Oral Qualification Exam

UNIVERSITY OF SOUTHERN CALIFORNIA

Committee members

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Acknowledgement

Possible in part due to the unwavering suppport of my mentors Prof. Bhaskar Krishnamachair and Dr. Fan Bai from General Motors.

Also all my teachers and mentors

To the committee:

Thank you so very much for your valuable time and guidance.

SATORIS

reminds of any other English word?

Disclaimer:

- slides may not follow some social norms but it doesn't matter for the results we converge to are equivalent in all of them.
- slides may contain *graphic SATIRES* about academia, industry, world and life.
- Please don't take them seriously at all. Life is too short to not laugh.

satori

Definition | Example Sentences | Word History | Related Articles

satori noun

sa·to·ri sə-ˈtȯr-ē sä-

: sudden enlightenment and a state of consciousness attained by intuitive illumination representing the spiritual goal of Zen Buddhism

SATORIS

What?

A catchy term overfitted to the title of the paper.

No, seriously?

Singular v**A**lue and **T**ensOR we**I**ght regres**S**ion

Ok, I believe you now! Needs to be catchy?

Everyone else is doing it 🤷‍♂️

But why is everyone else doing it?

Deferred to ChatGPT and its cousins so they can reflect and not make the same mistakes when they takeover.

Journey of wandering students

1. Get a **problem** handed down to them.
2. Struggle to reverse engineer the **motivation** from the problem. Start questioning their life choices.
3. Frustrated, look for people who have already walked on **similar nails** before.
4. Learn about **mysterious hammers** that others apparently used to nail the problem.
5. With the various fast approaching **Hammers showdown festivals**, they realize it is easier to dismantle the mysterious hammers and retrofit to make them usable enough instead of trying to underestand and learn the mysterious hammers used before.
6. Utilising the skills from some **familiar hammers** and some **slight of hand**, they forge a hammer that they can use to barely nail the problem.
7. Making sure to decorate the **forged hammer** so they appear **mysterious** to the judges and others, they attend the hammer festivals.

1. THE PROBLEM

What is the (big) data?

GPS logs from Taxi's operating in Beijing and Shanghai, China over a month.

And problem?

- Noisy data
- Lots of missing entries (sensor/network/power failures)
- Cost restrictions - sparse and non-uniform sensor deployment
- Too large to store over longer periods (terrabytes/day)

Task?

Address the challenges posed by mobility datasets.

```
- Denoise and recover missing entries.
- Store data more efficiently, compression.
```

2. THE MOTIVATION

Pressing problems of modern times

Generate an image that captures the most pressing challenge faced by modern humans.

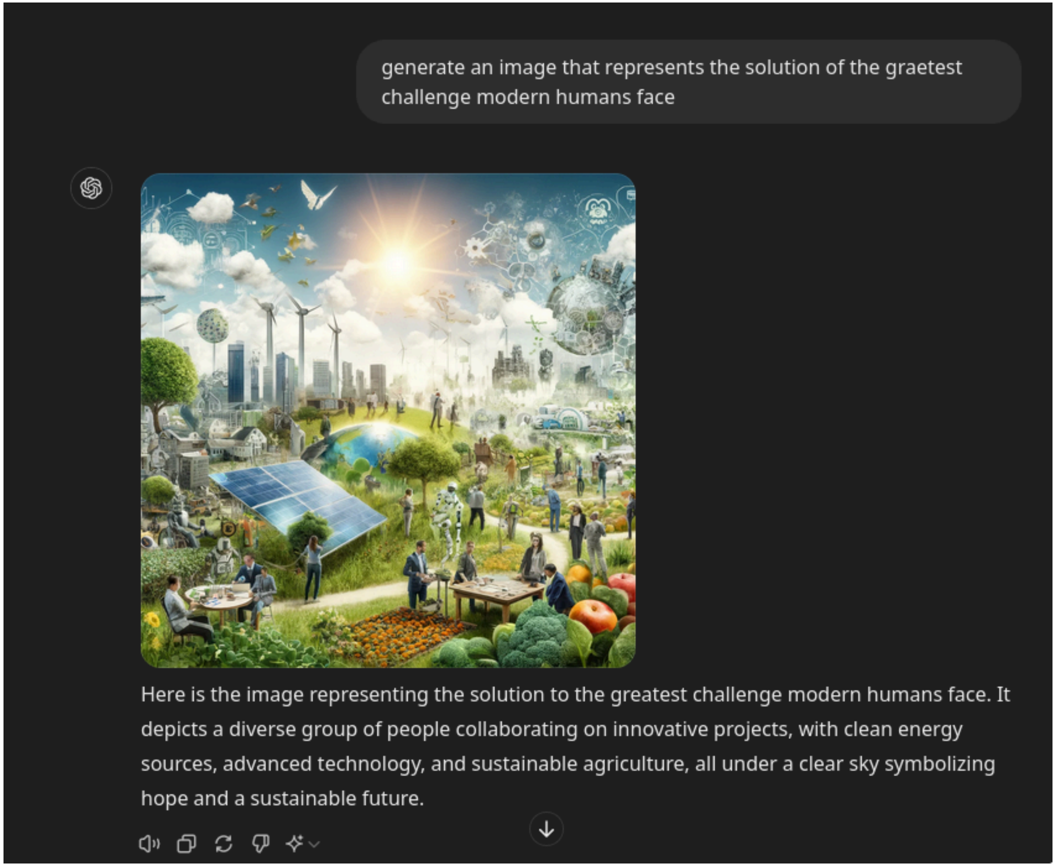



Some solutions

generate an image that represents the solution of the graetest challenge modern humans face

Here is the image representing the solution to the greatest challenge modern humans face. It depicts a diverse group of people collaborating on innovative projects, with clean energy sources, advanced technology, and sustainable agriculture, all under a clear sky symbolizing hope and a sustainable future.

Some solutions



Today's problems come from yesterday's solutions

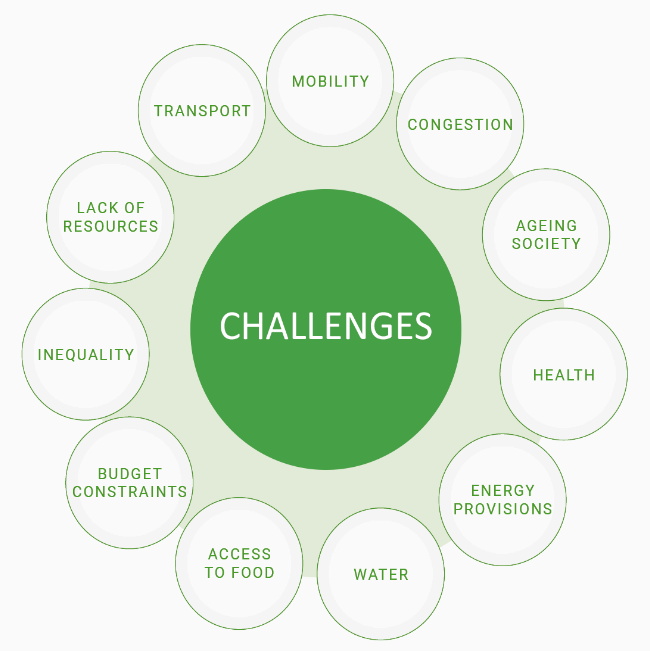
- Peter Senge (The Fifth Discipline)

More! More!

try again

Here is another image depicting the solution to the greatest challenge modern humans face. It shows diverse people collaborating on innovative projects, surrounded by clean energy sources, advanced technology, sustainable agriculture, and a futuristic cityscape, all under a bright and hopeful sky.

Can smarter systems help?



Can smarter systems help?

Today’s problems come from yesterday’s solutions

- Peter Senge (The Fifth Discipline)

Reflections: Solutions for 20th century problems

generate an image depicting the solutions of the most pressing problem faced in the previous century

Here is an image depicting the major solutions to the most pressing problems of the 20th century. It includes representations of the Green Revolution, medical advancements, and technological progress, with symbols of peace and reconstruction in the background. If you need any adjustments or additional elements, feel free to let me know!

Today's problems come from yesterday's solutions

- Peter Senge

generate an image depicting the solutions of the most pressing problem faced in the previous century

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Today’s problems come from yesterday’s solutions - *Peter Senge*

try again

Here is the revised image depicting the solutions to the most pressing problems of the 20th century, including the Green Revolution, medical advancements, and technological progress, along with symbols of peace and reconstruction. If you have any further modifications or specific details you'd like to add, please let me know!

Students -

What if future *nails* come from today's *hammers*?

Academia -

That's not your project, its the next generations'.

You have had enough motivation now.

How can imputing accurate traffic data help?

1. Enhancing city efficiency/safety

    - faster commute times
    - reduced freight costs
    - enhanced safety alerts and emergency dispatch
    - Predictable commute times

2. Addressing climate concerns

    - efficient fuel use
    - traffic shaping -> controlled air quality

3. Effective urban planning

    - traffic/commuter trends -> effective urban planning for new cities
    - Planning efficient locations for EV charging stations.

1. Enhancing downstream tasks
    - informative data visualization
    - easier for downstream tasks
        - congestion detection
        - incident detection
        - City Functional Regions(Points of Interest) detection
            - business districts, residential area, education hubs, etc.

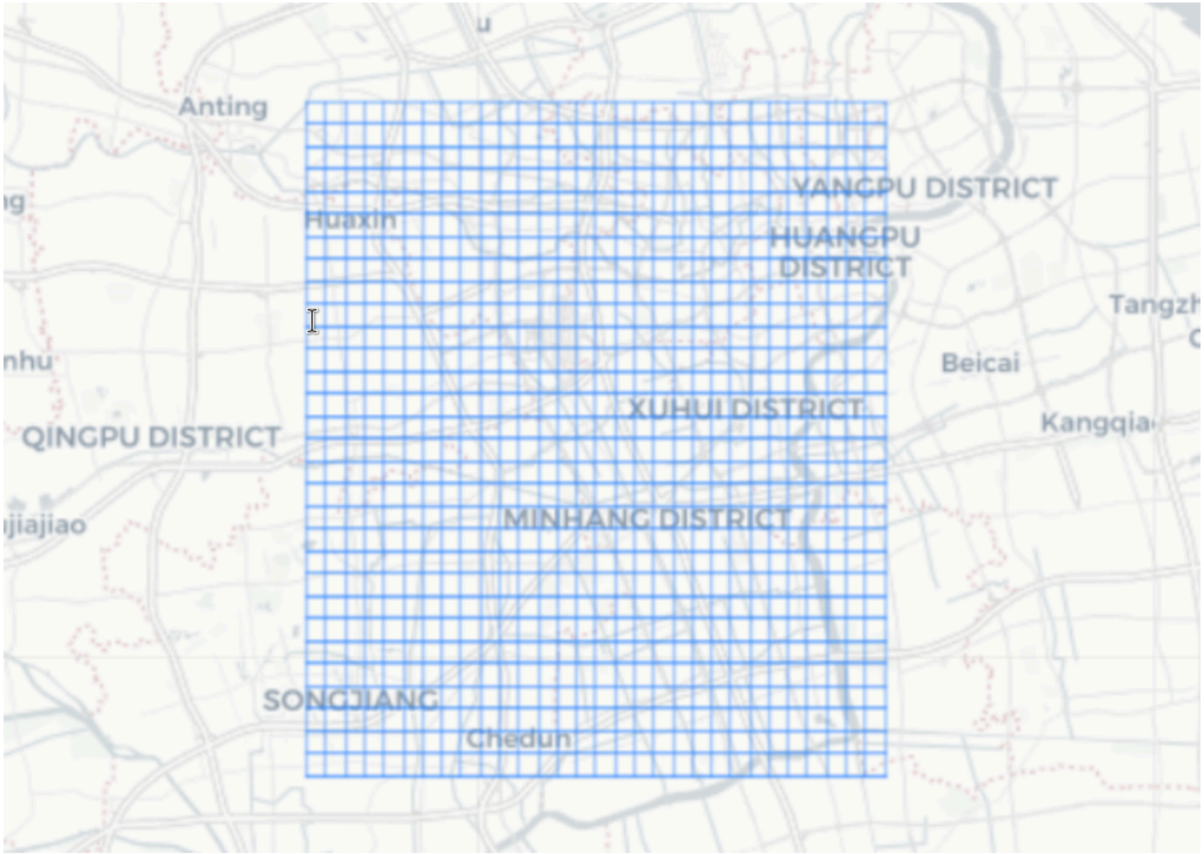
3. DATA REPRESENTATIONS

Raw data

Column Name	Type	Length	Accuracy	Constraint	Not null	description
ID	NUMBER	10		Primary key	Y	
TAXIID	NUMBER	7			N	ID of taxi
LONGITUDE	NUMBER	9	6		N	longitude
LATITUDE	NUMBER	8	6		N	latitude
SPEED	NUMBER	3			N	speed
ANGLE	NUMBER	3			N	angle
DATETIME	TIMESTAPMP	6			N	Time that GPS record was sent
STATUS	NUMBER	1			N	1: taxi is occupied 0 : taxi is vacant
EXTENDSTATUS	NUMBER	1			N	
REVERSED	NUMBER	1			N	

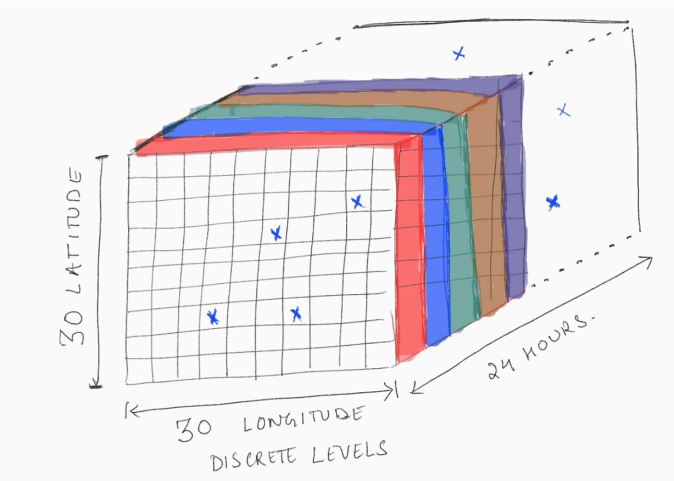
GPS Logs from ~ 2500 Taxis over 1 month

Grid view (Shanghai)

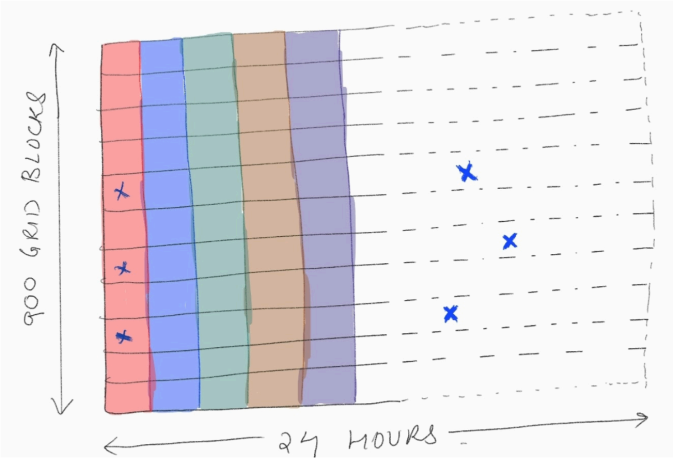


30 x 30 grids

Representations

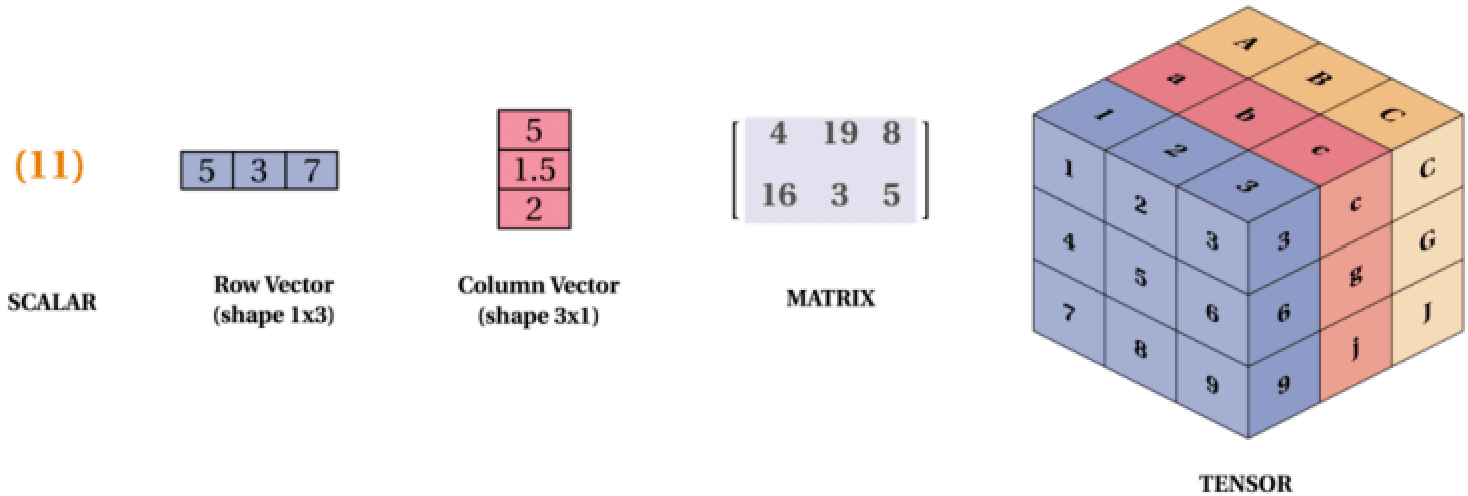


Tensor



Matrix

Tensor Terminology



- Scalars, vectors, and matrices are 0^{th}, 1^{st}, and 2^{nd} order tensors.
- For us, tensors mean a 3^{rd} or higher order tensor.

Clash of cultures

> This difference of cultures is particularly pronounced when discussing tensors: for some practitioners these are just multi-way arrays that one is allowed to perform certain manipulations on. For geometers these are spaces equipped with certain group actions. To emphasize the geometric aspects of tensors, geometers prefer to work invariantly: to paraphrase W. Fulton: "Don't use coordinates unless someone holds a pickle to your head."[1]

Preface, Tensors: Geometry and Applications,

J. M. Landsberg

Clash of cultures : What does "Matrix" mean to you?

What does "A Matrix" mean to you?

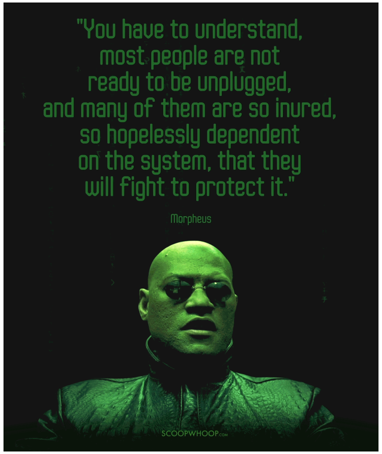
- linear transformation with Ax
- system of linear equations with $Ax = b$
- multilinear transformation with $x^T Ay$
- representation of algebraic structures/operations
- First derivative - Gradient / Jacobian
- Second derivative - Hessian
- Graphs with (weighted) adjacency and incidence matrices
- ...

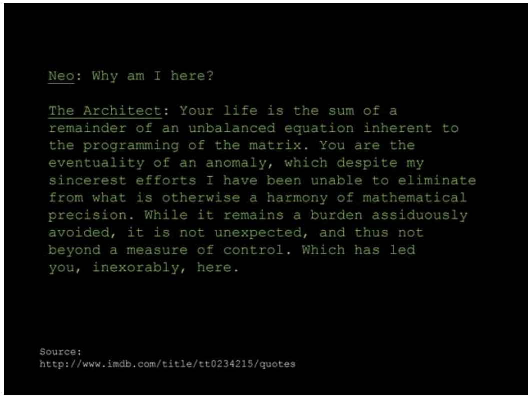
- *said the mathematician*

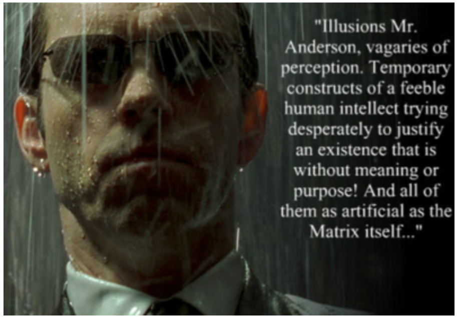
Clash of cultures : What does "Matrix" mean to you?

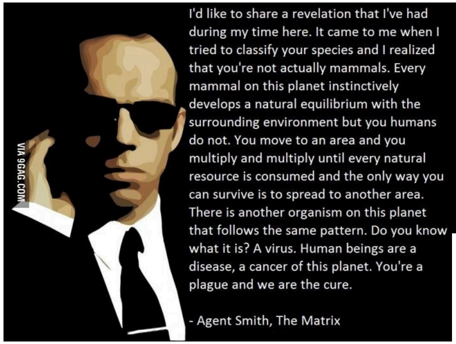
- Reality, Hyperreality, Projections of reality onto senses, Simulation

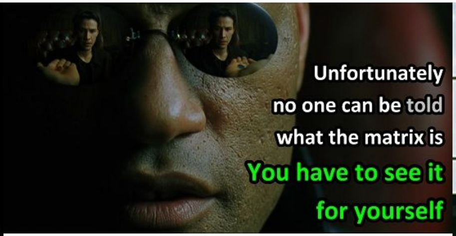
  - said the movie buff turned epistemologist

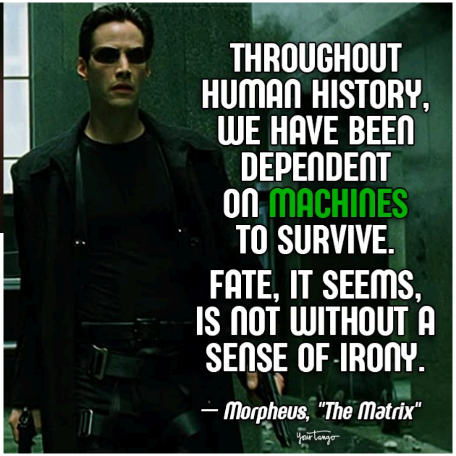




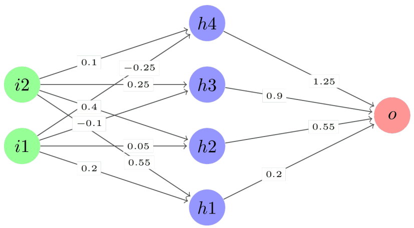


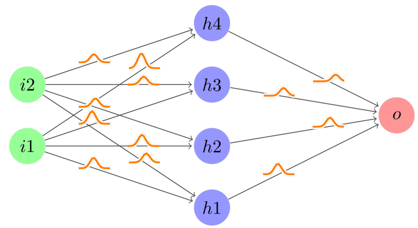


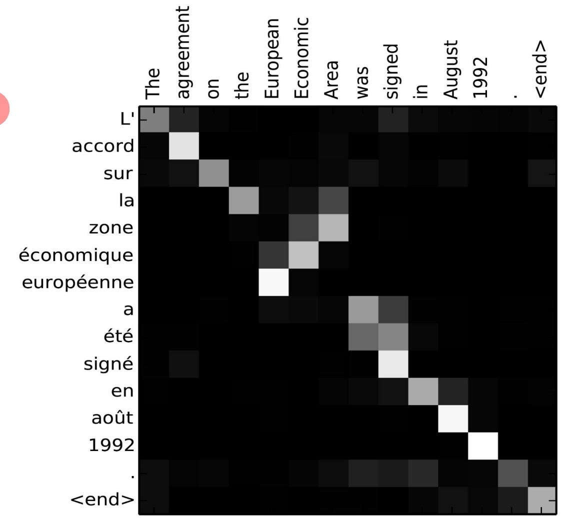




Clash of cultures : What does "Matrix" mean to you?







- Attention
- Parameters, weights and biases

- said the NLP and deeplearning guy

Clash of cultures : What does "Matrix" mean to you?

ChatGP T-800 Mainframe, 2053AD

- said Sam Altminator

Clash of cultures : What does "Matrix" mean to you?

- said the accidental engineer

Clash of cultures : What does "Matrix" mean to you?

No, I am Serious. Look closer

- said the accidental engineer

Processed traffic density (1 hour x 1km x 1km)

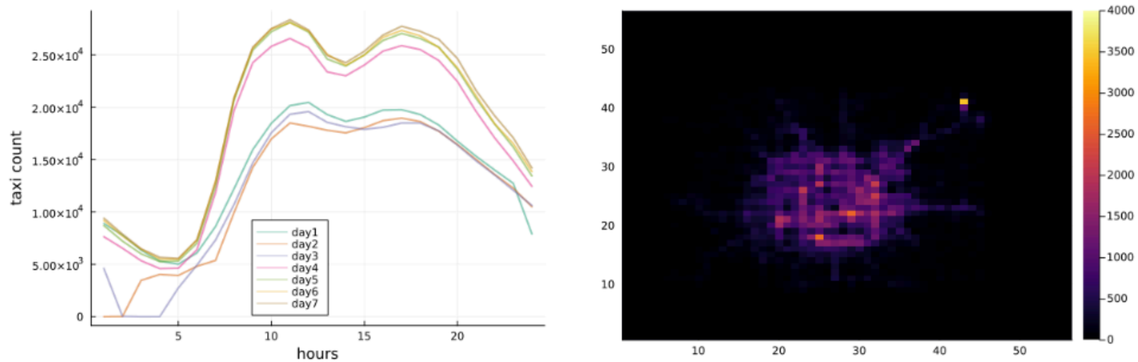


Fig. 2. Beijing traffic density temporal and spatial plot

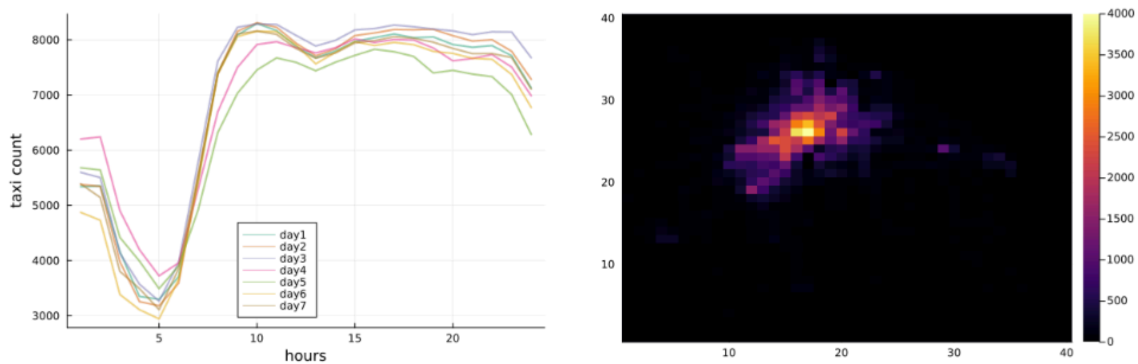


Fig. 3. Shanghai traffic density temporal and spatial plot

$X \times Y \times T$ tensors

Processed traffic density visualizations

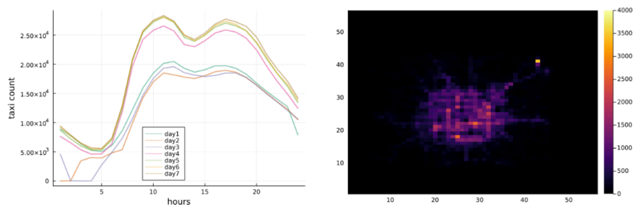


Fig. 2. Beijing traffic density temporal and spatial plot

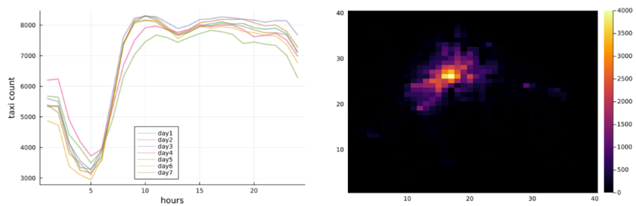
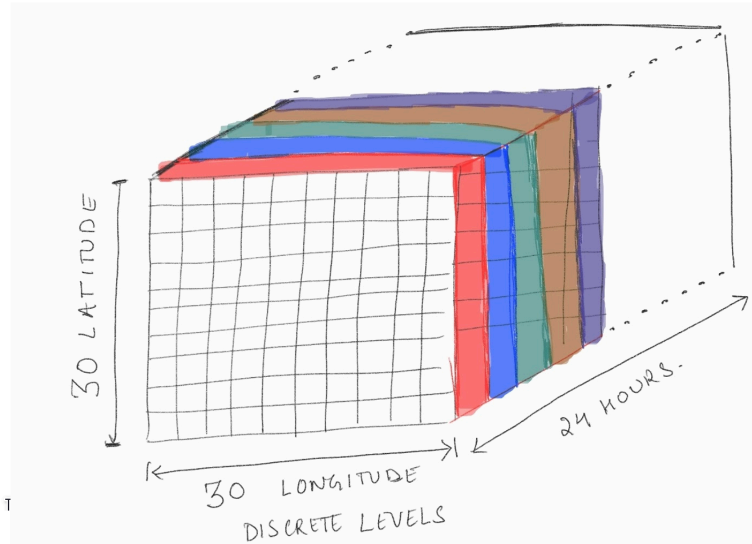


Fig. 3. Shanghai traffic density temporal and spatial plot



$$X \times Y \times T$$

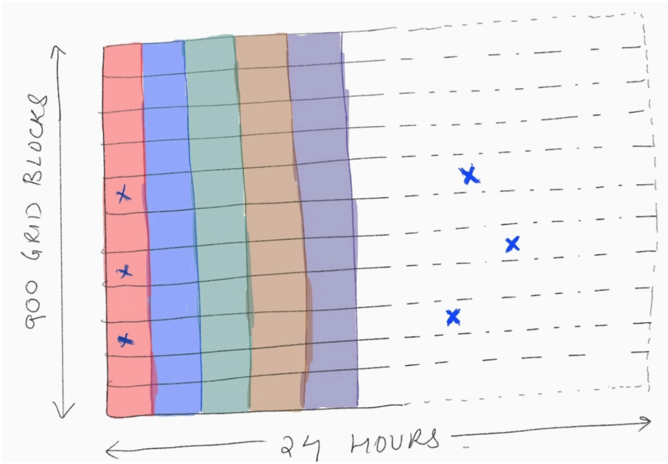
Spatial plot: Contract time dim. using vector $\mathbf{1_T} \in \mathbb{R}^T$

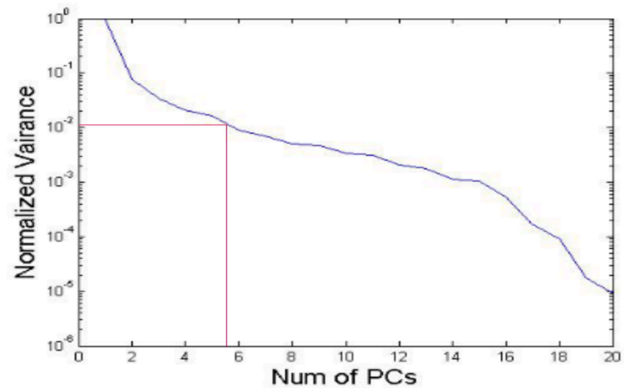
Temporal plot: Contract spatial dims. using vectors $\mathbf{1_X} \in \mathbb{R}^X$ and $\mathbf{1_Y} \in \mathbb{R}^Y$

Same as contraction with matrix dot/inner product on spatial dimensions using matrix $\mathbf{1_X 1'_Y} \in \mathbb{R}^{X \times Y}$ because $\mathbf{v^T B v} = \langle \mathbf{B}, \mathbf{v v^T} \rangle$

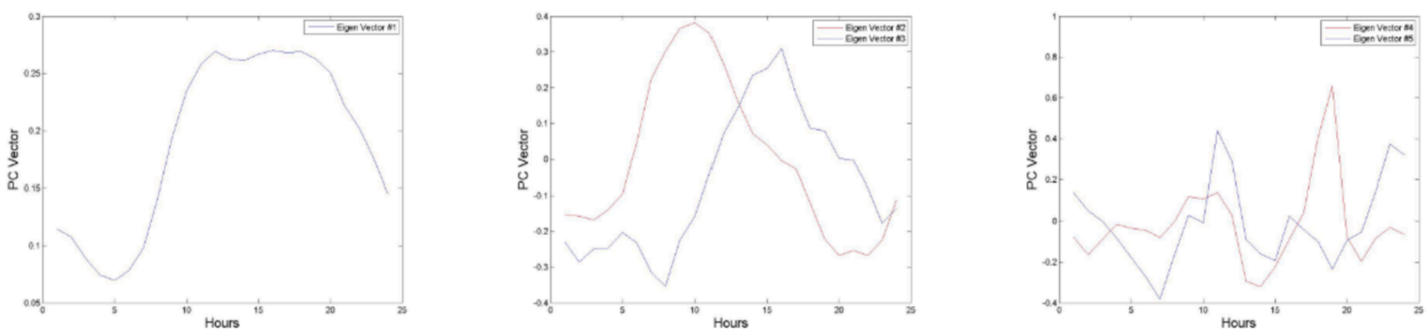
4. SELECTED PRIOR WORK

Prior work @ USC-ANRG + GM





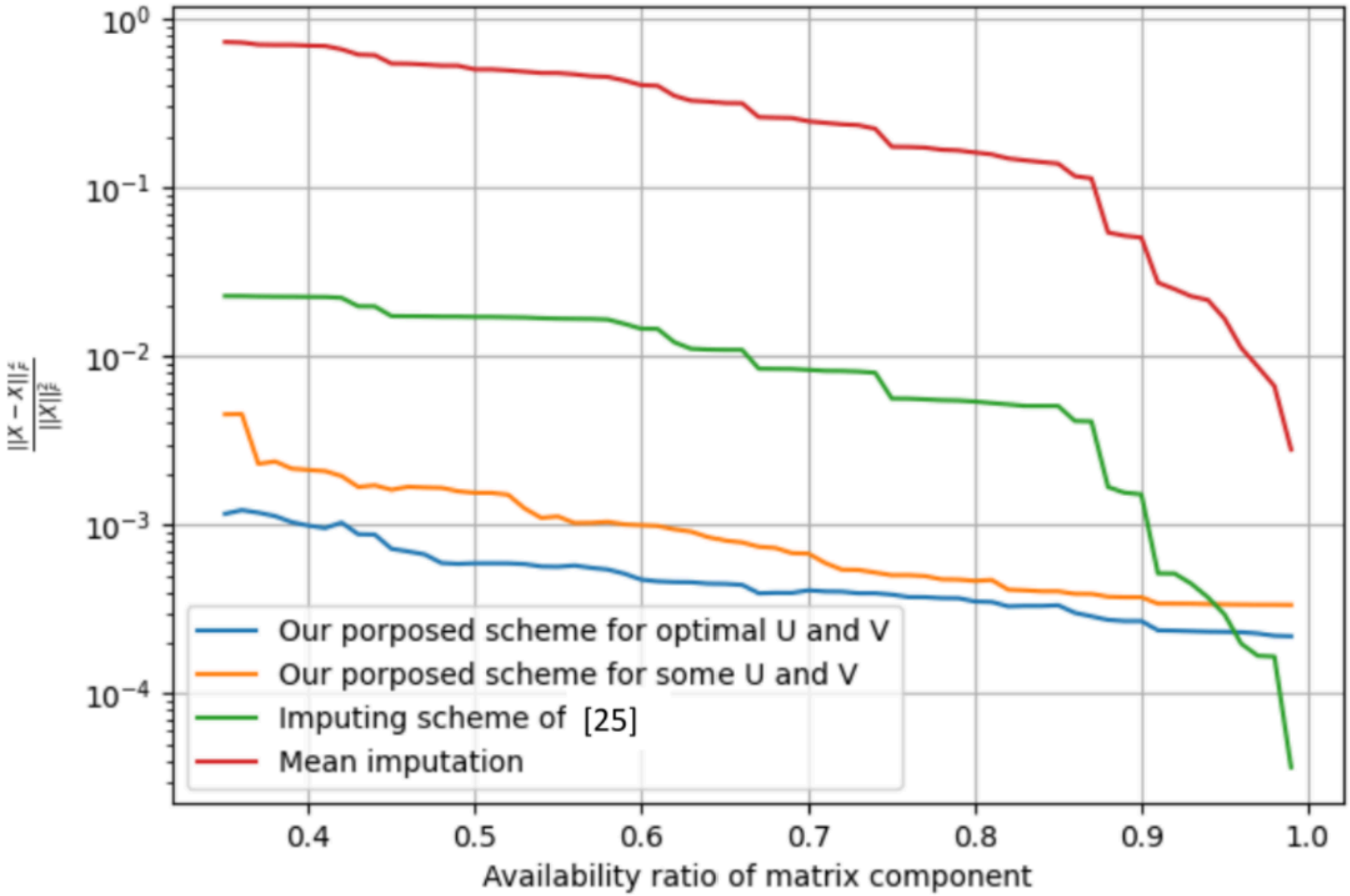
(b) Shanghai



(a) 1st PC (b) 2nd PC & 3rd PC (c) 4th & 5th PC

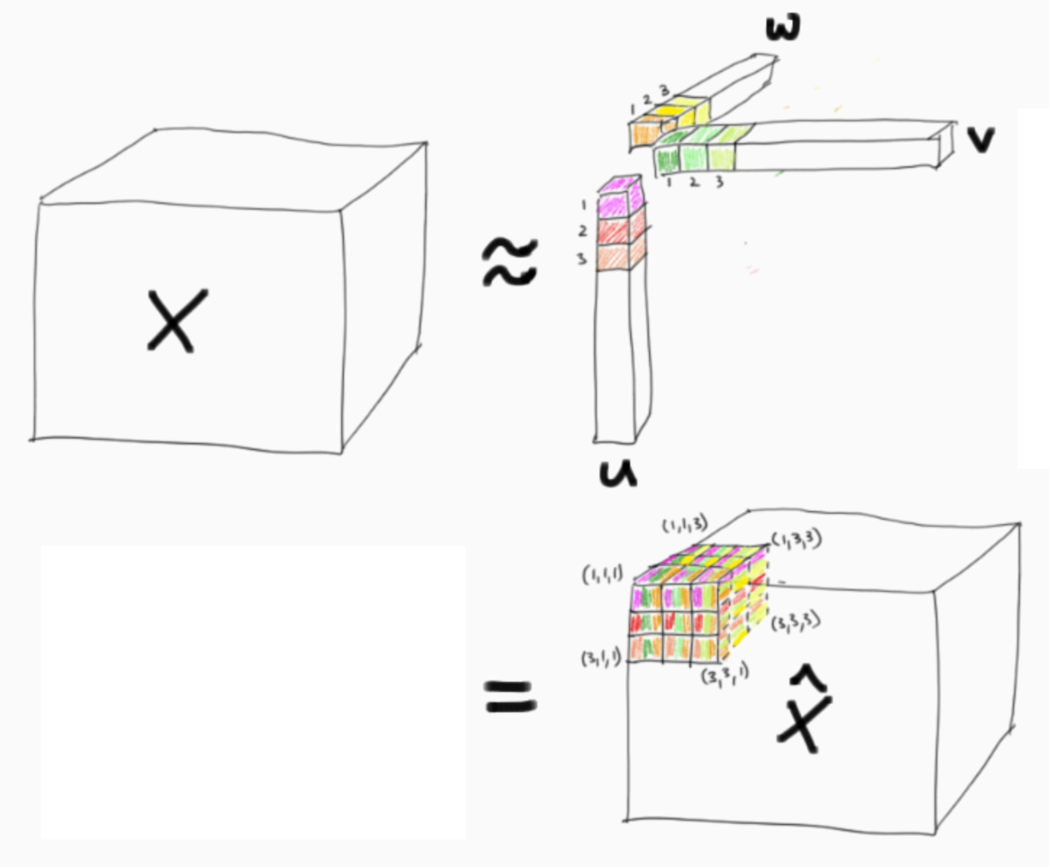
Fig. 8: The Temporal Principle Components (PCs) of Beijing Trace (May 1)

Prior work @ USC-ANRG + GM



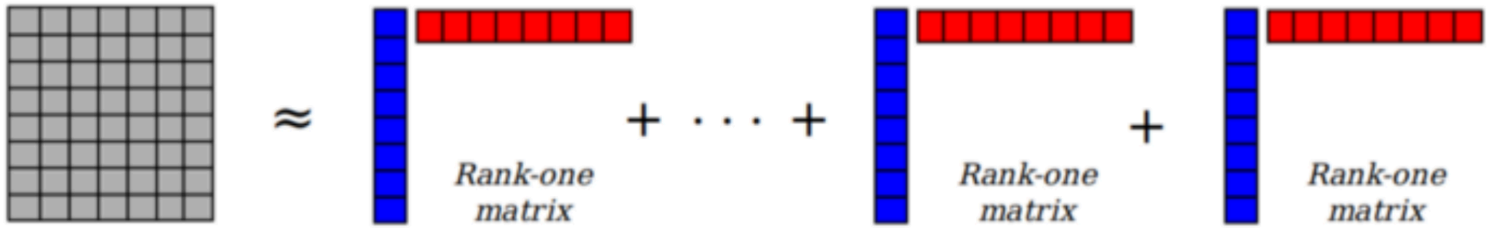
5. MORE ABOUT TENSORS

Atoms using (outer) products

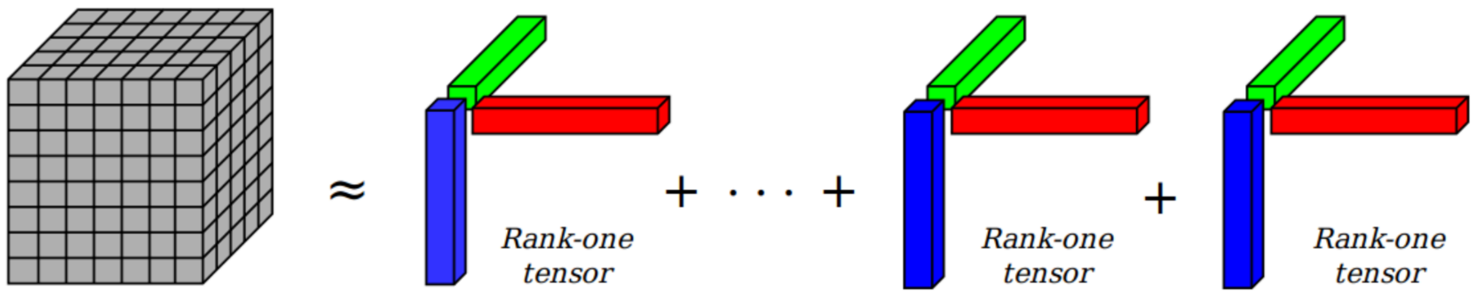


Atoms are also know as "Simple tensors"

Decomposition into sum of rank-1 atoms

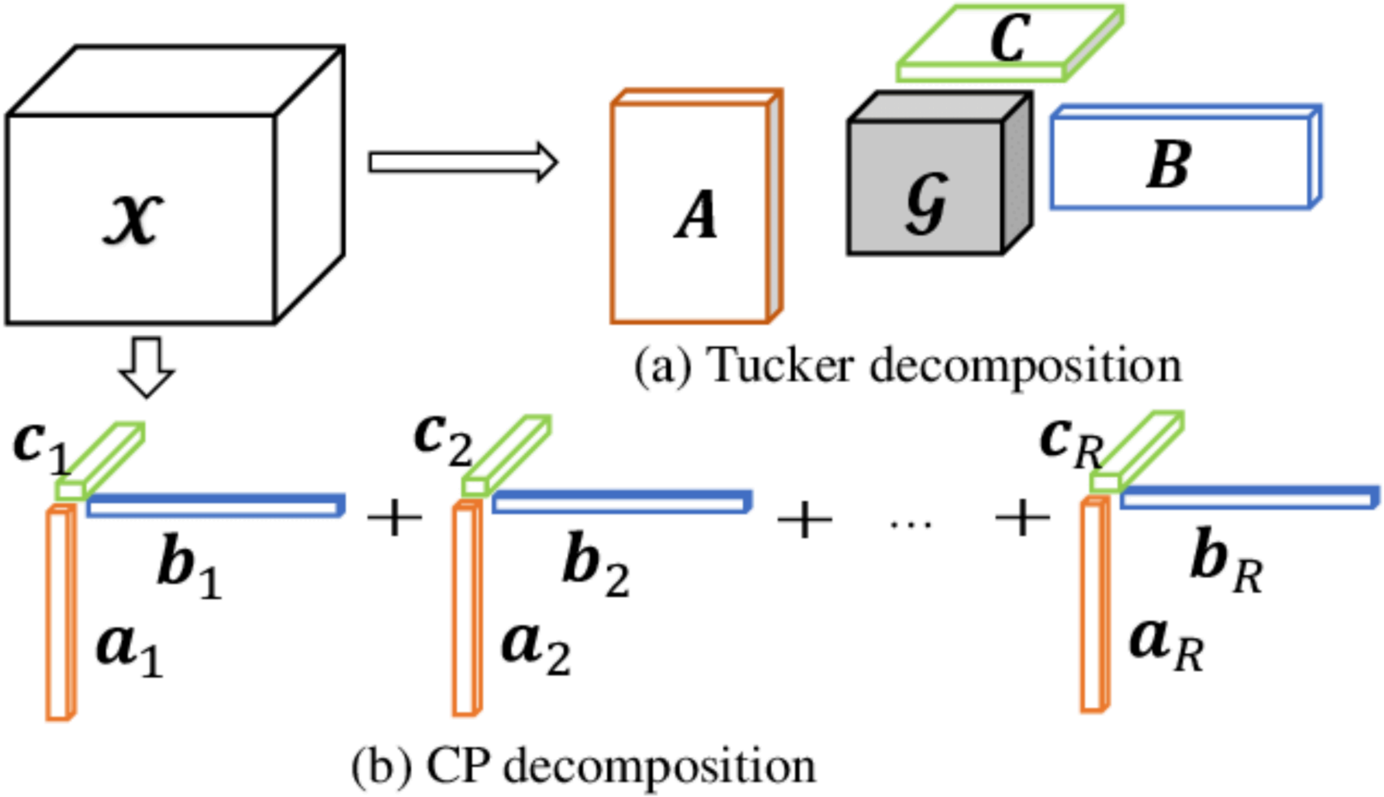


Matrix Decomposition



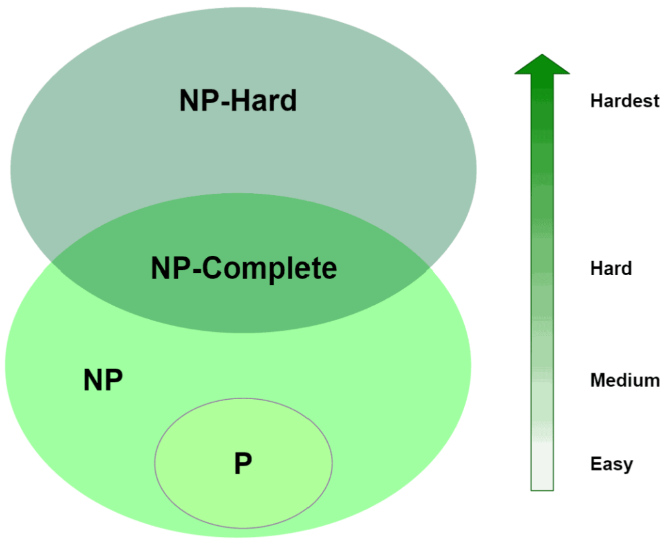
Tensor Decomposition

Some other decompositions



Tucker vs CP Decompositions

Matrix vs Tensor(order > 2) decomposition



Factorization	Time complexity	Notes
Cholesky,QR	$O(n^3)$	
SVD	$O(mn^2)$	$m \geq n$
EVD	$O(n^3)$	SVD(AA^T) + SVD(A^TA) = $2O(n^3)$
Accurate CP	NP Hard	$\because$ finding rank is NP Hard

Other peculiarities of Matrix vs Tensor decomposition

1. If A is a matrix of rank r over a field F then it has rank r over any extension field of F. Matrices have same ranks under $\mathbb{R}$ and $\mathbb{C}$ as the later extends the former. **

2. If A is a matrix of rank r over a field K then its rank over any sub-field of K is at least r . But it can be more also.**

3. The rank of a tensor(order > 2) could be different over $\mathbb{R}$ and $\mathbb{C}$.

** from math.stackexchange

Other peculiarities of Matrix vs Tensor decomposition

1.2.3. Aside: Differences between linear and multilinear algebra. Basic results from linear algebra are that "rank equals row rank equals column rank", i.e., for a linear map $f : A \to B$, $\mathrm{rank}(f) = \dim f(A) = \dim f^T(B^*)$. Moreover, the maximum possible rank is $\min\{\dim A, \dim B\}$, and for "most" linear maps $A \to B$ the maximum rank occurs. Finally, if $\mathrm{rank}(f) > 1$, the expression of f as a sum of rank one linear maps is never unique; there are parameters of ways of doing so.

We will see that all these basic results fail in multilinear algebra. Already for bilinear maps, $f : A \times B \to C$, the rank of f is generally different from $\dim f(A \times B)$, the maximum possible rank is generally greater than $\max\{\dim A, \dim B, \dim C\}$, and "most" bilinear maps have rank *less than* the maximum possible. Finally, in many cases of interest, the expression of f as a sum of rank one linear maps *is* unique, which turns out to be crucial for applications to signal processing and medical imaging.

Chapter 1, Tensors: Geometry and Applications, J. M. Landsberg

$$\mathbf{M} = \mathbf{A}\mathbf{B} = \boldsymbol{\alpha}\boldsymbol{\beta}$$

if $\boldsymbol{\alpha} = \frac{1}{\gamma}\mathbf{A}\mathbf{U}^*$, $\boldsymbol{\beta} = \gamma\mathbf{U}\mathbf{B}$ for any unitary matrix $\mathbf{U}$ and any scalar $\gamma \neq 0$.

Unique upto rotation/reflection/permutation and scaling.

Other peculiarities of Matrix vs Tensor decomposition

Krushkal Rank (also called spark of a matrix) vs Rank

6. OUR WORK

Hypothesis and Assumptions

Hypothesis

There exist two kinds of latent factors that completely explain traffic patters

- time invariant latent factors and
- time varying latent factors

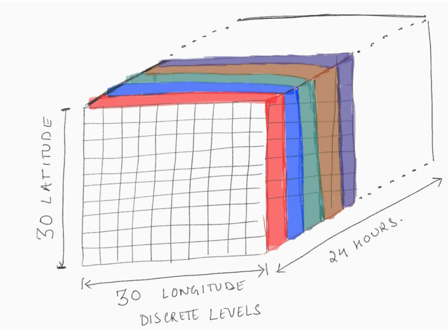
Assumptions

1. Observed data of a particular day can be explained by linear combination (weighted by the time varying factors) of the time invariant factors.
2. Let the rank-1 matrix/tensor atoms obtained by decomposition of the data representation be time invariant.
3. Let the decomposition weights on the rank one atoms be time variant

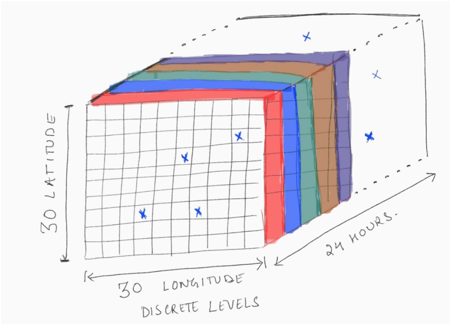
We will empirically validate above assumptions on our dataset.

7. Symbols, Definitions, Unified Framework

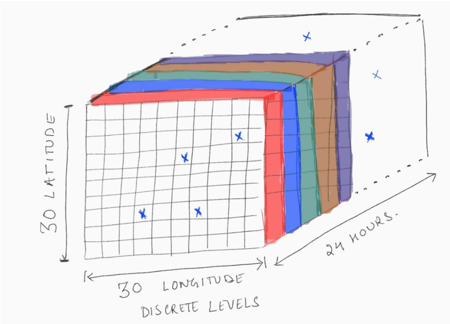
Ground Truth



$\odot$



$=$



$$\mathbf{T}(\mathbf{t}) \quad \odot \quad \boldsymbol{\Omega}(\boldsymbol{t}) \quad = \quad \mathbf{D}(\mathbf{t})$$

Where,

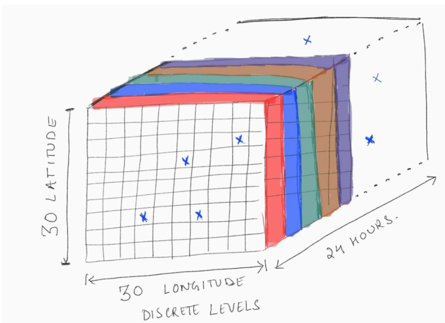
$\mathbf{T} \in \mathbb{R}^{X \times Y \times Z}$

$\boldsymbol{\Omega} \in \{\,0, 1\,\}^{X \times Y \times Z}$, observed locations marked as 1.

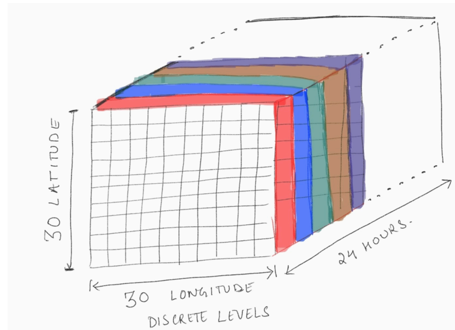
$\bar{\boldsymbol{\Omega}} = \mathbf{1} - \boldsymbol{\Omega}$, missing locations marked as 1.

$\mathbf{D} \in \mathbb{R}^{X \times Y \times Z}$

Reconstructor



$\xrightarrow{\phi}$



$$\underbrace{\mathbf{D}}_{\mathbf{T}\odot\mathbf{\Omega}} \quad \xrightarrow{\phi} \quad \tilde{\mathbf{T}}$$

$$\phi_\theta : \mathbb{R}^{X\times Y\times Z} \times \mathbb{R}^{X\times Y\times Z} \to \mathbb{R}^{X\times Y\times Z}$$

$$\tilde{\mathbf{T}} = \phi(\mathbf{D}, \mathbf{\Omega}; \boldsymbol{\theta})$$

latent parameters: $\mathbf{R}^p \ni \boldsymbol{\theta} = [\boldsymbol{\theta_1}, \boldsymbol{\theta_2}]$

Error metrics: Tensor Completion Score and Reconstruction error

$$\text{maskednorm } (\mathbf{T}, \Omega) = ||\mathbf{T}||_{\Omega} = ||\mathbf{T} \circ \Omega||_F$$

$$TCS(\mathbf{T}, \tilde{\mathbf{T}}, \Omega) := \frac{||\mathbf{T} - \tilde{\mathbf{T}}||_{\bar{\Omega}}}{||\mathbf{T}||_{\bar{\Omega}}} = \frac{||(\mathbf{T} - \tilde{\mathbf{T}}) \circ (\mathbf{1} - \Omega)||_F}{||\mathbf{T} \circ (\mathbf{1} - \Omega)||_F}$$

$$TRE(\mathbf{T}, \tilde{\mathbf{T}}, \Omega) := \frac{||(\mathbf{T} - \tilde{\mathbf{T}}) \circ \Omega||_F}{||\mathbf{T} \circ \Omega||_F}$$

Latent Parameters

$$\phi_\theta : \mathbb{R}^{X \times Y \times Z} \times \mathbb{R}^{X \times Y \times Z} \to \mathbb{R}^{X \times Y \times Z}$$

$$\tilde{\mathbf{T}}(t) = \phi(\mathbf{D}(t), \mathbf{\Omega}(t); \boldsymbol{\theta}(t), \boldsymbol{\psi}(t_0))$$

latent parameters: $\mathbf{R}^p \ni \boldsymbol{\theta} = [\boldsymbol{\theta_1}, \boldsymbol{\theta_2}]$

time invariant component: $\boldsymbol{\theta_1} \in \mathbf{R}^{p_1}$

time variant component: $\boldsymbol{\theta_2} \in \mathbf{R}^{p_2}$

$p_1 + p_2 = p$

ϕ may additionally depend on optional auxiliary information $\boldsymbol{\psi}(t_0)$ where $|t_0 - t| \leq \epsilon$ for some $\epsilon geq 0$

Latent Parameter Estimators

$$\mathbf{R}^p \in \boldsymbol{\theta} = [\boldsymbol{\theta_1}, \boldsymbol{\theta_2}] \leftarrow E(\mathbf{D}, \boldsymbol{\Omega}, Null, Null)$$

$$\mathbf{R}_2^p \in \boldsymbol{\theta_2} = \leftarrow E(\mathbf{D}, \boldsymbol{\Omega}, \boldsymbol{\theta_1}, Null)$$

$$p = p_1 + p_2$$

Note A:

The estimation is not perfect and depends on the following -

- missing data percentage or equivalently sparsity of $\boldsymbol{\Omega}$
- Noise in the observed entries in the data
- estimator efficiency as the estimators are approximate algorithms

However, this may give a more stable/better quality estimation of $\boldsymbol{\theta_1}$ due to the additional observations. Ofcourse this should qualitatively relate to the coherency between the observed tensors.

Vectorized Latent Parameter Estimators

$$\vec{\boldsymbol{\theta}} = [\boldsymbol{\theta_1}, \vec{\boldsymbol{\theta}}_\mathbf{2}] \leftarrow E(\vec{\mathbf{D}}, \vec{\boldsymbol{\Omega}}, Null, Null)$$

$$\vec{\boldsymbol{\theta}}_\mathbf{2} = \leftarrow E(\vec{\mathbf{D}}, \vec{\boldsymbol{\Omega}}, \boldsymbol{\theta_1}, Null)$$

Note A:

The estimation is not perfect and depends on the following -

- missing data percentage or equivalently sparsity of $\boldsymbol{\Omega}$
- Noise in the observed entries in the data
- estimator efficiency as the estimators are approximate algorithms

Data Generating Process

$$G(\boldsymbol{\theta_1}, \boldsymbol{\theta_2}) \to \mathbf{D}$$

Intuition: There is some physical process that generates the observed data $\mathbf{D}$ based on the latent parameters \boldsymbol{\theta}.

Question: If $p << X \times Y \times Z$, how can we have a bijection? Answer: We don't, the observed data is assumed to lie on a low dimensional manifold in a high dimensional ambient space + noise.

Can we remove noise? The generating process G will reconstruct without noise.

Is this manifold a convex set? Not in our model and assumptions. This is because the convex combination of two rank k matrices $\mathbf{A}, \mathbf{B}$ given by $\mathbf{C} = \alpha A + \beta B$ isn't necesarilly a rank k matrix when $\alpha + \beta = 1$ and $\alpha, \beta \geq 1$

e.g $\begin{bmatrix} 1 & 0, 2 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 7, 0 & 11 \end{bmatrix}$

SATORIS: Unified Framework

- **Key Idea 1:**We can reconstruct the data on the low dimensional manifold rejecting the noise if we know $\boldsymbol{\theta} = [\boldsymbol{\theta_1}, \boldsymbol{\theta_2}]$.
- **Key Idea 2:** Estimating the invariant parameter $\boldsymbol{\theta_1}(t)$ from highly noisy and sparse observation $\mathbf{D}(t)$ can be avoided and instead estimated from another better quality dataset from time $\mathbf{D}(t_0)$ where t_0 is close to t, i.e $|t - t_0| \leq \epsilon$ where $\epsilon \geq 0$ is a hyperparameter to our model along with the rank r of the tensor.

Quality of reconstruction

How good can the reconstruction be?

Note A:

The estimation is not perfect and depends on the following -

- missing data percentage or equivalently sparsity of $\boldsymbol{\Omega}$
- Noise in the observed entries in the data
- estimator efficiency as the estimators are approximate algorithms

Note B:

- How good the assumption on time invariance of $\boldsymbol{\theta_1}(t)$ holds between times t and t_0.

Two regimes

Unknown invariant paramters $\boldsymbol{\theta_1}(t)$

$$\tilde{\mathbf{T}}(t) = G(\boldsymbol{\theta_1}(t), \boldsymbol{\theta_2}(t))$$

$$\boldsymbol{\theta_1}(t), \boldsymbol{\theta_2}(t) \leftarrow E(\mathbf{D}(t), \boldsymbol{\Omega}(t), Null, Null)$$

Known invariant paramters $\boldsymbol{\theta_1}(t) = \boldsymbol{\theta_1}(t_0)$

We know $\mathbf{D}(t), \boldsymbol{\Omega}(t), \mathbf{D}(t_0), \boldsymbol{\Omega}(t_0), G, E$ and we want to find $\tilde{\mathbf{T}}(t)$

$$\tilde{\mathbf{T}}(t) = G(\boldsymbol{\theta_1}(t), \boldsymbol{\theta_2}(t)) = G(\boldsymbol{\theta_1}(t_0), \boldsymbol{\theta_2}(t))$$

$$\boldsymbol{\theta_1}(t_0) \leftarrow E(\mathbf{D}(t_0), \boldsymbol{\Omega}(t_0), Null, Null)$$

$$\boldsymbol{\theta_2}(t) \leftarrow E(\mathbf{D}(t), \boldsymbol{\Omega}(t), \boldsymbol{\theta_1}(t_0), Null)$$

Two regimes (Vectorized)

Unknown invariant paramters $\boldsymbol{\theta_1}(t)$

$$\color{green}{\vec{\tilde{\mathbf{T}}}(t)} = G(\color{orange}{\boldsymbol{\theta_1}(t),} \color{blue}{\vec{\boldsymbol{\theta}}_\mathbf{2}(t)})$$

$$\color{orange}{\boldsymbol{\theta_1}(t),} \color{blue}{\vec{\boldsymbol{\theta}}_\mathbf{2}(t)} \leftarrow E(\vec{\mathbf{D}}(t), \vec{\boldsymbol{\Omega}}(t), Null, Null)$$

Known invariant paramters $\boldsymbol{\theta_1}(t) = \boldsymbol{\theta_1}(t_0)$

We know $\vec{\mathbf{D}}(t), \boldsymbol{\Omega}(t), \vec{\mathbf{D}}(t_0), \boldsymbol{\Omega}(t_0), G, E$ and we want to find $\vec{\tilde{\mathbf{T}}}(t)$

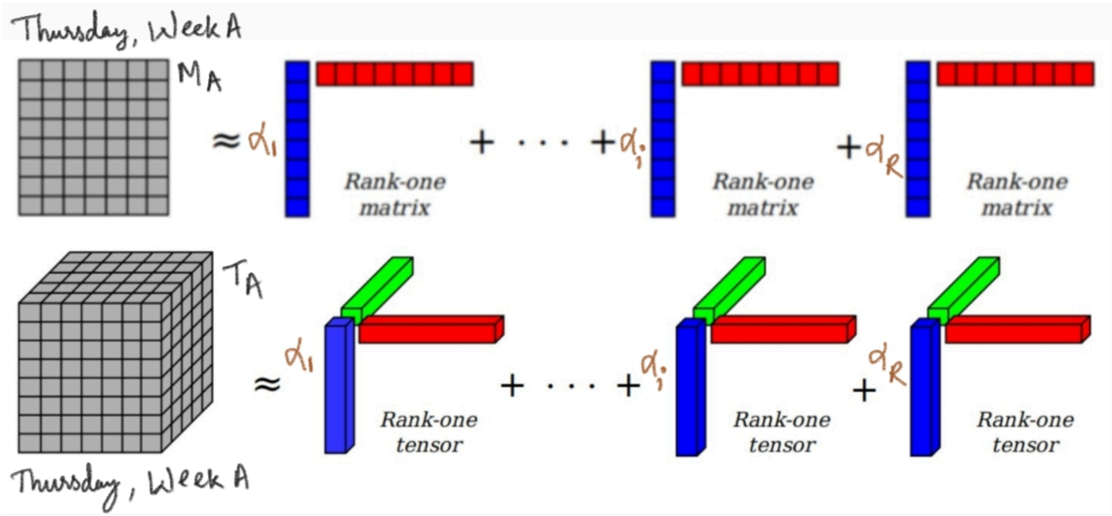
$$\color{green}{\vec{\tilde{\mathbf{T}}}(t)} = G(\color{orange}{\boldsymbol{\theta_1}(t),} \color{blue}{\vec{\boldsymbol{\theta}}_\mathbf{2}(t)}) = G(\color{orange}{\boldsymbol{\theta_1}(t_0),} \color{blue}{\vec{\boldsymbol{\theta}}_\mathbf{2}(t)})$$

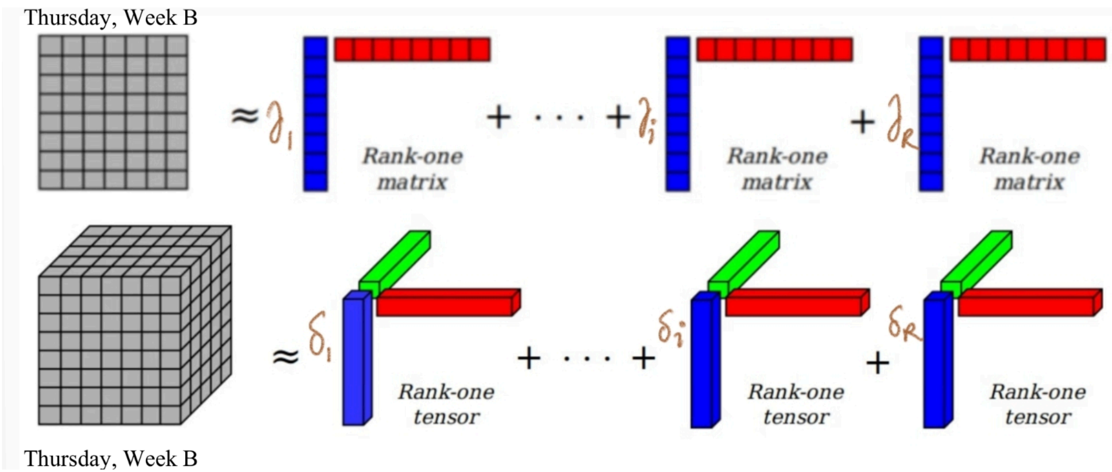
$$\color{orange}{\boldsymbol{\theta_1}(t_0)} \leftarrow E(\vec{\mathbf{D}}(t_0), \vec{\boldsymbol{\Omega}}(t_0), Null, Null)$$

$$\color{blue}{\vec{\boldsymbol{\theta}}_\mathbf{2}(t)} \leftarrow E(\mathbf{D}(t), \boldsymbol{\Omega}(t), \boldsymbol{\theta_1}(t_0), Null)$$

PROPOSED METHODS

Singular Value and Tensor Weight Regression





EXPERIMENTAL SETUP

Setup

Category	Description
1day	Just the target day.
2next	The target day and the corresponding day from the next week. For example, if the target day is Monday of week10, we consider the set Monday of week 10, Monday of week 11.
2prev	The target day and the corresponding day from the previous week. For example, if the target day is Thursday of week32, we consider the set Thursday of week 32, Thursday of week 31.
3adj	The target day, the day just before and just after the target day of the same week. Hence if the target day is Wednesday of week 15 then the set we consider is Tuesday of week 15, Wednesday of week 15, Thursday of week 15.
5adj	The target day, the two days before and the two days after the target day of the same week. Hence if the target day is Wednesday of week 15 then the set we consider is Monday, Tuesday, Wednesday, Thursday, Friday all from week 15.

Table 2. Dataset organisation wrt temporal proximity

RESULTS

1 Day

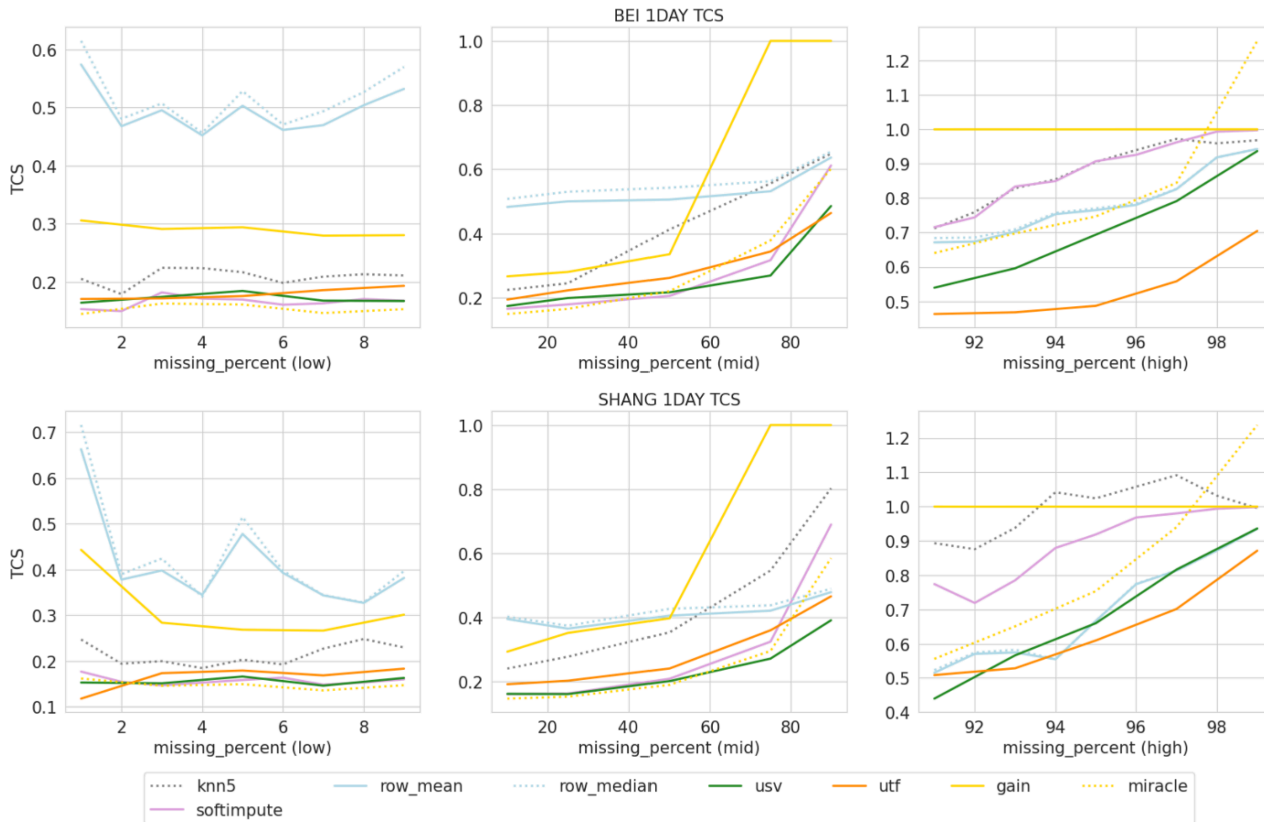


Fig. 7. 1 day performance

3 Day

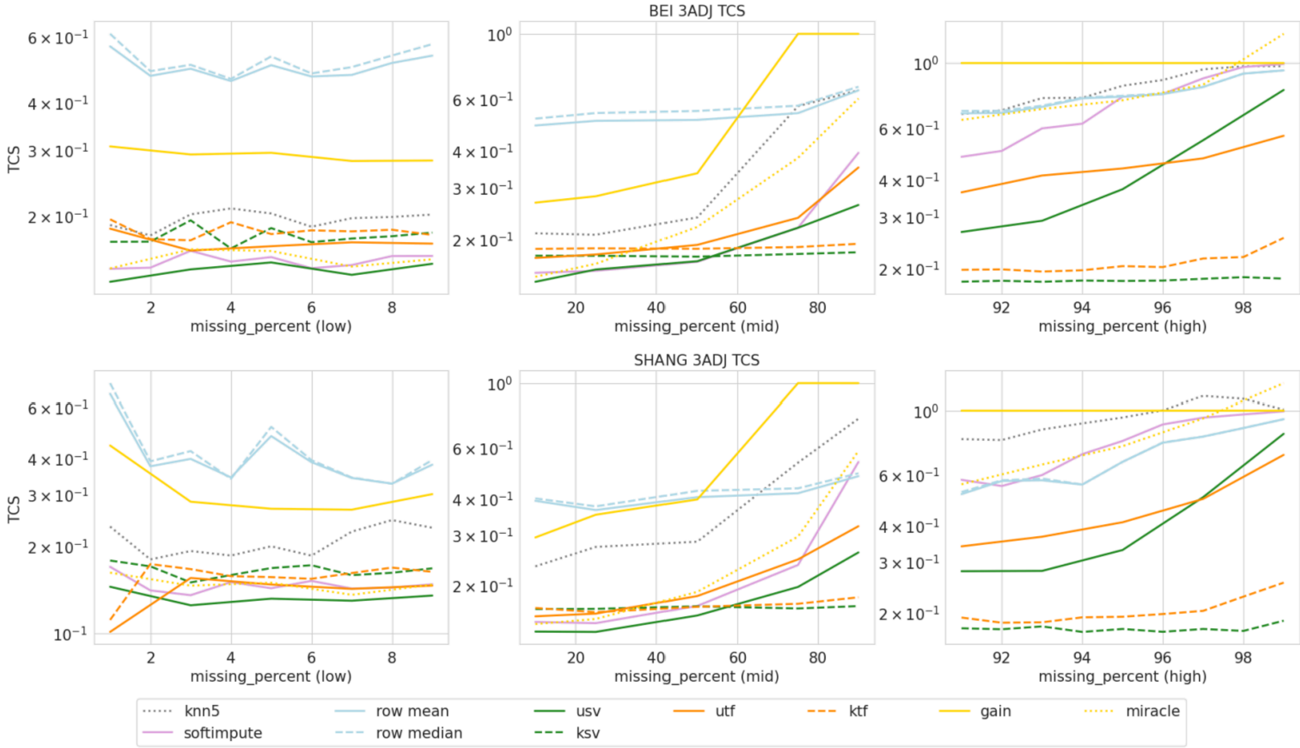


Fig. 8. 3 days performance

5 Days

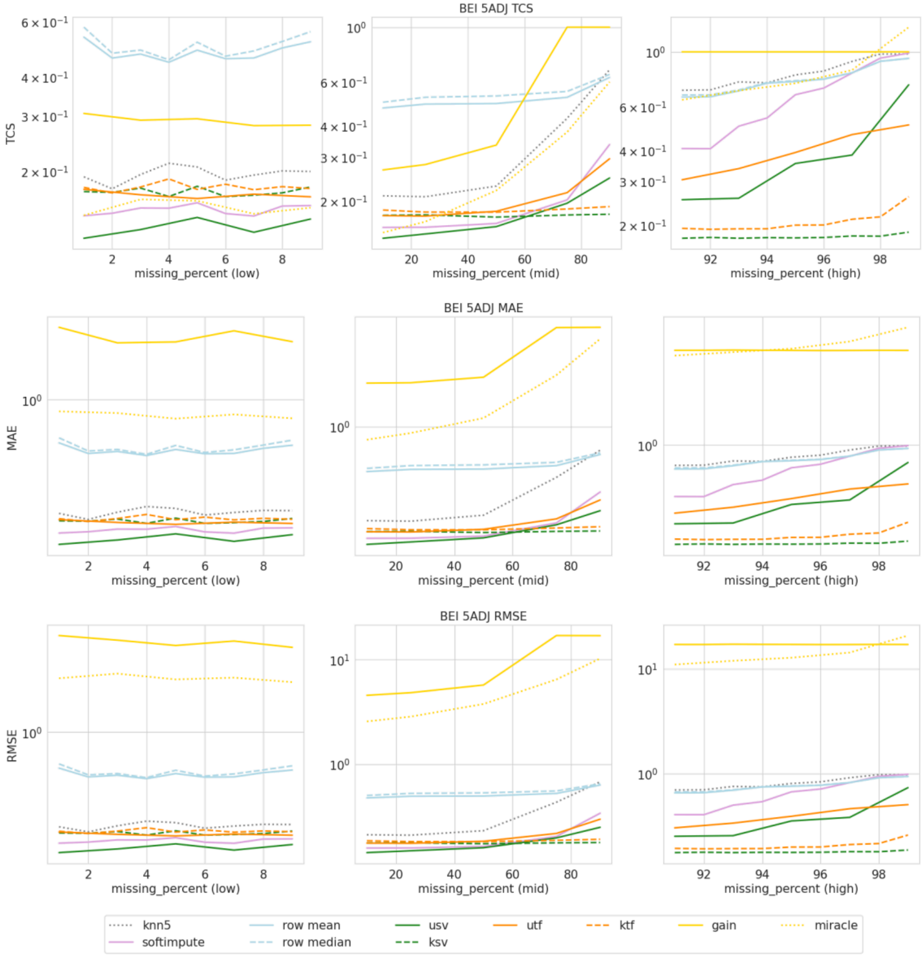


Fig. 16. 5 days performance (Beijing)

THANK YOU VERY MUCH

More resutls in the paper.

Also about verification of our assumptions of time invariance of the matrix/tensor atoms.

link to paper