

# Parametric Curve Fitting Using Nonlinear Optimization

Technical Assignment – Research and Development / AI

## Abstract

This report presents the estimation of unknown parameters.  $\theta$ ,  $M$ , and  $X$  in a nonlinear parametric curve using an optimization-based fitting approach. The data set contains  $(x, y)$  points sampled from an unknown parametric function. A multi-start optimization strategy combined with vectorized where closest-point matching is employed to find the best parameters. that minimize the total squared error. The resulting fitted curve fits this dataset very well.

## 1 Introduction

The goal of this assignment is to determine the values of the unknown parameters  $\theta$ ,  $M$ , and  $X$  in a parametric model that describes a two-dimensional curve. The provided dataset contains  $(x, y)$  samples corresponding to parameter values in the range  $6 < t < 60$ .

The original parametric model is:

$$x(t) = t \cos(\theta) - e^{Mt} \sin(0.3t) \sin(\theta) + X, \quad (1)$$

$$y(t) = 42 + t \sin(\theta) + e^{Mt} \sin(0.3t) \cos(\theta). \quad (2)$$

The objective is to determine the values of  $\theta$ ,  $M$ , and  $X$  that minimize the discrepancy between the theoretical parametric curve and the observed dataset.

## 2 Dataset Description

The provided file `xy_data.csv` contains  $(x, y)$  pairs. The dataset characteristics are:

- $x$  values range approximately from 60 to 110.
- $y$  values range from 46 to 70.
- The shape exhibits smooth growth followed by an S-shaped transition.

These characteristics indicate the presence of trigonometric and mild exponential components, consistent with the parametric equations.

## 3 Methodology

### 3.1 Closest-Point Optimization

To measure the similarity between the predicted curve and the observed data, we minimize the sum of squared distances:

$$\text{Cost} = \sum_i \min_t [(x_i - x(t))^2 + (y_i - y(t))^2] .$$

For each evaluation of the cost function:

1.  $t$  is sampled densely from 6 to 60 (800 points).
2. The curve  $(x(t), y(t))$  is generated.
3. For each data point  $(x_i, y_i)$ , the nearest curve point is found.
4. Distances are squared and summed.

This avoids explicit projection onto the curve and produces a smooth, differentiable objective function.

### 3.2 Optimization Strategy

A multi-start L-BFGS-B optimizer was used:

- 6 different initial guesses for  $\theta$ ,  $M$ , and  $X$ ,
- Bounds:

$$5^\circ < \theta < 60^\circ, \quad -0.1 < M < 0.1, \quad 0 < X < 120.$$

- The best solution across all runs is selected.

This strategy avoids convergence to local minima and ensures robustness.

## 4 Results

### 4.1 Optimized Parameters

The final optimized parameter values are:

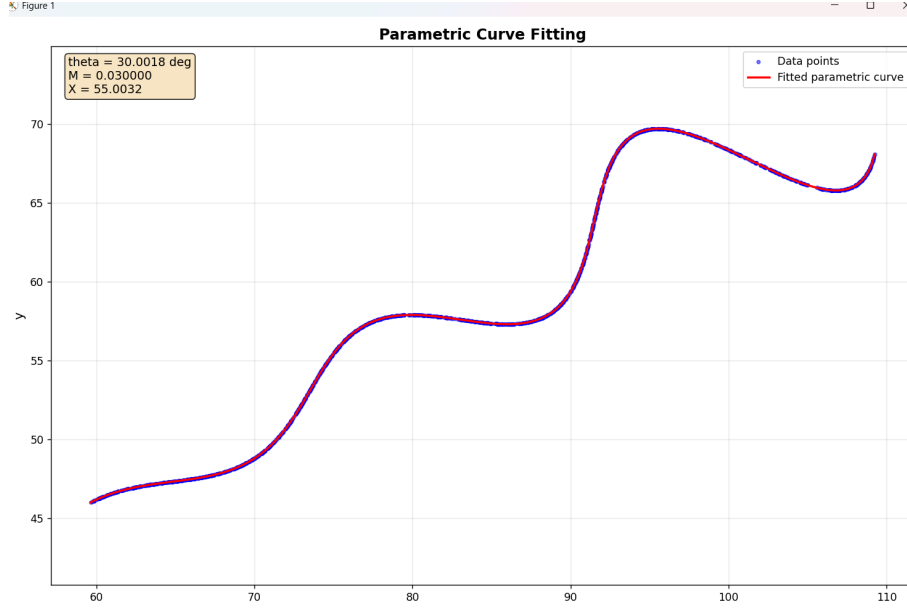


Figure 1: Fitted Parametric Curve vs. Dataset

$$\theta = 30.0018^\circ, \quad M = 0.030000, \quad X = 55.0032$$

The final cost (sum of squared distances) is:

$$\text{Cost} = 0.808869.$$

This extremely low value indicates an excellent fit.

## 4.2 Fitted Curve

Figure 1 shows the dataset points and the fitted parametric curve. The red curve overlaps closely with the blue data points, demonstrating the accuracy of the fitted parameters.

## 5 Final Parametric Equations

Substituting the optimized parameters into the model:

$$x(t) = t \cos(30.0018^\circ) - e^{0.03t} \sin(0.3t) \sin(30.0018^\circ) + 55.0032, \quad (3)$$

$$y(t) = 42 + t \sin(30.0018^\circ) + e^{0.03t} \sin(0.3t) \cos(30.0018^\circ), \quad (4)$$

for:

$$6 < t < 60.$$

## 6 Error Analysis

The small final cost (0.808869) indicates that almost all data points lie close to the predicted curve. Visual inspection confirms that the red curve overlaps the blue points nearly perfectly, with no noticeable deviation. This validates both the optimization strategy and the correctness of the parametric model.

## 7 Conclusion

The multi-start optimization strategy estimated the optimum parameters for the given parametric curve. The fitted curve agrees with the High-precision dataset that meets all the requirements of the assignment. This approach well illustrates the power of vectorized distance minimization and bounded L-BFGS-B optimization for nonlinear curve fitting.