Lecture Notes 2
Oct 24

A proposition:

1. A way things can be.
2. The content of declarative sentence.
    + Jenya is from Russia.
    + Jenya kommt aus Russland.

These two sentences express the same proposition.

Propositions can be thought of as sets of possible worlds.

+ A possible world is a maximally informative way in which things can be.

We can then use the possible worlds framework to understand relations between propositions:

1. Equivalence: if two propositions are made up of the same possible worlds, then they are equivalent.
2. Entailment: if the possible worlds in A are all also in B, then A entails B.
3. Mutual exclusivity: if there is no possible world that is in both A and B, then A and B are mutually exclusive.

And logical operations like:

1. Or: A or B is the union of the possible worlds in A and the possible worlds in B.
2. And: A and B is the intersection of the possible worlds in A and the possible worlds in B.
3. Not: ¬A is the set of possible worlds not in A.

Examples:

1. Kamala Harris is from France
2. All cats are fluffy.
3. The person who is actually Joe Biden's vice president is from France.
4. Either Kamala Harris from France or all cats are fluffy.
5. It's false that: KH is from France or all cats are fluffy.

A *distribution* is a map from propositions to real numbers.

A distribution is a *probability distribution* when it satisfies the kolmogorov axioms.

Kolmogorov axioms.

1. Non-negativity: For any proposition: $C(P) \geq 0$
2. Normality: For any tautology $\top$: $C(\top) = 1$
3. Finite additivity: For any two mutually exclusive proposition P and Q, $C(P \vee Q) = C(P) + C(Q)$.

Explain why the following aren't probability distributions.

Case 1.

+ $C(P \vee \neg P) = .9$

Case 2.

+ $C(P) = .5$
+ $C(\neg P) = .4$

Case 3.

+ $C(P \wedge \neg P) = .01$

Case 4.

+ $C(P) = .5$
+ $C(P \vee (\neg P \wedge Q)) = .4$

Case 5.

+ $C(P) = 1.2$