Probability

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1. Preliminaries

Definition 1.1. Discrete Random Variable:

Definition 1.2. Probability Mass Function of

$$X \text{ as } p_X : \Omega_X \to [0,1] \qquad p_X(k) = P(X = k)$$

- pmf of a d.r.v. X takes values from the same space Ω_X and a probability to each between 0 and 1 inclusive
$$p_X : \Omega_X \to [0,1]$$
- p_X is the PMF for random variable X, we use little p not big P
- $p_X \equiv p(x)$
- Ω_X is the sample space or support of X
- $[0,1]$ is the interval of real numbers from 0 to 1 inclusive
- $\to$ indicates the function maps elements from the domain Ω_X to the range $[0,1]$
- What can we tell from this definition? 1. the pmf is non-negative and at-most 1. $0 \leq p_X \leq 1$. This
$$p_X(k) = P(X = k)$$
- For each value k in Ω_X, $p_X(k)$ gives Pr that X takes on k

Properties

1. $0 \leq p_X \leq 1$: p_X is always non-negative and at most 1. This is basic intuitive understanding!
2. $\sum p(x) = 1$ the sum of all probabilities of all outcomes = 1, that is the sum taken over all x in Ω_X
This formalizes the idea that at least 1 outcome must occur, and all outcomes are accounted for. Importantly, it enables the complement probability! Because we sum to 1, then the complement probability e.g. $\Pr[\text{not } A] = 1 - \Pr[A]$.
3. For discrete random variables, $p(x) = 0$ for all x not in the sample space Ω_X
This formalizes the idea that impossible outcomes have zero probability, helps define the support/range of the distribution, allows us to extend the domain of the pmf.
E.g. for a dice roll: $\Omega_X = \{1,2,3,4,5,6\}$, if $p(x) = 0$ then $p(0), p(7), p(3.5) = 0$ the probability we roll a 0 is 0.

Mental Models

Binomial Distribution

‘’ Example: Flipping a fair coin 10 times and counting the number of heads. The number of heads is a binomial random variable. Characteristics: Fixed number of trials, 2 possible outcomes, constant probability of each trial, count successes Applications: number of defective items in a batch, number of customers making a purchase out of a total number in a store, number of opened emails in a campaign.

Poisson Distribution

Example: Number of customers arriving at a store per hour, number of calls received by a call center per day, defects in a length of fabric Characteristics:

$X\ Poi(\lambda)$ $X \in \{0, 1, \ldots, \}$ $\Pr[X = k] = e^{-\lambda}\frac{\lambda^k}{k!}$

insert distribution graph here

Gaussian

Bernoulli random variables

Moment Generating Function

Central Limit Theorem

If you want the math to appear in its own line, the standard way is to use:

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    \sqrt{x+y}
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