MATH-UA 252/MA-UY 3204 - Fall 2022 - Worksheet #2

Problem 1. Let $f(x, y) = x^3 + x^2 + xy - y^2 + \cos(x)\sin(y)$. Write down the general expression for the Newton step for minimizing f.

Problem 2. Consider the quadratic function $q : \mathbb{R}^n \to \mathbb{R}$ defined by $q(x) = \frac{1}{2}x^\top Qx - c^\top x$. For x^* to be a stationary point of q, $\nabla q(x^*) = 0$ is necessary. Write down $\nabla q(x^*)$ explicitly. Under what conditions does q have a local minimizer? Under what conditions does q have a stationary point but no local minimum or maximum?

Problem 3. Prove that if $p^\top \nabla f(x) < 0$, then $f(x + \epsilon p) < f(x)$ for $\epsilon > 0$ small enough.

Problem 4. Let:

$$\begin{aligned} f_1(x_1, x_2) &= x_1^2 + x_2^2 - 1 = 0, \\ f_2(x_1, x_2) &= 5x_1^2 - x_2^2 - 2 = 0. \end{aligned} \tag{1}$$

Write down the Newton step for solving this system of nonlinear equations. The system has four solutions. Can you find them using different initial guesses?

Problem 5. Prove that if Newton's method is applied to solve a system of nonlinear equations where *some* of the equations are actually linear, that the linear equations will be satisfied at each step of the iteration, except possibly at the first iteration.