Maths Work

Question: 01

(a)

$(abc.de)_x$

$$\begin{array}{cccccc} 2 & 1 & 0 & . & -1 & -2. \\ a & b & c & . & d & e \end{array}$$

Therefore,

$$\begin{array}{cccccc} a & b & c & . & d & e \\ \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow \\ ax^2 & bx^1 & cx^0 & & dx^{-1} & ex^{-2} \end{array}$$

Therefore, $(abc.de)_x$

$= ax^2 + bx + c + dx^{-1} + ex^{-2}$

$= (ax^2 + bx + c + dx^{-1} + ex^{-2})_{10}$

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(b)

(i) $(723)_8$ to hexa decimal system

<u>Solution:</u>

First we have to convert $(723)_8$ into decimal system.

$$(723)_8 = 3\times8^0 + 2\times8^1 + 7\times8^2$$
$$= 3 + 16 + 448$$
$$= (467)_{10}$$

16	467	
16	29	$(3)_{10} = (3)_{16}$
16	1	$(13)_{10} = (D)_{16}$
	0	$(1)_{10} = (1)_{16}$

$$(723)_8 = (467)_{10} = (1D3)_{16}$$

(ii) $(0.ABDF)_{16}$ to decimal system.

Solution:

$$(0.ABDF)_{16} = 10 \times \frac{1}{16} + 11 \times \frac{1}{(16)^2} + 13 \times \frac{1}{(16)^3} + 15 \times \frac{1}{(16)^4}$$

$$= \frac{10}{16} + \frac{11}{(16)^2} + \frac{13}{(16)^3} + \frac{15}{(16)^4}$$

$$= 0.625 + 0.0429 + 0.0031 + 0.0002$$

$$= (0.6712)_{10}$$

(iii) $(0.375)_{10}$ to binary system.

Solution:

$0.375 \times 2 = 0 + 0.750$

$0.750 \times 2 = 1 + 0.500$

$0.500 \times 2 = 1 + 0.000$

$(0.375)_{10} = (0.011)_2$

(iv) Digit "5" is not allowed in Quinary system.

Solution:

It has 5 digits. These are 0, 1, 2, 3 and 4.

(v) $(11010.1011)_2$ to hexadecimal.

First we have to convert $(11010.1011)_2$ into decimal.

$$(11010.1011)_2 = 0\times2^0+1\times2^1+0+1\times2^3+1\times2^4$$
$$+1\times2^{-1}+1\times2^{-3}+1\times2^{-4}$$
$$=0+2+0+8+16+0.5+0.125+0.0625$$
$$=(26.6875)_{10}$$

16	26	Remaider
16	1	$(10)_{10} = (A)_{16}$
	0	$(1)_{10} = (1)_{16}$

$26 = (1A)_{16}$

$0.6875\times16 = (11)_{10} = (B)_{16}$

$(26.6875)_{10} = (1A.B)_{16}$

(vi) $(257)_{10}$ to binary system.

2	257	Remainder
2	128	1
2	64	0
2	32	0
2	16	0
2	8	0
2	4	0
2	2	0
2	1	0
	0	1

$(257)_{10} = (100000001)_2$

(C)

(i) Consider the binary number 10.0011. Convert the above number to decimal system.

Solution:

$$(10.0011)_2 = 0\times2^0 + 1\times2^1 + 0 + 0 + 1\times2^{-3} + 1\times2^{-4}$$
$$= 0+2+0+0+0.125+0.0625$$
$$= (2.1875)_{10}$$

(ii) What are the place values of digits 1 in the number $(0.0011)_2$.

Place value of 1st 1 is,

$$= (1\times2^{-3})_{10}$$
$$= (0.125)_{10}$$

Place value of 2nd 1 is,

$$= (1\times2^{-4})_{10}$$
$$= (0.0625)_{10}$$

(iii) What is the sum of (1+1+1+1) in binary system.

Solution:

$$(1+1+1+1) = (4)_{10} = (100)_2$$

2	4	Remainder
2	2	0
	1	0
	0	1

$$(4)_{10} = (100)_2$$

(iv) Calculate 101 divided by 10 using long division.

Solution:

Using long division method.

$$\begin{array}{r} 10.1 \\ 10\overline{)\;101\;\;} \\ -10 \\ \hline 001 \\ -\;00 \\ \hline 10 \\ -10 \\ \hline .00 \end{array}$$

$$\frac{101}{10} = 10.1$$

(d)

Which one is the correct representation of a binary number from the following?

(i) 1101

In this, base is not declared.

(ii) $(214)_2$

In this, 4 is not possible.

(iii) $(0000)_2$

It is correct representation.

(iv) $(11)^2$

In this, 2 is power or index of 11

Question : 02

(a) Is $a_n = \frac{3n+2}{n-4}$ a general term of a sequence ? why ?

Solution:

For a general term of a sequence must have to be valid for all Natural numbers.

But a_n is not valid for $n = 4$. As, for $n = 4$, the denominator of a_n become zero. it implies that a_n is not define for $n = 4$. Hence a_n is not general term.

(b) Which term of the sequence with general term $\frac{3n-1}{5n+7}$ is $\frac{7}{12}$?

Solution:

As, $a_n = \frac{3n-1}{5n+7} = \frac{7}{12}$

$$12(3n-1) = 7(5n+7)$$

$$36n - 12 = 35n + 49$$

$$36n - 35n = 49 + 12$$

$$n = 61$$

Thus, 61^{th} term of the sequence is $7/12$.

(c) An arithematic sequence has its 4th term equal to 18 and its 12th term equal to 50. Find its 99th term.

Solution:

As, 4^{th} term $= 18$

12^{th} term $= 50$

Let, the first term be 'a' and common difference is 'd'. Then, 4th term is given by:

$$n_4 = a + (n-1)d$$

$$18 = a + (4-1)d$$

$$18 = a + 3d \longrightarrow (1)$$

Similarly,

$$n_{12} = a + (12-1)d$$

$$50 = a + 11d \longrightarrow (2)$$

Subtracting eq (2) from eq (1).

we have,

$$8d = 32$$

$$d = 4$$

Put the value of ~~eq~~ d in eq(1).

$$a = 18 - 4$$

$$a = 6$$

99th term will be $n_{99} = 6 + 98 \times 4$

$$= 398$$

(8) State whether the following sequences are arithematic, geometric or not any of them. Find the common Ratio if it is geometric sequence and the common difference if it is an arithematic sequence. Then, find next two terms.

(i) $-3, 3, -3, 3$

The above sequence is geometric sequence, with common ration $r = -1$.
Next two terms will be $-3, 3$.

(ii) $b_n = n^2 + 3$

$b_n = n^2 + 3$ is the n^{th} term of the sequence. Its not arithematic or geometric sequence

(iii) $\frac{-1}{2}, \frac{-5}{6}, \frac{-7}{6}$

The given series is arithematic.

Common Difference $= \frac{-5}{6} - \left(\frac{-1}{2}\right)$

$= \frac{-5}{6} + \frac{1}{2}$

$= \frac{-2}{6}$

$= \frac{-1}{3}$

Next two terms are $\frac{-9}{6}, \frac{-11}{6}$

because $\frac{-7}{6} + \left(\frac{-1}{3}\right) = \frac{-9}{6}$

$\frac{-9}{6} + \left(\frac{-1}{3}\right) = \frac{-11}{6}$

(e)

Consider the geometric sequence (b_n) with $b_1 = 1/9$ and $q = 3$. Is 243 a term of this sequence?

Solution:

(b_n) is geometric sequence.

$b_1 = 1/9$, $q = 3$

Let, $b_n = 243 = b_1 q^{n-1}$

$$= \frac{1}{9} 3^{n-1}$$

$$243 \times 9 = 3^{n-1}$$

$$3^{n-1} = 3^5 \times 3^2$$

$$3^{n-1} = 3^7$$

$$n-1 = 7$$

$$n = 8$$

Thus, 243 is term of sequence.

(f)

The nineteenth term of a sequence is -52 and the fourth term is -7. The difference between consecutive terms in the sequence is constant. Find 201^{st} term.

Solution:

$$a_{19} = -52 \quad , \quad a_4 = -7$$

As, difference between two consecutive terms is constant so, it is arithematic sequence.

$$a_1 + 18d = -52 \quad \text{and} \quad a_1 + 3d = -7$$

$$a + 3d + 15d = -52$$

$$-7 + 15d = -52$$

$$d = -3$$

$$a_{201} = a_1 + 200d = a_4 + 197d$$

$$a_{201} = -7 + 197(-3)$$

$$a_{201} = -598$$

(g)

Show whether the following sequence is convergent or divergent.

$$\lim_{n\to\infty}\left(\frac{n-1}{n}\right)$$

Solution:

$$\lim_{n\to\infty}\left(\frac{n-1}{n}\right) = \lim_{n\to\infty}\left[\frac{n}{n} - \frac{1}{n}\right]$$

$$= \lim_{n\to\infty}\left[1 - \frac{1}{n}\right]$$

$$= \lim_{n\to\infty} 1 - \lim_{n\to\infty}\frac{1}{n}$$

$$= 1.$$

As, $\lim_{n\to\infty}\frac{1}{n} \Rightarrow 0$

So, it is convergent.

(h)

Solution:

$1, -4, 9, -16, \ldots\ldots$

Yes, this is sequence and its general term is,

$a_n = (-1)^n (n)^2 \quad , n \geq 1$

(i)

Solution:

$$\frac{1}{2} + \frac{1}{2^2} + \ldots\ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

for $n = 1$

$$\frac{1}{2^1} = 1 - \frac{1}{2^1} \Rightarrow \frac{1}{2} = \frac{1}{2} \quad \text{Thats True}$$

let for $n = k$

$$\frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^k} = 1 - \frac{1}{2^k}$$

we will Prove for $n = k+1$

$$\frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^k} + \frac{1}{2^{k+1}} = 1 - \frac{1}{2^{k+1}}$$

$$\frac{1}{2} + \frac{1}{2^2} + \ldots\ldots + \frac{1}{2^k} + \frac{1}{2^{k+1}} = \frac{2^{k+1} - 1}{2^{k+1}}$$

$$L.H.S = \frac{1}{2} + \frac{1}{2^2} + \ldots\ldots + \frac{1}{2^k} + \frac{1}{2^{k+1}}$$

$$= 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}}$$

$$= \frac{2^{k+1} - 2 + 1}{2^{k+1}}$$

$$= \frac{2^{k+1} - 1}{2^{k+1}} = R.H.S \quad \text{Proved}$$

(j)

Find The remainder when 3^{123} is divided by 7

Sol:

Let

$$3^{6} \equiv 1 \pmod{7}$$

$$(3^{6})^{20} \equiv 1 \pmod{7}$$

$$3^{120} \equiv 1 \pmod{7}$$

$$3^{123} \equiv 3^{120} \times 3^{3} \equiv 1 \times 3^{3} \pmod{7}$$

$$3^{123} \equiv 3^{3} \equiv 27 \equiv 6 \pmod{7}$$

Remainder is 6 when 3^{123} is divided by 7

Question: 03

State whether the following statements are false, true Explain your answer:

(i) Given any integer a, b, c and any positive integer n if $a \equiv b \pmod n$ and $b \equiv c \pmod n$ and $a \equiv c \pmod n$.

Solution:

$$a \equiv b \pmod n$$

$$\frac{a-b}{n} = x \quad \text{(where } x \text{ be any number)}$$

$$a-b = nx$$

$$b \equiv c \pmod n$$

$$\frac{b-c}{n} = y \quad \text{(where } y \text{ be any number)}$$

$$b-c = ny$$

$$(a-b)+(b-c) = nx+ny$$

$$a-b+b-c = n(x+y)$$

$$a-c = n(x+y)$$

$$\frac{a-c}{n} = x+y$$

$$a \equiv c \pmod n$$

True.

(ii) Suppose $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then

$$a+c \equiv b+c \pmod{n}.$$

Solution:

$a \equiv b \pmod{n}$	$c \equiv d \pmod{n}$
$\frac{a-b}{n} = x$	$\frac{c-d}{n} = y$
$a-b = nx$	$c-d = ny$

$$(a-b)+(c-d) = n(x+y)$$

$$(a+c)-(b+d) = n(x+y)$$

$$\frac{(a+c)-(b+d)}{n} = x+y$$

$$(a+c) \equiv (b+d) \pmod{n}$$

Hence, This is False.

(iii) $7x \equiv 12 \pmod{7}$

$$\frac{7x-12}{7} = y \text{ (y is any number)}$$

$$7x-12 = 7y$$

When 7 is divided by $7x$, the result is x and not get any remainder. Hence this is False.

(b)

Find the least positive value of x such that $71 = x \pmod 8$.

Solution:

This means that when 8 is divided by 71, the remainder is x. To get x is least positive value, simply divide 71 by 8 and the remainder is the value of x.

$\therefore \quad x = 7$

(C)

Calculate the multiplicative inverse of 168 in modulo 83.

Solution:

Multiplicative inverse 'b' of 168 in modulo 83 can be written,

$$\frac{168b - 1}{83} = n$$

$$168b - 1 = 83n$$

$$168 \times 42 - 1 = 83 \times 85$$

Hence, $b = 42$, $n = 85$

So, multiplicative inverse of 168 in modulo 83 is 42.

(d)

Calculate the inverse of 4 modulo 15. Show your steps.

Solution:

$$x = \frac{1}{4} \text{ mod}(15)$$

$$\frac{4x-1}{15} = n$$

$$4 \times 4 - 1 = 15 \times 1$$

$$x = 4 \quad , \quad n = 1$$

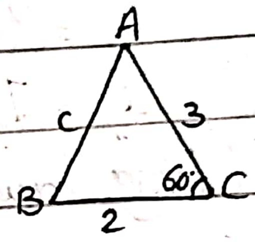
Inverse of 4 modulo 15 is 4.

Question : 04

(a). A triangle has sides $a=2, b=3$ and angle $C=60^\circ$. Find
(i) Length of side C.
(ii) Find the sine of angle B using sine rules.

Solution:

(i)



Using law of cosine.

$$c^2 = b^2 + a^2 - 2ab\cos C$$

$$\cos 60^\circ = \frac{(2)^2 + (3)^2 - c^2}{(2)(2)(3)}$$

$$\frac{1}{2} = \frac{13 - c^2}{12}$$

$$6 = 13 - c^2$$

$$c = \sqrt{7}$$

(ii) Using law of sine:

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{\sqrt{7}}{\sin 60^\circ} = \frac{3}{\sin B}$$

$$\sin B = \frac{3}{\sqrt{7}} \times \frac{\sqrt{3}}{2}$$

$$\sin B = \frac{3\sqrt{3}}{2\sqrt{7}}$$

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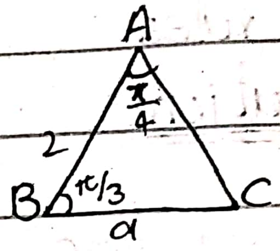
(b)

If we have a triangle which has one of its side $c = 2$ and angles $A = \pi/4$ and $B = \pi/3$. Workout the length a of the side opposite A.

Solution:



$$\angle C = 180° - 60° - 45°$$
$$= 180° - 105°$$
$$\angle C = 75°$$

Using law of sine

$$\frac{a}{\sin\frac{\pi}{4}} = \frac{2}{\sin 75°}$$

$$a = \frac{2\sqrt{2}}{\sqrt{2+\sqrt{3}}}$$

$$\cos 150° = 1 - 2\sin^2 75°$$
$$2\sin^2 75° = 1 - \cos 150°$$
$$2\sin^2 75° = 1 - \left(\frac{-\sqrt{3}}{2}\right)$$

$$\sin^2 75° = \frac{2+\sqrt{3}}{4}$$

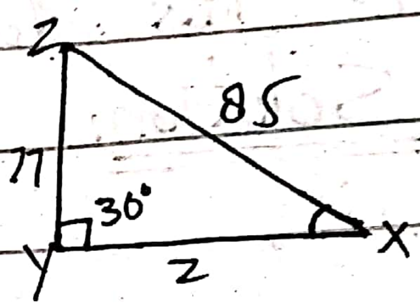
$$\sin 75° = \frac{\sqrt{2+\sqrt{3}}}{2}$$

(c)

XYZ is a right triangle with $Y = 90^\circ$. Given that $y = 85$, $\sin X = 77/85$. Find z, $\cos(Z)$ and the angle Z.

Solution:

Using Pathagorus theorem,



$z = \sqrt{(85)^2 - (77)^2}$

$= \sqrt{(85-77)(88+77)}$

$= \sqrt{8 \times 162}$

$z = 36$

Let, $z = \dfrac{77}{85}$

$z = \cos^{-1}\dfrac{77}{85}$

$z = 25.0576^\circ$

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(d)

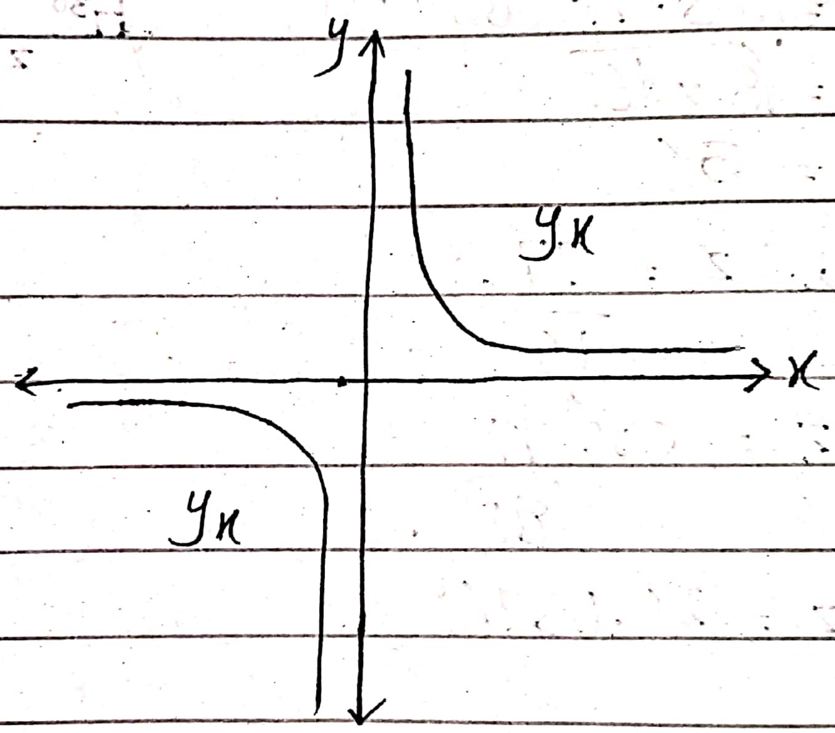
Let g be a function with its domain $(0,\infty)$, define by $g(x) = 1/x$

(i) Sketch the graph of g.

(ii) Is g continuous at other points of its domain?

Solution:

(i)



As $x \in (0,\infty)$, Only Quadrant I graph will be continuous.

(ii)

Yes g is continuous on its given domain of $(0,\infty)$.

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Question: 05

(i)

Let $f(x_1) = f(x_2)$

$x_1+1 = x_2+1$ if x is even.

$x_1 = x_2$

$x_1-3 = x_2-3$

$x_1 = x_2$ if x is odd.

For both cases, even and odd.

$f(x_1) = f(x_2)$

$x_1 = x_2$ Hence, given function is injective.

(ii). $f(x) = \begin{cases} x+1 & \text{if } x \text{ is even.} \\ x-3 & \text{if } x \text{ is odd} \end{cases}$

$g(y) = \begin{cases} y-1 & \text{if } y \text{ is even} \\ y+3 & \text{if } y \text{ is odd.} \end{cases}$

$g(y_1) = g(y_2)$

$y_1-1 = y_2-1$ if y is even.

$y_1 = y_2$

$y_1+3 = y_2+3$ if y is odd.

$y_1 = y_2$

For both the cases, even and odd inverse function g is one-one injective. Hence, the given function f is surjective. Hence, we say that every element of codomain has pre-image in domain. Hence surjective.

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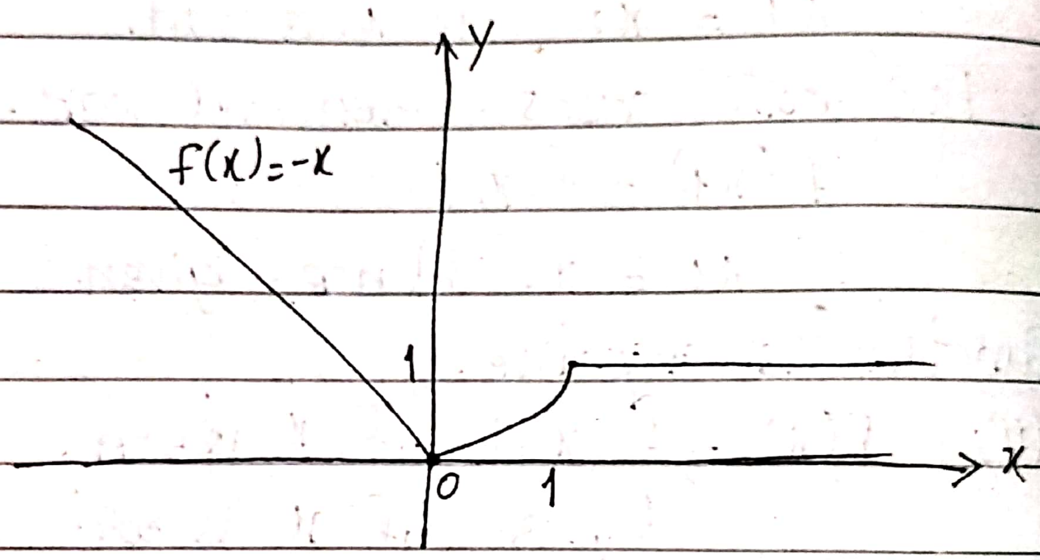
(b)

Solution:

$$f: \mathbb{R} \to \mathbb{R}$$

$$f(x) = \begin{cases} -x & , \text{ if } x < 0 \\ x^2 & , \text{ if } 0 \le x \le 1 \\ 1 & , \text{ if } x > 1 \end{cases}$$



Given function is not bijective we can see it in graph.

By the graph, the function is not one-one as $f(2) = f(3)$

but $2 \neq 3$

or 2 and 3 has same image. Hence not one-one.

The given function is not bijective.

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(c)

Solution:

Velocity $= V_x = 40 - 5t^2$

At, $t = 0$

$V_0 = 40\ M/s$

At, $t = 2$

$V_2 = 40 - 5(2)^2$

$V_2 = 20\ M/s$

Average acceleration.

$$\frac{V_2 - V_0}{t_2 - t_0} = \frac{20 - 40}{2 - 0}$$

$$\frac{-20}{2} = -10\ M/s$$

$$\frac{V_2 - V_0}{t_2 - t_0} = -10\ M/s$$

(d)

Solution:

$V_i = 12 cm/s$

$X_f = -5 cm$

$X_i = 3 cm$

$t = 2 sec$

$$X_f - X_i = V_i t + \frac{1}{2} a t^2$$

$$-5 - 3 = 12(2) + \frac{1}{2} a (2)^2$$

$$-8 = 24 + 2a$$

$$-8 - 24 = 2a$$

$$a = \frac{-32}{2}$$

$$a = -16 cm/s^2$$