NAME: **NET ID**:

MATH-UA 252/MA-UY 3204 - Fall 2022 - Quiz #3

Problem 1. A general iterative method for minimizing $f : \mathbb{R}^n \to \mathbb{R}$ has the form:

$$\boldsymbol{x}_{n+1} = \boldsymbol{x}_n + \boldsymbol{p}_n, \qquad n \geq 0.$$

What is the condition for $\boldsymbol{p}_n$ to be a descent direction?

$$\nabla f(\boldsymbol{x}_n)^\top \boldsymbol{p}_n < 0$$

Problem 2. Why is $\boldsymbol{p}_n = -\nabla f(\boldsymbol{x}_n)$ always a descent direction?

$$\nabla f(\boldsymbol{x}_n)^\top \boldsymbol{p}_n = -\nabla f(\boldsymbol{x}_n)^\top \nabla f(\boldsymbol{x}_n) = -\|\nabla f(\boldsymbol{x}_n)\|_2^2 \leq 0$$

$$\text{(Note: equality if and only if } \boldsymbol{x}_n = \boldsymbol{x}^*.\text{)}$$

Problem 3. What kind of functions does Newton's method minimize (or, at least, find a stationary point of) in exactly one step?

Quadratic functions, or "quadratic forms", e.g. functions of the form:

$$q(\boldsymbol{x}) = \boldsymbol{x}^\top \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}^\top \boldsymbol{x} + c$$