MATH-UA 252/MA-UY 3204 - Fall 2022 - Worksheet #4

Problem 1. Let M be a positive-definite matrix. Show that $p = -M^{-1}\nabla f(x)$ is a descent direction.

Problem 2. Let:

$$A = \begin{bmatrix} \epsilon & 1 \\ 1 & 1 \end{bmatrix}, \tag{1}$$

where $\epsilon > 0$ is small. Consider two ways of modifying A to make it positive definite. One where only A_{11} is modified, and one where both A_{11} and A_{22} are modified. Show that the norm of the modification (that is, any matrix norm of the matrix which needs to be added to A to do the correction) is $O(\epsilon^{-1})$ in the first case and $O(1)$ in the second case.

Problem 3. The vector d is a *direction of negative curvature* for the function f at the point x is $d^\top \nabla^2 f(x) d < 0$. Prove that a direction of negative curvature exists if and only if one of the eigenvalues of $\nabla^2 f(x)$ is negative. Furthermore, show that if a direction of negative curvature exists for f at the point x, then a direction of negative curvature which is also a descent direction for f at x exists.