CSCI698 - A written homework assignment

sbmohant

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1 Linear Algebra

a) If X is a $m \times n$ matrix, are $X^T X$ and $X X^T$ symmetric? What are their dimensions?

b) What does the matrix $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ do to the vector $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$?

c) Based on the previous question, what does the matrix $\mathbf{A}$ do to any arbitrary point $\begin{bmatrix} x \\ y \end{bmatrix}$.

d) What do you think is the physical interpretation of the transformation done by $\mathbf{A}$ on any generic point $\begin{bmatrix} x \\ y \end{bmatrix}$?

e) Let $\mathbf{B} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ be a matrix that rotates any vector $\mathbf{u} = \begin{bmatrix} x \\ y \end{bmatrix}$ by an angle θ counter-clockwise. Verify that after the operation of multiplication $\mathbf{B}$ on $\mathbf{u}$, the resultant vector $\mathbf{v} = \mathbf{B}\mathbf{u}$ has the same length as $\mathbf{u}$ and the term

$$\frac{\mathbf{u} \cdot \mathbf{v}}{(\mathbf{u} \cdot \mathbf{u})(\mathbf{v} \cdot \mathbf{v})}$$

evaluates to $\cos(\theta)$.

f) Find a 2×2 matrix that reflects any point $\begin{bmatrix} x \\ y \end{bmatrix}$ about the plane mirror given by $y = \tan(\theta)x$.

2 Probability

a) If a coin is tossed N times and we get t tails and h heads, then what is the probability p of obtaining a tails when we flip the coin based on the N observations intuitively?

b) Let's say we do not know p and let it be a parameter. Now, what is the

probability that we get a t tails in N flips of the coin?

c) The probability found above is called the likelihood L of observing t tails in N tosses. What is the partial of L with respect to p, i.e $\frac{\partial L}{\partial p}$

d) Find the value of p for which the likelihood L is maximized, i.e find the p such that $\frac{\partial L}{\partial p}$ is zero.