26. Electric energy cost changed least between which two consecutive months?

Indicate <u>two</u> such months.

- ☐ January

- ☐ February
- ☐ March
- ☐ April
- ☐ May
- ☐ June
- ☐ July
- ☐ August
- ☐ September
- ☐ October
- ☐ November
- ☐ December

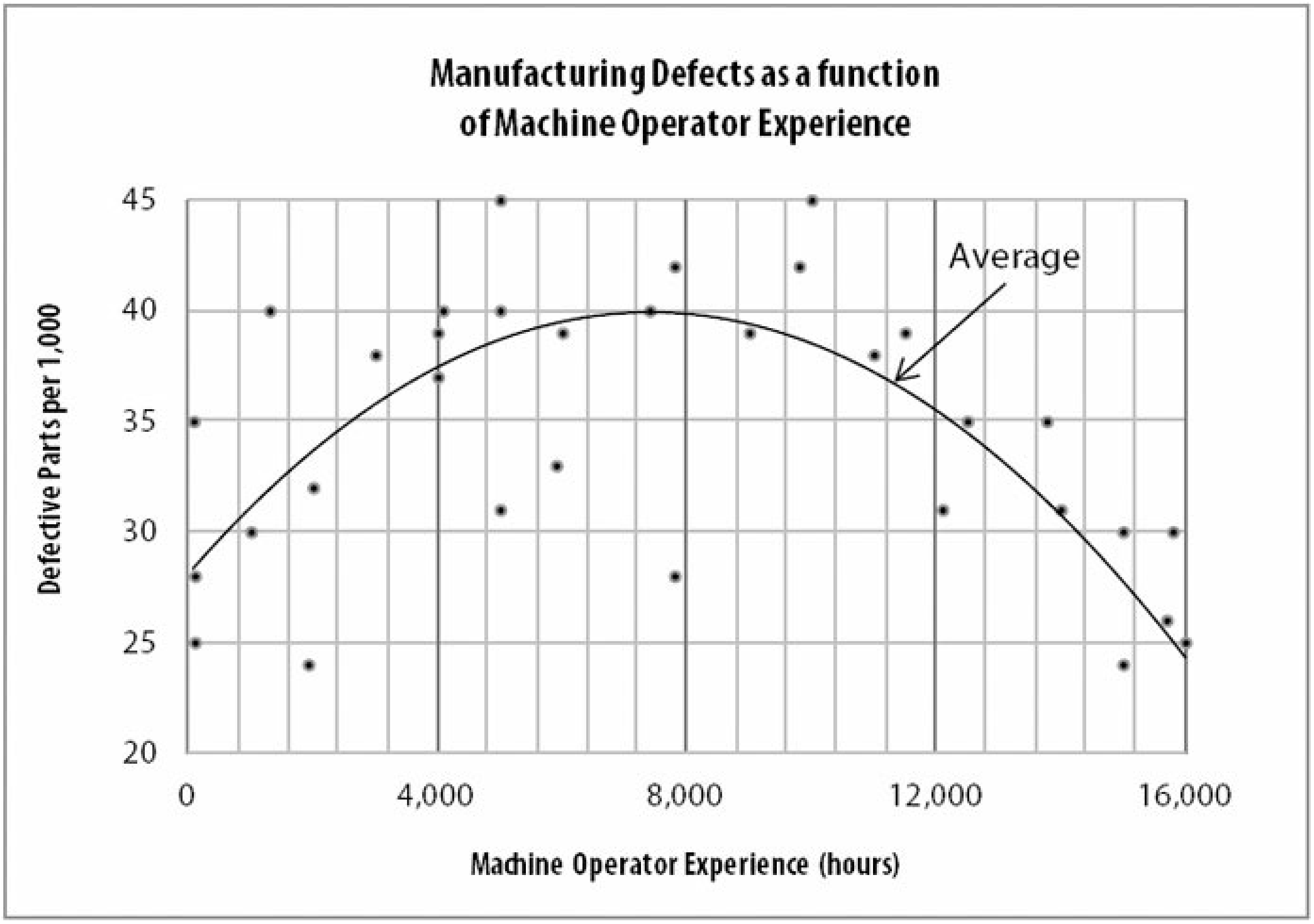
27. Approximately what was the average electric energy cost per month in the first half of the year?

    (A) $45
    (B) $50
    (C) $60
    (D) $70
    (E) $75

28. In what month was the electric energy cost per °F of average temperature least?

    (A) April
    (B) May
    (C) October
    (D) November
    (E) December

Problem Set H



29. On average, the machine operators that produce the fewest defective parts per 1,000 have how many hours of experience?

(A) 40
(B) 4,000
(C) 8,000
(D) 12,000
(E) 16,000

30. On average, the defective part rate is equal for machine operators with 12,000 hours and with approximately how many hours of experience?

(A) 2,000
(B) 2,700
(C) 4,400
(D) 8,400
(E) 12,800

31. At approximately what experience level, in hours, do machine operators produce the most defective parts per 1,000, on average?

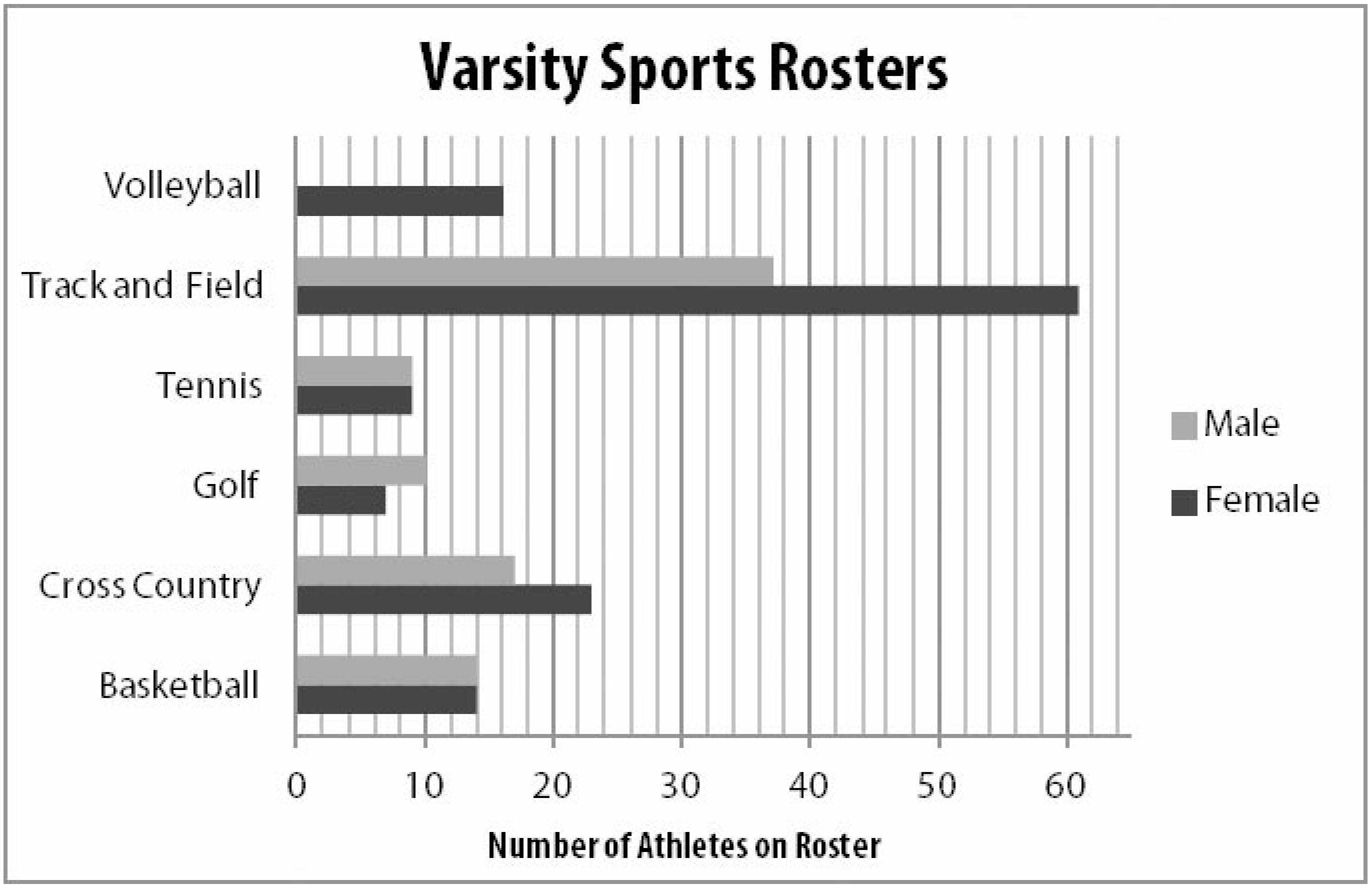
(A) 40

(B) 4,000
(C) 8,000
(D) 12,000
(E) 16,000

32. Of the two individual machine operators who had a defective part rate of 4.2%, approximately how many hours of experience did the less experienced operator have?

(A) 2,300
(B) 5,000
(C) 7,700
(D) 9,800
(E) 15,100

Problem Set I



33. What is the ratio of male athletes to female athletes on the track and field roster?

(A) $\frac{37}{61}$
(B) $\frac{9}{14}$
(C) $\frac{17}{23}$
(D) $\frac{14}{9}$
(E) $\frac{61}{37}$

34. All athletes are on only one varsity sports roster EXCEPT those who run on both the Track and Field team and the Cross Country team. If there are 76 male athletes total on the varsity sports rosters, how many male athletes are on both the Track and Field team and the Cross Country team?

(A) 11
(B) 17
(C) 37
(D) 54
(E) 76

35. On what varsity sports rosters do male athletes outnumber female athletes?

Indicate all such rosters.

- ☐ Volleyball
- ☐ Track and Field
- ☐ Tennis
- ☐ Golf
- ☐ Cross Country
- ☐ Basketball

36. What is the ratio of female tennis players to male basketball players on the varsity sports rosters?

(A) $\frac{5}{12}$

(B) $\frac{9}{14}$

(C) $\frac{7}{8}$

(D) $\frac{14}{9}$

(E) $\frac{12}{5}$

Problem Set J

	Change in Total Revenue (2011 to 2012)	Percent Change in Number of Distinct Customers (2011 to 2012)	Percent Change in Total Costs (2011 to 2012)
Store W	-$400,000	+2%	+15%
Store X	+$520,000	+14%	+4%
Store Y	-$365,000	+5%	+12%
Store Z	+$125,000	-7%	-20%

37. For which store was the revenue per distinct customer greatest in 2012?

(A) Store W
(B) Store X
(C) Store Y
(D) Store Z
(E) It cannot be determined from the information given.

38. Between 2011 and 2012, total costs per distinct customer increased by the greatest percent at which store?

(A) Store W
(B) Store X
(C) Store Y
(D) Store Z
(E) It cannot be determined from the information given.

39. Store profit in 2012 could have been less than same store's profit in 2011 at which of the following store(s)?

Indicate all such stores.

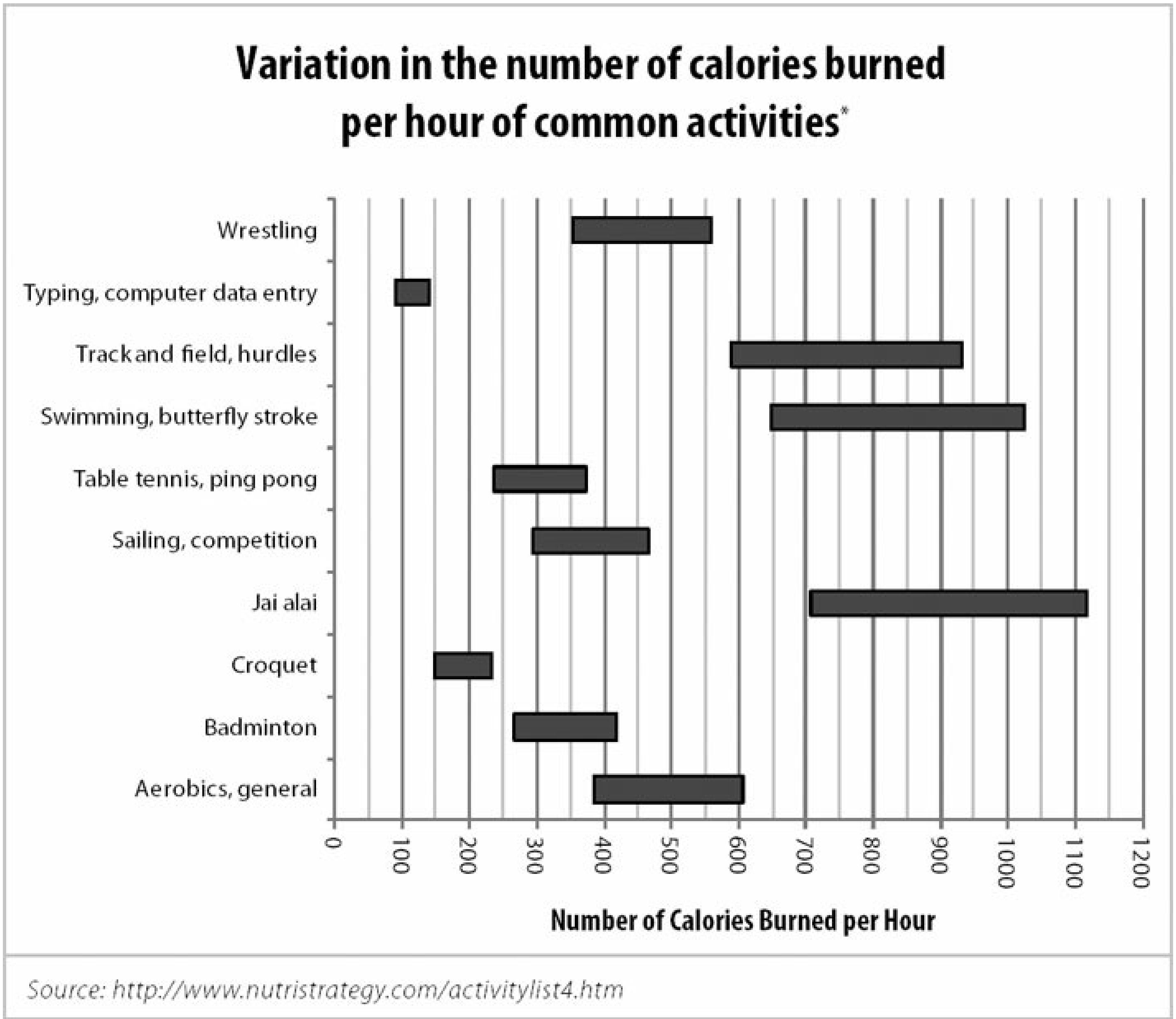
☐ Store W
☐ Store X
☐ Store Y
☐ Store Z
☐ None of the above.

40. Which of the following statements must be true?

(A) Of the four stores, Store X had the greatest percent increase in revenue from 2011 to 2012.
(B) Per customer revenue increased at Store Z from 2011 to 2012.
(C) Of the four stores, Store W had the greatest increase in total costs from 2011 to 2012.
(D) Of the four stores, Store Y had the highest percent of repeat customers.

(E) In 2012, Store W and Store Z combined had fewer distinct customers than did Store X.

Problem Set K



* Based on body weight of exercise subject. The lower limit represents the calories burned by a person weighing 130 pounds, while the upper limit represents the calories burned by a person weighing 205 pounds.

41. Which of the following statements could be true?

Indicate <u>all</u> such statements.

☐ A person weighing between 130 and 205 pounds performs one of the above activities for 10 hours yet burns fewer calories than another person in the same weight range performing another activity for only 1 hour.

☐ A 175 pound person playing jai alai for one hour burns fewer calories than a 180 pound person swimming the butterfly stroke for one hour.

☐ If all people in question weigh between 130 and 205 pounds, the average calories burned by two people playing table tennis for 1 hour is more than the total calories burned by 2 people typing for 3 hours.

42. Which combination of activities burns the fewest calories total?

(A) A 130 pound person playing badminton for 1 hour and a 205 pound person playing table tennis for 1 hour
(B) A 130 pound person wrestling for 1 hour and a 205 pound person running track and field, hurdles for 1 hour
(C) A 130 pound person typing for 1 hour and a 205 pound person swimming the butterfly stroke for 1 hour
(D) A 130 pound person sailing in a competition for 1 hour and a 205 pound person doing aerobics for 1 hour
(E) A 130 pound person typing for 1 hour and a 205 pound person playing croquet for 1 hour

Problem Set L

Population and GDP for 50 African Countries

Gross Domestic Product	Population: more than 50 million	20 to 50 million	10 to 20 million	2 to 10 million	less than 2 million	Total
more than $100 billion	3	2	0	0	0	5
$20 – 100 billion	1	7	1	1	0	10
$10 – 20 billion	1	3	3	3	3	13
less than $10 billion	0	0	7	8	7	22
Total	5	12	11	12	10	50

43. Among the 50 African countries represented in the chart above, how many countries have a population between 10 million and 50 million people and a GDP between $10 billion and $20 billion?

(A) 6
(B) 7
(C) 13
(D) 16
(E) 23

44. Among the 50 African countries represented in the chart above, what percent of the countries have a population less than 20 million people and a GDP of less than $20 billion?

(A) 38%
(B) 44%
(C) 62%
(D) 68%
(E) 90%

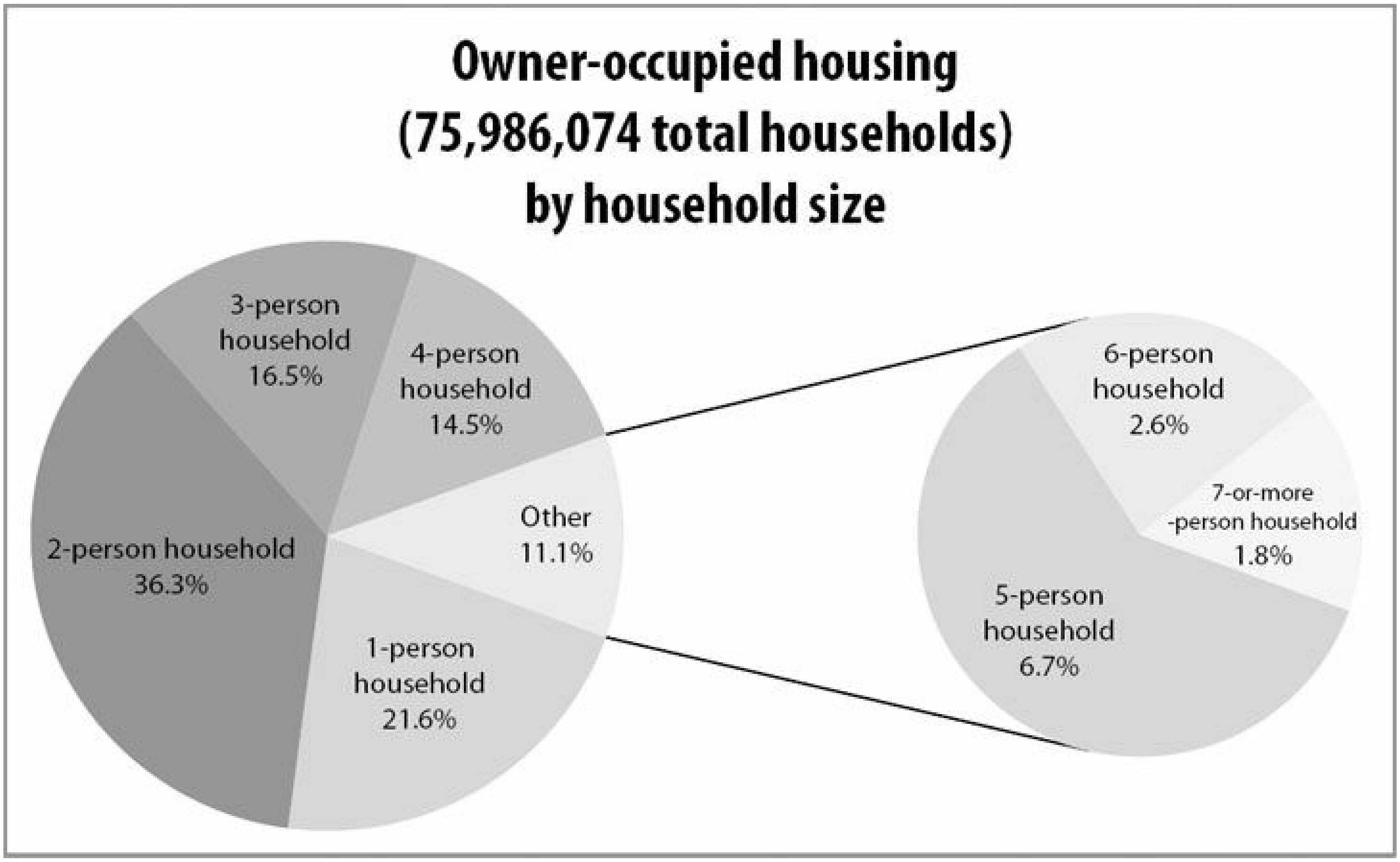
45. Approximately what percent of the African countries in the chart above with GDPs between $10 billion and $20 billion have populations between 10 million and 20 million?

(A) 6%
(B) 23%
(C) 26%
(D) 30%
(E) 51%

46. Referring to the 50 African countries represented in the chart above, which of the following is greatest?

(A) The number of countries with more than \$10 billion GDP and population less than 20 million
(B) The number of countries with less than \$20 billion GDP and population more than 10 million
(C) The number of countries with more than \$20 billion GDP
(D) The number of countries with less than \$100 billion GDP and population less than 10 million
(E) The number of countries with less than \$100 billion GDP and population between 10 million and 50 million

Problem Set M



47. What percent of owner-occupied housing units are households with fewer than four people?

(A) 11.1%
(B) 14.5%
(C) 25.6%
(D) 74.4%
(E) 88.9%

48. Among the owner-occupied housing units represented in the chart above, approximately how many households are 5-person households?

(A) 1 million
(B) 2 million
(C) 3 million
(D) 4 million
(E) 5 million

49. Which of the following is a correct ranking of 1-person households, 3-person households, and 5-person households based on the total number of people living in such households, from least to greatest number of people?

(A) 1-person households, 3-person households, 5-person households
(B) 1-person households, 5-person households, 3-person households
(C) 3-person households, 1-person households, 5-person households

(D) 3-person households, 5-person households, 1-person households
(E) 5-person households, 3-person households, 1-person households

50. Which range of household sizes, inclusive, accounts for more than 50% of all owner-occupied housing units?

(A) 2- to 3-person
(B) 3- to 4-person
(C) 4- to 5-person
(D) 5- to 6-person
(E) 6- to 7-person

Data Interpretation Answers

Problem Set A: First, read the title of the chart: everyone accounted for in the chart is a 9th grader at Millbrook Middle School. So, when problems mention "9th grade," you don't have to figure out how many people involved are 9th graders — everyone in the chart is.

When given a chart that depends on addition (boys plus girls = total students, and also those enrolled in Spanish plus those not enrolled in Spanish = total students), it can be helpful to sketch a quick version of the chart on your paper and to add a total column. (If the chart is large and this would be too time-consuming, just imagine that the "Total" row and "Total" column are present, and only calculate what you need.) For example:

	Boys	**Girls**	**TOTAL**
Enrolled in Spanish	12	13	
Not Enrolled in Spanish	19	16	
TOTAL			

Now add down and across:

	Boys	**Girls**	**TOTAL**
Enrolled in Spanish	12	13	25
Not Enrolled in Spanish	19	16	35
TOTAL	31	29	60

1. **(C).** There are 29 total girls and 13 are enrolled in Spanish. The fraction of girls enrolled in Spanish is $\frac{13}{29}$. Convert to a percent: $\left(\frac{13}{29}\times100\right)\%$, or about 45%.

2. **(A).** There are 60 total students and 12 boys enrolled in Spanish. The answer is 12/60, which reduces to 1/5. (Read carefully! "What fraction of the students … are boys who are enrolled in Spanish?" is NOT the same as "What fraction of the boys are enrolled in Spanish?")

3. **(C).** There are 16 girls not enrolled in Spanish and 60 total students. The ratio is 16/60, which reduces to 4/15 or 4 : 15.

4. **(D).** 35 students are not enrolled in Spanish and 25 are. The question can be rephrased as, "35 is what percent greater than 25?" Using the percent change formula:

$$\text{Percent Change} = \left(\frac{\textit{Difference}}{\textit{Original}} \times 100\right)\%$$

$$\text{Percent Change} = \left(\frac{10}{25} \times 100\right)\% = 40\%$$

Thus, x is 40.

5. **(E)**. Sketch a new chart to reflect the changes. Switch 2 of the boys from "not enrolled" to "enrolled." Then, add 8 new girls and 7 new boys to the "enrolled" groups, like this:

	Boys	**Girls**	**TOTAL**
Enrolled in Spanish	12 + 2 + 7 = 21	13 + 8 = 21	42
Not Enrolled in Spanish	19 - 2 = 17	16	33
TOTAL	38	37	75

Update the Total rows and columns as well. You will see that both "Boys" and "Girls", as well as "Enrolled in Spanish" and "Not Enrolled in Spanish," now sum to a total of 75.

What percent of 9th grade students at Millbrook would then be taking Spanish? Since 42 out of 75 students would be enrolled in Spanish, enter 42/75 into your calculator, then multiply by 100 to convert to a percent. The answer is 56%.

Problem Set B: This chart shows the frequency with which certain values — numbers of hours worked per week — appear. That is, the chart is a short way of showing a list of values that would begin: 15, 15, 15, 15, 25, 25, 25, 25, 25, 25, 25, 25, 25… (then the number 35 fifteen times, then the number 40 twenty-seven times, then the number 50 five times). Add the numbers in the lefthand column to determine that 60 people's work hours are being accounted for.

6. **(E).** The median of any list is the middle number or the average of the two middle numbers. However, you CANNOT assume that the middle number in the list 15, 25, 35, 40, 50 is the median number of hours worked per week, because this does not take into account the *frequency* with which each of those numbers occurs in the list. The actual list includes the value 15 *four times* (once for each of the 4 employees who works 15 hours per week), the value 25 *nine times* (once for each of the 9 employees who works 25 hours per week), etc.

To find the median, add the numbers of employees: 4 + 9 + 15 + 27 + 5 = 60. Thus, the middle of this list will be the average of the 30th and 31st values. Since 4 + 9 + 15 = 28, the 29th, 30th, 31st, etc. values fall into the next highest group — the group of 27 people who work 40 hours per week. The median number of hours worked per week per employee is 40.

7. **(D).** To average the number of hours worked per week, you CANNOT simply average 15, 25, 35, 40, and 50. You must take into account *how many people* work each of those numbers of hours. That is, you must average the values for all 60 workers:

$$\frac{4(15)+9(25)+15(35)+27(40)+5(50)}{60} = 35.\overline{6}\text{ hours}$$

The answer is $35.\overline{6}$ or $35\frac{2}{3}$.

8. **(C).** The mode is the number that appears in the list with the greatest frequency. Since 27 people worked 40 hours a week and every other group has fewer than 27 people, the mode is 40. The range is the highest value in the list minus the lowest value in the list. 50 - 15 = 35. The positive difference between 40 and 35 is 40 - 35 = 5.

Problem Set C: The two charts show how often daily temperature extremes occurred in each month of the year for 3 cities. For the sake of simplicity, you can think of the top chart as "cold" and the bottom chart as "hot."

Also note that there is no information about exactly how hot or how cold the days tallied are: a day with a minimum temperature of 27°F counts as a "cold" day, just as a day with minimum temperature of -10°F would. Therefore, it is likely that questions will just reference one or both of the two temperature categories broadly (= 90 and = 32), which you can simply think of as "hot" or "cold" days.

9. **(D).** From the "cold" chart, the black bar referring to Winnemucca rises above 20 in Jan, Feb, Mar, Oct, Nov, and Dec, for a total of 6 months.

10. **(B).** This question asks about number of days with both temperatures extremes in Galveston. Galveston had 1 "hot" day in June, 4 in July, 5 in August, and 2 in September, for a total of 12. It had 2 "cold" days in January and 1 each in February and December, for a total of 4. The total number of days with either extreme temperature is 16 days.

11. **(D).** From the grey bars on the "hot" day chart, St. Louis had a total of 1 + 8 + 15 + 12 + 4 + 4 = 44 days when the temperature reached at least 90°F, and 15 of those were in July. These July days account for $\left(\frac{15}{44}\times 100\right)\% \approx 34\%$ of all the hot days in St. Louis (approximately).

12. **(C).** In January, Winnemucca had 28 freezing days, while St. Louis had 25. So the question is asking, "28 is what percent more than 25?" Answer this with the percent change formula: $\left(\frac{Difference}{Original}\times 100\right)\% = \left(\frac{3}{25}\times 100\right)\% = 12\%$. It is true that 28 is 12% more than 25% (check: 25 × 0.12 = 3).

Problem Set D: This table tallies the number of households, according to number of pets in the household, and each column captures information about these households. For example, the left-most column with numbers indicates that there are 70 households that have 1 pet, and these households spend an average of $31.25 per month on pet supplies. In that group, the household(s) that spent the least spent $6.34 on pet supplies, while the household(s) that spent the most spent $57.32. Notice that the bars and the max/min/average range lines duplicate the information in the table. For exact calculations, rely on the chart numbers. For broader questions, such as "which is greater," the more visual representation of the data can often provide a quick answer.

13. **(A).** There are 557 households, of which 49 have four pets and 9 have five pets. Thus a total of 58 households have more than three pets. To express this as a percent, take this number and divide by the total number of households. 58/557 = 10% (approximately).

14. **(B).** Since there are 557 households, the median household would be midway between the 1st and 557th households on the list when the households are ranked according to how many pets they own. The midpoint between the 1st and 557th households is the (1 + 557)/2 = 279th household. Check: There are 278 households below this one, and 278 households above (because 279 + 278 = 557). Ranked by number of pets, households 1 through 70 (the first 70 households) have one pet, which means households 71 through 316 (in other words, the next 246 households) have two pets. Since the 279th household falls in this interval, the median household owns two pets.

15. **(B).** The group with the largest range of monthly spending is the group of households that own three pets. This can be seen by looking at the length of the vertical line between the maximum spending bar and the minimum spending triangle. Within this group, the maximum amount spent is $143.57 and the minimum is $45.84, so the range is $143.57 - $45.84 = $97.73.

16. **(D).** The group with one pet spent an average of $31.25 per pet, as indicated in the chart. The group with two pets spent an average of $56.42 on two pets, which is $28.21 per pet ($56.42/2). The third group spent an average of $83.11 on three pets, or $83.11/3 = $27.70 per pet (approximately). The fourth group spent an average of $127.74 on four pets, or $127.74/4 = $31.94 per pet (approximately). The fifth group spent an average of $147.38 on five pets, or $147.38/5 = $29.47 per pet (approximately). The highest average is among the group that has four pets.

As an alternative to using the calculator for all five groups, you could benchmark to $30 per pet. If that were the average spending per pet, the bottom row of the table would read $30, $60, $90, $120, and $150 from left to right. Only the households with one or four pets exceed these numbers, so the correct answer is one of them. For just those households, calculate average monthly spending per pet as before.

Problem Set E:

17. **(C).** To compare population in 2000 and 2010, look at the difference in the heights of the dark and light gray bars in the population bar chart. Population decreased in Town E, and was barely changed in Towns B and D, so focus on the remaining Towns A and C. This difference is about 5,000 for town A (45,000 - 40,000), but is more than 5,000 for Town C (more than 35,000 - less than 30,000). The question asks for the "population increase by the greatest *percent*," which requires comparison to the original (2000) population: Percent Change = $\left(\frac{\text{Difference}}{\text{Original}}\times 100\right)\%$. Not only is the population increase greatest for Town C, the population of Town C is smaller than for Town A. The percent increase in population for Town A was $\left(\frac{5{,}000}{40{,}000}\times 100\right)\% = 12.5\%$, but the percent increase in population for Town C was about $\left(\frac{7{,}000}{29{,}000}\times 100\right)\% \approx 24\%$.

18. **Town B and Town D.** Translating "the ratio of the population of one town to the population of another remained most unchanged between 2000 and 2010 for which two towns" literally:

$$\frac{\text{FirstTown}_{2000}}{\text{SecondTown}_{2000}} \text{ is most equal to } \frac{\text{FirstTown}_{2010}}{\text{SecondTown}_{2010}} \text{ for which two towns?}$$

$$\text{Or } \frac{\text{FirstTown}_{2000}}{\text{SecondTown}_{2000}} \approx \frac{\text{FirstTown}_{2010}}{\text{SecondTown}_{2010}} \text{ for which two towns?}$$

Cross multiplying, this could be rephrased as $\frac{\text{SecondTown}_{2010}}{\text{SecondTown}_{2000}} \approx \frac{\text{FirstTown}_{2010}}{\text{FirstTown}_{2000}}$ for which two towns? Going back to words, this is like asking "Which two towns had the most similar population increase or decrease (as a percent or fraction)?

The percent change in population for each town was:

Town A: $\frac{5{,}000}{40{,}000} \times 100\% = 12.5\%$

Town B: about $\frac{1{,}000}{47{,}000} \times 100\% = \text{a bit over } 2\%$

Town C: about $\frac{7{,}000}{29{,}000} \times 100\% \approx 24\%$

Town D: about $\frac{\text{at most } 500}{\text{at least } 14{,}000} \times 100\% = \text{at most } 3.5\%$

Town E: negative % (population decreased)

While the calculations were rough and the numbers are not quite equal for towns B and D, these two percents are closest to each other, and all of the percents calculated for the other towns are quite unique.

19. **Town D, Town E.** In the bar chart for area, dark gray represents the land area and light gray (stacked on top) represents the water area. Thus, to find water area, subtract the height of the dark gray land area bar from the total height of the stacked bars.

Water area of Town B is about 32– 21 = 11. Since the vertical scale is not that precise, you may want to consider a range. Water area for Town B is a little more than 10, as the top of the dark gray bar is slightly *closer* to the horizontal line for 20 than the top of the light gray bar is to the horizontal line for 30. Similarly, water area for Town B must be less than 12.5, as the top of the light gray bar is halfway between 30 and 35, but the top of the dark gray bar is clearly higher than 20.

Towns C, D, and E all have land area less than 10 square miles (i.e. all are below horizontal grid line for 10). Adding the land area of Town C (a bit less than 10) to that of Town D (about 4), the result is too high. The sum of land area in Towns D and E is about 4 + about 6 (certainly less than 7.5), for a result closest to 11.

20. **Town A and Town C.** In the elevation chart, the towns are on the x-axis and the elevation (in feet above sea level) is on the y-axis. Two towns have the same elevation if marked at the same y value, i.e. if their data points are on the same horizontal line. Towns A and C are both close to the horizontal line for 415.

Problem Set F: The dark gray bars indicate number of students in 2000, and the light gray bars indicate number of students in 1950, having various grade point averages. Although the title makes it unnecessary to do so, if you totaled the number of students in each bar color, you would get 3,000 students.

Note the general contrast between students in the two years. Connecting the top of each light gray bar with a smooth line, the result would be a sort-of bell curve that peaks at grade point average of 2.3. Similarly, the dark gray bars form a similar bell curve, but its peak is at grade point average of 3.3, so the grades in general are clustered at the higher end of the scale in 2000.

21. **(B).** The mode of a list of numbers is the number that occurs most frequently in the list. In the bar graph for grade point average, dark gray bars represent the students in 2000, and the mode of that data set is indicated by the tallest dark gray bar. This is at grade point average of 3.3. There were 625 students with a grade point average of 3.3 in the year 2000, more students than had any other grade point average.

22. **(D).** The median is the "middle value" of an ordered list of numbers, dividing the list into roughly two equal parts. For the 3,000 students in 1950, the median grade point average is the average of the 1,500th highest grade point average and the 1,501st highest grade point average. The students in 1950 are represented by the light gray bars.

150 students had a 4.0 grade point average.
225 students had a 3.7 grade point average. (Total with this GPA and higher = 150 + 225 = 375)
300 students had a 3.3 grade point average. (Total with this GPA and higher = 375 + 300 = 675)
450 students had a 3.0 grade point average. (Total with this GPA and higher = 675 + 450 = 1,125)
475 students had a 2.7 grade point average. (Total with this GPA and higher = 1,125 + 475 = 1,600)

The 1,500th and 1,501st students fall between the 1,125th and 1,600th students. Thus, the 1,500th and 1,501st highest grade point averages are both 2.7.

23. **(C).** The students in 2000 are represented by the dark gray bars.

350 students had a 4.0 grade point average.
525 students had a 3.7 grade point average.
625 students had a 3.3 grade point average.
500 students had a 3.0 grade point average.

There were 350 + 525 + 625 + 500 = 2000 students who earned at least a 3.0 grade point average in the year 2000, out of a total of 3000 students. This is $\frac{2}{3}$ of the students, or about 67% of the students.

24. **(D).** The students in 1950 are represented by the light gray bars.

150 students had a 4.0 grade point average.
225 students had a 3.7 grade point average.
300 students had a 3.3 grade point average.
450 students had a 3.0 grade point average.

In 1950, 150 + 225 + 300 + 450 = 1,125 students had a grade point average of 3.0 or higher. Thus, 3,000 – 1,125 = 1,875 students earned a grade point average *less than* 3.0. As a percent of the class, this was

$$\left(\frac{1{,}875}{3{,}000}\times 100\right)\% = 62.5\%$$

.

Problem Set G: The vertical number scale on the left side of the graph applies to both data sets, but for Average Temperature the units are °F and for Electric Energy Cost the units are dollars ($). For example, in January the average temperature was between 30°F and 40°F and the electric energy cost was about $70. Be careful to read data from the correct set; it would be easy to mix up which line is which. Consult the key frequently, and double-check your answers.

25. **July and August only.** Electric energy cost is represented by the light gray line and circular data points. A cost increase from one month to the next would mean a positive slope for the line segment between the two circular data points. The greater the slope of the light gray line segment, the greater the cost increase between those two months. There was an increase each month between May and September, and again between November and December. But the steepest positive slope is between July and August.

The cost increase from July to August was approximately \$145 - \$103 = \$42. For comparison, the cost increase from June to July was only about \$103 - \$70 = \$33. The correct answers are July and August.

26. **April and May only.** Electric energy cost is represented by the light gray line and circular data points. A cost increase from one month to the next would mean a positive slope for the line segment between the two data points, and a cost decrease would mean a negative slope. The steeper the slope of the line segment, the greater the cost change between two consecutive months. A cost change of \$0 would mean the line segment has a slope of 0 (i.e., it is horizontal).

To find the two consecutive months with the smallest electric energy cost change, look for the light gray line segment that is most horizontal. The line segment between April and May is nearly horizontal. The correct answers are April and May.

27. **(C).** There are two ways to approximate average electric energy cost per month in the first half of the year.

One way is to read the electric energy costs off of the chart and compute the average for the first 6 months, using the light gray circular data points:

Approximate average cost = $\frac{\$70+\$65+\$55+\$47+\$47+\$70}{6}=\frac{\$354}{6}=\$59$. Answer choice (C) \$60 is closest.

The other solution method is more visual. Consider choice (A) \$45, and imagine a horizontal line at \$45. All 6 cost data points for the first half of the year are above this horizontal line, so the average must be more than \$45. Similarly, imagine a horizontal line at \$75 for choice (E). All 6 cost data points for the first half of the year are below this horizontal line, so the average must be less than \$75. When a horizontal line at \$60 is considered, the 6 cost data points "balance": 3 are above the line and 3 are below, by approximately the same amount.

28. **(B).** To minimize $\frac{\text{Electric Energy Cost }(\$)}{\text{Average Temperature }(°\text{F})}$, minimize cost (light gray circular data points) while maximizing average temperature (black diamond data points). Only in April, May, October, and November is the black data point equal or greater than the gray data point (i.e., the $\frac{\text{Electric Energy Cost }(\$)}{\text{Average Temperature }(°F)}$ ratio is equal or less than 1). In April, October, and November, this ratio is close to 1. In May, the difference between the cost (\$) and the average temperature (°F) is greatest, so the electric energy cost per °F of average temperature is least. The correct answer is May.

Problem Set H: The chart shows defective parts per 1,000 (i.e., rate of making mistakes) as a function of machine operator experience. The dots indicate individual machine operators, and there is quite a bit of variance by individual. The line labeled "Average" shows the average performance of the group as a whole. A trend emerges: inexperienced machine operators and very experienced machine operators make fewer mistakes than those with medium level of experience. Also, certain individual machine operators are much "better" (i.e., they produce defective parts at a lower rate) than others, even with similar levels of experience.

29. **(E).** Because the question specifies "on average," refer to the curve marked "Average" rather than the individual data points. The fewest defective parts per 1,000 on the chart is where this average curve is lowest: operators with 16,000 hours of experience produce a little less than 25 defective parts per 1,000. Another low point is for operators with minimal experience, but even they produce between 25 and 30 defective parts per 1,000. In contrast, the defective part rate is maximized at the top of the curve: operators with 8,000 hours of experience produce about 40 defective parts per 1,000.

30. **(B).** Because the question specifies "on average," refer to the curve marked "Average" rather than the individual data points. Machine operators with 12,000 hours of experience produce an average of about 36 defective parts per 1,000.

The other group of machine operators that produces about 36 defective parts per 1,000 has a little less than 3,200 hours of experience. (Note that there are 5 grid lines for every 4,000 hours, so each vertical grid line is 800 hours apart. The grid mark to the left of the 4,000 mark represents 4,000 - 800 = 3,200 hours.) Choice (B) is close to and less than 3,200.

Alternatively, look up the average defective part rate for machine operators with the hours of experience listed in the choices:

(A) 2,000 hours (around 33 or 34 defective parts per 1,000).
(B) CORRECT. 2,700 hours (a bit over 35 defective parts per 1,000).
(C) 4,400 hours (around 38 defective parts per 1,000).
(D) 8,400 hours (a bit less than 40 defective parts per 1,000).
(E) 12,800 (around 34 defective parts per 1,000).

31. **(C).** Because the question specifies "on average," refer to the curve marked "Average" rather than the individual data points. The defective part rate is maximized at the top of the curve: operators with 8,000 hours of experience produce about 40 defective parts per 1,000.

32. **(C).** Because the question refers to "individual machine operators," refer to the individual data points rather than the curve marked "Average."

A defective part rate of 4.2% equates to $\frac{4.2}{100}\times 1{,}000 = 42$ defective parts per 1,000. On the chart, there are only 2 data points that fall between 40 and 45 defective parts per 1,000, and they do seem to be at 42 defective parts per 1,000. The less experienced of these two machine operators had just under 8,000 hours of experience.

Problem Set I:

33. **(A).** Note that there are 5 vertical grid lines for every 10 athletes, so each vertical grid line accounts for 2 people. On the track and field roster, there are between 36 and 38 men (so it must be 37) represented by the light gray bar. On the track and field roster, there are between 60 and 62 women (so it must be 61) represented by the dark gray bar. In fraction form, the "ratio of men to women" is $\frac{\text{men}}{\text{women}}$. The correct answer is $\frac{37}{61}$.

34. **(A).** Note that there are 5 vertical grid lines for every 10 athletes, so each vertical grid line accounts for 2 people. Male athletes are represented by the light gray bars for each sport. Sum the male athletes on each of the separate varsity sports rosters.

Males on Volleyball roster: 0
Males on Track and Field roster: between 36 and 38 (so it must be 37)
Males on Tennis roster: between 8 and 10 (so it must be 9)
Males on Golf roster: 10
Males on Cross Country roster: between 16 and 18 (so it must be 17)
Males on Basketball roster: 14

There are 0 + 37 + 9 + 10 + 17 + 14 = 87 male names on all of the rosters combined, but there are only 76 male athletes total. Since tennis, golf, and basketball players are all on one roster only, there must be 87 - 76 = 11 male athletes who are counted twice by being on both the Track and Field team and the Cross Country team. The correct answer is 11.

35. **Golf only.** Male athletes are represented by the light gray bars, female athletes by the dark gray bars. A sport in which male athletes outnumber female athletes will have a shorter dark gray bar than light gray bar.

This is only the case for golf, where there are 10 male athletes and 7 female athletes. Volleyball only has female athletes, so they outnumber the zero male athletes on the roster. In tennis and basketball, there are equal numbers of men and women. Female athletes outnumber male athletes on the Cross Country and Track and Field rosters.

36. **(B).** Note that there are 5 vertical grid lines for every 10 athletes, so each vertical grid line accounts for 2 people. There are between 8 and 10 female tennis players (so it must be 9) represented by the dark gray bar next to "Tennis." There are 14 male basketball players represented by the light gray bar next to "Basketball." In fraction form, the "ratio of female tennis players to male basketball players" is $\frac{\text{female tennis players}}{\text{male basketball players}}$. Thus, the answer is $\frac{9}{14}$.

Problem Set J: This chart compares four stores, providing information about change from 2011 to 2012 in three metrics: total revenue, number of distinct customers, and total costs. It is essential to note that change in total revenue is given in terms of dollars ($), whereas changes in number of distinct customers and in total costs are given only in percent terms. In general, percents provide less information than absolute numbers, as the total (i.e., percent of what?) is needed for context.

37. **(E).** It may be tempting to select Store Z, as revenue increased from 2011 to 2012 while number of distinct customers decreased, but be careful when mixing absolute numbers and percents. Without knowing the revenue in 2012 (only the change from the previous year is known) or the number of customers (only the percent change from the previous year is known) for any of the stores, you cannot determine the answer.

38. **(A).** Because the question is about costs per customer, both of which are given in percent change terms in the chart, and the question asks about percent change for this ratio, a comparison can be made among the stores. The percent change formula in general is $\left(\frac{\text{Difference}}{\text{Original}}\times 100\right)\%$. Thus, the percent change in total costs per distinct customer at a particular store is:

$$\left(\frac{\frac{\text{cost}_{2012}}{\text{customer}_{2012}}-\frac{\text{cost}_{2011}}{\text{customer}_{2011}}}{\frac{\text{cost}_{2011}}{\text{customer}_{2011}}}\times 100\right)\%$$

This looks like a mess, but remember that both cost_{2012} and customer_{2012} can be written in terms of cost_{2011} and customer_{2011}, respectively, based on the percent changes given in the table. Then cost_{2011} and customer_{2011} are in each term of the fraction and can be canceled. For example, for Store W, the percent change in total costs per distinct customer is:

$$\left(\frac{\frac{1.15\times \text{cost}_{2011}}{1.02\times \text{customer}_{2011}}-\frac{\text{cost}_{2011}}{\text{customer}_{2011}}}{\frac{\text{cost}_{2011}}{\text{customer}_{2011}}}\times 100\right)\%=\left(\frac{\frac{1.15}{1.02}-1}{1}\times 100\right)\%=\left(\left(\frac{1.15}{1.02}-1\right)\times 100\right)\%$$

In other words, the magnitude of percent change in total costs per distinct customer depends only on the ratio of (1 + Percent Change in Total Costs) to (1 + Percent Change in Number of Distinct Customers). Perform this comparison for all of the stores.

(A) Store W: $\frac{1.15}{1.02}=1.12745$. GREATEST

(B) Store X: $\frac{1.04}{1.14}=0.91228$

(C) Store Y: $\frac{1.12}{1.05}=1.06667$

(D) Store Z: $\frac{0.80}{0.93}=0.86022$

39. **Store W, Store X, and Store Y.** Because Profit = Revenue - Cost, start by thinking about whether the changes to Revenue and Costs were positive or negative for each of the stores.

(A) CORRECT. Store W: Revenue decreased by \$400,000, and costs increased by 15%. Both changes negatively affect profit.

(B) CORRECT. Store X: Revenue increased by \$520,000, but costs also increased by 4%. Profit in 2012 could be greater than, less than, or equal to profit in 2011, depending on the store's cost structure. Try sample numbers to show that profit could have decreased. If in 2011, revenue was \$20,000,000 and costs were \$15,000,000, the profit was

$5,000,000. In 2012, revenue would be $20,520,000 and costs $15,600,000, making profit $4,920,000, less than in the previous year.

(C) CORRECT. Store Y: Revenue decreased by $365,000, and costs increased by 12%. Both changes negatively affect profit.

(D) Store Z: Revenue increased by $125,000, and costs decreased by 20%. Both changes positively affect profit.

40. **(B)**. Consider each statement individually:

(A) Not necessarily true. While Store X had the greatest increase in revenue, in dollars, it is impossible to calculate percent change in revenue $\left(\frac{\text{Difference}}{\text{Original}}\times 100\right)\%$ for any of the stores without information about the actual dollar amount of their revenue in either year.

(B) TRUE. Per customer revenue is $\frac{\text{Revenue}}{\text{Number of customers}}$. Store Z experienced an increase in revenue and a decrease in number of distinct customers, both of which increase per customer revenue.

(C) Not necessarily true. While Store W had the greatest *percent* increase in total costs, it is impossible to say whether this was the greatest increase *in dollars* without knowing the actual dollar amount of total costs for each of the stores that experienced a cost increase.

(D) Not necessarily true. The chart says nothing about repeat customers, only “distinct” customers.

(E) Not necessarily true. The chart says nothing about absolute numbers of distinct customers at any of the stores, only percent change from 2011 to 2012.

Problem Set K: Much of the detail in this chart is given in the title and other text. According to the title and the * note below, the chart shows range of calories burned per hour by people in the 130 to 205 pound weight range when doing various activities.

41. **I and II only.** Consider each statement individually:

I. Could Be True. A 130 pound person typing for 10 hours would burn less than 1,000 calories, which is less than the number of calories burned by a 205 pound person doing one of several activities on the chart for 1 hour (certainly jai alai and swimming the butterfly stroke burn more than 1,000 calories).

II. Could Be True. In general, the range of calories burned per hour is greater for jai alai than for swimming the butterfly stroke. The people in question are about the same weight, but don’t make any assumptions about how number of calories burned might be a function of weight in this range (i.e., is the relationship linear?). All that matters is that the calorie burning ranges for the two activities overlap, and both people fall in the weight range, so it *could* be true that a 175 pound person playing jai alai for one hour burns fewer calories than a 180 pound person swimming the butterfly stroke for one hour.

III. Cannot Be True. The *average* calories burned by two people playing table tennis for 1 hour is a maximum of about 375. Two people typing for 3 hours burn as many calories *total* as one person typing for 2 × 3 = 6 hours, which is a minimum of about 550. “At most 375” cannot be greater than “at least 550.”

42. **(E).** For each combination of activities, look at the minimum value on the chart for the 130 pound person and the maximum value on the chart for the 205 pound person.

(A) Badminton (minimum) + Table tennis (maximum) = 275 + 375 = 650
(B) Wrestling (minimum) + Track and field, hurdles (maximum) = 350 + 925 = 1,275
(C) Typing (minimum) + Swimming, butterfly stroke (maximum) = under 100 + 1025 = under 1,125
(D) Sailing, competition (minimum) + Aerobics (maximum) = 300 + 600 = 900
(E) CORRECT. Typing (minimum) + Croquet (maximum) = under 100 + over 225 = about 325

Alternatively, note that typing and croquet are the two activities that burn the fewest calories per hour overall. A 130 pound person and a 205 pound person doing 1 hour of activity from the chart each would only burn fewer calories total if both people were typing.

Problem Set L: The table categorizes 50 African countries according to GDP (rows) and population (columns). Notice that each row sums to a subtotal number of countries with that GDP range, and each column sums to a subtotal number of countries with that population range. Both the subtotal row and subtotal column sum to 50, the grand total. Moreover, notice that both population and GDP are shown in descending order: high population/high GDP countries are in the upper left corner of the table, while low population/low GDP countries are in the lower right corner of the table.

43. **(A).** GDP between $10 billion and $20 billion is a single row in the table. Population between 10 and 50 million people includes two columns in the table. Look at the intersections between this row and two columns. There are 3 countries at the intersection of 10 to 20 million population and $10 billion to $20 billion GDP, and there are 3 countries at the intersection of 20 to 50 million population and $10 billion to $20 billion GDP, for a total of 6 countries.

44. **(C).** Adding the entries that are in the intersection of the bottom two rows (less than $20 billion GDP) and in the last three columns (population less than 20 million), the number of countries is 3 + 3 + 3 + 7 + 8 + 7 = 31. Out of 50 countries, 31 fit this description, so the percent is $\left(\frac{31}{50}\times 100\right)\%$ or 62%.

45. **(B).** There are 13 countries with GDPs between $10 billion and $20 billion, and of these, 3 have populations between 10 million and 20 million. Thus the percent is $\left(\frac{3}{13}\times 100\right)\%$ or approximately 23%.

46. **(D).** For each choice, carefully find the row(s)/column(s) that fit the description in, and sum all table entries that apply.

(A) More than $10 billion GDP (top 3 rows) intersecting with population less than 20 million (3 columns on right, before the subtotal column): 0 + 0 + 0 + 1 + 1 + 0 + 3 + 3 + 3 = 11
(B) Less than $20 billion GDP (bottom 2 rows, above the subtotal row) intersecting with population more than 10

million (3 columns on left): 1 + 3 + 3 + 0 + 0 + 7 = 14
(C) More than $20 billion GDP (entire top 2 rows, so sum the subtotal column in those rows): 5 + 10 = 15
(D) Less than $100 billion GDP (bottom 3 rows, above the subtotal row) intersecting with population less than 10 million (2 columns on right, before the subtotal column): 1 + 0 + 3 + 3 + 8 + 7 = 22
(E) Less than $100 billion GDP (bottom 3 rows, above the subtotal row) intersecting with population between 10 million and 50 million (2nd and 3rd column): 7 + 1 + 3 + 3 + 0 + 7 = 21

Choice (D), 22, is the greatest.

Problem Set M: This pie chart represents about 76 million owner-occupied housing units, categorized by how many people live in the household. Don't let the smaller pie chart on the right throw you off. It is just a way to "zoom in" on the relatively small categories of households with at least 5 people. These categories could have been shown as small slivers in the pie chart on the left (notice that 6.7% + 2.6% + 1.8% = 11.1%, the "Other" category in the chart on the left).

47. **(D).** Sum the households with one, two, or three people (i.e., "fewer than four people"). Together these account for 21.6% + 36.3% + 16.5% = 74.4% of the total.

48. **(E).** According to the chart, 6.7% of the 75,986,074 households are 5-person households. In the calculator, multiply 0.067 by 76 (keep "million" in mind). The result is about 5, so the answer is 5 million households.

49. **(B).** Approximate the total number of households as 76 million (close enough to 75,986,074). There are 21.6% of the total, or approximately 16.4 million, 1-person households. Since each such household has only 1 person, this represents about 16.4 million people.

There are 16.5% of the total, or approximately 12.5 million, 3-person households. Since each of these households has 3 people, that is about 37.5 million people.

There are 6.7% of the total, or approximately 5.1 million, 5-person households. Since each of these households has 5 people, that is about 25.5 million people.

Since 16.4 million < 25.5 million < 37.5 million, the correct ranking is 1-person households, 5-person households, 3-person households.

50. **(A).** The 2- to 3- person range contains 36.3% + 16.5% = 52.8% of households, so this is the correct answer. Quickly rule out the other choices as a check.
(B) The 3- to 4- person range contains 16.5% + 14.5% = 31.0% of households.
(C) The 4- to 5- person range contains 14.5% + 6.7% = 21.2% of households.
(D) The 5- to 6-person range contains 6.7% + 2.6% = 9.3% of households.
(E) The 6- to 7-person range contains at most 2.6% + 1.8% = at most 4.4% of households (remember that some of the 1.8% could consist of households with more than 7 people).

All of the choices other than (A) are less than 50%.

Chapter 25
of

Polygons and Rectangular Solids

In This Chapter…

Polygons and Rectangular Solids

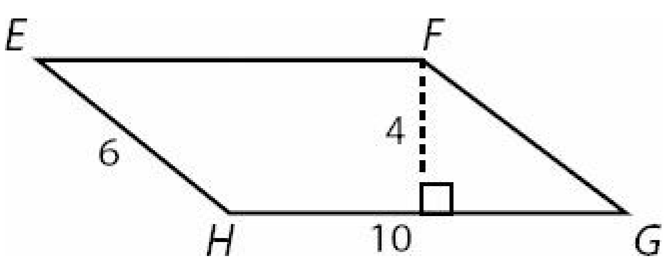
For questions in the Quantitative Comparison format (“Quantity A” and “Quantity B” given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box , you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes , you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

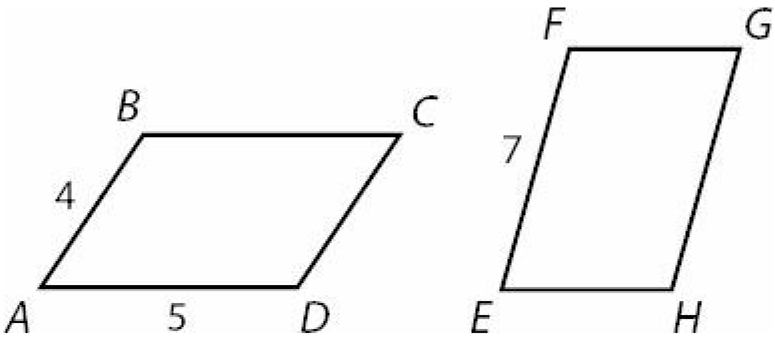
All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.



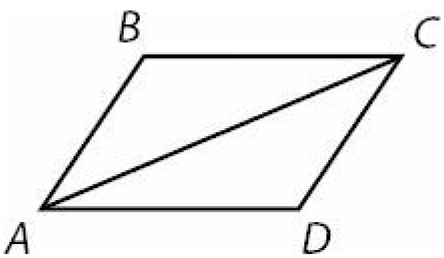
What is the area of parallelogram *EFGH*?

2.



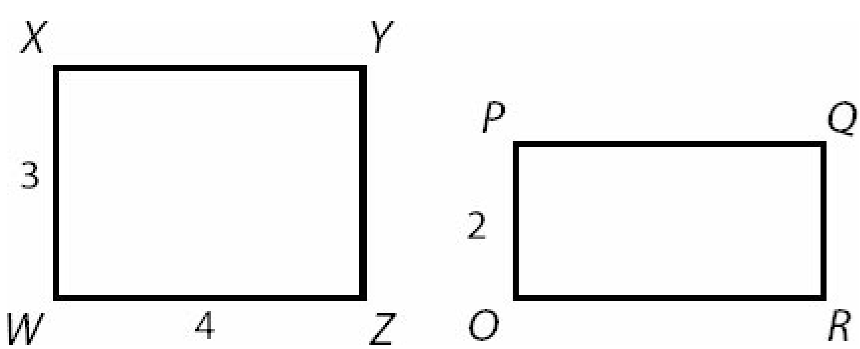
The two parallelograms pictured above have the same perimeter. What is the length of side *EH*?

3.



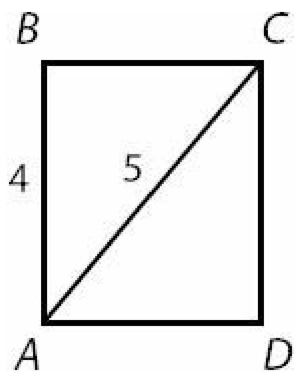
In Parallelogram *ABCD*, Triangle *ABC* has an area of 12. What is the area of Triangle *ACD*?

4.



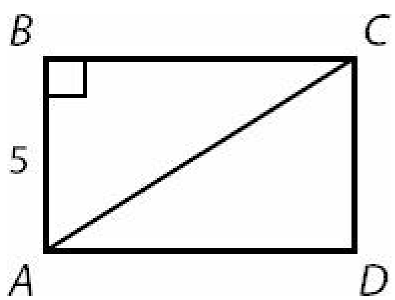
Rectangle *WXYZ* and Rectangle *OPQR* have equal areas. What is the length of side *PQ*?

5.



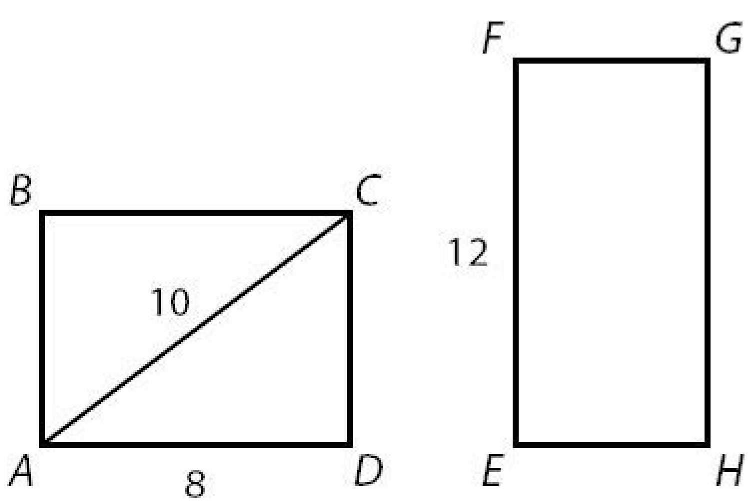
What is the area of Rectangle *ABCD*?

6.



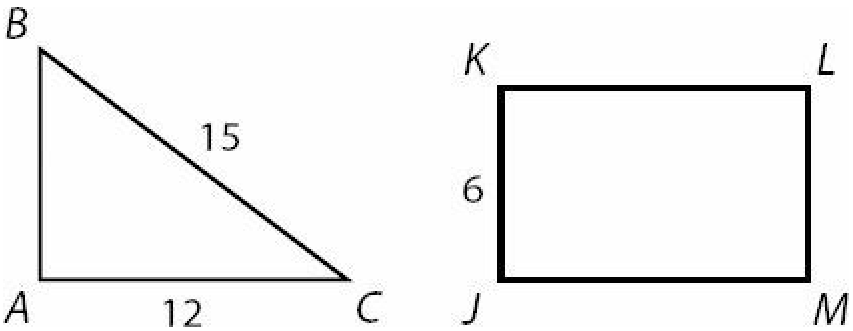
In Rectangle *ABCD*, the area of Triangle *ABC* is 30. What is the length of diagonal *AC*?

7.



Rectangles *ABCD* and *EFGH* have equal areas. What is the length of side *FG*?

8.



Triangle *ABC* and Rectangle *JKLM* have equal areas. What is the perimeter of Rectangle *JKLM*?

9.

Quantity A	Quantity B
The longest side of a rectangle with area 36	6

10.

Quantity A	Quantity B
The area of a rectangle with perimeter 40	110

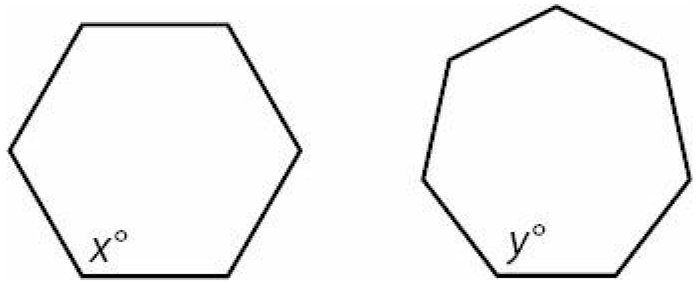
11. What is the area of a square with a diagonal measuring $4\sqrt{2}$ centimeters?

centimeters square

12.

Quantity A	Quantity B
The area of a parallelogram with a base of length 4 and height of 3.5	The area of a trapezoid with two parallel sides of lengths 5 and 9 and a height of 2

13.



Quantity A	Quantity B
x	y

14.

A trapezoid has an area of 42 and a height that is less than or equal to 6.

Quantity A	Quantity B
The height of the trapezoid	The length of the longer base of the trapezoid

15.

The perimeter of square W is 50% of the perimeter of square D.

Quantity A	Quantity B
The ratio of the area of square W to the area of square D	$\frac{1}{4}$

16. A 10 by 15 inch rectangular picture is displayed in a 16 by 24 inch rectangular frame. What is the area, in inches, of the part of the frame not covered by the picture?

(A) 150
(B) 234
(C) 244
(D) 264
(E) 384

17.

A rectangular box has edges of length 2, 3, and 4.

Quantity A	Quantity B
Twice the volume of the box	The surface area of the box

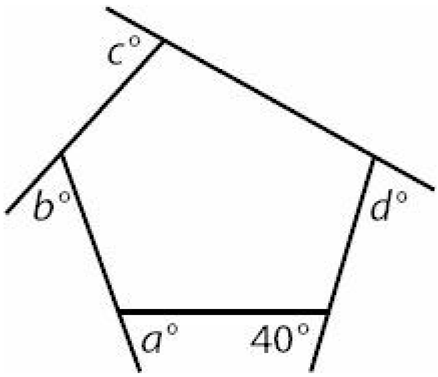
18. A perfect cube has surface area 96. What is its volume?

$\boxed{}$

19. How many 2 inch by 2 inch by 2 inch solid cubes can be cut from six solid cubes that are 1 foot on each side? (12 inches = 1 foot)

(A) 8
(B) 64
(C) 216
(D) 1,296
(E) 1,728

20.



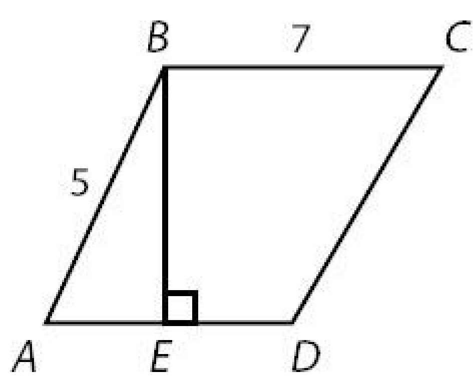
What is the value of $a + b + c + d$?

(A) 240
(B) 320
(C) 360
(D) 500
(E) 540

21. Garden A is a 225 meter by 180 meter rectangular vegetable garden, and Garden B is a rectangle with exactly half the length and width of Garden A. What is the ratio of the area of Garden A to the area of Garden B?

(A) 1 : 4
(B) 1 : 2
(C) 2 : 1
(D) 4 : 1
(E) 8 : 1

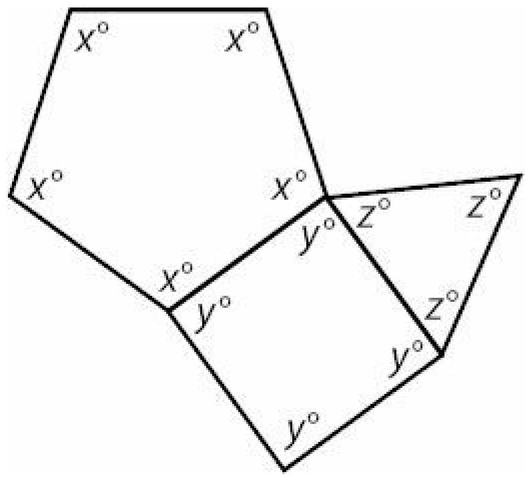
22.



In the trapezoid above, $AE = ED = 3$ and BC is parallel to AD.

Quantity A	Quantity B
The area of the trapezoid	35

23.



Quantity A	Quantity B
The value of $x + y + z$	270

24. A rectangle has an area of $54\sqrt{2}$ and a length of 6. What is the perimeter of the rectangle?

(A) $15\sqrt{2}$

(B) $30\sqrt{2}$

(C) $6 + 9\sqrt{2}$

(D) $12 + 18\sqrt{2}$

(E) $18 + 12\sqrt{2}$

25. A 1 meter by 1 meter by 1 meter sheet of paper is to be cut into 4 centimeter by 5 centimeter rectangles. How many such rectangles can be cut from the sheet of paper? (1 meter = 100 centimeters)

26.

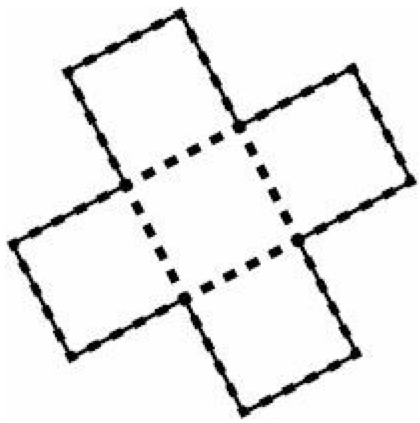
A parallelogram has two sides with length 10 and two sides with length 5.

Quantity A	Quantity B
The area of the parallelogram	30

27. What is the area of a regular hexagon with side length 2?

(A) $2\sqrt{3}$
(B) $2\sqrt{6}$
(C) $6\sqrt{2}$
(D) $6\sqrt{3}$
(E) $12\sqrt{3}$

28.



The figure above is composed of 5 squares of equal area, as indicated by the dotted lines. The total area of the figure is 45.

Quantity A	**Quantity B**
The perimeter of the figure	48

29.

Quantity A	**Quantity B**
The largest possible area of a rhombus with side 4.	The area of a square with side 4.

30. A 2 foot by 2 foot by 2 foot solid cube is cut into 2 inch by 2 inch by 4 inch rectangular solids. What is the ratio of the total surface area of all the resulting smaller rectangular solids to the surface area of the original cube? (1 foot = 12 inches)

(A) 2 : 1
(B) 4 : 1
(C) 5 : 1
(D) 8 : 1
(E) 10 : 1

31. If a cube has the same volume (in cubic units) as surface area (in square units), what is the length of one side?

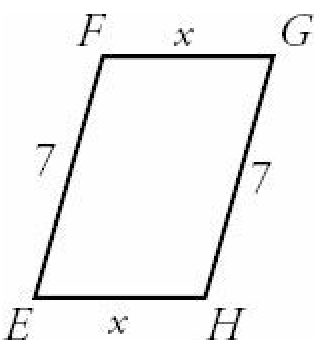
(A) 1
(B) 3
(C) $\frac{5}{3}$

(D) 6
(E) No such cube is possible.

Polygons and Rectangular Solids Answers

1. **40.** The area of a parallelogram is base × height. In this parallelogram, the base is 10 and the height is 4 (remember, base and height need to be perpendicular). So the area is 10 × 4 = 40.

2. **2.** First find the perimeter of Parallelogram *ABCD*. You know that 2 sides have a length of 4, and 2 sides have a length of 5. The perimeter is 2 × (4 + 5) = 18. That means Parallelogram *EFGH* also has a perimeter of 18. You know side *GH* also has a length of 7. You don't know the lengths of the other 2 sides, but you know they have the same length, so for now say the length of each side is *x*. Your parallelogram now looks like this:



So you know that $7 + x + 7 + x = 18 \rightarrow 2x + 14 = 18$ à $2x = 4 \rightarrow x = 2$

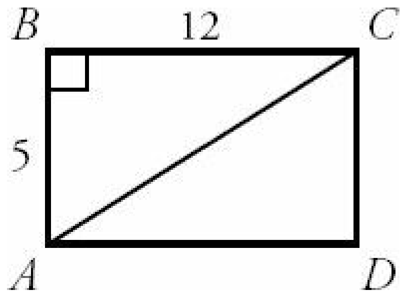
The length of side *EH* is 2.

3. **12.** One property that is true of any parallelogram is that the diagonal will split the parallelogram into two equal triangles. If Triangle *ABC* has an area of 12, then Triangle *ACD* must also have an area of 12.

4. **6.** You can start by finding the area of Rectangle *WXYZ*. Area of a rectangle is length × width, so the area of Rectangle *WXYZ* is 3 × 4 = 12. So Rectangle *OPQR* also has an area of 12. You know the length of side *OP*, so that is the width of Rectangle *OPQR*. So now you know the area, and you know the width, so you can solve for the length. $l \times 2 = 12 \rightarrow l = 6$. The length of side *PQ* is 6.

5. **12.** To find the area of Rectangle *ABCD*, you need to know the length of *AD* or *BC*. In a rectangle, every internal angle is 90 degrees, so Triangle *ABD* is actually a right triangle. That means you can use the Pythagorean Theorem to find the length of side *AD*. Actually, this right triangle is one of the Pythagorean Triplets—a 3-4-5 triangle. The length of side *AD* is 3. That means the area of Rectangle *ABCD* is 3 × 4 = 12.

6. **13.** You know the area of Triangle *ABC* and the length of side *AB*. Because side *BC* is perpendicular to side *AB*, you can use those as the base and height of Triangle *ABC*. So you know that 1/2(5) × (*BC*) = 30. That means the length of side *BC* is 12.



Now you can use the Pythagorean Theorem to find the length of diagonal *AC*, which is the hypotenuse of right triangle *ABC*. You can also recognize that this is a Pythagorean Triplet —a 5–12–13 triangle. The length of diagonal *AC* is 13.

7. **4.** The first thing to notice in this problem is that you can find the length of side *CD*. Triangle *ACD* is a right triangle, and you know the lengths of two of the sides. You can either use the Pythagorean Theorem or recognize that this is one of the Pythagorean Triplets—a 6–8–10 triangle. The length of side *CD* is 6. Now you can find the area of Rectangle *ABCD*. Side *AD* is the length and side *CD* is the width. $8 \times 6 = 48$.

That means that the area of Rectangle *EFGH* is also 48. You can use the area and the length of side *EF* to solve for the length of side *FG*. $12 \times (FG) = 48$. The length of side *FG* is 4.

8. **30.** If you can find the length of side *AB*, then you can find the area of Triangle *ABC*. You can use the Pythagorean Theorem to find the length of side *AB*. $(12)^2 + (AB)^2 = (15)^2 \rightarrow 144 + (AB)^2 = 225 \rightarrow (AB)^2 = 81 \rightarrow AB = 9$. (A 9–12–15 triangle is a 3–4–5 triangle, with all the measurements tripled.)

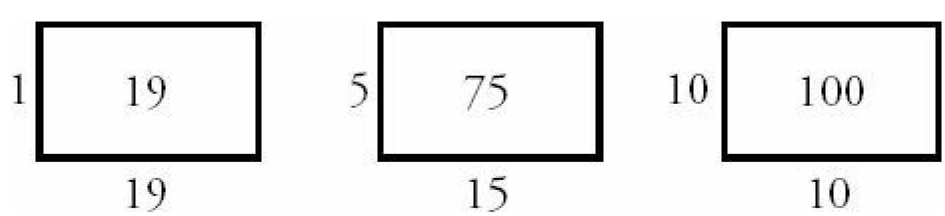
Now that you know *AB*, you can find the area of Triangle *ABC*. It’s $1/2(12) \times 9 = 54$.

That means that Rectangle *JKLM* also has an area of 54. You have one side of the rectangle, so you can solve for the other. $6 \times (JM) = 54$. So the length of side *JM* is 9. That means that the perimeter is $2 \times (6 + 9) = 30$.

9. **(D).** A rectangle with area 36 could have length of 36 and width of 1, or length of 9 and width of 4, or an infinite number of other values, since the problem does not say that the side lengths must be integers. In every example except one, though, the longer sides are longer than 6 (and the shorter sides are less than 6), and Quantity A is greater. The only exception occurs if the rectangle is actually a square. If length is 6 and width is 6, the two quantities are equal. A square is definitely a type of rectangle! It is, in fact, an equilateral rectangle. This exception means the correct answer is (D).

Note also that Quantitative Comparisons are very often more interested in testing weird cases and exceptions to rules than they are in testing your knowledge of straightforward cases.

10. **(B).** While a rectangle with perimeter 40 could have many different areas, all of these areas are less than 110:



How can you be sure this will always be the case? It would be helpful to know the rule that the area of a rectangle with constant perimeter increases as length and width become more similar, and is maximized when the rectangle is a square. Thus, the 10 by 10 version of the rectangle represents the maximum possible area, which is still less than 110.

11. **16.** When a square is cut by a diagonal, two 45–45–90 triangles are created. Use the 45–45–90 formula (sides in the ratio $1 : 1 : \sqrt{2}$) to determine that the sides are equal to 4, and thus the area is $4 \times 4 = 16$. Alternatively, you could label each side of the square x (since they're the same) and use the Pythagorean Theorem:

$$x + x = (4\sqrt{2})^2$$
$$2x^2 = 32$$
$$x^2 = 16$$
$$x = 4$$

Thus, area $= 4 \times 4 = 16$.

12. **(C).** The formula for area of a parallelogram is *base* × *height*, so Quantity A is $4 \times 3.5 = 14$.

The formula for area of a trapezoid is $A = \frac{(b_1 + b_2)}{2} \times h$, where b_1 and b_2 are the lengths of the parallel sides, so Quantity B is $\frac{(5+9)}{2} \times 2 = 14$.

The two quantities are equal.

13. **(D).** Do not assume that any polygon is a regular figure unless you are told this. (For instance, if *every* angle in the hexagon were labeled with the same variable, you could be sure the hexagon was regular).

Using the formula $(n - 2)(180)$ where n is the number of sides, you can calculate that the sum of the angles in the 6-sided figure is 720 and the sum of the angles in the 7-sided figure is 900. However, you do not know how those totals are distributed among the interior angles, so either x or y could be greater.

14. **(B).** The area formula for a trapezoid is $A = \frac{(b_1 + b_2)}{2} \times h$. For a fixed area, the average of the bases is minimized when the height is maximized, and vice versa. If the area is 42 and the maximum height is 6, then $\frac{(b_1 + b_2)}{2}$ is at least 7. Thus, the sum of the bases is at least 14. If two bases sum to 14, the longer base is greater than 7 (or else both bases are equal to 7).

Quantity A is less than or equal to 6.
Quantity B is greater than or equal to 7.

15. **(C).** If one square has twice the perimeter, it has twice the side length, it will have four times the area. Why is this? Doubling only the length doubles the area. Then, doubling the width doubles the area *again*.

You can also prove this with real numbers. Say square W has perimeter 8 and Square D has perimeter 16. Thus, square W has side 2 and Square D has side 4. The areas are 4 and 16, respectively. As a ratio, 4/16 reduces to 1/4.

16. **(B).** The area of the picture is $10 \times 15 = 150$. The area of the frame is $16 \times 24 = 384$. Subtract to get the answer: $384 - 150 = 234$.

17. **(B).** The volume of a rectangular box is $length \times width \times height = 2 \times 3 \times 4 = 24$. Quantity A is double this volume, or 48.

The surface area of a rectangular box is $2(length \times width) + 2(width \times height) + 2(length \times height) = 2(6) + 2(12) + 2(8) = 52$.

Quantity B is greater.

18. **64.** The surface area of a cube is given by the formula Surface Area = $6(side)^2$. Or just think about it logically: since all the faces are the same, the total surface area is 6 times the surface area of a single face. Since the Surface Area = 96:

$$96 = 6(side)^2$$
$$16 = (side)^2$$
$$4 = side$$

The volume of a cube is Volume = $(side)^3 = 4^3 = 64$.

19. **(D).** Each large solid cube is 12 inches × 12 inches × 12 inches. Each dimension (length, width, and height) is to be cut identically at 2 inch increments, creating 6 smaller cubes in each dimension. Thus, $6 \times 6 \times 6$ small cubes can be cut from each large cube. There are 6 large cubes to be cut this way, though, so the total number of small cubes that can be cut is $6(6 \times 6 \times 6) = 1{,}296$.

20. **(B).** The interior figure shown is a pentagon, although an irregular one. The sum of the interior angles of any polygon can be determined using the formula $(n - 2)(180)$, where n is the number of sides:

$$(5 - 2)(180) = (3)(180) = 540$$

Using the rule that angles forming a straight line sum to 180, the interior angles of the pentagon (starting at the top and going clockwise) are $180 - c$, $180 - d$, 140, $180 - a$, and $180 - b$. The sum of these angles can be set equal to 540.

$$540 = (180 - c) + (180 - d) + 140 + (180 - a) + (180 - b)$$
$$540 = 140 + 4(180) - a - b - c - d$$
$$540 - 140 - 720 = -(a + b + c + d)$$
$$-320 = -(a + b + c + d)$$

So, $a + b + c + d = 320$.

21. **(D).** Garden A has an area of $225 \times 180 = 40{,}500$. Garden B has an area of $112.5 \times 90 = 10{,}125$. The answer is 40,500/10,125, which reduces to 4/1, or a 4 : 1 ratio.

There is a more efficient solution, however. Halving only the length of a rectangle will divide the area by 2. Halving only the width will divide the area by 2. So halving both the length and width of the rectangle will divide the area by 4. The ratio is 4 : 1.

22. **(B).** First, note that while the figure may *look* like parallelogram, it is actually a trapezoid, as it has two parallel sides of unequal length ($AD = 6$ and $BC = 7$). A trapezoid has two parallel sides (the bases). The formula for the area of a trapezoid is $A = \frac{(b_1 + b_2)}{2} \times h$, where b_1 and b_2 are the lengths of the parallel sides and h is the height (BE in this figure).

Triangle ABE is a 3–4–5 special right triangle, so BE is 4. (You could also use the Pythagorean Theorem to determine this.)

Thus, the area is $\frac{(6+7)}{2} \times 4 = 26$. Quantity B is greater.

23. **(B).** Each angle in the pentagon is labeled with the same variable, so this is a regular pentagon. Using the formula (n - 2)(180), where n is the number of sides, the sum of all the interior angles of the pentagon is (3)(180) = 540 degrees. Divide by 5 to get $x = 108$.

Now, the quadrilateral. All four-sided figures have interior angles that sum to 360. If you didn't have that memorized, you could also use (n - 2)(180) to determine this. Divide 360 by 4 to get $y = 90$.

Now, the triangle. It is equilateral, so $z = 60$. (The sum of angle measures in a triangle is always 180; if the angles are equal, they will each equal 60.)

Thus, $x + y + z = 108 + 90 + 60 = 258$. Quantity B is greater.

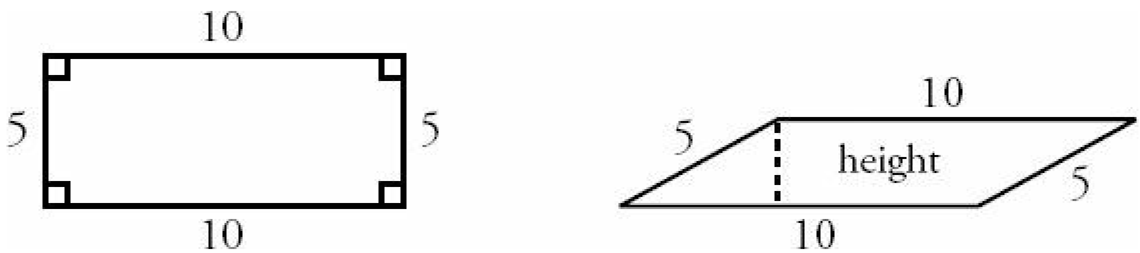
24. **(D).** Since the area of a rectangle is *length* × *width*:

$$54\sqrt{2} = 6 \times width$$
$$9\sqrt{2} = width$$

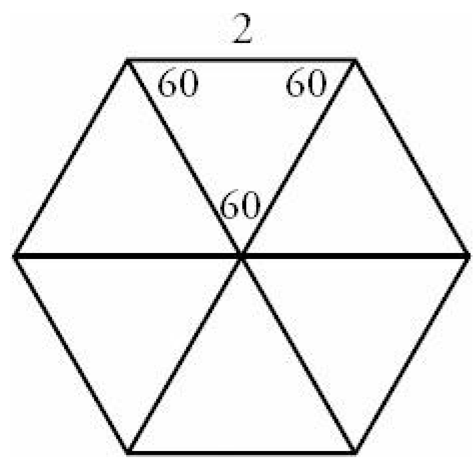
Since perimeter is 2*length* + 2*width*, the perimeter of the rectangle is $2(6) + 2(9\sqrt{2}) = 12 + 18\sqrt{2}$, or choice (D).

25. **500.** Since the sheet of paper is measured in meters and the small rectangles in centimeters, first convert the measures of the sheet of paper to centimeters. The large sheet of paper measures 100 cm by 100 cm. The most efficient way to cut 4 cm by 5 cm rectangles is to cut vertically every 4 cm and horizontally every 5 cm (or vice versa; the idea is that all the small rectangles should be oriented the same direction on the larger sheet). Doing so creates a grid of $\frac{100}{4} \times \frac{100}{5} = 25 \times 20 = 500$ small rectangles.

26. **(D).** The formula for the area of a parallelogram is *base* × *height*, where height is the perpendicular distance between the parallel bases, not necessarily the other side of the parallelogram. However, if the parallelogram is actually a rectangle, the height IS the other side of the parallelogram, and is thereby maximized. So, if the parallelogram is actually a rectangle, the area would be equal to 50, but if the parallelogram has more extreme angle measures, the height could be very, very small, making the area much less than 30.



27. **(D)**. Divide the hexagon with three diagonals (running through the center) to get six triangles. Since the sum of the angles in any polygon is (n - 2)(180), the sum for a hexagon is 720. Divide by 6 to get that each angle is 120. When you divide the hexagon into triangles, you split each 120 to make two 60 degree angles for each triangle. Any triangle that has two angles of 60 must have a third angle of 60 as well, since triangles always sum to 180. Thus, all six triangles are equilateral. Therefore, all three sides of each triangle are equal to 2.

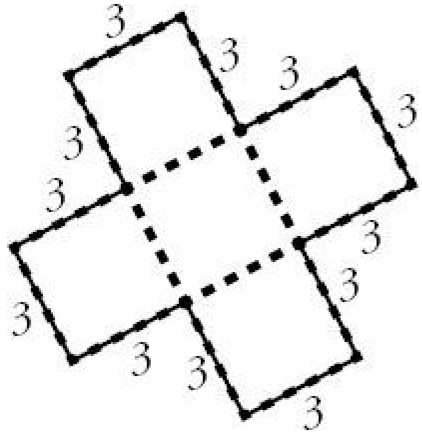


For any equilateral triangle, the height equals half the side times $\sqrt{3}$. Therefore, the height is simply $\sqrt{3}$. Since $A = \frac{bh}{2}$, the area of each equilateral is $\frac{2\sqrt{3}}{2} = \sqrt{3}$.

Since there are six such triangles, the answer is $6\sqrt{3}$.

28. **(B).** If a figure with area of 45 is composed of 5 equal squares, simply divide to get that the area of each square is 9 and thus the side of each square is 3.

Don't make the mistake of adding up EVERY side of every square to get the perimeter—make sure you only count lengths that are actually part of the perimeter of the overall figure. (Note that the central square does not have any lengths that are part of the perimeter).

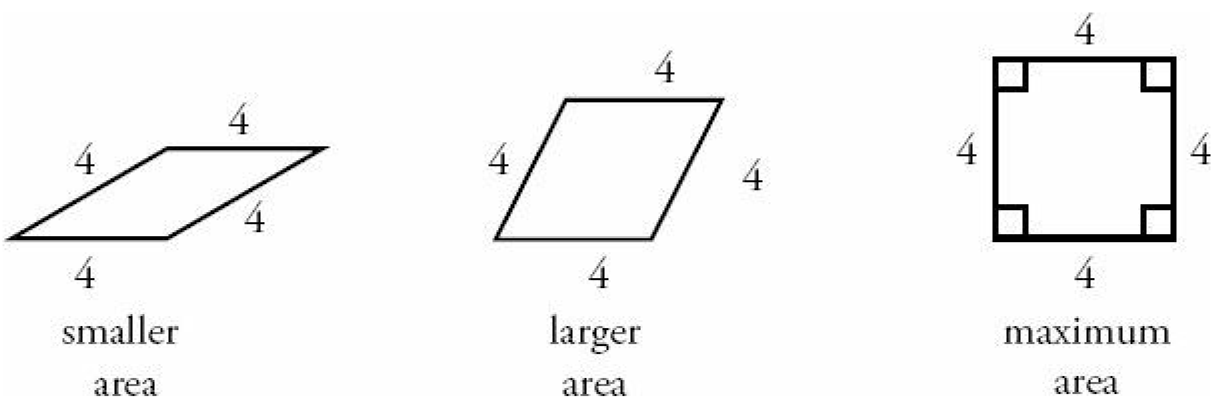


The perimeter is made of 12 segments, each with length 3. The perimeter is 36.

Incorrect choice (A) comes from reasoning that 5 squares have 20 total sides, each of length 3, and thus the combined length would be 60. You also cannot just subtract the four dotted line lengths, as each of these was actually counted twice, as part of the central square and one of the others. This mistake would incorrectly yield choice (C). The best approach here is to make a quick sketch of the figure, label the sketch with what you know, and count up the perimeter.

29. **(C).** First, note that Quantity B is simply 16.

For Quantity A, a rhombus is a parallelogram with 4 sides of equal length. A square is a type of rhombus (specifically, it is a rhombus that has four equal angles). The rhombus with the largest possible area would be a square—a square identical to the one described in Quantity B.



30. **(E).** To find the surface area of the original cube, first convert the side lengths to inches (it is NOT okay to find surface area or volume and then convert using 1 foot = 12 inches; this is only true for straight-line distances). The equation for surface area is $6s^2$. So, the surface area of the large original cube is 6(24 inches)^2 = 3,456 square inches.

Each large solid cube is 24 inches × 24 inches × 24 inches. To cut the large cube into 2 inch by 2 inch by 4 inch rectangular solids, two dimensions (length and width, say) will be sliced every 2 inches, while one dimension (height, say) will be sliced every 4 inches. Thus, $\frac{24}{2}\times\frac{24}{2}\times\frac{24}{4}=12\times12\times6=864$ small rectangular solids can be cut from the large cube.

The equation for the surface area of a rectangular solid is: $2lw + 2wh + 2lh$. In this case, that is 2(2 × 2) + 2(2 × 4) + 2(2 × 4) = 8 + 16 + 16 = 40 square inches per small rectangular solid. There are 864 small rectangular solids, so the total surface area is:

$40 \times 864 = 34{,}560$ square inches.

Finally, the ratio of the total surface area of all the resulting smaller rectangular solids to the surface area of the original cube is the ratio of 34,560 to 3,456. This ratio reduces to 10 to 1.

31. **(D)**. To solve this question, you need the equations for the volume and the surface area of a cube:

Volume = s^3 Surface area = $6s^2$

If a cube has the same volume as surface area, set these equal:

$s^3 = 6s^2$
$s = 6$

Chapter 26
of

Circles and Cylinders

In This Chapter…

Circles and Cylinders

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. A circle has an area of 16π. What is its circumference?

(A) 4π
(B) 8π
(C) 16π
(D) 32π
(E) It cannot be determined from the information given.

2. A circle has a circumference of 20π. What is its area?

(A) 10π
(B) 20π
(C) 40π
(D) 100π
(E) 400π

3. A circle has a circumference of 8. What is its area?

(A) $\frac{4}{\pi}$

(B) $\frac{4}{\pi^2}$

(C) $\frac{16}{\pi}$

(D) $\frac{16}{\pi^2}$

(E) 16π

4. A circle has a diameter of 5. What is its area?

(A) $\frac{25\pi}{4}$

(B) $\frac{25\pi}{2}$

(C) $\frac{25\pi^2}{2}$

(D) 10π

(E) 25π

5. A circle's area equals its circumference. What is its radius?

(A) 1
(B) 2
(C) 4
(D) 8
(E) 16

6.

Circle C has a radius r such that $1 < r < 5$

Quantity A	**Quantity B**
The area of Circle C	The circumference of Circle C

7.

Quantity A	**Quantity B**
The radius of a circle with area 36π	The radius of a circle with circumference 12π

8.

Quantity A	Quantity B
The area of a circle with radius 4	The circumference of a circle with radius 6

9. A circle has radius $\frac{5}{3}$. What is its area?

(A) $\frac{\sqrt{5}\pi}{3}$

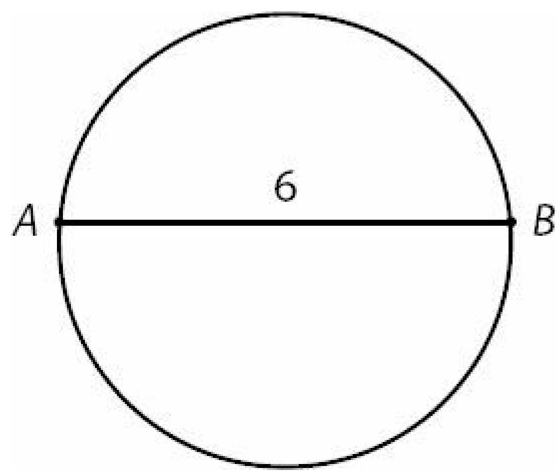
(B) $\frac{5\pi}{3}$

(C) $\frac{25\pi}{9}$

(D) $\frac{10\pi}{3}$

(E) $\frac{100\pi}{9}$

10.



AB is <u>not</u> a diameter of the circle

Quantity A	Quantity B
The area of the circle	9π

11. A circle has radius 0.01. What is its area?

(A) $\frac{\pi}{10}$

(B) $\frac{\pi}{100}$

(C) $\frac{\pi}{1,000}$

(D) $\frac{\pi}{10,000}$

(E) $\frac{\pi}{100,000}$

12. A circle has radius $\sqrt{x}$. What is its circumference?

(A) πx
(B) $2\pi x$
(C) $2\pi\sqrt{x}$
(D) $2\pi x^2$
(E) πx^2

13.

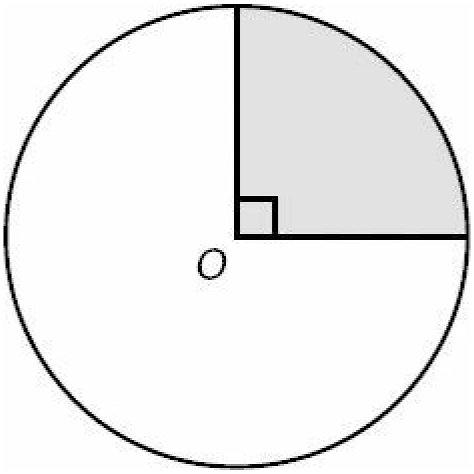
The circumference of a circle is greater than 7π.

Quantity A	**Quantity B**
The area of the circle	15π

14. A circle has an area of 4π. If the radius were doubled, the new area of the circle would be how many times the original area?

(A) 2
(B) 3
(C) 4
(D) 5
(E) It cannot be determined from the information given.

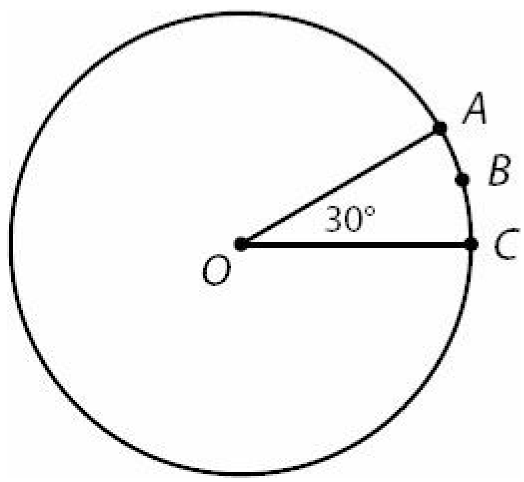
15.



In the figure above, point O is the center of the circle. If the radius of the circle is 8, what is the area of the shaded sector?

(A) 2π
(B) 4π
(C) 8π
(D) 16π
(E) 32π

16.



The radius of the circle with center O is 6.

Quantity A	**Quantity B**
The length of arc ABC	3

17. A sector of a circle has an arc length of 7π. If the diameter of the circle is 14, what is the measure of the central angle of the sector, in degrees?

(A) 45
(B) 60
(C) 90
(D) 120
(E) 180

18. A sector of a circle has a central angle of 270°. If the circle has a radius of 4, what is the area of the sector?

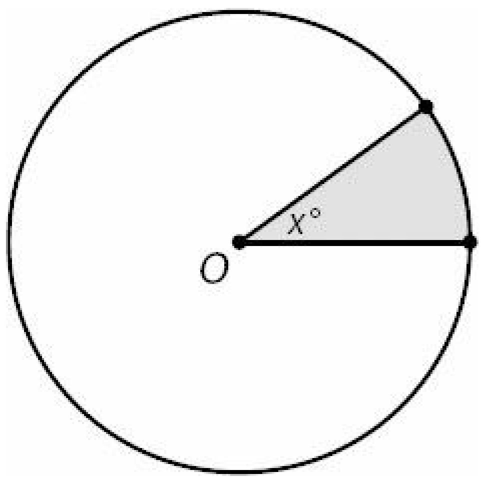
(A) 4π
(B) 8π
(C) 12π
(D) 16π
(E) 20π

19.

Within a circle with radius 12, a sector has an area of 24π.

Quantity A	**Quantity B**
The measure of the central angle of the sector, in degrees	90

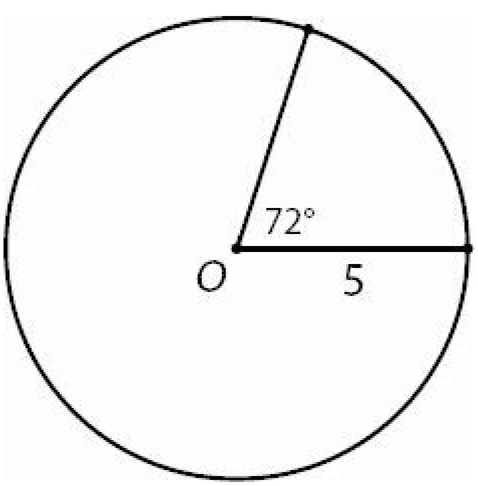
20.



The area of the shaded sector is $\frac{1}{10}$ of the area of the full circle.

Quantity A	**Quantity B**
$2x$	75

21.



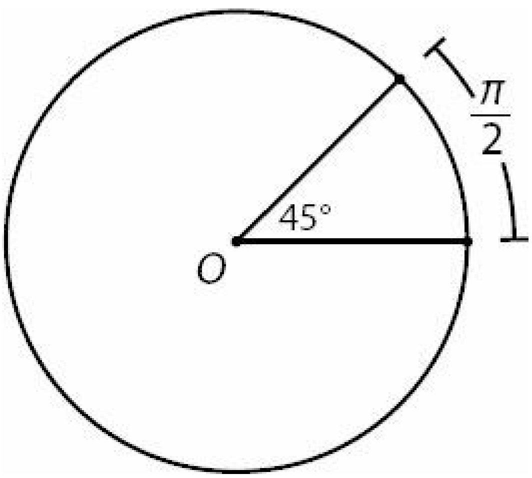
If O is the center of the circle, what is the perimeter of the sector with central angle 72°?

(A) $5 + 2\pi$
(B) $10 + 2\pi$
(C) $10 + 4\pi$
(D) $10 + 5\pi$
(E) $20 + 2\pi$

22. A sector of a circle has a radius of 8 and an area of 8π. What is the arc length of the sector?

(A) π
(B) 2π
(C) 4π
(D) 6π
(E) 8π

23.



If point O is the center of the circle in the figure above, what is the radius of the circle?

24.

Sector A and Sector B are sectors of two different circles.
Sector A has a radius of 4 and a central angle of 90°.
Sector B has a radius of 6 and a central angle of 45°.

Quantity A	**Quantity B**
The area of Sector A	The area of Sector B

25. What is the volume of a right circular cylinder with a radius of 2 and a height of 4?

(A) 8π
(B) 12π
(C) 16π
(D) 32π
(E) 72π

26. What is the height of a right circular cylinder with radius 1 and volume 16π?

27.

A right circular cylinder has volume 24π.

Quantity A	**Quantity B**
The height of the cylinder	The radius of the cylinder

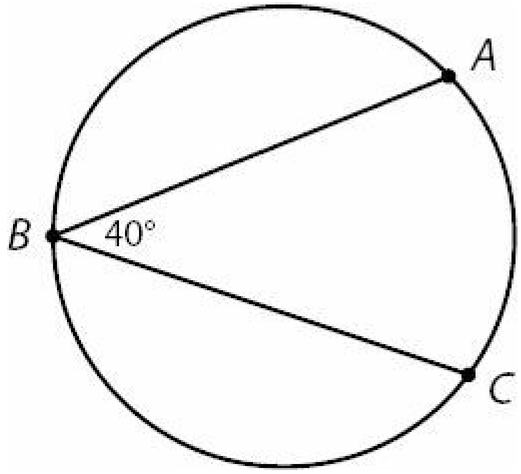
28. If a half-full 4-inch by 2-inch by 8-inch box of soymilk is poured into a right circular cylindrical glass with radius 2 inches, how many inches high will the soymilk reach? (Assume that the capacity of the glass is greater than the volume of the soymilk.)

(A) 8
(B) 16
(C) $\frac{4}{\pi}$
(D) $\frac{8}{\pi}$
(E) $\frac{16}{\pi}$

29. If a right circular cylinder's radius is halved and its height doubled, by what percent will the volume increase or decrease?

(A) 50% decrease
(B) no change
(C) 25% increase
(D) 50% increase
(E) 100% increase

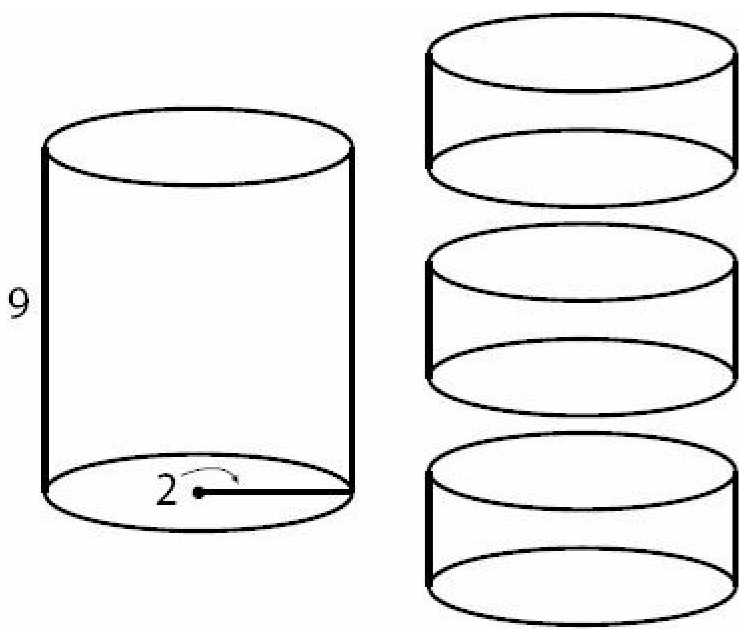
30.



If the diameter of the circle is 36, what is the length of arc *ABC*?

(A) 8
(B) 8π
(C) 28π
(D) 32π
(E) 56π

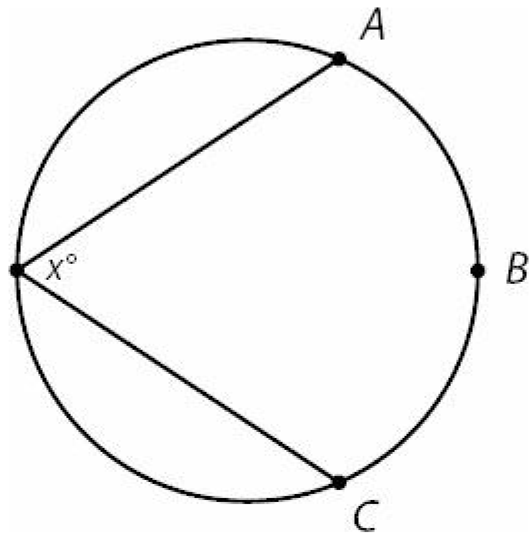
31.



If a solid right circular cylinder with height 9 and radius 2 is cut as shown into three new cylinders, each of equal and uniform height, how much new surface area is created?

(A) 4π
(B) 12π
(C) 16π
(D) 24π
(E) 36π

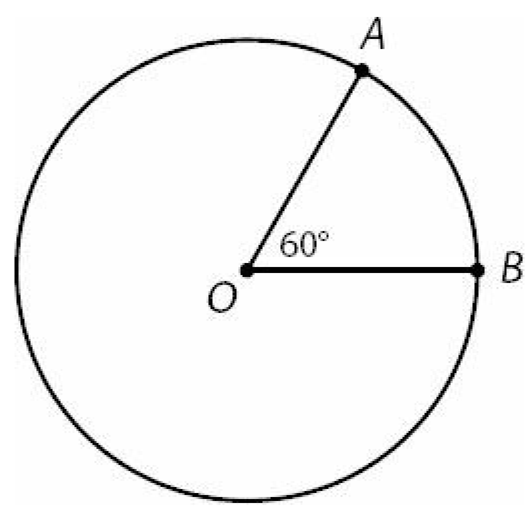
32.



$x > 60°$

Quantity A	**Quantity B**
The ratio of the length of arc *ABC* to the circumference of the circle	1/3

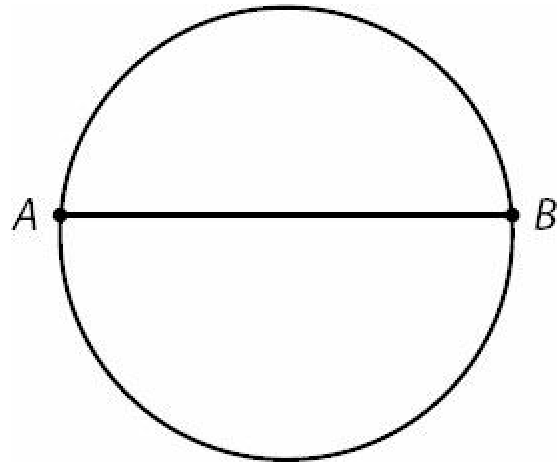
33.



Point *O* is the center of the circle above.

Quantity A	**Quantity B**
The ratio of the length of minor arc *AB* to major arc *AB*	1/6

34.



The circle above has area 25.

Quantity A	**Quantity B**
The length of chord *AB*	10

Circles and Cylinders Answers

1. **(B).** Since the area formula for a circle is $A = \pi r^2$:

$16\pi = \pi r^2$
$16 = r^2$
$4 = r$

Since the circumference formula is $C = 2\pi r$ and $r = 4$:

$C = 2\pi(4)$
$C = 8\pi$

2. **(D).** Since the circumference formula is $C = 2\pi r$:

$20\pi = 2\pi r$
$20 = 2r$
$10 = r$

Since the area formula for a circle is $A = \pi r^2$ and $r = 10$:

$A = \pi(10)^2$
$A = 100\pi$

3. **(C).** Since the circumference formula is $C = 2\pi r$:

$8 = 2\pi r$

Note that the circumference is just 8, not 8π, so the radius is going to look a bit unusual. First, divide both sides by 2:

$4 = \pi r$
$\frac{4}{\pi} = r$

Now, plug the radius $\frac{4}{\pi}$ into the area formula for a circle:

$$A = \pi\left(\frac{4}{\pi}\right)^2$$

$$A = \pi \times \frac{16}{\pi^2}$$

$$A = \frac{16}{\pi}$$

4. **(A).** If a circle's diameter is 5, its radius is $\frac{5}{2}$. Plug this into the area formula:

$$A = \pi\left(\frac{5}{2}\right)^2$$

$$A = \pi \times \frac{25}{4}$$

$$A = \frac{25\pi}{4}$$

5. **(B).** To find the radius that would make the area and the circumference of a circle equal, simply set the area and circumference formulas equal to one another:

$$\pi r^2 = 2\pi r$$

Since both sides have both r and π, divide both sides by πr:

$$r = 2$$

6. **(D).** Picking numbers is the easiest way to prove (D). If you begin with a radius of 3, the area is 9π and the circumference is 6π, so Quantity A is greater. If you try a radius of 4, the area is 16π and the circumference is 8π, so once again Quantity A is greater. But if you try a radius of 2, both the area and the circumference equal 4π. Therefore, Quantity A is not always greater, so the answer is (D). Note also that r is not required to be an integer. If you try a value close to the minimum, such as 1.1, Quantity B would be greater.

7. **(C).** Since the area formula for a circle is $A = \pi r^2$, calculate Quantity A by plugging 36π into the formula as the area:

$$36\pi = \pi r^2$$
$$36 = r^2$$
$$6 = r$$

Since the circumference formula for a circle is $C = 2\pi r$, calculate Quantity B by plugging 12π into the formula as the circumference:

$$12\pi = 2\pi r$$
$$12 = 2r$$
$$6 = r$$

The two quantities are equal. In other words, a circle with area 36π will also have circumference 12π.

8. **(A).** Since the area formula for a circle is $A = \pi r^2$, calculate Quantity A by plugging radius 4 into the formula:

$$A = \pi(4)^2$$
$$A = 16\pi$$

Since the circumference formula for a circle is $C = 2\pi r$, calculate Quantity B by plugging radius 6 into the formula:

$$C = \pi 2(6)$$
$$C = 12\pi$$

Quantity A is greater.

9. **(C).** Since the area formula for a circle is $A = \pi r^2$, plug radius $\frac{5}{3}$ into the formula:

$$A = \pi\left(\frac{5}{3}\right)^2$$

$$A = \frac{25\pi}{9}$$

10. **(A).** Since a diameter is the longest straight line you can draw from one point on a circle to another (that is, a diameter is the longest chord in a circle), the actual diameter must be *greater* than 6.

IF the diameter were 6, the radius would be 3, and the area would be:

$$A = \pi(3)^2$$
$$A = 9\pi$$

However, since the diameter must be greater than 6, the area must be greater than 9π. Do NOT make the mistake of picking (D) for Quantitative Comparison geometry questions in which you cannot "solve." There is often still a way to determine which quantity is greater.

11. **(D).** Since the formula for the area of a circle is $A = \pi r^2$, plug radius 0.01 into the formula. However, since the answers are in fraction format, it is probably easier to convert to fraction form now rather than at the end. Since

$0.01 = \frac{1}{100}$ (you can verify this in your calculator):

$$A = \pi\left(\frac{1}{100}\right)^2$$

$$A = \pi\left(\frac{1}{10,000}\right)$$

$$A = \frac{\pi}{10,000}$$

12. **(C).** Since the circumference formula for a circle is $C = 2\pi r$, plug $\sqrt{x}$ in for the radius:

$$C = 2\pi\sqrt{x}$$

This expression does not simplify — no more work is required. Note that incorrect answer choice (A) is the result of accidentally using the area formula rather than the circumference formula.

13. **(D).** The circumference is "greater than 7π." Do not make the mistake of thinking that it has to be at least 8π! There is NO rule that the number before the π must be an integer. If the circumference of the circle were 8π, the radius would be 4 and the area therefore 16π, making Quantity A greater.

However, the circumference of the circle could be 7.5π, in which case the radius would be 3.75 and the area would be 14.0625π, making Quantity B greater. Thus, the answer is (D).

14. **(C).** To begin, find the original radius of the circle: Area = $\pi r^2 = 4\pi$, so $r = 2$. Once doubled, the new radius is 4. A circle with a radius of 4 has an area of 16π. The new area of 16π is 4 times the old area of 4π.

15. **(D).** If the sector has a central angle of 90°, then the sector is 1/4 of the circle, because $\frac{90}{360} = \frac{1}{4}$. To find the area of the sector, first find the area of the whole circle. The radius is 8, which means the full circle area is $\pi(8)^2 = 64\pi$. If the circle area is 64π, then the sector's area is $1/4 \times 64\pi = 16\pi$.

16. **(A).** If the sector has a central angle of 30°, then it is 1/12th of the circle, because $\frac{30}{360} = \frac{1}{12}$. To find the arc length of the sector, first find the circumference of the entire circle. The radius of the circle is 6, so the circumference is $2\pi(6) = 12\pi$. That means that the arc length of the sector is $(1/12)(12\pi) = \pi$. Since π is about 3.14, Quantity A is greater.

17. **(E).** To find the central angle of the sector, first determine what fraction of the full circle the sector represents. The diameter of the circle is 14, so the circumference is $\pi(14) = 14\pi$. Since the arc length is 7π, the sector is 1/2 the full circle. That means that the central angle of the sector is 1/2 of 360°, or 180°.

18. **(C).** The sector is 3/4 of the circle, because $\frac{270}{360}=\frac{3}{4}$. To find the area of the sector, first find the area of the whole circle. The radius of the circle is 4, so the area is $\pi(4)^2 = 16\pi$. That means the area of the sector is $(3/4)(16\pi) = 12\pi$.

19. **(B).** First find the area of the whole circle. The radius is 12, which means the area is $\pi(12)^2 = 144\pi$. Since the sector has an area of 24π and $\frac{24\pi}{144\pi}=\frac{1}{6}$, the sector is 1/6th of the entire circle. That means that the central angle is 1/6th of 360, or 60°.

20. **(B).** If the area of the sector is $\frac{1}{10}$ of the area of the full circle, then the central angle is $\frac{1}{10}$ of the degree measure of the full circle, or $\frac{1}{10}$ of 360 = 36 = x. Thus, Quantity A = 2(36) = 72.

21. **(B).** To find the perimeter of a sector, you need the radius of the circle and the arc length of the sector. Begin by determining what fraction of the circle the sector is. The central angle of the sector is 72°, so the sector is $\frac{72}{360}=\frac{1}{5}$ of the circle. The radius is 5, so the circumference of the circle is $2\pi(5) = 10\pi$. The arc length of the sector is 1/5 of the circumference: $(1/5)(10\pi) = 2\pi$. The perimeter of the sector is simply this 2π, plus the two radii that make up the straight parts of the sector: $10 + 2\pi$.

22. **(B).** Compare the given area of the sector to the calculated area of the circle. The radius of the circle is 8, so the area of the circle is $\pi(8)^2 = 64\pi$. The area of the sector is 8π, or $\frac{8\pi}{64\pi}=\frac{1}{8}$ of the circle. The radius is 8, so the circumference of the whole circle is $2\pi(8) = 16\pi$. Since the sector is 1/8 of the circle, the arc length is $(1/8)(16\pi) = 2\pi$.

23. **2.** If the sector has a central angle of 45°, then the sector is $\frac{45}{360}=\frac{1}{8}$ of the circle. Thus, the arc length of the sector is 1/8 of the circumference of the circle, or stated differently, the circumference is 8 times the arc length of the sector. The circumference is $8(\pi/2) = 4\pi$. From the circumference formula, $4\pi = 2\pi r$, so $r = 2$. The radius of the circle is 2.

24. **(B).** Sector A is $\frac{90}{360}=\frac{1}{4}$ of the circle with radius 4. The area of this circle is $\pi(4)^2 = 16\pi$, so the area of Sector A is 1/4 of 16π, or 4π.

Sector B is $\frac{45}{360}=\frac{1}{8}$ of the circle with radius 6. The area of this circle is $\pi(6)^2 = 36\pi$, so the area of Sector B is 1/8 of 36π or 4.5π.

4.5π is greater than 4π, so the area of Sector B is greater than the area of Sector A.

25. **(C).** Use the formula for volume of a right circular cylinder, $V = \pi r^2 h$. (This formula is easy to memorize as it is simply the area of a circle, multiplied by height). $V = \pi(2)^2(4) = 16\pi$.

26. **16.** From the formula for volume of a right circular cylinder, $V = \pi r^2 h$:

$$16\pi = \pi(1)^2 h$$
$$16 = h$$

27. **(D).** Plugging into the formula for volume of a right circular cylinder, $V = 24\pi = \pi r^2 h$. However, there are many combinations of r and h that would make the volume 24π. For instance, $r = 1$ and $h = 24$, or $r = 4$ and $h = 1.5$. Keep in mind that the radius and height don't even have to be integers, so there truly are an infinite number of possibilities, some for which h is greater and some for which r is greater.

28. **(D).** A box is a rectangular solid whose volume formula is simply V = length × width × height. So, the volume of the box is 4 inches × 2 inches × 8 inches = 64 inches^3. Since the box is half full, there are 32 inches^3 of soymilk. This volume will not change when the soymilk is poured from the box into the cylinder. The formula for the volume of a cylinder is $V = \pi r^2 h$, so:

$32 = \pi(2)^2 h$, where r and h are in units of inches.

$$\frac{32}{4\pi} = h$$

$$\frac{8}{\pi} = h$$

The height is $8/\pi$ inches. Note that the height is "weird" (divided by π) because the volume of the cylinder did *not* have a π.

29. **(A).** According to the formula for the volume of a right circular cylinder, the original volume is $V = \pi r^2 h$. To halve the radius, simply replace r with $r/2$. To double the height, simply replace h with $2h$. The only caveat: be sure to use parentheses!

$$V = \pi\left(\frac{r}{2}\right)^2 (2h) = \frac{2\pi r^2 h}{2^2} = \frac{\pi r^2 h}{2}$$

Thus, the volume, which was once $\pi r^2 h$, is now $\frac{\pi r^2 h}{2}$. In other words, it has been cut in half, or reduced by 50%.

Alternatively, plug in numbers. If the cylinder originally had radius 2 and height 1, the volume would be $V = \pi(2)^2(1) = 4\pi$. If the radius were halved to become 1 and the height were doubled to become 2, the volume would be $V = \pi(1)^2(2) = 2\pi$. Again, the volume is cut in half, or reduced by 50%.

30. **(C).** Note that a MINOR arc is the "short way around" the circle from one point to another, and a MAJOR arc is the "long way around." Arc *ABC* is thus the same as major arc *AC*.

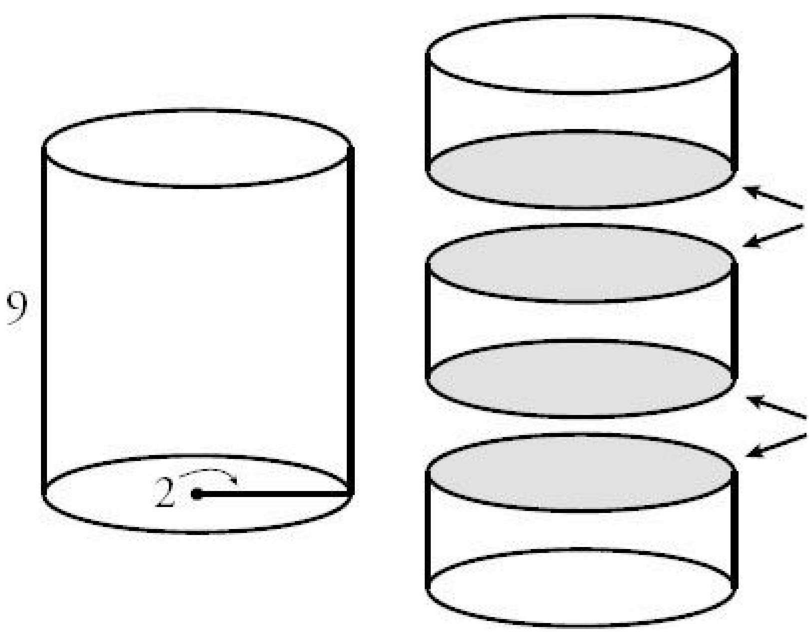
For a given arc, an inscribed angle is always half the central angle, which would be 80° in this case. The minor arc *AC* is thus $\frac{80}{360} = \frac{2}{9}$ of the circle. Since the circumference is 36π:

$$\text{minor arc } AC = \frac{2}{9}(36\pi) = 8\pi$$

Arc *ABC*, or major arc *AC*, is the entire circumference minus the minor arc:

$$36\pi - 8\pi = 28\pi$$

31. **(C).** You *could* find the surface area of the large cylinder, then the surface areas of the three new cylinders, then subtract the surface area of the large cylinder from the combined surface areas of the three new cylinders. However, there is a much faster way. When the large cylinder is cut into three smaller ones, only a few *new* surfaces are created — the bottom base of the top cylinder, the top and bottom bases of the middle cylinder, and the top surface of the bottom cylinder.



Thus, these four circular bases represent the new surface area created. Since the radius of each base is 2, use the area formula for a circle, $A = \pi r^2$:

$$A = \pi(2)^2$$
$$A = 4\pi$$

Since there are 4 such bases, multiply by 4 to get 16π.

32. **(A).** If x were equal to 60°, arc ABC would have a central angle of 120°. (Inscribed angles, with the vertex at the far side of the circle, are always half the central angle.) A 120° arc is 120/360 = 1/3 of the circumference of the circle. Since x is actually greater than 60°, the arc is actually greater than 1/3 of the circumference. Thus, the ratio of the arc length to the circumference is greater than 1/3.

33. **(A).** Since the angle that determines the arc is equal to 60 and 60/360 = 1/6, minor arc AB is 1/6 of the circumference of the circle. (There are always 360 degrees in a circle. Minor arc AB is the "short way around" from A to B, while major arc AB is the "long way around.")

Since minor arc AB is 1/6 of the circumference, major arc AB must be the other 5/6. Therefore, the ratio of the minor arc to the major arc is 1 to 5 (NOT 1 to 6!) You could calculate this as $\frac{1/6}{5/6} = \frac{1}{6} \times \frac{6}{5} = \frac{1}{5}$, or you could just reason the ratio of 1 of *anything* (such as sixths) to 5 of the same thing (again, sixths) is a 1 to 5 ratio.

Of course, the trap answer here is (C). This is a common mistake. 1/6 of the total is not the same as a 1 to 6 ratio of two parts.

34. **(B).** The equation for the area of a circle is $A = \pi r^2$. Note that the given area is just 25, *not* 25π! So:

$\pi r^2 = 25$
$r^2 = \frac{25}{\pi} \approx 8$
r = a bit less than 3.

So the diameter of the circle is a bit less than 6. The diameter is the chord with maximum length, so wherever AB is on this circle, it's significantly shorter than 10.

Chapter 27
of

Triangles

In This Chapter…

Triangles

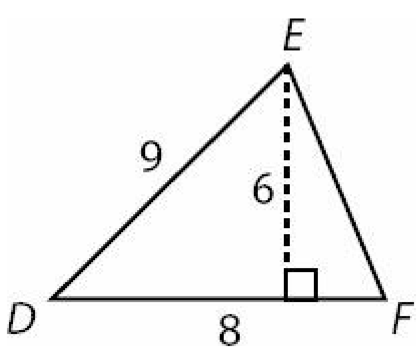
For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

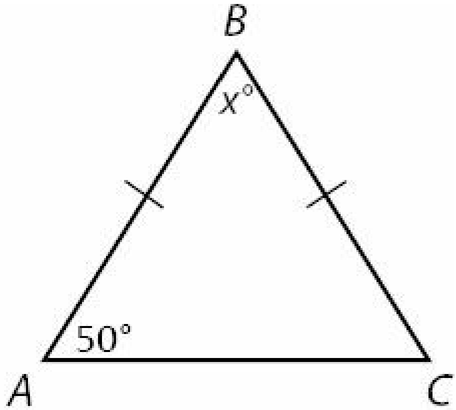
All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. What is the area of Triangle *DEF*?



(A) 23
(B) 24
(C) 48
(D) 56
(E) 81

2. What is the value of *x*?



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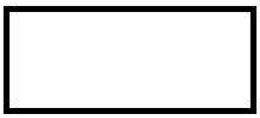
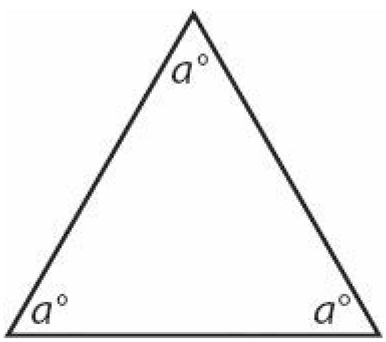
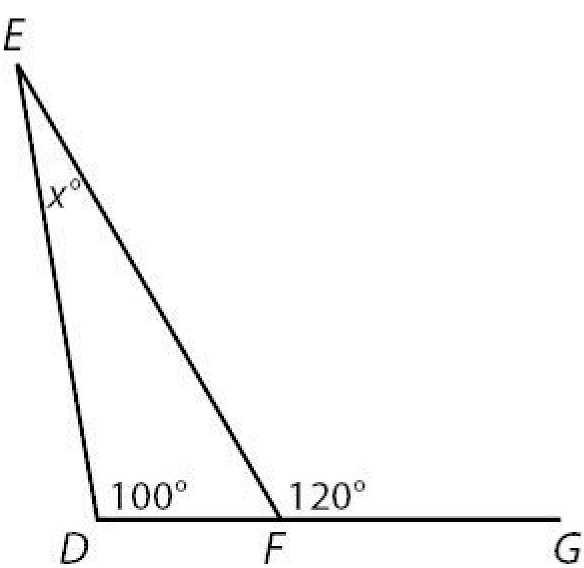


3.



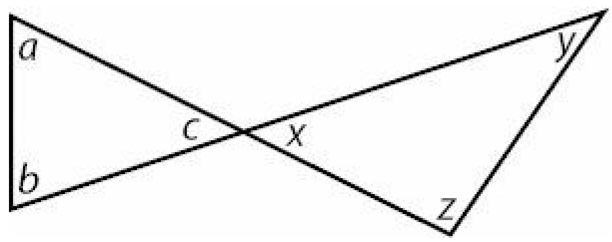
Quantity A	Quantity B
$2a + b$	$3a + \frac{b}{3}$

4.



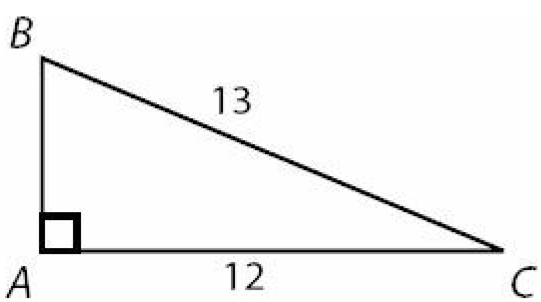
DFG is a straight line. What is the value of *x*?

5



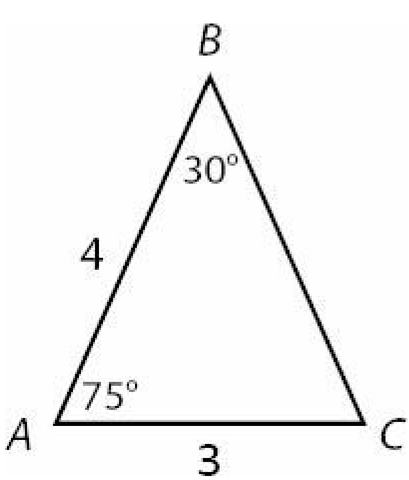
Quantity A	**Quantity B**
$a + b + x$	$c + y + z$

6.



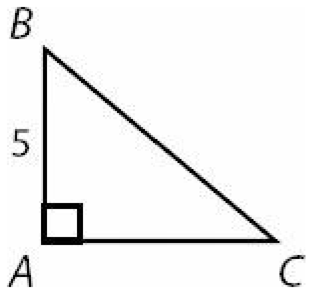
What is the area of right triangle *ABC*?

7.



What is the perimeter of triangle *ABC*?

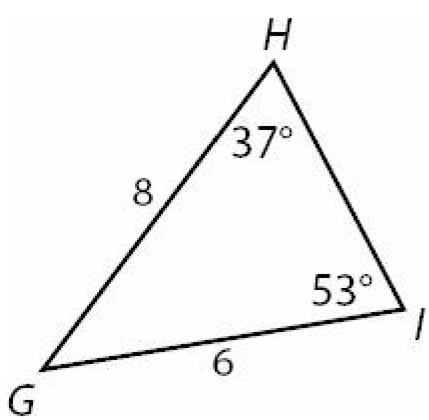
8.



The area of right triangle *ABC* is 15. What is the length of hypotenuse *BC*?

(A) $\sqrt{34}$
(B) 6
(C) $\sqrt{51}$
(D) $\sqrt{61}$
(E) $\sqrt{71}$

9.

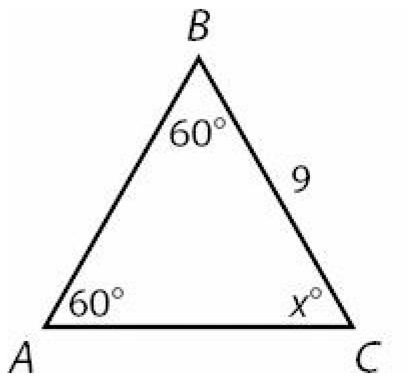


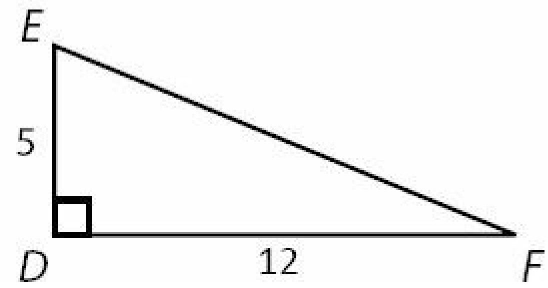
What is the length of side *HI*?

10. If the hypotenuse of an isosceles right triangle is $7\sqrt{2}$, what is the area of the triangle?

(A) 14
(B) 18
(C) 24.5
(D) 28
(E) 49

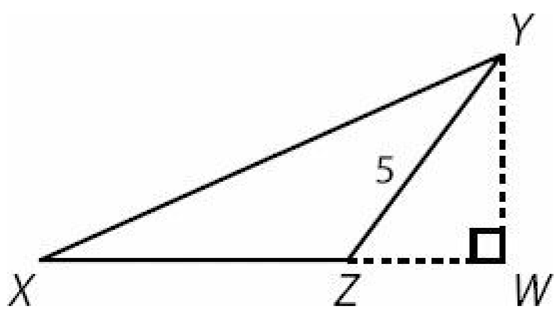
11.





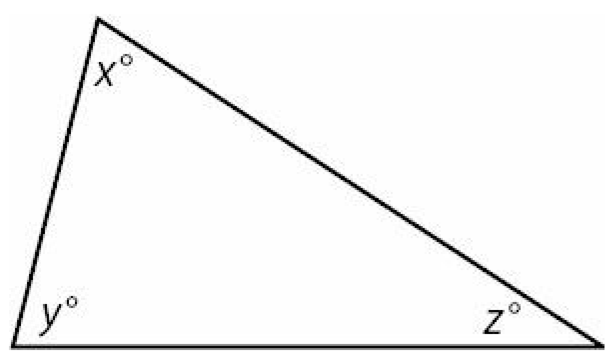
Quantity A	**Quantity B**
Perimeter of triangle *ABC*	Perimeter of triangle *DEF*

12.



WZ has a length of 3 and *ZX* has a length of 6. What is the area of Triangle *XYZ*?

13.



In the figure shown, $z + x$ is 110 degrees.

Quantity A	**Quantity B**
x	y

14.

Isosceles triangle *ABC* has two sides with lengths 8 and 5.

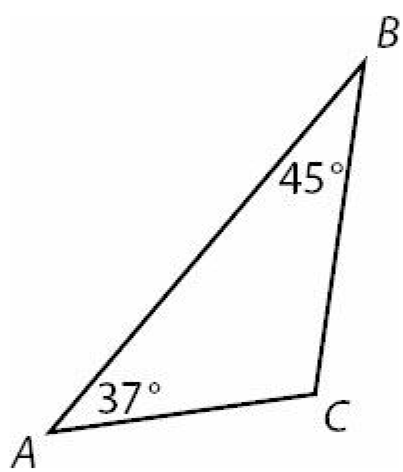
Quantity A	**Quantity B**
The length of the third side	8

15.

Isosceles triangle *ABC* has two sides with lengths 2 and 11.

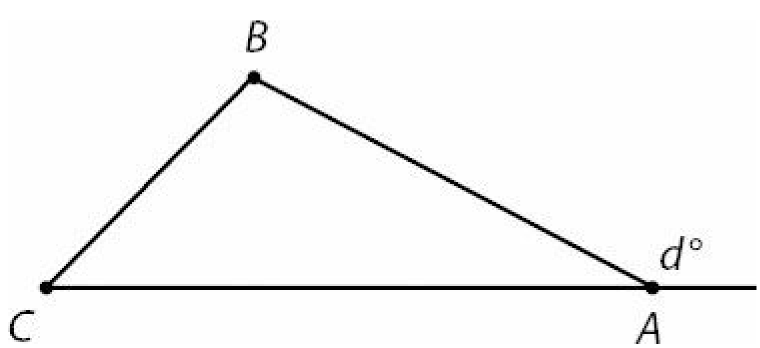
Quantity A	**Quantity B**
The length of the third side	11

16.



Quantity A	**Quantity B**
Side length *AC*	Side length *BC*

17.



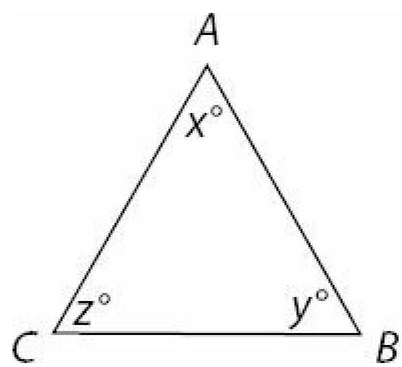
Quantity A	**Quantity B**
The sum of the measures of angles *B* and *C*	*d*

18.

The sides of a right triangle are 3, 4, and z.

Quantity A	**Quantity B**
z	5

19.

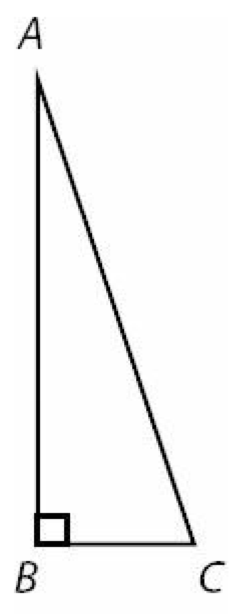


Note: Figure NOT drawn to scale

$x > z$
$z > 60$

Quantity A	**Quantity B**
The length of side AC	The length of side AB

20.



$AC = 4\sqrt{10}$

BC is 1/3 the length of AB

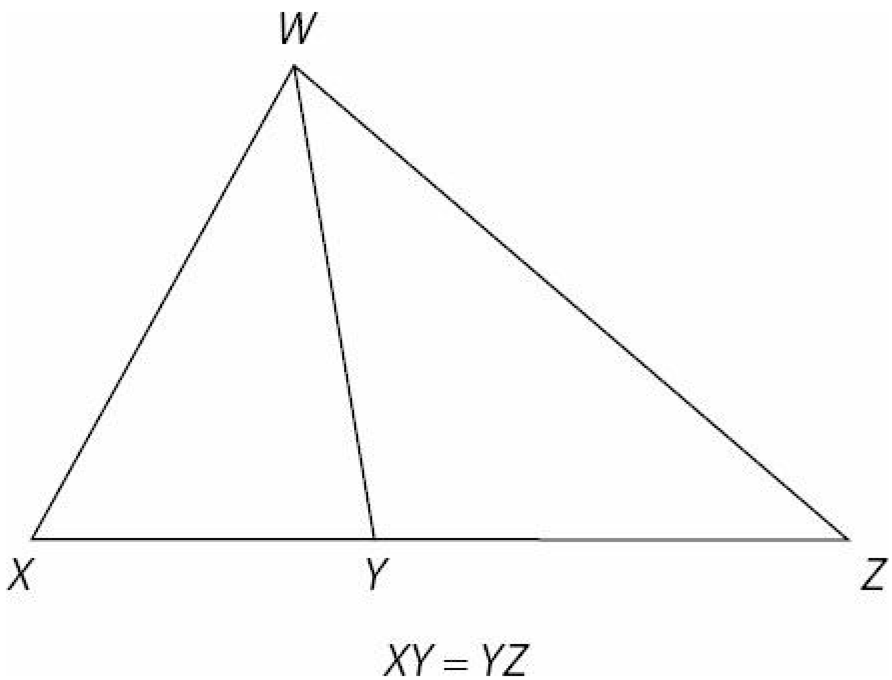
Quantity A	**Quantity B**
The length of *AB*	10

21.

A triangle has perimeter 24.

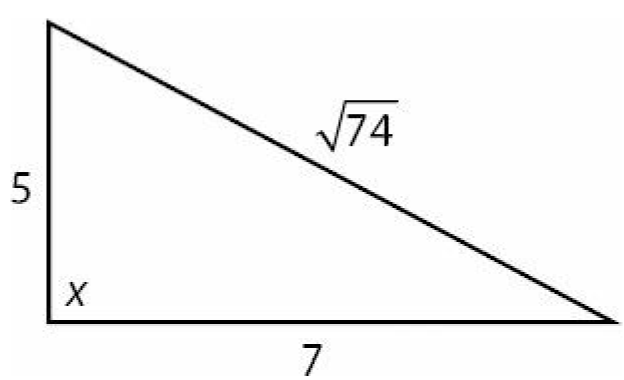
Quantity A	**Quantity B**
The area of the triangle	20

22.



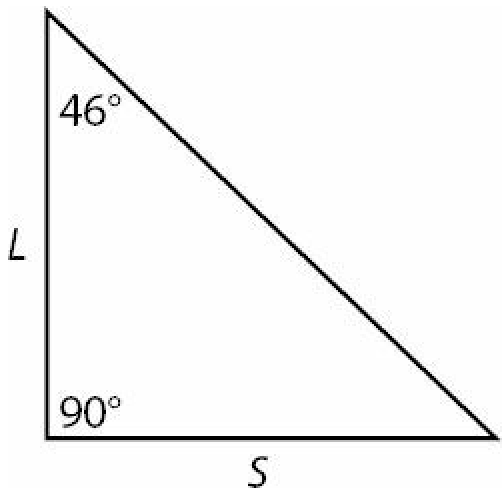
Quantity A	**Quantity B**
The area of *WYX*	The area of *ZYW*

23.



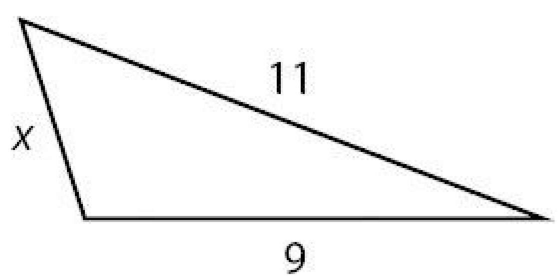
Quantity A	Quantity B
x	90

24.



Quantity A	Quantity B
$\frac{L}{S}$	1

25.



x is an integer

Quantity A	Quantity B
The number of possible values of x	17

26. If p is the perimeter of a triangle with one side of 6 and another side of 9, what is the range of possible values for p?

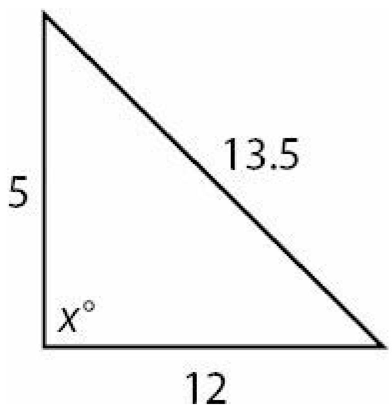
(A) $3 < p < 15$
(B) $15 < p < 24$
(C) $18 < p < 30$
(D) $18 < p < 42$
(E) $21 < p < 42$

27.

A right triangle has hypotenuse 8 and legs of 6 and x.

Quantity A	**Quantity B**
x	10

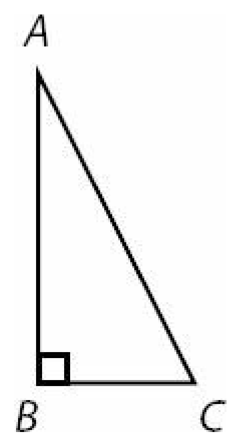
28.



Note: Figure NOT drawn to scale

Quantity A	**Quantity B**
x	90

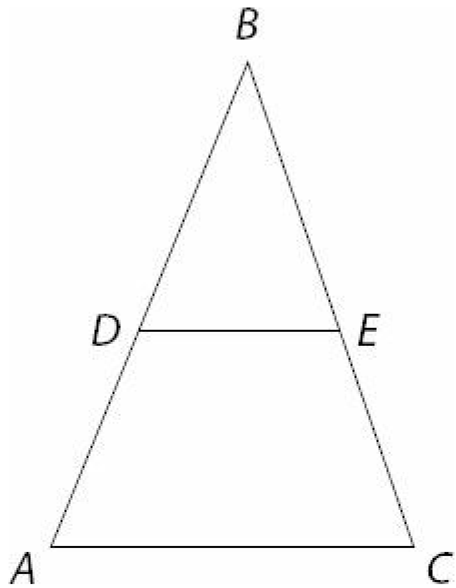
29.



The length of BC is equal to x AB is twice as long as BC

Quantity A	**Quantity B**
The length of AC	$x\sqrt{3}$

30.



DE is parallel to *AC*
BE = *EC*
AC = 14

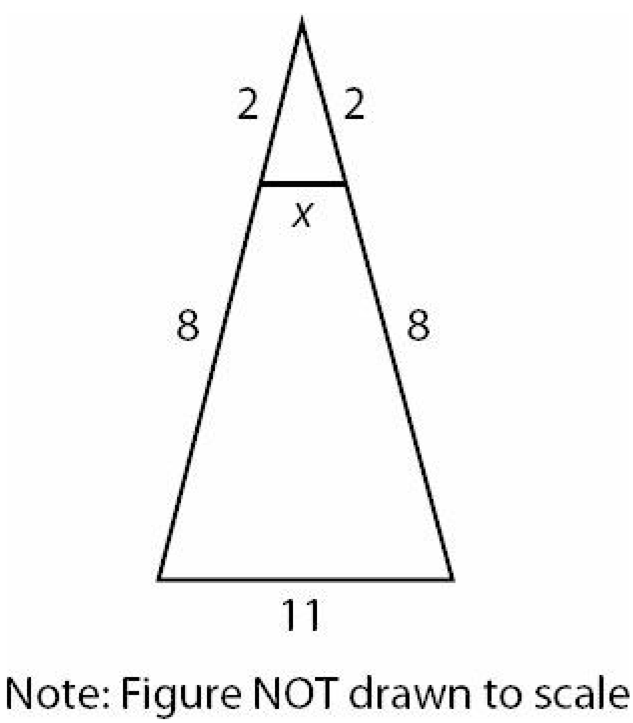
Quantity A	**Quantity B**
DE	7

31.

A triangle has sides of 8, *m,* and *n*.

Quantity A	**Quantity B**
$\lvert m - n \rvert$	8

32.



If the line segment with length x is parallel to the line segment with length 11, what is the value of x?

(A) 1

(B) $\sqrt{2}$

(C) $\frac{11}{5}$

(D) $\frac{11}{4}$

(E) 5.5

33.

Two sides of a triangle have measures 13 and 9.

Quantity A	**Quantity B**
The measure of the third side of the triangle.	$\sqrt{226}$

34. What is the area of an equilateral triangle with side length 4?

(A) $2\sqrt{3}$

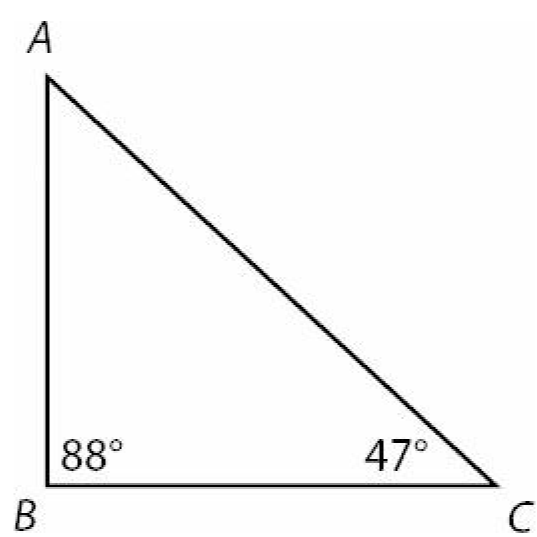
(B) 4.5

(C) $4\sqrt{2}$

(D) $4\sqrt{3}$

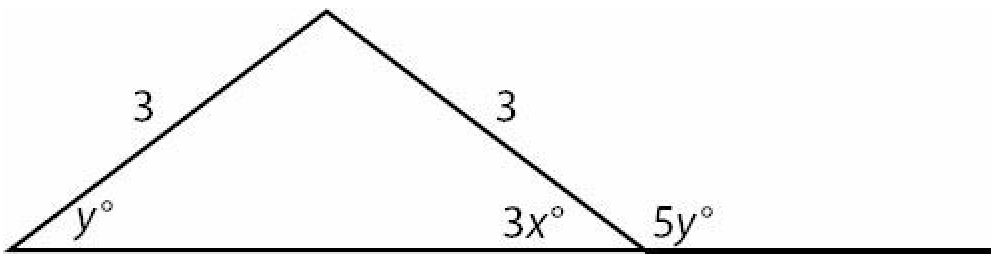
(E) 8

35. Triangle *ABC* is given below with angle measures *B* and *C* given.



Quantity A	Quantity B
The length of side *AB*	The length of side *BC*

36.



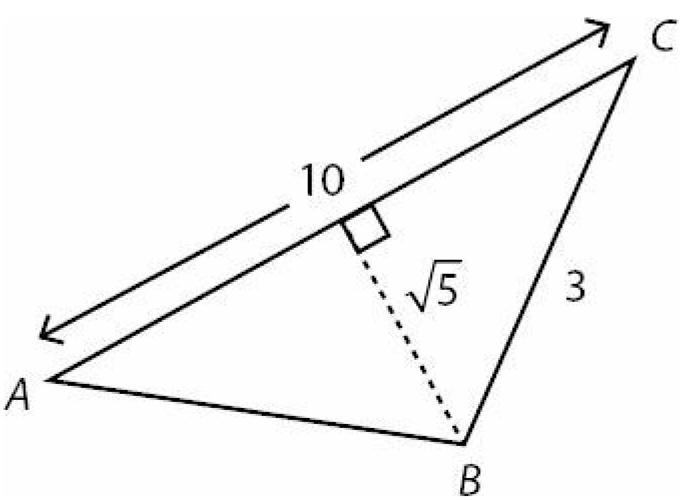
What is the value of x in the figure above?

(A) 5
(B) 10
(C) 18
(D) 30
(E) 54

37. An isosceles right triangle has an area of 50. What is the length of the hypotenuse?

(A) 5
(B) $5\sqrt{2}$
(C) $5\sqrt{3}$
(D) 10
(E) $10\sqrt{2}$

38.

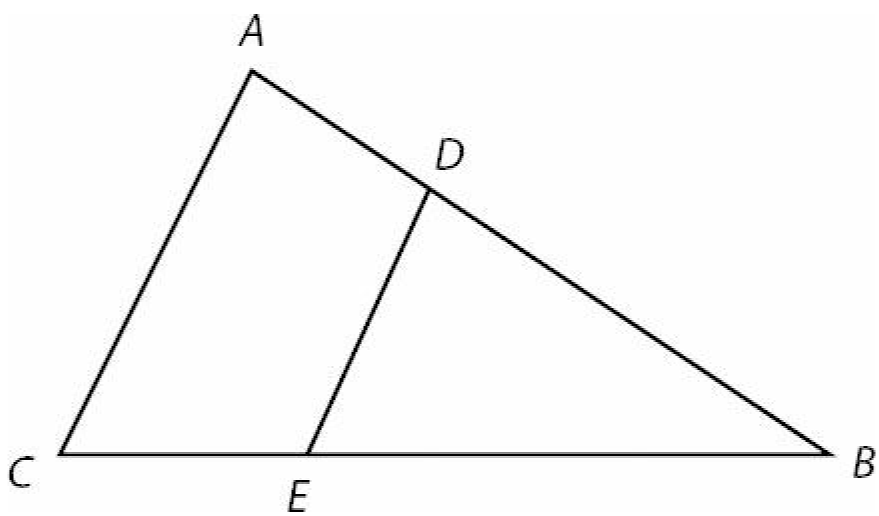


Note: Figure NOT drawn to scale

In the figure above, what is the length of side *AB*?

(A) 5
(B) $\sqrt{30}$
(C) $5\sqrt{2}$
(D) 8
(E) $\sqrt{69}$

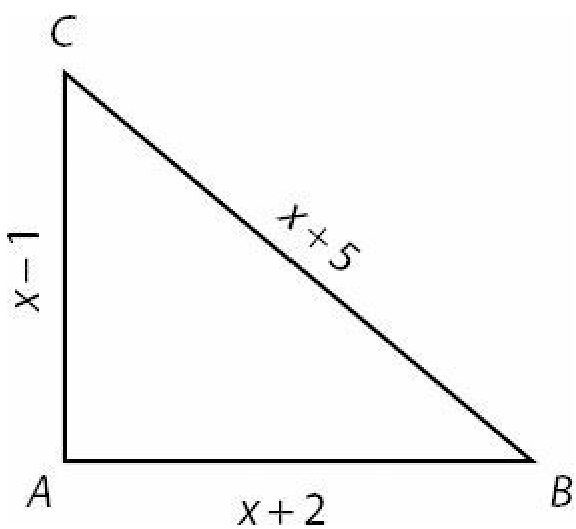
39. In the figure below, *AC* is parallel to *DE* and the length of *DE* is equal to the length of *EB*.



Note: Figure NOT drawn to scale

Quantity A	**Quantity B**
The length of side *AC*	The length of side *CB*

40.

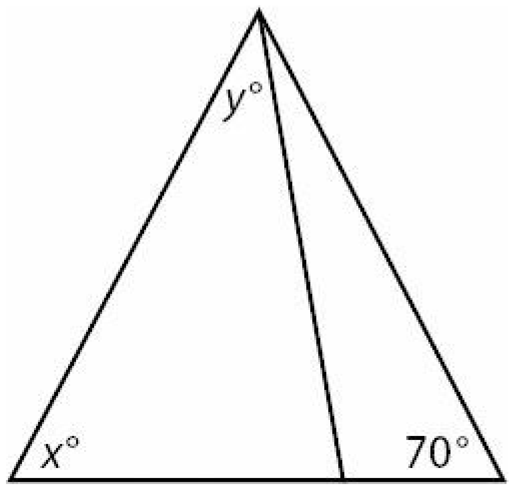


In the right triangle above, what is the length of *AB*?

(A) 9
(B) 10
(C) 12
(D) 13

(E) 15

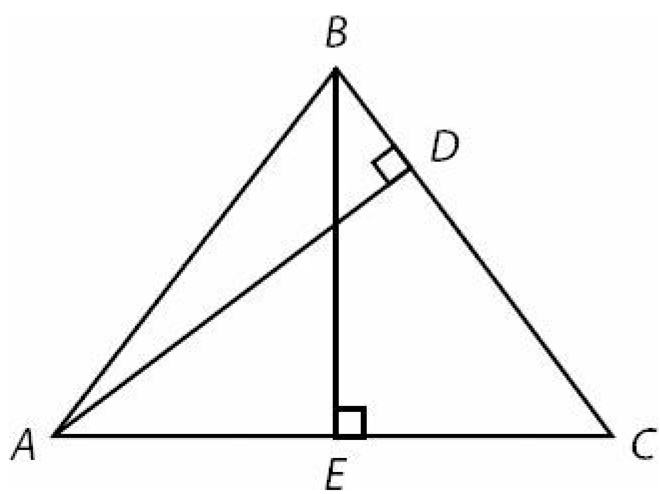
41.



Note: Figure NOT drawn to scale

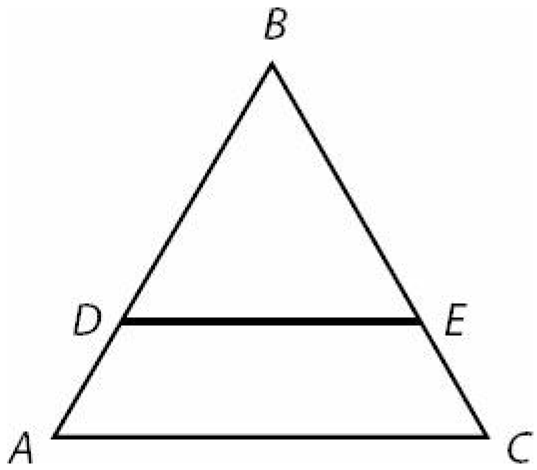
Quantity A	Quantity B
$x + y$	110

42.



Quantity A	Quantity B
The product of *BE* and *AC*	The product of *BC* and *AD*

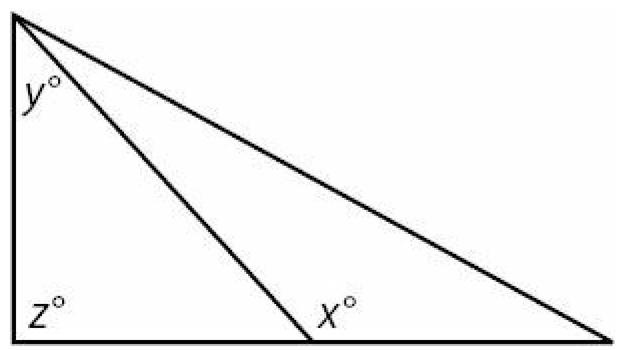
43.



In the figure above, *DE* and *AC* are parallel lines. If $AC = 10$, $DE = 6$, and $CE = 2$, what is the length of side *BC*?

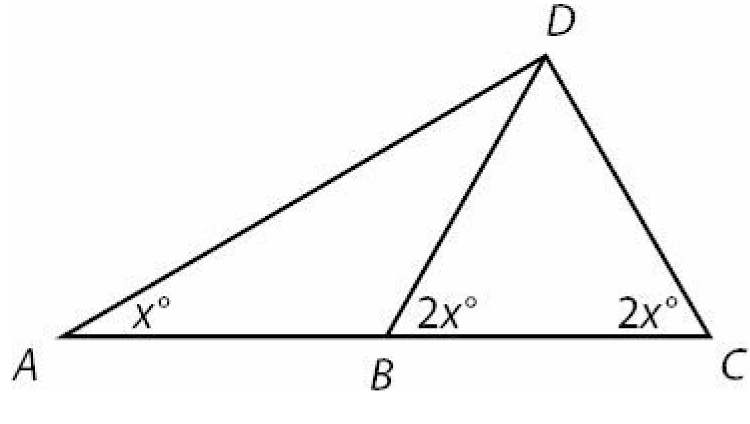
(A) 2
(B) 3
(C) 5
(D) 6
(E) 8

44.



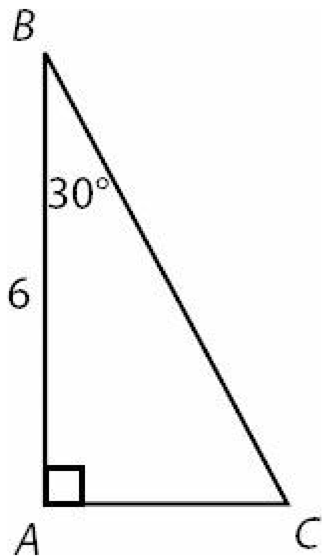
Quantity A	**Quantity B**
x	$y + z$

45.



Quantity A	Quantity B
The length of side *DC*	The length of side *AB*

46.



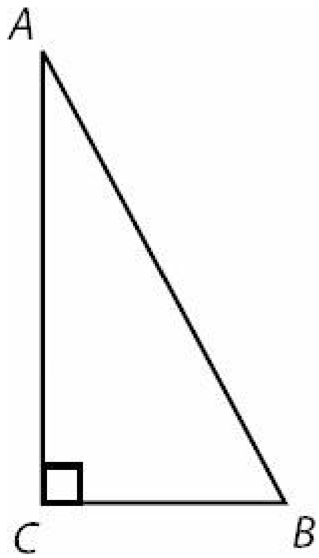
What is the perimeter of right triangle *ABC* above?

(A) $6 + 4\sqrt{3}$
(B) $6 + 6\sqrt{3}$
(C) $6 + 8\sqrt{3}$
(D) $9 + 6\sqrt{3}$
(E) $18 + 6\sqrt{3}$

47. A 10 foot ladder leans against a vertical wall and forms a 60 degree angle with the floor. Assuming the ground below the ladder is perfectly horizontal, how far above the ground is the top of the ladder?

(A) 5 feet
(B) $5\sqrt{3}$ feet
(C) 7.5 feet
(D) 10 feet
(E) $10\sqrt{3}$ feet

48.

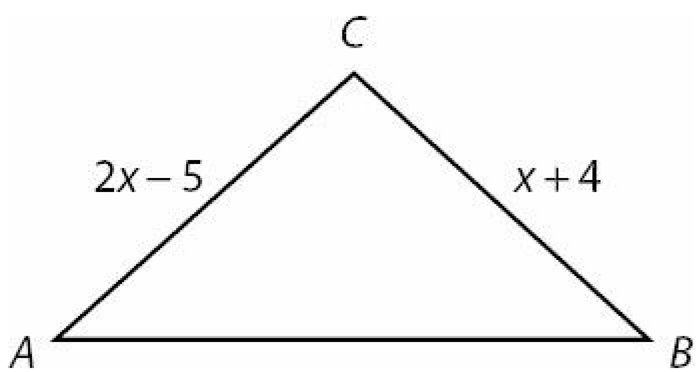


Note: Figure NOT drawn to scale

Triangle *ABC* has area 36. If side *AC* is twice as long as side *CB*, what is the length of side *AB*?

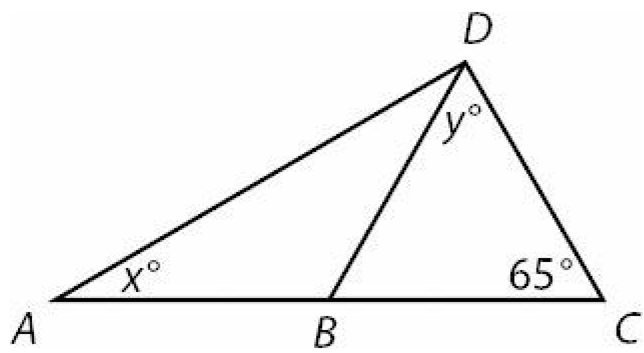
(A) 6
(B) 12
(C) $6\sqrt{5}$
(D) 18
(E) $12\sqrt{5}$

49. In the figure shown, the measure of angle *A* is equal to the measure of angle *B*.



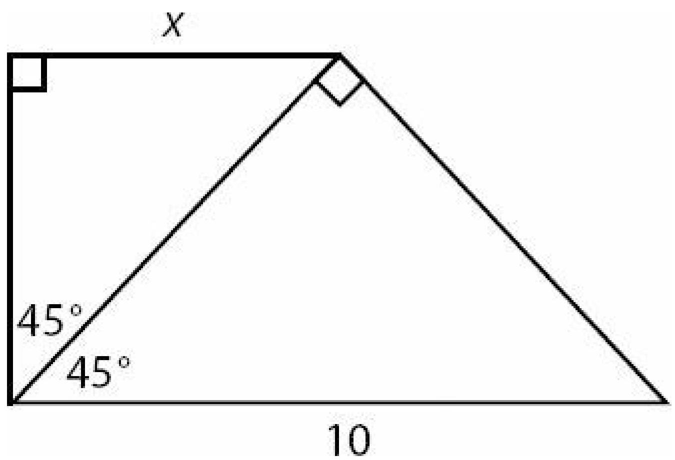
Quantity A	Quantity B
Side length *CB*	7

50. In the figure shown, side lengths *AB*, *BD*, and *DC* are all equal.



Quantity A	**Quantity B**
x	y

51.



In the figure shown, what is the value of x?

(A) 2.5

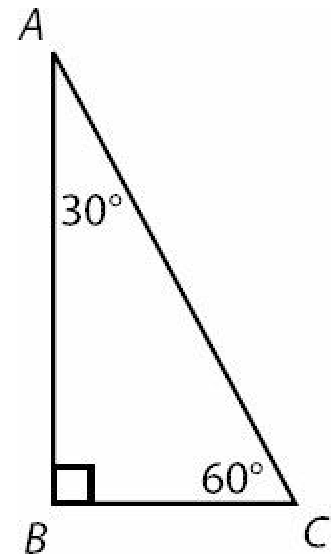
(B) $\frac{5}{\sqrt{2}}$

(C) 5

(D) $5\sqrt{2}$

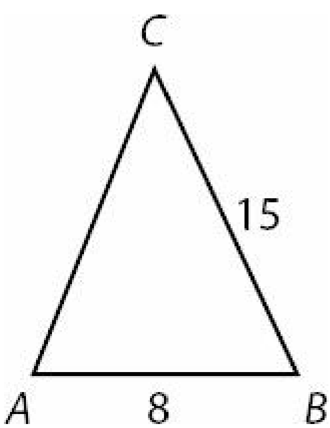
(E) $\frac{10}{\sqrt{2}}$

52.



Quantity A	Quantity B
The ratio of the length of side *BC* to the length of side *AB*	$\frac{10}{17}$

53.

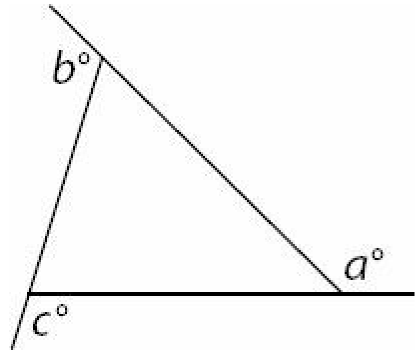


Note: Figure NOT drawn to scale

Which of the following statements individually provide sufficient information to calculate the area of triangle *ABC*?

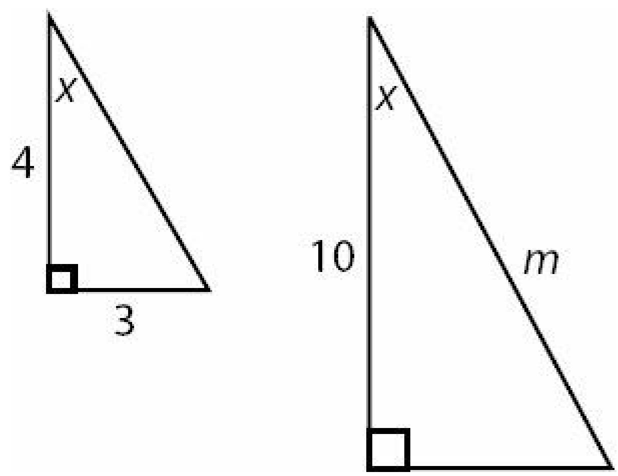
☐ Angle *B* equals 90.
☐ Side *AC* equals 17.
☐ *ABC* is a right triangle

54.



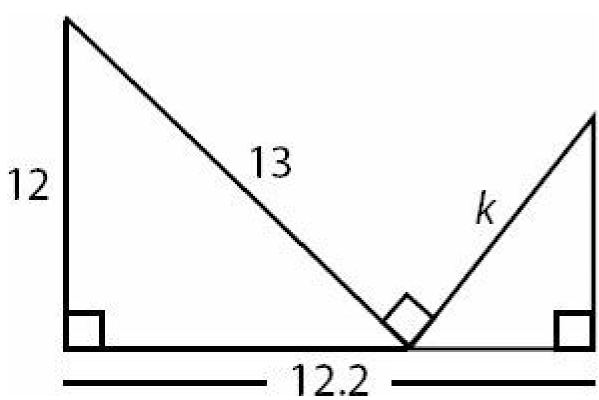
Quantity A	Quantity B
$a + b + c$	180

55.



Quantity A	Quantity B
m	15

56.

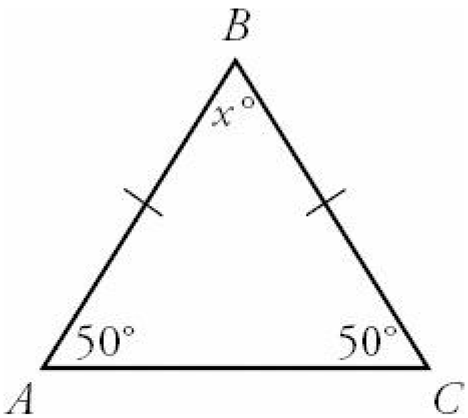


What is the length of hypotenuse k?

Triangles Answers

1. **(B).** The area of a triangle is equal to $\frac{bh}{2}$. Base and height must always be perpendicular. Use 8 as the base and 6 as the height. $A = \frac{(8)(6)}{2} = 24$.

2. **80.** If you know the other 2 angles in a triangle, then you can find the third, because all 3 angles must add up to 180. In Triangle *ABC*, sides *AB* and *BC* are equal. That means their opposite angles are also equal. That means that angle *ACB* is also 50°.

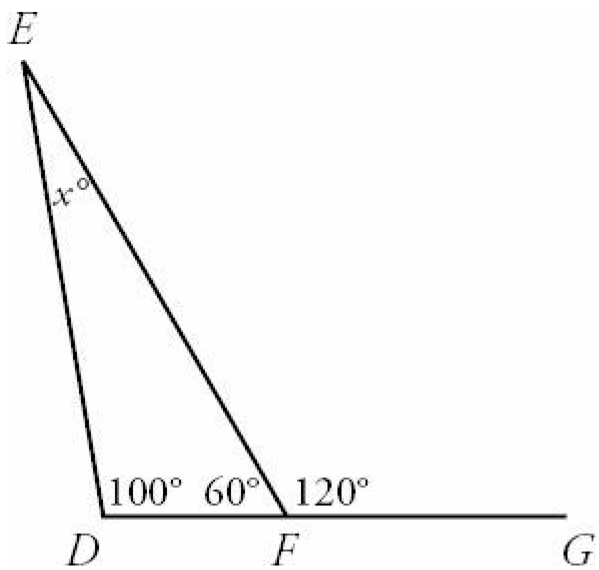


Now that you know the other 2 angles, you can find angle x. You know that $50 + 50 + x = 180$, so $x = 80$.

3. **(C).** The three angles in a triangle must add up to 180°, so $3a = 180$ and $a = 60$ (the triangle is equilateral). The four angles in a quadrilateral must add up to 360°, so $4b = 360$ and $b = 90$ (the angles are right angles, so the figure is a rectangle).

Substitute the values of a and b into Quantity A to get $2(60) + 90 = 210$. Likewise, substitute into Quantity B to get $3(60) + \frac{90}{3} = 210$. The quantities are equal.

4. **20.** To find the value of x, you need to find the degree measures of the other two angles in Triangle *DEF*. You can make use of the fact that *DFG* is a straight line. Straight lines have a degree measure of 180, so angle *DFE* + 120 = 180, which means angle *DFE* = 60.



Now you can solve for x, because $100 + 60 + x = 180$. Solving for x, you get $x = 20$.

5. **(C).** Since c and x are vertical angles, they are equal. So you can swap their positions in the quantities, to put all the angles in the same triangle together.

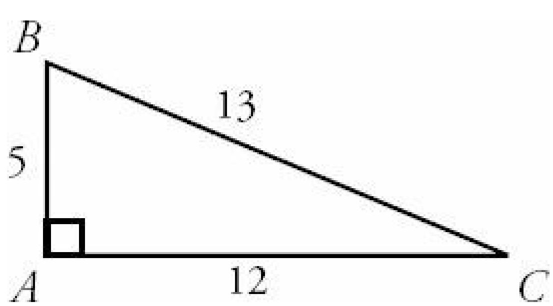
Quantity A	**Quantity B**
$a + b + c$	$x + y + z$

The three angles inside a triangle add up to 180°, so both sides are equal to 180. The quantities are equal.

6. **30.** To find the area, you need a base and a height. If you can find the length of side AB, then AB can be the height and AC can be the base, because the two sides are perpendicular to each other.

You can use the Pythagorean Theorem to find the length of side AB. $(a)^2 + (12)^2 = (13)^2$. $a^2 + 144 = 169$. $a^2 = 25$. $a = 5$. Alternatively, you could recognize that the triangle is a Pythagorean triplet 5–12–13.

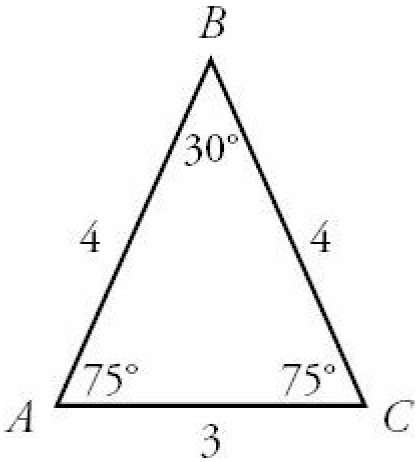
Now that you know the length of side AB you can find the area. $\text{Area} = \dfrac{(12)(5)}{2} = 30.$



7. **11.** To find the perimeter of Triangle ABC, you need the lengths of all 3 sides. There is no immediately obvious way to find the length of side BC, so let's see what inferences you can make from the information the question gave you.

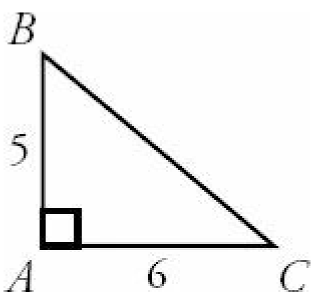
You know the degree measures of two of the angles in Triangle ABC, so you can find the degree measure of the third. You'll label the third angle x. You know that $30 + 75 + x = 180$. Solving for x you find that $x = 75$.

Angle *BAC* and angle *BCA* are both 75, which means Triangle *ABC* is an isosceles triangle. If those two angles are equal, you know that their opposite sides are also equal. Side *AB* has a length of 4, so you know that *BC* also has a length of 4.



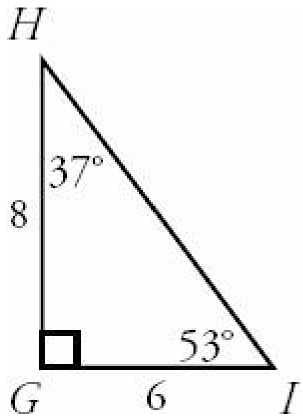
To find the perimeter, you add up the lengths of the three sides. 4 + 4 + 3 = 11.

8. **(D).** To find the length of the hypotenuse, you need the lengths of the other two sides. Then you can use the Pythagorean Theorem to find the length of the hypotenuse. You can use the area formula to find the length of *AC*. $\text{Area} = \frac{bh}{2}$, and you know the area and the height. So $15 = \frac{(\text{base})(5)}{2}$. When you solve this equation, you find that the base = 6.



Now you can use the Pythagorean Theorem. $(5)^2 + (6)^2 = c^2$. $25 + 36 = c^2$. $61 = c^2$. $\sqrt{61} = c$. Since 61 is not a perfect square, you know that c will be a decimal. 61 is also prime, so you cannot simplify $\sqrt{61}$ any further. (It will be a little less than $\sqrt{64} = 8$.)

9. **10.** There is no immediately obvious way to find the length of side *HI*, so let's see what you can infer from the picture. You know two of the angles of Triangle *GHI*, so you can find the third. You'll label the third angle x. $37 + 53 + x = 180$. That means $x = 90$. So really your triangle looks like this:



You should definitely redraw once you discover the triangle is a right triangle!

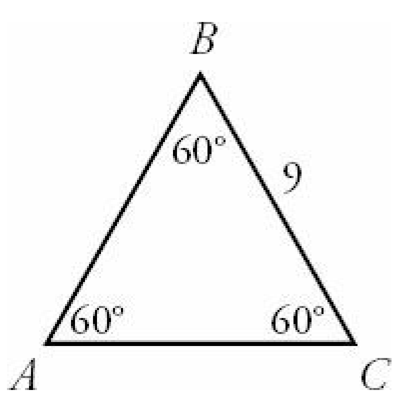
Now that you know Triangle *GHI* is a right triangle, you can use the Pythagorean Theorem to find the length of *HI*. *HI* is the hypotenuse, so $(6)^2 + (8)^2 = c^2$. $36 + 64 = c^2$. $100 = c^2$. $10 = c$. The length of *HI* is 10.

Alternatively, you could have recognized the Pythagorean triplet. Triangle *GHI* is a 6–8–10 triangle.

10. **(C) 24.5.** All isosceles right triangles (or 45–45-90 triangles) have sides in the ratio $1 : 1 : \sqrt{2}$. Thus, an isosceles right triangle with hypotenuse $7\sqrt{2}$ has all its sides in the ratio $7 : 7 : 7\sqrt{2}$. The base and height are each 7. From $A = \frac{bh}{2}$, you get $A = \frac{(7)(7)}{2} = 24.5$.

11. **(B).** To determine which triangle has the greater perimeter, you need to know the side lengths of all three sides of both triangles. Begin with Triangle *ABC*.

There's no immediate way to find the lengths of the missing sides, so let's start by seeing what you can infer from the picture. You know two of the angles, so you can find the third. You'll label the unknown angle x. $60 + 60 + x = 180$. $x = 60$.



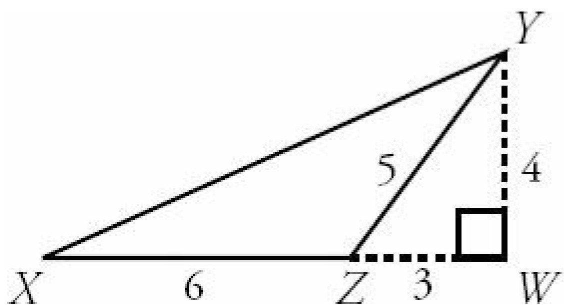
All three angles in Triangle *ABC* are 60°. If all three angles are equal, that means all three sides are equal in this equilateral triangle. So every side of Triangle *ABC* has a length of 9. That means the perimeter = 9 + 9 + 9 = 27. Now look at Triangle *DEF*. Triangle *DEF* is a right triangle, so you can use the Pythagorean Theorem to find the length of side *EF*. *EF* is the hypotenuse, so $(5)^2 + (12)^2 = c^2$. $25 + 144 = c^2$. $169 = c^2$. $13 = c$. That means the perimeter is 5 + 12 + 13 = 30. Alternatively, 5–12–13 is a Pythagorean triplet.

$30 > 27$, so Triangle *DEF* has a greater perimeter than Triangle *ABC*.

12. **12.** Start by filling in everything you know about Triangle *XYZ*.

To find the area of Triangle *XYZ*, you need a base and a height. If Side *XZ* is a base, then *YW* can act as a height. You can find the length of *YW* because Triangle *ZYW* is a right triangle, and you know the lengths of two of the sides. *YZ* is the hypotenuse, so $(a)^2 + (3)^2 = (5)^2$. $a^2 + 9 = 25$. $a^2 = 16$. $a = 4$.

Alternatively, you could recognize the Pythagorean triplet: *ZYW* is a 3–4–5 triangle.



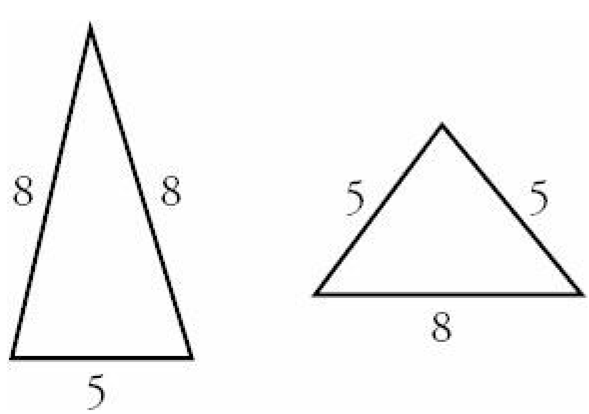
Now you know that the area of Triangle *XYZ* is $\frac{bh}{2} = \frac{(6)(4)}{2} = 12$.

13. **(D).** The problem tells you that $z + x = 110$ degrees. Given that angles of a triangle must sum to 180 degrees, you also know that $x + y + z = 180$. Substitute 110 for $x + z$ on the left side:

$y + 110 = 180$
$y = 70$ degrees

The problem asks you to compare angles x and y. Although you have solved for y, you are still uncertain about the measure of angle x (all you know is that it must be greater than 0 and less than 110 degrees). Therefore, you cannot determine which quantity is greater.

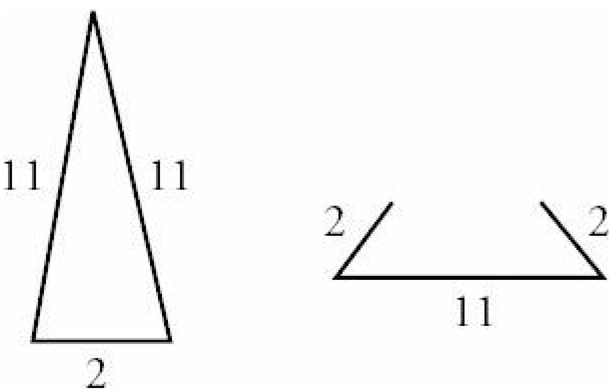
14. **(D).** An isosceles triangle has 2 equal sides, so this triangle must have a third side of either 8 or 5. Use the Third Side Rule (any side of a triangle must be greater than the difference of the other two sides and less than their sum) to check whether both options are actually possible.



Since $8 - 5 = 3$ and $8 + 5 = 13$, the third side has to be greater than 3 and less than 13. Therefore, that side could indeed be either 5 or 8. You don't know which it is, though, so you cannot determine which quantity is greater.

15. **(C).** An isosceles triangle has 2 equal sides, so this triangle must have a third side of either 2 or 11. Because one side is so long and the other so short, it is worth testing via the Third Side Rule (any side of a triangle must be greater than the difference of the other two sides and less than their sum) to see whether both possibilities are really possible.

From the Third Side Rule, a triangle with sides of 2 and 11 must have a third side greater than 11 - 2 = 9 and less than 11 + 2 = 13. Since 2 is not between 9 and 13, you simply cannot have a triangle with sides of length 2, 2, and 11. However, you *can* have a third side of length 11: a 2–11–11 triangle is possible. So the third side must be 11.



Thus, the two quantities are equal.

16. **(A).** In this triangles problem, you are asked to compare the relative lengths of two sides of a triangle. In any triangle, the following rule is true: the larger the angle, the longer the side opposite that angle.

Therefore, since angle B is larger than angle A, the side opposite angle B must be longer than the side opposite angle A. Side length AC is longer than side length BC. Quantity A is greater.

17. **(C).** By definition, the exterior angle d is supplementary to the adjacent interior angle and equal to the sum of the two non-adjacent angles. Thus, d = angle B + angle C.

Alternatively, try plugging in numbers. If $d = 120$, angle A equals 180 - 120 = 60. How many degrees are left for angles B and C to share? 180 - 60 = 120, so angles B and C must add up to 120—the same as d. As long as your example obeys the rules of triangles (the 3 angles in the triangle add to 180) and straight lines (the 2 angles on the line also add to 180), your example will show that d = angle B + angle C. The two quantities are equal.

18. **(D).** If the question had said the two LEGS of the triangle were 3 and 4, then the hypotenuse would be 5 (as you know from the 3–4–5 special right triangle). However, you can't assume that 3 and 4 are the legs. In fact, 4 could be the *hypotenuse*. In that case, 3 would still be a leg, and the length of the other leg would be $\sqrt{16-9}=\sqrt{7}$, as you could show from the Pythagorean theorem. You don't need to calculate this value—just recognize that it must be less than 4 (because 4 is the hypotenuse in this case). Since z could be either equal to or less than 5, you cannot determine which quantity is greater.

19. **(B).** Since $z > 60$ and $x > z$, x must also be greater than 60. Thus, $x + z$ must be greater than 120, which leaves less than 60 degrees for the third angle y. You can now order the angles by size: $y < z < x$.

The smallest side is across from the smallest angle, which is y, so the smallest side must be AC. The middle side is across from the middle angle, which is z, so the middle side must be AB. At this point, you know that the length of AB

is greater than the length of AC. Quantity B is greater.

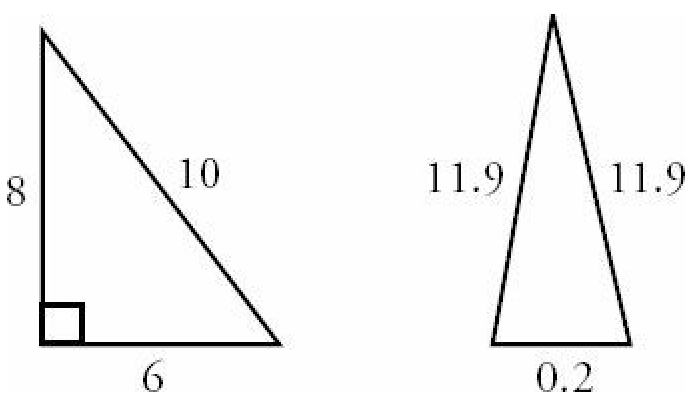
20. **(A).** Since BC is 1/3 the length of AB, relate the two sides using a variable. The easiest way to do this is to label AB "$3x$" and label BC "x." (Calling the smaller side x lets you avoid using a fraction.)

Now apply the Pythagorean Theorem:

$$x^2 + (3x)^2 = (4\sqrt{10})^2$$
$$x^2 + 9x^2 = 160$$
$$10x^2 = 160$$
$$10x^2 = 16$$
$$x = 4$$

You are looking for AB, which equals $3x = 3(4) = 12$. Quantity A is greater.

21. **(D).** The perimeter of a triangle is not enough to find its area, and vice-versa. A triangle with perimeter 24 could be a right triangle with sides of 6, 8, and 10, in which case the area would be $A = \frac{bh}{2} = \frac{6(8)}{2} = 24$, which is larger than 20. Or the triangle could have sides of 11.9, 11.9, and 0.2, in which case the area would be incredibly small.



You should note that you don't have *complete* freedom with the area. There is a maximum area that a triangle with perimeter 24 can have—namely, the area of an equilateral triangle with side lengths of 8. Such a triangle would have an area of $\frac{s^2\sqrt{3}}{4} = \frac{8^2\sqrt{3}}{4} = 16\sqrt{3} \approx 16(1.7)$, which is greater than 20. (You can derive that formula for the area of an equilateral triangle by dropping a height, making two 30–60–90 triangles.) But any positive area less than this maximum is possible. You can drastically shrink the area of a triangle with perimeter 24, making that area as close to zero as you wish. Thus, you cannot determine which quantity is greater.

22. **(C).** The area of a triangle is equal to $\frac{bh}{2}$. You know that the two triangles have equal bases, since $XY = YZ$. They also have the same height, since they both have the same height as the larger triangle WXZ. The two quantities are equal.

23. **(C).** You certainly cannot *assume* that $x = 90$. But since you have three values for the sides of the triangle, you can test whether the triangle is a right triangle by applying the Pythagorean Theorem to the three values and seeing whether

you get a true statement.

$$5^2 + 7^2 = (\sqrt{74})^2$$
$$25 + 49 = 74$$
$$74 = 74$$

Since 74 obviously equals 74, the Pythagorean theorem does apply to this triangle. So the triangle is a right triangle. Notice also that you used the side across from x as the hypotenuse. Thus, you can be sure that $x = 90$. The two quantities are equal.

24. **(B).** Within a single triangle, there is a direct relationship between the side length and the opposite angle. That is, the biggest side is opposite the biggest angle; the smallest side is opposite the smallest angle; and the middle side is opposite the middle angle.

Since the angles in a triangle sum to 180, the angle opposite L is 44. So L is smaller than S, and $\frac{L}{S}$ is less than 1. Quantity B is greater.

25. **(C).** From the Third Side Rule, any side of a triangle must be greater than the difference of the other two sides and less than their sum. Since 11 - 9 = 2 and 11 + 9 = 20, x must be between 2 and 20, *not* inclusive. Note that x is an integer, so x must be between 3 and 19, *inclusive*.

Now you can list the possibilities and count: x can be 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, or 19. Or you can subtract the *inclusive* endpoints and "add 1 before you're done:" 19 - 3 + 1 = 17. Either way, there are 17 total possibilities. The two quantities are equal.

26. **(C) 18 < p < 30.** From the Third Side Rule, any side of a triangle must be greater than the difference of the other two sides and less than their sum. Since 9 - 6 = 3 and 9 + 6 = 15, the unknown third side must be between 3 and 15, *not* inclusive. To get the lower boundary for the perimeter, add the lower boundary of the third side to the other two sides: 3 + 6 + 9 = 18. To get the upper boundary for the perimeter, add the upper boundary for the third side to the other two sides: 15 + 6 + 9 = 30. Thus, p must be between 18 and 30, not inclusive—in other words, $18 < p < 30$.

27. **(B).** You may have memorized the 6–8–10 Pythagorean triple, a fact that this problem is trying to exploit—don't be tricked into thinking that x equals 10! In a 6–8–10 triangle, 10 would have to be the *hypotenuse*. In any right triangle, the hypotenuse must be the longest side.

Since the given triangle has 8 as the hypotenuse, the leg of length x must be less than 8. So x must also be less than 10. At this point, you can safely choose (B). If you really want the actual value of x, simply apply the Pythagorean theorem:

$$6^2 + x^2 = 8^2$$
$$36 + x^2 = 64$$
$$x^2 = 28$$

$x = \sqrt{28}$, which is between 5 and 6 (so it is less than 10).

Quantity B is greater.

28. **(A).** One good approach here is to test the value in Quantity B. If angle x equals 90°, then you have a right triangle. Use the legs of 5 and 12 to find the hypotenuse. $5^2 + 12^2 = c^2$ will tell you that c equals 13 in this case. (Or, memorize the 5–12–13 Pythagorean triple, since it appears often on the GRE.)

Since the hypotenuse is slightly *longer* than 13, the angle across from the 13 must actually be slightly *larger* than 90°. Therefore, x is larger than 90. Quantity A is greater.

29. **(A).** Since the figure is a right triangle, set up a Pythagorean theorem to solve for the hypotenuse in terms of x:

$$x^2 + (2x)^2 = c^2$$
$$x^2 + 4x^2 = c^2$$
$$5x^2 = c^2$$
$$\sqrt{5x^2} = c$$
$$x\sqrt{5} = c$$

The hypotenuse is equal to $x\sqrt{5}$, which is greater than $x\sqrt{3}$ (as long as x is positive, which of course it is since x is a distance).

The "trick" in this problem is that if you accidentally simplify $(2x)^2$ as $2x^2$ rather than as $4x^2$, you end up with incorrect choice (C).

Quantity A is greater.

30. **(C).** If $AC = 14$ and is parallel to DE, then triangles DBE and ABC are similar.

Since $BE = EC$, $BC = 2BE$ (that is, the side of the big triangle is twice the side of the small triangle). Since the two triangles are similar, *all* the sides of the small triangle will equal half of the corresponding sides of the big triangle. Thus, $DE = 7$.

Alternatively, you can write out a proportion:

$$\frac{BE}{BC} = \frac{DE}{AC}$$

You don't know the exact lengths of BE and BC, but you do know that they are in a 1 to 2 ratio, which is all you need (even if you did know the exact lengths, you'd be able to reduce that fraction to 1/2 anyway).

$$\frac{1}{2} = \frac{DE}{14}$$

$$DE = 7$$

The two quantities are equal.

31. **(B).** From the Third Side Rule, any side of a triangle must be less than the sum of the other two sides and greater than their difference.

Since one side equals 8, the other two sides must differ by less than 8. Thus:

$|m - n| < 8$

The absolute value signs are not totally necessary here, in fact—all they do is guarantee that you're taking the positive difference of m and n (it doesn't matter which one is longer).

Since $|m - n|$ must be less than 8, Quantity B is greater.

32. **(C).** The two triangles—the small triangle and the large triangle (which encompasses the small triangle)—share one angle, the angle at the top of both triangles.

Furthermore, because the parallel lines create equal angles when cut by transversals (in this case, the sides of the triangle), the other two angles of both of the triangles are also in equal measure. Therefore, the two triangles are similar.

Because the two triangles are similar, they are in proportion to one another. The small triangle has two sides of 2, while the large triangle has corresponding sides of 10 (NOT 8! The lengths labeled 8 are NOT the sides of an actual triangle—the large triangle has two sides of 8 + 2 = 10). So the two triangles are in a 2 : 10 proportion to one another. Reduce 2 : 10 to 1 : 5 and write a proportion:

$$\frac{1}{5} = \frac{x}{11}$$

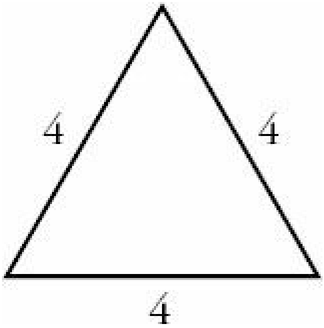
$$5x = 11$$

$$x = \frac{11}{5}$$

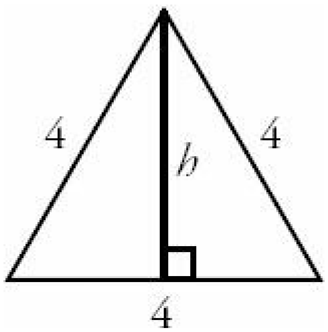
33. **(D).** From the Third Side Rule, a triangle with sides of 13 and 9 must have a third side greater than 13 - 9 = 4 and less than 13 + 9 = 22.

Use the square root button on the calculator to see that $\sqrt{226} \approx 15.033$, or note that since $\sqrt{225} = 15$, $\sqrt{226}$ must be just a little more than 15 (but certainly less than 16). Since the third side's length is between 4 and 22, it could be more or less than $\sqrt{226}$. You cannot determine which quantity is greater.

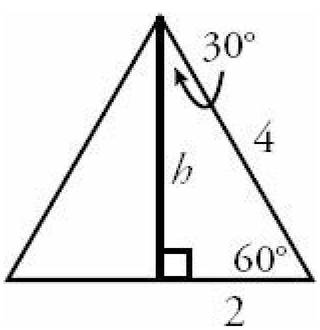
34. **(D).** An equilateral triangle with side length 4 can be drawn as:



In order to find the area, recall that the area of a triangle is given by the formula $A = \frac{bh}{2}$. The base of the triangle is already known to be 4, so you must find the height in order to solve for area. The height is the straight line from the highest point on the triangle dropped down perpendicular to the base:



The angle opposite *h* must be 60°, since it is one of the three angles of the original equilateral triangle. Thus, the triangle formed by *h* is a 30–60–90 triangle as shown below in red.



Using the properties of 30–60–90 triangles, you know that *h* is equal to the shortest side multiplied by $\sqrt{3}$. Thus, $h = 2\sqrt{3}$ and the area is given by

$$A = \frac{bh}{2} = \frac{4 \times 2\sqrt{3}}{2} = 4\sqrt{3}$$

The shortcut formula for the area of an equilateral triangle is $A = \frac{s^2\sqrt{3}}{4}$, which can be derived by the same logic as shown above.

35. **(A).** Label angle *A* with the variable *x*. Since, the three angles of a triangle must add up to 180 degrees, you know

that

$x + 88 + 47 = 180$
$x + 135 = 180$
$x = 45$

Therefore, angle A has a measure of 45 degrees. Now, although you do not know the lengths of the sides, the largest side is opposite the largest angle, and the smallest side is opposite the smallest angle. Because angle C is larger than angle A, you know that the side opposite angle C is longer than the side opposite angle A. In other words, side AB is longer than side BC. Quantity A is greater.

36. **(B).** You are asked for the value of x. Since there are two unknowns, look for two equations to help you solve. The first equation comes from the fact that $3x$ and $5y$ make a straight line, so they must add to 180:

$3x + 5y = 180$

The second equation comes from the isosceles triangle theorem, which states that angles across from sides of a triangle with equal length are equal. In this case, the two sides with length 3 are equal, so the angles across from them (y and $3x$) must also be equal:

$y = 3x$

Substitute for y in the first equation:

$3x + 5(3x) = 180$
$3x + 15x = 180$
$18x = 180$
$x = 10$

37. **(E).** You are told that the area is 50, so $\frac{bh}{2} = 50$. In an isosceles right triangle, base = height, so you can substitute another b in for h:

$\frac{b^2}{2} = 50$
$b^2 = 100$
$b = 10$

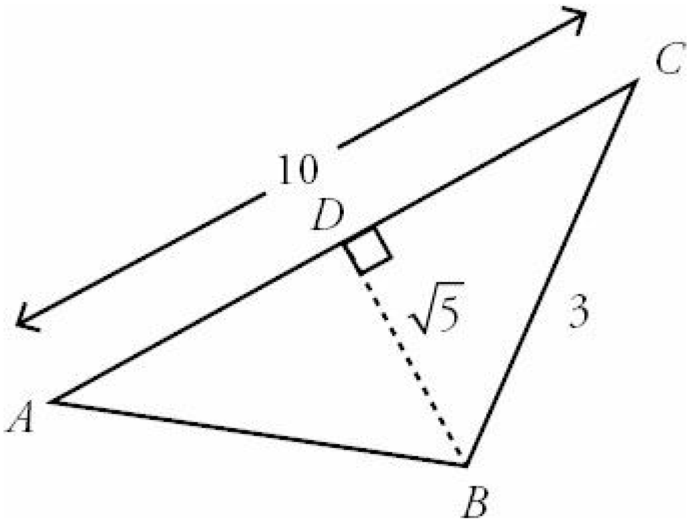
An isosceles right triangle follows the 45–45–90 triangle formula, so the hypotenuse is $10\sqrt{2}$.

Alternatively, use the Pythagorean Theorem to find the hypotenuse:

$10^2 + 10^2 = c^2$
$200 = c^2$

Thus, $c = \sqrt{200} = \sqrt{100 \times 2} = 10\sqrt{2}$.

38. **(E).** You are asked for the length of segment *AB*. For convenience, put the letter *D* on the point at the right angle between *A* and *C*, as shown:



Solve this multi-step problem by working backwards from your goal. To find the length of *AB*, you can use the Pythagorean theorem on triangle *ADB*, since angle *ADB* must be a right angle. In order to use the Pythagorean theorem, you need the lengths of the two legs. *BD* is known, so you just need *AD*. Since *AD* and *DC* add up to a line segment of length 10, you know that *AD* = 10 - *DC*.

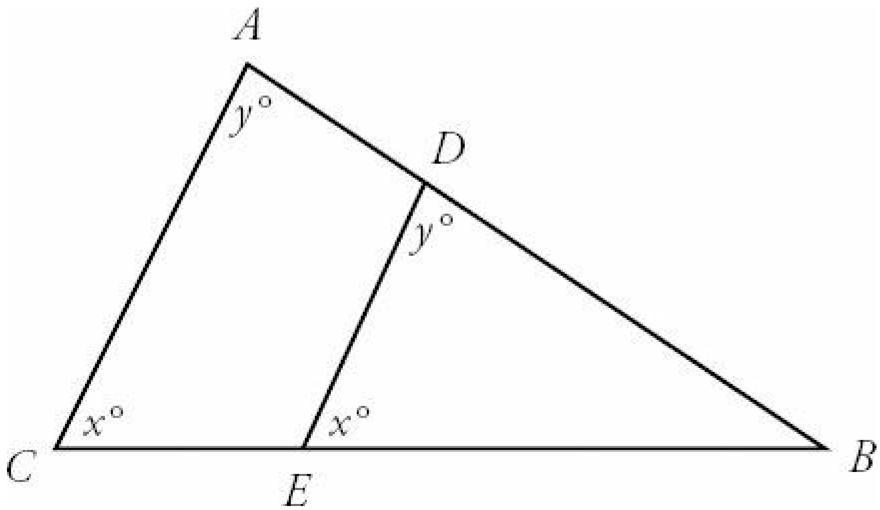
Finally, to find *DC*, apply the Pythagorean theorem to triangle *BDC*:

$$(\sqrt{5})^2 + (DC)^2 = 3^2$$
$$5 + (DC)^2 = 9$$
$$(DC)^2 = 4$$
$$DC = 2$$

So *AD* = 10 - *DC* = 10 - 2 = 8. Apply the Pythagorean theorem to *ADB*:

$$(\sqrt{5})^2 + 8^2 = (AB)^2$$
$$5 + 64 = (AB)^2$$
$$69 = (AB)^2$$
$$AB = \sqrt{69}$$

39. **(C).** You are asked to compare the lengths of *AC* and *CB*. Call angle *DEB* $x°$. *DE* and *AC* are parallel, and they are both cut by transversal *CB*. So angles *DEB* and *ACB* are corresponding angles—they have the same measure, and angle *ACB* will also be $x°$. Similarly, if angle *EDB* is labeled as $y°$, then angle *A* will also be $y°$. At this point, the diagram looks like this:



Now you have two triangles with all three angles the same, *ACB* and *DEB* (angle *B* will obviously be the same in both triangles). So these triangles are similar, and the sides of *ACB* will be in the same proportions as the corresponding sides of *DEB*.

You are told that the length of *DE* is equal to the length of *EB*. (When the figure is not to scale, don't trust your eyes—trust what the problem tells you!) To maintain similarity, the corresponding sides on the larger triangle (*AC* and *CB*) must also be equal. Thus, although you do not know the lengths of *AC* and *CB*, you know they must be the same. The two quantities are equal.

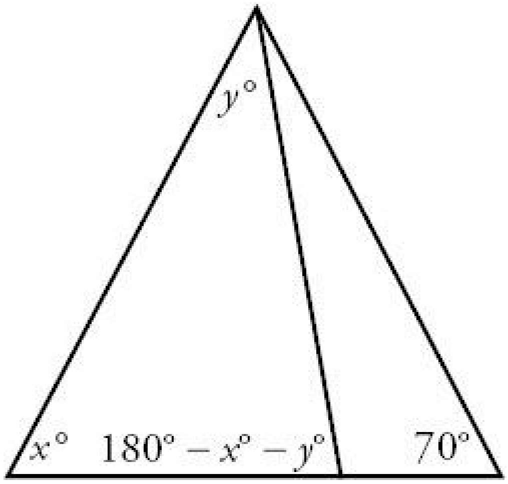
40. **(C).** Because you are told that this is a right triangle, you can use the Pythagorean theorem to solve for the lengths of the sides. The Pythagorean theorem states that $a^2 + b^2 = c^2$ where c is the hypotenuse and a and b are the legs of a right triangle. In this case, $x + 5$ must be the length of the hypotenuse (the longest side), because $x + 5$ is definitely greater than both $x + 2$ and $x - 1$. Plug the expressions into the theorem and simplify:

$$(x - 1)^2 + (x + 2)^2 = (x + 5)^2$$
$$(x^2 - 2x + 1) + (x^2 + 4x + 4) = x^2 + 10x + 25$$
$$2x^2 + 2x + 5 = x^2 + 10x + 25$$
$$x^2 - 8x - 20 = 0$$
$$(x - 10)(x + 2) = 0$$
$$x = 10 \text{ or } x = -2$$

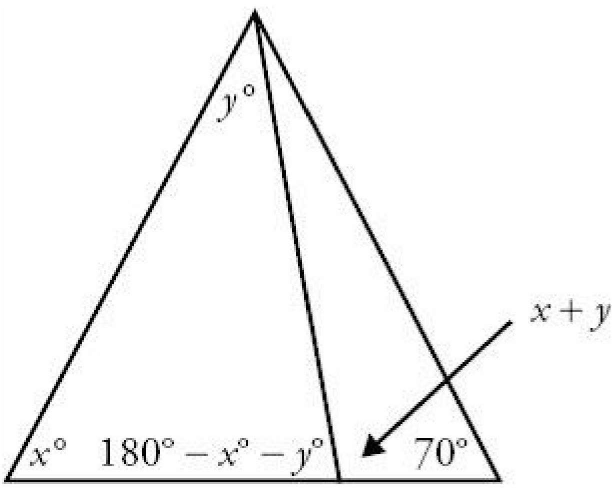
However, $x = -2$ is not an option; side lengths can't be negative. So x must equal 10. This is NOT the final answer, however. You are ultimately asked for side length *AB*:

$$AB = x + 2 = 10 + 2 = 12.$$

41. **(B).** You are asked to compare $x + y$ with 110. To do so, fill in the missing angles on the triangles. In the triangle on the left, all three angles must add up to 180 degrees. Therefore, the missing angle must be $(180 - x - y)$, as shown here:



Now consider the angle next to the one you just solved for. These two angles add up to 180, forming a straight line. So the adjacent angle must be $x + y$.



Alternatively, you could notice also that $x + y$ is the exterior angle to the triangle on the left, so it must be the sum of the two non-adjacent angles (namely, x and y).

Now, the three angles of a triangle must add up to 180, and no angle can equal 0. So any *two* angles in a triangle must add up to *less* than 180. Consider the triangle on the right side, which contains angles of $x + y$ and 70. Then their sum is less than 180.

$$(x + y) + 70 < 180$$

Subtract 70 from both sides:

$$x + y < 110$$

Quantity B is greater.

42. **(C).** First determine how Quantities A and B relate to the triangle. For instance, examine Quantity A, the product of BE and AC. Notice that BE is the height of the triangle, while AC is the base. This product should remind you of the formula for area: $A = \frac{bh}{2}$.

With $b = AC$ and $h = BD$, and moving the 2, you get

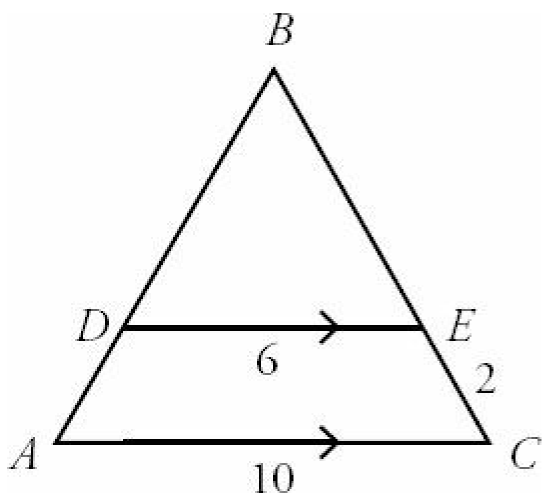
$$2 \times \text{Area} = (AC)(BD)$$

What about BC and AD? Well, you can consider BC the base—and if you do so, then AD is the height to that base. So you can put $b = AC$ and $h = BD$ into the area formula, yielding

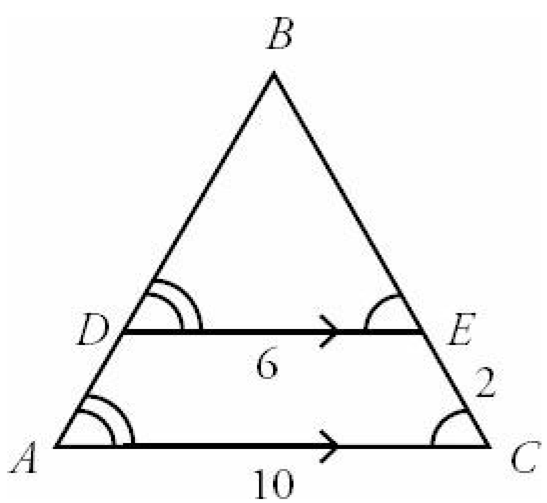
$$2 \times \text{Area} = (BC)(AD)$$

Both Quantity A and Quantity B are twice the area of the triangle. The two quantities are equal.

43. **(C)** First, label the diagram with the information given:



where the arrows represent parallel lines. Because DE and AC are parallel, angle DEB must be the same as angle ACB, as they are formed by the same transversal (line segment BC). Similarly, angle BDE must be the same as angle BAC.



Triangle BDE has all the same angles as triangle BAC (angle B is shared), so the two triangles are similar. The ratio of any two corresponding sides on similar triangles is the same, whichever pair of sides you pick. You can find this constant ratio from DE and AC: $\frac{DE}{AC} = \frac{6}{10} = \frac{3}{5}$. So all other corresponding sides must also be in the ratio of 3 to 5. Assign x to the unknown length of BE. Then BC will be $x + 2$. Apply the ratio above:

$$\frac{BE}{BC} = \frac{x}{x+2} = \frac{3}{5}$$

Cross multiply and solve for *x:*

$5x = 3(x + 2)$
$5x = 3x + 6$
$2x = 6$
$x = 3$

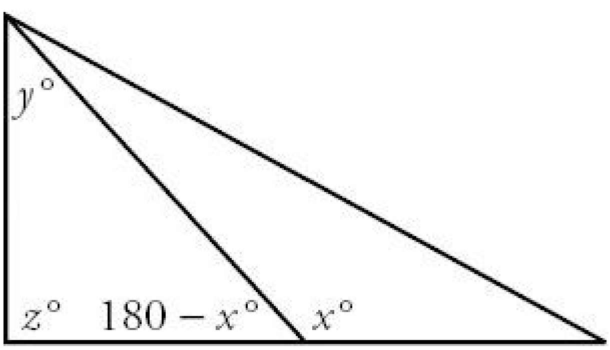
Remember that x is the length of side BE, but you want the length of BC.

$BC = x + 2 = 3 + 2 = 5$

44. **(C).** You can solve this problem in two ways. The quick way is to notice that x is the exterior angle to the smaller triangle on the left (which contains y and z). Since y and z are the non-adjacent interior angles, you can immediately apply the rule that the exterior angle (x) is equal to the sum of the two non-adjacent interior angles (y and z).

The longer way is to derive that relationship, essentially, from two rules: (1) the three angles in a triangle add up to 180, and (2) the two angles formed by a segment running into a straight line (such as x and its unlabeled neighbor) also add up to 180.

First, label that missing angle as $180 - x$ as shown:

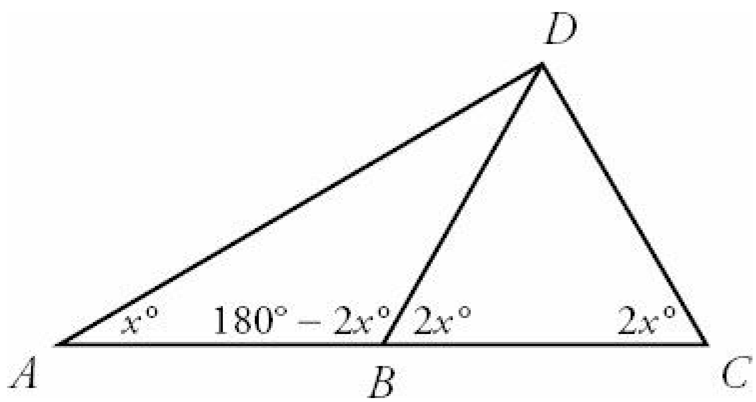


Now apply the rule that the three angles in a triangle add up to 180:

$y + z + (180 - x) = 180$
$y + z + 180 - x = 180$
$y + z - x = 0$
$y + z = x$

The two quantities are equal.

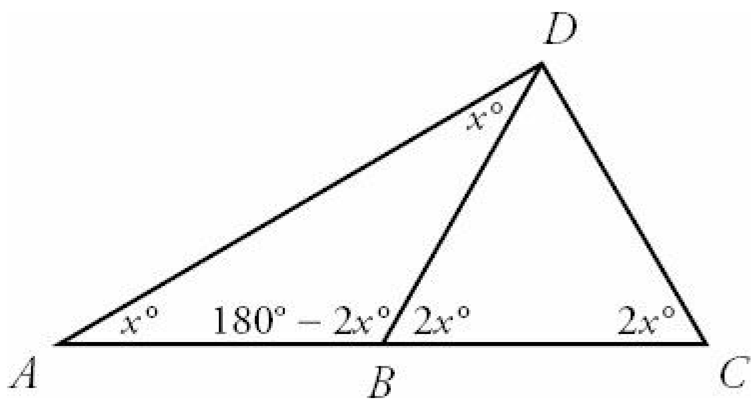
45. **(C).** To compare DC and AB, first solve for the unlabeled angles in the diagram. The two angles at point B make a straight line, so they add up to 180, and the unlabeled angle is $180 - 2x$, as shown:



Now make the angles of triangle *ABD* add up to 180:

(Angle *A*) + (Angle *B*) + (Angle *D*) = 180
x + (180 - 2x) + (Angle *D*) = 180
x + 180 - 2x + (Angle *D*) = 180
-x + (Angle *D*) = 0
Angle *D* = x

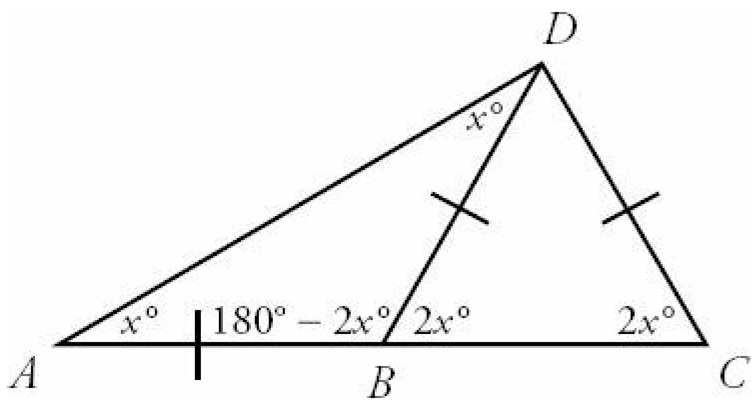
Therefore, the figure becomes



(You could have also gotten here if you noticed that angle *DBC* (equal to 2x) is the exterior angle to the triangle on the left, and so it equals the sum of the two non-adjacent angles in that triangle. One of those angles, namely angle *A*, is x, so the other one must be x as well.)

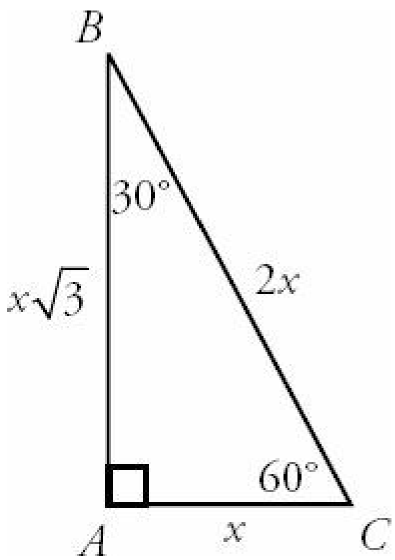
Now apply the properties of isosceles triangles. The two angles labeled x are equal, so the triangle that contains them (triangle *ABD*) is isosceles, and the sides opposite those equal angles are also equal. Put a slash through those sides (*AB* and *DB*) to mark them as the same length.

Likewise, the two angles labeled 2x are equal, so the triangle that contains them (triangle *DBC*) is isosceles, and the sides opposite those angles (*DB* and *DC*) are equal. Adding one more slash through *DC*, you get



Thus, sides *AB* and *DC* have the same length. The two quantities are equal.

46. **(B).** To compute the perimeter of this triangle, you need the lengths of all three sides. Because *A* is a right angle and angle *B* is 30°, right triangle *ABC* is a 30–60–90 triangle. For any 30–60–90 triangle, the sides are in these proportions:



Now you must match up this universal 30–60–90 triangle to your given triangle, so that you can find *x* in this particular case. The only labeled side in the given triangle (6) matches the $x\sqrt{3}$ side in the universal triangle (they're both opposite the 60° angle), so set them equal to each other:

$$6 = x\sqrt{3}$$
$$x = \frac{6}{\sqrt{3}}$$

Rationalize the denominator by multiplying by $\frac{\sqrt{3}}{\sqrt{3}}$ (which does not change the value of *x*, as $\frac{\sqrt{3}}{\sqrt{3}}$ is just a form of 1):

$$x = \frac{6}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$x = \frac{6\sqrt{3}}{3}$$

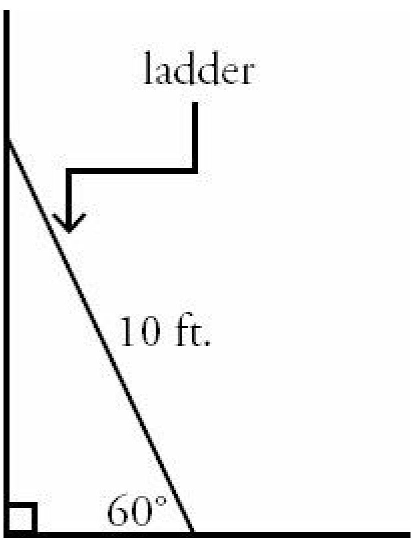
$$x = 2\sqrt{3}$$

Now figure out all the sides in your given triangle. The length of side *AC* (x) is $2\sqrt{3}$, the length of side *AB* is given by 6, and the length of side *BC* is $2x = 2(2\sqrt{3}) = 4\sqrt{3}$.

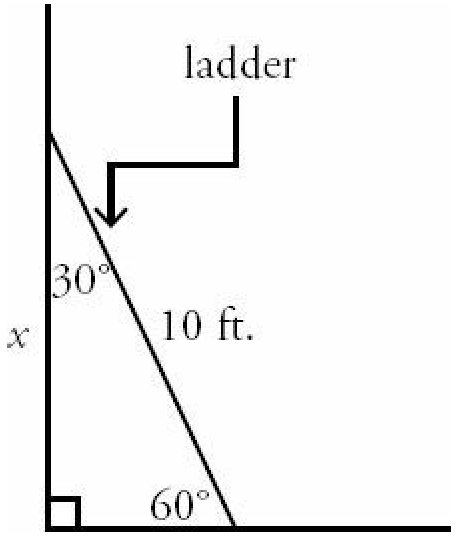
Finally, add up all the sides to get the perimeter:

$$\text{Perimeter} = 6 + 2\sqrt{3} + 4\sqrt{3}$$
$$\text{Perimeter} = 6 + 6\sqrt{3}$$

47. **(B).** First, draw a diagram and label all the givens:

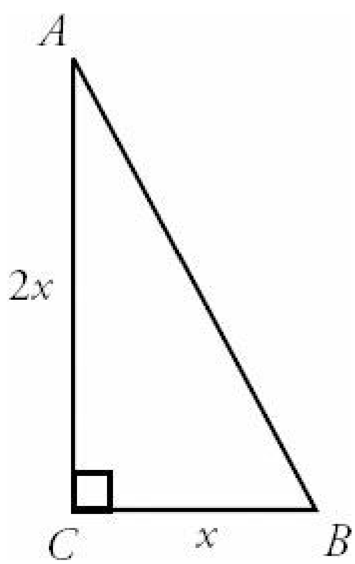


Since the wall is vertical and the floor is perfectly horizontal, the angle where they meet is 90°. So the triangle is 30–60–90. You are asked to find the vertical distance from the top of the ladder to the floor, so represent this length as x.



In any 30–60–90 triangle, the short leg (opposite the 30° angle) is the hypotenuse divided by 2, making the floor-side $10 \div 2 = 5$ feet. The longer leg (opposite the 60° angle) is $\sqrt{3}$ times the short leg. So $x = 5\sqrt{3}$ feet.

48. **(C).** Draw a diagram and label the sides of the triangle with the information given. Since *AC* is twice as long as *CB*, label *CB* as *x* and *AC* as 2*x*, as shown



You are given the area of the triangle, and you can use *x* as the base and 2*x* as the height in the formula for area (which equals $\frac{bh}{2}$). Plug in and solve for *x*:

$$36 = \frac{x(2x)}{2} = \frac{2x^2}{2}$$
$$36 = x^2$$
$$x = 6$$

So *CB* = 6 and *AC* = (2)(6) = 12. Use the Pythagorean Theorem to find *AB*:

$$(AB)^2 = 6^2 + 12^2$$

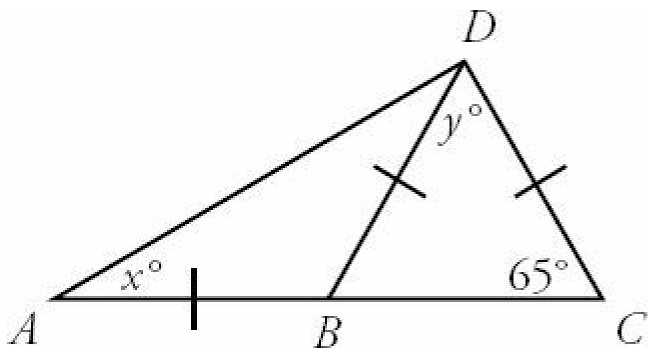
$(AB)^2 = 36 + 144$
$(AB)^2 = 180$
$AB = 6\sqrt{5}$

49. **(A).** In order to make the comparison, you need the length of *CB*. Since angles *A* and *B* are equal, the triangle is isosceles, and the sides opposite those angles (*AC* and *CB*) must also be equal. Write the equation:

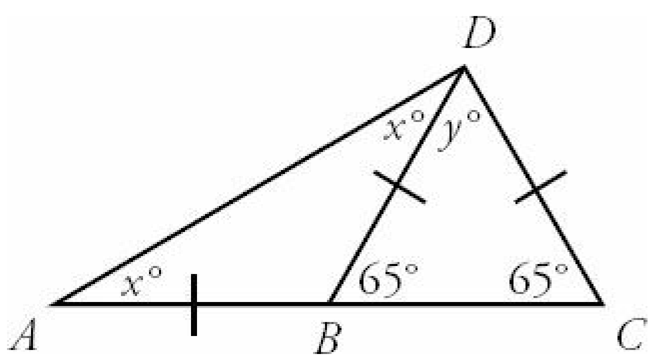
$2x - 5 = x + 4$
$x - 5 = 4$
$x = 9$

CB is therefore equal to 9 + 4 = 13. Quantity A is greater.

50. **(B).** Redraw the figure and label the equal sides:



The two small triangles (on the left and on the right) are each isosceles, because they each contain two equal sides. From the isosceles triangle theorem, the angles opposite equal sides are also equal. Fill in more angles on the figure:



The three angles in the triangle on the right must add up to 180 degrees:

$65 + 65 + y = 180$
$130 + y = 180$
$y = 50$

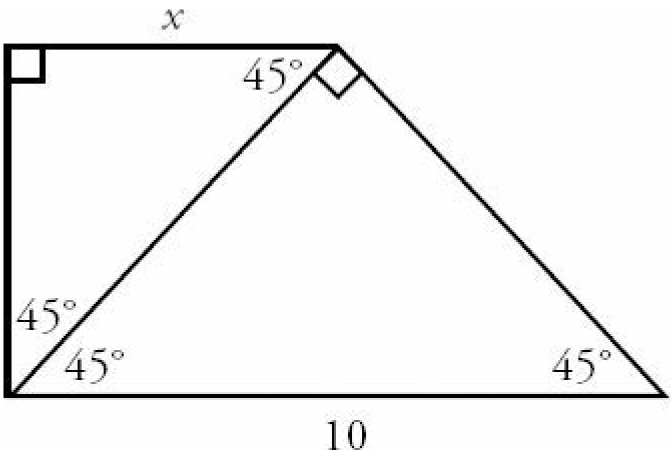
The two angles at point *B* make a straight line, so they add up to 180 degrees. So the unlabeled angle must be 180 - 65 = 115 degrees.

Finally, the three angles in the triangle on the left must sum to 180 degrees:

$x + x + 115 = 180$
$2x = 65$
$x = 32.5$

So y is greater than x. Quantity B is greater.

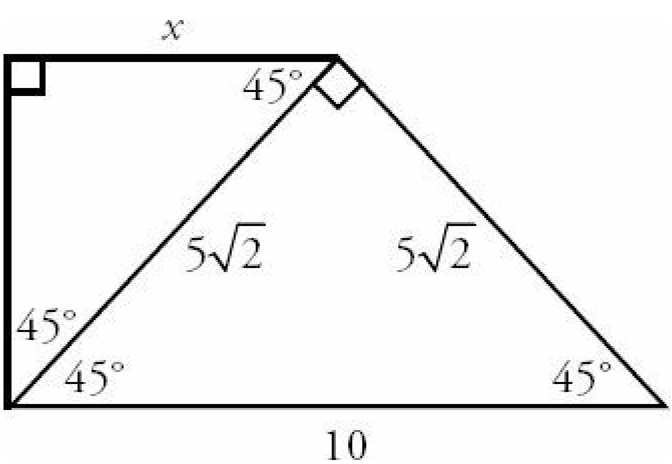
51. **(C).** Redraw the figure and label all angles, applying the rule that the angles in a triangle add up to 180:



You can now see that you have two separate 45–45–90 triangles. In a 45–45–90 triangle, the sides are in a 1 : 1 : $\sqrt{2}$ ratio. Thus, the length of each leg equals the length of the hypotenuse divided by $\sqrt{2}$. The hypotenuse of the larger triangle is 10, so each leg of that triangle is $\frac{10}{\sqrt{2}}$. Rationalize the denominator by multiplying by $\frac{\sqrt{2}}{\sqrt{2}}$:

$$\frac{10}{\sqrt{2}} = \frac{10}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

Add more labels to your figure:



Now you know that the hypotenuse of the smaller triangle is $5\sqrt{2}$. Apply the 45–45–90 triangle ratio once more (1 : 1 : $\sqrt{2}$) to see that $x = 5$.

52. **(B).** The sides of a 30–60–90 triangle are always in a $1:\sqrt{3}:2$ ratio. Since *BC* is across from the 30° angle and *AB* is across from the 60° angle, Quantity A is equal to $\frac{1}{\sqrt{3}}$.

In the calculator, this is 0.578…. Use the calculator to see that Quantity B = 10/17 = 0.588… and is slightly larger than Quantity A.

Alternatively, set the two fractions next to each other and cross multiply to compare them:

$$\frac{1}{\sqrt{3}} \ ? \ \frac{10}{17}$$
$$17 \ ? \ 10\sqrt{3}$$
$$1.7 \ ? \ \sqrt{3}$$
$$1.7^2 \ ? \ 3$$

$1.7^2 = 2.89$. Note that 1.7 is a good approximation of $\sqrt{3}$, but $\sqrt{3}$ is actually a bit bigger.

Quantity B is larger.

53. **I and II only.** If Angle *B* = 90°, then 8 and 15 are the base and height, and you can calculate the area. Statement I is sufficient.

If side *AC* = 17, you can plug 8, 15, and 17 into the Pythagorean theorem to see whether you get a true statement. Use 17 as the hypotenuse in the Pythagorean theorem because 17 is the longest side:

$$8^2 + 15^2 = 17^2$$
$$64 + 225 = 289$$
$$289 = 289$$

Since this is obviously true, the triangle is a right triangle with the right angle at *B*. If Angle *B* = 90°, then 8 and 15 are the base and height, and you can calculate the area. (8–15–17 is a Pythagorean triplet, so if you had that fact memorized, you could skip the step above.) Statement II is sufficient.

Knowing that *ABC* is a right triangle (Statement III) is *not* sufficient to calculate the area because you don't know which angle is the right angle. A triangle with sides of 8 and 15 could have hypotenuse 17, as you've already seen, but another scenario is possible: perhaps 15 is the hypotenuse. In this case, the third side is smaller than 15, and the area is smaller than in the 8–15–17 scenario.

54. **(A).** The three interior angles of the triangle add up to 180. Try an example: say each interior angle is 60°. In that case, *a*, *b*, and *c* would each equal 120° (since two angles that make up a straight line add to 180), and Quantity A would equal 360.

You can prove this result in general by expressing each interior angle in terms of a, b, and c, and then setting their sum equal to 180:

$$(180 - a) + (180 - b) + (180 - c) = 180$$
$$540 - a - b - c = 180$$
$$360 = a + b + c$$

Quantity A is greater.

55. **(B).** Since both triangles have a 90° angle and an angle x, the third angle of each is the same as well (because the three angles in each triangle add up to 180). All the corresponding angles are equal, so the triangles are similar, and the ratio of corresponding sides is constant.

The smaller triangle is a 3–4–5 Pythagorean triple (the missing hypotenuse is 5). Set up a proportion that includes two pairs of corresponding sides. The words "4 is to 10 as 5 is to m" become this equation:

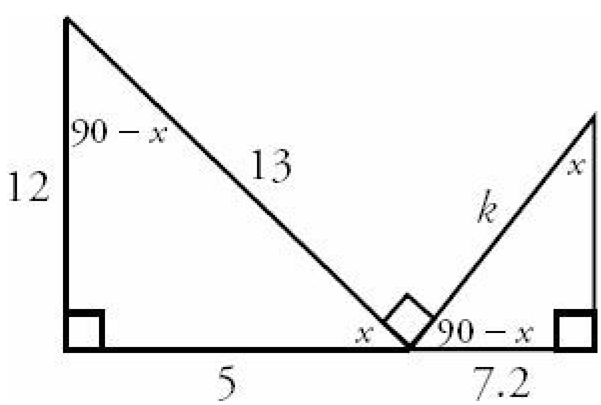
$$\frac{4}{10} = \frac{5}{m}$$
$$4m = 50$$
$$m = 12.5$$

Quantity B is larger.

56. **7.8.** Begin by noting that the triangle on the left is a 5–12–13 Pythagorean triple, so the bottom side is 5. Subtract $12.2 - 5 = 7.2$ to get the bottom side of the triangle on the right.

Next, the two unmarked angles that "touch" at the middle must add up to 90, because they form a straight line together with the right angle of 90° between them, and all three angles must add up to 180. Mark the angle on the left x. The angle on the right must then be $90 - x$.

Now the other angles that are still unmarked can be labeled in terms of x. Using the rule that the angles in a triangle add up to 180, the angle between 12 and 13 must be $90 - x$, while the last angle on the right must be x, as shown:



Since each triangle has angles of 90, x, and $90 - x$, the triangles are similar. This observation is the key to the problem. Now you can make a proportion, carefully tracking which side corresponds to which. 7.2 corresponds to 12, since each side is across from angle x. Likewise, k corresponds to 13, since each side is the hypotenuse. Write the equation and solve for k:

$$\frac{7.2}{12} = \frac{k}{13}$$

$$\frac{(13)7.2}{12} = k$$

$$k = 7.8$$

Chapter 28
of

Coordinate Geometry

In This Chapter…

Coordinate Geometry

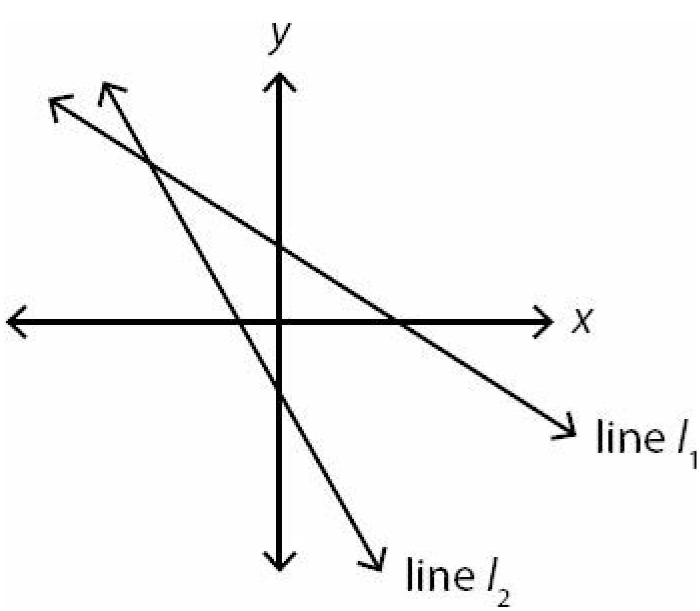
For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

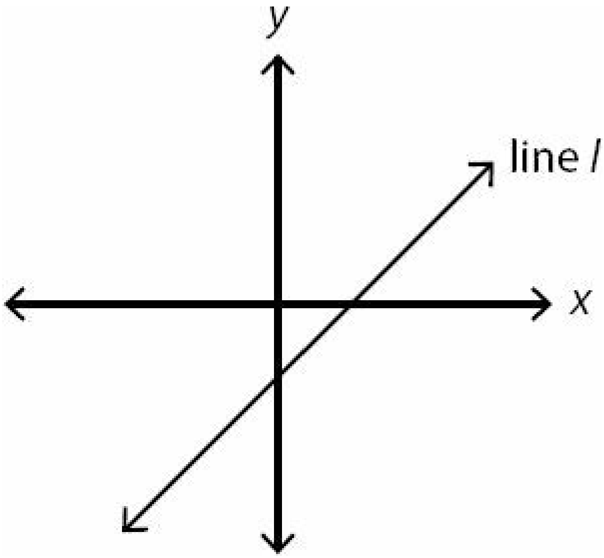
1.



Quantity A	**Quantity B**

The slope of line l_1 The slope of line l_2

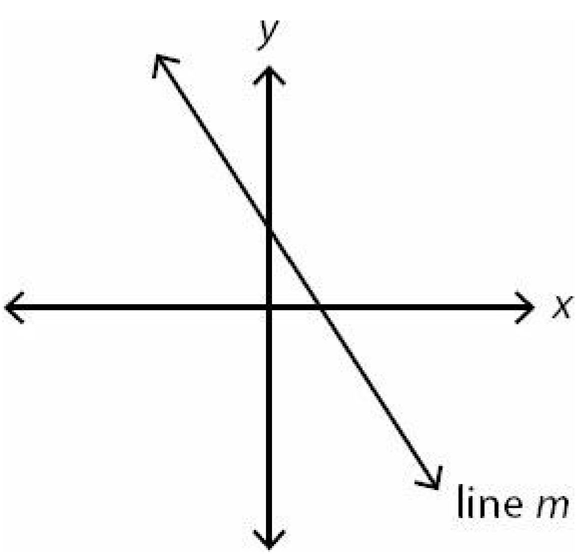
2.



Which of the following is most likely to be the equation of line *l*?

(A) $y = 4x + 4$
(B) $y = 4x - 4$
(C) $y = x - 6$
(D) $y = x + 1/2$
(E) $y = -x - 3$

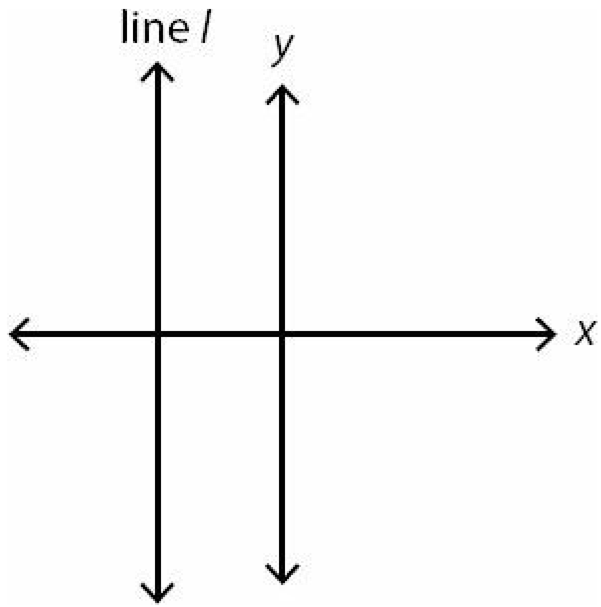
3.



Which of the following could be the equation of line *m*?

(A) $6y + 6x = 7$
(B) $3y = -4x - 3$
(C) $5y + 10 = -4x$
(D) $y = 2$
(E) $x = -2$

4.



If line l is parallel to the y-axis, what could be the equation of line l?

(A) $x = 2$
(B) $x = -2$
(C) $y = 2$
(D) $y = -2$
(E) $y = -2x$

5. What is the equation of the line that passes through (-1, -3) and has a slope of -2?

(A) $y = -2x - 1$
(B) $y = -2x - 2$
(C) $y = -2x - 5$
(D) $y = -4x - 2$
(E) $y = -5x + 2$

6. What is the slope of a line that passes through the points (-4, 5) and (1, 2)?

(A) $-\frac{3}{5}$

(B) -1

(C) $-\frac{5}{3}$

(D) $-\frac{7}{3}$

(E) -3

7. Which of the following could be the slope of a line that passes through the point (-2, -3) and crosses the y-axis above the origin?

Indicate <u>all</u> such values.

☐ $-\frac{2}{3}$

☐ $\frac{3}{7}$

☐ $\frac{3}{2}$

☐ $\frac{5}{3}$

☐ $\frac{9}{4}$

☐ 4

8. If a line has slope -2 and passes through the points (4, 9) and (6, y), what is the value of y?

☐

9. What is the distance between the points (-1, -1) and (5, 6)?

(A) 6
(B) 7
(C) $\sqrt{79}$
(D) $\sqrt{85}$
(E) 11

10. If the longest distance between any two of the points (-1, -2), (6, -2), and (7, 10) is $p\sqrt{13}$, what is the value of p?

☐

11.

A line has the equation $2y - 4x - 8 = 0$.

Quantity A	**Quantity B**
The slope of the line	4

12. Which of the following points lies on the line $y = 2x - 8$?

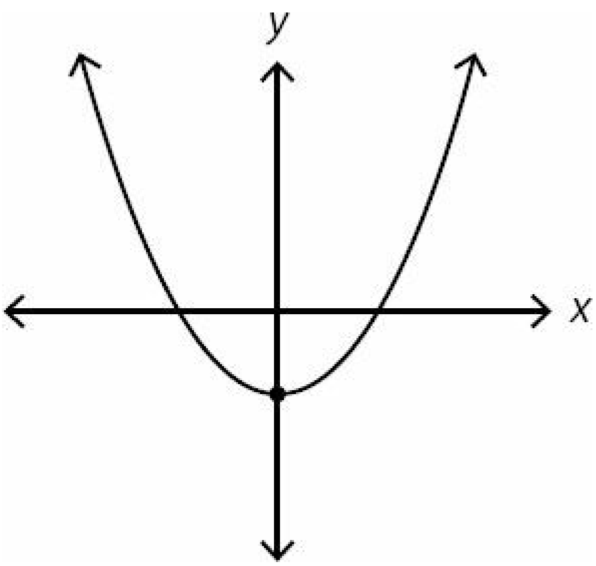
Indicate <u>all</u> such values.

☐ (3, -2)
☐ (-8, 0)
☐ (1/2, -7)

13. Which of the following points does NOT lie on the curve $y = x^2 - 3$?

(A) (3, 6)
(B) (-3, 6)
(C) (0, -3)
(D) (-3, 0)
(E) (0.5, -2.75)

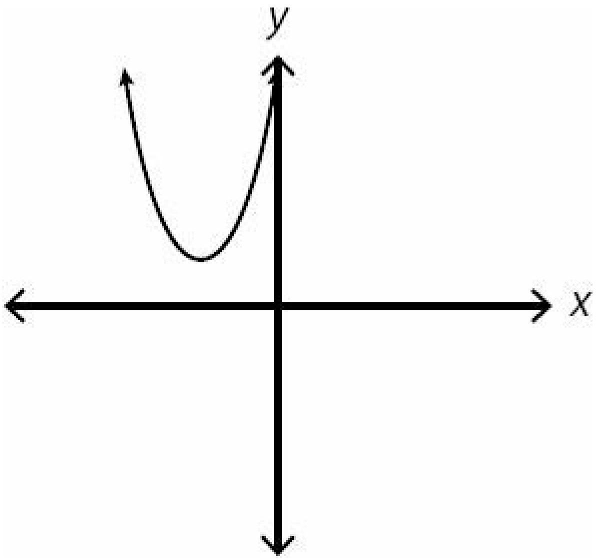
14.



Which of the following could be the equation of the figure above?

(A) $y = x - 2$
(B) $y = x^2 - x$
(C) $y = x^2 - 2$
(D) $y^2 = x^2$
(E) $y = x^3 - 2$

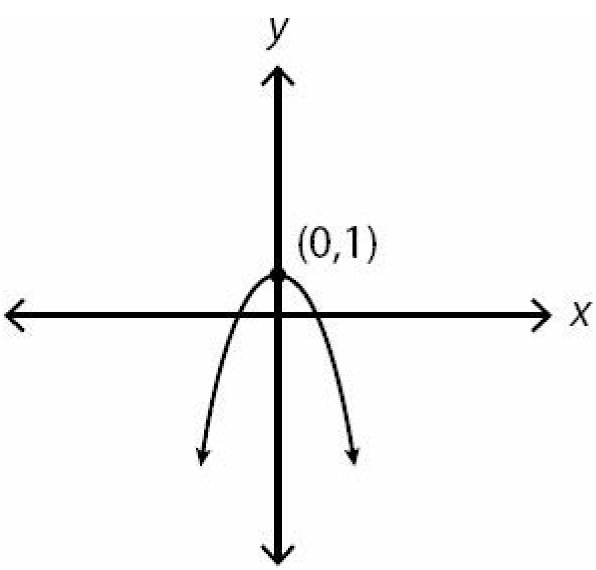
15.



Which of the following could be the equation of the parabola pictured above?

(A) $y = x^2 + 3$
(B) $y = (x - 3)^2 + 3$
(C) $y = (x + 3)^2 - 3$
(D) $y = (x - 3)^2 - 3$
(E) $y = (x + 3)^2 + 3$

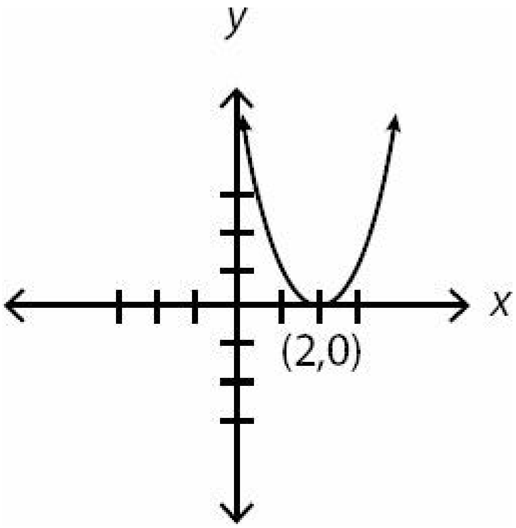
16.



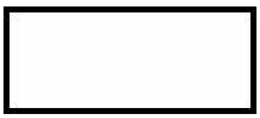
Which of the following could be the equation of the parabola pictured above?

(A) $y = -x - 1$
(B) $y = x^2 + 1$
(C) $y = -x^2 - 1$
(D) $y = -x^2 + 1$
(E) $y = -(x - 1)^2$

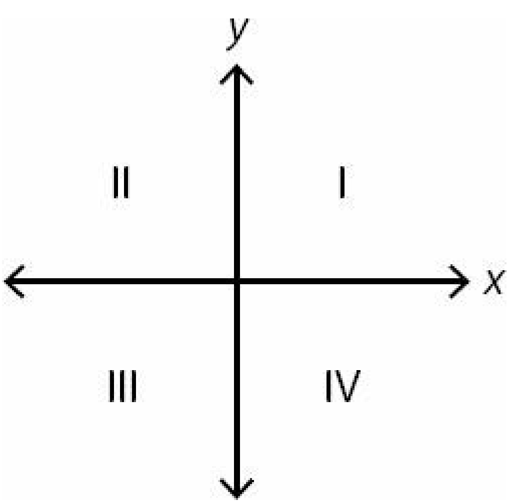
17.



If the equation of the parabola pictured above is $y = (x - h)^2 + k$ and $(-3, n)$ is a point on the parabola, what is the value of n?



18.



Which quadrant, if any, contains no point (x, y) that satisfies the inequality $y \geq (x - 3)^2 - 1$?

(A) I
(B) II
(C) III
(D) IV
(E) All quadrants contain at least one point that satisfies the given inequality.

19.

In the coordinate plane, line p has an equation of $3y - 9x = 9$.

Quantity A	**Quantity B**

The slope of line p	The x-intercept of line p

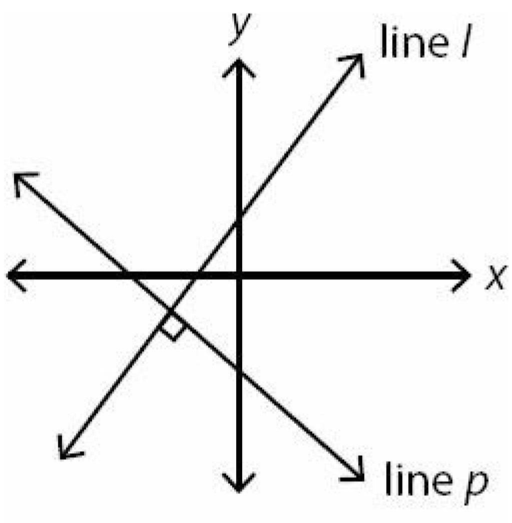
20. In the xy coordinate plane, lines l_1 and l_2 intersect at $(2, 4)$. If the equation of l_1 is $y = px + 16$ and the equation of l_2 is $y = mx + p$, where m and p are constants, what is the value of m?

21. If (3, 5) and (4, 9) are points on line L, which of the following is also a point on that line?

Indicate <u>all</u> such values.

☐ (2, 1)
☐ (5, 12)
☐ (6, 17)

22.



Line l has slope > 1.

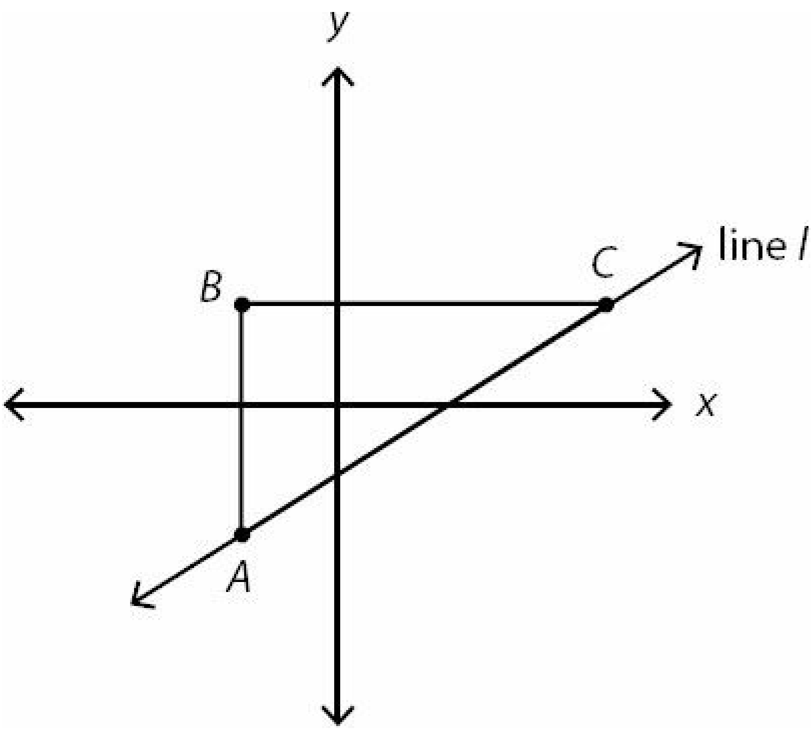
Quantity A	**Quantity B**
Slope of line p	-1

23.

Lines l_1 and l_2 are parallel and have slopes that sum to less than 1.

Quantity A	**Quantity B**
The slope of a line perpendicular to lines l_1 and l_2	$-\frac{1}{2}$

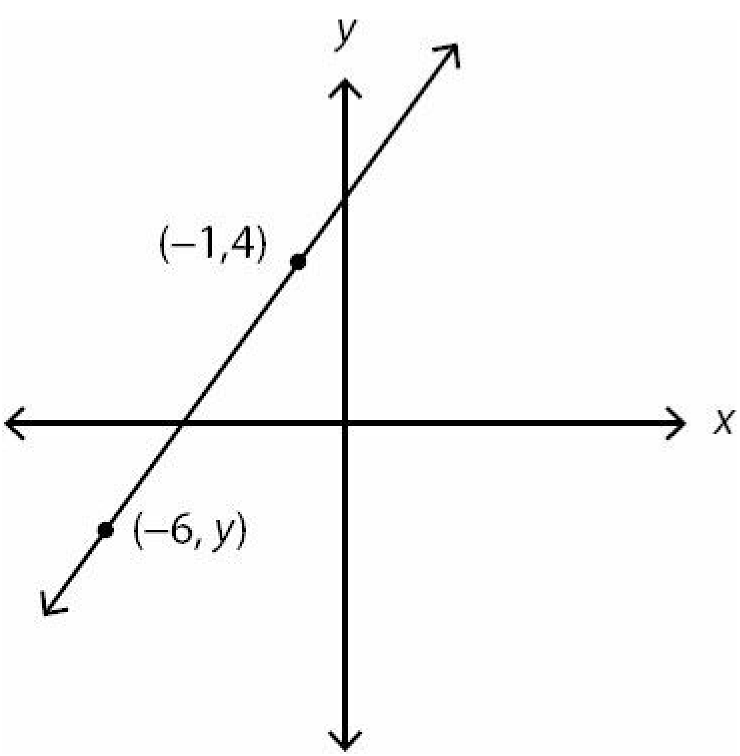
24.



If the slope of line *l* is 1/3 and the length of line segment *BC* is 4, how long is line segment *AB*?

(A) 3/4
(B) 4/3
(C) 3
(D) 4
(E) 12

25.



If the slope of the line is $\frac{15}{14}$, what is the value of *y*?

(A) $\frac{2}{7}$

(B) $\frac{7}{2}$

(C) $-\frac{7}{2}$

(D) $-\frac{14}{19}$

(E) $-\frac{19}{14}$

26. What is the area of a triangle with vertices (-2, 4), (2, 4) and (-6, 6)?

27.

Lines k and p are perpendicular, neither is vertical, and p passes through the origin.

Quantity A	**Quantity B**
The product of the slopes of lines k and p	The product of the y-intercepts of lines k and p

28.

In the coordinate plane, points (a, b) and (c, d) are equidistant from the origin.

$|a| > |c|$

Quantity A	**Quantity B**
$\|b\|$	$\|d\|$

29.

In the coordinate plane, lines j and k are parallel. The x-intercept of line j is greater than that of line k and the product of their slopes is positive.

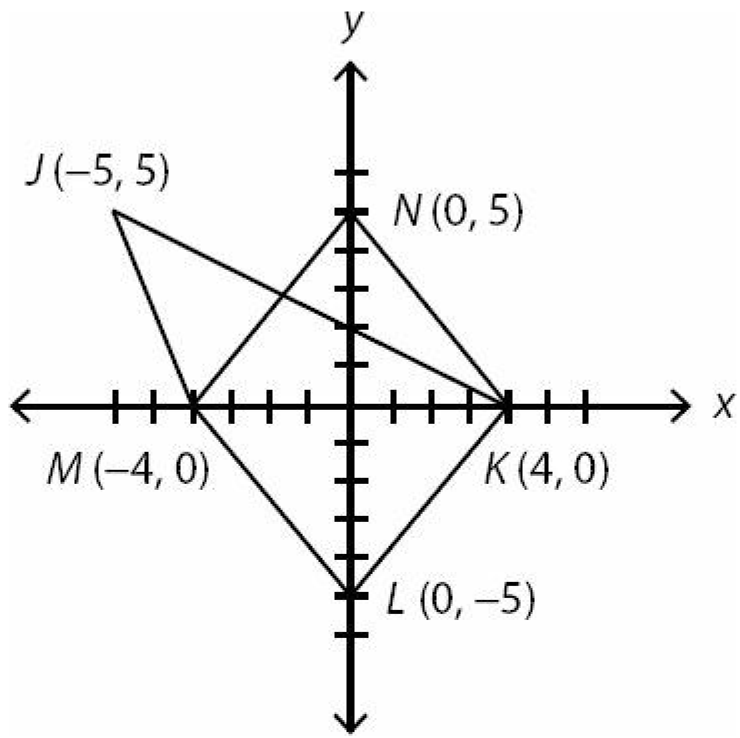
Quantity A	**Quantity B**
The y-intercept of line j	The y-intercept of line k

30. In the xy plane, which of the statements below individually provide enough information to determine whether line z passes through the origin?

Indicate all such statements.

- ☐ The equation of line z is $y = mx + b$ and $b = 0$.
- ☐ The sum of the slope and the y-intercept of line z is 0.
- ☐ For any point (a, b) on line z, $|a| = |b|$.

31.



Quantity A	**Quantity B**
The area of parallelogram *KLMN*	The area of quadrilateral *JKLM*

32. Which of the following could be the equation of a line parallel to the line $5x - 6y = 9$?

(A) $y = -\frac{5}{6}x + 1$

(B) $y = \frac{6}{5}x + 1$

(C) $y = \frac{5}{6}x + 1$

(D) $y = \frac{3}{2}x - 1$

(E) $y = \frac{2}{3}x - 1$

33. Which of the following could be the equation of a line perpendicular to the line $y = -6x + 4$?

(A) $6y - x = 12$

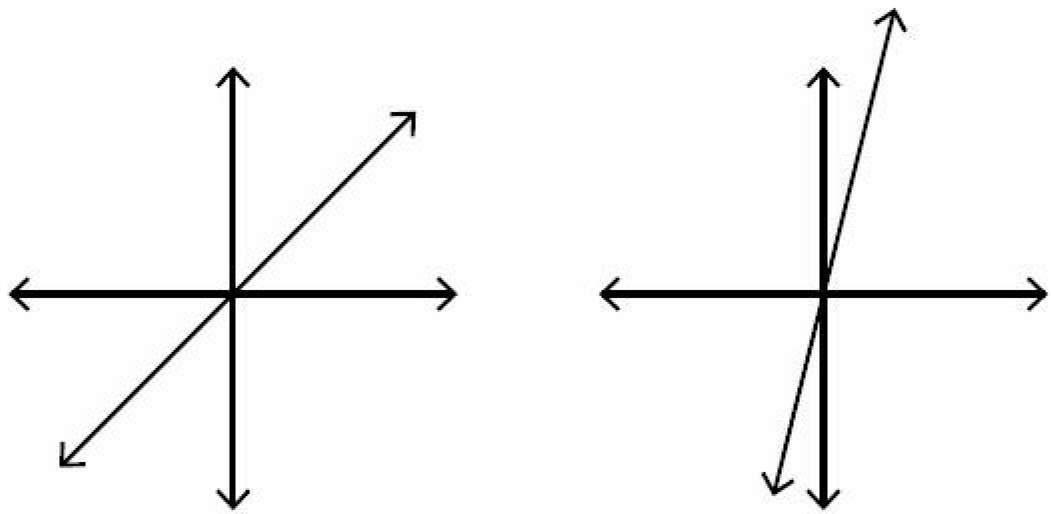
(B) $x = -6y - 12$
(C) $y + 4x = 2$
(D) $\frac{y}{2} = -3x + 5$
(E) $y + 1 = 6x$

Coordinate Geometry Answers

1. **(A).** Both slopes are negative (pointing down when reading from left to right), and line l_2 is clearly steeper than line l_1. Thus, the slope of l_2 has a larger *absolute value*. But since the values are both negative, the slope of l_1 is a larger number. For instance, the slope of l_1 could be -1 and the slope of l_2 could be -2. Whatever the actual numbers are, the slope of l_1 is closer to 0 and therefore larger.

2. **(C).** Since there are no numbers on the graph, you can't determine the actual equation of the line, but the line clearly has a positive slope (it points upward when reading from left to right) and a negative y-intercept (it crosses the y-axis below the origin). All of the answers are already in slope-intercept form ($y = mx + b$ where m = slope and b = y-intercept). Choices (A), (B), (C), and (D) have positive slope. Of those, only choices (B) and (C) have a negative y-intercept.

Now, is the slope closer to positive 4 or positive 1? A slope of 1 makes 45° angles when it cuts through the x and y axes, and this figure looks very much like it represents a slope of 1. A slope of 4 would look much steeper than this picture. Note that xy-planes are drawn to scale on the GRE, and units on the x-axis and on the y-axis are the same, unless otherwise noted.



slope = 1 slope = 4

The correct answer is (C). Note that the GRE would only give questions like this where the answers are far enough apart that you can clearly determine the intended answer.

3. **(A).** Since there are no numbers on the graph, you can't determine the actual equation of the line, but the line clearly has a negative slope (it points down when reading from left to right) and a positive y-intercept (it crosses the y-axis above the origin).

Change the answers to slope-intercept form ($y = mx + b$ where m = slope and b = y-intercept). First note that (D) and (E) cannot be the answers—choice (D) represents a horizontal line crossing through (0, 2), and choice (E) represents a vertical line passing through (-2, 0). Now, look at choice (A):

$$6y + 6x = 7$$

$6y = -6x + 7$

$y = -x + \frac{7}{6}$

This line (choice A) has slope -1 and y-intercept $\frac{7}{6}$. Next, test choice (B):

$3y = -4x - 3$

$y = -\frac{4}{3}x - 1$

This line (choice B) has slope $-\frac{4}{3}$ and y-intercept. Finally, look at (C):

$5y + 10 = -4x$
$5y = -4x - 10$

$y = -\frac{4}{5}x - 2$

This line (choice C) has slope $-\frac{4}{5}$ and y-intercept -2.

The only choice with a negative slope and a positive y-intercept is choice (A).

4. **(B).** Since the line is vertical, it always has the same x-coordinate. That is what the correct answer, $x = -2$, expresses. No matter what the y-coordinate is, x is always some negative value. All equations of vertical lines have the form x = [number]. Similarly, all equations of horizontal lines have the form y = [number].

5. **(C).** In $y = mx + b$ form, m is the slope and b is the y-intercept. Since the slope is -2:

$y = -2x + b$

Now, plug in the point (-1, -3) to determine b:

$-3 = -2(-1) + b$
$-3 = 2 + b$
$-5 = b$

Since $b = -5$, the equation of the line is:

$y = -2x - 5$

Note that the coordinates (-1, -3) do not belong in the final answer. The point (-1, -3) was merely an example of one of the infinite number of points along the line.

6. **(A).** The slope formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$. It doesn't matter which point is first; just be consistent. Using (-4, 5) as x_1 and y_1 and (1, 2) as x_2 and y_2:

$$m = \frac{2-5}{1-(-4)} = -\frac{3}{5}$$

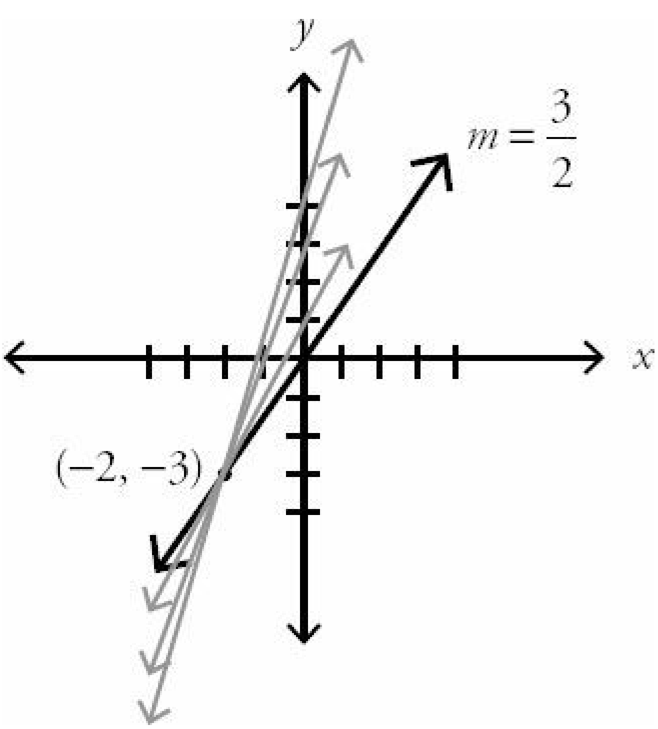
The slope is $\frac{-3}{5}$, which appears in the choices as $-\frac{3}{5}$ (these are identical).

7. **IV, V, and VI only.** The line must hit a point on the y-axis above (0, 0). That means the line could include (0, 0.1), (0, 25), or even (0, 0.00000001). Since the x-intercept could get very, very close to (0, 0), use the point (0, 0) to calculate the slope—and then reason that since the line can't *actually* go through (0, 0), the slope will actually have to be steeper than that.

The slope formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Using (0, 0) as x_1 and y_1 and (-2, -3) as x_2 and y_2 (you can make either pair of points x_1 and y_1, so make whatever choice is most convenient):

$$m = \frac{-3-0}{-2-0} = \frac{-3}{-2} = \frac{3}{2}$$

Since the slope is positive and the line referenced in the problem needs to hit the x-axis above (0, 0), the slope of that line will have to be greater than $\frac{3}{2}$, as in the gray lines below:



Select all answers with a slope greater than $\frac{3}{2}$. Thus, $\frac{5}{3}$, $\frac{9}{4}$, and 4 are correct.

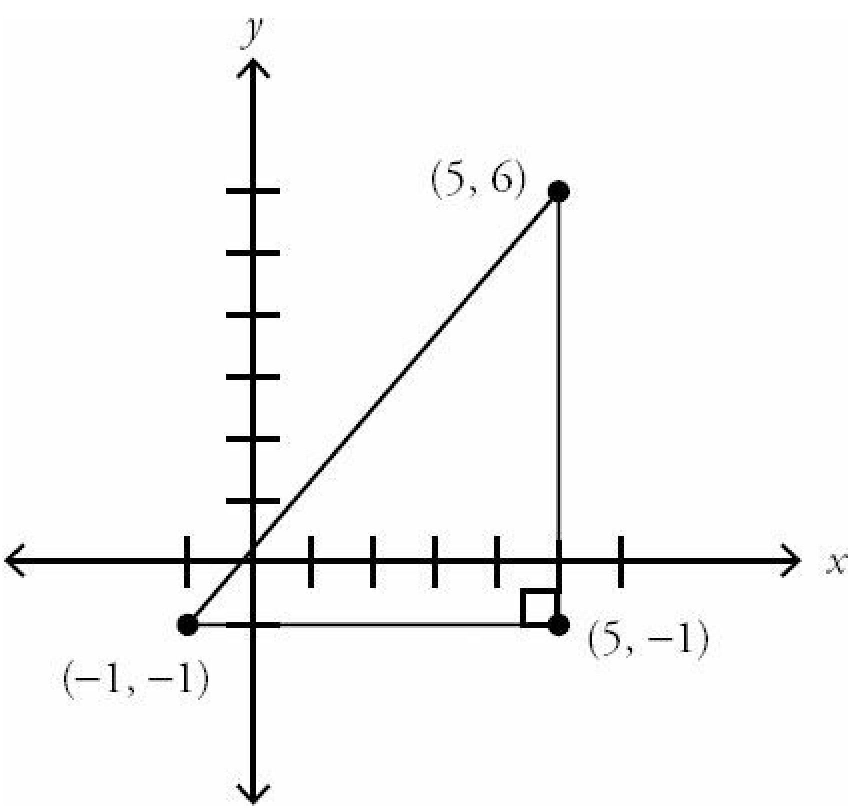
8. **5.** The slope formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Using (4, 9) as x_1 and y_1 and (6, y) as x_2 and y_2, and plugging in -2 for the slope:

$$-2 = \frac{y-9}{6-4}$$
$$-2 = \frac{y-9}{2}$$
$$-4 = y = -9$$
$$5 = y$$

9. **(D).** You could use the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$:

$$d = \sqrt{(6-(-1))^2 + (5-(-1))^2}$$
$$d = \sqrt{(7)^2 + (6)^2}$$
$$d = \sqrt{85}$$

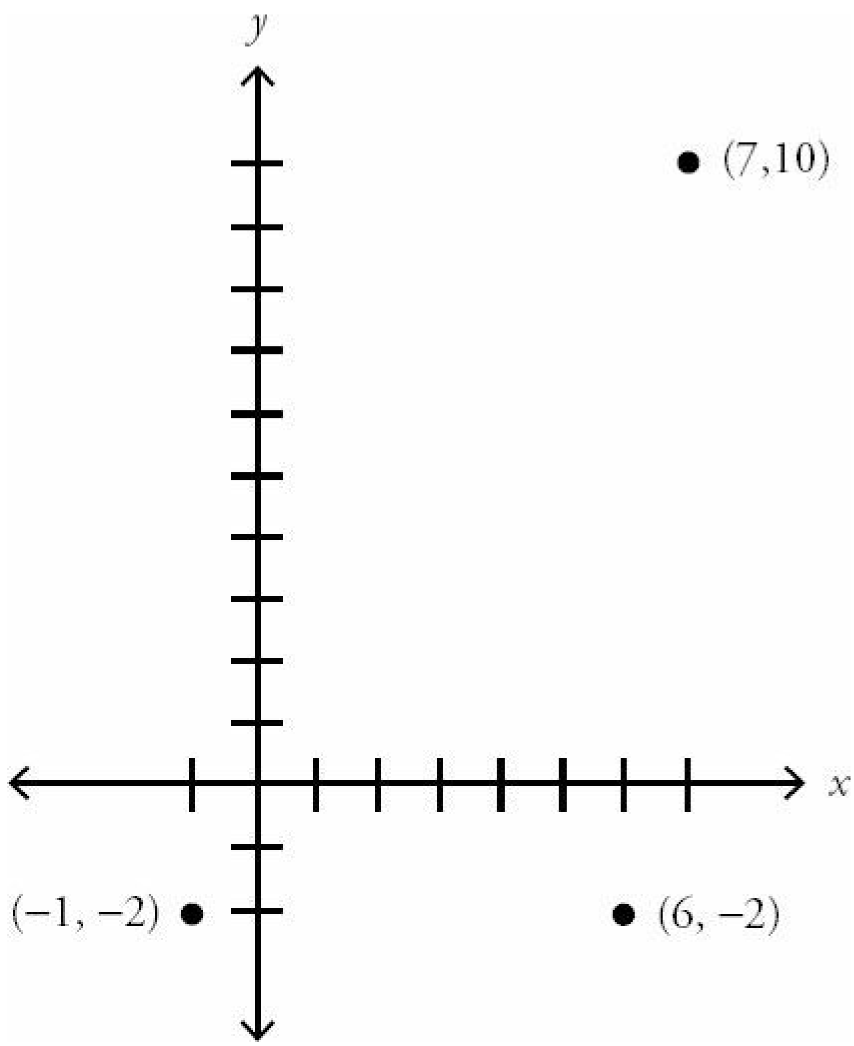
Or you could construct a right triangle and use the Pythagorean theorem:



The third point, (5, -1) lets you construct a right triangle with the distance *d* as the hypotenuse. The distance between (-1, -1) and (5, -1) is 6. The distance between (5, -1) and (5, 6) is 7. From the Pythagorean theorem:

$$6^2 + 7^2 = d^2$$
$$36 + 49 = d^2$$
$$85 = d^2$$
$$d = \sqrt{85}$$

10. **4.** Make a quick sketch:



It should be obvious that the two furthest points are (-1, -2) and (7, 10). Also keep in mind that since (-1, -2) and (6, -2) share a *y*-coordinate, the distance between the two points is just the distance between their *x*-coordinates: 6 - (-1) = 7. The correct answer should definitely be longer than 7.

To find the distance between (-1, -2) and (7, 10), you could use the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(10 - (-2))^2 + (7 - (-1))^2}$$
$$d = \sqrt{(12)^2 + (8)^2}$$

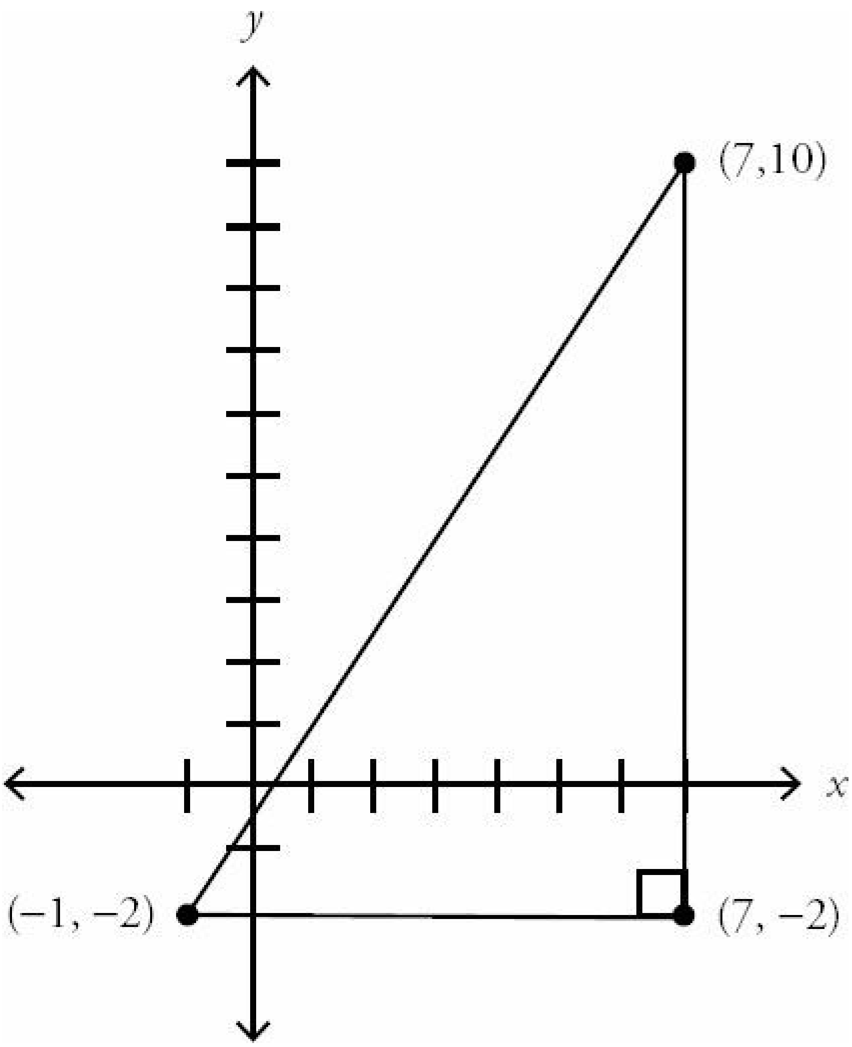
$d = \sqrt{208}$

Use your calculator, if needed, to find the biggest perfect square that goes into 208. It is 16:

$$d = \sqrt{208} = \sqrt{16 \times 13} = 4\sqrt{13}$$

Since the distance $p\sqrt{13}$ is equal to $4\sqrt{13}$, $p = 4$.

Alternatively, you could construct a right triangle and use the Pythagorean theorem.



The point (7, -2) lets you construct a right triangle with the distance *d* as the hypotenuse. (Don't get confused—this point has nothing to do with the (6, -2) from the problem; that point was irrelevant to the question being asked.) You create the point (7, -2) by dropping a line straight down from (7, 10) and stopping at (7, -2), which has the same *y*-coordinate as the point (-1, -2).

The distance between (-1, -2) and (7, -2) is 8. The distance between (7, 10) and (7, -2) is 12. From the Pythagorean Theorem:

$$8^2 + 12^2 = d^2$$
$$64 + 144 = d^2$$

$208 = d^2$
$d = \sqrt{208} = \sqrt{16 \times 13} = 4\sqrt{13}$

Again, since the distance $p\sqrt{13}$ is equal to $4\sqrt{13}$, $p = 4$.

11. **(B).** Manipulate $2y - 4x - 8 = 0$ into slope intercept form ($y = mx + b$, where m is the slope and b is the y-intercept):

$2y - 4x = 8$
$2y = 4x + 8$
$y = 2x + 4$

The slope of the line is 2. (The y-intercept is 4, which may be the "trick" intended in Quantity B). Quantity B is larger.

12. **I and III only.** For the point (3, -2) to lie on the line $y = 2x - 8$, y needs to equal -2 when you plug in 3 for x.

$y = 2(3) - 8$
$y = 6 - 8 = -2$

y does equal -2 when x equals 3, so the point does lie on the line and statement I is correct.

However, when you plug in -8 for x, y does not equal 0, so statement II is not correct. When you plug in 1/2 for x, y does equal -7, so statement III is correct.

13. **(D).** The problem asks for the point that does NOT lie on the curve. $y = x^2 - 3$ is the equation of a parabola, but you don't need to know that fact in order to answer this question. For each choice, just plug in the coordinates for x and y. For instance, try choice (A):

$6 = (3)^2 - 3$
$6 = 6$

Since this is a true statement, choice (A) lies on the curve. The only choice that yields a false statement when plugged in is choice (D), the correct answer.

For the point (-3, 0) to lie on the curve $y = x^2 - 3$, y needs to equal 0 when you plug in -3 for x.

$y = (-3)^2 - 3$
$y = 9 - 3 = 6$

y does not equal 0 when x equals -3, so the point does not lie on the curve.

14. **(C).** The figure is a parabola. Choices (A), (D), and (E) do not represent parabolas. An equation for a parabola should look something like this: $y = 5x^2 + 4x + 3$, where the 5, 4, and 3 can be other numbers. When the left side is just y, the right side has to have an x^2 term (and no higher power). There can be an x term and/or a constant term.

Both (B) and (C) represent parabolas, but the parabola in (B) is not centered on the y-axis (there must be no x term). The equation in (C), $y = x^2 - 2$, represents a parabola that opens upward, that is centered on the y-axis, and that has a y-intercept of -2. These features are consistent with the diagram.

15. **(E).** The standard equation of a parabola in vertex form is $y = a(x - h)^2 + k$, where the vertex is (h, k). Here is the vertex of the parabola described by each answer choice:

(A) (0, 3) On the axis
(B) (3, 3) Incorrect Quadrant
(C) (-3, -3) Incorrect Quadrant
(D) (3, -3) Incorrect Quadrant
(E) (-3, 3) Correct

Only choice (E) places the vertex in the correct quadrant.

16. **(D).** The standard equation of a parabola in vertex form is $y = a(x - h)^2 + k$, where the vertex is (h, k). Eliminate choice (A), as it is not the equation of a parabola. Here is the vertex of the parabola described by each remaining answer choice:

(B) (0, 1) Correct
(C) (0, -1) Incorrect
(D) (0, 1) Correct
(E) (1, 0) Incorrect

Both (B) and (D) have the correct vertex. However, choice (B) describes a parabola pointing upward from that vertex, because the x^2 term is positive. The negative in front of choice (D) indicates a parabola pointing downward from that vertex.

17. **25.** The equation of the parabola is $y = (x - h)^2 + k$. The standard equation of a parabola in vertex form is $y = a(x - h)^2 + k$, where the vertex is (h, k). (Since the equation of this particular parabola does not have constant a, a must be equal to 1.)

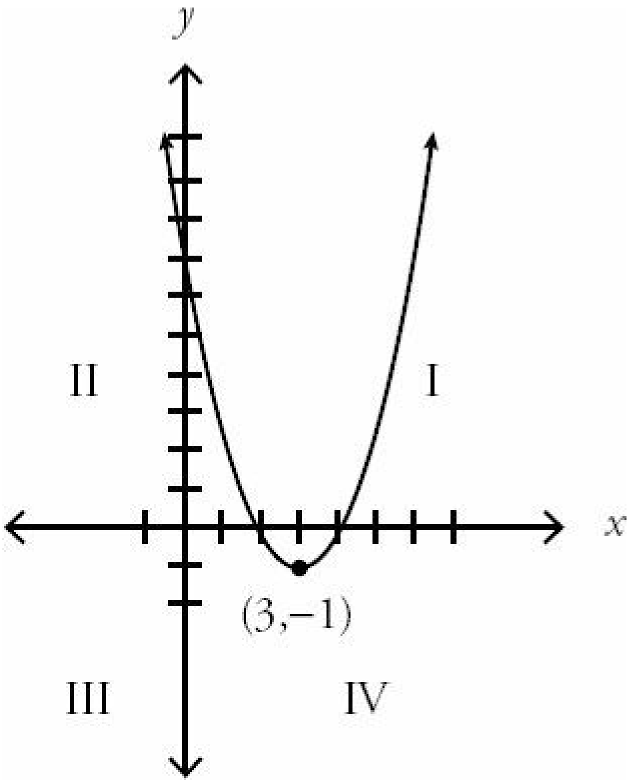
Using $y = (x - h)^2 + k$ and the vertex (2, 0) shown in the graph:

$$y = (x - 2)^2 + 0$$
$$y = (x - 2)^2$$

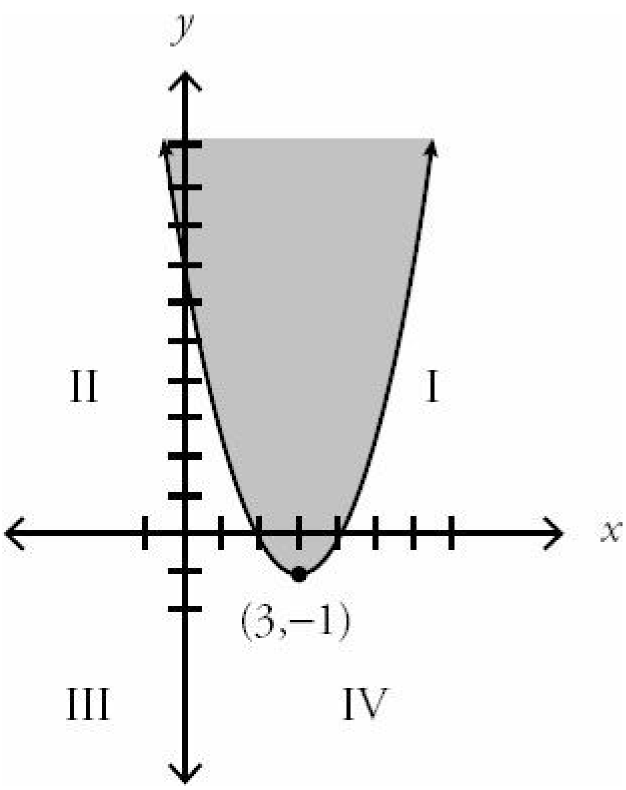
Since $(-3, n)$ is a point on the parabola, plug in -3 and n for x and y:

$$n = (-3 - 2)^2$$
$$n = (-5)^2$$
$$n = 25$$

18. **(C).** One method for solving this problem is to graph the curve. To do this, first graph the parabola as though the $\geq$ sign were an equals sign. The standard equation of a parabola in vertex form is $y = a(x - h)^2 + k$, where the vertex is (h, k). Thus, $y = (x - 3)^2 - 1$ is the graph of a parabola with its vertex at (3, -1). (3, 1)



Since $y \geq (x - 3)^2 - 1$ is actually an inequality in which y (i.e., all the y-coordinates) are greater than the graph, shade above the curve.



The inequality has no points in Quadrant III.

Alternatively, you could use algebra to prove this. Coordinates in Quadrant III have negative x-coordinates and negative y-coordinates. There is no such pair of coordinates that will satisfy $y \geq (x - 3)^2 - 1$.

Specifically, the only way the y-coordinate could be negative is if $(x - 3)^2$ were less than 1 (so that subtracting 1 from it yielded a negative). The only way for $(x - 3)^2$ to be less than 1 is for x to be less than 1 away from 3. That is, y is only negative when $2 < x < 4$. You can also demonstrate this by setting $(x - 3)^2 - 1$ in an inequality with 0:

$(x - 3)^2 - 1 < 0 \longleftarrow$ Note: this is NOT necessarily a true statement; you are investigating what would have to be true about x in order for this to be true.

$(x - 3)^2 < 1$
$x - 3 < 1$ or $x - 3 > -1$
$x < 4$ or $x > 2$
$2 < x < 4$

Thus, y is only negative when $2 < x < 4$. Therefore, no coordinate pair in Quadrant III will satisfy the inequality.

19. **(A).** In slope intercept form ($y = mx + b$, where m is the slope and b is the y-intercept):

$3y - 9x = 9$
$3y = 9x + 9$
$y = 3x + 3$

The slope is 3. The y-intercept is also 3, but the problem asks for the x-intercept. To get an x-intercept, substitute 0 for y:

$0 = 3x + 3$
$-3 = 3x$
$-1 = x$

Thus, the slope is 3 and the x-intercept is -1. Quantity A is greater.

20. **5.** If l_1 and l_2 intersect at (2, 4), then 2 can be plugged in for x and 4 plugged in for y in either equation. Equation l_1:

$y = px + 16$
$4 = p(2) + 16$
$-12 = 2p$
$-6 = p$

Now, plug (2, 4) as well as $p = -6$ into Equation l_2 to get m, the final answer:

$y = mx + p$
$4 = m(2) - 6$
$10 = 2m$
$5 = m$

21. **I and III only.** To be on the same line as (3, 5) and (4, 9), the slope between any given point and either (3, 5) or (4, 9) must be the same as the slope between (3, 5) and (4, 9).

The (very) long way to do this problem would be to find the slope of (3, 5) and (4, 9). Using “change in y” divided by “change in x,” you get a slope of 4/1, or 4. Test the choice (2,1) with either (3, 5) or (4, 9) to see if the slope is the same —for instance, the slope of the line segment between (2,1) and (3,5) is clearly 4/1, since the difference between the y-coordinates is 4 and the difference between the x-coordinates is 1. Since the slopes of these connecting line segments are the same, they are in fact parts of the same line.

It is possible to do this procedure for each choice. However, like most GRE problems, this problem has a "trick": Using the original two points, notice that to get from (3, 5) to (4, 9), the x-coordinate goes up 1, while the y-coordinate goes up 4. Now just continue that pattern upward from (4, 9), adding 1 to the x and 4 to the y. You get (4 + **1**, 9 + **4**), or (5, 13). This point is not in the choices, and in fact you can now eliminate (5, 12) since the line passes above that point.

Keep going up on the line. (5 + **1**, 13 + **4**) is (6, 17), so this point is on the line.

You can do the same trick going *down*. Start from (3, 5) and instead of adding 1 and 4, *subtract* 1 and 4. (3 - **1**, 5 - **4**) is (2, 1), so this point is on the line.

Thus, (2, 1) and (6, 17) are on the line, and (5, 12) is not.

22. **(A).** If the slope of line l is > 1 and line p is perpendicular (you know this because of the right angle symbol on the figure), then line p will have a slope greater than -1 because perpendicular lines have negative reciprocal slopes—that is, the product of the two slopes is -1.

Try a few examples to better illustrate this: line l could have slope 2, in which case line p would have slope -1/2. Line l could have slope 3/2, in which case line p would have slope -2/3. Or, line l could have slope 100, in which case line p would have slope -1/100.

All of these values (-1/2, -2/3, and -1/100) are greater than -1. This will work with any example you try; since line l has a slope greater than 1, line p will have a slope with an absolute value less than 1. Since that value will also be negative, it will always be the case that -1 < (slope of line p) < 0.

23. **(D).** Since lines l_1 and l_2 are parallel, they have the same slope. Call that slope m. Since the slopes add to less than 1:

$$m + m < 1$$
$$2m < 1$$
$$m < 1/2$$

Thus, lines l_1 and l_2 each have slopes less than 1/2. A line perpendicular to those lines would have a negative reciprocal slope. However, there isn't much more you can do here. Lines l_1 and l_2 could have slopes of 1/4 (in which case a perpendicular line would have slope = -4), or slopes of -100 (in which case a perpendicular line would have slope = 1/100). Thus, the slope of the perpendicular line could be less than or greater than $-\frac{1}{2}$.

24. **(B).** You are told that the slope is 1/3. Since slope = rise/run (or "change in y" divided by "change in x"), for every 1 unit the line moves up, it will move 3 units to the right.

Since the *actual* move to the right is equal to 4, you can now create a proportion:

$$\frac{1}{3} = \frac{n}{4}$$

Here, n is the distance from A to B (which is also the change in the y-coordinates).

Cross multiply to get $3n = 4$ or $n = 4/3$.

25. **(E).** The slope formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Using $\frac{15}{14}$ as the slope, (-6, y) as x_1 and y_1, and (-1, 4) as x_2 and y_2:

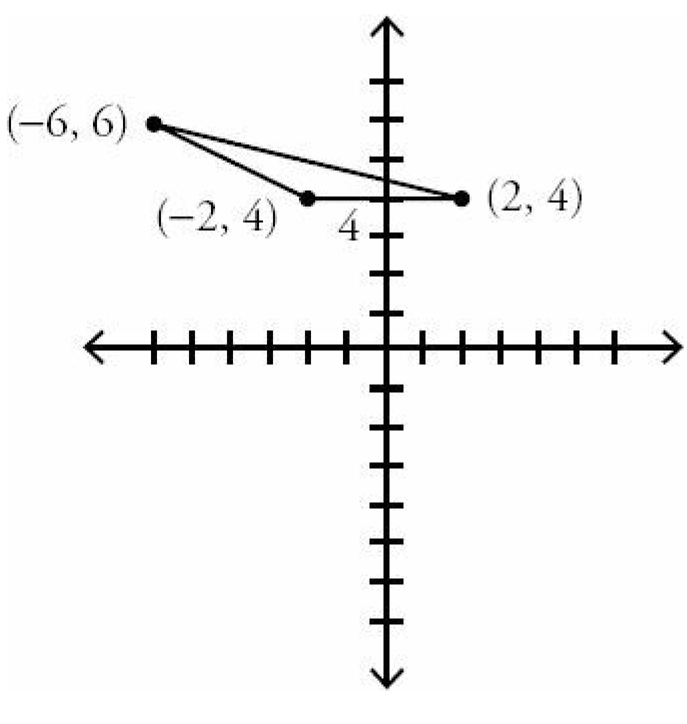
$$\frac{15}{14} = \frac{4 - y}{-1 - (-6)}$$

$$\frac{15}{14} = \frac{4 - y}{5}$$

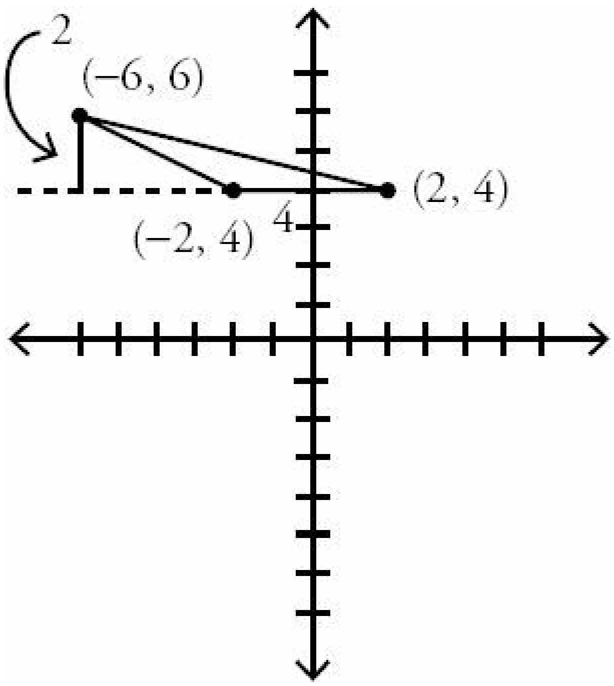
Cross multiply and solve for y:

$15(5) = 14(4 - y)$
$75 = 56 - 14y$
$19 = - 14y$
$-\frac{19}{14} = y$

26. **4.** Make a quick sketch of the three points, joining them to make a triangle. Since (-2, 4) and (2, 4) make a horizontal line, use this line as the base. Since these two points share a y-coordinate, the distance between them is simply the distance between their x-coordinates: 2 - (-2) = 4.



The height of a triangle is always perpendicular to the base. Drop a height vertically from (-6, 6). Subtract the y-coordinates to get the distance: 6 - 4 = 2.

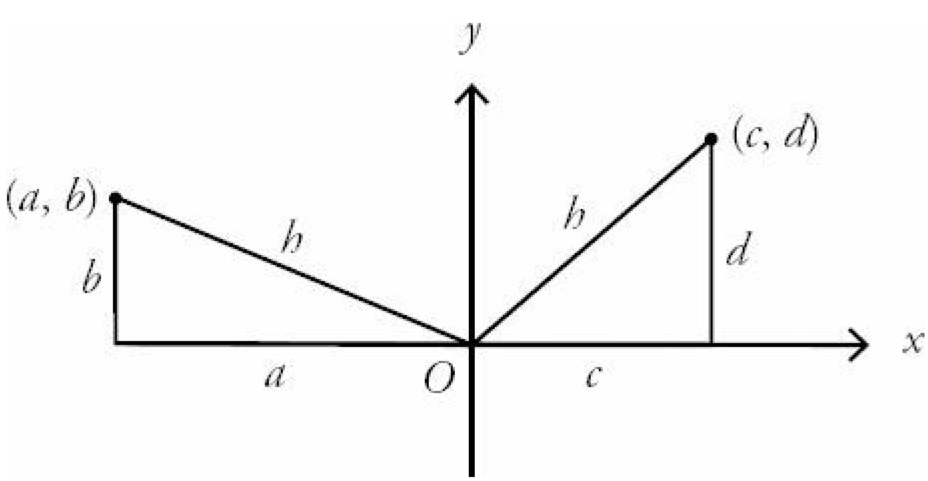


The formula for area of a triangle is $\frac{bh}{2}$. Thus, the area is $\frac{(4)(2)}{2}$, or 4.

27. **(B).** The slopes of perpendicular lines are the negative inverse of each other, so their product is -1. For example, perpendicular lines could have slopes of 2 and -1/2, or -5/7 and 7/5. In all of these cases, Quantity A is equal to -1. (The only exception is when one of the lines has an undefined slope because it's vertical, but that case has been specifically excluded.)

If line p passes through the origin, its y-intercept is 0, so regardless of the y-intercept of line k, Quantity B is equal to zero.

28. **(B).** A point's distance from the origin can be calculated by constructing a right triangle in which the legs are the vertical and horizontal distances. Sketch a diagram in which you place (a, b) and (c, d) anywhere in the coordinate plane that you wish; then construct two right triangles using (0, 0) as a vertex.



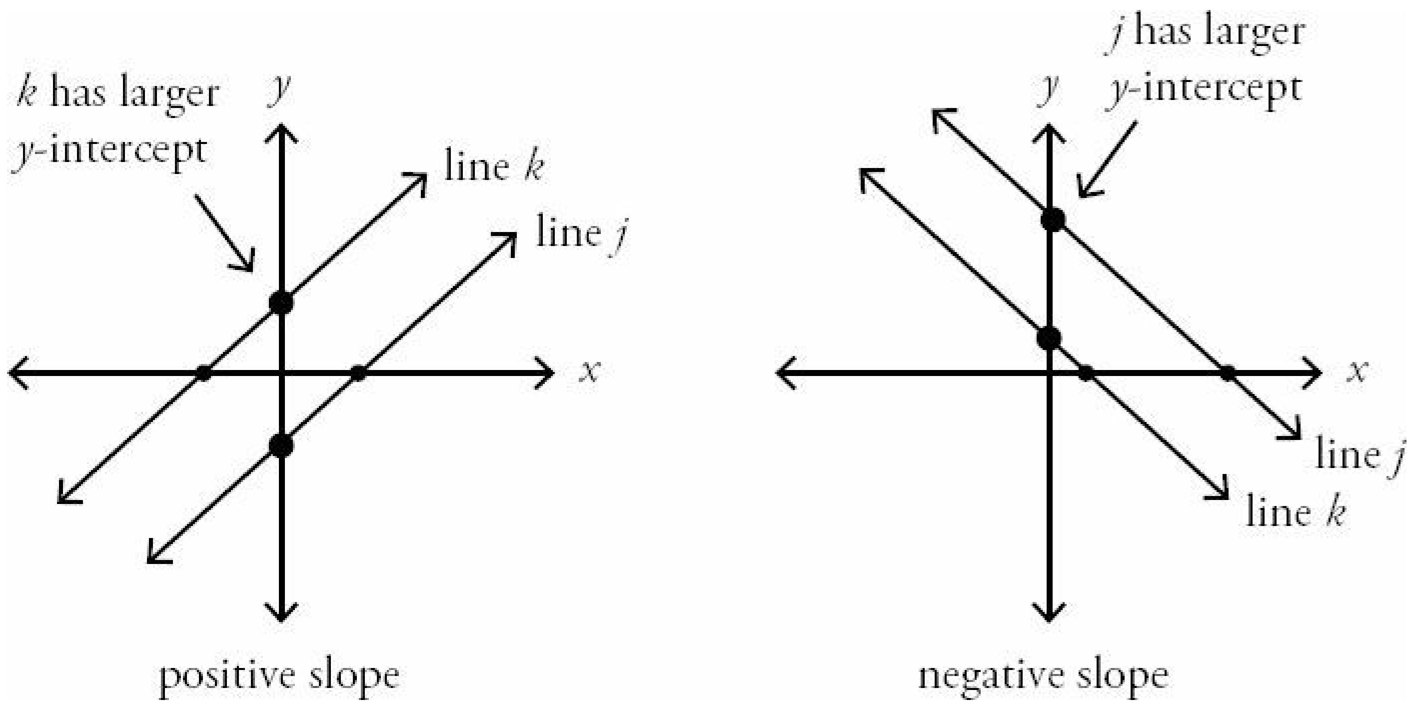
Both hypotenuses are labeled h, since the points are equidistant from the origin. Set up two Pythagorean theorems:

$a^2 + b^2 = h^2$
$c^2 + d^2 = h^2$

So $a^2 + b^2 = c^2 + d^2$.

Since $|a| > |c|$, you know that $a^2 > c^2$. (Try it with test numbers.) To make the equation $a^2 + b^2 = c^2 + d^2$ true, you must have $b^2 < d^2$. This means that $|b| < |d|$, and Quantity B is larger.

29. **(D).** Parallel lines have the same slope. Since the product of the two slopes is positive, either both slopes are positive or both slopes are negative. Here are two examples in which line *j* has a larger *x*-intercept, as specified by the problem:



If the slopes are positive, *k* will have the greater *y*-intercept, but if the slopes are negative, *l* will have the greater *y*-intercept.

30. **I and III only.** Statement I tells you directly that *b*, the *y*-intercept, is equal to 0. Thus, the line passes through the origin.

For statement II, both the slope and the *y*-intercept could be 0, in which case line *z* is a horizontal line lying on the *x*-axis and therefore passes through the origin. Or, the slope and *y*-intercept could simply be opposites, such as 2 and -2. A line with a *y*-intercept of -2 and a slope of 2 would not pass through the origin. Therefore, this statement is not sufficient to determine whether line *z* passes through the origin.

As for statement III, since $|a| = |b|$ must hold for every point on the line, then (0, 0) is a point on the line, since $|0| = |0|$.

31. **(C).** Both figures share triangle *MLK*, so you don't need to calculate anything for this part of the figure. Parallelogram *KLMN* and quadrilateral *JKLM* each have a "top" (the part above the *x*-axis) that is a triangle with base *MK* (= 8) and height 5. If two triangles have the same base and equal heights, their areas are equal. No calculation is needed to pick (C).

32. **(C).** Rearrange the equation to get it into $y = mx + b$ format where m is the slope:

$$5x - 6y = 9$$
$$-6y = -5x + 9$$
$$y = \frac{5}{6}x - \frac{3}{2}$$

The slope is $\frac{5}{6}$. Parallel lines have the same slope, so only choice (C) is parallel to the given line.

33. **(A)**. The line $y = -6x + 4$ is already in $y = mx + b$ format, so the slope is -6. Perpendicular lines have negative reciprocal slopes, so you are looking for a line with slope $\frac{1}{6}$. Rearrange each choice into $y = mx + b$ format, if needed, to find a match.

Choice (A):

$$6y - x = 12$$
$$6y = x + 12$$
$$y = \frac{1}{6}x + 2$$

The slope of this line is $\frac{1}{6}$. Choice (A) works, so it is not necessary to try the other choices.

Chapter 29
of

Mixed Geometry

In This Chapter…

Mixed Geometry

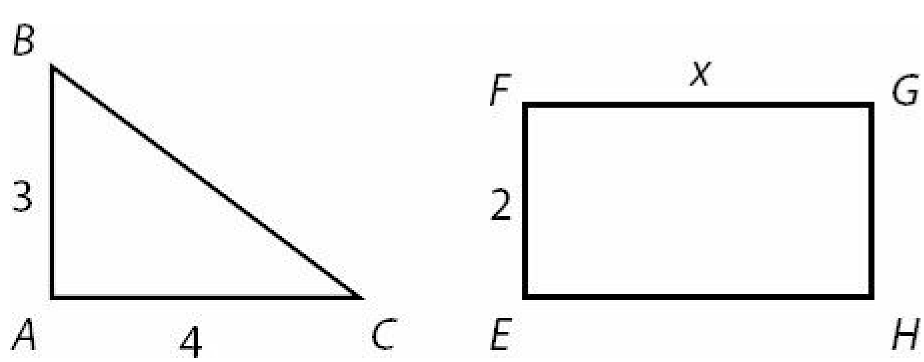
For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

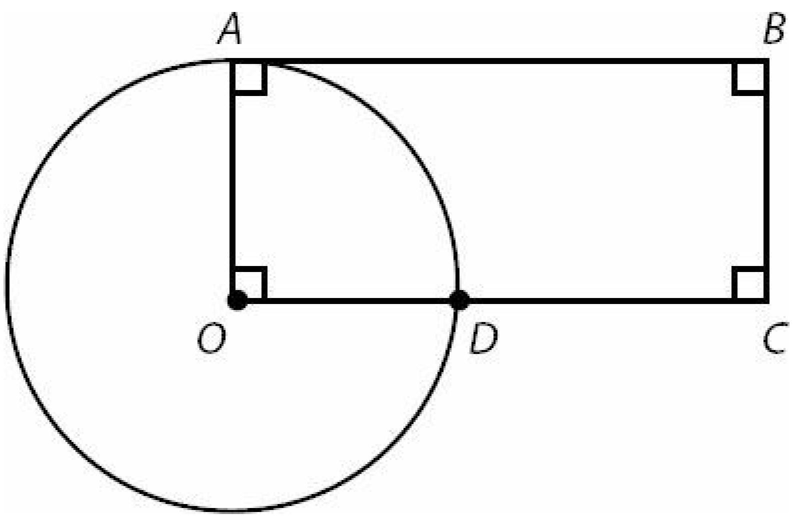
All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.



Right Triangle *ABC* and Rectangle *EFGH* have the same perimeter. What is the value of x?

2.



Point *O* is the center of the circle.

If the area of the circle is $36p$ and the area of the rectangle is 72, what is the length of *DC*?

3. The center of a circle is (5, -2). (5, 7) is outside the circle, and (1, -2) is inside the circle. If the radius, *r*, is an integer, how many possible values are there for *r*?

(A) 4
(B) 5
(C) 6
(D) 7
(E) 8

4.

A square’s perimeter in inches is equal to its area in square inches.
A circle’s circumference in inches is equal to its area in square inches.

Quantity A	**Quantity B**
The side length of the square.	The diameter of the circle.

5.

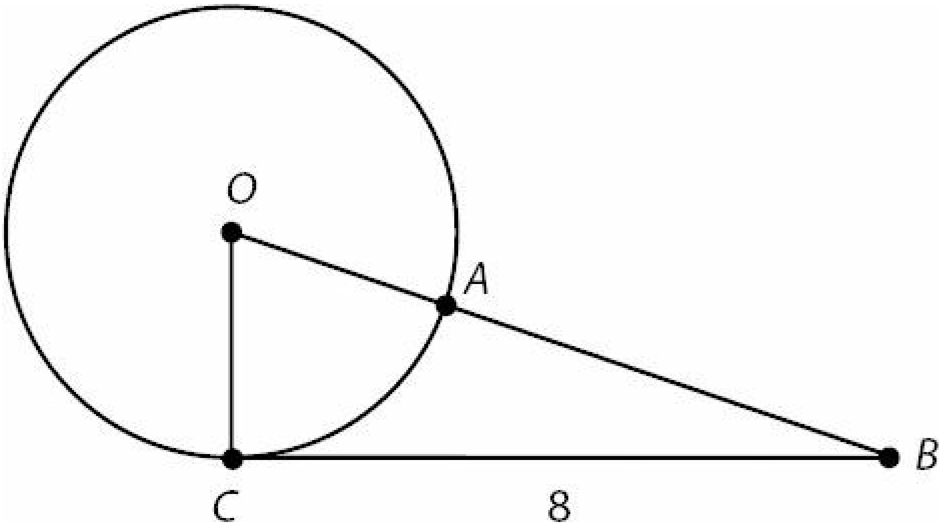
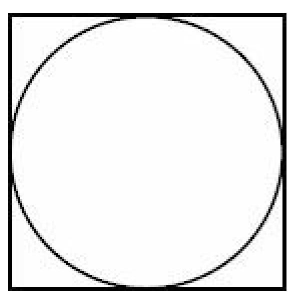


Image NOT to scale

In the figure above, point O is the center of the circle, points A and C are located on the circle, and line segment BC is tangent to the circle. If the area of triangle OBC is 24, what is the length of AB?

(A) 2
(B) 4
(C) 6
(D) 8
(E) 10

6.

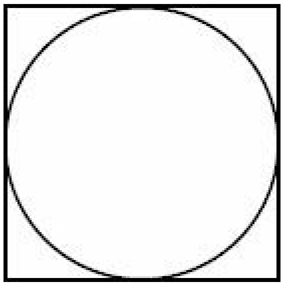


The circle is inscribed in the square.
The area of the circle is 25π.

Quantity A	**Quantity B**
The area of the square	50

7. If a circle is inscribed in a square with area 16, the area of the circle is equal to how many π?

8.



If the circle is inscribed in the square above, and the area of the square is 50, what is the area of the circle?

(A) $\frac{25\pi}{4}$

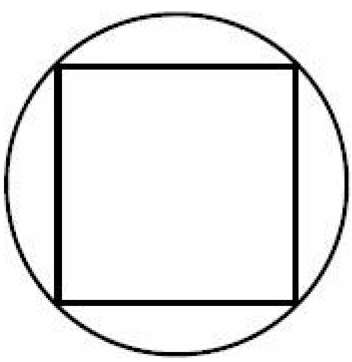
(B) $\frac{25\pi}{2}$

(C) 25π

(D) 50π

(E) $\frac{625\pi}{16}$

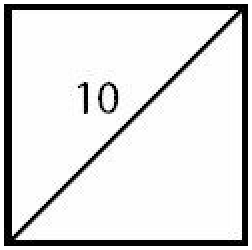
9.



In the figure above, a square is inscribed in a circle. If the area of the square is 4, what is the area of the circle?

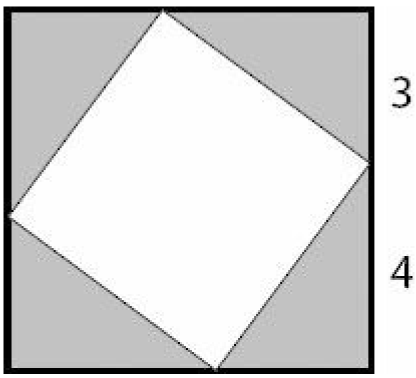
(A) π
(B) 2π
(C) 4π
(D) 6π
(E) 8π

10.



What is the area of the square in the figure above?

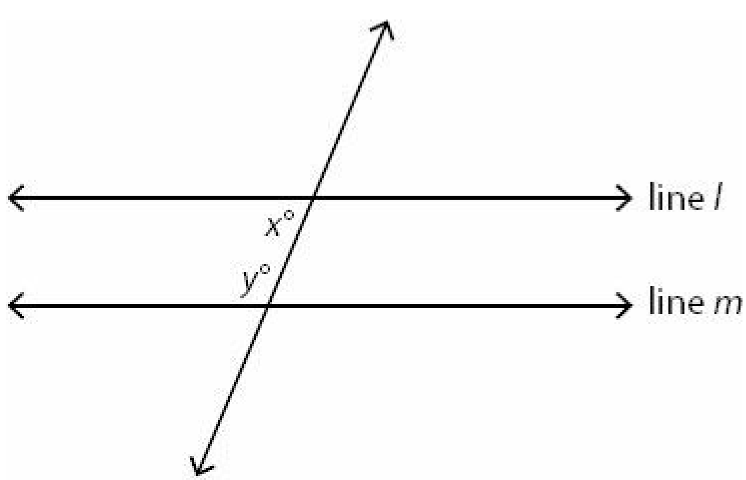
11.



In the 7-inch square above, another square is inscribed. What fraction of the larger square is shaded?

(A) 3/12
(B) 24/49
(C) 1/2
(D) 25/49
(E) 7/12

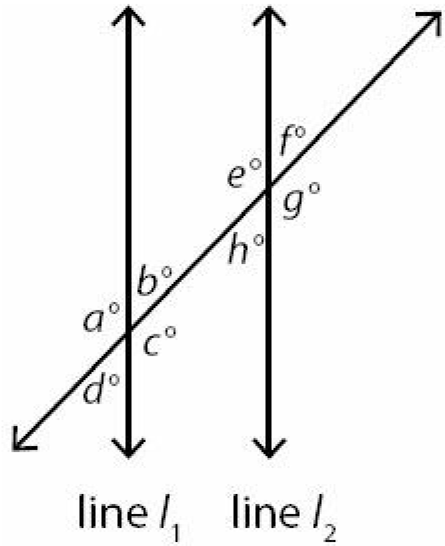
12.



Lines *l* and *m* are parallel.

Quantity A	Quantity B
$x°$	$180 - y°$

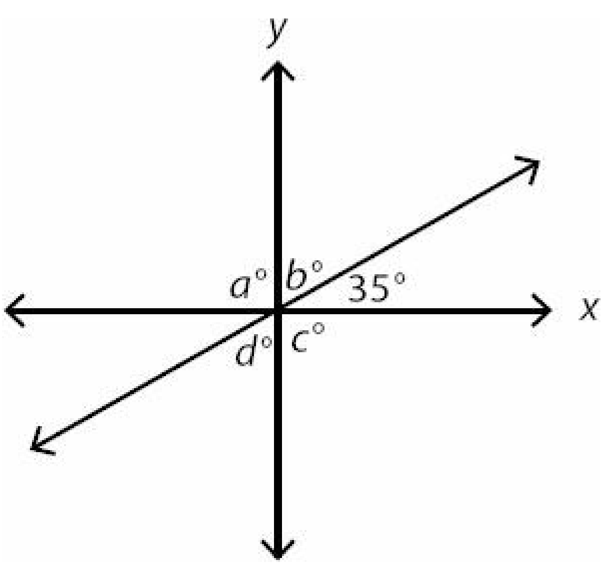
13.



Lines l_1 and l_2 are parallel.
$a > 90$

Quantity A	Quantity B
$a + g + f$	$e + b + h$

14.



What is the value of $a + b + c + d$?

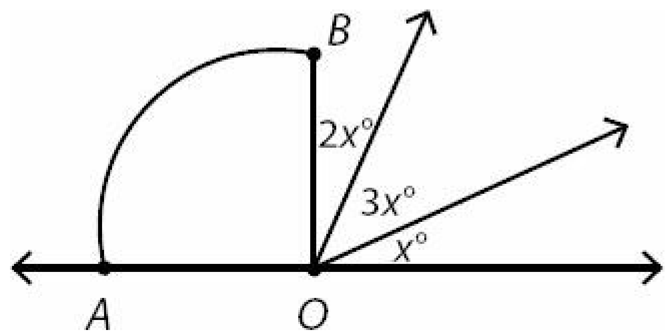
$\boxed{}$

15.

A right isosceles triangle with a leg of length f has the same area as a square with a side of length 5.

Quantity A	**Quantity B**
f	s

16.



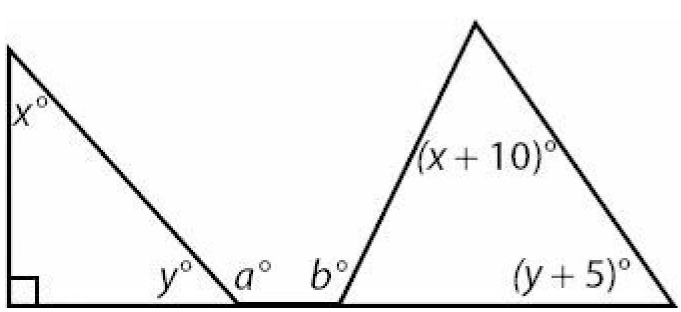
Sector *OAB* is a quarter-circle.

Quantity A	**Quantity B**
x	15

17. In the *xy*-plane, an equilateral triangle has vertices at (0, 0) and (9, 0). What could be the coordinates of the third vertex?

(A) (0, 4.5)
(B) (4.5, 4.5)
(C) $\left(\frac{9\sqrt{3}}{2}, \frac{9\sqrt{3}}{2}\right)$
(D) $(4.5,\ 9\sqrt{3})$
(E) $(4.5,\ \frac{9\sqrt{3}}{2})$

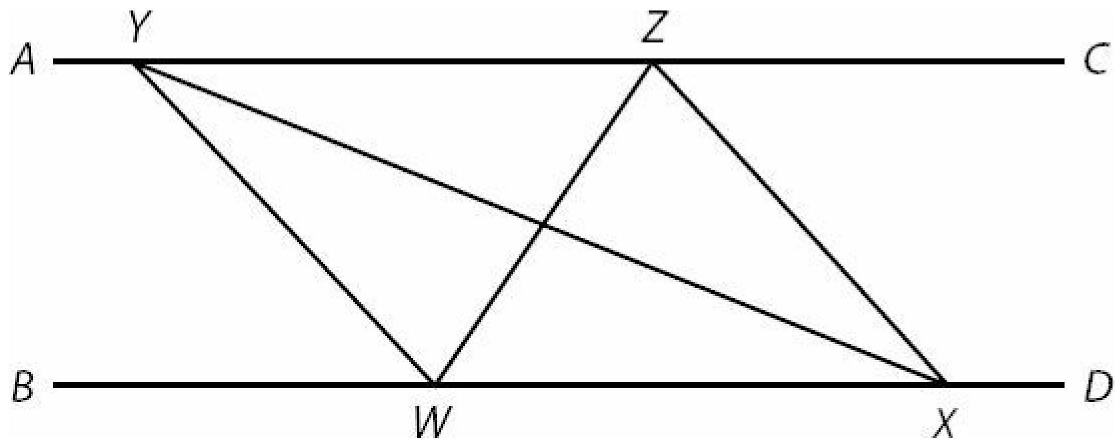
18.



What is a in terms of b and y?

(A) $b + y + 65$
(B) $b - y + 65$
(C) $b + y + 75$
(D) $b - 2y + 45$
(E) $b - y + 75$

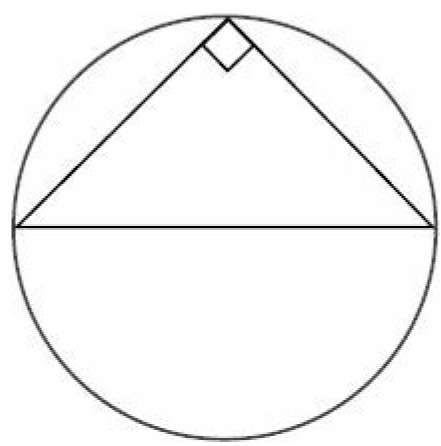
19.



In the figure above, line segments AC and BD are parallel.

Quantity A	**Quantity B**
The area of triangle WYX	The area of triangle WZX

20.



A right triangle is inscribed in a circle with an area of 16π centimeters^2 as shown above.

Quantity A	**Quantity B**
The hypotenuse of the triangle, in centimeters	8

21. A rectangular box has a length of 6 cm, a width of 8 cm, and a height of 10 cm. What is the length of the diagonal of the box, in cm?

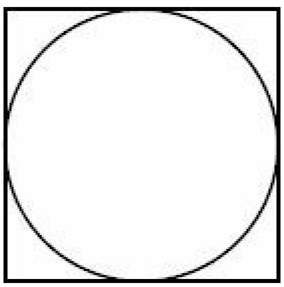
(A) 10
(B) 12
(C) $10\sqrt{2}$

(D) $10\sqrt{3}$
(E) 24

22. If the diagonal of a square garden is 20 feet long, what is the perimeter of the garden?

(A) $10\sqrt{2}$ feet
(B) $20\sqrt{2}$ feet
(C) 40 feet
(D) $40\sqrt{2}$ feet
(E) 80 feet

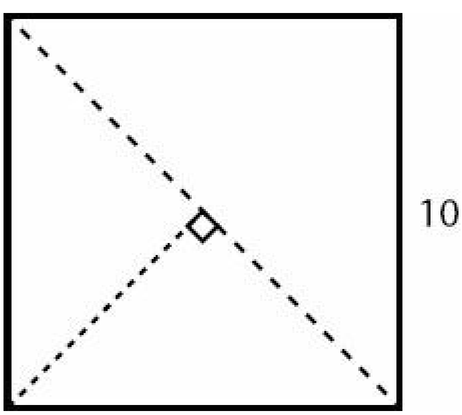
23.



In the figure above, if the diagonal of the square is 12, what is the radius of the circle?

(A) $3\sqrt{2}$
(B) 6
(C) $6\sqrt{2}$
(D) 9
(E) 18

24.



Julian takes a 10- by 10-inch square piece of paper and cuts it in half along the diagonal. He then takes one of the halves and cuts it in half again from the corner to the midpoint of the opposite side. All cuts are represented in the figure with dotted lines. What is the perimeter of one of the smallest triangles, in inches?

(A) 10
(B) $10\sqrt{2}$

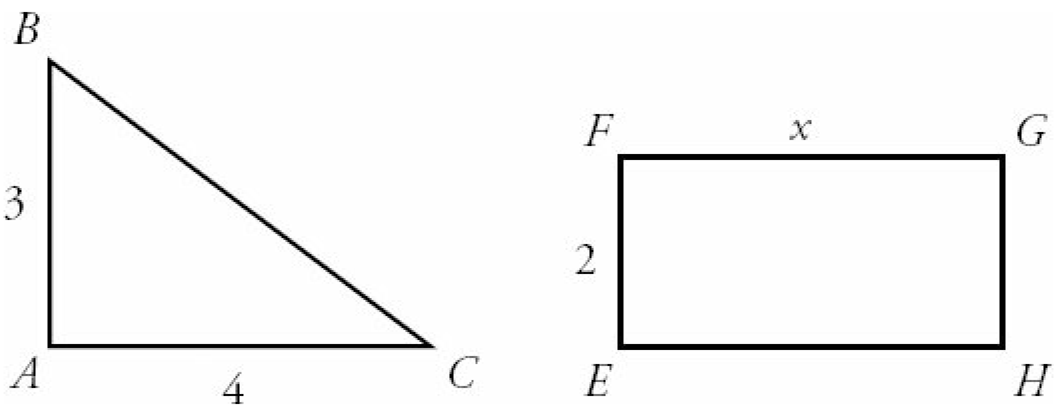
(C) 20

(D) 10 + $10\sqrt{2}$

(E) 10 + $20\sqrt{2}$

Mixed Geometry Answers

1. **4.**



Triangle *ABC* is a right triangle, so you can find the length of hypotenuse *BC*. This is a 3–4–5 triangle, so the length of side *BC* is 5. That means the perimeter of Triangle *ABC* is 3 + 4 + 5 = 12.

That means the perimeter of Rectangle *EFGH* is also 12. That means that $2 \times (2 + x) = 12$.
So $4 + 2x = 12 \rightarrow 2x = 8 \rightarrow x = 4$.

2. **6.** The area of this circle is 36π and the area of any circle is πr^2, so the radius of this circle is 6. Label both radii (*OA* and *OD*) as 6. Because *ABCO* is a rectangle, its area is base times height, where radius *OA* is the height.

> Area of a rectangle = bh
> $72 = b(6)$
> $b = 12$

Since *OC* is a base of the rectangle, it is equal to 12. Subtract radius *OD* from base *OC* to get the length of segment *DC*: 12 - 6 = 6.

3. **(A).** This problem does not actually require any special formulas regarding circles. Calculate the distance between (5, -2) and (5, 7). Since the *x*-coordinates are the same and 7 - (-2) = 9, the two points are 9 apart. Because (5, 7) is outside the circle, the radius must be less than 9.

Similarly, (1, -2) is inside the circle. Calculate the distance between (5, -2) and (1, -2). Since the *y*-coordinates are the same, the distance is 5 - 1 = 4. Because (1, -2) is inside the circle, the radius must be more than 4.

The radius must be an integer that is greater than 4 and less than 9, so it can only be 5, 6, 7, or 8. Thus, there are 4 possible values for *r*.

4. **(C).** The perimeter of a square is $4s$ and the area of a square is s^2 (where *s* is a side length). If the square's perimeter equals its area, set the two expressions equal to each other and solve:

$4s = s^2$
$0 = s^2 - 4s$
$0 = s(s - 4)$
$s = 4$ or 0

Only $s = 4$ would result in an actual square, so $s = 0$ is not a valid solution.

The circumference of a circle is $2\pi r$ and the area of a circle is πr^2 (where r is the radius). If the circle's circumference equals its area, set the two expressions equal to each other and solve:

$2\pi r = \pi r^2$
$2r = r^2$
$0 = r^2 - 2r$
$0 = r(r - 2)$
$r = 2$ or 0

Only $r = 2$ would result in an actual circle, so $r = 0$ is not a valid solution.

If the radius of the circle is 2, then the diameter is 4. Thus, the two quantities are each equal to 4.

5. **(B).** Because BC is tangent to the circle, angle OCB is a right angle. Thus, radius OC is the height of the triangle. If the area of the triangle is 24, use the area formula for a triangle (and 8 as the base, from the figure) to determine the height:

$$A = \frac{bh}{2}$$
$$24 = \frac{8 \times OC}{2}$$
$$48 = 8 \times OC$$
$$6 = OC$$

Thus, the radius of the circle is 6 (note that you have TWO radii on the diagram, OC and OA). Since you now have two sides of a right triangle, use the Pythagorean theorem to find the third:

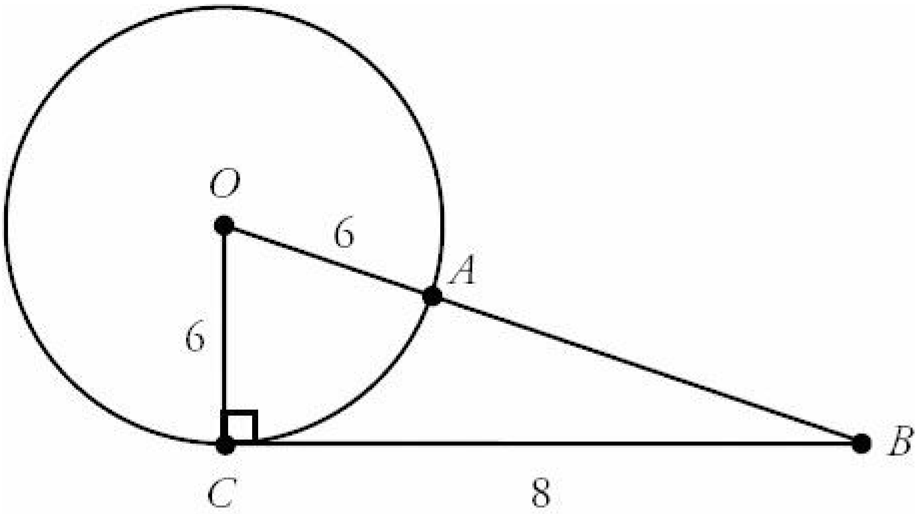


Image NOT to scale

$6^2 + 8^2 = (OB)^2$
$36 + 64 = (OB)^2$
$100 = (OB)^2$
$10 = OB$

(Of course, the 6–8–10 triangle is one of the special right triangles you should memorize for the GRE!)

Since the hypotenuse OB is equal to 10 and the radius OA is equal to 6, subtract to get the length of AB. The answer is $10 - 6 = 4$.

6. **(A).** The area of the circle $= 25\pi = \pi r^2$, so the radius is 5 and therefore the diameter of the circle is 10. The diameter of the circle is equal to the side of the square (the circle and square are "equally tall"), so the area of the square is $10 \times 10 = 100$.

Alternatively, the area of the circle is 25π, which is approximately 25(3.14), or greater than 75. The square is clearly larger than the circle, so the area of the square is greater than 75, which is greater than 50.

7. **4.** If the square has area 16, its sides equal 4. If the square is 4 "tall," so is the circle. That is, the side of the square is equal to the diameter of the circle. Since the diameter of the circle is 4, the radius is 2. For the circle, area $A = \pi r^2 = \pi(2)^2$ or 4π. Since the question asks "how many π?" and π is already written next to the box, type only 4 in the box.

8. **(B).** If the area of the square is 50, the sides of the square are $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$.

If the square is $5\sqrt{2}$ "tall", so is the circle. That is, the side of the square is equal to the diameter of the circle. Since the circle diameter is $5\sqrt{2}$, the radius is $\frac{5\sqrt{2}}{2}$. Using the formula for the area of a circle, $A = \pi r^2$:

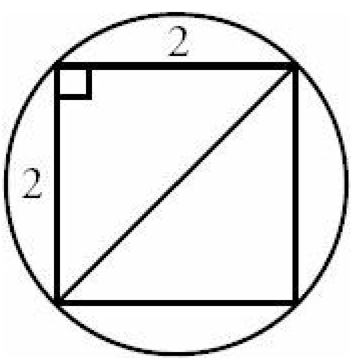
$$A = \pi\left(\frac{5\sqrt{2}}{2}\right)^2$$

$$A = \pi\left(\frac{25 \times 2}{4}\right)$$

$$A = \frac{25\pi}{2}$$

Note that even if you got a bit lost in the math, you could estimate quite reliably! The square is clearly a bit larger than the circle, so the circle area should be a bit less than 50. Put all the answers in your calculator, using 3.14 as an approximate value for π, and you will quickly see that choice (A) = 19.625, which is too small, and choice (B) = 39.25, while the other three choices are much too large (larger than the square!)

9. **(B).** If the area of the square is 4, then the side length is 2. To find the area of the circle, you need the circle's radius, which is not obvious yet. Draw a diagonal in the square—this line segment is also a diameter of the circle. Then use the Pythagorean theorem (or the 45–45–90 angle formula) to find the diagonal length:



$2^2 + 2^2 = d^2$ (where d is the diagonal of the square and the diameter of the circle)
$8 = d^2$
$\sqrt{8} = d$

Because $\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$ is the diameter of the circle, the radius is $1\sqrt{2}$ or just $\sqrt{2}$. The area of the circle is:

$A = \pi r^2$
$A = \pi(\sqrt{2})^2$
$A = 2\pi$

Note that even if you got a bit lost in this problem, you could just use common sense to estimate. The circle is obviously larger than the square, so the answer should be somewhat larger than 4. Plug in $\pi = 3.14$ using your calculator to see which choices are reasonable. Choice (A) is too small. Choice (B) is about 6.28. Choice (C) is *twice* as big, and (D) and (E) are even larger. Only (B) is reasonable.

10. **50.** One way to solve this problem is by using the Pythagorean theorem. All sides of a square are equal to s, so:

$s^2 + s^2 = 10^2$
$2s^2 = 100$
$s^2 = 50$

Note that you *could* solve for s ($s = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$), but the area of the square is s^2, which is already calculated above. The area of the square is 50.

11. **(B).** Each of the shaded triangles is a 3–4–5 Pythagorean triple. (Or, just note that each shaded triangle has legs of 3 and 4; the Pythagorean theorem will tell you that each hypotenuse = 5).

Since each hypotenuse is also a side of the square, the square has area $5 \times 5 = 25$.

The larger square (the overall figure) has area $7 \times 7 = 49$.

Subtract to find the area of the shaded region: 49 - 25 = 24.

The fraction of the larger square that is shaded is therefore 24/49.

12. **(C).** When two parallel lines are cut by a transversal, same-side interior angles are supplementary. Thus, $x + y = 180$, and $x = 180 - y$.

13. **(A).** While the exact measures of any of the angles are not given, when parallel lines are cut by a transversal, only two angle measures are created: all the "big" angles are the same, and all the "small" angles are the same. Since $a > 90$ (i.e., the picture is, indeed, the way it looks) and $a = c = e = g$, infer that a, c, e, and g are all the same "big" angle measure, which is greater than 90.

Similarly, $b = d = f = h$, so these are the same "small" angle measure, which is less than 90.

Quantity A is the sum of two "big" angles and one "small."
Quantity B is the sum of one "big" angle and two "small."
Quantity A is greater.

If you wish to try this with real numbers, plug in $a = 100$ (for example), and you will see that a, c, e, and g are all equal to 100, and b, d, f, and h are all equal to 80, so Quantity A would equal 280 and Quantity B would equal 260. For any example with $a > 90$, Quantity A will be larger.

14. **290.** Angles that "go around in a circle" sum to 360 degrees. It may be tempting to simply subtract 35 from 360 and answer 325, but don't overlook the unlabeled angle, which is opposite and therefore equal to 35°. So, subtract 35 + 35 = 70 from 360 to get the answer, 290.

15. **(A).** In the right isoceles triangle, the base and height are the perpendicular sides, which are each of length f. Thus, Area = $\frac{f^2}{2}$. The square, of course, has area s^2. Thus, $\frac{f^2}{2} = s^2$, or $f^2 = 2s^2$. Since f and s are definitely positive, f is larger than s.

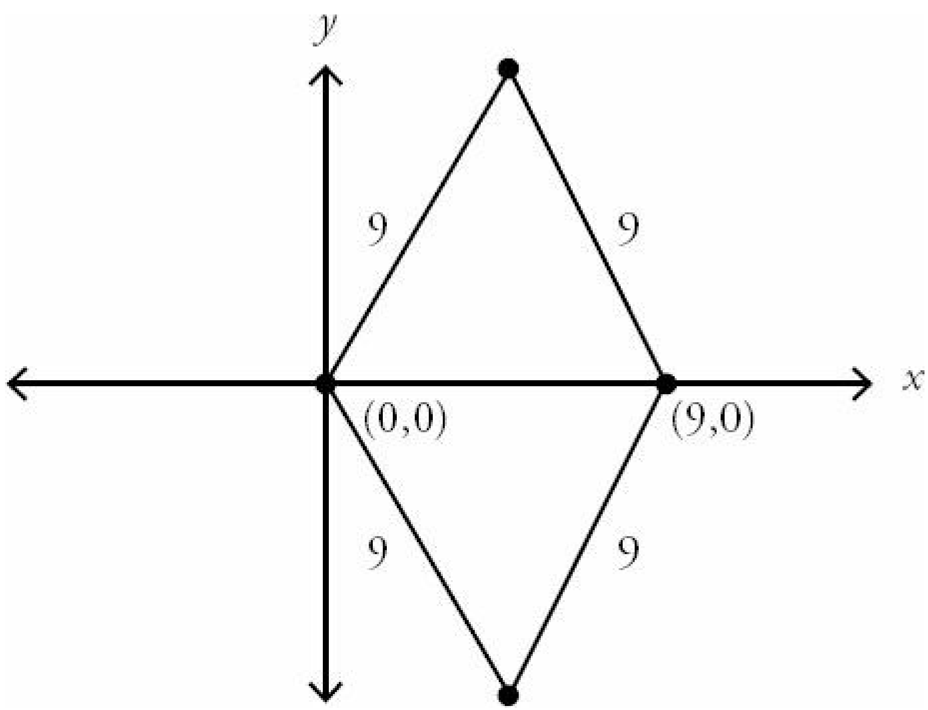
Alternatively, you could envision that if the areas were equal, the triangle sides would have to be much longer. After all, an isosceles right triangle is only half of a square.

16. **(C).** If sector *OAB* is a quarter-circle, then the angle at *O* measures 90°. Thus, since angles that make up a straight line must sum to 180, $2x + 3x + x$ must sum to 90:

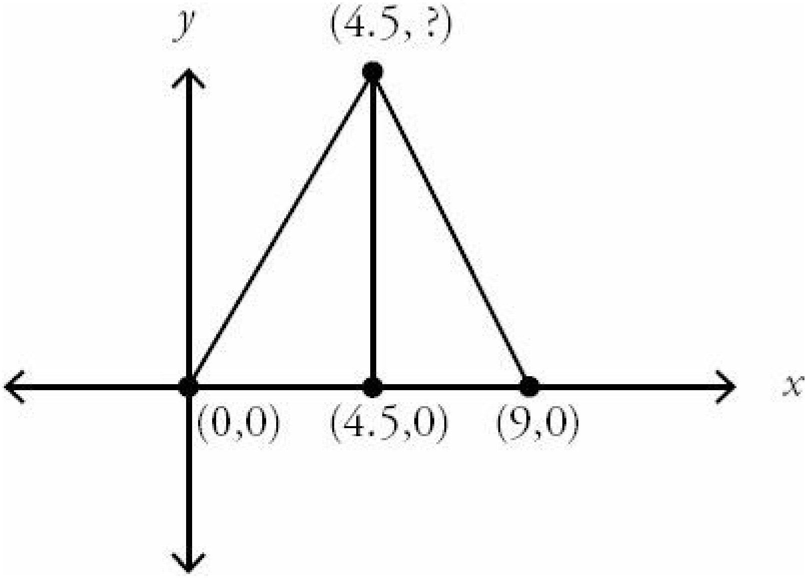
$$2x + 3x + x = 90$$
$$6x = 90$$
$$x = 15$$

The two quantities are equal.

17. **(E).** Given the two vertices at (0, 0) and (9, 0), there are only two possible locations for the third coordinate: (9,0)



All of the answer choices are positive, so focus on the "upper" option. Since equilateral triangles are symmetrical, the x-coordinate of the third vertex will be halfway between 0 and 9, or 4.5. Thus, the answer must be (B), (D), or (E). Draw the height of the triangle:



The height of an equilateral triangle is given by $h = \frac{s\sqrt{3}}{2}$. Alternatively, note that the height cuts the 60–60–60 triangle into two 30–60–90 triangles. Use the side length ratios for a 30–60–90 triangle (1 : $\sqrt{3}$: 2) to determine that, since the side across from the 30-degree angle is equal to 4.5, the side across from the 60-degree angle will be equal to 4.5 $\sqrt{3}$ or $\frac{9\sqrt{3}}{2}$.

This height is the *y*-coordinate of the third vertex. The answer is (4.5, $\frac{9\sqrt{3}}{2}$).

18. **(E).** An exterior angle of a triangle is equal to the sum of the two opposite interior angles. From the left triangle, $a = x + 90$. From the right triangle, $b = (x + 10) + (y + 5) = x + y + 15$.

Alternatively, you could use the facts that the interior angles of a triangle sum to 180, as do angles that form a straight line. From the left triangle, $x + y + 90$ and $y + a$ both equal 180, so $x + y + 90 = y + a$, or $x + 90 = a$. From the right triangle, $180 = (x + 10) + (y + 5) + (180 - b)$, or $b = x + y + 15$.

The question asks for a in terms of b and y, so x is the variable that needs to be eliminated. Do so by solving one equation for x, and substituting this expression for x in the other equation.

From the right triangle: $b = x + y + \longrightarrow x = b - y - 15$

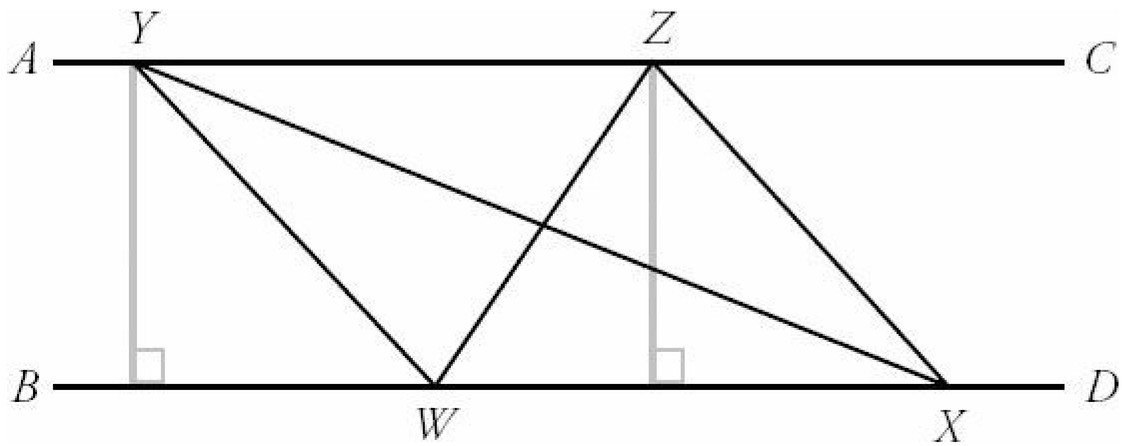
From the left triangle:

$$a = x + 90$$
$$a = (b - y - 15) + 90$$
$$a = b - y + 75$$

19. **(C).** Both triangles, *WYX* and *WZX*, share a common base of segment *WX*. As the area of a triangle is given by

Area = (1/2)(base)(height)

and both triangles have equal bases, you can determine which has a greater area by determining which has a greater height. The height is a perpendicular line drawn from the highest point on the triangle to the base. In this case, the heights would be given in gray below:



By the definition of parallel lines, *AC* and *BD* are uniform distance apart. Therefore, the heights shown are the same. Because these triangles have equal bases and heights, they must have equal area.

20. **(C).** To solve this problem, recall that a triangle inscribed in a semi-circle will be a right triangle *if and only if* one side of the triangle is the diameter (i.e., the center of the circle must lie on one side of the triangle). Because this is a right triangle, the hypotenuse must be the diameter of the circle.

To find the diameter of the circle, recall the formula for area: Area = πr^2.

$16\pi \text{ cm}^2 = \pi r^2$
$16 \text{ cm}^2 = r^2$
$r = 4$ cm

Given that diameter is twice the radius, the diameter (i.e., the hypotenuse of the triangle) is 8 cm. Quantity A is 8, making the two quantities equal.

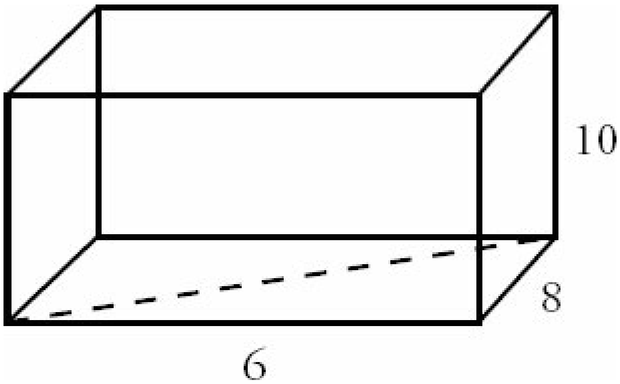
21. **(C).** The fastest approach to solving this problem is to use the "Super Pythagorean Theorem," which states that the diagonal of any rectangular box is given by:

$d^2 = l^2 + w^2 + h^2$

Where *l*, *w*, and *h* are the length, width, and height of the box, respectively. Plugging in yields

$d^2 = 6^2 + 8^2 + 10^2$
$d^2 = 36 + 64 + 100$
$d^2 = 200$
$d = 10\sqrt{2}$

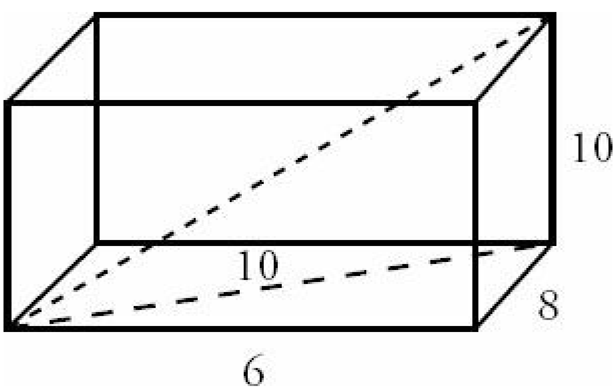
Alternatively, one could avoid the Super Pythagorean Theorem by applying the normal Pythagorean theorem twice. To find the diagonal of the box, you must first find the diagonal of one of the sides. Choosing this side as the base,



where the dashed line represents the diagonal of the base. Applying the Pythagorean theorem:

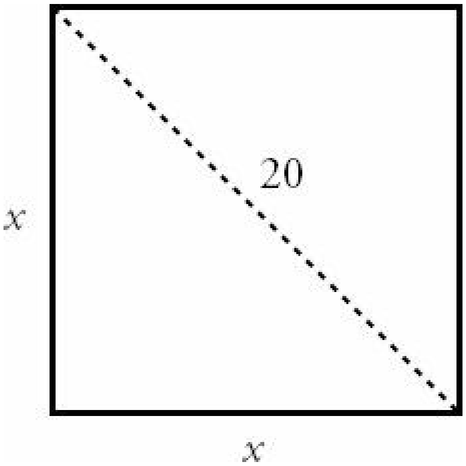
$$c^2 = a^2 + b^2$$
$$c^2 = 6^2 + 8^2$$
$$c^2 = 36 + 64$$
$$c^2 = 100$$
$$c = 10$$

From here, draw the diagonal of the box and apply the Pythagorean theorem again as shown.



$$d^2 = 10^2 + 10^2$$
$$d^2 = 100 + 100$$
$$d^2 = 200$$
$$d = 10\sqrt{2}$$

22. **(D).** Draw the following figure to represent the square garden.



Label the diagonal with the given length of 20 and the sides with the variable x. The diagonal forms the hypotenuse of a right triangle with legs of length x. Using Pythagorean theorem, solve for the length of the legs as

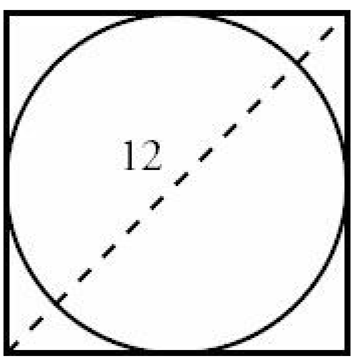
$$x^2 + x^2 = 20^2$$
$$2x^2 = 400$$
$$x^2 = 200$$
$$x = 10\sqrt{2}$$

The perimeter of a square is 4 × (side length).

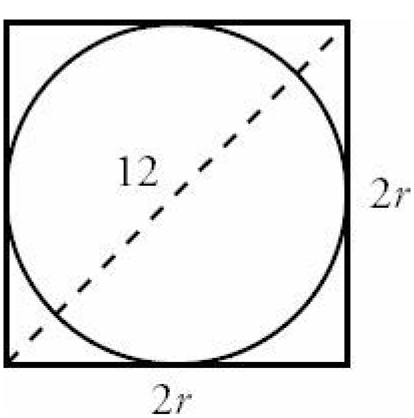
Perimeter = $4 \times (10\sqrt{2})$

Perimeter = $40\sqrt{2}$

23. **(A).** Begin by diagramming the figure, labeling the diagonal of the circle as 12.



From here, recognize that the square is as "tall" as the circle, or the side length of the square equals the diameter of the circle, which is $2r$.



By the Pythagorean theorem:

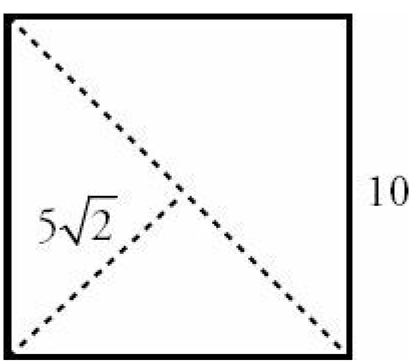
$$(2r)^2 + (2r)^2 = 12^2$$
$$4r^2 + 4r^2 = 144$$
$$8r^2 = 144$$
$$r^2 = 18$$
$$r = 3\sqrt{2}$$

24. **(D).** In order to compute the perimeter of one of the smaller triangles, first compute the length of the diagonal. For a square with side length 10 inches, the length of the diagonal can be computed by the Pythagorean theorem:

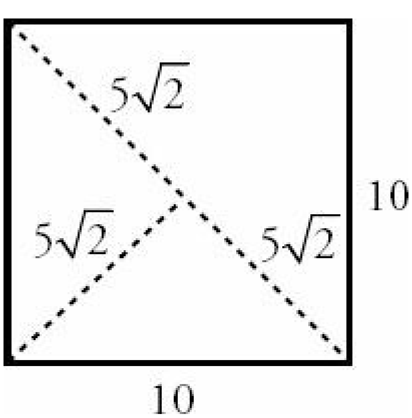
$$(\text{diagonal})^2 = (\text{side})^2 + (\text{side})^2$$
$$(\text{diagonal})^2 = 10^2 + 10^2$$
$$(\text{diagonal})^2 = 200$$
$$\text{diagonal} = 10\sqrt{2}$$

Alternatively, recognize that the diagonal of a square is always $\sqrt{2}$ times the side length.

The second cut goes from the corner to the midpoint of the diagonal, so that slice is half as long as the diagonal of the square: $\frac{10\sqrt{2}}{2} = 5\sqrt{2}$. This can be seen as



Similarly, because the remaining line in each of the smaller triangles is half of a diagonal, each is of length $5\sqrt{2}$ inches.



Adding up the lengths of the sides, the perimeter of the smallest triangle is

Perimeter = $10 + 5\sqrt{2} + 5\sqrt{2}$

Perimeter = $10 + 10\sqrt{2}$

Chapter 30
of

Advanced Quant

In This Chapter…

Advanced Quant

The following questions are *extremely* advanced for the GRE. We have included them by popular demand — students who are aiming for perfect math GRE scores often wish to practice on problems that may even be harder than any they see on the real GRE. We estimate that a GRE test taker who does well on the e first math section and therefore is given a difficult second section might see one or two problems, at most, of this level of difficulty.

If you are NOT aiming for a perfect math score, we absolutely recommend that you skip these problems!

If you are taking the GRE for business school or another quantitative program, you may wish to attempt some of these problems. For instance, you might do one or two of these problems — think of them as "brain teasers" — to cap off a study session from elsewhere in the book. (For reference, getting 50% of these problems correct would be a pretty incredible performance!)

Even if you *are* aiming for a perfect math score, though, please make sure you are *flawless* at the types of math problems in the *rest* of this book before you work on these. You will gain far more points by reducing silly mistakes (through practice, steady pacing, and good organization) on easy and medium questions than by focusing on ultra-hard questions.

For more such problems, visit the Manhattan Prep GRE blog for our weekly Challenge Problem. (Access to the archive of over 100 Challenge Problems is available to our course and Guided Self-Study students for free and to the public for a small fee.)

That said, attempt these Advanced Quant problems — if you dare!

1. The probability of rolling any number on a weighted 6-sided die, with faces numbered 1 through 6, is directly proportional to the number rolled. What is the expected value of a single roll of the die?

    (A) $4\frac{1}{6}$

    (B) $4\frac{1}{3}$

    (C) $4\frac{1}{2}$

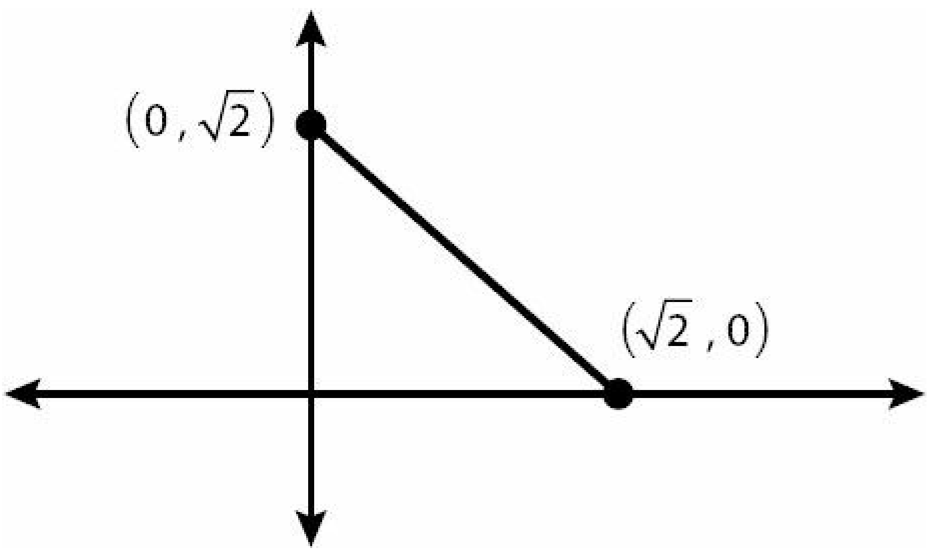
    (D) $4\frac{2}{3}$

    (E) $4\frac{5}{6}$

2. 21 people per minute enter a previously empty train station beginning at 7:00:00 p.m. (7 o'clock and zero seconds). Every 9 minutes beginning at 7:04:00 p.m., a train comes and everyone who has entered the station in the last 9 minutes gets on the train. If the last train comes at 8:25:00, what is the average number of people who get on each of the trains leaving from 7:00:00 to 8:25:00?

(A) 84
(B) 136.5
(C) 178.5
(D) 189
(E) 198.5

3. The random variable X has the following continuous probability distribution in the range $0 \leq X \leq \sqrt{2}$, as shown in the coordinate plane with X on the horizontal axis:



The probability that $X < 0$ = the probability that $X > \sqrt{2} = 0$.

What is the median of X?

(A) $\frac{\sqrt{2}-1}{2}$
(B) $\frac{\sqrt{2}}{4}$
(C) $\sqrt{2}-1$
(D) $\frac{\sqrt{2}+1}{4}$
(E) $\frac{\sqrt{2}}{2}$

4.

$x < 0$

Quantity A	**Quantity B**
$x^2 - 5x + 6$	$x^2 - 9x + 20$

5. If x is a positive integer, what is the units digit of $(24)^{5+2x}(36)^6(17)^3$?

(A) 2
(B) 3
(C) 4
(D) 6
(E) 8

6. A rectangular solid is changed such that the width and length are increased by 1 inch apiece and the height is decreased by 9 inches. Despite these changes, the new rectangular solid has the same volume as the original rectangular solid. If the width and length of the original rectangular solid are equal and the height of the new rectangular solid is 4 times the width of the original rectangular solid, what is the volume of the rectangular solid?

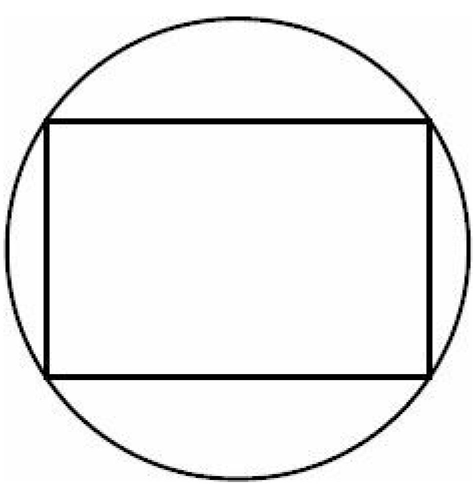
(A) 18
(B) 50
(C) 100
(D) 200
(E) 400

7. The sum of all solutions for x in the equation $x^2 - 8x + 21 = |x - 4| + 5$ is equal to:

(A) -7
(B) 7
(C) 10
(D) 12
(E) 14

8. In the figure shown, the circumference of the circle is 10π. Which of the following is NOT a possible value for the area of the rectangle?

(A) 30
(B) 40
(C) $20\sqrt{2}$
(D) $30\sqrt{2}$
(E) $40\sqrt{2}$



9. The length of one edge of a cube equals 4. What is the distance between the center of the cube and one of its

vertices?

(A) 2
(B) $2\sqrt{2}$
(C) $2\sqrt{3}$
(D) $4\sqrt{2}$
(E) $4\sqrt{3}$

10. If c is randomly chosen from the integers 20 to 99, inclusive, what is the probability that $c^3 - c$ is divisible by 12?

11. If x and y are positive integers greater than 1 such that $x - y$ and x/y are both even integers, which of the following numbers must be non-prime integers?

Indicate <u>all</u> such statements.

☐ x
☐ $x + y$
☐ y/x

12. The remainder when 120 is divided by single-digit integer m is positive, as is the remainder when 120 is divided by single-digit integer n. If $m > n$, what is the remainder when 120 is divided by $m - n$?

13. A circular microchip with a radius of 2.5 centimeters is manufactured following a blueprint scaled such that a measurement of 1 centimeter on the blueprint corresponds to a measurement of 0.05 millimeters on the microchip. What is the area of the blueprint, in square centimeters? (1 centimeter = 10 millimeters)

π

14.

For a certain quantity of a gas, pressure P, volume V, and temperature T are related according to the formula $PV = kT$, where k is a constant.

Quantity A	**Quantity B**
The value of P if $V = 20$ and $T = 32$	The value of T if $V = 10$ and $P = 78$

15.

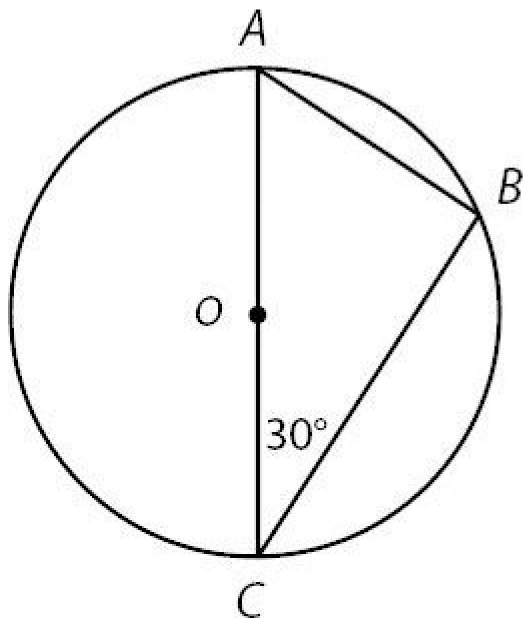


Figure not drawn to scale.

The circle with center O has a circumference of $6\pi\sqrt{3}$. If AC is a diameter of the circle, what is the length of line segment BC?

(A) $\frac{3}{\sqrt{2}}$
(B) 3
(C) $3\sqrt{3}$
(D) 9
(E) $9\sqrt{3}$

16. A batch of widgets costs $p + 15$ dollars for a company to produce and each batch sells for $p(9 - p)$ dollars. For which of the following values of p does the company make a profit?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

17. If K is the sum of the reciprocals of the consecutive integers from 41 to 60 inclusive, which of the following is less than K?

Indicate <u>all</u> such statements.

☐ 1/4
☐ 1/3
☐ 1/2

18. Triplets Adam, Bruce, and Charlie enter a triathlon. There are nine competitors in the triathlon. If every competitor has an equal chance of winning, and three medals will be awarded, what is the probability that at least two of the triplets will win a medal?

(A) 3/14
(B) 19/84
(C) 11/42
(D) 15/28
(E) 3/4

19. The expression $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}}}$ extends to an infinite number of roots. Which of the following choices most closely approximates the value of this expression?

(A) $\sqrt{3}$
(B) 2
(C) 1+ $\sqrt{2}$
(D) 1+ $\sqrt{3}$
(E) $2\sqrt{3}$

20. Half an hour after Car A started traveling from Newtown to Oldtown, a distance of 62 miles, Car B started traveling along the same road from Oldtown to Newtown. The cars met each other on the road 15 minutes after Car B started its trip. If Car A traveled at a constant rate that was 8 miles per hour greater than Car B's constant rate, how many miles had Car B driven when they met?

(A) 14
(B) 12
(C) 10
(D) 9
(E) 8

21.

x and y are positive integers such that $x^2 5^y = 10{,}125$

Quantity A	Quantity B
x^2	5^y

22. If $x = 2^b - (8^8 + 8^6)$, for which of the following b values is x closest to zero?

(A) 20
(B) 24
(C) 25
(D) 30
(E) 42

23. If $k > 1$, which of the following must be equal to $\dfrac{2}{\sqrt{k+1}+\sqrt{k-1}}$?

(A) 2
(B) $2\sqrt{2k}$
(C) $2\sqrt{k+1}+\sqrt{k-1}$

(D) $\dfrac{\sqrt{k+1}}{\sqrt{k-1}}$

(E) $\sqrt{k+1}-\sqrt{k-1}$

24. Bank account A contains exactly *x* dollars, an amount that will decrease by 10% each month for the next two months. Bank account B contains exactly *y* dollars, an amount that will increase by 20% each month for the next two months. If A and B contain the same amount at the end of two months, what is the ratio of $\sqrt{x}$ to $\sqrt{y}$?

(A) 4 : 3
(B) 3 : 2
(C) 16 : 9
(D) 2 : 1
(E) 9 : 4

25. Let *a* be the sum of *x* consecutive positive integers. Let *b* be the sum of *y* consecutive positive integers. For which of the following values of *x* and *y* is it NOT possible that *a* =*b*?

(A) $x = 2; y = 6$
(B) $x = 3; y = 6$
(C) $x = 6; y = 4$
(D) $x = 6; y = 7$
(E) $x = 7; y = 5$

26.

Body Mass Index (BMI) is calculated by the formula $\dfrac{703w}{h^2}$, where *w* is weight in pounds and *h* is height in inches. (12 inches = 1 foot.)

Quantity A	**Quantity B**
The number of pounds gained by a 6 foot, 2 inch tall person whose BMI increased by 1.0.	The number of pounds lost by a 5 foot, 5 inch tall person whose BMI decreased by 1.2.

27. Bag A contains 3 white and 3 red marbles. Bag B contains 6 white and 3 red marbles. One of the two bags will be chosen at random, and then two marbles will be drawn from that bag at random without replacement. What is the probability that the two marbles drawn will be the same color?

(A) 7/20
(B) 9/10
(C) 9/20
(D) 11/20
(E) 13/20

28. How many positive four-digit integers contain the digit grouping “62” (in that order) at least once? For instance,

2628 and 6244 are two such integers to include, but 2268 and 5602 do not meet the restrictions.

(A) 180
(B) 190
(C) 279
(D) 280
(E) 360

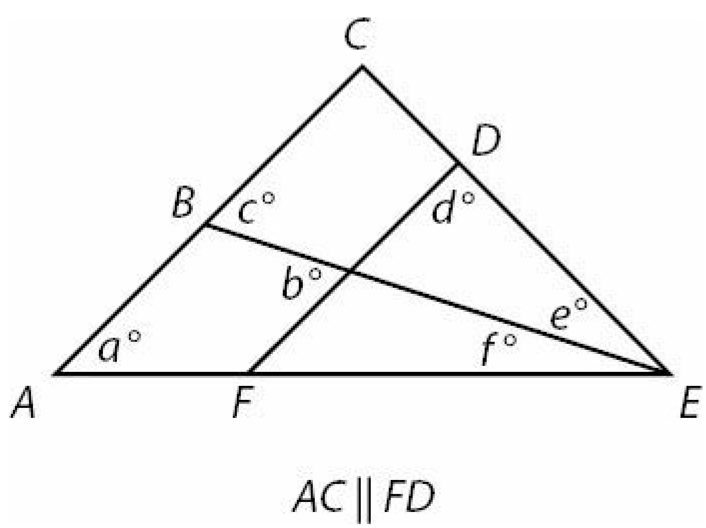
29. How many 5 digit numbers that are divisible by 9 can be formed using the digits 0, 1, 2, 4, 5, 6 if repeats are not allowed?

(A) 66
(B) 120
(C) 360
(D) 488
(E) 720

30.

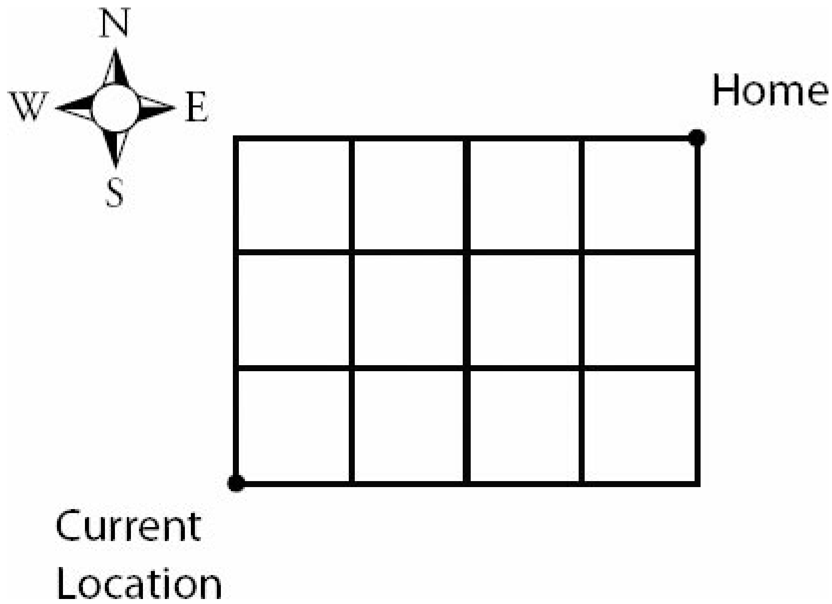
Quantity A	**Quantity B**
The average of all the multiples of 3 between 101 and 598	The average of all the multiples of 4 between 101 and 598

31.



Quantity A	**Quantity B**
$a + d - c - 90$	$90 - e - b - f$

32.



A man walks to his home from his current location on the rectangular grid shown. If he may choose to walk north or east at any corner, but may never move south or west, how many different paths can the man take to get home?

(A) 12
(B) 24
(C) 32
(D) 35
(E) 64

33. A bag contains 3 white, 4 black, and 2 red marbles. Two marbles are drawn from the bag. What is the probability that the second ball drawn will be red if replacement is NOT allowed?

(A) 1/36
(B) 1/12
(C) 7/36
(D) 2/9
(E) 7/9

34.

$x < 0$

Quantity A	Quantity B
$\left(\left(25^{x}\right)^{-2}\right)^{3}$	$\left(\left(5^{-3}\right)^{2}\right)^{-x}$

35.

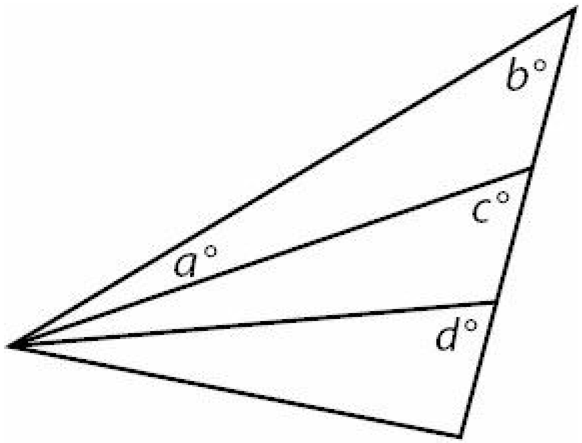
Quantity A	Quantity B
The sum of the multiples of 3 between -93 and 252, inclusive	9,162

36.

x is an integer

Quantity A	Quantity B
$(-1)^{x^2} + (-1)^{x^3} + (-1)^{x^4}$	$(-1)^{x} + (-1)^{2x} + (-1)^{3x} + (-1)^{4x}$

37.



$d > c$

Quantity A	Quantity B
a	$d - b$

38.

Sequence S is such that $S_n = S_{n-1} + \frac{5}{2}$ and $S_1 = 1$
Sequence A is such that $A_n = A_{n-1} - 2.5$ and $A_1 = 36$

Quantity A	Quantity B
The sum of the terms in S from S_1 to S_{14}, inclusive	The sum of the terms in A from A_1 to A_{14}, inclusive

39.

Quantity A	Quantity B
$\dfrac{a^{64} - 1}{(a^{32}+1)(a^{16}+1)(a^{8}+1)^2}$	1

40.

The circumference of a circle is 7/8 the perimeter of a square.

Quantity A	Quantity B
The area of the square	The area of the circle

41.

Per Serving of:	**Calories**	**Cost**
Snack A	320	$1.50
Snack B	110	$0.45

Choosing from the snacks in the table above, a group of people consumes 2,370 calories of snacks that cost a total of $10.65.

Quantity A	**Quantity B**
The number of servings of Snack A the group consumed	5

42.

$a_1, a_2, a_3, \ldots, a_n,$

… In the sequence above, each term after the first is equal to the average of the preceding term and the following term.

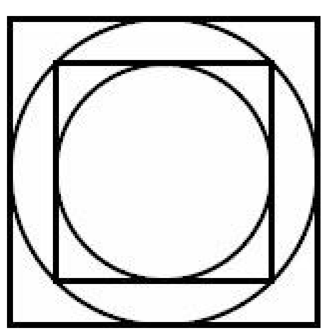
Quantity A	**Quantity B**
$a_{51} - a_{48}$	$a_{37} - a_{34}$

43.

The greatest common factor of $12x$ and $35y$ is $5y$
x and y are positive integers

Quantity A	**Quantity B**
The remainder when $12x$ is divided by 10	The greatest common factor of x and y

44.



Quantity A	**Quantity B**
The ratio of the area of the larger square to the area of the smaller square	Twice the ratio of the area of the smaller circle to the area of the larger circle

45.

$$m = 2^{16}3^{17}4^{18}5^{19}$$
$$n = 2^{19}3^{18}4^{17}5^{16}$$

Quantity A	Quantity B
When integer m is multiplied out, the number of zeroes at the end of m	When integer n is multiplied out, the number of zeroes at the end of n

46.

The sequence of numbers $a_1, a_2, a_3, \ldots, a_n, \ldots$ is defined by $a_n = 2^n - \frac{1}{2^{n-33}}$ for each integer $n \geq 1$.

Quantity A	Quantity B
The sum of the first 32 terms of this sequence	The sum of the first 31 terms of this sequence

47. Set $S = \{-1, 4, 30, -21\}$. If the mean = 4, then the standard deviation, rounded to the nearest tenth, is equal to

48. Each of 100 balls has an integer value from 1 to 8, inclusive, painted on the side. The number n_x of balls representing integer x is given by the formula $n_x = 18 - (x - 4)^2$. The interquartile range of the 100 integers is

(A) 1.5
(B) 2.0
(C) 2.5
(D) 3.0
(E) 3.5

49. When x is divided by 13 the answer is y with a remainder of 3. When x is divided by 7 the answer is z with a remainder of 3. If x, y, and z are all positive integers, what is the remainder of $\frac{yz}{13}$?

(A) 0
(B) 3
(C) 4
(D) 7
(E) 10

50. In a certain sequence, the term a_n is defined as the value of x that satisfies the equation $2 = (x/2) - a_{n-1}$. If $a_6 = 156$, what is the value of a_2?

(A) 1
(B) 6
(C) 16

(D) 26
(E) 106

51. The operator ! is defined such that $a!b = a^b \times b^{-a}$

Quantity A	Quantity B
$(x!4) \div (4!x)$	$\frac{x^8}{16^x}$

52. What is the ratio of the sum of the even positive integers between 1 and 100 (inclusive) and the sum of the odd positive integers between 100 and 150?

(A) 102 to 125
(B) 50 to 51
(C) 51 to 56
(D) 202 to 251
(E) 2 to 3

53. For integer $n \geq 3$, a sequence is defined as $a_n = a_{n-1}^2 - a_{n-2}^2$ and $a_n > 0$ for all positive integer n. The first term, a_1, is 2, and the fourth term is equal to the first term multiplied by the sum of the second and third terms. What is the third term, a_3?

(A) 0
(B) 3
(C) 5
(D) 10
(E) 16

54. In a certain sequence, each term beyond the second term is equal to the average of the previous two terms. If a_1 and a_3 are positive integers, which of the following is NOT a possible value of a_5

(A) -9/4
(B) 0
(C) 9/4
(D) 75/8
(E) 41/2

55. The operator ? is defined by the following expression: $a?b = \left|\frac{a+1}{a}\right| - \frac{b+1}{b}$ where $ab \neq 0$. What is the sum of the solutions to the equation $x?2 = \frac{x?(-1)}{2}$?

(A) -1
(B) -0.75
(C) -0.25
(D) 0.25
(E) 0.75

56. X is a non-negative number and the square root of $(10 - 3X)$ is greater than X.

Quantity A	Quantity B
$\|X\|$	2

57. The area of an equilateral triangle is greater than $25\sqrt{3}$ but less than $36\sqrt{3}$

Quantity A	Quantity B
The length of one of the sides of the triangle	9

58. The inequality $|8 - 2x| < 3y - 9$ is equivalent to which of the following?

(A) $2x < (17 - 3y)/2$
(B) $3y + 2x > 1$
(C) $6y - 2 < 2x$
(D) $1 - y < 2x < 17 + y$
(E) $3y - 1 > 2x > 17 - 3y$

59. In the sport of mixed martial arts (MMA), more than 30% of all fighters are skilled in both the Muy Thai and Brazilian Jiu Jitsu styles of fighting. 20% of the fighters who are not skilled in Brazilian Jiu Jitsu are skilled in Muy Thai. 60% of all fighters are skilled in Brazilian Jiu Jitsu.

Quantity A	Quantity B
The percent of fighters who are skilled in Muy Thai	37%

60. The rate of data transfer, r, over a particular network is directly proportional to the bandwidth, b, and inversely proportional to the square of the number of networked computers, n.

Quantity A	Quantity B
The resulting rate of data transfer if the bandwidth is quadrupled and the number of networked computers is more than tripled	$\frac{4}{9}r$

Advanced Quant Answers

1. **(B).** First, figure out the probability of each outcome. The die has six faces, numbered 1 through 6. Since the probability of rolling any particular number is directly proportional to that number, you can write each probability with an unknown multiplier x like so:

Probability of rolling a 1 = $1x = x$
Probability of rolling a 2 = $2x$
Probability of rolling a 3 = $3x$
Probability of rolling a 4 = $4x$
Probability of rolling a 5 = $5x$
Probability of rolling a 6 = $6x$

These are the only possible outcomes, so the probabilities must sum to 1:

$$x + 2x + 3x + 4x + 5x + 6x = 1$$
$$21x = 1$$
$$x = \frac{1}{21}$$

Now you can find all the probabilities, since they are just multiples of x.

The expected value, or mean, of a roll of the die is found this way:

1. Multiply each outcome (1, 2, 3, 4, 5, and 6) by its corresponding probability.
2. Sum up all those products.

So the mean equals the following sum:

$$(1)\left(\frac{1}{21}\right)+(2)\left(\frac{2}{21}\right)+(3)\left(\frac{3}{21}\right)+(4)\left(\frac{4}{21}\right)+(5)\left(\frac{5}{21}\right)+(6)\left(\frac{6}{21}\right)$$
$$= (1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2)\left(\frac{1}{21}\right)$$
$$= (1 + 4 + 9 + 16 + 25 + 36)\left(\frac{1}{21}\right)$$
$$= \frac{91}{21} = \frac{13}{3} = 4\frac{1}{3}$$

2. **(C).** From 7pm to 7:04, 84 people enter the station (21 per minute). These 84 people will get on the 7:04 train.

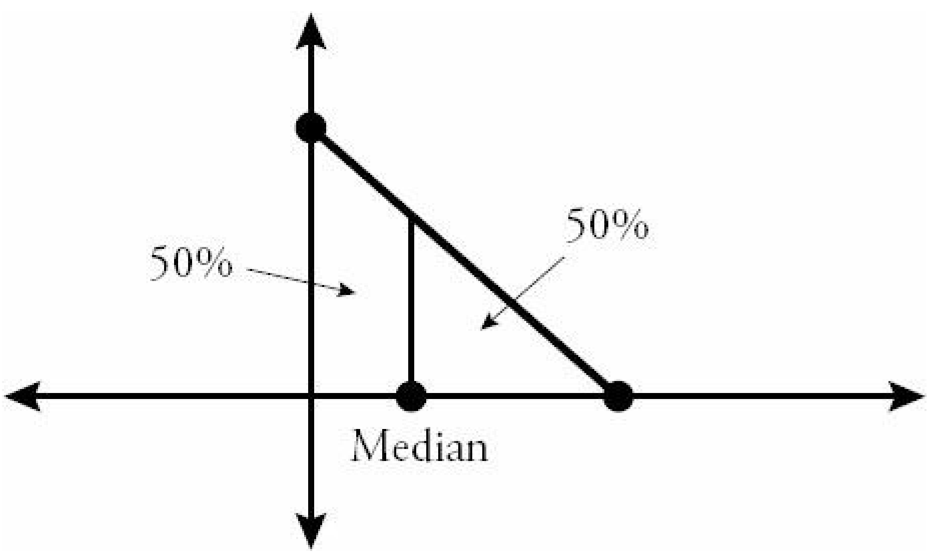
After that, for each 9 minute period, $9(21) = 189$ people will enter the station and then get on a train. These trains will leave at 7:13, 7:22, 7:31, 7:40, 7:49, 7:58, 8:07, 8:16, and 8:25.

Since 9 trains each have 189 people and the first train has 84 people, the average is:

$$\frac{9(189)+1(84)}{10} = 178.5$$

Note that the strange time format (minutes and seconds) doesn't make the problem any harder — the problem is actually more clear if you know that the train comes at 7:04 and zero seconds, rather than 7:04 and 30 seconds, at which point more people would have entered the station.

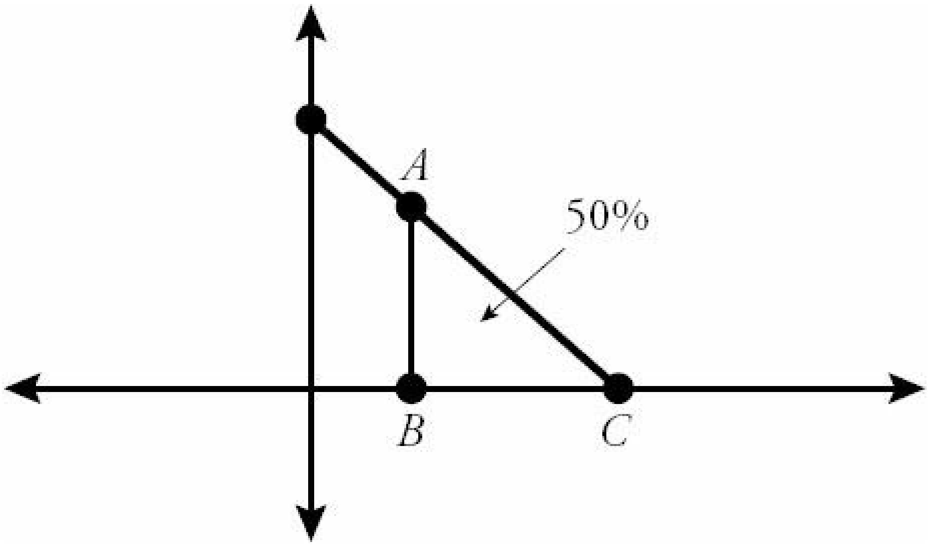
3. **(C).** A continuous probability distribution has a total area of 100%, or 1, underneath the entire curve. The median of such a distribution splits the area into two equal halves, with 50% of the area to the left of the median and the other 50% to the right of the median:



In simpler terms, the random variable *X* has a 50% chance of being above the median and a 50% chance of being below the median. You can ignore the regions to the right or the left of this triangle, since the probability that *X* could fall in either of those regions is zero. So the question becomes this: what point on the *X*-axis will divide the large right triangle into two equal areas?

One shortcut is to note that the area of the large isosceles right triangle must be 1, which equals the total area under any probability distribution curve. You can easily confirm this fact, though, by finding the area of this right triangle: $\frac{1}{2}bh = \frac{1}{2}\left(\sqrt{2}\right)\left(\sqrt{2}\right) = \frac{2}{2} = 1$.

The quickest way to find the median is to consider the *small* isosceles right triangle, ΔABC as shown:



Triangle *ABC* must have an area of $\frac{1}{2}$. So what must be the length of each of its legs, *AB* and *BC*? From the formula $\frac{1}{2}bh = \frac{1}{2}$, and noting that the base *BC* equals the height *AB*, you can see that the base *BC* must be 1 (the same as the height). Since the coordinates of point *C* are ($\sqrt{2}$, 0), the coordinates of point *B* must be ($\sqrt{2}$, 0). That is, the median is $\sqrt{2} - 1$.

4. **(B).** One way to solve is to set up an implied equation or inequality, then make the same changes to both quantities, and finally compare after simplifying.

Quantity A		**Quantity B**	
$x^2 - 5x + 6$	?	$x^2 - 9x + 20$	
$\underline{-(x^2 - 5x + 6)}$		$\underline{-(x^2 - 5x + 6)}$	
0	?	$-9x - (-5x) + 20 - 6$	Notice that x^2 is common to both quantities, so it can be ignored (i.e. it cancels).
0	?	$-9x + 5x + 14$	
0	$<$	$-4x + 14$	

Because x is negative, $-4x + 14 = -4(neg) + 14 = pos + 14$, which is greater than 0.

Another way to solve is to factor and then compare based on number properties. Quantity A factors to $(x - 2)(x - 3)$. Quantity B factors to $(x - 4)(x - 5)$. Because x is negative, "x minus a positive number" is also negative. Each quantity is the product of two negative numbers, which is positive.

Quantity A: $(x - 2)(x - 3) = (neg)(neg) = pos$

Quantity B: $(x - 4)(x - 5) = (\text{more } neg)(\text{more } neg) = \text{more } pos$

Thus, Quantity B is larger.

5. **(A).** 24 to any power will end in the same units digit as 4 to the same power (it is always true that, if you only need the last digit of the product, you only need the last digits of the numbers being multiplied).

4 to any power ends in either 4 or 6 ($4^1 = 4$, $4^2 = 16$, $4^3 = 64$, etc.) If the power is odd, the answer will end in 4; if the power is even, the answer will end in 6.

Since the exponent "5 + 2*x*" will be odd for any integer power of *x,* $24^{5 + 2x}$ will end in 4.

36 to any power will end in the same units digit as 6 to the same power. Interestingly, powers of 6 always end in 6, so 36^6 will end in 6.

17 to any power will end in the same units digit as 7 to the same power. While the units digits of the powers of 7 do indeed create a pattern, 7^3 is just 343, which ends in 3.

Thus:

$24^{5 + 2x}$ ends in 4
36^6 end in 6
7^3 ends in 3

Multiplying three numbers that end in 4, 6, and 3 will yield answer that ends in 2, because (4)(6)(3) = 72, which ends in 2.

6. **(E).** The old solid has:

Width = w
Length = l
Height = h

But then you are told that width and length are equal, so substitute right away to reduce the number of variables:

The old solid has:

Width = w
Length = w
Height = h

After the changes detailed in the problem, the new solid has:

Width = $w + 1$
Length = $w + 1$
Height = $h - 9$

You are then told that the old and new solids have equal volume. Since volume = length × width × height:

$$w^2h = (w + 1)^2(h - 9)$$

Before you get too far into simplifying this, there is one more fact yet to be considered: the height of the new solid is four times the width of the original solid. Thus:

$$h - 9 = 4w$$

or

$$h = 4w + 9$$

Substitute into both spots in $w^2h = (w + 1)^2(h - 9)$ where h appears:

$$w^2(4w + 9) = (w + 1)^2(4w)$$

Distribute $w^2(4w + 9)$ and FOIL $(w + 1)^2$:

$$4w^3 + 9w^2 = (w + 1)^2(4w)$$
$$4w^3 + 9w^2 = (w^2 + 2w + 1)(4w)$$
$$4w^3 + 9w^2 = 4w^3 + 8w^2 + 4w$$

Fortunately, you can now subtract $4w^3$ from both sides and simplify from there:

$$4w^3 + 9w^2 = 4w^3 + 8w^2 + 4w$$
$$9w^2 = 8w^2 + 4w$$
$$w^2 = 4w$$
$$w = 4 \text{ (since } w \text{ cannot be 0)}$$

You now need the *volume* of the original solid. The old solid has:

Width = w
Length = w
Height = h

You also know that $h = 4w + 9$.

Thus, width = 4, length = 4, and height = 4(4) + 9 = 25, and the volume of the original solid is (4)(4)(25) = 400.

7. **(D).** Since the part of the equation inside the absolute value could have a positive value (in which case the absolute value is irrelevant) or a negative one, solve the equation twice, once for each scenario:

Scenario 1:	**Scenario 2:**
$x - 4 \geq 0$	$x - 4 \leq 0$
$x^2 - 8x + 21 = \lvert x - 4\rvert + 5$	$x^2 - 8x + 21 = \lvert x - 4\rvert + 5$
$x^2 - 8x + 21 = x - 4 + 5$	$x^2 - 8x + 21 = -(x - 4) + 5$

$$x^2 - 9x + 20 = 0$$
$$(x - 5)(x - 4) = 0$$
$$x = 5 \text{ or } 4$$

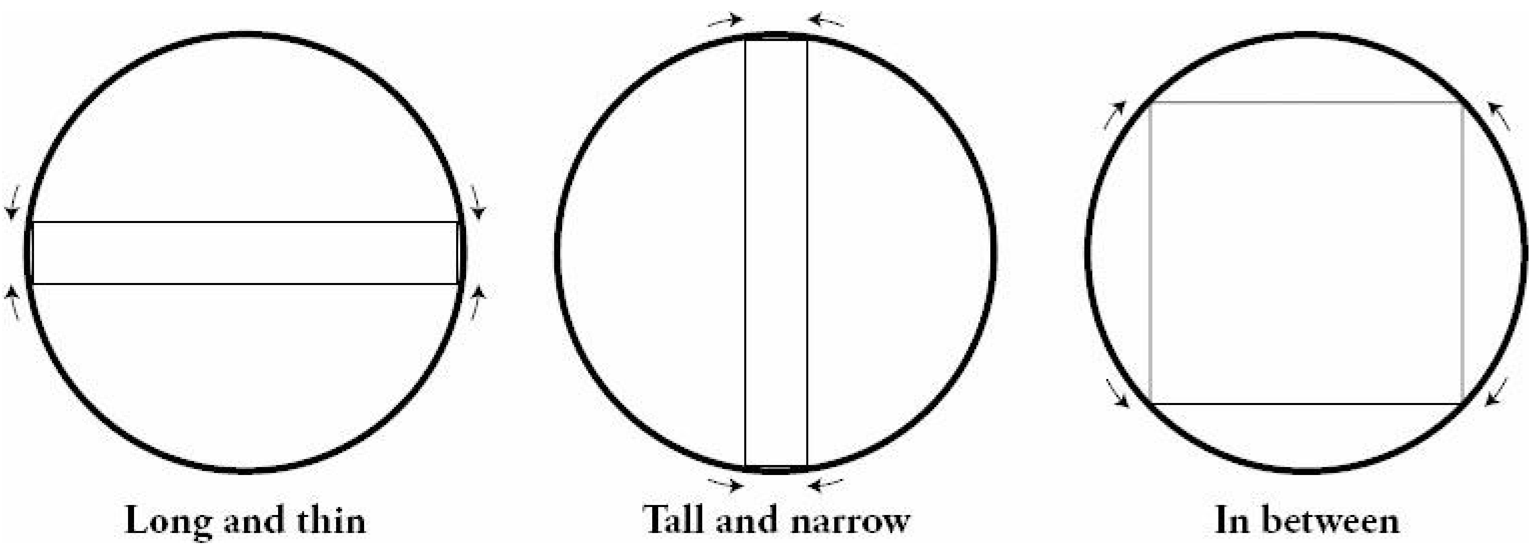
$$x^2 - 8x + 21 = -x + 4 + 5$$
$$x^2 - 7x + 21 = 0$$
$$(x - 4)(x - 3) = 0$$
$$x = 4 \text{ or } 3$$

Sum of the different solutions: 5 + 4 + 3 = 12.

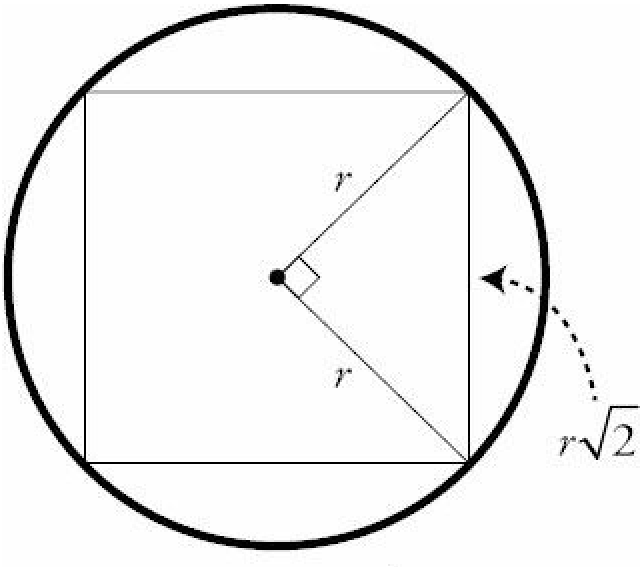
The repeated solution (4 occurs twice) may simply be ignored — the work shows two different ways of achieving the solution 4, but that is still just one solution to the equation. There are three total solutions that sum to 12.

8. **(E).** This question asks you to determine which answer choice lists an area for the inscribed rectangle that is not feasible. What makes one possible area feasible and another one infeasible?

The inscribed rectangle can be stretched and pulled to extremes: extremely long and thin, extremely tall and narrow, and somewhere in between:



The "long and thin" and "tall and narrow" rectangles will have a very small area, and the "in between" rectangle will have the largest possible area. In fact, the largest possible rectangle inscribed inside a circle will be a square:

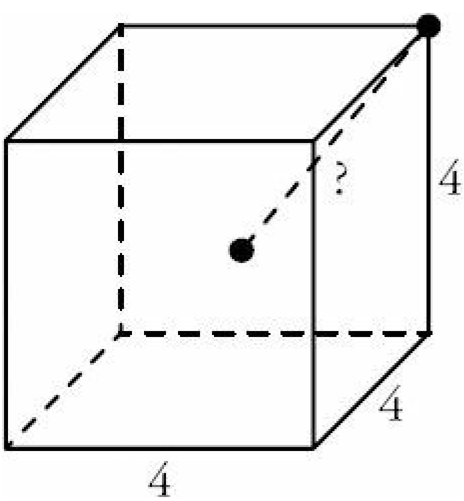


Square: maximal area

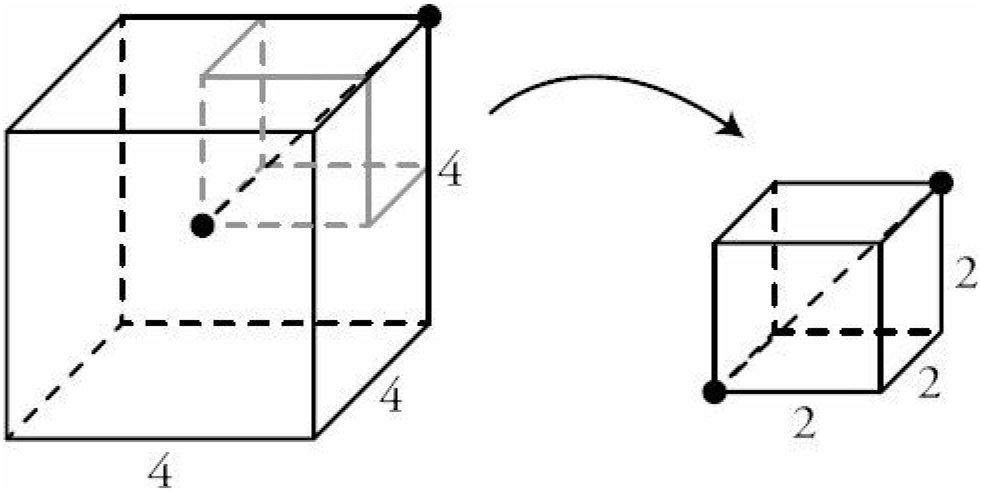
In this problem, the circumference = $10\pi = 2\pi r$. Thus $r = 5$, and the diagonal of the square is $2r = 10$. The square then has a side length of $5\sqrt{2}$ and an area of $\left(5\sqrt{2}\right)^2 = 50$

Only answer choice (E) is larger than 50. The correct answer is (E).

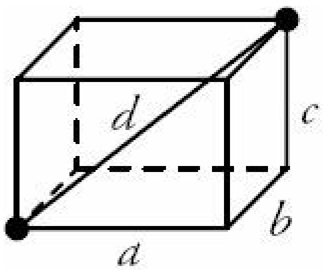
9. **(C).** First, you should represent the object with pictures, as is good practice with any 3-dimensional situation.



The length of any side of the cube is 4, and you are asked for the distance between the center of the cube and any of its vertices (corners). If you chop up the cube into 8 smaller cubes, you can see that the distance from the center of the 4 × 4 × 4 cube to any corner is the diagonal of a 2 × 2 × 2 cube.



You can find the diagonal of a cube in a variety of ways. Probably the fastest (besides applying a memorized formula) is to use the "super-Pythagorean" Theorem, which extends to three dimensions:



$a^2 + b^2 + c^2 + = d^2$

In the special case when the three sides of the box are equal, as they are in a cube, then you have this equation, letting s represent any side of the cube:

$s^2 + s^2 + s^2 = d^2$
$3s^2 = d^2$
$s\sqrt{3} = d$

Since $s = 2$, you know that $d = 2\sqrt{3}$.

10. **3/4.** Probability is (favorable outcomes)/(total # of possibilities). There are 99 – 20 + 1 = 80 possible values for c, so the unknown is how many of these c values yield a $c^3 - c$ that is divisible by 12.

The prime factorization of 12 is 2 × 2 × 3. There are several ways of thinking about this: numbers are divisible by 12 if they are divisible by 3 and by 2 twice, or if they are multiples of both 4 and 3, or if half of the number is an even multiple of 3, etc.

The expression involving c can be factored.

$c^3 - c = c(c^2 - 1) = c(c - 1)(c + 1)$

These are consecutive integers. It may help to put them in increasing order: $(c - 1)c(c + 1)$. Thus, this question has a lot to do with *Consecutive Integers*, and not only because the integers 20 to 99 themselves are consecutive.

In any set of three consecutive integers, a multiple of 3 will be included. Thus, $(c - 1)c(c + 1)$ is always divisible by 3 for any integer c. This takes care of part of the 12. So the question simply becomes "How many of the possible $(c - 1)c(c + 1)$ values are divisible by 4?" Since the prime factors of 4 are 2's, it makes sense to think in terms of odds and evens.

$(c - 1)c(c + 1)$ could be (E)(O)(E), which is definitely divisible by 4, because the two evens would each provide at least one separate factor of 2. Thus, $c^3 - c$ is divisible by 12 whenever c is odd, which are the cases $c = 21, 23, 25, \ldots, 95, 97, 99$. That's $((99 – 21)/2) + 1 = (78/2) + 1 = 40$ possibilities.

Alternatively, $(c - 1)c(c + 1)$ could be (O)(E)(O), which will only be divisible by 4 when the even term itself is a multiple of 4. Thus, $c^3 - c$ is also divisible by 12 whenever c is a multiple of 4, which are the cases $c = 20, 24, 28, \ldots, 92, 96$. That's $((96 - 20)/4) + 1 = (76/4) + 1 = 20$ possibilities.

The probability is thus $(40 + 20)/80 = 60/80 = 3/4$.

11. **I and II only.** x cannot equal y, as that would make $x/y = 1 \neq$ even. So either $x > y$ or $y > x$.

x and y are both positive, and x/y is an integer, so $x > y$.

If $x - y$ is even, either x and y are both even, or they are both odd.

Since x/y = an even integer, $x = y \times$ an even integer.

Odd $\neq$ Odd $\times$ an even integer, so x and y can't be odd.

Even = Even $\times$ an even integer, so x and y must be even.

I. TRUE. x and y are both even, and x/y is an even integer. The smallest value of x is 4, when y is 2, and $x/y = 4/2 = 2$. No even number greater than 2 is prime, so x can't be prime.

II. TRUE. x and y are each positive even numbers and $x \neq y$. Thus, $x + y$ is even, and the smallest possible value of $x + y = 4 + 2 = 6$. All even numbers greater than or equal to 6 are non-prime.

III. FALSE. It could be that $x = 4$ and $y = 2$, so $y/x = 1/2$, which is technically non-prime, but is not an integer. In fact, if x/y = an even integer, y/x = 1/an even integer = positive fraction.

12. **0.** Since the remainder is defined as what is left over after one number is divided by another, it makes sense that the leftover amount would be positive. So why is this information provided, if the remainder is "automatically" positive? Because there is a third possibility: that the remainder is 0! So when you are told here that the remainder when 120 is divided by m is positive, you are really being told that $120/m$ does not have a remainder of 0. In other words, 120 is not divisible by m, or m is not a factor of 120. Similarly, n is not a factor of 120.

Another constraint on both m and n is that they are single-digit positive integers. So m and n are integers between 1 and 9, inclusive, that are not factors of 120. Only two such possibilities exist: 7 and 9.

Since $m > n$, $m = 9$ and $n = 7$. Thus, $m - n = 2$, and the remainder when 120 is divided by 2 is 0.

13. **225,000.** Microchip radius = (2.5 cm)(10 mm/cm) = 25 mm

Blueprint radius = 1 cm per every 0.05 mm on the microchip
= 10 mm per every 0.05 mm on the microchip
= (10 mm/0.05 mm on microchip)(25mm on microchip)
= (10 mm/0.05)(25)
= (1,000 mm/5)(25)
= (1,000 mm)(5)
= 5,000 mm
= (5,000 mm)(cm/10 mm)
= 500 cm

Blueprint area = $\pi \times r^2$
= $\pi \times (500 \text{ cm})^2$
= $250{,}000\pi \text{ cm}^2$

14. **(D).** If $PV = kT$, then $P = \dfrac{kT}{V}$. Quantity A is $P = \dfrac{k(32)}{(20)} = \dfrac{8}{5}k$.

If $PV = kT$, then $T = \dfrac{PV}{k}$. Quantity B is $T = \dfrac{(78)(10)}{k} = \dfrac{780}{k}$.

Don't rush to judgment, thinking that $780 > \dfrac{8}{5}$ means that Quantity B is greater. Notice that the k term is in the numerator of one quantity (so Quantity A increases with k) and the denominator of the other (so the larger k is, the smaller Quantity B is).

If $k = 1$, then Quantity B is greater $\left(780 > \dfrac{8}{5}\right)$. But if $k = 100$, Quantity A is greater ($160 > 7.8$). Thus, (D) is the answer.

15. **(D).** Some intuitive recollection of geometry rules and a picture drawn to scale can help you determine reasonable answer choices. If AC is a diameter of the circle, then Triangle ABC is a right triangle, with angle $ABC = 90$ degrees. The shortest side of a triangle is across from its smallest angle, and the longest side of a triangle is across from its largest angle. Therefore, $AC > BC > AB$.

The circumference of the circle = $\pi d = 6\pi\sqrt{3}$, so $d = 6\sqrt{3} \approx 6(1.7) = 10.2$. Thus, $AC \approx 10.2$ and $BC < 10.2$. But you can clearly see from the picture drawn to scale that BC is longer than half the diameter, so you can conservatively determine that $BC > 5.1$.

(A) $\frac{3}{\sqrt{2}} \approx \frac{3}{1.4} = \frac{30}{14} = 2\,1/7$ TOO LOW

(B) 3 TOO LOW

(C) $3\sqrt{3} \approx 3(1.7) = 5.1$ TOO LOW

(D) 9 OK

(E) $9\sqrt{3} \approx 9(1.7) = 15.3$ TOO HIGH

Alternatively, you could use rules of geometry to solve directly for the answer. Line *AC* passes through the center of the circle, so the inscribed Triangle *ABC* is a right triangle with angle *ABC* = 90°. Since angle *ACB* is 30°, angle *CAB* is 60°.

The sides in a 30–60–90 triangle have the ratio 1 : $\sqrt{3}$: 2, so given any side, you can compute the other two sides.

First, use the circumference to solve for *AC* (the diameter):

$$6\pi\sqrt{3} = \pi d = \text{Circumference}$$
$$\frac{6\pi\sqrt{3}}{\pi} = d$$
$$6\sqrt{3} = d$$

Now you can use ratios (specifically, the unknown multiplier) to find *BC*.

	AB	***BC***	***AC***
Basic Ratio	$1x$	$\sqrt{3}x$	$2x$
Known Side			$6\sqrt{3}$
Unknown Multiplier			$x = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$
Computer Sides	$3\sqrt{3}$	$(\sqrt{3})(3\sqrt{3}) = 9$	$2(3\sqrt{3}) = 6\sqrt{3}$

Line segment *BC* has length 9.

16. **(B).** Profit equals revenue minus cost. The company's profit is:

$$p(9 - p) - (p + 15) = 9p - p^2 - p - 15$$
$$= -p^2 + 8p - 15$$

$= -(p^2 - 8p + 15)$
$= -(p - 5)(p - 3)$

Profit will be zero if $p = 5$ or $p = 3$, which eliminates answers (A) and (C). For $p > 5$, both $(p - 5)$ and $(p - 3)$ are positive. In that case, the profit is negative (i.e., the company loses money). The profit is only positive if $(p - 5)$ and $(p - 3)$ have opposite signs, which occurs when $3 < p < 5$.

The correct answer is (B).

17. **I and II only.** The sum (1/41 + 1/42 + 1/43 + 1/44 + … + 1/57 + 1/58 + 1/59 + 1/60) has 20 fractional terms. It would be nearly impossible to compute if you had to find a common denominator and solve without a calculator and a lot of time. Instead, look at the maximum and minimum possible values for the sum.

Maximum: The largest fraction in the sum is 1/41. K is definitely smaller than $20 \times 1/41$, which is itself smaller than $20 \times 1/40 = 1/2$.

Minimum: The smallest fraction in the sum is 1/60. K is definitely larger than $20 \times 1/60 = 1/3$.

Therefore, $1/3 < K < 1/2$.

I. YES: $1/4 < 1/3 < K$
II. YES: $1/3 < K$
III. NO: $1/2 > K$

18. **(B).** First, make some observations. With 9 competitors and only 3 medals awarded, only 1/3 of the competitors will win overall. Although a simplification, it is reasonable for each competitor to see his or her chance of winning a medal as 1/3, or to expect to win 1/3 of a medal (pretending for a moment that medals can be "shared").

You are asked for the probability *at least* 2 of the triplets will win a medal. In other words, you want 2/3 to 3/3 of the triplets to win medals, or for each triplet to win 2/3 to 3/3 of a medal. Since 2/3 and 3/3 are both greater than 1/3, you are looking for the probability that the triplets will win medals at a rate greater than that expected for competitors overall. This would certainly be an unusual outcome. Thus, the probability should be less than 1/2. Eliminate (D) and (E). You could then at least make an educated guess from among the remaining choices with at least a 1 in 3 shot at success.

To solve, use the probability formula and combinatorics:

$$\text{Probability} = \frac{\text{specified outcome}}{\text{all possible outcomes}} = \frac{\text{\# of ways at least 2 triplets win medal}}{\text{\# of ways 3 medals can be awarded}}$$

First, find the total number of outcomes for the triathlon. There are nine competitors; three will win medals and six will not. Set up an anagram grid where Y represents a medal, N no medal:

competitor:	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
medal:	Y	Y	Y	N	N	N	N	N	N

# of ways 3 medals can be awarded= $\frac{9!}{3!6!} = \frac{(9)(8)(7)}{(3)(2)(1)} = (3)(4)(7) = 84$

Now, you need to determine the number of instances when *at least two* brothers win a medal. Practically speaking, this could happen when (1) exactly three brothers win or (2) exactly two brothers win.

Start with *all three* triplets winning medals, where Y represents a medal:

triplet:	A	B	C		non-triplet:	C_1	C_2	C_3	C_4	C_5	C_6
medal:	Y	Y	Y		medal:	N	N	N	N	N	N

The number of ways this could happen is $\frac{3!}{3!} \times \frac{6!}{6!} = 1$. This makes sense, as there is only one instance in which all three triplets would win medals and all of the other competitors would not.

Next, calculate the instances when *exactly two* of the triplets win medals:

triplet:	A	B	C		non-triplet:	C_1	C_2	C_3	C_4	C_5	C_6
medal:	Y	Y	N		medal:	Y	N	N	N	N	N

Since both triplets and non-triplets win medals in this scenario, we need to consider possibilities for both sides of the grid. For the triplets, the number of ways that two could win medals is $\frac{3!}{2!1!} = 3$.

For the non-triplet competitors, the number of ways that one could win the remaining medal is $\frac{6!}{1!5!} = 6$.

Multiply these two numbers to get the total number of instances: 3 × 6 = 18.

The brothers win *at least two* medals in 18 + 1 = 19 cases. The total number of cases is 84, so the probability is 19/84.

The correct answer is (B).

19. **(B).** Since the answer asks for an approximation, you should use decimal approximations for all square roots in the question and answer choices.

(A) $\sqrt{3} \approx 1.7$
(B) 2
(C) $1 + \sqrt{2} \approx 1 + 1.4 = 2.4$

(D) $1 + \sqrt{3} \approx 1 + 1.7 = 2.7$

(E) $2\sqrt{3} \approx 2(1.7) = 3.4$

Note that there is a minimum difference of 0.3 between answer choices. This implies that you must be *reasonably* careful when approximating, but will have no trouble choosing an answer if you approximate every square root to the nearest tenth.

$$\sqrt{2} \approx 1.4$$
$$\sqrt{2+\sqrt{2}} \approx \sqrt{2+1.4} \approx \sqrt{3.4} \approx 1.8$$
$$\sqrt{2+\sqrt{2+\sqrt{2}}} \approx \sqrt{2+1.8} \approx \sqrt{3.8} \approx 1.9$$
$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}} \approx \sqrt{2+1.9} \approx \sqrt{3.9} \approx 2$$
$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}} \approx \sqrt{2+2} \approx \sqrt{4} = 2$$

At this point, you can see that the expression is converging on 2.

Alternatively, an algebraic solution is possible if you recognize that the infinite expression is nested within itself:

$$x = \sqrt{2+\left(\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}}\right)} = \sqrt{2+x}$$

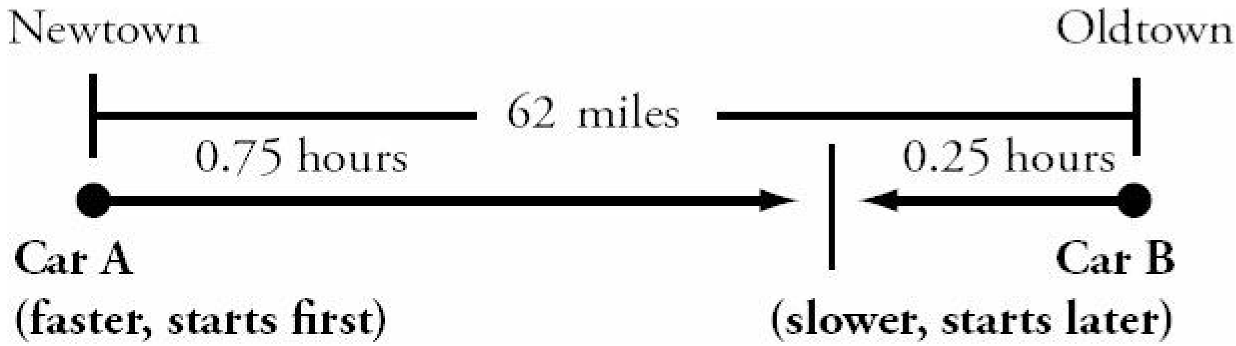
You can solve for x as follows:

$x = \sqrt{2+x}$
$x^2 = 2 + x$
$x^2 - x - 2 = 0$
$(x - 2)(x + 1) = 0$

This implies that $x = 2$ or $x = -1$. Since x is the square root of a real, positive number, it must be positive, and you can conclude that $x = 2$.

The correct answer is (B).

20. **(A).** Draw a diagram to illustrate the moment at which A and B pass each other moving in opposite directions:



You could test the answer choices:

	B's distance (miles)	B's rate (mph) = D/T = $D/0.25$	A's rate (mph) = B's rate + 8	A's distance (miles) = $R \times T$ = $R \times 0.75$	Total distance
(A)	**14**	**56**	**64**	**48**	**62**
(B)	12	48	56	42	54
(C)	10	40	48	36	46
(D)	9	36	44	33	42
(E)	8	32	40	30	38

Or you could solve algebraically, using an *RTD* chart. Note that you must convert 15 minutes to 1/4 (or 0.25) hours:

	Rate	**Time**	**Distance**
Car A	$(r + 8)$ mph	0.75 hours	$(0.75)(r + 8)$ miles
Car B	r mph	0.25 hours	$0.25r$ miles
Total			62 miles

Set up and solve an equation for the total distance:

$$(0.75)(r + 8) + (0.25r) = 62$$
$$0.75r + 6 + 0.25r = 62$$
$$r = 56$$

Therefore, Car B traveled a distance of $0.25r = (0.25)(56) = 14$ miles.

The correct answer is (A).

21. **(D).** Factor 10,125 to its prime factors: $10{,}125 = 3^4 5^3$.

So, $x^2 5^y = 3^4 5^3$.

In order to have 5^3 on the right side, there have to be three factors of 5 on the left side. All three could be in the 5^y term (i.e., y could equal 3). Or, one of the 5's could be in the 5^y term, and two of the 5's in the x^2 term; i.e. y could equal 1 and x could have a single factor of 5.

In order to have 3^4 on the right side, x^2 must have $3^4 = (3^2)^2$ as a factor. In other words, x must have 3^2 as a factor, because 3^2 is certainly not a factor of 5. Thus, x is a multiple of 9.
The possibilities:

Quantity A: x^2		Quantity B: 5^y	Check: The product must be 10,125	Check: Quantity A must be a perfect square	Check: Quantity B must be a power of 5
$x^2 = 9^2 = 81$	<	$5^y = 5^3 = 125$	(81)(125) = 10,125	yes	yes
$x^2 = (9 \times 5)^2 = 2{,}025$	>	$5^y = 5^1 = 5$	(2,025)(5) = 10,125	yes	yes

In one case, Quantity A is greater. In the other, Quantity B is greater. The correct answer is (D).

22. **(B).** Testing the choices would be a natural way to solve this problem, since the question doesn't ask you to solve for b in general, but rather "for which of the following is x closest to zero?" However, numbers between 2^{20} and 2^{42} are too large to plug and compute. You must manipulate the terms with base 8 to see how they might balance with 2^b:

$$x = 2^b - (8^8 + 8^6)$$
$$0 \approx 2^b - (8^8 + 8^6)$$
$$2^b \approx (8^8 + 8^6)$$
$$2^b \approx (8^6)(8^2 + 1)$$
$$2^b \approx ((2^3)^6)\,((2^3)^2 + 1)$$
$$2^b \approx (2^{18})(2^6 + 1)$$

Since 1 is very small in comparison to 2^6, we can approximate $(2^6 + 1) \approx (2^6)$. Therefore,

$$2^b \approx (2^{18})(2^6)$$
$$2^b \approx 2^{24}$$
$$b \approx 24$$

The correct answer is (B).

23. **(E).** Since there are variables in the answer choices, we should pick a number and test the choices. If $k = 2$, then $\frac{2}{\sqrt{k+1}+\sqrt{k-1}} = \frac{2}{\sqrt{3}+\sqrt{1}} \approx \frac{2}{1.7+1} = \frac{2}{2.7}$, which is less than 1. Now test the answer choices and try to match the target:

(A) 2 TOO HIGH

(B) $2\sqrt{2k} = 2\sqrt{4} = 4$ TOO HIGH

(C) $2\sqrt{k+1} + \sqrt{k-1} = 2\sqrt{3} + \sqrt{1} \approx 2(1.7) + 1 = 4.4$ TOO HIGH

(D) $\dfrac{\sqrt{k+1}}{\sqrt{k-1}} = \dfrac{\sqrt{3}}{\sqrt{1}} \approx 1.7$ TOO HIGH

(E) $\sqrt{k+1} - \sqrt{k-1} = \sqrt{3} - \sqrt{1} \approx 1.7 - 1 = 0.7$ OK

Alternatively, you could solve this problem algebraically. The expression given is of the form $\dfrac{2}{a+b}$, where $a = \sqrt{k+1}$ and $b = \sqrt{k-1}$.

You need to either simplify or cancel the denominator, as none of the answer choices have the denominator you start with, and most of the choices have no denominator at all. To be able to manipulate a denominator with radical signs, you must first try to eliminate the radical signs entirely, leaving only a^2 and b^2 in the denominator. To do so, multiply by a fraction that is a convenient form of 1:

$$\frac{2}{(a+b)} = \frac{2}{(a+b)} \times \frac{(a-b)}{(a-b)} = \frac{2(a-b)}{a^2 - b^2}$$

Notice the "difference of two squares" special product created in the denominator with your choice of (a - b).

Substituting for a and b,

$$\frac{2}{\left(\sqrt{k+1}+\sqrt{k-1}\right)} \times \frac{\left(\sqrt{k+1}-\sqrt{k-1}\right)}{\left(\sqrt{k+1}-\sqrt{k-1}\right)} = \frac{2\left(\sqrt{k+1}-\sqrt{k-1}\right)}{(k+1)-(k-1)} = \frac{2\left(\sqrt{k+1}-\sqrt{k-1}\right)}{2} = \sqrt{k+1}-\sqrt{k-1}$$

The correct answer is (E).

24. **(A).** First, note the answer pairs (A)&(C) and (B)&(E), in which one ratio is the square of the other. This represents a likely trap in a problem that asks for the ratio of $\sqrt{x}$ to $\sqrt{y}$ rather than the more typical ratio of x to y. You can eliminate (D), as it is not paired with a trap answer and therefore probably not the correct answer. You should also suspect that the correct answer is (A) or (B), the "square root" answer choice in their respective pairs.

For problems involving successive changes in amounts — such as population-growth problems, or compound interest problems — it is helpful to make a table:

	Account A	**Account B**
Now	x	y
After 1 month	$\left(\frac{9}{10}\right)x$	$\left(\frac{12}{10}\right)y$

After 2 month	$\left(\frac{9}{10}\right)\left(\frac{9}{10}\right)x = \left(\frac{81}{100}\right)x$	$\left(\frac{12}{10}\right)\left(\frac{12}{10}\right)y = \left(\frac{144}{100}\right)y$

If the accounts have the same amount of money after two months, then:

$$\left(\frac{81}{100}\right)x = \left(\frac{144}{100}\right)y$$

$$81x = 144y$$

This can be solved for $\frac{\sqrt{x}}{\sqrt{y}}$:

$$\frac{x}{y} = \frac{144}{81}$$

$$\frac{\sqrt{x}}{\sqrt{y}} = \frac{\sqrt{144}}{\sqrt{81}} = \frac{12}{9} = \frac{4}{3}$$

The correct answer is (A).

25. **(C).** Consecutive integers have two key characteristics: they differ by a known, constant value (i.e., 1), and they alternate odd, even, odd, even, etc. You should use the odd/even property to evaluate these choices. This general approach is usually faster than considering specific values. It works particularly well for very general questions about whether something CANNOT or MUST be true.

(A)	*a* = E + O = O, or *a* = O + E = O	*b* = 3 pairs (E + O) = O, or *b* = 3 pairs (O + E) = O	*a* could equal *b* (both Odd)
(B)	*a* = E + O + E = O, or *a* = O + E + O = E	*b* = 3 pairs (E + O) = O, or *b* = 3 pairs (O + E) = O	*a* could equal *b* (both Odd)
(C)	***a* = 3 pairs (E + O) = O, or** ***a* = 3 pairs (O + E) = O**	***b* = 2 pairs (E + O) = E, or** ***b* = 2 pairs (O + E) = E**	$a \neq b$ **(Odd ≠ Even)**
(D)	*a* = 3 pairs (E + O) = O, or *a* = 3 pairs (O + E) = O	*b* = O + 3 pairs (E + O) = E, or *b* = E + 3 pairs (O + E) = O	*a* could equal *b* (both Odd)
(E)	*a* = O + 3 pairs (E + O) = E, or *a* = E + 3 pairs (O + E) = O	*b* = O + 2 pairs (E + O) = O, or *b* = E + 2 pairs (O + E) = E	*a* could equal *b* (both Odd or both Even)

The correct answer is (C).

26. **(A).** First of all, note that the height of each person in question is fixed (no one grew taller or shorter); only

weights changed. Second, note that BMI is always positive, and is proportional to w; as weight increases, BMI increases, and vice versa. So the language of the quantities — pounds gained … BMI increased" and "pounds lost … BMI decreased" — is aligned with this proportionality. Both quantities are a positive number of pounds.

Since $\text{BMI} = \dfrac{703w}{h^2}$, change in $\text{BMI} = \dfrac{703w_{before}}{h^2} - \dfrac{703w_{after}}{h^2} = \dfrac{703}{h^2}\left(w_{before} - w_{after}\right)$.

To simplify things, you can write this in terms of ΔBMI and Δw, the positive change in BMI and weight, respectively:

$$\Delta BMI = \frac{703}{h^2}\Delta w$$

(The triangle symbol indicating positive change in a quantity does not appear on the GRE — it is used here for convenience in notating an explanation.)

Since the quantities both refer to Δw, rewrite the relationship as $\Delta w = \dfrac{h^2}{703}\Delta BMI$. Both ΔBMI and h are given in each quantity, so Δw can be calculated and the relationship between the two quantities determined. (The answer is definitely not (D).)

Quantity A:
A 6' 2" tall person is 6(12) + 2 = 74 inches tall.

$$\Delta w = \frac{h^2}{703}\Delta BMI = \frac{74^2}{703}(1.0) = \frac{74^2}{703}$$

Quantity B:
A 5' 5" tall person is 5(12) + 5 = 65 inches tall.

$$\Delta w = \frac{h^2}{703}\Delta BMI = \frac{65^2}{703}(1.2)$$

Since the 703 in the denominator is common to both quantities, the comparison is really between 74^2 = 5,476 and 65^2(1.2) = 4,225(1.2) = 5,070. Quantity A is greater.

27. **(C).** There are four different outcomes that can yield two balls of the same color: Bag A with white, Bag A with red, Bag B with white, or Bag B with red. The first decision that must be made is to choose a bag. Because the problem states that one of the two bags will be chosen at random, you are no more likely to choose one bag than the other. Therefore, the probability of choosing Bag A, P(A), and the probability of choosing Bag B, P(B), must be the same, i.e. P(A) = P(B) = 1/2.

If Bag A is chosen, what is the probability of a matched pair? First, compute the probability of two whites. The probability of the first white is 3/6 and the probability of the second white is 2/5, so the probability of a first AND second white is (3/6)(2/5) = 1/5. Similarly, the probability of two reds is (3/6)(2/5) = 1/5. If Bag A is chosen, you can obtain a match by either grabbing a pair of white OR a pair of red, so you must add their probabilities to get the total chance of a pair. This gives P(Bag A Pair) = 1/5 + 1/5 = 2/5.

Similarly, if Bag B is chosen the probability of a pair of white marbles is (6/9)(5/8) = 5/12 and the probability of a pair of red marbles is (3/9)(2/8) = 1/12. Therefore, the probability of a pair is P(Bag B pair) = 5/12 + 1/12 = 6/12 = 1/2. The probability of choosing Bag A AND a pair from Bag A is the product of the two events, (1/2)(2/5) = 1/5. Similarly, the probability of choosing Bag B AND a pair from Bag B is (1/2)(1/2) = 1/4. The total probability of choosing a pair will be the probability of choosing Bag A and a pair from Bag A OR choosing Bag B and a pair from bag B, meaning you must sum these two events. This gives: P(pair) = 1/5 + 1/4 = 4/20 + 5/20 = 9/20.

28. **(C).** There are three different cases in which you must count: 62_ _, _62_, and _ _62. In the case of 62_ _, any digits from 00 to 99 will work, which gives you 100 numbers. In the case of _62_, you have 9 choices for the first digit as you are allowed to use any number from 1–9 inclusive, but not zero because you must meet the requirement of using a four digit positive integer. For the last digit you still allow any number from 0–9, which is 10 choices. Thus, by the fundamental counting principle, for _62_ you have (9)(10) = 90 choices. For the case of _ _62 you again have 1–9 inclusive for the first digit and 0–9 inclusive for the second digit for a total of 90 choices. However, in this case you are double counting one number, since 6262 already appeared in the 62_ _ case. Therefore there are only 89 new numbers that meet the criteria. Since you could create the case 62_ _ OR _62_ OR _ _62, you must add the number of possibilities together for each case to achieve the total. This gives you 100 + 90 + 89 = 279.

29. **(B).** In order for a number to be divisible by 9 the sum of the digits must be a multiple of 9. The lowest number that can be made by summing 5 of the digits is given by 0 + 1 + 2 + 4 + 5 = 12 and the highest number that can be made is 1 + 2 + 4 + 5 + 6 = 18. The only number in this range that sums to a multiple of 9 is 18, and thus the only possible combination of numbers you can use is {1, 2, 4, 5, 6}. In other words, no combination of numbers using the number 0 will ever yield a multiple of 9. The question can now be rephrased as, "How many different 5 digit numbers can be made using the digits {1, 2, 4, 5, 6} without repetition?" as these numbers will always sum to 18 and thus will always be divisible by 9. In this case, the answer is simply 5!, as there are 5 choices for the first number, 4 for the second, 3 for the third, and so on. Thus, there are 5! = 120 possible 5 digit numbers that are divisible by 9.

30. **(B).** This problem is greatly simplified if you realize that you do not need to sum all the multiples of 4 (or of 3) in the given range and divide by the number of such multiples, using the typical average formula. The average for a set of evenly spaced integers is equal to the average of the first and last term.

Quantity A: The multiples of 3 between 101 and 598 are 102, 105, 108, ..., 591, 594, 597. The average of the whole set is $\frac{102+597}{2}=\frac{699}{2}=349.5$

Quantity B: The multiples of 4 between 101 and 598 are 104, 108, 112, ..., 588, 592, 596. The average of the whole set is $\frac{104+596}{2}=\frac{700}{2}=350$

31. **(C).** Set up an implied inequality and perform identical operations on each quantity, grouping variables.

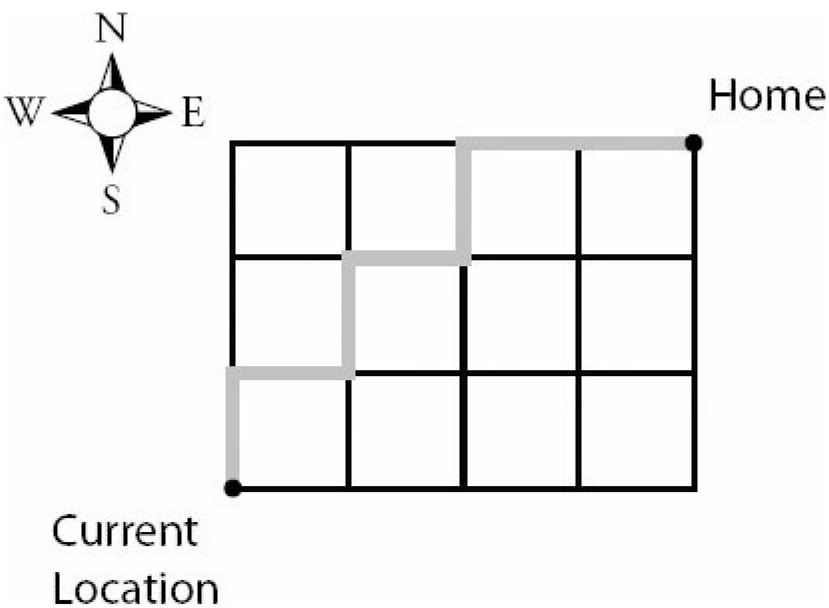
Quantity A		Quantity B
$a + d - c - 90$	?	$90 - e - b - f$
$a + d - c$	?	$180 - e - b - f$
$a + d - c + e + b + f$	?	180
$(a + d + e + f) + (b - c)$	?	180

In the last step above, only the order of the variables was changed, and parentheses added to group certain terms. Notice that the angle at point C and point D is the same, as AC and FD are parallel lines intersected by the transversal CE. So, the first set of parentheses holds the sum of the interior angles of the biggest triangle ACE, which is 180. Also because AC and FD are parallel lines intersected by transversal BE, $b = c$, so $b - c = 0$ in the second set of parentheses.

Quantity A		**Quantity B**
$(a + d + e + f) + (b - c)$	?	180
$(180) + (0)$	=	180

Thus, the quantities each equal 180 and the correct answer is (C).

32. **(D).** Given that the man can only move north and east, he must advance exactly 7 blocks from his current location to get home regardless of which path he takes. Of these 7 blocks, 4 must be moving east and 3 must be moving north. An example path is given below:



The problem can then be rephrased as follows: "Of the 7 steps, when does the man choose to go east and when does he choose to go north?" Labeling each step as N for north and E for east, you can see the problem as the number of unique rearrangements of NNNEEEE (for example, this arrangement corresponds to going north 3 times and then east 4 times straight to home). This is given by $\frac{total!}{repeats!} = \frac{7!}{3!4!} = 35$.

33. **(D).** Denote red as R, white as W, and black as B. There are exactly three ways in which the selections may occur where you get red as the second marble, either RR, WR, or BR. Since you may have any of these options the problem is an OR, so calculate the probability of each event and then add them together. First, for RR, you have the probability of the first red as (2/9) and the second red as (1/8), yielding a probability of red AND red as (2/9)(1/8) = 1/36. Similarly, the probability of first white AND second red is (3/9)(2/8) = 1/12. Finally, the probability of first black and second red is (4/9)(2/8) = 1/9. Thus, the total probability of the second marble being red is P(RR) + P(WR) + P(BR) = 1/36 + 1/12 + 1/9 = 1/36 + 3/36 + 4/36 = 8/36 = 2/9. Of course, an easier way to solve this problem is to consider that the first draw is completely irrelevant, so you may consider the second draw alone. For the second draw, there are 2 red marbles out of a total of 9 marbles, giving you 2/9. Keep in mind that even though there are often difficult solution methods, sometimes a clever insight can greatly simplify the problem.

34. **(A).** Simplify both quantities, remembering that a power to a power means you multiply the exponents. Also, 25 is 5 squared, so you can substitute, putting both quantities in terms of a base of 5.

Quantity A: $\left((25^x)^{-2}\right)^3 = 25^{-6x} = (5^2)^{-6x} = 5^{-12x}$

Quantity B: $\left((5^{-3})^2\right)^{-x} = 5^{6x}$

Typically, when you are comparing exponents with the same base, the one with the larger exponent is greater. It might be tempting to conclude that $6x > -12x$, but be careful with negative variables.

If $x = -1$, Quantity A $= 5^{12}$ and Quantity B $= 5^{-6}$, or $\frac{1}{5^6}$. In this case, Quantity A is much larger.

If $x = -\frac{1}{2}$, Quantity A $= 5^6$ and Quantity B $= 5^{-3}$, or $\frac{1}{5^3} = \frac{1}{125}$. Again, Quantity A is much larger.

If $x = -10$, Quantity A $= 5^{120}$ and Quantity B $= 5^{-60}$. You can see that, the more negative x gets, the larger the difference between Quantity A and Quantity B becomes. Quantity A will always be larger.

Another way to look at it:

Quantity A: $5^{-12x} = 5^{-12 \times negative} = 5^{negative}$

Quantity B: $5^{6x} = 5^{6 \times negative} = 5^{negative}$

Even if $|x|$ is a tiny fraction, i.e. you are taking some high order root of 5 such as $\sqrt[8]{5}$ or $\sqrt[100]{5}$, these quantities would approach 1 such that

Quantity A: $51^{\text{positive}} > 1$

Quantity B: $51^{\text{negative}} < 1$

Since Quantity A is greater than 1 and Quantity B is less than 1, Quantity A is larger.

35. **(A).** First, notice that -93 and 252 are both multiples of 3 and "inclusive" means they should be included in the sum with all the multiples of 3 in between them. Listing and adding the numbers would be time consuming and error prone, so some strategies are useful.

Quantity A: (-93) + (-90) + (-87) + … + (-6) + (-3) + 0 + 3 + 6 + … + 87 + 90 + 93 + 96 + 99 + … 246 + 249 + 252

Since 252/3 = 84, 252 is the 84th positive multiple of 3. By the same logic, minus 93 is the 31st negative multiple of 3 (because -93/3 = -31). So the sum in question is 3 times the sum of the integers from -31 to +84, inclusive.

Quantity A: $3 \times [(-31) + (-30) + (-29) + \ldots + (-2) + (-1) + 0 + 1 + 2 + \ldots + 29 + 30 + 31 + 32 + 33 + \ldots 82 + 83 + 84]$

Notice that all of the negative integers have an additive inverse elsewhere in the sum that cancels them out. For example, (-31) + 31 = 0, and (-30) + 30 = 0, etc. So the sum in question is really 3 times the sum of just the integers from 32 to 84, inclusive.

Quantity A: $3 \times [32 + 33 + \ldots 82 + 83 + 84]$

Now, apply the formula for summing consecutive integers: $\frac{First + Last}{2} \times Number\ of\ terms$

The number of terms is Last - First + 1 = 84 - 32 + 1 = 53.

Quantity A: $3 \times [32 + 33 + \ldots 82 + 83 + 84] = 3\times\left[\frac{32+84}{2}\times 53\right] = 3\times\left[\frac{116}{2}\times 53\right] = 3 \times [58 \times 53]$
$= 3 \times 3{,}074 = 9{,}222$

Thus, Quantity A is larger.

36. **(B).** When you see a negative base raised to an integer power, the question is about positives and negatives: $(-1)^{odd} = -1$ and $(-1)^{even} = +1$.

If *x* is even, all of the exponents in this question are even.

Quantity A: $(-1)^{even^2} + (-1)^{even^3} + (-1)^{even^4} = (-1)^{even} + (-1)^{even} + (-1)^{even} = 1 + 1 + 1 = 3$

Quantity B: $(-1)^{even} + (-1)^{2\times even} + (-1)^{3\times even} + (-1)^{4\times even} = 1 + 1 + 1 + 1 = 4$

If *x* is odd, *some* of the exponents in this question are odd.

Quantity A: $(-1)^{odd^2} + (-1)^{odd^3} + (-1)^{odd^4} = (-1)^{odd} + (-1)^{odd} + (-1)^{odd} = (-1) + (-1) + (-1) = -3$

Quantity B: $(-1)^{odd} + (-1)^{2\times odd} + (-1)^{3\times odd} + (-1)^{4\times odd} = (-1)^{odd} + (-1)^{even} + (-1)^{odd} + (-1)^{even}$
$= (-1) + 1 + (-1) + 1$
$= 0$

In both cases, Quantity B is greater than Quantity A.

37. **(B).** Because an exterior angle of a triangle is equal to the sum of the two opposite interior angles of the triangle (in this case, the top small triangle), $c = a + b$.

Since $d > c$ and you can substitute $a + b$ for c, you know:

$$d > a + b$$

Subtract b from both sides:

$$d - b > a$$

Thus, Quantity B is larger.

38. **(B).** Many students find sequence notation intimidating, but it doesn't have to be. Let's rephrase each sequence in normal language.

$S_n = S_{n-1} + \frac{5}{2}$ is just saying that *every term in S is equal to the term before it, plus 5/2.*

$A_n = A_{n-1} - 2.5$ is just saying that *every term in A is equal to the term before it, minus 2.5.*

Of course, 5/2 and 2.5 are equal, which is a good clue that there's probably some simple way to solve this problem without actually summing up a sequence.

Since $S_1 = 1$ and every term in S is just 2.5 greater than the term before it, Sequence S begins like this: 1, 3.5, 6, 8.5, 11, 13.5, 16, 18.5, 21, 23.5, …

Since $A_1 = 36$ and every term in A is just 2.5 less than the term before it, Sequence A begins like this: 36, 33.5, 31, 28.5, 26, 23.5, 21, 18.5…

At this point, it looks as though the two sequences are going to have a lot of terms in common! Remember, any common elements appearing in both Quantity A and Quantity B can just be canceled out.

You could just write out all 14 terms for each column, or you could "skip up" to S_{14} by noting that S_{14} is just going to be S_1 plus 2.5, thirteen times (since it takes thirteen "jumps" to get from 1 to 14). Take the first term, 1, plus 13(2.5) = 32.5 to get $S_{14} = 33.5$.

You can also "skip up" to the final term in A. To get to A_{14}, take A_1 and subtract 2.5 thirteen times (since it takes thirteen "jumps" to get from 1 to 14). Take the first term, 36, minus 13(2.5) = 32.5 to get $A_{14} = 3.5$.

So, Quantity A looks like this: The sum of 1, 3.5, 6, …, 28.5, 31, 33.5

And Quantity B looks like this: The sum of 36, 33.5, 31, …, 8.5, 6, 3.5

That is, all the terms from 3.5 to 33.5, inclusive, are held in common by both sets, so you can safely subtract them out. Here's what's left.

Quantity A: 1

Quantity B: 36

Quantity B is greater.

39. **(B).** This problem depends on knowing how to factor the "difference of squares." The basic formula that you learned in the Manhattan Prep's *Algebra GRE® Strategy Guide* is this:

$x^2 - y^2 = (x - y)(x + y)$

The important part about learning this formula is that *anything* can be "subbed in" for x and y. Another way to think about it is that two perfect squares can be "subbed in" for x^2 and y^2. For instance, a^{64} and 1 can serve as x^2 and y^2 (remember, 1 is a perfect square — if it helps, think of it as 1^2).

So, you can factor $a^{64} - 1$ in the numerator according to the pattern above:

Quantity A: $$\frac{(a^{32}+1)(a^{32}-1)}{(a^{32}+1)(a^{16}+1)(a^{8}+1)^2}$$

In order to do this, you also need to know how to take the square root of a^{64}. To take the square root of any number with an even exponent, just cut the exponent in half. Do not take the square root of the exponent itself! So, $\sqrt{a^{64}} = a^{32}$, not a^8.

Now, notice that $a^{32} - 1$ also matches the pattern (a perfect square minus a perfect square). $a^{32} + 1$, however, cannot be factored. Let's factor $a^{32} - 1$ only:

Quantity A: $$\frac{(a^{32}+1)(a^{16}+1)(a^{16}-1)}{(a^{32}+1)(a^{16}+1)(a^{8}+1)^2}$$

Looking good. But wait! $a^{16} - 1$ ALSO matches the pattern! You can factor again:

Quantity A: $$\frac{(a^{32}+1)(a^{16}+1)(a^{8}+1)(a^{8}-1)}{(a^{32}+1)(a^{16}+1)(a^{8}+1)^2}$$

You actually could factor $a^8 - 1$, but nothing on the bottom is going to cancel with terms broken down further on top. Instead, cancel common terms on top and bottom.

Quantity A: $$\frac{\cancel{(a^{32}+1)}\cancel{(a^{16}+1)}(a^8+1)(a^8-1)}{\cancel{(a^{32}+1)}\cancel{(a^{16}+1)}(a^8+1)^2}=\frac{(a^8+1)(a^8-1)}{(a^8+1)^2}=\frac{\cancel{(a^8+1)}(a^8-1)}{\cancel{(a^8+1)}(a^8+1)}=\frac{(a^8-1)}{(a^8+1)}$$

Whatever a^8 is, the numerator of Quantity A is less than the denominator of Quantity A (which is also definitely positive, since $a^8 \geq 0$). Thus, Quantity A < 1. The correct answer is (B).

40. **(A).** This problem introduces a square and a circle, and stating that the circumference of the circle is 7/8 the perimeter of the square.

This is license to plug in. Since both a square and a circle are regular figures — that is, all squares are in the same proportion as all other squares, and all circles are in the same proportion as all other circles — you can be certain that plugging only *one* set of values will give the same result you'd get from plugging *any* set of values. Because the figures are regular and related in a known way (circumference = 7/8 × square perimeter), there is no need to repeatedly try different values as is often necessary on Quantitative Comparisons.

You could say the radius of the circle is 2, so the circumference is 4π. Then, the perimeter of the square is $(8/7)(4\pi) = 32\pi/7$. This isn't ideal, because then you are stuck with π in the calculations for the square, where it is unnecessarily awkward.

It is best to pick values for the square. If the side of the square is 2, the perimeter is 4(2) = 8 and the area is (2)(2) = 4. Then, circumference of the circle is (7/8)(8) = 7. Since circumference is $2\pi r = 7$, the radius of the circle is

$$r=\frac{7}{2\pi}$$

Using these numbers:

Quantity A: The area of the square = 4

The area of the circle =

Quantity B: $$\pi r^2=\pi\left(\frac{7}{2\pi}\right)^2=\pi\left(\frac{49}{4\pi^2}\right)=\frac{49}{4\pi}\approx 3.9$$

(Use the calculator and the approximation 3.14 for π to determine that Quantity A is larger.)

41. **(C).** You can set up and simplify two equations, one for the calories consumed and the other for cost, using variables A and B for the number of servings of Snacks A and B, respectively.

Calories:
$320A + 110B = 2{,}370$
$32A + 11B = 237$ {divided by 10}

Cost:
$\$1.50A + \$0.45B = \$10.65$
$150A + 45B = 1{,}065$ {multiply by 100 to eliminate decimals}

$30A + 9B = 213$ {divided by 5}

So you now have a system of two equations and two variables, which could be solved for A and B. But even in their simplified form, these two equations have awkward coefficients that will make solving messy.

Since this is a Quantitative Comparison question, it would be smarter to "cheat off of the easy statement." That means to plug in the 5 from Quantity B as a possible number of servings of Snack A, and see what happens.

Plug $A = 5$ in to each equation.

Calories: $32(5) + 11B = 237$, so $160 + 11B = 237$, and $11B = 77$. Therefore $B = 7$.

Cost: $30(5) + 9B = 213$, so $150 + 9B = 213$, and $9B = 63$. Therefore $B = 7$.

This shows that $A = 5$ and $B = 7$ is the solution you would get by solving the system of equations yourself.

Thus, Quantity A and Quantity B are both 5.

42. **(C).** In this recursive function, each term is dependent on two others:

$$a_2 = (a_1 + a_3)/2$$
$$a_3 = (a_2 + a_4)/2$$
$$a_4 = (a_3 + a_5)/2$$

… and so on. Without actual numbers to plug in, it will be difficult to compare Quantity A and Quantity B.

You could try to put all a_n in terms of a_1 algebraically, and hope to find a pattern. If you haven't already, go ahead, try it! It's a mess.

The best way to make sense of the sequence definition is to list some (randomly made-up) actual numbers that follow the sequence rules.

If $a_1 = 1$ and $a_2 = 3$, you can extrapolate that the sequence is: 1, 3, 5, 7, 9, 11, etc.
If $a_1 = -100$ and $a_2 = 50$, you can continue the sequence: -100, 50, 200, 350, 500, 650, etc.
If $a_1 = 0$ and $a_2 = -4$, the sequence is: 0, -4, -8, -12, -16, -20, etc.

No matter the value of a_1 and a_2, the pattern is the same. After the first term, each term in the sequence is equal to the preceding term plus some constant. The constant in the test sequences was equal to $a_2 - a_1$, according to the numbers you started with.

Now you can more easily put all a_n in terms of a_1:

$$a_1 = a_1$$
$$a_2 = a_1 + c$$

$a_3 = a2 + c = a_1 + 2c$
$a_4 = a_1 + 3c$
$a_5 = a_1 + 4c$
…
$a_n = a_1 + (n - 1)c$

Thus:

Quantity A	**Quantity B**
$a_{51} - a_{48} = (a_1 + 50c) - (a_1 + 47c) = 3c$	$a_{37} - a_{34} = (a_1 + 36c) - (a_1 + 33c) = 3c$

The two quantities are the same.

43. **(B).** The term $12x$ has prime factors 2, 2, 3, and x (actually, you don't know whether x is prime, but since you don't know anything else about it right now, leave it as x).

The term $35y$ has prime factors 5, 7, and y. (Again, you don't know whether y is prime, but you can't do anything more with it right now.)

You are told that the greatest factor held in common between $12x$ and $35y$ is $5y$. Therefore, $12x$ and $35y$ each contain both 5 and y. Of course, you already knew that $35y$ contained both 5 and y, but you have definitely just learned something new about $12x$ - it also contains $5y$. You also now know that y CANNOT contain 2 and/or 3, since, if it did, the greatest common factor would also contain the 2 and/or 3 (since $12x$ and $35y$ would then *both* contain the 2 and/or 3). Similarly, x cannot contain a 7 — if it did, the 7 would appear in the greatest common factor (which it does not).

Thus, so far, you know:

$12x$ contains 2, 2, 3, 5, y, and possibly other factors but NOT a 7
$35y$ contains 5, 7, y and y does NOT contain 2 or 3

Take a look at some examples. If $x = 55$ and $y = 11$, then x correctly contains both 5 and y, and the GCF of 12(55) and 35(11) would indeed be $5y$, or 55. Alternatively, if $x = 5$ and $y = 1$, then the GCF of 12(5) and 35(1) would again be $5y$, which in this case would be 5.

In both examples, the remainder when $12x$ is divided by 10 is 0 and thus Quantity A is equal to 0. You can be certain that this will always be true because $12x$ definitely contains both 2 and 5. Any integer with 2 and 5 in its prime factors will always be a multiple of 10.

In the first example, $x = 55$ and $y = 11$, the GCF of x and y is 11. In the second example, $x = 5$ and $y = 1$, the GCF of x and y is 1. Since x and y will always be integers, their greatest common factor will always be 1 or more. Thus, Quantity B is larger.

44. **(A).** One good way to work through this problem is to pick a number, ideally starting with the innermost shape, the small circle. Let's say this circle has radius 1 and diameter 2, which would also make the side of the smaller square equal to 2.

If the small square has side 2, its diagonal would be $2\sqrt{2}$ (based on the 45-45-90 triangle ratios, or you could do the Pythagorean Theorem using the legs of 2 and 2). If the diagonal is $2\sqrt{2}$, then the diameter of the larger circle is also $2\sqrt{2}$ (and the radius of the larger circle is one-half of that, or $\sqrt{2}$), making the side of the larger square also equal to $2\sqrt{2}$. Therefore:

Small circle: radius = 1, area = π

Large circle: radius = $\sqrt{2}$, area = 2π

Small square: side = 2, area = 4

Large square: side = $2\sqrt{2}$, area = 8

Thus, the large circle has twice the area of the small circle, and the large square has twice the area of the small square. This will work for any numbers you choose. In fact, you may wish to memorize this as a shortcut: if a circle is inscribed in a square that is inscribed in a circle, the large circle has twice the area of the small circle; similarly, if a square is inscribed in a circle that is inscribed in a square, the large square has twice the area of the small square.

In Quantity A, the ratio of the area of the larger square to the smaller square is 2/1 = 2.

In Quantity B, twice the ratio of the area of the smaller circle to the area of the larger circle = 2(1/2) = 1.

45. **(A).** This problem is much easier than it looks! Of course, the integers are much too large to fit in your calculator. However, all you need to know is that a pair consisting of one 2 and one 5 multiplies to 10 and therefore adds a zero to the end of a number. For instance, a number with two 2's and two 5's in its prime factors will end with two zeroes, because the number is a multiple of 100.

Quantity A has 19 5's and many more 2's (since 2^{16} and 4^{18} together is obviously more than 19 2's — if you really want to know, it's 2^{16} and $(2^2)^{18}$, or 2^{16} and 2^{36}, or 2^{52}, or 52 2's). Since you need *pairs* made up of one 2 and one 5, you can make exactly 19 pairs (the leftover 2's don't matter), and the number ends in 19 zeroes.
Quantity B has 16 5's and many more 2's (specifically, there are 53 2's, but you should be able to tell at a glance that there are obviously more than 16 2's, so you don't need to calculate this). Since you need *pairs* made up of one 2 and one 5, you can make exactly 16 pairs (the leftover 2's don't matter), and the number ends in 16 zeroes. Thus, Quantity A is larger.

46. **(A).** Calculate several terms of the sequence defined by $a_n = 2^n - \frac{1}{2^{n-33}}$ and look for a pattern.

$$a_1 = 2^1 - \frac{1}{2^{-32}} = 2^1 - 2^{32}$$

$$a_2 = 2^2 - \frac{1}{2^{-31}} = 2^2 - 2^{31}$$

...

$$a_{16} = 2^{16} - \frac{1}{2^{-17}} = 2^{16} - 2^{17}$$

$$a_{17} = 2^{17} - \frac{1}{2^{-16}} = 2^{17} - 2^{16}$$

...

$$a_{31} = 2^{31} - \frac{1}{2^{-2}} = 2^{31} - 2^2$$

$$a_{32} = 2^{32} - \frac{1}{2^{-1}} = 2^{32} - 2^1$$

Notice that the 16th and 17th terms (the two middle terms in a set of 32 terms) are arithmetic inverses, that is, their sum is zero. Likewise, the 1st and 32nd terms sum to zero, as do the 2nd and 31st terms. In the first 32 terms of the sequence, there are 16 pairs that each sum to zero. Thus, Quantity A is zero.

For the sum of the first 31 terms, you could either

1. Subtract a^{32} from the sum of the first 32 terms: $0 - (2^{32} - 2^1) = 2^1 - 2^{32} = 2$ - (a very large number) = negative, or
2. Realize that in the first 31 terms, all terms except a_1 can be paired such that the pair sums to zero, so the sum of the first 19 terms = $a_1 = 2^1 - 2^{32} = 2$ - (a very large number) = negative.

Thus, Quantity B is negative, which is less than zero. Quantity A is larger.

47. **36.6.** To calculate first find the squared differences between the average and the terms. For example, the difference between the average, 4, and the first term, -1, is 5. $5^2 = 25$. Do this for all four terms:

-1: $(4 - (-1))^2 = 5^2 = 25$
4: $(4 - (0))^2 = 4^2 = 16$
30: $(4 - (30))^2 = (-26)^2 = 676$
-21: $(4 - (-21))^2 = (25)^2 = 625$

Add all of these terms together: 25 + 16 + 625 + 676 = 1,342.

Take the square root (use the calculator!): Square root of 1342 = 36.633.

The question asked you to round to the nearest tenth, so the answer is 36.6.

Note: Thus far, all of the questions we've seen on the real GRE dance around the issue of calculating standard deviation — no question has actually asked you to calculate it. However, we've included this example here, just in case the GRE ups the ante on us in future.

48. **(C).** Before figuring out how many balls you have of each integer value, consider what the question is asking: the "interquartile range" of a group of 100 integers. To find this range, split the 100 integers into two groups, a lower 50 and an upper 50. Then find the median of each of those groups. The median of the lower group is the first quartile (Q_1), while the median of the upper group is the third quartile (Q_3). Finally, Q_3-Q_1 is the interquartile range.

The median of a group of 50 integers is the average (arithmetic mean) of the 25th and the 26th integers when ordered from smallest to largest. Out of the ordered list of 100 integers from smallest to largest, then, find #25 and #26 and average them to get the first quartile. Likewise, find #75 and #76 and average them to get the third quartile. Then perform the subtraction.

Each ball has an integer value painted on the side — either 1, 2, 3, 4, 5, 6, 7, or 8. Figure out how many balls there are for each integer by applying the given formula, starting with the lowest integer in the list (1) and going up from there.

Number of balls labeled number 1 = 18 - $(1 - 4)^2$ = 18 - (-3)2 = 18 - 9 = 9 balls. These represent balls #1 through #9.

Number of balls labeled number 2 = 18 - $(2 - 4)^2$ = 18 - $(-2)^2$ = 18 - 4 = 14 balls, representing balls #10 through #23. Be careful when counting; the 14th ball is #23, not #24, because #10 is the first, #11 is the second, and so on.

Number of balls labeled number 3 = 18 - $(3 - 4)^2$ = 18 - $(-1)^2$ = 18 - 1 = 17 balls, representing #24 through #40.

At this point, you can tell that balls #25 and #26 both have a 3 on them. So the first quartile Q_1 is the average of 3 and 3, namely 3. Now keep going!

Number of balls labeled number 4 = 18 - $(4 - 4)^2$ = 18 - $(0)^2$ = 18 balls, representing balls #41 through #58.

Number of balls labeled number 5 = 18 - $(5 - 4)^2$ = 18 - $(1)^2$ = 18 - 1 = 17 balls, representing #59 through #75.

You can stop here. Ball #75 has a 5 on it (in fact, the last 5), while ball #76 must have a 6 on it (since 6 is the next integer in the list). Thus, the third quartile Q_1 is the average of 5 and 6, or 5.5. Notice that you have to count carefully — if you are off by even just one either way, you'll get a different number for the third quartile.

Finally, Q_3-Q_1 = 5.5 - 3 = 2.5, the interquartile range of this list of integers.

49. **(A).** Setting up the information in the question in the form of an equation, you see that:

$x/13 = y + 3/13$

$x = 13y + 3$

and

$x/7 = z + 3/7$
$x = 7z + 3$

Setting the two values for x equal to one another you see that

$13y + 3 = 7z + 3$
$13y = 7z$

Because y and z must be whole numbers, y must have 7 as a factor and z must have 13 as a factor. y and z can share an unlimited number of factors, but y must have a 7 in its prime box and z must have a 13 in its prime box.

The question now asks what is the remainder of $\frac{yz}{13}$. Since 13 is in the numerator, it can be canceled out of the fraction, leaving a 1 in the denominator and resulting in a whole number which has a remainder of 0.

50. **(B).** First solve for x in the equation to reveal the recursive formula for calculating a_n: $2 = (x/2) - a_{n-1}$

$4 = x - 2(a_{(n-1)})$
$x = 4 + 2(a_{(n-1)})$

Since "the term a_n is defined as the value of x that satisfies the equation," substitute a_n for x to get the real formula you are being asked to use:

$a_n = 4 + 2(a_{(n-1)})$

Since you have a_6 and want to calculate a previous term, a_4, it may be useful to rewrite the equation in a form that allows you to solve for the previous term. That is, solve for $(a_{(n-1)})$:

$a_{(n-1)} = (a_n - 4)/2$

Now let a_b be a_n and a_5 be $a_{(n-1)}$:

$a_5 = (156 - 4)/2 = 76$
$a_4 = (76 - 4)/2 = 36$
$a_3 = (36 - 4)/2 = 16$
And $a_2 = (16 - 4)/2 = 6$

51. **(C).** Compute the expressions for each of the terms:

$x!4 = x^4 \times 4^{-x}$ and $4!x = 4^x \times x^{-4}$

Dividing the first by the second yields

$$\frac{x^4 4^{-x}}{4^x x^{-4}} = \frac{x^4}{x^{-4}} \times \frac{4^{-x}}{4^x} = x^8 4^{-2x}$$

There are a number of ways we could write $x^8 4^{-2x}$:

$$x^8 4^{-2x} = \frac{x^8}{4^{2x}} = \frac{x^8}{16^x}$$

The two quantities are equal.

52. **(A).** First, calculate the sum of the even integers between 1 and 100 (2, 4, 6… 98, 100). You can think of this list as 50 even integers to be summed or, more usefully, you can think of it as 25 integer pairs each of which sum to 102 (2 + 100, 4 + 98, …, 50 + 52). The sum of these 25 pairs is simply 25 × 102. This is an alternate approach to the usual formula that tells you to compute the average value times the number of terms. Next, calculate the sum of the odd integers between 100 and 150 (101, 103, 105, …, 147, 149). Like before, you can turn this list into pairs, though you need to be careful because there are an odd number of integers in this list and the middle number (125) will not get a pair: (101 + 149, 103 + 147, …, 123 + 127). There are 12 pairs that each add to 250 and one leftover integer, 125. The sum is 12 × 250 + 125 or 12.5 × 250. The ratio be asked for in the question is (25 × 102) to (12.5 × 250). You can divide both sides by 25 to yield the ratio 102 to 125.

53. **(C).** The problem gives two ways to calculate the fourth term: (1) the definition of the sequence tells you that $a_4 = a_3^2 - a_2^2$ and (2) you are told that $a_4 = a_1(a_2 + a_3) = 2(a_2 + a_3)$. Setting these two equal gives $a_3^2 - a_2^2 = 2(a_2 + a_3)$. Factor the left side: $(a_3 + a_2)(a_3 - a_2) = 2(a_2 + a_3)$. Since $a_n > 0$ for all possible n's, you know that $(a_3 + a_2)$ does not equal 0 and you can divide both sides by it: $a_3 - a_2 = 2$ and $a_3 = a_2 + 2$. Using the definition of a_3, you know $a_3 = a_2^2 - a_1^2 = a_2^2 - 4$. Substituting for a_3 yields: $a_2 + 2 = a_2^2 - 4$ and $a_2^2 - a_2 - 6 = 0$. Factor and solve: $(a_2 - 3)(a_2 + 2) = 0$; $a_2 = 3$ or -2. a_n must be positive, so $a_2 = 3$ and $a_3 = a_2 + 2 = 3 + 2 = 5$.

54. **(D).** Since a_1 and a_3 are integers, a_2 must also be an integer: $a_3 = (a_1 + a_2)/2$ or INT = (INT + a_2)/2 so 2(INT) = INT + a_2 and a_2 = 2(INT) - INT which is itself an integer. a_4 will be the average of two integers. If $a_2 + a_3$ is even, a_4 will be an integer. If $a_2 + a_3$ is odd, a_4 will be a decimal ending in 0.5. If a_4 is an integer, a_5 can be an integer or can be a decimal ending in 0.5. If a_4 is a decimal ending in 0.5, a_5 must be a decimal ending in 0.25 or 0.75. a_5 cannot be a decimal ending in 0.375 such as 75/8 = 9.375. Note that a_5 can be negative: Even if a_1 and a_3 are positive, that does not rule out the possibility that a_2 (and subsequent terms) could be negative.

55. **(D).** Use the definition of ? to rewrite the equation: $\left|\frac{x+1}{x}\right| - \frac{2+1}{2} = \frac{1}{2}\left(\left|\frac{x+1}{x}\right| - \frac{-1+1}{-1}\right)$. Simplifying yields: $\left|\frac{x+1}{x}\right| - \frac{3}{2} = \frac{1}{2}\left|\frac{x+1}{x}\right|$. Let $z = \left|\frac{x+1}{x}\right|$. Substitute z into the equation: $z - 3/2 = (1/2)z$ or $z = 3$. To solve $\left|\frac{x+1}{x}\right| = 3$, take two cases

(1) $\frac{x+1}{x} > 0$. $\frac{x+1}{x} = 3$ or $x = 0.5$.

(2) $\frac{x+1}{x} < 0$. $\frac{x+1}{x} = -3$ or $x = -0.25$. The sum of the solutions is $0.5 + (-0.25) = 0.25$.

56. **(B).** Expressed algebraically, $\sqrt{10-3X} > X$. Because both sides of this inequality are non-negative, you can square both sides to result in the following:

$$10 - 3X > X^2$$
$$0 > X^2 + 3X - 10$$
$$0 > (X + 5)(X - 2)$$

Now, because the product of $(X + 5)$ and $(X - 2)$ is negative, you can deduce that the larger of the two expressions, $(X + 5)$, must be positive and the smaller expression, $(X - 2)$, must be negative. Therefore, $X > -5$ and $X < 2$. Combining these yields $-5 < X < 2$.

However, because the question indicates that X is non-negative, X must be 0 or greater. Therefore, $0 \leq X < 2$. The absolute value sign in Quantity A doesn't change anything — X is still greater than or equal to zero and less than 2, and Quantity B is larger.

Alternatively, plug the value from Quantity B into $\sqrt{10-3X} > X$:

$$\sqrt{10-3(2)} > 2$$
$$\sqrt{4} > 2$$
$$2 > 2$$

This is FALSE, so X cannot be 2.

Now, plug in a smaller or larger value to determine whether X needs to be greater than or less than 2. If $x = 1$:

$$\sqrt{10-3(1)} > 1$$
$$\sqrt{7} > 1$$

$\sqrt{7}$ is between 2 and 3, so this is true.

Trying values will show that only values greater than or equal to zero and less than 2 make the statement true, so Quantity A must be smaller than 2.

57. **(A).** The area of an equilateral triangle is $\frac{b^2\sqrt{3}}{4}$ where b is the length of one side. Since this area is between 25

$\sqrt{3}$ and $36\sqrt{3}$, you can substitute to get $25\sqrt{3} < \frac{b^2\sqrt{3}}{4} < 36\sqrt{3}$. Dividing all sides by $\sqrt{3}$ yields $25 < b^2/4 < 36$. Multiplying all sides by 4 yields $100 < b^2 < 144$, and taking the square root of all sides, one gets $10 < b < 12$. Since every possibility for b is greater than 9, Quantity A is larger.

58. **(E).** When dealing with absolute values, you must typically consider two outcomes. First determine the outcome if the expression within the absolute value sign is positive. So, if $8 - 2x > 0$, then $|8 - 2x| = 8 - 2x$, and therefore $8 - 2x < 3y - 9$ or $2x > 17 - 3y$.

You also must determine the outcome if the expression within the absolute value sign is negative. So, if $8 - 2x < 0$, then $|8 - 2x| = 2x - 8$, and therefore $2x - 8 < 3y - 9$ or $2x < 3y - 1$. Combining these two inequalities, one arrives at $3y - 1 > 2x > 17 - 3y$.

Now a quick sanity check to make sure the inequality makes sense: $3y - 9$ must be greater than 0 or the absolute value could not be less than $3y - 9$. So $y > 3$. This means $17 - 3y < 8$, and $3y - 1 > 8$, so there is definitely room for $2x$ to fit between those values. If the potential values of $17 - 3y$ and $3y - 1$ had overlapped, this would be an indication either that a mistake had been made or that the problem required further investigation to refine the result. As it is, (E) will work as an answer for this problem.

59. **(A).** This is an overlapping set problem. Matrix 1 shows an initial setup for a double-set matrix. The columns are headed "Skilled in BJJ" and "Not Skilled in BJJ." The rows are headed "Skilled in Muy Thai" and "Not Skilled in Muy Thai." There is also a total row and a total column.

When dealing with overlapping sets, consider whether the question is giving information regarding the population as a whole or regarding a subset of the population. While the first statement ("30% of all fighters") refers to the whole population, the second statement ("20% of the fighters who are not skilled in Brazilian Jiu Jitsu") refers to a subset of the population, in this case the 40% who are not skilled in Brazilian Jiu Jitsu. Thus, 8% are skilled in Muy Thai but not in Brazilian Jiu Jitsu, as seen in Matrix 1.

Matrix 1

	Skilled in BJJ	**Not Skilled in BJJ**	**Total**
Skilled in Muy Thai	> 30	8	
Not Skilled in Muy Thai			
Total	60	40	100

Matrix 2 shows how to fill out additional cells. Notably, there are some ranges of values that are possible for the cells in the first column. These ranges are limited by 0 on the low end and 60 on the high end.

Matrix 2

	Skilled in BJJ	**Not Skilled in BJJ**	**Total**
Skilled in Muy Thai	> 30 but ≤ 60	8	
Not Skilled in Muy Thai	≤ 0 but < 30	32	
Total	60	40	100

Matrix 3 shows the ranges of values that are possible for the percentage of people skilled in Muy Thai and the percentage of people not skilled in Muy Thai. Particularly, the percent of fighters who are skilled in Muy Thai is greater than 38 but less than or equal to 68. Thus, Quantity A is larger.

Matrix 3		**Skilled in BJJ**	**Not Skilled in BJJ**	**Total**
	Skilled in Muy Thai	> 30 but ≤ 60	8	> 38 but ≤ 68
	Not Skilled in Muy Thai	≤ 0 but < 30	32	≤ 32 but < 62
	Total	60	40	100

60. **(B).** This is a good example of a problem where one can use the idea of extreme values. You can express this situation with the equation $r = \frac{kb}{n^2}$, where k is a constant. Quadrupling b and more than tripling n yields the following equation: $r_1 = \frac{k \times 4b}{(\text{"greater than } 3n\text{"})^2}$, where r_1 represents the new rate of data transfer.
Squaring a value that is greater than 3 produces a value that is greater than 9, allowing one to rewrite the equation as $r_1 = \frac{k \times 4b}{\text{"greater than } 9n^2\text{"}}$. Rearranging this equation yields $r_1 = \frac{4}{\text{"greater than 9"}} \times \frac{k \times b}{n^2}$ divided by a value greater than 9 is a value that is less than $\frac{4}{9}$. Thus, Quantity B is greater.

Chapter 33

of

Math Practice Sections

In This Chapter…

Math Practice Section 1: Easy Difficult

Math Practice Section: Easy
20 Questions
35 Minutes

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{\quad\quad}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\dfrac{\boxed{\quad\quad}}{\boxed{\quad\quad}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

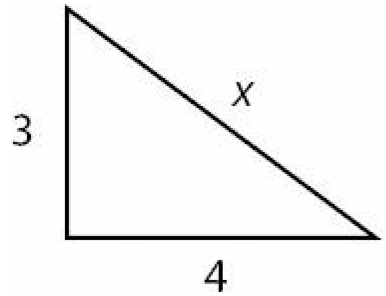
x, y, and z are consecutive integers such that $x < y < z$

Quantity A	Quantity B
y	$\dfrac{x+z}{2}$

2.

Quantity A	Quantity B
$(-3)^4$	$(-3)^{-3}$

3.



Quantity A	Quantity B
x	5

4.

Quantity A	Quantity B
$y^7 \times y^8 \times y^{-6}$	$3y^9$

5.

$xy > 0$ and $yz < 0$

Quantity A	Quantity B
xz	0

6. In 2011, it cost Tammy \$1.30 to manufacture each copy of her magazine, which she sold for \$2.30. In 2012, it cost Tammy \$1.50 to manufacture each copy of the same magazine, which she sold for \$3.00.

Quantity A	Quantity B
The percent by which Tammy's profit per copy of the magazine changed from 2011 to 2012	$33\frac{1}{3}\%$

7. List X: 4, 7, 9, 11, 24, 32

List Y (not shown) consists of 6 unique numbers, each computed from the corresponding term in List X by dividing the number in List X by 2, then adding 5 to the result.

Quantity A	Quantity B
The range of List Y	6 less than the greatest number in List Y

8. Which of the following represents the length of the diagonal d of a square with area a?

(A) $d = a^2$

(B) $d = \sqrt{2a}$

(C) $d = 2\sqrt{a}$

(D) $d = a\sqrt{2}$

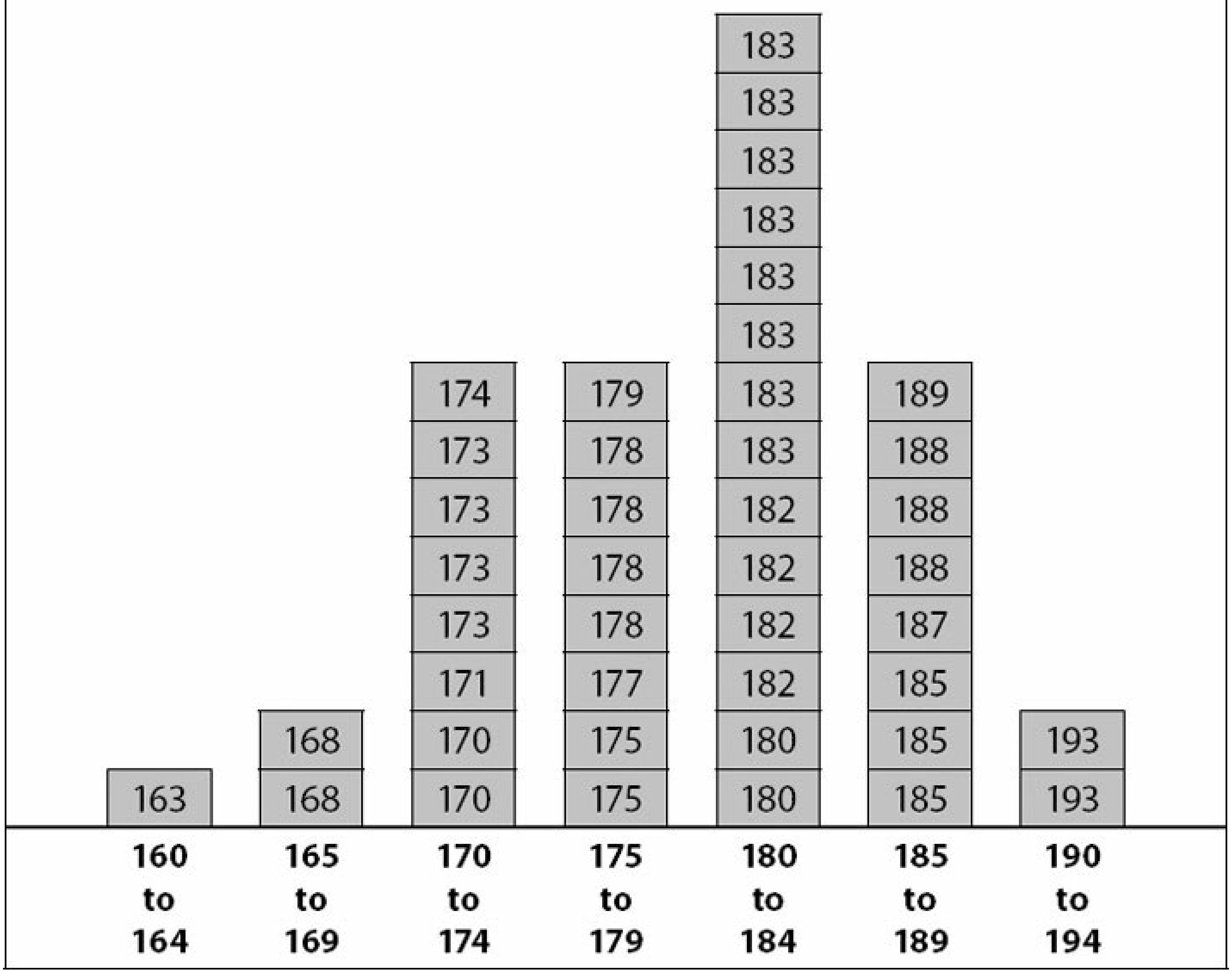
(E) $d = a\sqrt{3}$

9. In an apartment complex, 60 percent of the apartments contain at least one television, and 20 percent of these apartments are equipped with cable. If every apartment that is equipped with cable contains at least one television, what percent of the apartments in the complex are not equipped with cable?

(A) 8%
(B) 12%
(C) 16%
(D) 88%
(E) 92%

Questions 10–12 are based on the following chart.

Heights of 43 Presidents of the United States of America (in cm)



10. What is the range of heights of the 43 U.S. Presidents in the chart?

(A) 30 cm
(B) 34 cm
(C) 35 cm
(D) 163 cm
(E) 178 cm

11. What is the median height of the 43 U.S. Presidents in the chart, in centimeters?

(A) 175
(B) 177
(C) 178
(D) 180
(E) 182

12. Approximately what percent of U.S. Presidents have been 185 cm or taller?

(A) 10%
(B) 23%
(C) 29%
(D) 43%
(E) 50%

13. If $m + 5 < \frac{3}{2}$, which of the following scould be the value of *m*?

(A) $-\frac{15}{4}$

(B) $-\frac{7}{2}$

(C) -2

(D) $\frac{7}{2}$

(E) 2

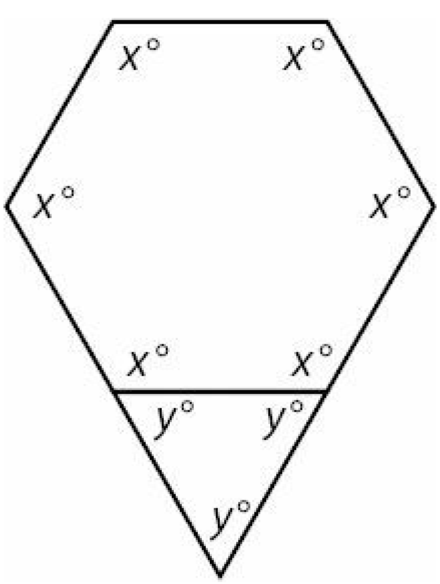
14. List M consists of the numbers 10, 20, 30, 40, 50.

Which of the following lists of numbers have an average (arithmetic mean) that is equal to the average of the numbers in List M?

Indicate <u>all</u> such lists.

☐ 0, 30, 60
☐ 10, 20, 30, 35, 50
☐ 10, 22, 30, 38, 50
☐ 0, 0, 0, 0, 150

15.



What is the value of xy?

16.

Buying Habits Of Customers Buying Toothpaste X At Chan's Grocery Store

Discount Type	Manufacturers' Coupon	Store Coupon	No Coupon
Percent of Customers	54%	43%	x%

The table above summarizes all possible discount types for customers buying Toothpaste X at a certain grocery store. No one used both types of coupon. If a person is selected randomly from among the customers buying Toothpaste X at Chan's Grocery Store, what is the probability that this customer did not use a coupon?

(A) 0.003
(B) 0.03
(C) 0.3
(D) 0.33
(E) 3.3

17. Company A can pave 500 feet of sidewalk in 6 hours, and Company B can pave 1,000 feet of sidewalk in 8 hours. At these rates, how many more *yards* of sidewalk can Company B pave in 9 hours than Company A can pave in 9 hours? (3 feet = 1 yard)

(A) 125
(B) 166
(C) 333
(D) 375
(E) 500

18. If the three sides of an equilateral triangle are equal to $4x$, $6y$, and 24, respectively, what is the ratio of x to y?

Give your answer as a fraction.

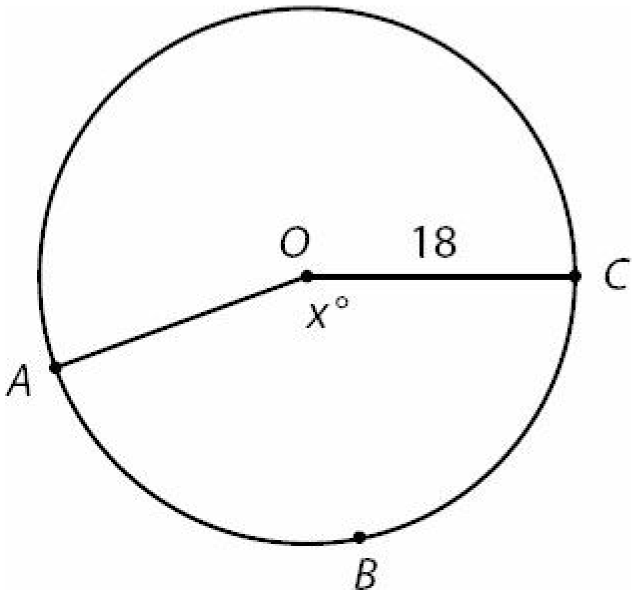
19. If the ratio of undergraduate students to graduate students is 7 to 4 and the ratio of graduate students to professors is 2 to 1, which could be the total number of undergraduate students, graduate students, and professors?

Indicate <u>all</u> such numbers.

☐ 520

- ☐ 640
- ☐ 2,600
- ☐ 10,000

20.



What is the perimeter of sector *ABCO* if $x = 160$?

(A) $18 + 8\pi$
(B) $18 + 16\pi$
(C) $36 + 8\pi$
(D) $36 + 16\pi$
(E) $36 + 24\pi$

Answers to Math Practice Section 1

1. **(C).** The average of three consecutive integers is always equal to the middle value, and is always equal to the average of the smallest and largest terms. Since Quantity B represents the average of the smallest and largest terms, it is equal to the middle term *y*.

Alternatively, pick numbers. If *x*, *y*, and *z* are 1, 2, and 3, the Quantity A = 2 and Quantity $B = \frac{1+3}{2} = 2$ quantities are equal. Any other example of three consecutive numbers will also yield equal quantities.

2. **(A).** In Quantity A, $(-3)^4 = (-3)(-3)(-3)(-3) = 81$. In Quantity B, $(-3)^{-3} = \frac{1}{(-3)^3} = \frac{1}{-27}$. Quantity A is greater. Note that you can stop calculating as soon as you realize that one quantity is positive and one is negative. The negative base in both quantities suggests that you should check whether the exponents are odd or even. Even exponents "hide the sign" of the base, so a negative base to a even exponent is positive. On the other hand, a negative base to an odd exponent remains negative (even if the exponent is a *negative* odd).

3. **(D).** If this were a right triangle, the Pythagorean theorem would indicate that

$$3^2 + 4^2 = x^2$$
$$9 + 16 = x^2$$
$$25 = x^2$$
$$5 = x$$

However, the triangle is not known to be right (the Pythagorean theorem only applies to right triangles), as none of the angles are labeled. The Third Side Rule, which applies to all triangles regardless of angle measures, states that the third side of any triangle must be greater than the difference between the other two sides and less than the sum of the other two sides. So, *x* must be greater than 4 - 3 = 1 and less than 4 + 3 = 7. *x* could be less than, greater than, or equal to 5, so it cannot be determined which quantity is greater.

4. **(D).** Since the terms in Quantity A have the same base and are multiplied together, simplify by adding the exponents:

$$y^7 \times y^8 \times y^{-6} = y^9$$

While y^9 may *seem* smaller than $3y^9$, this is only true if *y* is positive. If $y = 0$, the two quantities are equal. If *y* is negative, so is y^9, and $3y^9$ is more negative than y^9. Thus, it cannot be determined which quantity is greater.

5. **(B).** Since $xy > 0$, *x* and *y* have the same sign. Since $yz < 0$, *y* and *z* have opposite signs. Therefore, *x* and *z* have opposite signs. If *x* and *z* have opposite signs, their product is negative, which is less than 0. Quantity B is greater.

6. **(A).** In order to calculate the percent change in profit from 2011 to 2012, first calculate the profits in each year

based on the formula:

Profit = Revenues - Costs

Therefore,

Profit per each copy of the magazine in 2011 = \$2.30 - \$1.30 = \$1.00
Profit per each copy of the magazine in 2012 = \$3.00 - \$1.50 = \$1.50

To find the percent increase, use the percent change formula:

$$\text{Percent Change} = \left(\frac{\textit{Difference}}{\textit{Original}} \times 100\right)\%$$

$$\text{Percent Change} = \left(\frac{0.50}{1.00} \times 100\right)\% = 50\%$$

Be careful not to put the 2012 profit in the denominator. Mistakenly doing so would lead you to pick (C) erroneously. The “Original” profit is that for 2011.

Quantity A is greater.

7. **(B).** Since the terms in List Y are “each computed from the corresponding term in List X by dividing the number in List X by 2, then adding 5 to the result,” List Y consists of 7, 8.5, 9.5, 10.5, 17, 21.

Quantity A: The range is 21 - 7 = 14.
Quantity B: 6 less than the greatest number in Set Y = 21 - 6 = 15.

Quantity B is greater.

8. **(B).** A square with area a has sides of $\sqrt{a}$. Use the Pythagorean theorem with $\sqrt{a}$ for each leg and d for the hypotenuse:

$$(\sqrt{a})^2 + (\sqrt{a})^2 = d^2$$
$$a + a = d^2$$
$$2a = d^2$$
$$\sqrt{2a} = d$$

This is a match with choice (B). Alternatively, plug in numbers. If a square has side length 4, the area a equals 16 and the diagonal would be:

$$4^2 + 4^2 = d^2$$
$$32 = d^2$$
$$\sqrt{32} = d$$

Plug a = 16 into each choice to see which yields $d = \sqrt{32}$. Only choice (B) works.

9. **(D).** The easiest way to solve this problem is to choose a smart number for the total number of apartments in the apartment complex. As this is a percents problem, choose a total of 100 apartments. Since 60% of these apartments have a television, 60 apartments contain a television (or more than one television—it doesn't matter how many—only television at all vs. no television matters) and 40 apartments do not contain a television.

Because 20% of the apartments that contain a television are equipped with cable, 20% of 60 = 12 apartments have both television and cable. By extension, 60 - 12 = 48 apartments have television, but are not equipped with cable.

"Every apartment that is equipped with cable contains at least one television" means that none of the 40 apartments without a television are equipped with cable. Thus, 40 apartments have neither a television nor cable.

In summary:

No TV, no cable = 40
TV, no cable = 48
No TV, cable = 0
TV and cable = 12

Only 12 apartments are equipped with cable, meaning 100 - 12 = 88 are not. Alternatively, 48 + 40 = 88 apartments are not equipped with cable.

Since 88 out of 100 apartments are not equipped with cable, the answer is 88%.

Alternatively, you can solve this problem by assigning the variable x to the total number of apartments in the apartment complex. Following the steps from above, $0.6x$ apartments contain a television and $(0.2)(0.6x) = 0.12x$ apartments are equipped with cable. From here, $x - 0.12x = 0.88x$ apartments, or 88% of the apartments in the complex, do not have cable.

10. **(A).** The shortest US President was 163 centimeters tall, and the tallest was 193 centimeters tall. The range is the difference between the highest and lowest value, and 193 cm - 163 cm = 30 cm.

11. **(E).** The median is the middle value if all the data points are arranged from least to greatest. With 43 data points, the median is the 22nd data point, because there are 21 data points that are less than or equal, and 21 data points that are greater than or equal, this median. Counting up from the least value (or down from the greatest value), the 22nd data point is 182 cm.

12. **(B).** From the chart, 10 U.S. Presidents have been 185 cm or taller, out of a total of 43. As a percent, this is $\left(\frac{10}{43} \times 100\right)\%$, or approximately 23%.

13. **(A).** Solve the inequality:

$$m+5<\frac{3}{2}$$
$$m<\frac{3}{2}-5$$
$$m<\frac{3}{2}-\frac{10}{2}$$
$$m<-\frac{7}{2}$$

(A) is the only answer choice that is less than $-\frac{7}{2}$. If needed, plug each answer choice into the calculator and compare decimal values to -3.5.

14. **I, III, and IV only.** Certainly, you could average the list 10, 20, 30, 40, 50 (the average is 30) and then average the lists in all the answer choices to see which also average to 30. However, you cannot afford to waste any time on the GRE.
Instead, note that the average of an evenly-spaced set is equal to the median. Thus, the average of 10, 20, 30, 40, 50 is the median, or middle term, 30. In Statement I, the list 0, 30, 60 is also evenly-spaced, so the average is 30.

In Statement II, the list 10, 20, 30, 35, 50 is the same as the original list (10, 20, 30, 40, 50) except for one number — the 40 has been changed to 35. Thus, the averages cannot be the same.

In Statement III, the list 10, 22, 30, 28, 50 is the same as the original list (10, 20, 30, 40, 50), but with 2 taken away from the fourth number and added to the second number. Since the sum didn't change, the average doesn't either.

In Statement IV, the average is simply the sum divided by the number of items, or 150/5 = 30.

15. **7,200.** Since every angle in the hexagon is labeled $x°$, the hexagon is equiangular. To find the sum of the degree measures in a polygon, use the formula $(n - 2)(180)$, where n is the number of sides. Since $n = 6$, $(6 - 2)(180) = 720$, and the sum of the degrees in the hexagon is 720. Thus, $6x = 720$ and $x = 120$.

Since the triangle is equiangular, $3y = 180$ and $y = 60$.

Thus, the value of $xy = 120 \times 60 = 7{,}200$.

16. **(B).** Add 54% + 43% = 97% to get the percent of customers who used a coupon. Only 100% - 97% = 3% of customers did not use a coupon. Thus, for a person selected randomly from among the customers buying Toothpaste X at Chan's Grocery Store, there is a 3%, or 0.03, probability that he or she did not use a coupon.

17. **(A).** Company A can pave 500 feet of sidewalk in 6 hours, and thus $\frac{500}{6}$ feet per hour. In 9 hours, Company A can pave $\frac{500}{6} \times 9 = 750$ feet of sidewalk.

Company B can pave 1,000 feet of sidewalk in 8 hours, and thus $\frac{1000}{8} = 125$ feet per hour. In 9 hours, Company B can pave $125 \times 9 = 1{,}125$ feet of sidewalk.

Thus, in 9 hours, Company B can pave 1,125 - 750 = 375 feet of sidewalk more than Company A. Since 3 feet = 1 yard, divide by 3 to get the answer in the correct units: 375 feet divided by 3 feet per yard = 125 yards.

18. $\frac{3}{2}$ **(or any equivalent fraction).** Since the sides of an equilateral triangle are all equal, $4x = 6y = 24$. With a three part equation, you can equate any two parts you wish.

For instance:

$4x = 24$
$x = 6$
$6y = 24$
$y = 4$

Thus, the ratio of x to y is 6 to 4, which reduces to 3 to 2. On the GRE, you do not need to reduce the answers to fraction numeric entry questions.

19. **I and III only.** If the ratio of undergraduate students to graduate students is 7 to 4 and the ratio of graduate students to professors is 2 to 1:

Undergraduate	Graduate	Professors
7	4	
	2	1

Equate the ratios by making the two numbers under “Graduate” equal. To do this, double the second ratio. (If you change one number in the ratio 2 : 1, you must perform the same operation to the other number in that ratio.)

Undergraduate	Graduate	Professors
7	4	
	4	2

Now, collapse the ratios onto one line:

Undergraduate	Graduate	Professors
7	4	2

The ratio is 7 to 4 to 2. Since $7 + 4 + 2 = 13$ and numbers of people must be integers, the total number of people must be a multiple of 13. Only 520 and 2,600 qualify.

20. **(D).** If $x = 160$, then the sector is $\frac{160}{360} = \frac{4}{9}$ of the circle. Thus, arc *ABC* is $\frac{4}{9}$ of the circumference. Since the circumference $= 2\pi r = 2\pi(18) = 36\pi$, take $\frac{4}{9}(36\pi) = 16\pi$

Thus, the perimeter of the sector is equal to two radii plus 16π, or $36 + 16\pi$.

Math Practice Section 2: Medium Difficult

Math Practice Section: Medium
20 Questions
35 Minutes

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

Set M consists of all the integers between -2 and 12, inclusive
Set N consists of all the integers between 9 and 15, inclusive

Quantity A	**Quantity B**
The smallest integer in Set M that is also in Set N	9

2.

17% of p is equal to 18% of q, where p and q are positive

Quantity A	**Quantity B**

p q

3.

Circle A has area a

Semicircle B has area $\frac{a}{2}$

Quantity A	**Quantity B**
The circumference of Circle A	Twice the perimeter of semicircle B

4.

Quantity A	**Quantity B**
The standard deviation of the set 1, 5, 7, 19	The standard deviation of the set 0, 5, 7, 20

5.

An isosceles triangle has a perimeter of 28. The shortest side has length 8.

Quantity A	**Quantity B**
The length of the longest side of the triangle	12

6.

$$(3 - z)(z + 4) = 0$$

Quantity A	**Quantity B**
z	5

7.

$$a > b > c > d$$
$$ab > 0$$
$$ad < 0$$

Quantity A	**Quantity B**
ac	cd

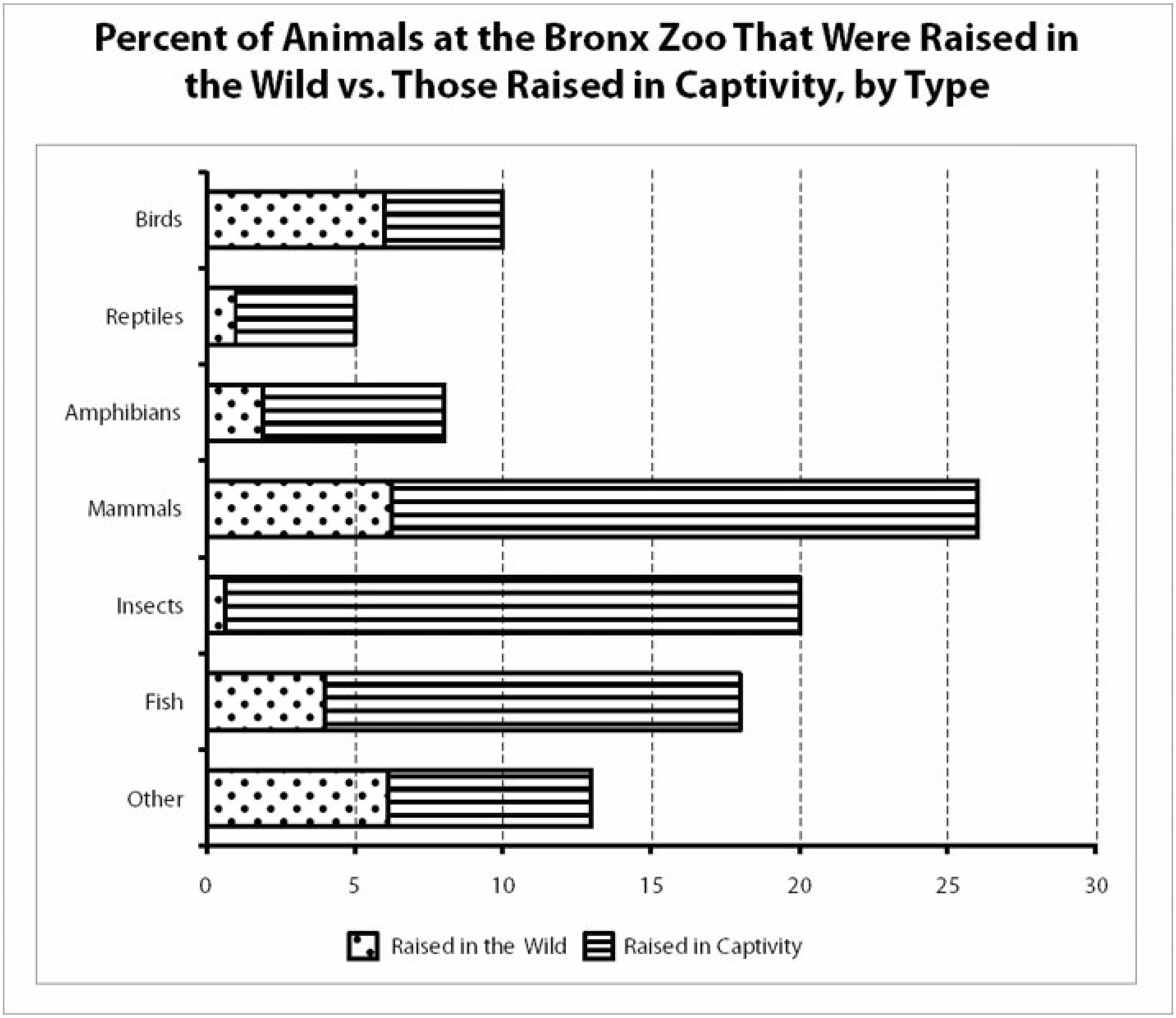
8. If $12b = 2g$ and $4g - 3b = 63$, what is the value of $\frac{g}{b}$?
Give your answer as a fraction.

9. $81^3 + 27^4$ is equivalent to which of the following expressions?

Indicate <u>all</u> such expressions.

- ☐ $3^7(2)$
- ☐ $3^{12}(2)$
- ☐ $9^6(2)$
- ☐ 9^{12}
- ☐ 3^{24}

Questions 10–12 are based on the following chart.



10. Approximately what percent of all the zoo's animals are either mammals that were raised in the wild or amphibians raised in captivity?

(A) 8
(B) 12
(C) 18
(D) 34
(E) 100

11. If the Bronx Zoo donated all of its insects and fish to other zoos, approximately what percent of the animals in the zoo would be birds raised in the wild?

(A) 5
(B) 9
(C) 24
(D) 32
(E) 60

12. If the zoo currently has 80 total birds, what is the smallest number of birds that could be added such that at least 20% of the animals at the zoo would be birds?

(A) 10
(B) 80
(C) 100
(D) 125
(E) 200

13. Trail mix is made by combining 3 pounds of nuts that cost x dollars per pound with 1 pound of chocolate that costs y dollars per pound and 2 pounds of dried fruit that costs z dollars per pound. What is the cost in dollars per pound for the trail mix?

(A) $\frac{3x+y+2z}{xyz}$

(B) $3x+y+2z$

(C) $\frac{3x+y+2z}{6}$

(D) $6(3x+y+2z)$

(E) $\frac{x}{3}+y+\frac{2}{z}$

14. If $z = 3^4$, then $\left(3^z\right)^z =$

(A) 3^{16}
(B) 3^{81}
(C) 3^{324}
(D) 3^{405}
(E) $3^{6,561}$

15. Maurice entered a number into his calculator and erroneously divided the number by 0.03 instead of 0.0003, resulting in an incorrect result. Which of the following is a single operation that Maurice could perform on his calculator to correct the error?

Indicate all such operations.

☐ Multiply the incorrect product by 100
☐ Divide the incorrect product by 100
☐ Multiply the incorrect product by 0.01
☐ Divide the incorrect product by 0.01

16. A company's annual expenses are composed entirely of a fixed amount in costs, plus a variable amount that is directly proportional to the number of clients served. In 2009, the company served 450 clients and its total expense was \$830,000. In 2010, the company served 510 clients and its total expense was \$896,000. What is the company's fixed annual expense, in dollars?

(A) 1,844
(B) 1,757
(C) 335,000

(D) 485,000
(E) 830,000

17. Which of the following lines is perpendicular to $4x + 5y = 9$ on the xy plane?

(A) $y = \frac{5}{4}x + 2$

(B) $y = -\frac{5}{4}x + 9$

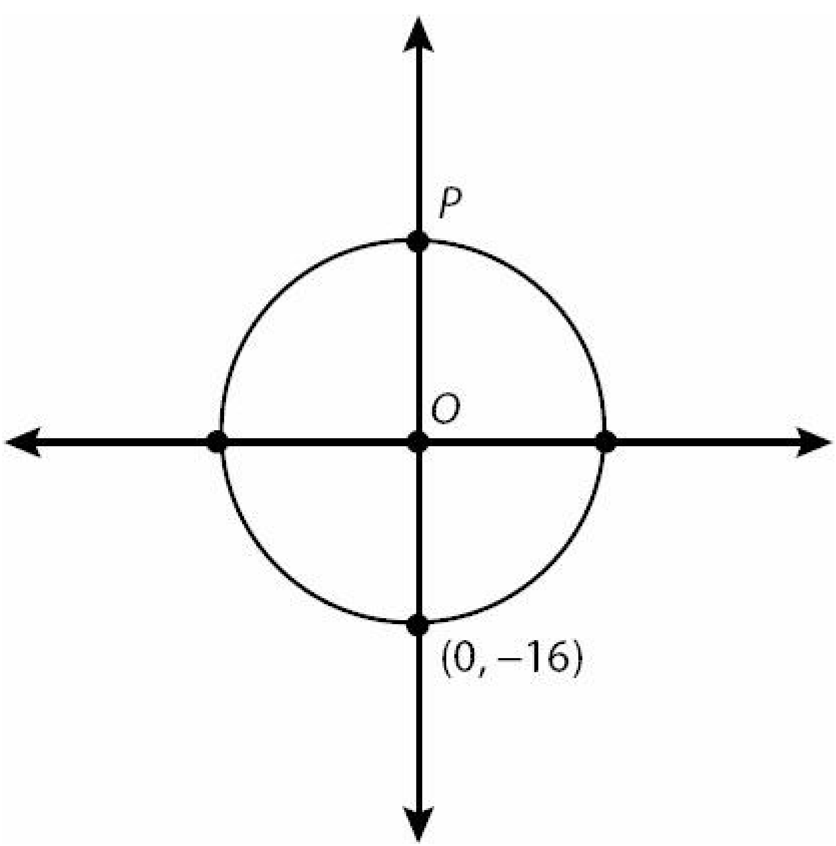
(C) $y = -4x + \frac{9}{5}$

(D) $y = \frac{4}{5}x - \frac{4}{5}$

(E) $y = -\frac{4}{5}x$

18. The tens digit is missing from the three-digit number 8 __ 9. If the tens digit is to be randomly selected from the ten different digits from 0 to 9, what is the probability that the resulting three-digit number will be a multiple of 9?

(A) 0.1
(B) 0.2
(C) 0.4
(D) 0.9
(E) 1



19. In the figure above, the circle is centered at (0, 0). What is the distance between point P and the point (-10, -8) (not shown on the graph)?

(A) 18
(B) 20
(C) 22
(D) 24
(E) 26

20. If $f(-0.5) = 0$, which of the following could be $f(x)$?

(A) $2x + 2$
(B) $4x - 2$
(C) $4x^2 - 1$
(D) $x^2 - 1$
(E) $(-x)^2 - 2.5$

Answers to Math Practice Section 2

1. **(C).** Set M consists of -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12. Quantity A is the *least* of these integers that is also in Set N. The smallest integer in Set N is 9, which is also in Set M, so Quantity A is 9. The two quantities are equal.

2. **(A).** As algebra, "17% of p is equal to 18% of q" is:

$$\frac{17}{100}p = \frac{18}{100}q$$

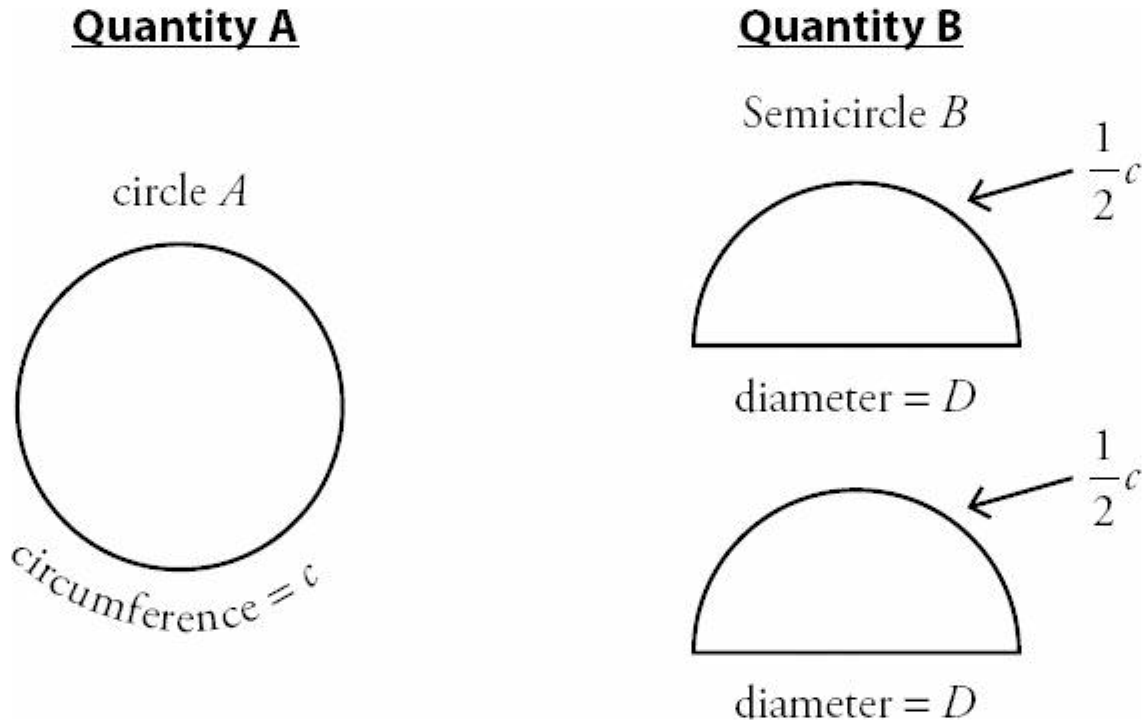
Solve for p. The easiest way to do this is to first multiply both sides of the equation by 100, then divide both sides by 17:

$$p = \frac{18}{17}q$$

Since $\frac{18}{17}$ is greater than 1 and both variables are positive, p is greater than q.

(Note that it was necessary to know that both variables were positive! If they were negative, $p = \frac{18}{17}q$ would imply that p is more negative than q, so q would have been greater than p. Without information about sign, the answer would have been (D).)

3. **(B).** If the given semicircle has half the area of the circle, then the Semicircle B is simply equal to half of Circle A. However, that does *not* mean that the semicircle has half the perimeter. Observe:



The semicircle is drawn twice, as Quantity B refers to "twice the perimeter of Semicircle *B*." Note that Quantity A is equal to the circumference *c*, while Quantity B is equal to this same circumference, plus twice the length of the diameter. Quantity B is greater.

4. **(B).** Standard deviation measures the variance from the mean; the more spread out a set is, the higher the deviation. The set in Quantity B is the same as the one in Quantity A, but with the smallest number *even smaller* and the largest number *even larger*, so the set in Quantity B is more spread out, and has a greater standard deviation.

5. **(D).** An isosceles triangle has two sides that are equal and a third side that is a different length. The isosceles triangle in this question has a perimeter of 28 and shortest side of length 8. Now, suppose that the shortest side is the one that is repeated, such that the triangle has two sides of length 8 and one other side of length x. This would mean:

$8 + 8 + x = \text{Perimeter}$
$16 + x = 28$
$x = 12$

So, this triangle would have lengths 8, 8, and 12 as the three legs. Test this triangle via the Third Side Rule: the length of any side of a triangle must be greater than the difference between the other two sides and less than the sum of the other two sides. The third side (x) must be greater than $8 - 8 = 0$ and less than $8 + 8 = 16$. Since 12 is between 0 and 16, this is a legal triangle.

On the other hand, consider the possibility that the other side, x, is repeated and the length 8 is used only once. In this case:

$x + x + 8 = 28$
$2x = 20$
$x = 10$

The sides of this triangle are 10, 10, and 8. Test this triangle via the Third Side Rule: the third side (8) must be greater than $10 - 10 = 0$ and less than $10 + 10 = 20$. Since 8 is between 0 and 20, this is a legal triangle.

For one triangle, the quantities in Quantity A and Quantity B would be equal, but for the other, Quantity B would be greater than Quantity A. Therefore, the relationship cannot be determined from the given information.

6. **(B).** $(3 - z)(z + 4) = 0$, so either $(3 - z)$ or $(z + 4)$ must equal 0:

$3 - z = 0$
$z = 3$

OR

$z + 4 = 0$
$z = -4$

z is either 3 or -4. Either way, Quantity B is greater.

7. **(D).** If $ad < 0$, a and d have opposite signs. Because $a > d$, a must be positive and d must be negative. Similarly, if $ab > 0$, a and b have the same sign, so a and b are both positive. The remaining variable c can be positive, 0, or negative and still fall between b and d. If c is 0, the two quantities are equal. If c is positive, Quantity A is positive and Quantity B is negative. If c is negative, Quantity B is positive and Quantity A is negative. The relationship cannot be determined from the information given.

Alternatively, pick numbers. If $a = 4$, $b = 3$, $c = 2$, and $d = -1$, then all the criteria of the problem are fulfilled, and Quantity A is greater. But if $a = 4$, $b = 3$, $c = -5$, and $d = -10$, then all the criteria of the problem are still fulfilled, but Quantity B is greater.

8. $\frac{6}{1}$ **(or any equivalent fraction).** Solve one equation for a single variable, and substitute into the other equation:

Eq. (1): $12b = 2g$	Eq. (2): $4g - 3b = 63$
$12b = 2g$	
$6b = g$	Isolate g in Eq. (1). Divide by 2.
$4(6b) - 3b = 63$	Substitute $(6b)$ for g in Eq. (2).
$24b - 3b = 63$	Solve for b. Simplify.
$21b = 63$	Combine like terms.
$b = 3$	Divide by 21.
$12(3) = 2g$	Substitute (3) for b in Eq. (1). Solve for g.
$36 = 2g$	Simplify.
$g = 18$	Divide by 2.

$b = 3$ and $g = 18$, so $\frac{g}{b} = 6$.

9. **II and III only.** To simplify $81^3 + 27^4$, note that both bases are powers of 3. Rewrite the bases and combine.

$81^3 + 27^4 =$
$(34)^3 + (3^3)^4 =$
$3^{12} + 3^{12} =$
$3^{12}(1 + 1) =$
$3^{12}(2)$

Since $3^{12}(2)$ appears in the choices, this is one answer. However, this is an "indicate all" question, so you should check whether any other choices are equivalent. One other choice, $9^6(2)$, also qualifies, since $9^6(2) = (3^2)^6(2) = 3^{12}(2)$.

10. **(B).** 26% of the animals are mammals, and about a quarter *of those* were raised in the wild: $\frac{1}{4}$ of 26% = about 6.5%. 8% of all the animals are amphibians, and about three quarters *of those* were raised in captivity: $\frac{3}{4}$ of 8% = about 6 %. In total, these two categories account for about 12% of all the zoo's animals.

11. **(B).** To solve this question, imagine that there were originally 100 animals in the zoo. If the zoo gives away all the insects and fish, then there are 38 fewer animals (20 + 18) in the zoo, or 62. But there are still 10 birds, which now make up about 16% of the zoo's animals (use your calculator to find this if you don't feel comfortable estimating). Of those, a little more than half were raised in the wild. Among the choices, only 9% is a little more than half of 16%.

12. **(C).** If the zoo has 80 birds, which make up 10% of the total number of animals at the zoo, then there are 800 animals total. To correctly calculate how many birds must be added, realize that any birds added increases not only the subtotal of 80 birds but also the total of 800 animals. If adding new animals (rather than trading reptiles for birds, for example), you cannot simply double the number of birds to double the percent of the animals that are birds!

Thus, use the following inequality: 80

$$\frac{80+x}{800+x} \geq \frac{20}{100}$$

$100(80 + x) \geq 20(800 + x)$

$8{,}000 + 100x \geq 160{,}000\ 20x$
$80x \geq 8{,}000$
$x \geq 100$

At least 100 birds must be added such that at least 20% of the animals at the zoo would be birds (check: There would be 180 birds among 900 animals, or 20% of the total).

13. **(C).** This question is a tricky one, because even though it never uses the word *average* or the word *ratio*, it's more or less a combined ratio and averages question. The trail mix is nuts, chocolate, and dried fruit in a ratio of 3 : 1 : 2. For every 6 pounds of trail mix, there are 3 pounds of nuts, 1 pound of chocolate, and 2 pounds of dried fruit.

The cost of 6 pounds of trail mix is $3x + y + 2z$. However, to solve for the cost of one pound, divide by 6. You could also think of this as a kind of average:

Average = (Sum)/(# of terms) = $(3x + y + 2z)/6$, where each "term" is a pound.

This is choice (C). Alternatively, pick numbers. For example:

$x = 6$
$y = 5$
$z = 2$

In this example, 3 lbs. of nuts that cost $x = 6$ dollars per pound plus 1 lb. chocolate that costs $y = 5$ dollars per pound plus 2 lbs. dried fruit that costs $z = 2$ dollars per pound would cost:

$$3(6) + 1(5) + 2(2) = 27$$

Thus, 6 pounds of trail mix (3 lbs. nuts + 1 lb. chocolate + 2 lbs. dried fruit) would cost \$27. So, 1 pound would cost one-sixth of that: 27/6 or 9/2 dollars, which is \$4.50.

Now, plug $x = 6$, $y = 5$, and $z = 2$ into the choices to see which answer yields \$4.50. Only (C) works.

14. **(E).** Since $3^4 = 81$, $z = 81$. So, $(3^z)^z = (3^{81})^{81} = 3^{81\times81} = 3^{6,561}$.

15. **I and IV only.** Since 0.03 is 100 times greater than 0.0003, when Maurice accidentally divided by 0.03 instead of 0.0003, he divided by a number 100 times too big. Thus, multiplying by 100 will correct the error. Thus, Statement I is correct.

However, dividing by any quantity is the same as multiplying by its reciprocal. So, multiplying by 100 is the same as dividing by 0.01. Thus, Statement IV is also correct.

Alternatively, pick a number. Divide by both 0.03 and 0.0003, and then check each answer to see which correct the error. For instance, suppose the original number were 12.

12 divided by 0.03 = 400 ⟵ INCORRECT RESULT
12 divided by 0.0003 = 40,000 ⟵ CORRECT RESULT

Now, perform the operation in each answer choice on the incorrect product, 400, to see which operations turn that product into 40,000. Operations I and IV work.

16. **(C).** Begin by constructing a function describing the situation in the problem. Using E for expenses, x for the number of clients, c for the expense per client, and f for fixed costs:

$$E(x) = xc + f$$

In words, expense as a function of the number of clients equals the number of clients multiplied by the variable cost per client, plus the fixed cost.

In 2009, the company served 450 clients and its total expense was \$830,000. Thus:

$830{,}000 = 450c + f$

In 2010, the company served 510 clients and its total expense was $896,000. Thus:

$896{,}000 = 510c + f$

Since it is easier to isolate *f* than *c* in each equation, get *f* by itself for each equation and then set the opposite sides equal:

$830{,}000 = 450c + f$
$f = 830{,}000 - 450c$

$896{,}000 = 510c + f$
$f = 896{,}000 - 510c$

$830{,}000 - 450c = 896{,}000 - 510c$
$830{,}000 + 60c = 896{,}000$
$60c = 66{,}000$
$c = 1{,}100$

Plug $c = 1{,}100$ into either equation to find *f*:

$f = 830{,}000 - 450(1{,}100)$
$f = 335{,}000$

Alternatively, subtract $896,000 - $830,000 to get $66,000, which must be the cost difference between serving 450 clients and serving 510 clients (a difference of 60 clients). Divide $66,000 by 60 clients to get $1,100, the variable cost per client. Then, multiply $1,100 × 450 = $495,000 to get the variable cost of serving 450 clients, not counting the fixed cost. Finally, subtract this figure from the total cost of serving 450 clients to get the fixed cost. $830,000 - $495,000 = $335,000. The numbers should look familiar; the point is that you can "reason through it" without strictly setting up equations.

17. **(A).** First, algebraically manipulate $4x + 5y = 9$ into $y = mx + b$ format, where *m* is the slope and *b* is the *y*-intercept.

$4x + 5y = 9$
$5y = -4x + 9$
$$y = -\frac{4}{5}x + \frac{9}{5}$$

Since $m = -\frac{4}{5}$, the slope is $-\frac{4}{5}$. Perpendicular lines have negative reciprocal slopes. Thus, the correct answer has a slope of $\frac{5}{4}$. Only choice (A) qualifies.

18. **(A).** If the tens digit is to be randomly selected from the digits 0 to 9, there are ten possibilities for the completed number. Using your calculator, divide each by 9 to see which ones are multiples of 9:

809	$\longleftarrow$	not a multiple of 9
819	$\longleftarrow$	MULTIPLE OF 9
829	$\longleftarrow$	not a multiple of 9
839	$\longleftarrow$	not a multiple of 9
849	$\longleftarrow$	not a multiple of 9
859	$\longleftarrow$	not a multiple of 9
869	$\longleftarrow$	not a multiple of 9
879	$\longleftarrow$	not a multiple of 9
889	$\longleftarrow$	not a multiple of 9
899	$\longleftarrow$	not a multiple of 9

The answer is 1/10, or 0.1.

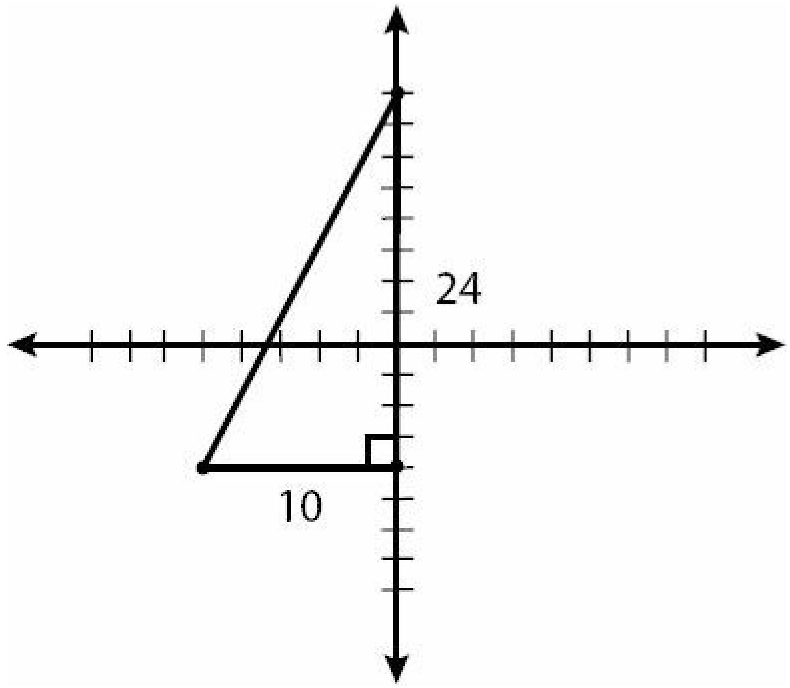
Alternatively, a number is divisible by 9 if the sum of its digits is a multiple of 9. The existing digits sum to 8 + 9 = 17, so the addition of 0 through 9 means that the sum of all three digits could be 17 through 26, inclusive. Only one multiple of 9 (i.e., 18) is found in this range.

19. **(E).** Because the circle is centered at (0, 0) and passes through (0, -16), the radius of the circle is 16. Point *P* lies on the circle and the *y*-axis, so it lies exactly one radius above the origin. Point *P*'s coordinates are therefore (0, 16). To find the distance between (0, 16) and (-10, -8), either use the distance formula, or draw a graph and make a right triangle on which you can use the Pythagorean theorem.

From the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$d = \sqrt{(-10-0)^2 + (-8-16)^2}$$
$$d = \sqrt{(-10)^2 + (-24)^2}$$
$$d = \sqrt{676}$$
$$d = 26$$

To use the triangle method, plot (0, 16) and (-10, -8), then drop a line down from (0, 16) to make a right triangle. To do so, you will need to add the third point (0, -8).



Use the coordinates to determine the lengths of the legs, then use the Pythagorean theorem (the hypotenuse is d):

$$24^2 + 10^2 = d^2$$
$$576 + 100 = d^2$$
$$676 = d^2$$
$$d = 26$$

20. **(C).** If $f(-0.5) = 0$, then the answer is 0 when $x = -0.5$. For each choice, plug in -0.5 for x. Only if the result is 0 could the choice be $f(x)$.

(A) $2x + 2 = 2(-0.5) + 2 = -1 + 2 = 1$
(B) $4x - 2 = 4(-0.5) - 2 = -2 - 2 = -4$
(C) CORRECT. $4x^2 - 1 = 4(-0.5)^2 - 1 = 4(0.25) - 1 = 1 - 1 = 0$
(D) $x^2 - 1 = (-0.5)^2 - 1 = 0.25 - 1 = -0.75$
(E) $(-x)^2 - 2.5 = (-(-0.5))^2 - 2.5 = (0.5)^2 - 2.5 = 0.25 - 2.5 = -2.25$

Math Practice Section 3: Hard Difficult

Math Practice Section: Hard
20 Questions
35 Minutes

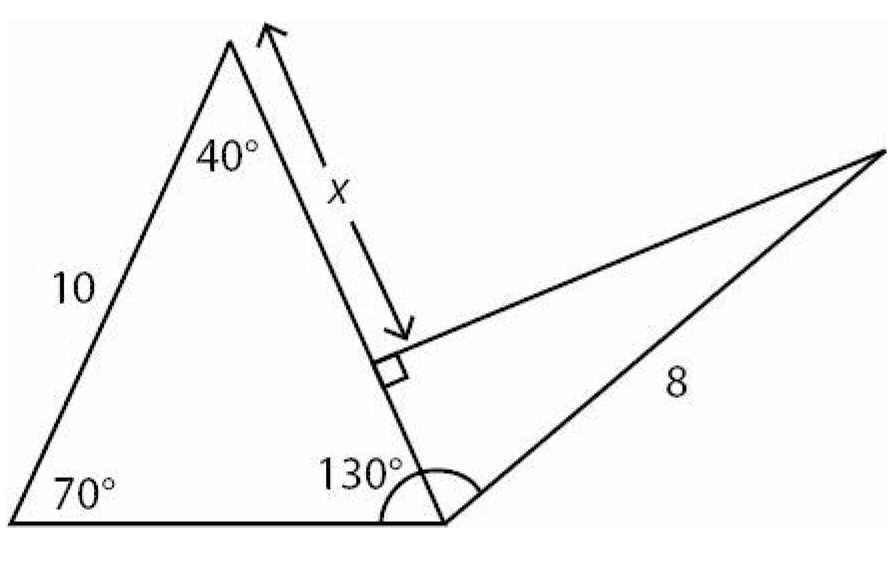
For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.



Quantity A	Quantity B
x	6

2.

Quantity A	Quantity B
$(z^6)^x \times z^{3x}$	z^{9x}

3.

For a group of test takers, the scores on an aptitude test were normally distributed, had a mean of 154, and a standard deviation of 3.

Quantity A	Quantity B
The fraction of test takers in the group who scored greater than 158	$\frac{1}{3}$

4.

$$3x + 5y + 2z = 20$$
$$6x + 4z = 10$$

Quantity A	Quantity B
y by itself	2

5.

Romero Automobiles sells cars only from Manufacturer X and Manufacturer Y. The range of the list prices of the cars from Manufacturer X is \$22,000. The range of the list prices of the cars from Manufacturer Y is \$15,000.

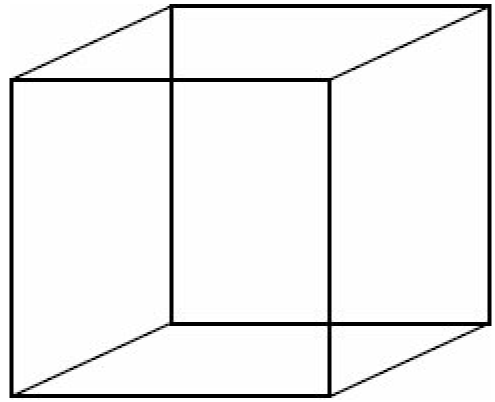
Quantity A	Quantity B
The range of the list prices of all automobiles sold by Romero Automobiles	\$22,000

6.

The operation # is defined by $x\# = \frac{1}{x} + x$

Quantity A	Quantity B
(4#)#	4.5

7.



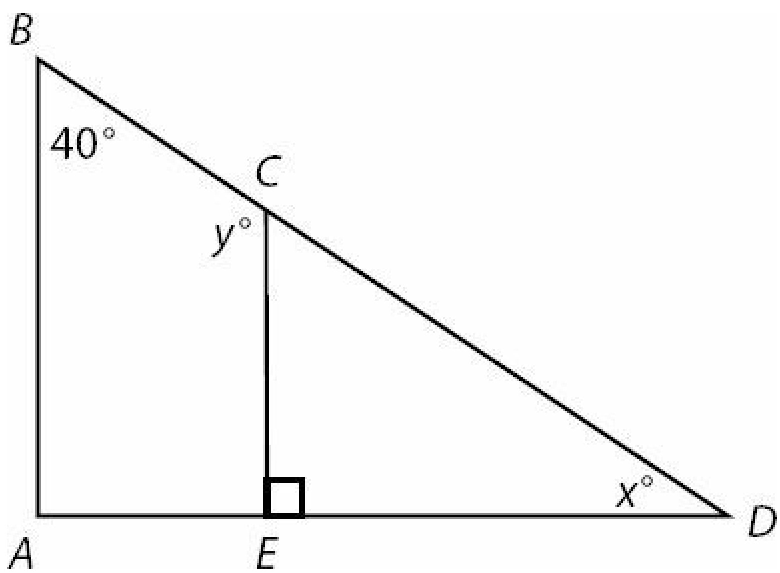
The cube above has side length of 4

Quantity A	Quantity B
After selecting one vertex of the cube, the number of straight line segments longer than 4 that can be drawn from that vertex of the cube to another vertex of the cube	When the cube is placed on a flat surface, the maximum number of edges of the cube that can be touching the flat surface at once

8. If $160^2 = 16x$, then x is equivalent to which of the following?

(A) 10
(B) $2^3 5$
(C) $2^2 5^2$
(D) $2^6 5^2$
(E) $2^6 5^3$

9.



In the triangle shown above, BA is parallel to CE. What is the value of $x + y$?

10. If the tick marks on the number line below are evenly spaced, what value is represented by x?

(A) 2^0
(B) 2
(C) $(-2)2^5$
(D) $(-3)2^5$
(E) $(-4)2^5$

11. If the volume of a cube is v, what is the surface area of the cube in terms of v?

(A) $6\sqrt{v}$

(B) $\left(\sqrt[2]{v}\right)^3$

(C) $6\left(\sqrt[2]{v}\right)^3$

(D) $\left(\sqrt[3]{v}\right)^2$

(E) $6\left(\sqrt[3]{v}\right)^2$

12. What is the area of an equilateral triangle with vertices at (-1, -3), (9, -3), and (m, n) where m and n are both positive numbers?

(A) $25\sqrt{2}$

(B) $50\sqrt{2}$

(C) $10\sqrt{3}$

(D) $25\sqrt{3}$

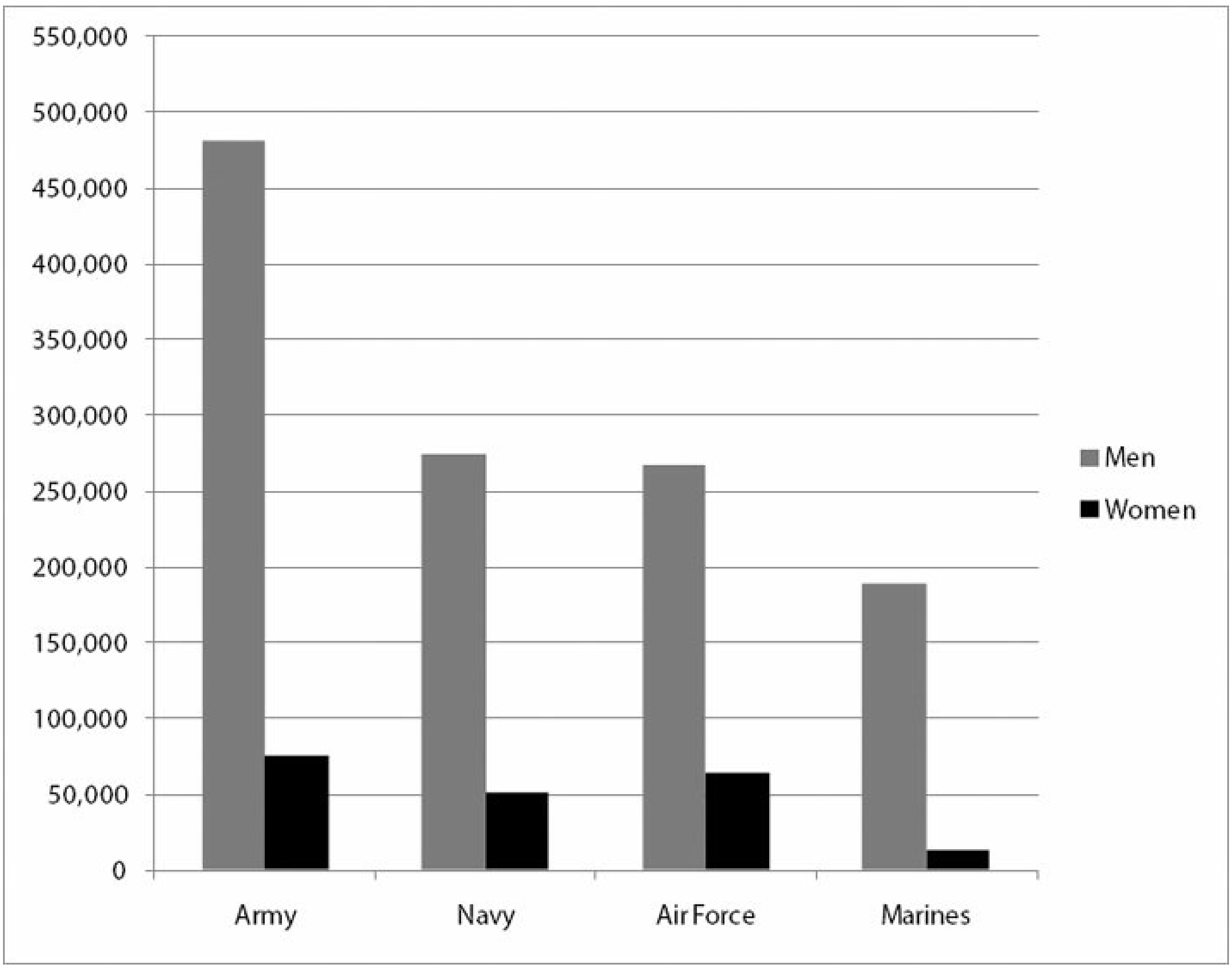
(E) $50\sqrt{3}$

Questions 13–15 are based on the following charts.

Marital Status of Military Personnel by Gender and Branch

Marital Status		Army	Navy	Air Force	Marines
Single, no children less than 18 years old	men	164,513	107,349	94,800	90,949
	women	27,492	24,757	25,247	6,338
Single, with children less than 18 years old	men	26,571	10,506	9,544	4,807
	women	11,037	5,859	6,313	1,263
Married, spouse is also military personnel or retired military	men	15,058	8,638	20,760	4,719
	women	14,633	8,832	18,574	3,676
Married, spouse is a civilian	men	275,953	147,255	142,573	88,233
	women	21,687	11,175	13,982	1,858

Number of Military Personnel by Gender and Branch



13. Which military branch has the greatest percentage of women who are single and have children under the age of 18?

(A) Army
(B) Navy

(C) Air Force
(D) Marines
(E) It cannot be determined from the information given.

14. If a man whose spouse is also military personnel or retired military were to be selected at random, what would the probability be that he was NOT in the Air Force?

(A) 72%
(B) 58%
(C) 42%
(D) 24%
(E) 13%

15. Which of the following expressions is equal to the approximate number of women who would have to enlist in the Army to make the fraction of Army personnel who are women equal the fraction of Air Force personnel who are women?

(Assume that the number of men in the Army and the number of men and women in the Air Force remain unchanged from what is shown in the tables above.)

(A) $\dfrac{482{,}000-268{,}000}{75{,}000-64{,}000}$

(B) $\dfrac{(482{,}000)(64{,}000)-(75{,}000)(268{,}000)}{482{,}000}$

(C) $\dfrac{(482{,}000)(75{,}000)-(64{,}000)(268{,}000)}{482{,}000}$

(D) $\dfrac{(482{,}000)(75{,}000)}{268{,}000}-64{,}000$

(E) $\dfrac{(482{,}000)(64{,}000)}{268{,}000}-75{,}000$

16. A cable car travels from City X to Resortville, making two stops in between. Between City X and the first stop, the cable car travels $\frac{1}{3}$ of the total distance between City X and Resortville. Between the first stop and the second stop, the cable car travels $\frac{3}{5}$ of the remaining distance between the first stop and Resortville. What fraction of the entire distance from City X to Resortville remains between the second stop and Resortville?

(A) $1-\frac{1}{3}-\frac{3}{5}$

(B) $1-\frac{1}{3}-\frac{3}{5}\left(\frac{1}{3}\right)$

(C) $1-\frac{1}{3}-\frac{3}{5}\left(1-\frac{1}{3}\right)$

(D) $1-\frac{1}{3}-\frac{1}{3}\left(1-\frac{3}{5}\right)$

(E) $1-\frac{1}{3}-\frac{1}{5}\left(1-\frac{1}{3}-\frac{1}{5}\right)$

17. If p and q are integers and $20p + 3q$ is odd, which of the following must be odd?

(A) $p - q$
(B) $p + 2q$
(C) $3p + q$
(D) $2p + q^2$
(E) $3p + 3q$

18. 4,400 participants in a study were surveyed regarding side effects of a new medication, and x percent reported experiencing drowsiness. If x is rounded to the nearest integer, the result is 8. Which of the following could be the number of survey participants who reported experiencing drowsiness?

Indicate all such values.

☐ 325
☐ 330
☐ 352
☐ 375

19. $\dfrac{5^3(4^{45}-4^{43})27}{225^2}$ is equivalent to which of the following?

(A) 4^{43}
(B) 4^{45}
(C) $4^{90}5^3$
(D) $4^{86}5^3 3^3$
(E) $4^{90}5^3\ 3^3$

20. Price of Plane Ticket for an April 1 Flight Based on Date of Purchase

Price	When Purchased By
$210	March 31

\$168	March 15
\$140	March 1

Harpreet purchased a ticket on March 1st. If he had purchased the ticket on March 2nd, he would have paid x percent more. If he had purchased the ticket on March 16th, he would have paid y percent more than he would have paid on March 2nd. What is the positive difference between x and y?

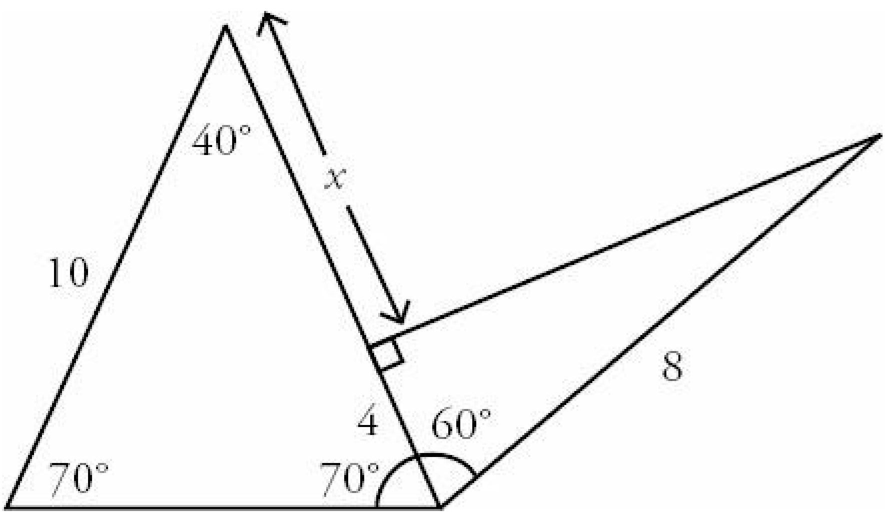
(A) 5
(B) 14
(C) 20
(D) 25
(E) 28

Answers to Math Practice Section 3

1. **(C).** The leftmost triangle has two angles labeled 40 and 70. Subtract these from 180 (the sum of the angles in any triangle) to determine that the third angle is 70. Subtract 130 - 70 = 60 to get the measure of the adjoining angle in the rightmost triangle.

Since the leftmost triangle is isosceles, the two long sides are each equal to 10.

Since the rightmost triangle is a 30–60–90 triangle, the sides are in the proportion $x : \sqrt{3}\,x : 2x$. Because the hypotenuse is 8 and also the $2x$ in the ratio, the shortest leg of this triangle is $x = 4$.



To calculate x, subtract: 10 - 4 = 6. The two quantities are equal.

2. **(C).** The terms in Quantity A have the same base, so add the exponents: $(z^6)^x \times z^{3x} = z^{6x} \times z^{3x} = z^{9x}$. The two quantities are equal. Note that $(z^6)^x$ is interchangeable with$(z^x)^6$ and z^{6x}.

3. **(B).** For a normal distribution, approximately two thirds of the values are within one standard deviation of the mean. Thus, roughly 1/6 of the population is more than a deviation above the mean, and 1/6 is more than a deviation below. Thus, about 1/6 of the test takers would score greater than 157 (154 + 3 = 157, one standard deviation above the mean), so an even smaller fraction of the test takers would score greater than 158.

4. **(A).** In order to isolate y, eliminate both x and z. Because there are only two equations, both x and z must be eliminated at the same time if the value of y is to be determined.

Notice that the coefficients for x and z in the second equation (6 and 4, respectively) are exactly double their coefficients in equation 1 (3 and 2, respectively). Divide the second equation by 2, making the coefficients the same.

$$3x + 5y + 2z = 20 \quad \longrightarrow \quad 3x + 5y + 2z = 20$$
$$6x + 4z = 10 \quad \longrightarrow \quad 3x + 2z = 5$$

Now subtract the second equation from the first.

$$\begin{array}{l} 3x + 5y + 2z = 20 \\ \underline{-(3x + 2z = 5)\qquad\qquad} \\ 5y = 15 \\ y = 3 \end{array}$$

Quantity A is greater.

5. **(D).** The range of list prices of automobiles is found by subtracting the price of the least expensive automobile from the price of the most expensive automobile. Given just the range, there is not enough information to determine the maximum and minimum list price vehicles from either manufacturer. Before selecting (D), though, you should try to prove (D). Construct two examples in which the list prices of the cars from Manufacturer X have a range of \$22,000 and the list prices of the cars from Manufacturer Y have a range of \$15,000, but the overall ranges are drastically different.

EXAMPLE 1:
List prices of Manufacturer X's cars range from \$10,000 to \$32,000
List prices of Manufacturer Y's cars range from \$10,000 to \$25,000
Here, the overall range is the same as X's range, which is \$32,000 - \$10,000 = \$22,000

EXAMPLE 2:
List prices of Manufacturer X's cars range from \$10,000 to \$32,000
List prices of Manufacturer Y's cars range from \$100,000 to \$115,000
Here, the overall range is \$115,000 - \$10,000 = \$105,000

In Example 1, the range = \$22,000 and the quantities are equal. In Example 2, Quantity A is much greater than Quantity B. It is not possible to make the range any smaller than \$22,000 (the minimum range of all the prices cannot be *smaller* than the larger of the two ranges of each manufacturer's prices), but it can get much, much larger.

Note that the testing done above was very important! If Quantity B had read "\$21,999," the answer would be (A) rather than (D).
The correct answer is (D).

6. **(B).** Start inside the parentheses (according to PEMDAS, always deal with parentheses first).

$$4\# = \frac{1}{4} + 4, \text{ or } \frac{17}{4}.$$

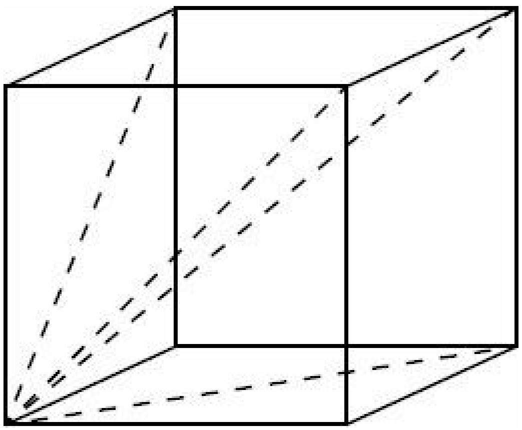
Since $4\# = \frac{17}{4}$, plug $\frac{17}{4}$ in for x to get $(4\#)\#$.

$$\text{Thus, } (4\#)\# = \frac{1}{\frac{17}{4}} + \frac{17}{4} = \frac{4}{17} + \frac{17}{4}$$

While you could find a common denominator, it is more efficient to ballpark the value or simply use the calculator.

Ballparking, $\frac{4}{17}$ is less than 0.25 and $\frac{17}{4}$ is exactly 4.25, so the sum is less than 4.5. Using the calculator, $\frac{4}{17}+\frac{17}{4}$ is about 4.485. Quantity B is greater.

7. **(C).** If a cube has side length of 4, all of the "straight line segments" connecting vertices of the cube *along an edge of the cube* will have length of 4. The only straight line segments between vertices that are longer than 4 are those that go diagonally through the cube or diagonally across a face. From a selected vertex of the cube, there are 3 diagonals across the adjacent faces of the cube, and 1 diagonal through the cube to the opposite vertex.



Thus, Quantity A is 4.

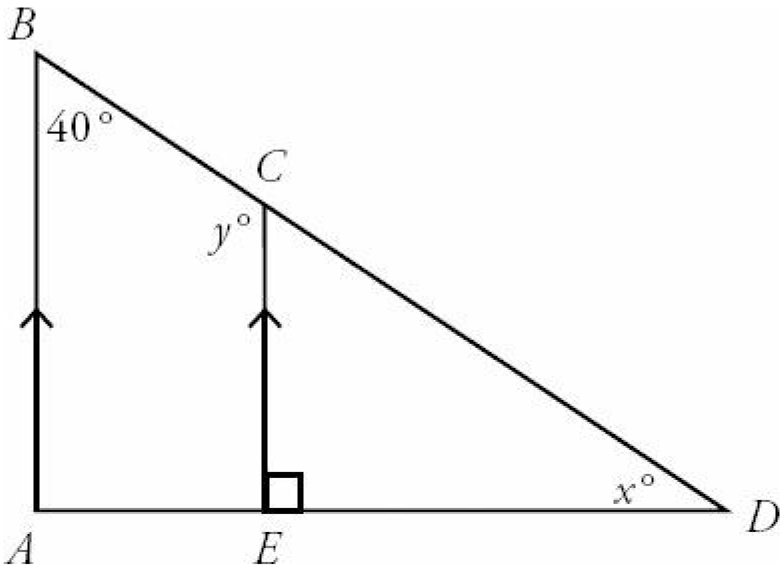
If a cube is placed on a flat surface, the maximum contact occurs when one cube face abuts the surface—and thus 4 cube edges touch the surface. There is no way to make more than 4 cube edges touch the flat surface at once. The two quantities are equal.

8. **(D).** The easiest first step is to divide both sides by 16. To do that, make sure you separate out 160^2 first. Notice that $160^2 = 160 \times 160 = 16 \times 10 \times 160$:

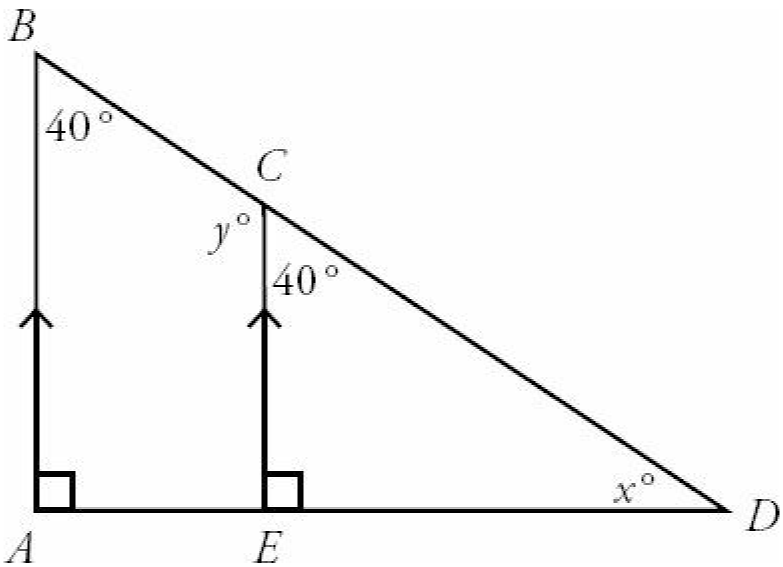
$16 \times 10 \times 160 = 16x$
$10 \times 160 = x$
$1{,}600 = x$

None of the answer choices match this, so break 1,600 down into its primes ($1{,}600 = 100 \times 16 = 25 \times 4 \times 16 = 5^2 \times 2^2 \times 2^4 = 5^2 \times 2^6$) and see which choice is equivalent. Alternatively, multiply out the answer choices to see which equals 1,600. The correct choice is (D).

9. **190.** Redraw the figure, labeling all information given:



Since BA and CE are parallel, angle B and minor angle C are equivalent, as shown:



The two angles that meet at C make up a straight line, so they sum to 180 degrees:

$$180 = y + 40$$
$$y = 140$$

The three angles of triangle CDE must sum to 180 degrees, and so

$$180 = 40 + 90 + x$$
$$180 = 130 + x$$
$$x = 50$$

Therefore, $x + y = 140 + 50 = 190$.

10. **(D).** At first glance, you might be tempted to think that each tick mark on this number line corresponds to a power of 2, but remember that powers grow exponentially (i.e. the distance between 2^5 and 2^6 is not the same as the distance between 2^1 and 2^2), whereas the tick marks in the diagram are evenly spaced. So, start by finding the distance between 2^5 and 2^6.

$2^5 = 32$, and $2^6 = 64$. The difference between them is 32. That means the distance between each tick mark on the number line is 32. So to get from 2^5 to x, "walk back" or subtract four intervals of 32: 32 - 4(32) = -96.

Multiply out the answers to see which one equals -96. Only choice (D) works.

11. **(E).** There are two ways to solve this question, with smart numbers or algebra. Start with plugging-in. First, set a value for the volume. In this case, pick a perfect cube, so the side length and all other values will be integers. The smallest perfect cube (other than 1, which you should try never to use when doing plug-in questions) is 8.

A cube with a volume of 8 has a side length of 2, meaning each side has an area of 4. A cube has 6 sides, making the total surface area 24. (The equation for surface area is Surface Area = $6s^2$). The answer to this question is 24, based on these numbers.

Immediately eliminate any answer choices that have the square root of 8, as the result will not be an integer. The answer must be either (D) or (E). The cube root of 8 is 2. Answer choice (D) simply squares it, yielding 4. In answer choice (E), that result is multiplied by 6, producing 24, which is the required answer. Thus, the answer is (E).

If you wanted to solve with algebra, you'd need to start by solving for a side of a cube with volume v:

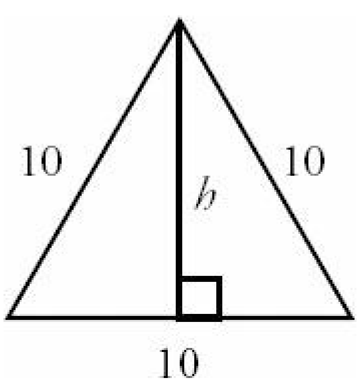
$$V = s^3 \quad \text{so} \quad s = (\sqrt[3]{v})$$

The equation for the surface area of a cube is $6s^2$. In this case, substitution for s results in exactly the expression written in answer choice (E).

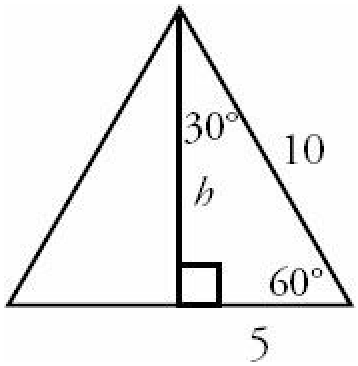
12. **(D).** To find the area of an equilateral triangle with vertices at (-1, -3), (9, -3), and (m, n), you do not need to find the values of m and n. To find the area of an equilateral triangle, you only need one side. So, you should first find the distance between (-1, -3) and (9, -3).

Since these two points are on a horizontal line together (they share a y-coordinate), the distance is just the difference between their x-coordinates: 9 - (-1) = 10.

An equilateral triangle with side 10 will have the same area regardless of where it is placed on an xy-coordinate plane, so the location of m and n is irrelevant. Instead, draw an equilateral triangle with sides equal 10. Drop a height down the middle. 10



Dividing a 60–60–60 triangle in this way creates two 30–60–90 triangles. The bottom side of the triangle is bisected by the height:



Using the properties of 30–60–90 triangles, h is equal to the shortest side multiplied by the square root of 3. Thus, $h = 5\sqrt{3}$. (You may also wish to memorize that the height of an equilateral triangle is *always* equal to half the side multiplied by $\sqrt{3}$.)
Find the area of the triangle, using 10 as the base:

$$A = \frac{bh}{2} = \frac{10(5\sqrt{3})}{2} = 25\sqrt{3}$$

13. **(A).** The percentage of women in a service who are single mothers is:

$$\frac{\text{\# single women with children}}{\text{\# women}}$$

Find the number of single mothers in each of the four services by looking at the first table, ***Martial Status of Military Personnel by Gender and Branch.*** The number of women who are single with children is given as:

Army	11,037
Navy	5,859
Air Force	6,313
Marines	1,263

There are two ways to find the total number of women in each service, though. Either sum the exact number of women in each branch of the service across each of the marital status' given in the first chart, or read an approximate number of women from the second bar chart, ***Number of Military Personnel by Gender and Branch***, then only bother to sum from the detailed chart if two answers are very close to each other.

Since using the chart will be faster and GRE problems are designed to be solved quickly, try approximating from the bar chart first. The total number of women in each of the four services is approximately

Army	75,000
Navy	50,000
Air Force	60,000

Marines	10,000

Calculate the approximate percent of women who are single mothers in each branch of the service.

Army	11,000/75,000 = about 14.7%
Navy	5,900/50,000 = about 11.8%
Air Force	6,300/60,000 = about 10.5%
Marines	1,300/10,000 = about 13%

The percent looks highest in the Army. At least, reason that the number of single mothers in the Army is about double the number of single women in either the Navy or Air Force, yet the total number of women in the Army is definitely less than double the total number of women in either the Navy or Air Force, making their percentage of women greater in the Army.

Just quickly check the actual totals for the Army and the Marines.

Army: single mothers = 11,037 and total women = 74,849. The percent is 14.7%.
Marines: single mothers = 1,263 and total women = 13,135. The percent is 9.6%.

Thus, the Army has the greatest percentage of women who are single and have dependents under the age of 18.

14. **(B).** The probability that a man whose spouse is also military personnel or retired military is NOT in the Air Force is given by the formula:

$$\frac{\text{\# of men in the "married, military spouse" category who are NOT in the Air Force}}{\text{total \# men in the "married, military spouse" category}}$$

All of the information needed to calculate both of these numbers is in the first table, ***Martial Status of Military Personnel by Gender and Branch.***

The total number of men married to a military spouse or retired military in each of the four services:

15,058 + 8,638 + 20,760 + 4,719 = 49,175

Then just subtract the number of Air Force men in this category to get the number of men in such marriages who are not in the Air Force:

49,175 - 20,760 = 28,415

And finally:

$$\frac{28,415}{49,175} = 0.5778 \approx 58\%$$

15. **(E).** In order to solve this problem, make the two ratios equal. The ratio in question is Women/Total, but Women/Men is simpler and works also, because Total depends only on Women and Men):

$$\frac{\text{resulting \# of women in Army}}{\text{\# of men in Army}} = \frac{\text{\# of women in Air Force}}{\text{\# of men in Air Force}}$$

There are two ways to find the number of women and men in the Army and Air Force. Either sum the exact number of women and men in each marital status for each branch of the service in question, or read an approximate number from the second bar chart, ***Number of Military Personnel by Gender and Branch.***

Since the problem says to approximate and gives numbers in the answer choices that can serve as guidelines, approximation from the bar chart will be good enough.

The important thing is to focus on the *structure* of the math. Since adding women to the Army will change the number of women in the Army, use a variable to represent the additional women. Let x represent the number of women who would have to enlist in the Army in order to make the ratios equal.

$$\frac{\text{current \# of women in Army} + x}{\text{\# of men in Army}} = \frac{\text{\# of women in Air Force}}{\text{\# of men in Air Force}}$$

From the bar chart, look up the approximate numbers:

	Army	AF
# of women	75,000	60,000
# of men	475,000	270,000

The next step is to plug these approximate numbers into the equation and solve for x.

$$\frac{75{,}000 + x}{475{,}000} = \frac{60{,}000}{270{,}000}$$

$$\rightarrow 75{,}000 + x = 475{,}000 \times \frac{60{,}000}{270{,}000}$$

$$\rightarrow x = 475{,}000 \times \frac{60{,}000}{270{,}000} - 75{,}000$$

Looking at the answer choices, structurally, the answer must be (D) or (E), and the numbers in (E) are a better fit to the numbers approximated from the chart.

16. **(C).** Since the question concerns the "fraction of the entire distance from City X to Resortville," think of the entire distance as equal to 1. Between City X and the first stop, the cable car travels $\frac{1}{3}$, leaving $\frac{2}{3}$ left to travel.

Between the first stop and the second stop, the cable car travels $\frac{3}{5}$ of the remaining $\frac{2}{3}$, or $\frac{3}{5} \times \frac{2}{3} = \frac{2}{5}$.

So far, the cable car has gone $\frac{1}{3} + \frac{2}{5} = \frac{11}{15}$. Thus, the remaining distance is $1 - \frac{11}{15} = \frac{4}{15}$. Only choice (C) is equal to $\frac{4}{15}$, although this takes some manipulation of the choices to check.

Alternatively, construct a formula. The first leg of the journey leaves $1 - \frac{1}{3}$ left to travel. The second leg of the journey subtracts another $\frac{3}{5}$ of the remaining $1 - \frac{1}{3}$, or $\frac{3}{5}\left(1 - \frac{1}{3}\right)$. Thus, the correct expression is $1 - \frac{1}{3} - \frac{3}{5}\left(1 - \frac{1}{3}\right)$.

17. **(D).** If p and q are integers, then $20p$ is even regardless of whether p is even or odd. Since $20p + 3q$ is odd, $3q$ must be odd. If $3q$ is odd, then q is odd. Thus, q is odd, but p could be odd or even. The correct answer must be odd regardless of whether p is odd or even.

If p is odd, (A) is even, (B) is odd, (C) is even, (D) is odd, and (E) is even. Since the correct answer choice is the one that *must* be odd, only (B) and (D) are possibilities.

If p is even, (B) is even and (D) is odd. Thus, choice (D) is definitely odd and is the correct answer.

18. **II and III only (330 and 352).** Using your calculator, convert each choice to a percent, and determine whether that percent would round up or down to 8%.

The first choice $= \frac{325}{4,400} \times 100 = 7.386\ldots\%$ This number would round down to 7%, not up to 8%.

The second choice $= \frac{330}{4,400} \times 100 = 7.5\%$ This number rounds up to 8%, and thus this choice is correct.

The third choice $= \frac{352}{4,400} \times 100 = 8\%$ exactly, and thus this choice is correct.

The fourth choice $= \frac{375}{4,400} \times 100 = 8.522\ldots\%$ This number would round up to 9%, not down to 8%.

19. **(A).** When dealing with exponents, try to get (almost) everything in terms of common prime bases. Since all the answer choices have a base 4, leave those terms alone for now.

5^3 is already simplified
$27 = 3^3$
$225^2 = 25^2 \times 9^2 = (5^2)^2 \times (3^2)^2 = 5^4 \times 3^4$

Replacing all of these in the equation, you get this: $\dfrac{5^3(4^{45}-4^{43})3^3}{5^4 3^4}$

Cancel 5's and 3's in the top and bottom: $\dfrac{(4^{45}-4^{43})}{5\times 3}=\dfrac{(4^{45}-4^{43})}{15}$

Factor 4^{43} out of both terms in the numerator and simplify: $\dfrac{4^{43}(4^2-4^0)}{15}=\dfrac{4^{43}(16-1)}{15}=\dfrac{4^{43}(15)}{15}=4^{43}$

20. **(A).** On March 1st, the ticket cost $140. If he had purchased it on March 2nd, Harpreet would have paid $168, which is $28 more. To find *x*, use the percent change formula:

$$\text{Percent Change }=\left(\frac{\textit{Difference}}{\textit{Original}}\times 100\right)\%=\left(\frac{28}{140}\times 100\right)\%=20\%$$

Thus, $x = 20$. If he had purchased the ticket on March 16th, he would have paid $210, which is $42 more than the $168 he would have paid on March 2nd.

$$\text{Percent Change }=\left(\frac{\textit{Difference}}{\textit{Original}}\times 100\right)\%=\left(\frac{42}{168}\times 100\right)\%=25\%$$

Thus, $y = 25$ and the positive difference between *x* and *y* is 5. ("Positive difference" just means to subtract the smaller one from the bigger one, or to subtract either one from the other and then take the absolute value.)