auxiliary and marginal particle filters

the why, how, and how much

sam power, thursday 16 april 2020, straight outta quarantine

overview

- review of HMMs (set notation)
- general particle filtering
    - bootstrap PF, guided PF, auxiliary PF
- marginal particle filtering
    - aside: multiple importance sampling
    - standard MargPF
    - auxiliary MargPF

hidden markov models

$\mu_0(dx_0)$ prior on location at t = 0

$f_t\left(x_{t-1} \to x_t\right)$ transition kernel / propagation / dynamics / …

$g_t\left(x_t, y_t\right)$ emission / observation / potential / weights / …

recipe for *a* particle filter

(based on presentation by N. Chopin)

- a base HMM (`Feynman-Kac model')
- fake prior w/ same support as the real prior (simulate, evaluate density)
- fake dynamics w/ same support as the real dynamics (simulate, density)
- weights (*usually* specified by other stuff)
- everything you would need for *(sequential)* <u>importance sampling</u>

pf1 - bootstrap

(various authors)

- fake prior = real prior
- fake dynamics = real dynamics
- weights = original weights
- requires that real dynamics are still a good proposal under filtering distribution
- ~ importance sampling the posterior, using the prior

pf2 - guided

(don't know source)

- fake prior ≠ real prior
- fake dynamics ≠ real dynamics
- weights ≠ original weights
- idea: use dynamics that push particles into good regions

$$w_t\left(x_{t-1}, x_t\right) = \frac{f_t(x_{t-1} \to x_t)}{\tilde{f}_t(x_{t-1} \to x_t)} \cdot g_t(x_t, y_t)$$

pf3 - auxiliary

(pitt-shephard; johansen-doucet)

- different prior, dynamics
- also different *target*
- idea: select particles with promising future, then propagate

$$\gamma_t\left(x_{0:t}\right) = \mu_0(x_0) \cdot \prod_{s=1}^{t} f_s\left(x_{s-1} \to x_s\right) \cdot \hat{p}(y_{t+1} \,|\, x_t)$$

some comments

- for { guided, auxiliary } PF, there are (locally) `optimal’ choices
  - usually (not always though!) intractable, but a useful abstraction (c.f. IS)
  - nested SMC: use *SMC sampler* to approximate optimal proposal
- c.f. lookahead schemes, { twisted / controlled / … } SMC
- **can** still be used in particle MCMC (Finke-Doucet-Johansen)
  - ~ just need the IS argument to work

multiple importance sampling

(veach, guibas)

- suppose we do IS targeting p, using q_1, …, q_N.
    - which of the following should we use?

$$\mathbf{E}_p\left[f(x)\right] \approx \frac{1}{N}\sum_{i=1}^{N}\frac{p(x_i)}{q_i(x_i)}$$

$$\mathbf{E}_p\left[f(x)\right] \approx \frac{1}{N}\sum_{i=1}^{N}\frac{p(x_i)}{\frac{1}{N}\sum_{j=1}^{N} q_j(x_i)}$$

marginal particle filtering

(klaas, doucet, de freitas)

- roughly: same idea as MIS, but for particle filters

$$\pi(x_{t-1} \mid y_{0:t}) \approx \sum_{i=1}^{N} w_{t-1}^{i} \delta(x_{t-1}^{i}, dx_{t-1})$$

$$\pi(x_t \mid y_{0:t}) \approx \sum_{i=1}^{N} w_{t-1}^{i} f_t(x_{t-1}^{i} \to x_t) \cdot g_t(x_t, y_t)$$

$$Q_t(x_t \mid y_{0:t}) = \sum_{i=1}^{N} w_{t-1}^{i} q_t(x_{t-1}^{i} \to x_t \mid y_t)$$

weights for standard marginal PF

$$w_t^i = \frac{\sum_{j=1}^{N} w_{t-1}^j f_t(x_{t-1}^j \to x_t^i) \cdot g_t(x_t^i, y_t)}{\sum_{j=1}^{N} w_{t-1}^j q_t(x_{t-1}^j \to x_t^i \,|\, y_t)}$$

- things to consider
    - cost of evaluating weights
    - (optimal?) choice of q
    - what if q = f?

auxiliary marginal particle filter

$$\pi(x_{t-1} \mid y_{0:t}) \approx \sum_{i=1}^{N} w_{t-1}^{i} \delta(x_{t-1}^{i}, dx_{t-1})$$

$$\pi(x_t \mid y_{0:t}) \approx \sum_{i=1}^{N} w_{t-1}^{i} f_t(x_{t-1}^{i} \to x_t) \cdot g_t(x_t, y_t)$$

$$\pi(x_t \mid y_{0:t}) \approx \sum_{i=1}^{N} w_{t-1}^{i} \hat{g}_t(x_{t-1}^{i}, y_t) \cdot \hat{f}_t(x_{t-1}^{i} \to x_t \mid y_t)$$

$$Q_t(x_t \mid y_{0:t}) = \sum_{i=1}^{N} \hat{w}_{t-1}^{i} q_t(x_{t-1}^{i} \to x_t \mid y_t)$$

some more comments

- can show that marginal APF has `better’ weights than standard APF
- can (sometimes) reduce O(N^2) cost to O(N * logN), O(N * log(1/eps))
  - usually: incur some bias, via ~ low-rank approximation
  - can be worthwhile, useful technique for many-body systems in general
- extension to non-markovian models not totally clear
- not clear whether it can be used in PMCMC