Problem 1

(i) if $B_{k+1} = B_k + \dfrac{(y_k - B_k S_k)v^T}{v^T S_k}$

$$(v^T S_k \neq 0)$$

then

$$B_{k+1} S_k = B_k S_k + \frac{(y_k - B_k S_k)v^T S_k}{v^T S_k}$$

$$= B_k S_k + (y_k - B_k S_k)$$

$$= y_k$$

therefore
scant condition holds

(ii) As for BFGS update

$$B_{k+1} = B_k - \frac{B_k S_k S_k^T B_k}{S_k^T B_k S_k} + \frac{y_k y_k^T}{y_k^T S_k}$$

$\Downarrow$

$$B_{k+1} S_k = B_k S_k - B_k S_k \cdot \frac{S_k^T B_k S_k}{S_k^T B_k S_k}$$

$$+ \frac{y_k (y_k^T S_k)}{(y_k^T S_k)}$$

$$= B_k S_k - B_k S_k + y_k$$

$$= y_k$$

$\Downarrow$

the scant condition holds