Midterm #2 Study Guide

11.6: Directional derivatives and the gradient

In 2D and 3D, understand:

- the limit definition of the directional derivative
- how to compute a directional derivative using Theorem 3
- the definition of the gradient
- the fact that the directional derivative is just the dot product between $\vec{u}$ and the ∇f

Also, understand:

- that the gradient $\nabla f(x)$ is a vector which points in the direction of steepest ascent in f at the point x (Theorem 15)
- the difference between the direction of steepest increase and the rate of steepest increase (Example 6)
- that the gradient of a level set function is a normal to the tangent plane of the level set at that point (Equation 19)
- that the gradient of that same level set function spans the normal line of the level set at that point

See Example 7 regarding the last two points.

11.7: Maximum and minimum values

Understand:

- the definition of local maxima and minima
- the definition of absolute maxima and minima
- the first-order necessary condition for optimality (Theorem 2)
- the definition of a critical point
- the conditions under which a critical point is a local max or min
- how to find the critical points of a function
- the second-sufficient condition for optimality of a function in two variables (the "2D second derivative test")—see Theorem 3 and Example 3
- how to model and solve simple geometry problems by minimizing or maximizing functions—see Example 4 and Example 5