CSCI698 - A Programming Assignment

sbmohant

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1 Prelude to Gradient Descent

Let us revisit some of the calculus concepts you have learnt during high-school/freshmen years. For a differentiable function (x), we can find the minima/maxima of the function by setting the derivative $f'(x)$ to zero.

The minima of the quadratic $f(x) = (x - a)^2 + b$ occurs at $x = a$. We can see that by setting $f'(x) = 2(x - a) = 0$ or $x = a$.

However, there is another way in which we can find the minimum of this function. It is called sliding along the slope aka gradient descent.

In the figure 1 below, we can see that a=5 and b=-10.

a) Let us look at $f(x) = (x-5)^2-10$. The function is implemented in python below. Please fill out the python implementation for the derivative $df(x)$.

```
def f(x):
    return (x-5)**2 - 10

def df(x):
    # fill in your code here
    # return ...

print(df(1))
```

Now, let us evaluate the $df(1)$ which equals -8.

Now, observe that we can slide along the slope at $(1, f(1))$ along the slope slightly to get closer to the minima. Let us try that. The slope at $x = 1$ is $df(1) = -8$ which is negative. Of course as evident from the figure 1, we need to move to the right of $x = 1$ to reach the minima $x = 5$. Observe that to achieve a small slide to the right we can do the following

$$x = x - \frac{df(x)}{10}$$

If $x = 1$, after the above update, we get $x = 1 + 8/10$ which is a slight slide to the right which takes us closer to the minima $x = 5$.

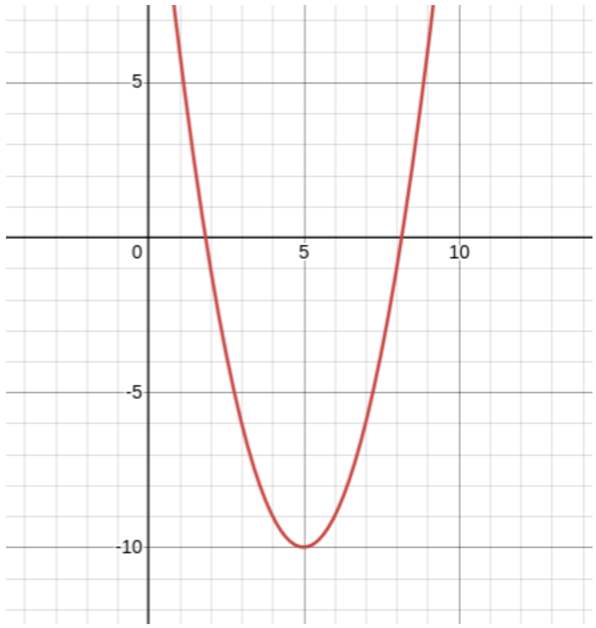

Figure 1: $(x - 5)^2 - 10$

b) Let us take 30 such small steps using a for loop and print the value of x we obtain after 30 such steps. Complete the code below to achieve this

```
x=1
for i in range(30):
    # .... write your code here ....
print("found approximate minima:",x)
```

The approximate minima must be 4.995048239842858. Figure 2 plots the vasrious x over the 30 steps.

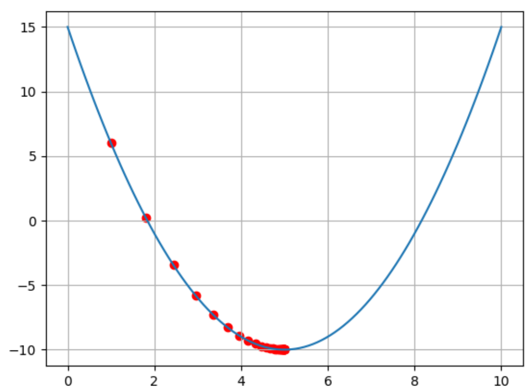

Figure 2: sliding along the slope