The First-order Autoregression Model

We'll now look at theoretical properties of the AR(1) model. Recall from Lesson 1.1, that the 1st order autoregression model is denoted as AR(1). In this model, the value of x at time t is a linear function of the value of x at time $t-1$. The algebraic expression of the model is as follows:

$$x_t = \delta + \phi_1 x_{t-1} + w_t$$

Assumptions

- $w_t \overset{iid}{\sim} N(0, \sigma_w^2)$, meaning that the errors are independently distributed with a normal distribution that has mean 0 and constant variance.
- Properties of the errors w_t are independent of x_t.
- The series x_1, x_2, ... is (weakly) stationary. A requirement for a stationary AR(1) is that $|\phi_1| < 1$. We'll see why below.

Properties of the AR(1)

Formulas for the mean, variance, and ACF for a time series process with an AR(1) model follow.

- The (theoretical) **mean** of x_t is

$$E(x_t) = \mu = \frac{\delta}{1-\phi_1}$$

- The **variance** of x_t is

$$\text{Var}(x_t) = \frac{\sigma_w^2}{1-\phi_1^2}$$

- The **correlation** between observations h time periods apart is

$$\rho_h = \phi_1^h$$

This defines the theoretical ACF for a time series variable with an AR(1) model.

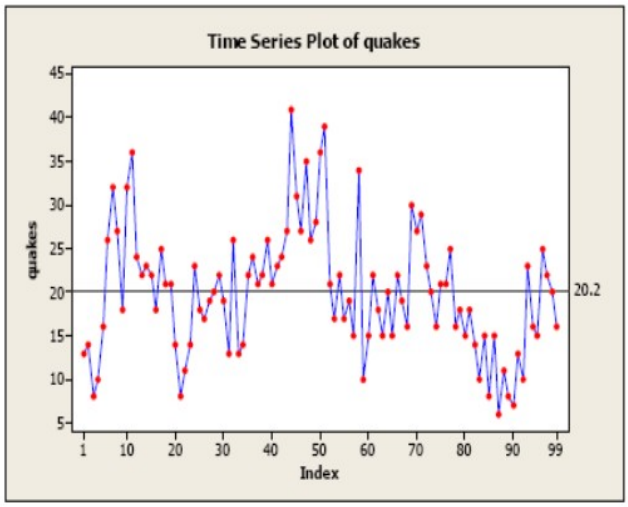
> **Note!**
> ϕ_1 is the slope in the AR(1) model and we now see that it is also the lag 1 autocorrelation.

Pattern of ACF for AR(1) Model

The ACF property defines a distinct pattern for the autocorrelations. For a positive value of ϕ_1, the ACF exponentially decreases to 0 as the lag h increases. For negative ϕ_1, the ACF also exponentially decays to 0 as the lag increases, but the algebraic signs for the autocorrelations alternate between positive and negative.

Problem 1

The following plot is a **time series plot** of the annual number of earthquakes in the world with seismic magnitude over 7.0, for 99 consecutive years. By a time series plot, we simply mean that the variable is plotted against time.



Problem 3

Comment on the features of the above seismic data

Problem 4

Based on the seismic data of 98 cases the AR(1) model with one missing case is obtained as

quakes = 9.19 + 0.543 lag1

Find the coefficients and examine the significance at lag 1. Plot the ACF at 5% LOS and and draw your conclusions.