Simulation Results

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1 Results

Problem 1.1. Simulate a simple symmetric random walk in 2 dimensions. Then let time T grow and rescale space by a factor of $\sqrt{T}$. The resulting trace of the random walk should approximate 2-dimensional Brownian motion. Try to compute, though repeated simulations, the maximal displacement from the origin. How does it grow as a function of T?

My simulation code is attached to this email (BrownWalk.cpp). Here is one example of a walk generated from the code (with $T = 100000$):

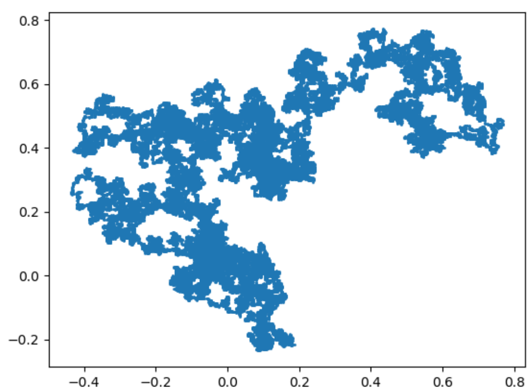
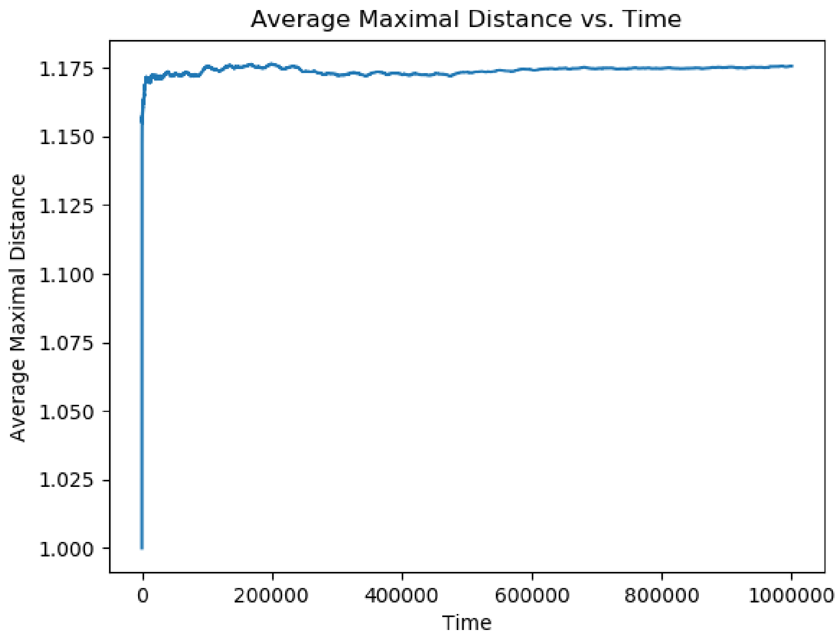


Figure 1: Sample Random Walk

I simulated 10000 walks of length 1000000, then graphed the average maximal distance vs. T. Here are the results:

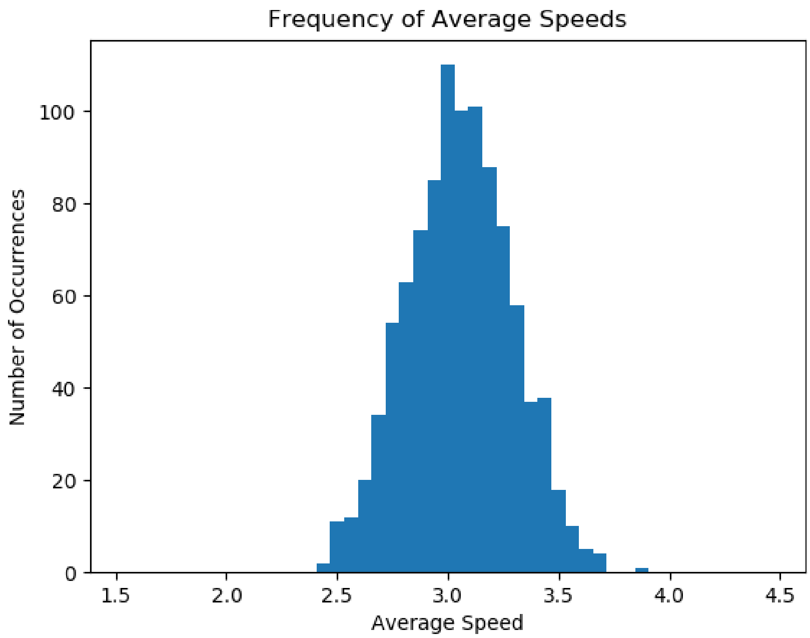


We see that the average maximal distance approaches a value that is about 1.18 asymptotically, as evidenced by the sharp plateau after the initial increase.

Problem 1.2. Simulate a continuous time 1 dimensional random walk which increases by 1 at rate p and decreases by 1 at rate q (here, "rate" means that after an exponential random variable waiting time, the event in question occurs). As time T grows, what can you say about the trajectory? Is there an overall "speed" that it moves at – how does that relate to p and q? How do the fluctuations around that grow (i.e., repeat the simulation many times and give a histogram for the time T location, perhaps centering by the overall speed).

Again, the code for this simulation is attached (ExpDis.cpp). As expected, we see that if $p > q$ then the particle moves to the right on average, whereas if $p < q$ then it moves to the left. Furthermore, through repeated simulation, we find that the average rate of motion is $p - q$. This makes sense because in one time unit, the particle is expected to move to the right p times and to the left q times. By linearity of expectation, the average displacement in one second would therefore be $p - q$.

I simulated a random walk for 100 time units with $p = 4$ and $q = 1$. After 1000 trials, I made a histogram with the frequencies of the average speeds. Here are the results:



The distribution appears to be normal (the K^2 test gives a $p = 0.134 > 0.05$), centered around the average. Note that the actual location of the particle at time 100 can be found by multiplying the speed by 100.