Probability

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1. Preliminaries

Definition 1.1. Discrete Random Variable:

Definition 1.2. Probability Mass Function of

$$X \text{ as } p_X : \Omega_X \to [0,1] \qquad p_X(k) = P(X = k)$$

- pmf of a d.r.v. X takes values from the same space Ω_X and a probability to each between 0 and 1 inclusive
$$p_X : \Omega_X \to [0,1]$$
- p_X is the PMF for random variable X, we use little p not big P
- $p_X \equiv p(x)$
- Ω_X is the sample space or support of X
- $[0,1]$ is the interval of real numbers from 0 to 1 inclusive
- $\to$ indicates the function maps elements from the domain Ω_X to the range $[0,1]$
- What can we tell from this definition? 1. the pmf is non-negative and at-most 1. $0 \leq p_X \leq 1$. This
$$p_X(k) = P(X = k)$$
- For each value k in Ω_X, $p_X(k)$ gives Pr that X takes on k

Properties

1. $0 \leq p_X \leq 1$: p_X is always non-negative and at most 1. This is basic intuitive understanding!
2. $\sum p(x) = 1$ the sum of all probabilities of all outcomes = 1, that is the sum taken over all x in Ω_X
This formalizes the idea that at least 1 outcome must occur, and all outcomes are accounted for. Importantly, it enables the complement probability! Because we sum to 1, then the complement probability e.g. $\Pr[\text{not } A] = 1 - \Pr[A]$.
3. For discrete random variables, $p(x) = 0$ for all x not in the sample space Ω_X
This formalizes the idea that impossible outcomes have zero probability, helps define the support/range of the distribution, allows us to extend the domain of the pmf.
E.g. for a dice roll: $\Omega_X = \{1,2,3,4,5,6\}$, if $p(x) = 0$ then $p(0), p(7), p(3.5) = 0$ the probability we roll a 0 is 0.

Example 1.3. adf

If you want the math to appear in its own line, the standard way is to use:

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    \sqrt{x+y}
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