Right Eigenvector.

$$\text{square} \leftarrow \left.\begin{aligned} A\vec{r}_1 &= \lambda_1 \vec{r}_1 \\ A\vec{r}_2 &= \lambda_2 \vec{r}_2 \end{aligned}\right\} \text{ Assume } \lambda_1 \neq \lambda_2 \text{ distinct.}$$

Left Eigenvector.

$$\left.\begin{aligned} l_1^T A &= \lambda_1 l_1^T \\ l_2^T A &= \lambda_2 l_2^T \end{aligned}\right\} \vec{l}_1, \vec{l}_2 \text{ are left eigenvecs}$$

Prove: $\vec{l}_2$ is $\perp$ $\vec{r}_1$

$$A\vec{r}_1 = \lambda_1 \vec{r}_1 \Rightarrow l_2^T A r_1 = \vec{l}_2^T \lambda_1 r_1 = \lambda_1 l_2^T r_1$$

$$l_2^T A = \lambda_2 l_2^T \Rightarrow \underbrace{l_2^T A r_1 = l_2^T \lambda_2 r_1 = \lambda_2 l_2^T r_1}$$

$$0 = \underbrace{(\lambda_1 - \lambda_2)}_{\text{distinct}} l_2^T r_1$$

$$\Rightarrow \frac{0}{\lambda_1 - \lambda_2} = l_2^T r_1$$

$$\Rightarrow l_2^T r_1 = 0$$

$$\Rightarrow \langle l_2, r_1 \rangle = 0$$

$$\Rightarrow \vec{l}_2 \perp \vec{r}_1$$