# A FIVE-BOX MODEL OF EARTH HISTORY

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## 1. Model Overview

The goal of this exercise is to quantify the mass, compositional and isotopic evolution of the Earth system from 4.5 Ga to the present using a box-modeling approach. We are particularly interested in constraining five global reservoirs throughout Earth History, namely: the mantle (reservoir #1), continental crust (#2), oceanic crust (#3), seawater-atmosphere (#4), and sediments (#5). The relationships of these reservoirs, in addition to characteristic fluxes and enrichment factors, are illustrated below in Figure 1.

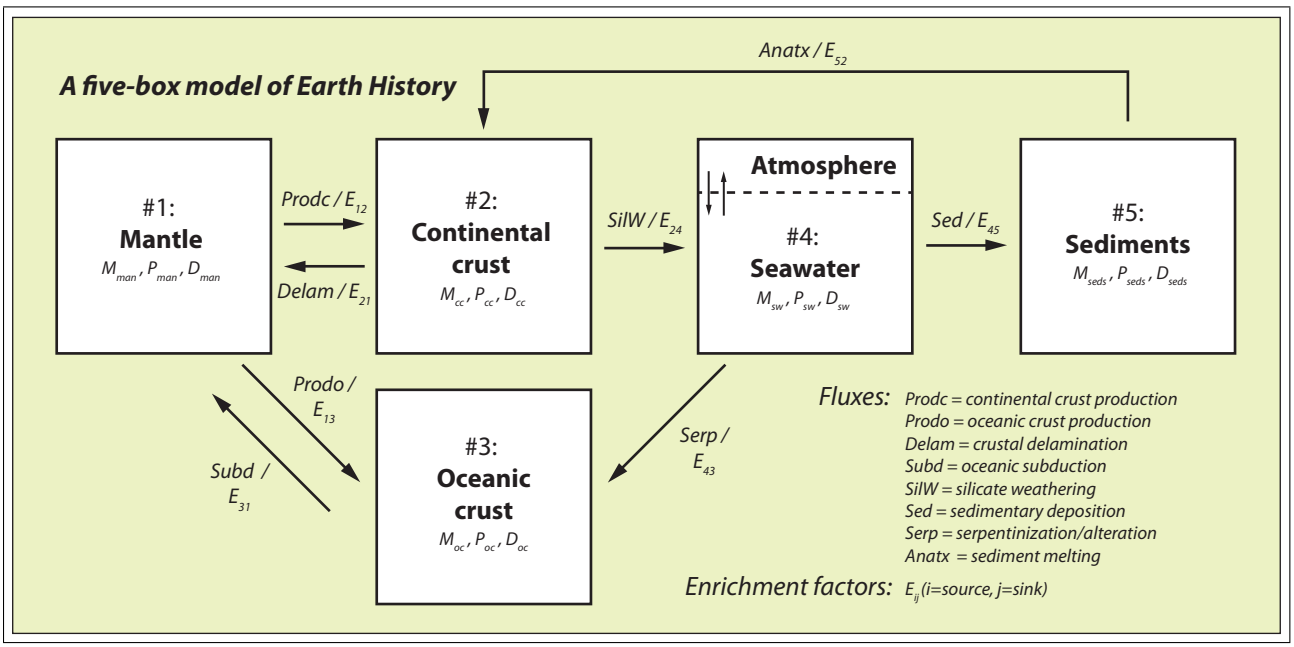


Figure 1. Schematic diagram of the Earth system as a five-reservoir box model, including the mantle, continental crust, oceanic crust, seawater-atmosphere and sediments.

## 2. Conservation of reservoir masses

The following ordinary differential equations (ODEs) balance the mass evolution of the mantle ($man$), continental crust ($cc$), oceanic crust ($oc$), seawater-atmosphere ($sw$), and sediments ($seds$) in terms of the mass fluxes into/out of each reservoir. Masses of individual reservoirs are denoted as $M_{res}$.

$$\frac{\mathrm{d}M_{man}}{\mathrm{d}t} + \frac{\mathrm{d}M_{cc}}{\mathrm{d}t} + \frac{\mathrm{d}M_{oc}}{\mathrm{d}t} + \frac{\mathrm{d}M_{sw}}{\mathrm{d}t} + \frac{\mathrm{d}M_{seds}}{\mathrm{d}t} = 0 \tag{1}$$

$$\frac{\mathrm{d}M_{man}}{\mathrm{d}t} = -Prodc - Prodo + Delam + Subd \tag{2}$$

$$\frac{\mathrm{d}M_{cc}}{\mathrm{d}t} = -Delam - SilW + Prodc + Anatx \tag{3}$$

$$\frac{\mathrm{d}M_{oc}}{\mathrm{d}t} = -Subd + Prodo + Serp \tag{4}$$

$$\frac{\mathrm{d}M_{sw}}{\mathrm{d}t} = -Sed - Serp + SilW \tag{5}$$

$$\frac{\mathrm{d}M_{seds}}{\mathrm{d}t} = -Anatx + Sed \tag{6}$$

## 3. Conservation of radiogenic parent/stable daughter isotopic abundances

The following ODEs describe the evolution of parent ($P$) and daughter isotopes ($D$) of an arbitrary radiogenic decay system characterized by decay constant $\lambda_P$. Note that these equations can be adapted for non-radiogenic species by excluding the first term on the right hand side of each equation (i.e., *in situ* radiogenic decay/growth term, $\pm\lambda_P P^*_{res}$):

$$\frac{\mathrm{d}P_{man}}{\mathrm{d}t} = -\lambda_P P^*_{man} - \left(\frac{Prodc}{M_{man}} P_{man} E^P_{12}\right) - \left(\frac{Prodo}{M_{man}} P_{man} E^P_{13}\right) + \left(\frac{Delam}{M_{cc}} P_{cc} E^P_{21}\right) + \left(\frac{Subd}{M_{oc}} P_{oc} E^P_{31}\right) \tag{7}$$

$$\frac{\mathrm{d}D_{man}}{\mathrm{d}t} = \lambda_P P^*_{man} - \left(\frac{Prodc}{M_{man}} D_{man} E^D_{12}\right) - \left(\frac{Prodo}{M_{man}} D_{man} E^D_{13}]\right) + \left(\frac{Delam}{M_{cc}} D_{cc} E^D_{21}\right) + \left(\frac{Subd}{M_{oc}} D_{oc} E^D_{31}\right) \tag{8}$$

$$\frac{\mathrm{d}P_{cc}}{\mathrm{d}t} = -\lambda_P P^*_{cc} - \left(\frac{SilW}{M_{cc}} P_{cc} E^P_{24}\right) - \left(\frac{Delam}{M_{cc}} P_{cc} E^P_{21}\right) + \left(\frac{Prodc}{M_{man}} P_{man} E^P_{12}\right) + \left(\frac{Anatx}{M_{seds}} P_{seds} E^P_{52}\right) \tag{9}$$

$$\frac{\mathrm{d}D_{cc}}{\mathrm{d}t} = \lambda_P P^*_{cc} - \left(\frac{SilW}{M_{cc}} D_{cc} E^D_{24}\right) - \left(\frac{Delam}{M_{cc}} D_{cc} E^D_{21}\right) + \left(\frac{Prodc}{M_{man}} D_{man} E^D_{12}\right) + \left(\frac{Anatx}{M_{seds}} D_{seds} E^D_{52}\right) \tag{10}$$

$$\frac{\mathrm{d}P_{oc}}{\mathrm{d}t} = -\lambda_P P^*_{oc} - \left(\frac{Subd}{M_{oc}} P_{oc} E^P_{31}\right) + \left(\frac{Prodo}{M_{man}} P_{man} E^P_{13}\right) + \left(\frac{Serp}{M_{sw}} P_{sw} E^P_{43}\right) \tag{11}$$

$$\frac{\mathrm{d}D_{oc}}{\mathrm{d}t} = \lambda_P P^*_{oc} - \left(\frac{Subd}{M_{oc}} D_{oc} E^D_{31}\right) + \left(\frac{Prodo}{M_{man}} D_{man} E^D_{13}\right) + \left(\frac{Serp}{M_{sw}} D_{sw} E^D_{43}\right) \tag{12}$$

$$\frac{\mathrm{d}P_{sw}}{\mathrm{d}t} = -\lambda_P P^*_{sw} - \left(\frac{Serp}{M_{sw}} P_{sw} E^P_{43}\right) - \left(\frac{Sed}{M_{sw}} P_{sw} E^P_{45}\right) + \left(\frac{SilW}{M_{cc}} P_{cc} E^P_{24}\right) \tag{13}$$

$$\frac{\mathrm{d}D_{sw}}{\mathrm{d}t} = \lambda_P P^*_{sw} - \left(\frac{Serp}{M_{sw}} D_{sw} E^D_{43}\right) - \left(\frac{Sed}{M_{sw}} D_{sw} E^D_{45}\right) + \left(\frac{SilW}{M_{cc}} D_{cc} E^D_{24}\right) \tag{14}$$

$$\frac{\mathrm{d}P_{seds}}{\mathrm{d}t} = -\lambda_P P^*_{seds} - \left(\frac{Anatx}{M_{seds}} P_{seds} E^P_{52}\right) + \left(\frac{Sed}{M_{sw}} P_{sw} E^P_{45}\right) \tag{15}$$

$$\frac{\mathrm{d}D_{seds}}{\mathrm{d}t} = \lambda_P P^*_{seds} - \left(\frac{Anatx}{M_{seds}} D_{seds} E^D_{52}\right) + \left(\frac{Sed}{M_{sw}} D_{sw} E^D_{45}\right) \tag{16}$$