Mathematical Quantification and Note Sequence Generation of Hindustani and Carnatic Ragas Using Transitional Probability Matrices

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Abstract

Music has been an integral part of human culture for centuries, with Hindustani and Carnatic music emerging as two distinct classical traditions in India. These styles, rich in complexity and nuance, continue to influence modern music. While listening to and appreciating these forms may appear straightforward, quantifying the intricate structures of their notes and rhythms presents a considerable challenge. Numerous methods have been developed to analyze and quantify the raga structures inherent in these traditions. This paper explores the use of transitional probability matrices(TPMs) as a tool for quantifying the similarities between Hindustani and Carnatic ragas and introduces an initial algorithm designed to generate note sequences based on a given transitional probability matrix. Such an approach holds potential applications in raga identification and could also serve as a foundational system for music sampling in the context of classical Indian music.

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1 Introduction

Indian classical music is primarily divided into two schools: Hindustani and Carnatic, with roots in the northern and southern regions of India, respectively. While both are now practiced and celebrated across the country, each style reflects unique historical and cultural influences.

Hindustani music is segmented into a progression of movements within a single raga. This includes an introductory section, *raag parichaya*, where the raga is introduced, followed by *alaap*, a slow and non-rhythmic improvisation. The piece then moves into the structured *bada khyal* (major theme) and *chota khyal* (minor theme), interspersed with *bol taans* (melodic syllables) and *taans* (fast melodic passages). Optional lighter elements such as folk songs, devotional songs may also be included to enhance the composition. Instrumental support often features the *tabla*, *harmonium*, and *flute*, which can be played in unison or *jugalbandi* (a duet style) with other instruments or vocalists, enriching the texture of the performance.

Ragas in Hindustani music are categorized into ten foundational scales, or *thaats*, which provide the structural basis for other ragas. Each raga is associated with a specific time for performance and expresses a particular mood or sentiment, known as *rasa* or *bhava*, that performers aim to evoke through nuanced interpretation and style.

Carnatic music, a classical genre from South India, has developed over centuries with a strong emphasis on structured melodic and rhythmic frameworks. Central to its system are *ragas*, which provide the melodic structure for compositions and improvisations. These *ragas* are divided into two categories: *Janaka ragas*, which include all seven musical notes in their structure, and *Janya ragas*, which are derived from these primary frameworks to allow greater expressive flexibility.

Rhythmic cycles, or *talas*, are equally important in Carnatic music. The seven primary *talas* form the basis for rhythmic patterns, each having a unique structure that guides timing and tempo in performances.

The instrumental ensemble in Carnatic music is varied and complements the vocal tradition. Instruments are categorized into four main types:

- String instruments, which produce sound through vibrating strings.
- Wind instruments, where sound is created by controlled airflow.
- Percussion instruments, which provide rhythm through striking or tapping.
- Idiophones, which naturally resonate without tuning.

Each element, from *ragas* and *talas* to instrumental diversity, contributes to the intricate, expressive quality of Carnatic music, blending structured compositions with rich improvisation, defining its enduring appeal and complexity.

2 Literature Review

In this study, all notations and methodologies adhere to the framework established in Bhattacharjee and Srinivasan (2011). The process for constructing Transition Probability Matrices (TPMs) aligns with the approach detailed in Bhattacharjee and Srinivasan (2011), ensuring consistency in our analysis. Our findings serve to corroborate the conclusions drawn in Bhattacharjee and Srinivasan (2011), while the same analytical logic is applied to both Hindustani and Carnatic music traditions. By grounding our research in the established literature, we aim to provide a robust comparison and deepen the understanding of raga similarities across these two prominent Indian classical music styles.

Table 1: Swaras and their corresponding notes [Bhattacharjee and Srinivasan (2011)]

Name	Pronunciation	Symbol	Solfège	Western
Sharaj	Sa	S	do	C
Komal Rishabh	Re	r	re (flat)	Db
Shuddha Rishabh	Re	R	re	D
Komal Gandhar	Ga	g	mi (flat)	Eb
Shuddha Gandhar	Ga	G	mi	E
Shuddha Madhyam	Ma	m	fa	F
Tivra Madhyam	Ma	M	fa (sharp)	F#
Pancham	Pa	P	sol	G
Komal Dhaivat	Dha	d	la (flat)	Ab
Shuddha Dhaivat	Dha	D	la	A
Komal Nishad	Ni	n	si (flat)	Bb
Shuddha Nishad	Ni	N	si	B

3 Methodology

The process of creating a Transition Probability Matrix (TPM) involves structured steps to analyze the movement between musical notes. In this study, both Hindustani and Carnatic compositions are transcribed into sequences of notes. Following the notation guidelines established in Bhattacharjee and Srinivasan (2011), we transcribe compositions of renowned Hindustani and Carnatic artists, into a standardized format compatible with MathWorks (2022) for further processing.

To ensure comparability, we selected three sets, each consisting of a Hindustani and a Carnatic raga that share identical notes. The resulting raga sequences for each set are then processed to facilitate a direct analysis of note transitions and similarities within and across the classical music systems of Hindustani and Carnatic traditions.

The following are the notes of the ragas analyzed in this paper.

Raag Bilaval & Shankarabharanam

- **Aarohan**: S R G m P D N S
- **Avarohan**: S N D P m G R S

Raag Bharirav & Mayamalagowa

- **Aarohan**: S r G m P d N S
- **Avarohan**: S N d P m G r S

Raag Kafi & Kharaharapriya

- **Aarohan**: S R g m P D n S
- **Avarohan**: S n D P m g R S

Generation of Transition Probability Matrix (TPM)

The note sequences are first processed to create a count matrix that tracks transitions between consecutive notes. This count matrix is normalized by dividing each row by its total, resulting in values between 0 and 1 for each row. This normalized matrix thus represents the relative probability of each transition occurring from a given note.

The note sequences are also used to calculate the percentage occurrences of individual notes, allowing for a detailed comparison of the ragas. This process facilitates both a visual and quantitative understanding of the similarities and differences between Hindustani and Carnatic ragas, enhancing our insights into the structural relationships within and between these two classical musical traditions.

The TPMs generated in this study are 12×12 matrices, each representing transitions between musical notes, with each row and column corresponding to a unique note. However, strictly speaking, these matrices do not qualify as proper TPMs, as the rows corresponding to non-existent notes (those not used in a specific raga) contain only zero values. In a standard TPM, the sum of each row must equal 1 to represent the full probability distribution across potential transitions.

To adapt these matrices for proper probabilistic analysis, a modification is applied. Specifically, the intersection cell where both the row and column represent a non-existent note (i.e., the "zero" entries) is assigned a value of 1. This ensures that the matrix mathematically satisfies the requirement of row sums equaling 1, thus aligning with the formal definition of a TPM while maintaining the structure representative of the raga's actual note transitions.

This adjustment allows for consistency in subsequent analyses and comparisons, even though it is a workaround rather than a strict adherence to typical TPM construction principles.

Postprocessing of Transition Probability Matrix

The Transition Probability Matrix (TPM) undergoes a postprocessing step to transform each row into a vector of cumulative intervals. This step is essential for enabling a random number generator to map effectively to specific transitions within the matrix. Each element in a row, except for the first element, is modified by adding the value of the preceding element, resulting in a cumulative sum. This cumulative structure transforms the row into a set of intervals.

For instance, if the first element of a row has a value of 0.1 and the second element is also 0.1, the cumulative value for the second element becomes 0.2. This restructuring allows each row's values to function as cumulative intervals, which facilitates probabilistic note selection.

Note Selection Process

A random number generator is used to simulate the progression of notes (*swaras*) in the raga sequence. A random number is generated within the interval $[0, 1]$ and used to select a corresponding *swara* based on the cumulative intervals established during postprocessing.

The process begins by selecting an initial *swara* that has a non-zero probability in the TPM, ensuring that the note is indeed part of the chosen raga. This note serves as the starting point for the sequence.

Generation of Swara Sequence

After selecting the initial note, an empty list named `notes` is initialized to store the generated note sequence. The random number generator then selects each subsequent note by referencing the cumulative intervals in the row of the preprocessed matrix that corresponds to the most recently selected note.

For example, if the initial note is denoted as a_i where $1 \leq i \leq 12$, it is stored in the `notes` array. The next note, a_j, is chosen based on the intervals defined in the row corresponding to a_i, where $1 \leq j \leq 12$. This iterative selection continues until the desired sequence length is achieved. Each new note is appended to the `notes` array, resulting in a sequence that reflects the probabilistic structure of the raga and maintains musical coherence.The TPM system uses 12 distinct notes without an inherent understanding of octaves. To manage this limitation, specific rules are applied for octave transitions during note generation. When moving from the notes d, D, n, N to S, r, R, g, G, an upward octave shift occurs to ensure a smooth melodic transition. Conversely, when transitioning from S, r, R, g, G to d, D, n, N, a downward octave shift takes place to maintain musical continuity. These rules help avoid abrupt jumps and preserve the natural flow of the melody across octaves.The TPMs used in this study naturally gravitate towards higher notes and, consequently, higher octaves. This tendency arises because the TPMs are derived from the *alaap* of the raga, which represents the gradual unfolding and development of the raga as it expands to encompass the full octave.To ensure that the note generation sequence remains centered around the middle octave, an external parameter called *bias* is introduced into the transition probabilities. This bias encourages the sequence to stay within the middle octave, and its value can be fine-tuned within the code.The code is provided in the supplementary information[GenT-Note (2025)].

This process yields a generated note sequence that embodies the structural patterns of the raga while adhering to its characteristic transitions.

4 Results

In this study, we aim to assess the similarities between ragas using simple mathematical tools such as difference heat maps, singular value decomposition (SVD), Frobenius norms, and note percentage graphs[Bhattacharjee and Srinivasan (2011)], and provide a preliminary code to generate note sequences from a given TPM[GenT-Note (2025)]. These tools serve distinct purposes: the difference heat map quantifies the differences in the transitions of the notes used in each raga, the singular value decomposition (SVD) measures the dominance of each note in the overall transitions, and the note percentage graphs identify the dominant notes and reveal similarities between ragas based on their frequency of usage.

Figure 1 illustrates the similarities between the ragas Shankarabharanam and Bilaval, which exhibit the highest degree of similarity in this study.

Figures 2 and 3 provide graphical representations of the similarities between the ragas Kharaharapriya and Kafi, and Mayamalagowla and Bhairav, respectively.

The actual vaadi and samvaadi of Raag Bilaval are D and G, respectively. However, from Figure 1(c), it can be observed that the most frequently used notes, in descending order, are G, S, N, and D. This discrepancy arises because the TPM for Raag Bilaval was generated using only a 30-minute excerpt of the alaap from a 2.5-hour-long performance. Since the alaap primarily emphasizes lower notes, this bias is reflected in the analysis.

Figure 1(a) provides an insightful interpretation of the Transition Probability Matrix (TPM) difference for both the Bilaval and Shankarabharanam ragas. The rank of the TPM matrix corresponds to the number of distinct notes present in the raga, which, in this case, is 7.

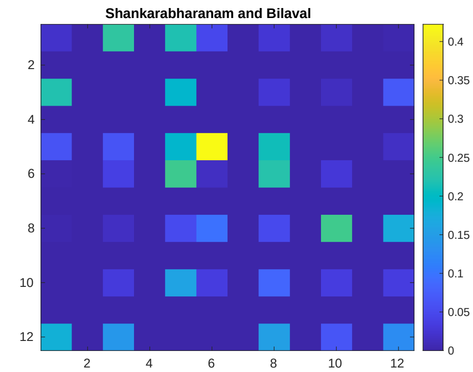
A notable feature of the singular values, as depicted in Figure 1(b), is their gradual decline in magnitude with increasing index. This trend highlights that lower-index singular values dominate the transitions, indicating that the most significant transitions are largely governed by the most dominant notes in the raga. Specifically, in both Bilaval and Shankarabharanam, the most dominant starting notes are R (Rishabh) and G (Gandhar), further underlining the similarities between these two ragas.

Figure 2(b) reveals that the number of significant notes in Kharaharapriya is one less than in Kafi (significant notes are those with non-zero values). This suggests a higher influence of the lower notes of the octave in Kharaharapriya, indicating more transitions to and from the lower notes. Figure 2(c) highlights the affinity towards higher notes in Kharaharapriya, with a linear increase in note percentages, which is referred to as Uttaranga Pradhana. In contrast, Kafi exhibits a stable note percentage distribution, reflecting its calm and steady nature.

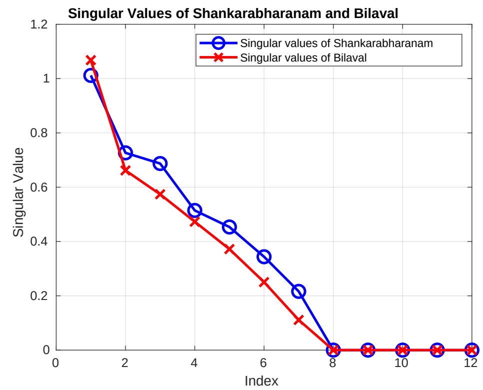
Figure 3(b) presents the singular values for the ragas Bhairav and Mayamalagowla. The rank of the TPM is the same for both ragas, 7. However, Bhairav displays a smoother trend in the decline of singular values compared to Mayamalagowla, which may indicate the ease with which Bhairav is presented. Known as a morning raga, this smooth trend aligns with its serene character.

Figure 3(c) illustrates the note percentages of both ragas, showing a clear difference in note usage. Bhairav tends to emphasize the mid notes of the octave, aligning with its representation in many famous compositions, while Mayamalagowla seems to evenly distribute its emphasis across all notes. Figure 3(a) further emphasizes these observations, particularly highlighting the differences in notes G, P, and N.

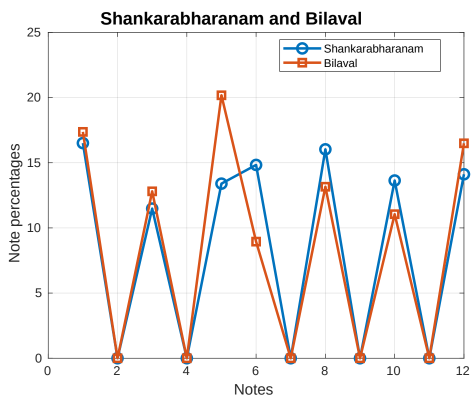
Together, these visualizations provide valuable insights into the structural and statistical differences across these ragas, offering a mathematical perspective on their inherent musical characteristics.



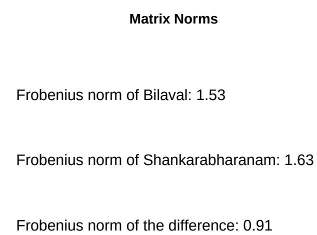
(a) Difference Heat Map



(b) Singular Value Decomposition

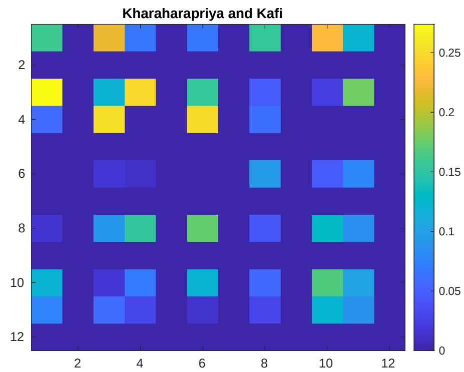


(c) Note Percentage

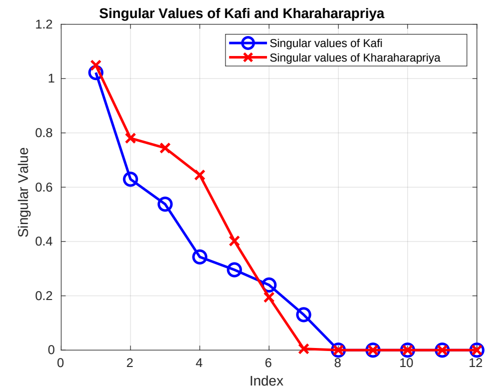


(d) Frobenius Norm

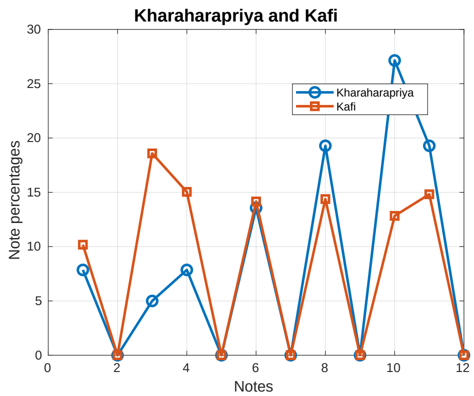
Figure 1: Shankarabharanam and Bilaval. **Alt Text:**The figure presents a 2×2 subplot comparing key properties of the Transition Probability Matrices (TPM) for the ragas Shankarabharanam and Bilaval. The first subplot displays the Difference Heat Map of TPMs, where the values range from 0 to 0.4, highlighting variations between the two ragas. The most significant difference is observed in the note 'm' with a value of 0.4, while the remaining values are generally in the range of 0.1–0.25. The second subplot shows the Singular Value Plot of TPMs, revealing 7 non-zero singular values that decrease in a linearly declining fashion with the same slope, followed by 5 zero values for both ragas. The third subplot illustrates the Note Percentages of the Note Sequence, showing that the note sequences of both ragas align closely, so much so that they might have been derived from the same raga. Finally, the fourth subplot presents the Frobenius Norm of the TPM, with values of 1.53 for Bilaval and 1.63 for Shankarabharanam.



(a) Difference Heat Map



(b) Singular Value Decomposition



(c) Note Percentage

Matrix Norms

Frobenius norm of Kafi: 1.42

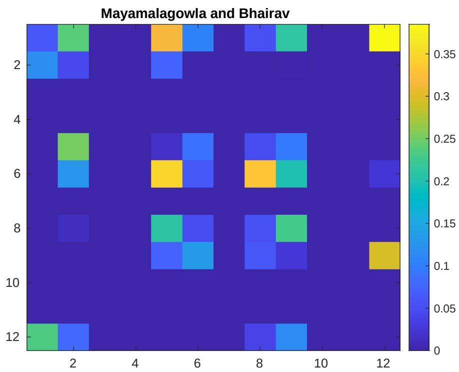
Frobenius norm of Kharaharapriya: 1.70

Frobenius norm of the difference: 0.86

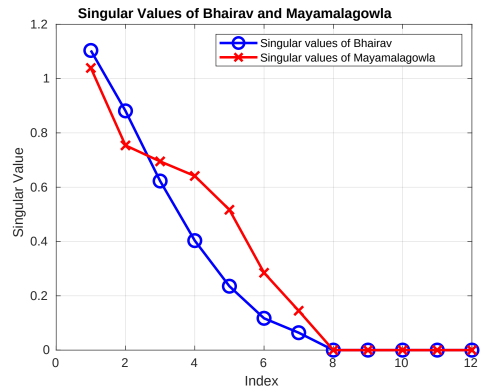
(d) Frobenius Norm

Figure 2: Kharaharapriya and Kafi. **Alt Text:** The figure presents a 2×2 subplot comparing key properties of the Transition Probability Matrices (TPM) for the ragas Kharaharapriya and Kafi. The first subplot displays the Difference Heat Map of the TPMs, with values ranging from 0 to 0.25. Significant differences are observed in the lower half of the octave — in the notes S, R, g, and m, where values approach 0.25. The remaining differences are generally within the range of 0.05–0.15. The second subplot shows the Singular Value Plot of the TPMs, where Kafi has 7 non-zero singular values and Kharaharapriya has 6 non-zero singular values. These values decrease almost linearly before reaching zero, with Kharaharapriya exhibiting a steeper slope compared to Kafi. The third subplot illustrates the Note Percentages of the Note Sequence, revealing distinct patterns. In Kharaharapriya, the note percentages increase almost linearly from 7% to 28%, while in Kafi, the values remain relatively steady around 15%. Finally, the fourth subplot presents the Frobenius Norm of the TPMs, with values of 1.70 for Kharaharapriya and 1.42 for Kafi.

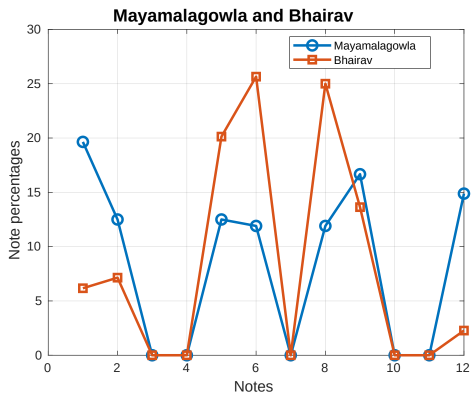
While more patterns and similarities can be uncovered by analyzing additional ragas using multiple analytical techniques, the differences between these ragas become particularly evident when examining their dominant notes, as well as the intricate time transitions and pauses within their structures. Although Transition Probability Matrices (TPMs) effectively capture the general flow and frequency of note transitions, they fail to fully represent the temporal intricacies—such as duration, pauses, and rhythmic emphasis—that are essential to the expressive nature of Indian classical music. These temporal elements, which play a crucial role in conveying the emotional and aesthetic essence of a raga, are inherently lost in matrix-based analyses, resulting in a simplified representation of the ragas' true characteristics.



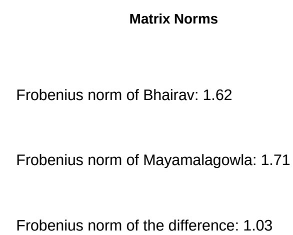
(a) Difference Heat Map



(b) Singular Value Decomposition



(c) Note Percentage



(d) Frobenius Norm

Figure 3: Mayamalagowla and Bhairav. **Alt Text:** The figure presents a 2×2 subplot comparing key properties of the Transition Probability Matrices (TPMs) for the ragas Bhairav and Mayamalagowla. The first subplot displays the difference heat map of the TPMs, where the values range from 0 to 0.35. Significant differences are observed in the notes G, m, and N, with values around 0.3, while the remaining values generally range between 0.1 and 0.25. The second subplot illustrates the singular value plots of both ragas, showing 7 non-zero singular values followed by 5 zero singular values. These values exhibit an almost linear decline before reaching zero, with Mayamalagowla displaying a steeper slope compared to Bhairav, suggesting a difference in how transitions are distributed across the dominant notes. The third subplot represents the note percentages for both ragas. Bhairav shows higher percentages for the notes G, m, and P, while the other notes display relatively lower percentages. In contrast, Mayamalagowla maintains a more even distribution across the octave. Finally, the Frobenius norm of the TPM for Bhairav is 1.62, while for Mayamalagowla, it is slightly higher at 1.71.

The generated note sequence from the code consists of sinusoidal notes, which lack the nuanced tonal variations and expressive dynamics that are integral to capturing the true essence of a raga. Despite this limitation, the sequence effectively preserves the fundamental note phrases and characteristic patterns of the raga, allowing it to remain identifiable. The clarity of the note arrangement ensures that the core melodic structure is conveyed, even if the finer details are absent. The code, named GenT-Note, which was used to generate note sequences from TPMs, has been provided in the references. GenT-Note (2025)

5 Conclusion and Future Directions

The present study provides valuable insights into the differences in the presentation of ragas across different styles. While each method of quantification offers a unique perspective, none of them can fully capture the essence of a raga. A raga is not only defined by its notes, but also by nuances such as the halts, pauses, and silences, which are crucial to its interpretation. In this study, Bilaval from the Hindustani system and Shankarabharanam from the Carnatic system emerge as the most similar in terms of singing style across all the comparison methods employed. The algorithm developed for generating note sequences shows significant potential in applications such as robotics and synthesizers. With further advancements, this algorithm could improve the quality of generated sequences and may one day be capable of replicating the styles of renowned singers, offering new forms of entertainment.

Several enhancements and extensions to the current project are proposed for future research and development:

- **Variable Time Intervals:** The introduction of variability in the time intervals between notes is proposed to create more dynamic and expressive musical compositions. This could involve varying the duration between notes based on the transition probabilities or introducing rhythmic variations.
- **Alternative Sampling Techniques:** Exploring different sampling methods to generate the transition probability matrix is suggested, which could potentially enhance the musical quality and diversity of the generated compositions. For example, instead of relying solely on

observed transition frequencies, machine learning techniques could be employed to model the transition probabilities.

- **Diverse Audio Forms:** The current implementation uses piano notes to represent the swaras. Future work could involve experimenting with various forms of audio, such as string instruments, wind instruments, or synthesized sounds, to broaden the auditory palette of the generated music.

- **Application to Other Instruments:** The methodology can be extended to include other musical instruments, such as percussion, string instruments, or wind instruments. By generating transition probability matrices for these instruments, the approach could be adapted to create music using a wider range of instrumental sounds.

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