

# Probability

July 30, 2024

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## 1. week 1

Experiment Shuffle a deck of cards, go through in order. How many times do 2 consecutive cards have the same suit?

### 1.1. Linearity of expectation

...The sum of each little thing

$$\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y]$$

No assumption of independence or anything. Surprisingly useful.

Prove Expectation of selecting a card of type 1 1/13

sum of all expectations  $X_i$  depends on 2 cards. What's Pr

Probability of 52 C 13  $\mathbb{E}[X_i] = \Pr[X_i = 1]$

Note: difference between Expectation and Probability. Probability = the likelihood of the event e.g. selecting 2 consecutive suit cards from a deck of 52 Expectation = the average outcome. Multiply each outcome by its probability.

Monte Carlo, Las Vegas Running time Output quality

Question I have an array with  $n = 100$  index of an even number what is time complexity of getting even numbers Theta  $n$

- why isn't this constant? because you can create an algorithm that only selects

"on expectation", the las Vegas

expected time For all expectation  $[T_a] = \sum_{i=1}^{\infty} i \cdot \Pr[\text{takes } i \text{ attempts to find an even number}]$

$$\mathbb{E}[T_a] = \sum_{i=1}^{\infty} i \cdot \Pr[\text{takes } i \text{ attempts to find an even number}]$$

$$= \sum_{i=1}^{\infty} \frac{i}{2^i} = O(1)$$

need to do

discuss diff

Why Randomization?

Faster, Simpler Algo'S - miller rabin, it's a monte carlo algo. Runs in  $O(n^2)$

Algos Quicksort Expected running time Is it Las Vegas or Monte Carlo? It's always going to return the sorted array, so it's Las Vegas. Proof:  $T(n) = \text{Expectations}[\text{runtime on array size } n] = E[T(|A1|)] + E[T(|A2|)] + O(n)$  We know  $|A1| + |A2| = n-1$  Why  $n-1$ ?

## 2. Tutorial 1

Problem 1 Consider a deck of  $4n$  cards with 'S', 'H', 'D', 'C', after shuffled randomly, what's the expected number of consecutive pairs of the same suit.

## 3. Quiz 0

## 4. week 2

```
\begin{math}  
  \sqrt{x+y}  
\end{math}
```

If you want the math to appear in its own line, the standard way is to use:

```
\[  
  \sqrt{x+y}  
\]
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## Acknowledgments

Thank everyone.