MATH-UA 252/MA-UY 3204 - Fall 2022 - Worksheet #5

Problem 1. Let $f(x, y) = (x - 2y)^2 + x^4$. Compute the Newton step at $(x, y) = (2, 1)$. Suppose we use a backtracking line search at this point. For what values of μ does $\alpha = 1$ satisfy the Armijo condition?

Problem 2. Let $f(x) = \frac{1}{2}x^\top Qx - c^\top x$. Let p be a descent direction for f at x. If we apply exact line search, show that:

$$\alpha = \frac{-p^\top \nabla f(x)}{p^\top Qp}. \tag{1}$$

Problem 3. Assume that we're minimizing a function using exact line search, so that:

$$x_{k+1} = x_k + \alpha_k p_k, \qquad \alpha_k = \arg\min_{\alpha>0} f(x_k + \alpha p_k). \tag{2}$$

Prove that $\nabla f(x_{k+1})$ and p_k are orthogonal.

Problem 4. Let C be a symmetric matrix of rank 1. Show that it must have the form $C = \gamma ww^\top$, where $\|w\|_2 = 1$ and $\gamma \neq 0$. (*Hint*: consider the eigenvalue decomposition of C.)

Problem 5. Let $f(x) = \alpha x^2$, where $\alpha, x \in \mathbb{R}$ and $\alpha > 0$. Show that the secant method minimizes $f(x)$ in exact one step from any starting point.