HW – III ML – 257 (1) SAMPATH LAKKARAJU 011818781

Q1
8.2 Given data set $X = \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ -2 & 0 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}$ $y = \begin{bmatrix} -1 \\ -1 \\ +1 \end{bmatrix} \begin{matrix} y_1 \\ y_2 \\ y_3 \end{matrix}$ solve eq (8.4) to get hyperplane & Margin.

Ans :-
(8.4) $\min_{b,w} \frac{1}{2} w^T w$; $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_d \end{pmatrix}$ $w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_d \end{bmatrix}$ $w_0 = b$

subjected to : $y_n(w^T x_n + b) \geq 1$;

Solving $y_n(w^T x_n + b) \geq 1$ for w_1, w_2 given x_1, x_2, x_3 ; y_1, y_2, y_3

$x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $y_1 = [-1]$

$-1\left([w_1 \; w_2]\begin{bmatrix} 0 \\ 0 \end{bmatrix} + b\right] \geq 1$

$\boxed{-b \geq 1}$

$x_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$, $y_2 = -1$

$-1\left([w_1 \; w_2]\begin{bmatrix} 0 \\ -1 \end{bmatrix} + b\right) \geq 1$

$\rightarrow -1(-w_2 + b) \geq 0$

$\boxed{w_2 - b \geq 1}$

$x_3 = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$ $y_3 = 1$

$1(-2w_1 + b) \geq 1$

$\boxed{-2w_1 + b \geq 1}$

$\rightarrow$ we have 3 conditions

$-b \geq 1$; $w_2 - b \geq 1$; $-2w_1 + b \geq 1$

Solving for w_1

$$\begin{array}{r} -2w_1 + b \geq 1 \\ + \quad -b \geq 1 \\ \hline -2w_1 \geq 2 \end{array}$$

$\boxed{w_1 \leq -1}$

$\Rightarrow$ Solving for w_2

$w_2 - b \geq 1$

$w_2 \geq 1 + b$ [but $b \leq -1$]

max value $\boxed{w_2 \geq 0}$

From Solving the equations we get

$b \leq -1$; $w_1 \leq -1$;

$w_2 \geq 0$

we will be equating this rather than the using inequalities.

So for $\frac{1}{2}w^T w \Rightarrow \frac{1}{2}[w_1, w_2]\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \frac{1}{2}(w_1^2 + w_2^2)$ the

with equality $w_1 = -1$ & $w_2 = 0$ will have optimal solution

at $b^* = -1$; $w_1^* = -1$; $w_2^* = 0$ satisfies all the constraint

of the given data set & $\frac{1}{2}(w_1^2 + w_2^2)$

The optimal hyperplane can be produced using

$$g(x) = \text{sing}(w^T x + b)$$

$$= \text{sign}\left([-1 \ 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - 1\right) = \text{sign}\left[-x_1 + 0(x_2) - 1\right)$$

$$\boxed{g(x) = \text{sign}(-1 - x_1)}$$

The margin will be $\frac{1}{\|w^*\|} = \frac{1}{\sqrt{1^2 + 0^2}} = \frac{1}{\sqrt{1}} = 1$

margin = 1

Q2
8.4
Ans) From Ex 8.2 the toy data set

$$X = \begin{bmatrix} 0 & 0 \\ 2 & 2 \\ 2 & 0 \end{bmatrix} \quad y = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \quad w = \begin{bmatrix} 1.2 \\ -3.2 \end{bmatrix} \quad b = -0.5$$

From the dual formulae of Hard-margin SVM :-

$$L(b, w, \alpha) = \frac{1}{2} w^T w + \sum_{n=1}^{N} \alpha_n y_n w^t x_n - b \sum_{n=1}^{n} \alpha_n y_n + \sum_{n=1}^{n} \alpha \quad --(8.17)$$

and derivating this wrt w & d we get

$$\frac{dL}{dw} = w - \sum_{n=1}^{n} \alpha_n y_n x_n \quad \& \quad \frac{dL}{db} = -\sum_{n=1}^{N} \alpha_n y_n$$

equating thee equations to zero.

$$W = \sum_{n=1}^{n} \alpha_n y_n x_n \qquad \sum_{n=1}^{N} \alpha_n y_n = 0$$

Substituting

$$\begin{bmatrix} 1.2 \\ -3.2 \end{bmatrix} = \alpha_1 (-1) \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \alpha_2 (-1) \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \alpha_3 (1) \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1.2 \\ -3.2 \end{bmatrix} = \begin{bmatrix} -2\alpha_2 \\ -2\alpha_2 \end{bmatrix} + \begin{bmatrix} 2\alpha_3 \\ 0 \end{bmatrix} = \begin{bmatrix} -2\alpha_2 + 2\alpha_3 \\ -2\alpha_2 \end{bmatrix}$$

equating

$$-2\alpha_2 + 2\alpha_3 = 1.2 \quad ; \quad -2\alpha_2 = -3.2 \Rightarrow \boxed{\alpha_2 = 1.6}$$

substituting α_2

$$2\alpha_3 = 1.2 + 3.2 \Rightarrow \alpha_3 = \frac{4.4}{2} \Rightarrow \boxed{\alpha_3 = 2.2}$$

From $\sum_{n=1}^{n} \alpha_n y_n = 0$ we get

$$-\alpha_1 - \alpha_2 + \alpha_3 = 0 \Rightarrow \alpha_1 = \alpha_3 - \alpha_2$$

Substituting $\alpha_2 = 1.6$, $\alpha_3 = 2.2$ we get

$$\alpha_1 = 2.2 - 1.6 = 0.6$$

so the $\alpha_1 = 0.6$, $\alpha_2 = 1.6$, $\alpha_3 = 2.2$

we get $\alpha^* = \begin{bmatrix} 0.6 \\ 1.6 \\ 2.2 \end{bmatrix}$