

[\[back to article\]](#)

The Matrix Cheatsheet by Sebastian Raschka is licensed under a Creative Commons Attribution 4.0 International License.

(last updated: June 22, 2018)

Task

MATLAB/Octave

Python NumPy

R

Julia

Task

CREATING MATRICES

Creating Matrices

(here: 3x3 matrix)

```
M> A = [ 1 2 3; 4 5 6; 7 8 9]
A =
     1     2     3
     4     5     6
     7     8     9
```

```
P> A = np.array([ [1,2,3], [4,5,6],
                 [7,8,9] ])
P> A
array([[1, 2, 3],
       [4, 5, 6],
       [7, 8, 9]])
```

```
R> A =
matrix(c(1,2,3,4,5,6,7,8,9),nrow=3,byrow=T)
# equivalent to
# A = matrix(1:9,nrow=3,byrow=T)
R> A
[,1] [,2] [,3]
[1,] 1 2 3
[2,] 4 5 6
[3,] 7 8 9
```

```
J> A=[1 2 3; 4 5 6; 7 8 9]
3x3 Array{Int64,2}:
1 2 3
4 5 6
7 8 9
```

Creating Matrices

(here: 3x3 matrix)

Creating an column vector (nx1 matrix)

```
M> a = [1; 2; 3]
a =
     1
     2
     3
```

```
P> a = np.array([1,2,3]).reshape(3,1)
P> a.shape
(3, 1)
```

```
R> a = matrix(c(1,2,3), nrow=3, byrow=T)
R> a
[,1]
[1,] 1
[2,] 2
[3,] 3
```

```
J> a=[1; 2; 3]
3-element Array{Int64,1}:
1
2
3
```

Creating a column vector (nx1 matrix)

Creating an row vector (1xn matrix)

```
M> b = [1 2 3]
b =
     1     2     3
```

```
P> b = np.array([1,2,3]).reshape(1, 3)
P> b
array([[1],
       [2],
       [3]])
# note that in numpy, 1D arrays
# can be multiplied
# with 2d arrays, too
```

```
R> b = matrix(c(1,2,3), ncol=3)
R> b
[,1] [,2] [,3]
[1,] 1 2 3
```

```
J> b=[1 2 3]
1x3 Array{Int64,2}:
1 2 3
# note that this is a 2D array.
```

Creating an row vector (1xn matrix)

Creating a random m x n matrix

```
M> rand(3,2)
ans =
```

```
P> np.random.rand(3,2)
array([[ 0.29347865,  0.17920462],
```

```
R> matrix(runif(3*2), ncol=2)
[,1] [,2]
```

```
J> rand(3,2)
3x2 Array{Float64,2}:
```

Creating a

random m x n matrix

Creating a

zero m x n matrix

```
M> zeros(3,2)
```

```
ans =
```

```
0 0
```

```
0 0
```

```
0 0
```

Creating an

m x n matrix of ones

```
M> ones(3,2)
```

```
ans =
```

```
1 1
```

```
1 1
```

```
1 1
```

Creating an

identity matrix

```
M> eye(3)
```

```
ans =
```

```
Diagonal Matrix
```

```
1 0 0
```

```
0 1 0
```

```
0 0 1
```

Creating a

diagonal matrix

```
M> a = [1 2 3]
```

```
M> diag(a)
```

```
ans =
```

```
Diagonal Matrix
```

```
1 0 0
```

```
0 2 0
```

```
0 0 3
```

```
P> np.zeros((3,2))
```

```
array([[ 0.,  0.],
```

```
[ 0.,  0.],
```

```
[ 0.,  0.]])
```

```
P> np.ones((3,2))
```

```
array([[ 1.,  1.],
```

```
[ 1.,  1.],
```

```
[ 1.,  1.]])
```

```
P> np.eye(3)
```

```
array([[ 1.,  0.,  0.],
```

```
[ 0.,  1.,  0.],
```

```
[ 0.,  0.,  1.]])
```

```
P> a = np.array([1,2,3])
```

```
P> np.diag(a)
```

```
array([[1, 0, 0],
```

```
[0, 2, 0],
```

```
[0, 0, 3]])
```

```
[1,] 0.5675127 0.7751204
```

```
[2,] 0.3439412 0.5261893
```

```
[3,] 0.2273177 0.223438
```

```
R> mat.or.vec(3, 2)
```

```
[,1] [,2]
```

```
[1,] 0 0
```

```
[2,] 0 0
```

```
[3,] 0 0
```

```
R> matrix(1L, 3, 2)
```

```
[,1] [,2]
```

```
[1,] 1 1
```

```
[2,] 1 1
```

```
[3,] 1 1
```

```
R> diag(3)
```

```
[,1] [,2] [,3]
```

```
[1,] 1 0 0
```

```
[2,] 0 1 0
```

```
[3,] 0 0 1
```

```
R> diag(1:3)
```

```
[,1] [,2] [,3]
```

```
[1,] 1 0 0
```

```
[2,] 0 2 0
```

```
[3,] 0 0 3
```

```
0.36882 0.267725
```

```
0.571856 0.601524
```

```
0.848084 0.858935
```

```
J> zeros(3,2)
```

```
3x2 Array{Float64,2}:
```

```
0.0 0.0
```

```
0.0 0.0
```

```
0.0 0.0
```

```
J> ones(3,2)
```

```
3x2 Array{Float64,2}:
```

```
1.0 1.0
```

```
1.0 1.0
```

```
1.0 1.0
```

```
J> eye(3)
```

```
3x3 Array{Float64,2}:
```

```
1.0 0.0 0.0
```

```
0.0 1.0 0.0
```

```
0.0 0.0 1.0
```

```
J> a=[1, 2, 3]
```

```
# added commas because julia
```

```
# vectors are columnar
```

```
J> diagm(a)
```

```
3x3 Array{Int64,2}:
```

```
1 0 0
```

```
0 2 0
```

```
0 0 3
```

Creating a

zero m x n matrix

Creating an

m x n matrix of ones

Creating an

identity matrix

Creating a

diagonal matrix

ACCESSING MATRIX ELEMENTS

Getting the dimension
of a matrix
(here: 2D, rows x cols)

```
M> A = [ 1 2 3; 4 5 6]
A =
     1     2     3
     4     5     6
```

```
M> size(A)
ans =
     2     3
```

Selecting rows

```
M> A = [ 1 2 3; 4 5 6; 7 8 9]
```

```
% 1st row
```

```
M> A(1,:)
ans =
     1     2     3
```

```
% 1st 2 rows
```

```
M> A(1:2,:)
ans =
     1     2     3
     4     5     6
```

Selecting columns

```
M> A = [ 1 2 3; 4 5 6; 7 8 9]
```

```
% 1st column
```

```
M> A(:,1)
ans =
     1
     4
     7
```

```
% 1st 2 columns
```

```
M> A(:,1:2)
ans =
     1     2
     4     5
     7     8
```

Extracting rows and columns by
criteria

(here: get rows that have value 9 in
column 3)

```
M> A = [ 1 2 3; 4 5 9; 7 8 9]
A =
     1     2     3
     4     5     9
     7     8     9
```

```
P> A = np.array([ [1,2,3], [4,5,6] ])
P> A
array([[1, 2, 3],
       [4, 5, 6]])
P> A.shape
(2, 3)
P> A = np.array([ [1,2,3], [4,5,6], [7,8,9] ])
# 1st row
P> A[0,:]
array([1, 2, 3])
# 1st 2 rows
P> A[0:2,:]
array([[1, 2, 3], [4, 5, 6]])
P> A = np.array([ [1,2,3], [4,5,6], [7,8,9] ])
# 1st column (as row vector)
P> A[:,0]
array([1, 4, 7])
# 1st column (as column vector)
P> A[:,[0]]
array([[1],
       [4],
       [7]])
# 1st 2 columns
P> A[:,0:2]
array([[1, 2],
       [4, 5],
       [7, 8]])
P> A
array([[1, 2, 3],
```

```
# 1st row
```

```
P> A[0,:]
array([1, 2, 3])
```

```
# 1st 2 rows
```

```
P> A[0:2,:]
array([[1, 2, 3], [4, 5, 6]])
```

```
P> A = np.array([ [1,2,3], [4,5,6], [7,8,9] ])
# 1st column (as row vector)
P> A[:,0]
array([1, 4, 7])
# 1st column (as column vector)
P> A[:,[0]]
array([[1],
       [4],
       [7]])
# 1st 2 columns
P> A[:,0:2]
array([[1, 2],
       [4, 5],
       [7, 8]])
P> A = np.array([ [1,2,3], [4,5,9], [7,8,9] ])
P> A
array([[1, 2, 3],
```

```
# 1st column (as row vector)
```

```
P> A[:,0]
array([1, 4, 7])
```

```
# 1st column (as column vector)
```

```
P> A[:,[0]]
array([[1],
       [4],
       [7]])
```

```
# 1st 2 columns
```

```
P> A[:,0:2]
array([[1, 2],
       [4, 5],
       [7, 8]])
```

```
P> A = np.array([ [1,2,3], [4,5,9], [7,8,9] ])
P> A
array([[1, 2, 3],
```

```
P> A
```

```
array([[1, 2, 3],
```

```
R> A = matrix(1:6,nrow=2,byrow=T)
R> A
[,1] [,2] [,3]
[1,] 1 2 3
[2,] 4 5 6
R> dim(A)
[1] 2 3
R> A = matrix(1:9,nrow=3,byrow=T)
# 1st row
R> A[1,]
[1] 1 2 3
# 1st 2 rows
R> A[1:2,]
[,1] [,2] [,3]
[1,] 1 2 3
[2,] 4 5 6
R> A = matrix(1:9,nrow=3,byrow=T)
# 1st column as row vector
R> t(A[,1])
[,1] [,2] [,3]
[1,] 1 4 7
# 1st column as column vector
R> A[,1]
[1] 1 4 7
# 1st 2 columns
R> A[,1:2]
[,1] [,2]
[1,] 1 2
[2,] 4 5
[3,] 7 8
R> A = matrix(1:9,nrow=3,byrow=T)
R> A
[,1] [,2] [,3]
```

```
# 1st row
```

```
R> A[1,]
[1] 1 2 3
```

```
# 1st 2 rows
```

```
R> A[1:2,]
[,1] [,2] [,3]
[1,] 1 2 3
[2,] 4 5 6
```

```
R> A = matrix(1:9,nrow=3,byrow=T)
# 1st column as row vector
R> t(A[,1])
[,1] [,2] [,3]
[1,] 1 4 7
# 1st column as column vector
R> A[,1]
[1] 1 4 7
# 1st 2 columns
R> A[,1:2]
[,1] [,2]
[1,] 1 2
[2,] 4 5
[3,] 7 8
R> A = matrix(1:9,nrow=3,byrow=T)
R> A
[,1] [,2] [,3]
```

```
# 1st column as row vector
```

```
R> t(A[,1])
[,1] [,2] [,3]
[1,] 1 4 7
```

```
# 1st column as column vector
```

```
R> A[,1]
[1] 1 4 7
```

```
# 1st 2 columns
```

```
R> A[,1:2]
[,1] [,2]
[1,] 1 2
[2,] 4 5
[3,] 7 8
```

```
R> A = matrix(1:9,nrow=3,byrow=T)
```

```
R> A
```

```
[,1] [,2] [,3]
```

```
J> A=[ 1 2 3; 4 5 6]
2x3 Array{Int64,2}:
 1 2 3
 4 5 6
J> size(A)
(2,3)
J> A=[ 1 2 3; 4 5 6; 7 8 9];
#semicolon suppresses output
#1st row
J> A[1,:]
1x3 Array{Int64,2}:
 1 2 3
#1st 2 rows
J> A[1:2,:]
2x3 Array{Int64,2}:
 1 2 3
 4 5 6
J> A=[ 1 2 3; 4 5 6; 7 8 9];
#1st column
J> A[:,1]
3-element Array{Int64,1}:
 1
 4
 7
#1st 2 columns
J> A[:,1:2]
3x2 Array{Int64,2}:
 1 2
 4 5
 7 8
J> A=[ 1 2 3; 4 5 9; 7 8 9]
3x3 Array{Int64,2}:
 1 2 3
 4 5 9
```

```
#1st row
```

```
J> A[1,:]
1x3 Array{Int64,2}:
 1 2 3
```

```
#1st 2 rows
```

```
J> A[1:2,:]
2x3 Array{Int64,2}:
 1 2 3
 4 5 6
```

```
J> A=[ 1 2 3; 4 5 6; 7 8 9];
```

```
#1st column
```

```
J> A[:,1]
3-element Array{Int64,1}:
 1
 4
 7
```

```
#1st 2 columns
```

```
J> A[:,1:2]
3x2 Array{Int64,2}:
 1 2
 4 5
 7 8
```

```
J> A=[ 1 2 3; 4 5 9; 7 8 9]
```

```
3x3 Array{Int64,2}:
 1 2 3
 4 5 9
```

```
4 5 9
```

Getting the dimension
of a matrix
(here: 2D, rows x cols)

Selecting rows

Selecting columns

Extracting rows and columns by criteria

(here: get rows that have value 9 in column
3)

Accessing elements
(here: 1st element)

```
7 8 9

M> A(A(:,3) == 9,:)
ans =

4 5 9
7 8 9

M> A = [1 2 3; 4 5 6; 7 8 9]

M> A(1,1)

ans = 1
```

```
[4, 5, 9],
[7, 8, 9]])

P> A[A[:,2] == 9]
array([[4, 5, 9],
       [7, 8, 9]])

P> A = np.array([ [1,2,3], [4,5,6],
                 [7,8,9] ])

P> A[0,0]

1
```

```
[1,] 1 2 3
[2,] 4 5 9
[3,] 7 8 9

R> A[A[,3]==9,]

[1] 7 8 9

R> A =
matrix(c(1,2,3,4,5,9,7,8,9),nrow=3,byrow=T)

R> A[1,1]

[1] 1
```

```
7 8 9

# use '==' for
# element-wise check

J> A[ A[:,3] .==9, :]

2x3 Array{Int64,2}:
4 5 9
7 8 9

J> A=[1 2 3; 4 5 6; 7 8 9];

J> A[1,1]

1
```

Accessing elements
(here: 1st element)

MANIPULATING SHAPE AND DIMENSIONS

Converting
a matrix into a row vector (by column)

```
M> A = [1 2 3; 4 5 6; 7 8 9]

M> A(:)

ans =

1
2
3
4
5
6
7
8
9

M> b = [1 2 3]

M> b = b'
b =

1
2
3
```

Converting
row to column vectors

```
P> A = np.array([[1,2,3],[4,5,6],
                 [7,8,9]])

P> A.flatten(1) # returns a copy

array([1, 4, 7, 2, 5, 8, 3, 6, 9])

1# alternatively A.ravel()
4# ravel() returns a view
7
2
5
8
3
6
9

P> b = np.array([1, 2, 3])

P> b = b[np.newaxis].T
# alternatively
# b = b[:,np.newaxis]

P> b
```

```
R> A = matrix(1:9,nrow=3,byrow=T)

R> as.vector(A)

[1] 1 4 7 2 5 8 3 6 9

R> b = matrix(c(1,2,3), ncol=3)

R> t(b)

[1,] 1
[2,] 2
[3,] 3
```

```
J> A=[1 2 3; 4 5 6; 7 8 9]

J> vec(A)

9-element Array{Int64,1}:

1
4
7
2
5
8
3
6
9

J> b=vec([1 2 3])

3-element Array{Int64,1}:

1
2
3
```

Converting
a matrix into a row vector (by column)

1
4
7
2
5
8
3
6
9

Converting
row to column vectors

Reshaping Matrices

(here: 3x3 matrix to row vector)

Concatenating matrices

Stacking
vectors and matrices

```
M> A = [1 2 3; 4 5 6; 7 8 9]
A =
     1     2     3
     4     5     6
     7     8     9

M> total_elements = numel(A)

M> B = reshape(A,1,total_elements)
% or reshape(A,1,9)
B =
     1     4     7     2     5     8     3     6     9

M> A = [1 2 3; 4 5 6]

M> B = [7 8 9; 10 11 12]

M> C = [A; B]
     1     2     3
     4     5     6
     7     8     9
    10    11    12

M> a = [1 2 3]

M> b = [4 5 6]

M> c = [a' b']
c =
     1     4
     2     5
     3     6

M> c = [a; b]
c =
```

```
array([1,
        2,
        3])

P> A = np.array([[1,2,3],[4,5,6],
[7,8,9]])

P> A
array([[1, 2, 3],
       [4, 5, 9],
       [7, 8, 9]])

P> total_elements = np.prod(A.shape)

P> B = A.reshape(1, total_elements)

# alternative shortcut:
# A.reshape(1,-1)

P> B
array([[1, 2, 3, 4, 5, 6, 7, 8, 9]])

P> A = np.array([[1, 2, 3], [4, 5, 6]])

P> B = np.array([[7, 8, 9],[10,11,12]])

P> C = np.concatenate((A, B), axis=0)

P> C
array([[ 1, 2, 3],
       [ 4, 5, 6],
       [ 7, 8, 9],
       [10, 11, 12]])

P> a = np.array([1,2,3])
P> b = np.array([4,5,6])

P> np.column_stack([a,b])
array([[1, 4],
       [2, 5],
       [3, 6]])

P> np.row_stack([a,b])
array([[1, 2, 3],
       [4, 5, 6]])
```

```
R> A = matrix(1:9,nrow=3,byrow=T)

R> A
[,1] [,2] [,3]
[1,] 1 2 3
[2,] 4 5 6
[3,] 7 8 9

R> total_elements = dim(A)[1] * dim(A)[2]

R> B = matrix(A, ncol=total_elements)

R> B
[,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
[1,] 1 4 7 2 5 8 3 6 9

R> A = matrix(1:6,nrow=2,byrow=T)

R> B = matrix(7:12,nrow=2,byrow=T)

R> C = rbind(A,B)

R> C
[,1] [,2] [,3]
[1,] 1 2 3
[2,] 4 5 6
[3,] 7 8 9
[4,] 10 11 12

R> a = matrix(1:3, ncol=3)

R> b = matrix(4:6, ncol=3)

R> matrix(rbind(A, B), ncol=2)
[,1] [,2]
[1,] 1 5
[2,] 4 3

R> rbind(A,B)
[,1] [,2] [,3]
[1,] 1 2 3
```

```
J> A=[1 2 3; 4 5 6; 7 8 9]
3x3 Array{Int64,2}:
1 2 3
4 5 6
7 8 9

J> total_elements=length(A)
9

J> B=reshape(A,1,total_elements)
1x9 Array{Int64,2}:
1 4 7 2 5 8 3 6 9

J> A=[1 2 3; 4 5 6];

J> B=[7 8 9; 10 11 12];

J> C=[A; B]
4x3 Array{Int64,2}:
1 2 3
4 5 6
7 8 9
10 11 12

J> a=[1 2 3];

J> b=[4 5 6];

J> c=[a' b']
3x2 Array{Int64,2}:
1 4
2 5
3 6

J> c=[a; b]
2x3 Array{Int64,2}:
```

Reshaping Matrices

(here: 3x3 matrix to row vector)

Concatenating matrices

Stacking
vectors and matrices

1 2 3 4 5 6	[2,] 4 5 6	1 2 3 4 5 6
----------------	------------	----------------

BASIC MATRIX OPERATIONS

M> A = [1 2 3; 4 5 6; 7 8 9]

M> A * 2

ans =

2 4 6

8 10 12

14 16 18

M> A + 2

M> A - 2

M> A / 2

Note that NumPy was optimized for

in-place assignments

e.g., A += A instead of

A = A + A

M> A = [1 2 3; 4 5 6; 7 8 9]

M> A * A

ans =

30 36 42

66 81 96

102 126 150

M> A = [1 2 3; 4 5 6; 7 8 9]

M> b = [1; 2; 3]

M> A * b

ans =

14

32

50

P> A = np.array([[1,2,3], [4,5,6], [7,8,9]])

P> A * 2

array([[2, 4, 6], [8, 10, 12], [14, 16, 18]])

P> A + 2

P> A - 2

P> A / 2

Note that NumPy was optimized for

in-place assignments

e.g., A += A instead of

A = A + A

P> A = np.array([[1,2,3], [4,5,6], [7,8,9]])

P> np.dot(A,A) # or A.dot(A)

array([[30, 36, 42], [66, 81, 96], [102, 126, 150]])

P> A = np.array([[1,2,3], [4,5,6], [7,8,9]])

P> b = np.array([[1], [2], [3]])

P> np.dot(A,b) # or A.dot(b)

array([[14], [32], [50]])

R> A = matrix(1:9, nrow=3, byrow=T)

R> A * 2

[,1] [,2] [,3]

[1,] 2 4 6

[2,] 8 10 12

[3,] 14 16 18

R> A + 2

R> A - 2

R> A / 2

R> A = matrix(1:9, nrow=3, byrow=T)

R> A %% A

[,1] [,2] [,3]

[1,] 30 36 42

[2,] 66 81 96

[3,] 102 126 150

R> A = matrix(1:9, ncol=3)

R> b = matrix(1:3, nrow=3)

R> t(b %% A)

[,1]

[1,] 14

[2,] 32

[3,] 50

J> A=[1 2 3; 4 5 6; 7 8 9];

elementwise operator

J> A .* 2

3x3 Array{Int64,2}:

2 4 6

8 10 12

14 16 18

J> A .+ 2;

J> A .- 2;

J> A ./ 2;

J> A=[1 2 3; 4 5 6; 7 8 9];

J> A * A

3x3 Array{Int64,2}:

30 36 42

66 81 96

102 126 150

J> A=[1 2 3; 4 5 6; 7 8 9];

J> b=[1; 2; 3];

J> A*b

3-element Array{Int64,1}:

14

32

50

Matrix-scalar operations

Matrix-matrix multiplication

Matrix-vector multiplication

<div>Element-wise</div> <div>matrix-matrix operations</div>	<pre>M> A = [1 2 3; 4 5 6; 7 8 9] M> A .* A ans = 1 4 9 16 25 36 49 64 81 M> A .+ A M> A .- A M> A ./ A</pre>	<pre>P> A = np.array([[1,2,3], [4,5,6], [7,8,9]]) P> A * A array([[1, 4, 9], [16, 25, 36], [49, 64, 81]]) P> A + A P> A - A P> A / A # Note that NumPy was optimized for # in-place assignments # e.g., A += A instead of # A = A + A P> A = np.array([[1,2,3], [4,5,6], [7,8,9]])</pre>	<pre>R> A = matrix(1:9, nrow=3, byrow=T) R> A * A [,1] [,2] [,3] [1,] 1 4 9 [2,] 16 25 36 [3,] 49 64 81 R> A + A R> A - A R> A / A R> A = matrix(1:9, nrow=3, byrow=T)</pre>	<pre>J> A=[1 2 3; 4 5 6; 7 8 9]; J> A .* A 3x3 Array{Int64,2}: 1 4 9 16 25 36 49 64 81 J> A .+ A; J> A .- A; J> A ./ A;</pre>	<div>Element-wise</div> <div>matrix-matrix operations</div>
<div>Matrix elements to power n</div> <div>(here: individual elements squared)</div>	<pre>M> A = [1 2 3; 4 5 6; 7 8 9] M> A.^2 ans = 1 4 9 16 25 36 49 64 81</pre>	<pre>P> np.power(A,2) array([[1, 4, 9], [16, 25, 36], [49, 64, 81]])</pre>	<pre>R> A ^ 2 [,1] [,2] [,3] [1,] 1 4 9 [2,] 16 25 36 [3,] 49 64 81 R> A = matrix(1:9, ncol=3) # requires the 'expm' package R> install.packages('expm') R> library(expm) R> A %^% 2 [,1] [,2] [,3] [1,] 30 66 102 [2,] 36 81 126 [3,] 42 96 150</pre>	<pre>J> A=[1 2 3; 4 5 6; 7 8 9]; J> A .^ 2 3x3 Array{Int64,2}: 1 4 9 16 25 36 49 64 81</pre>	<div>Matrix elements to power n</div> <div>(here: individual elements squared)</div>
<div>Matrix to power n</div> <div>(here: matrix-matrix multiplication with itself)</div>	<pre>M> A = [1 2 3; 4 5 6; 7 8 9] M> A ^ 2 ans = 30 36 42 66 81 96 102 126 150</pre>	<pre>P> A = np.array([[1,2,3], [4,5,6], [7,8,9]]) P> np.linalg.matrix_power(A,2) array([[30, 36, 42], [66, 81, 96], [102, 126, 150]])</pre>	<pre>R> A = matrix(1:9, ncol=3) # requires the 'expm' package R> install.packages('expm') R> library(expm) R> A %^% 2 [,1] [,2] [,3] [1,] 30 66 102 [2,] 36 81 126 [3,] 42 96 150</pre>	<pre>J> A=[1 2 3; 4 5 6; 7 8 9]; J> A ^ 2 3x3 Array{Int64,2}: 30 36 42 66 81 96 102 126 150</pre>	<div>Matrix to power n</div> <div>(here: matrix-matrix multiplication with itself)</div>
<div>Matrix transpose</div>	<pre>M> A = [1 2 3; 4 5 6; 7 8 9] M> A' ans = 1 4 7 2 5 8 3 6 9</pre>	<pre>P> A = np.array([[1,2,3], [4,5,6], [7,8,9]]) P> A.T array([[1, 4, 7], [2, 5, 8], [3, 6, 9]])</pre>	<pre>R> A = matrix(1:9, nrow=3, byrow=T) R> t(A) [,1] [,2] [,3] [1,] 1 4 7 [2,] 2 5 8 [3,] 3 6 9</pre>	<pre>J> A=[1 2 3; 4 5 6; 7 8 9] 3x3 Array{Int64,2}: 1 2 3 4 5 6 7 8 9 J> A' 3x3 Array{Int64,2}:</pre>	<div>Matrix transpose</div>

Determinant of a matrix:
A -> |A|

```
M> A = [6 1 1; 4 -2 5; 2 8 7]
A =
     6     1     1
     4    -2     5
     2     8     7

M> det(A)
ans = -306
```

Inverse of a matrix

```
M> A = [4 7; 2 6]
A =
     4     7
     2     6

M> A_inv = inv(A)
A_inv =
     0.60000    -0.70000
    -0.20000     0.40000
```

```
P> A = np.array([[6,1,1],[4,-2,5],
[2,8,7]])

P> A
array([[ 6,  1,  1],
       [ 4, -2,  5],
       [ 2,  8,  7]])

P> np.linalg.det(A)
-306

P> A = np.array([[4, 7], [2, 6]])

P> A
array([[4, 7],
       [2, 6]])

P> A_inverse = np.linalg.inv(A)

P> A_inverse
array([[ 0.6, -0.7],
       [-0.2,  0.4]])
```

```
R> A = matrix(c(6,1,1,4,-2,5,2,8,7), nrow=3,
byrow=T)

R> A
[,1] [,2] [,3]
[1,] 6 1 1
[2,] 4 -2 5
[3,] 2 8 7

R> det(A)
[1] -306

R> A = matrix(c(4,7,2,6), nrow=2, byrow=T)

R> A
[,1] [,2]
[1,] 4 7
[2,] 2 6

R> solve(A)
[,1] [,2]
[1,] 0.6 -0.7
[2,] -0.2 0.4
```

```
1 4 7
2 5 8
3 6 9

J> A=[6 1 1; 4 -2 5; 2 8 7]
3x3 Array{Int64,2}:
6 1 1
4 -2 5
2 8 7

J> det(A)
-306

J> A=[4 7; 2 6]
2x2 Array{Int64,2}:
4 7
2 6

J> A_inv=inv(A)
2x2 Array{Float64,2}:
0.6 -0.7
-0.2 0.4
```

Determinant of a matrix:
A -> |A|

Inverse of a matrix

ADVANCED MATRIX OPERATIONS

Calculating the covariance matrix
of 3 random variables

(here: covariances of the means
of x1, x2, and x3)

```
M> x1 = [4.0000 4.2000 3.9000 4.3000
4.1000]'

M> x2 = [2.0000 2.1000 2.0000 2.1000
2.2000]'

M> x3 = [0.60000 0.59000 0.58000
0.62000 0.63000]'
```

```
P> x1 = np.array([ 4, 4.2, 3.9, 4.3,
4.1])

P> x2 = np.array([ 2, 2.1, 2, 2.1, 2.2])

P> x3 = np.array([ 0.6, 0.59, 0.58, 0.62,
0.63])
```

```
R> x1 = matrix(c(4, 4.2, 3.9, 4.3, 4.1),
ncol=5)

R> x2 = matrix(c(2, 2.1, 2, 2.1, 2.2),
ncol=5)

R> x3 = matrix(c(0.6, 0.59, 0.58, 0.62,
0.63), ncol=5)
```

```
J> x1=[4.0 4.2 3.9 4.3 4.1]';

J> x2=[2. 2.1 2. 2.1 2.2]';

J> x3=[0.6 .59 .58 .62 .63]';
```

Calculating the covariance matrix
of 3 random variables

(here: covariances of the means
of x1, x2, and x3)

Calculating eigenvectors and eigenvalues

```
M> cov( [x1,x2,x3] )

ans =

    2.5000e-02    7.5000e-03    1.7500e-03
    7.5000e-03    7.0000e-03    1.3500e-03
    1.7500e-03    1.3500e-03    4.3000e-04

M> A = [3 1; 1 3]
A =

     3     1
     1     3

M> [eig_vec,eig_val] = eig(A)
eig_vec =

   -0.70711    0.70711
    0.70711    0.70711

eig_val =

    2     0
    0     4

Diagonal Matrix

     2     0
     0     4

% requires statistics toolbox package
% how to install and load it in Octave:
% download the package from:
% http://octave.sourceforge.net/packages/eigen
% pkg install
% ~/Desktop/io-2.0.2.tar.gz
% pkg install
% ~/Desktop/statistics-1.2.3.tar.gz

M> pkg load statistics

M> mean = [0 0]

M> cov = [2 0; 0 2]
cov =
```

```
P> np.cov([x1, x2, x3])

Array([[ 0.025 ,  0.0075 ,  0.00175],
       [ 0.0075 ,  0.007 ,  0.00135],
       [ 0.00175,  0.00135,  0.00043]])

P> A = np.array([[3, 1], [1, 3]])

P> A
array([[3, 1],
       [1, 3]])

P> eig_val, eig_vec = np.linalg.eig(A)

P> eig_val
array([ 4.,  2.])

P> eig_vec
Array([[ 0.70710678, -0.70710678],
       [ 0.70710678,  0.70710678]])

P> mean = np.array([0,0])

P> cov = np.array([[2,0],[0,2]])

P> np.random.multivariate_normal(mean, cov, 5)

Array([[ 1.55432624, -1.17972629],
       [-2.01185294,  1.96081908],
       [-2.11810813,  1.45784216],
       [-2.93207591, -0.07369322],
       [-1.37031244, -1.18408792]])
```

```
R> cov(matrix(c(x1, x2, x3), ncol=3))

[,1] [,2] [,3]

[1,] 0.02500 0.00750 0.00175
[2,] 0.00750 0.00700 0.00135
[3,] 0.00175 0.00135 0.00043

R> A = matrix(c(3,1,1,3), ncol=2)

R> A
[,1] [,2]
[1,] 3 1
[2,] 1 3

R> eigen(A)
$values
[1] 4 2

$vectors
[,1] [,2]
[1,] 0.7071068 -0.7071068
[2,] 0.7071068 0.7071068

# requires the 'mass' package

R> install.packages('MASS')

R> library(MASS)

R> mvrnorm(n=10, mean, cov)

[,1] [,2]
[1,] -0.8407830 -0.1882706
[2,] 0.8496822 -0.7889329
[3,] -0.1564171 0.8422177
[4,] -0.6288779 1.0618688
[5,] -0.5103879 0.1303697
[6,] 0.8413189 -0.1623758
[7,] -1.0495466 -0.4161082
[8,] -1.3236339 0.7755572
```

```
J> cov([x1 x2 x3])

3x3 Array{Float64,2}:

0.025 0.0075 0.00175
0.0075 0.007 0.00135
0.00175 0.00135 0.00043

J> A=[3 1; 1 3]
2x2 Array{Int64,2}:

3 1
1 3

J> (eig_vec,eig_val)=eig(a)
([2.0,4.0],
2x2 Array{Float64,2}:
-0.707107 0.707107
0.707107 0.707107)

# requires the Distributions package from
https://github.com/JuliaStats/Distributions.jl

J> using Distributions

J> mean=[0., 0.]
2-element Array{Float64,1}:

0
0

J> cov=[2. 0.; 0. 2.]
2x2 Array{Float64,2}:

2.0 0.0
0.0 2.0

J> rand( MvNormal(mean, cov), 5)
2x5 Array{Float64,2}:
```

Calculating eigenvectors and eigenvalues

Generating a Gaussian dataset:

creating random vectors from the multivariate normal distribution given mean and covariance matrix

(here: 5 random vectors with mean 0, covariance = 0, variance = 2)

Generating a Gaussian dataset:

creating random vectors from the multivariate normal distribution given mean and covariance matrix

(here: 5 random vectors with mean 0, covariance = 0, variance = 2)

```
2 0
0 2
```

```
M> mvnrnd(mean,cov,5)
```

```
2.480150 -0.559906
-2.933047 0.560212
0.098206 3.055316
-0.985215 -0.990936
1.122528 0.686977
```

```
[9,] 0.2771013 1.4900494
[10,] -1.3536268 0.2338913
```

```
-0.527634 0.370725 -0.761928
-3.91747 1.47516
-0.448821 2.21904 2.24561
0.692063 0.390495
```