Worksheet -02

Outputs Screenshots

```python
print("Dataset Preview:")
print(df.head())
print("\nDataset Information:")
print(df.info())
```

```
Dataset Preview:
   label  pixel_0  pixel_1  pixel_2  pixel_3  pixel_4  pixel_5  pixel_6  \
0      5        0        0        0        0        0        0        0
1      0        0        0        0        0        0        0        0
2      4        0        0        0        0        0        0        0
3      1        0        0        0        0        0        0        0
4      9        0        0        0        0        0        0        0

   pixel_7  pixel_8  ...  pixel_774  pixel_775  pixel_776  pixel_777  \
0        0        0  ...          0          0          0          0
1        0        0  ...          0          0          0          0
2        0        0  ...          0          0          0          0
3        0        0  ...          0          0          0          0
4        0        0  ...          0          0          0          0

   pixel_778  pixel_779  pixel_780  pixel_781  pixel_782  pixel_783
0          0          0          0          0          0          0
1          0          0          0          0          0          0
2          0          0          0          0          0          0
3          0          0          0          0          0          0
4          0          0          0          0          0          0

[5 rows x 785 columns]

Dataset Information:
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 60000 entries, 0 to 59999
Columns: 785 entries, label to pixel_783
dtypes: int64(785)
memory usage: 359.3 MB
None
```

```python
X = df.iloc[:, 1:-1].values
y = df.iloc[:, -1].values

label_encoder = LabelEncoder()
y_encoded = label_encoder.fit_transform(y)

one_hot_encoder = OneHotEncoder(sparse_output=False)
y_one_hot = one_hot_encoder.fit_transform(y_encoded.reshape(-1, 1))

print("\nUnique Classes:", np.unique(y))
print("Encoded Labels:", np.unique(y_encoded))
print("One-Hot Encoded Labels:\n", y_one_hot[:5])
```

```
Unique Classes: [0]
Encoded Labels: [0]
One-Hot Encoded Labels:
 [[1.]
 [1.]
 [1.]
 [1.]
 [1.]]
```

```
[ ] X_train, X_test, y_train, y_test = train_test_split(X, y_one_hot, test_size=0.2, random_state=

    print("\nShapes:")
    print("X_train:", X_train.shape, "y_train:", y_train.shape)
    print("X_test:", X_test.shape, "y_test:", y_test.shape)
```

```
Shapes:
X_train: (48000, 783) y_train: (48000, 1)
X_test: (12000, 783) y_test: (12000, 1)
```

Softmax Function:

```
import numpy as np

def softmax(z):
    """
    Compute the softmax probabilities for a given input matrix.

    Parameters:
    z (numpy.ndarray): Logits (raw scores) of shape (m, n), where
                       - m is the number of samples.
                       - n is the number of classes.

    Returns:
    numpy.ndarray: Softmax probability matrix of shape (m, n), where
                   each row sums to 1 and represents the probability
                   distribution over classes.

    Notes:
    - The input to softmax is typically computed as: z = XW + b.
    - Uses numerical stabilization by subtracting the max value per row.
    """
    z_shifted = z - np.max(z, axis=1, keepdims=True)
    exp_z = np.exp(z_shifted)
    return exp_z / np.sum(exp_z, axis=1, keepdims=True)
```

Softmax Test Case:

This test case checks that each row in the resulting softmax probabilities sums to 1, which is the fundamental property of softmax.

```
[ ] z_test = np.array([[2.0, 1.0, 0.1], [1.0, 1.0, 1.0]])
    softmax_output = softmax(z_test)

    row_sums = np.sum(softmax_output, axis=1)

    assert np.allclose(row_sums, 1), f"Test failed: Row sums are {row_sums}"

    print("Softmax function passed the test case!")
```

```
Softmax function passed the test case!
```

Prediction Function:

```
[ ] def predict_softmax(X, W, b):
        """
        Predict the class labels for a set of samples using the trained softmax model.

        Parameters:
        X (numpy.ndarray): Feature matrix of shape (n, d), where n is the number of samples and d
        W (numpy.ndarray): Weight matrix of shape (d, c), where c is the number of classes.
        b (numpy.ndarray): Bias vector of shape (c,).

        Returns:
        numpy.ndarray: Predicted class labels of shape (n,), where each value is the index of the
        """
        z = np.dot(X, W) + b
        y_pred = softmax(z)

        predicted_classes = np.argmax(y_pred, axis=1)

        return predicted_classes
```

Test Function for Prediction Function:

The test function ensures that the predicted class labels have the same number of elements as the input samples, verifying that the model produces a valid output shape.

```
X_test = np.array([[0.2, 0.8], [0.5, 0.5], [0.9, 0.1]])
W_test = np.array([[0.4, 0.2, 0.1], [0.3, 0.7, 0.5]])
b_test = np.array([0.1, 0.2, 0.3])

y_pred_test = predict_softmax(X_test, W_test, b_test)

assert y_pred_test.shape == (3,), f"Test failed: Expected shape (3,), got {y_pred_test.shape}"

print("Predicted class labels:", y_pred_test)
```

```
Predicted class labels: [1 1 0]
```

Loss Function:

```
[ ] def loss_softmax(y_pred, y):
        """
        Compute the cross-entropy loss for a single sample.

        Parameters:
        y_pred (numpy.ndarray): Predicted probabilities of shape (c,) for a single sample,
                                where c is the number of classes.
        y (numpy.ndarray): True labels (one-hot encoded) of shape (c,), where c is the number of c

        Returns:
        float: Cross-entropy loss for the given sample.
        """

        epsilon = 1e-12
        y_pred = np.clip(y_pred, epsilon, 1.0 - epsilon)
        n = y.shape[0]
        loss = -np.sum(y * np.log(y_pred)) / n
        return loss
```

Test case for Loss Function:

This test case Compares loss for correct vs. incorrect predictions.

- Expects low loss for correct predictions.
- Expects high loss for incorrect predictions.

```
import numpy as np

y_true_correct = np.array([[1, 0, 0], [0, 1, 0], [0, 0, 1]])
y_pred_correct = np.array([[0.9, 0.05, 0.05],
                           [0.1, 0.85, 0.05],
                           [0.05, 0.1, 0.85]])

y_pred_incorrect = np.array([[0.05, 0.05, 0.9],
                             [0.1, 0.05, 0.85],
                             [0.85, 0.1, 0.05]])

loss_correct = loss_softmax(y_pred_correct, y_true_correct)
loss_incorrect = loss_softmax(y_pred_incorrect, y_true_correct)

assert loss_correct < loss_incorrect, f"Test failed: Expected loss_correct < loss_incorrect, b

print(f"Cross-Entropy Loss (Correct Predictions): {loss_correct:.4f}")
print(f"Cross-Entropy Loss (Incorrect Predictions): {loss_incorrect:.4f}")
```

```
Cross-Entropy Loss (Correct Predictions): 0.1435
Cross-Entropy Loss (Incorrect Predictions): 2.9957
```

Cost Function:

```
[ ] def cost_softmax(X, y, W, b):
        """
        Compute the average softmax regression cost (cross-entropy loss) over all samples.

        Parameters:
        X (numpy.ndarray): Feature matrix of shape (n, d), where n is the number of samples and d
        y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c), where n is the number of
        W (numpy.ndarray): Weight matrix of shape (d, c).
        b (numpy.ndarray): Bias vector of shape (c,).

        Returns:
        float: Average softmax cost (cross-entropy loss) over all samples.
        """

        n = X.shape[0]
        z = np.dot(X, W) + b
        y_pred = softmax(z)
        cost = loss_softmax(y_pred, y)
        return cost
```

Test Case for Cost Function:

The test case assures that the cost for the incorrect prediction should be higher than for the correct prediction, confirming that the cost function behaves as expected.

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```
import numpy as np

X_correct = np.array([[1.0, 0.0], [0.0, 1.0]])
y_correct = np.array([[1, 0], [0, 1]])
W_correct = np.array([[5.0, -2.0], [-3.0, 5.0]])
b_correct = np.array([0.1, 0.1])

X_incorrect = np.array([[0.1, 0.9], [0.8, 0.2]])
y_incorrect = np.array([[1, 0], [0, 1]])
W_incorrect = np.array([[0.1, 2.0], [1.5, 0.3]])
b_incorrect = np.array([0.5, 0.6])

cost_correct = cost_softmax(X_correct, y_correct, W_correct, b_correct)

cost_incorrect = cost_softmax(X_incorrect, y_incorrect, W_incorrect, b_incorrect)

assert cost_incorrect > cost_correct, f"Test failed: Incorrect cost {cost_incorrect} is not gr

print("Cost for correct prediction:", cost_correct)
print("Cost for incorrect prediction:", cost_incorrect)

print("Test passed!")
```

```
Cost for correct prediction: 0.0006234364133349324
Cost for incorrect prediction: 0.29930861359446115
Test passed!
```

Computing Gradients:

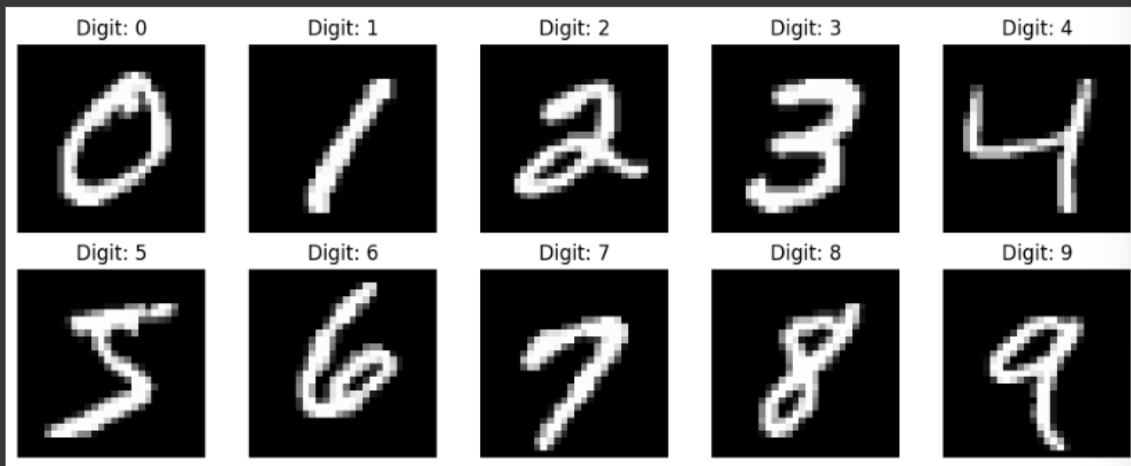
```
def compute_gradient_softmax(X, y, W, b):
    """
    Compute the gradients of the cost function with respect to weights and biases.

    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d).
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c).
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).

    Returns:
    tuple: Gradients with respect to weights (d, c) and biases (c,).
    """


    n, d = X.shape
    z = np.dot(X, W) + b
    y_pred = softmax(z)

    grad_W = np.dot(X.T, (y_pred - y)) / n
    grad_b = np.sum(y_pred - y, axis=0) / n

    return grad_W, grad_b
```

The test checks if the gradients from the function are close enough to the manually computed gradients using np.allclose, which accounts for potential floating-point discrepancies.

```
[ ] import numpy as np

    X_test = np.array([[0.2, 0.8], [0.5, 0.5], [0.9, 0.1]])
    y_test = np.array([[1, 0, 0], [0, 1, 0], [0, 0, 1]])

    W_test = np.array([[0.4, 0.2, 0.1], [0.3, 0.7, 0.5]])
    b_test = np.array([0.1, 0.2, 0.3])

    grad_W, grad_b = compute_gradient_softmax(X_test, y_test, W_test, b_test)

    z_test = np.dot(X_test, W_test) + b_test
    y_pred_test = softmax(z_test)

    grad_W_manual = np.dot(X_test.T, (y_pred_test - y_test)) / X_test.shape[0]
    grad_b_manual = np.sum(y_pred_test - y_test, axis=0) / X_test.shape[0]

    assert np.allclose(grad_W, grad_W_manual), f"Test failed: Gradients w.r.t. W are not equal.\nE
    assert np.allclose(grad_b, grad_b_manual), f"Test failed: Gradients w.r.t. b are not equal.\nE

    print("Gradient w.r.t. W:", grad_W)
    print("Gradient w.r.t. b:", grad_b)

    print("Test passed!")
```

```
Gradient w.r.t. W: [[ 0.1031051   0.01805685 -0.12116196]
 [-0.13600547  0.00679023  0.12921524]]
Gradient w.r.t. b: [-0.03290036  0.02484708  0.00805328]
Test passed!
```

```python
csv_file_path = "/content/drive/MyDrive/Sem6/AI and ML Workshop/Week-2/mnist_dataset.csv"
X_train, X_test, y_train, y_test = load_and_prepare_mnist(csv_file_path)
```



A Quick debugging Step:

```python
assert len(X_train) == len(y_train), f"Error: X and y have different lengths! X={len(X_train)}
print("Move forward: Dimension of Feture Matrix X and label vector y matched.")
```

```
Move forward: Dimension of Feture Matrix X and label vector y matched.
```

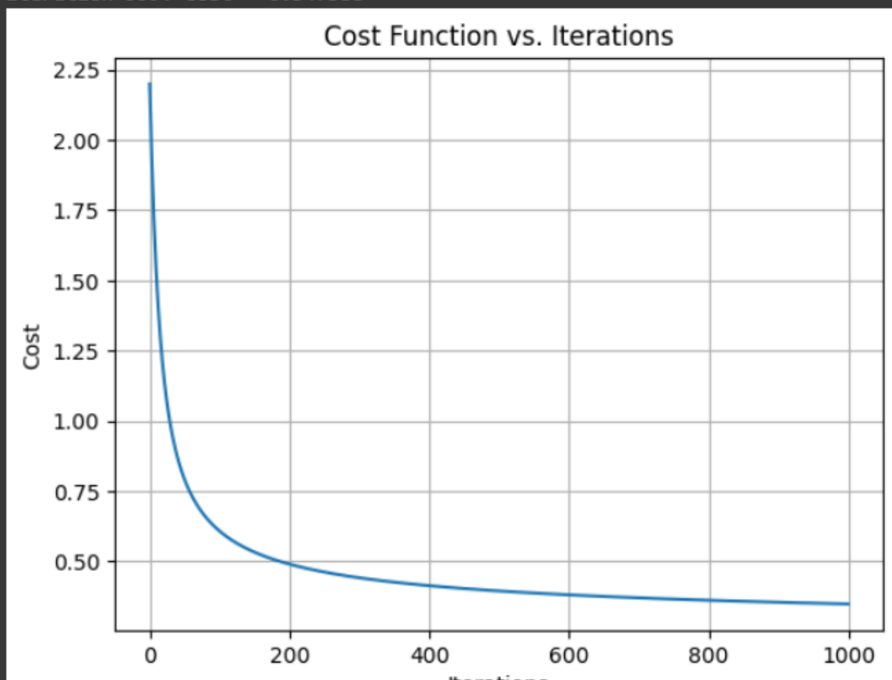
Train the Model:

```
print(f"Training data shape: {X_train.shape}")
print(f"Test data shape: {X_test.shape}")
```

```
Training data shape: (48000, 784)
Test data shape: (12000, 784)
```

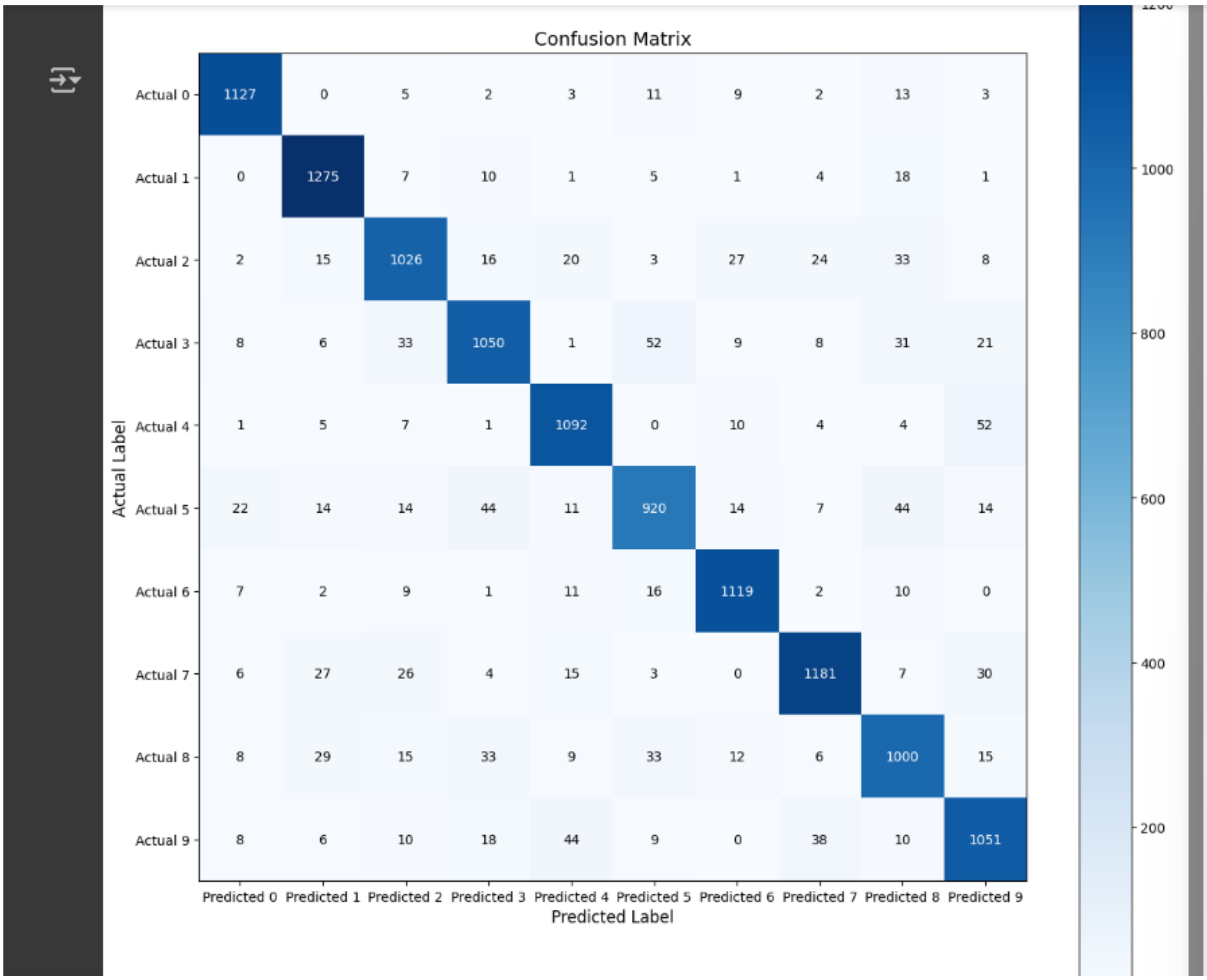
```
from sklearn.preprocessing import OneHotEncoder

if len(y_train.shape) == 1:
    encoder = OneHotEncoder(sparse_output=False)
    y_train = encoder.fit_transform(y_train.reshape(-1, 1))
    y_test = encoder.transform(y_test.reshape(-1, 1))

d = X_train.shape[1]
c = y_train.shape[1]

W = np.random.randn(d, c) * 0.01
b = np.zeros(c)

alpha = 0.1
n_iter = 1000

W_opt, b_opt, cost_history = gradient_descent_softmax(X_train, y_train, W, b, alpha, n_iter, s

plt.plot(cost_history)
plt.title('Cost Function vs. Iterations')
plt.xlabel('Iterations')
plt.ylabel('Cost')
plt.grid(True)
```

```
Iteration 0: Cost = 2.199553
Iteration 100: Cost = 0.607084
Iteration 200: Cost = 0.489327
Iteration 300: Cost = 0.440739
Iteration 400: Cost = 0.412682
Iteration 500: Cost = 0.393835
Iteration 600: Cost = 0.380030
Iteration 700: Cost = 0.369336
Iteration 800: Cost = 0.360723
Iteration 900: Cost = 0.353583
Iteration 999: Cost = 0.347588
```



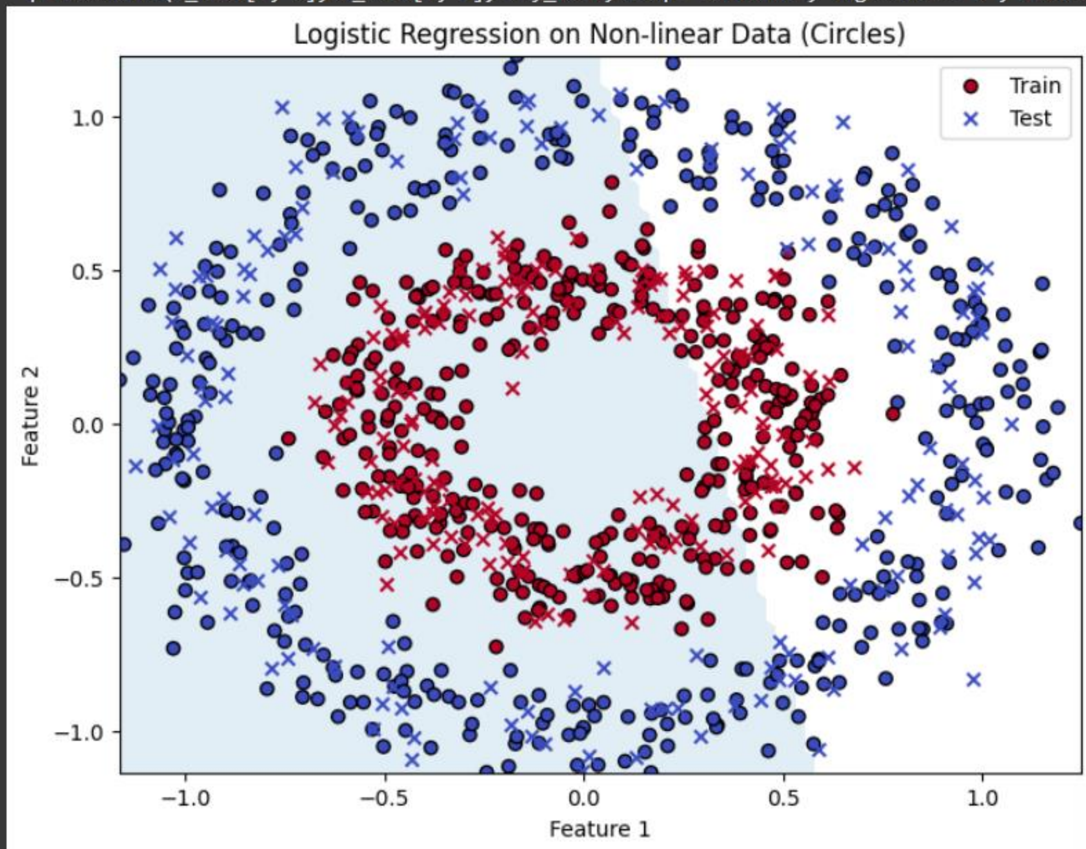
```
[ ] ax.set_xticks(range(num_classes))
    ax.set_yticks(range(num_classes))
    ax.set_xticklabels([f'Predicted {i}' for i in range(num_classes)])
    ax.set_yticklabels([f'Actual {i}' for i in range(num_classes)])

    for i in range(cm.shape[0]):
        for j in range(cm.shape[1]):
            ax.text(j, i, cm[i, j], ha='center', va='center', color='white' if cm[i, j] > np.max(

    ax.grid(False)
    plt.title('Confusion Matrix', fontsize=14)
    plt.xlabel('Predicted Label', fontsize=12)
    plt.ylabel('Actual Label', fontsize=12)

    plt.tight_layout()
    plt.colorbar(cax)
    plt.show()
```

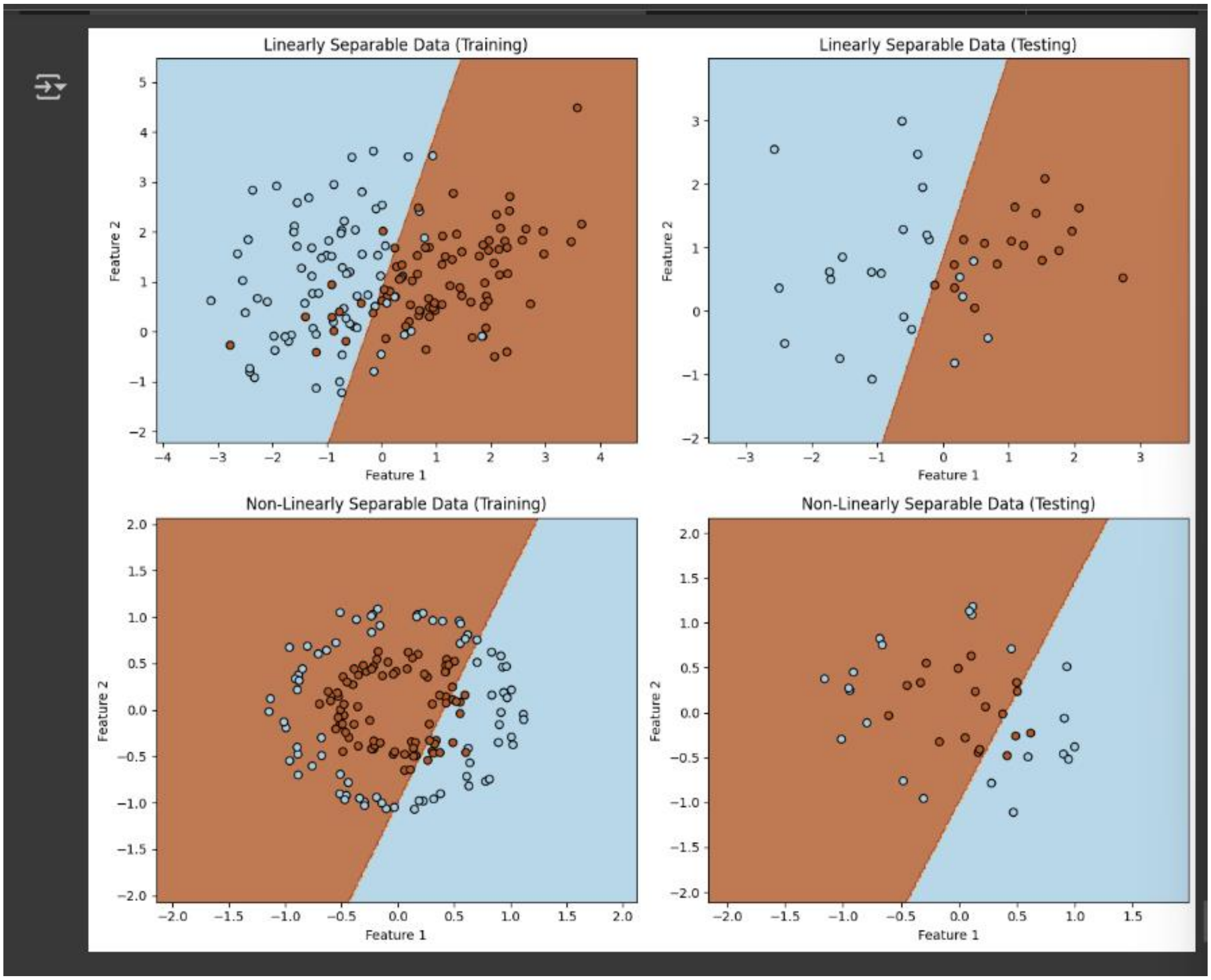
```
Confusion Matrix:
[[1127    0    5    2    3   11    9    2   13    3]
 [   0 1275    7   10    1    5    1    4   18    1]
 [   2   15 1026   16   20    3   27   24   33    8]
 [   8    6   33 1050    1   52    9    8   31   21]
 [   1    5    7    1 1092    0   10    4    4   52]
 [  22   14   14   44   11  920   14    7   44   14]
 [   7    2    9    1   11   16 1119    2   10    0]
 [   6   27   26    4   15    3    0 1181    7   30]
 [   8   29   15   33    9   33   12    6 1000   15]
 [   8    6   10   18   44    9    0   38   10 1051]]
Precision: 0.90
Recall: 0.90
F1-Score: 0.90
```



Confusion Matrix
Actual Label
Predicted Label
Actual 0
Actual 1
Actual 2
Actual 3
Actual 4
Actual 5
Actual 6
Actual 7
Actual 8
Actual 9
Predicted 0
Predicted 1
Predicted 2
Predicted 3
Predicted 4
Predicted 5
Predicted 6
Predicted 7
Predicted 8
Predicted 9
1127 0 5 2 3 11 9 2 13 3
0 1275 7 10 1 5 1 4 18 1
2 15 1026 16 20 3 27 24 33 8
8 6 33 1050 1 52 9 8 31 21
1 5 7 1 1092 0 10 4 4 52
22 14 14 44 11 920 14 7 44 14
7 2 9 1 11 16 1119 2 10 0
6 27 26 4 15 3 0 1181 7 30
8 29 15 33 9 33 12 6 1000 15
8 6 10 18 44 9 0 38 10 1051
1000
800
600
400
200

```
<ipython-input-24-6a5a1fca6a1b>:19: UserWarning: You passed a edgecolor/edgecolors ('k') for an
  plt.scatter(X_test[:, 0], X_test[:, 1], c=y_test, cmap='coolwarm', edgecolors='k', marker='x'
```





Linearly Separable Data (Training)
Linearly Separable Data (Testing)
Non-Linearly Separable Data (Training)
Non-Linearly Separable Data (Testing)
Feature 1
Feature 2