NAME: NET ID:

MATH-UA 252/MA-UY 3204 - Fall 2022 - Quiz #1

Problem 1. What does $f(x) = O(g(x))$ as $x \to x_0$ mean?

$$\lim_{x \to x_0} \frac{f(x)}{|g(x)|} < \infty$$

Problem 2. Write the Taylor expansion of the single-variable function $f : \mathbb{R} \to \mathbb{R}$ about the base point x with increment h to three terms (constant, linear, and quadratic). Write the remainder using big-O notation.

$$f(x+h) = f(x) + f'(x)h + \frac{1}{2} f''(x)h^2 + O(h^3)$$

Problem 3. Consider the least-squares problem:

$$\text{minimize} \quad \|Ax - b\|_2^2$$

in the variable $x \in \mathbb{R}^n$. Write down the exact solution.

$$x = A^\dagger b = (A^T A)^{-1} A^T b$$

Problem 4. Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a vector-valued function. What is the Jacobian of f?

$$Df = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$