CSE 242 Homework 4

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December 14, 2020

1 Kernel methods with noisy setting

1.1 (a)

[1]:
```
import scipy.stats as stats
import numpy as np
np.random.seed(100)
plus_train = stats.multivariate_normal.rvs(mean=[1,1], cov=[[1,0],[0,1]],␣
↪size=1000)
minus_train = stats.multivariate_normal.rvs(mean=[-1,-1], cov=[[3,0],[0,3]],␣
↪size=1000)
train = np.concatenate((plus_train,minus_train))
plus_test = stats.multivariate_normal.rvs(mean=[1,1], cov=[[1,0],[0,1]],␣
↪size=1000)
minus_test = stats.multivariate_normal.rvs(mean=[-1,-1], cov=[[3,0],[0,3]],␣
↪size=1000)
test = np.concatenate((plus_test,minus_test))

train_labels = np.array([1] * 1000 + [-1] * 1000)
test_labels = np.array([1] * 1000 + [-1] * 1000)
```

[2]:
```
def flip_labels(labels):
    to_minus_prob = np.concatenate((stats.uniform.rvs(size=1000), np.
↪ones(1000)))
    to_plus_prob = np.concatenate((np.ones(1000),stats.uniform.rvs(size=1000)))
    randomized_labels = labels.copy()
    randomized_labels[to_minus_prob <= 0.35] = -1
    randomized_labels[to_plus_prob <= 0.2] = 1
    return(randomized_labels)
```

[3]:
```
from sklearn.svm import SVC

np.random.seed(2020)
a_accuracy = []
a_sums = []
for i in range(20):
    noisy_labels = flip_labels(train_labels)
```

```
    a_sums.append(sum(noisy_labels))
    svm_a = SVC(kernel="rbf", C=1, gamma=0.01)
    svm_a.fit(train,noisy_labels)
    a_accuracy.append(svm_a.score(test, test_labels))
np.mean(a_accuracy)
```

[3]: 0.8102499999999999

The average is around 0.81

1.2 (b)

[4]:
```
from sklearn.neighbors import KNeighborsClassifier

np.random.seed(2020)
b_accuracy = []
b_sums = []
for i in range(20):
    noisy_labels = flip_labels(train_labels)
    KNN = KNeighborsClassifier(n_neighbors=19)
    KNN.fit(train, noisy_labels)
    fixed_labels = KNN.predict(train)
    b_sums.append(sum(fixed_labels))
    svm_b = SVC(kernel="rbf", C=1, gamma=0.01)
    svm_b.fit(train,fixed_labels)
    b_accuracy.append(svm_b.score(test, test_labels))
np.mean(b_accuracy)
```

[4]: 0.8528249999999999

[5]:
```
np.transpose(b_accuracy)
```

[5]:
```
array([0.8435, 0.8535, 0.8545, 0.8485, 0.845 , 0.851 , 0.861 , 0.8505,
       0.8495, 0.855 , 0.8395, 0.855 , 0.8525, 0.862 , 0.858 , 0.8575,
       0.858 , 0.8675, 0.8465, 0.848 ])
```

k was chosen empirically and 19 seems to give the highest accuracy. It looks like there is a definite improvement, around 5%. Though we repeated the process 20 times to make sure the improvement is not by chance, it seems that each time is higher than 80% anyway.

1.3 (c)

[6]:
```
from sklearn.cluster import KMeans

np.random.seed(2020)
c_accuracy = []
c_sums = []
```

```
for i in range(20):
    noisy_labels = flip_labels(train_labels)
    KM = KMeans(n_clusters=2)
    KM.fit(train)
    fixed_labels_KM = KM.predict(train)
    fixed_labels_KM[fixed_labels_KM == 0] = -1
    c_sums.append(sum(fixed_labels_KM))
    svm_c = SVC(kernel="rbf", C=1, gamma=0.01)
    svm_c.fit(train, fixed_labels_KM)
    c_accuracy.append(svm_c.score(test, test_labels))
np.mean(c_accuracy)
```

[6]: 0.39635

[7]:
```
KM.cluster_centers_
```

[7]:
```
array([[ 0.97198463,  0.91039327],
       [-1.54659148, -1.46298677]])
```

k was chosen to be 2 because we know that there are 2 clusters. There is a marked decrease in performance. One of the cluster centers (0.972, 0.910) was close to the mean we used to generate the data, but the other is not, which explains why. Also, increasing this parameter seems to make the accuracy worse. Overall, this method seems to make the data noisier and would not be recommended as a way to "fix" the labels.

2 Fairness

[8]:
```
import sqlalchemy
engine = sqlalchemy.create_engine('sqlite:///compas.db')
inspector = sqlalchemy.inspect(engine)
inspector.get_table_names()
```

[8]:
```
['casearrest',
 'charge',
 'compas',
 'jailhistory',
 'people',
 'prisonhistory',
 'summary']
```

[9]:
```
cursor = engine.execute('SELECT * FROM compas LIMIT 10')
cursor.fetchall()
```

[9]:
```
[(1, 'miguel', 'hernandez', 56418, 58393, 64352, 'PRETRIAL', 'Risk and
Prescreen', 'Intake', 'Pretrial', 'Jail Inmate', 'Single', '2013-08-14
00:00:00.000000', 1, 'Low', 'Low', 7, 'Risk of Violence', -4.31, 1, 1),
 (2, 'miguel', 'hernandez', 56418, 58393, 64352, 'PRETRIAL', 'Risk and
```

```
Prescreen', 'Intake', 'Pretrial', 'Jail Inmate', 'Single', '2013-08-14
00:00:00.000000', 1, 'Low', 'Low', 8, 'Risk of Recidivism', -2.78, 1, 1),
 (3, 'miguel', 'hernandez', 56418, 58393, 64352, 'PRETRIAL', 'Risk and
Prescreen', 'Intake', 'Pretrial', 'Jail Inmate', 'Single', '2013-08-14
00:00:00.000000', 1, 'Low', 'Low', 18, 'Risk of Failure to Appear', 13, 1, 1),
 (4, 'michael', 'ryan', 68603, 72042, 79669, 'PRETRIAL', 'Risk and Prescreen',
'Intake', 'Pretrial', 'Jail Inmate', 'Single', '2014-12-31 00:00:00.000000', 1,
'Low', 'Low', 7, 'Risk of Violence', -2.75, 2, 2),
 (5, 'michael', 'ryan', 68603, 72042, 79669, 'PRETRIAL', 'Risk and Prescreen',
'Intake', 'Pretrial', 'Jail Inmate', 'Single', '2014-12-31 00:00:00.000000', 1,
'Low', 'Medium', 8, 'Risk of Recidivism', -0.34, 5, 2),
 (6, 'michael', 'ryan', 68603, 72042, 79669, 'PRETRIAL', 'Risk and Prescreen',
'Intake', 'Pretrial', 'Jail Inmate', 'Single', '2014-12-31 00:00:00.000000', 1,
'Low', 'Low', 18, 'Risk of Failure to Appear', 16, 2, 2),
 (7, 'kevon', 'dixon', 51601, 52796, 58102, 'PRETRIAL', 'Risk and Prescreen',
'Intake', 'Pretrial', 'Jail Inmate', 'Single', '2013-01-27 00:00:00.000000', 1,
'Low', 'Low', 7, 'Risk of Violence', -3.07, 1, 3),
 (8, 'kevon', 'dixon', 51601, 52796, 58102, 'PRETRIAL', 'Risk and Prescreen',
'Intake', 'Pretrial', 'Jail Inmate', 'Single', '2013-01-27 00:00:00.000000', 1,
'Low', 'Low', 8, 'Risk of Recidivism', -0.76, 3, 3),
 (9, 'kevon', 'dixon', 51601, 52796, 58102, 'PRETRIAL', 'Risk and Prescreen',
'Intake', 'Pretrial', 'Jail Inmate', 'Single', '2013-01-27 00:00:00.000000', 1,
'Low', 'Medium', 18, 'Risk of Failure to Appear', 25, 6, 3),
 (10, 'ed', 'philo', 38864, 55421, 61042, 'PRETRIAL', 'Risk and Prescreen',
'Intake', 'Pretrial', 'Jail Inmate', 'Single', '2013-04-14 00:00:00.000000', 1,
'Low', 'Low', 7, 'Risk of Violence', -2.26, 3, 4)]
```

[10]:
```python
# Import the table "people" from the database into pandas
import pandas
people = pandas.read_sql('people', engine)
people.head()
```

[10]:
```
   id               name    first       last   sex              race  \
0   1  miguel hernandez   miguel  hernandez  Male             Other
1   2       michael ryan  michael       ryan  Male         Caucasian
2   3        kevon dixon    kevon      dixon  Male  African-American
3   4           ed philo       ed      philo  Male  African-American
4   5        marcu brown    marcu      brown  Male  African-American

         dob  age           age_cat  juv_fel_count  …  r_offense_date  \
0 1947-04-18   69  Greater than 45              0  …             NaT
1 1985-02-06   31           25 - 45              0  …             NaT
2 1982-01-22   34           25 - 45              0  …      2013-07-05
3 1991-05-14   24     Less than 25              0  …      2013-06-16
4 1993-01-21   23     Less than 25              0  …             NaT

                 r_charge_desc          r_jail_in          r_jail_out  \
```

```
0                    None              NaT              NaT
1                    None              NaT              NaT
2  Felony Battery (Dom Strang)         NaT              NaT
3  Driving Under The Influence 2013-06-16 09:05:47 2013-06-16 07:18:55
4                    None              NaT              NaT

  is_violent_recid num_vr_cases  vr_case_number vr_charge_degree  \
0                0         None            None             None
1                0         None            None             None
2                1         None   13009779CF10A             (F3)
3                0         None            None             None
4                0         None            None             None

  vr_offense_date              vr_charge_desc
0             NaT                        None
1             NaT                        None
2      2013-07-05  Felony Battery (Dom Strang)
3             NaT                        None
4             NaT                        None

[5 rows x 41 columns]
```

[11]:
```
people.columns
```

[11]:
```
Index(['id', 'name', 'first', 'last', 'sex', 'race', 'dob', 'age', 'age_cat',
       'juv_fel_count', 'juv_misd_count', 'juv_other_count',
       'compas_screening_date', 'decile_score', 'score_text', 'violent_recid',
       'priors_count', 'days_b_screening_arrest', 'c_jail_in', 'c_jail_out',
       'c_case_number', 'c_days_from_compas', 'c_arrest_date',
       'c_offense_date', 'c_charge_degree', 'c_charge_desc', 'is_recid',
       'num_r_cases', 'r_case_number', 'r_charge_degree', 'r_days_from_arrest',
       'r_offense_date', 'r_charge_desc', 'r_jail_in', 'r_jail_out',
       'is_violent_recid', 'num_vr_cases', 'vr_case_number',
       'vr_charge_degree', 'vr_offense_date', 'vr_charge_desc'],
      dtype='object')
```

[12]:
```
# Import the table "compas" from the database into pandas
# COMPAS is the risk assessment tool used in the article and is the subject of␣
↪study
compas = pandas.read_sql('compas', engine)
compas.head()
```

[12]:
```
   id    first       last  compas_person_id  compas_case_id  \
0   1   miguel  hernandez             56418           58393
1   2   miguel  hernandez             56418           58393
2   3   miguel  hernandez             56418           58393
3   4  michael       ryan             68603           72042
```

```
4   5  michael       ryan             68603          72042

   compas_assessment_id agency_text           scale_set assessment_reason  \
0                 64352    PRETRIAL  Risk and Prescreen            Intake
1                 64352    PRETRIAL  Risk and Prescreen            Intake
2                 64352    PRETRIAL  Risk and Prescreen            Intake
3                 79669    PRETRIAL  Risk and Prescreen            Intake
4                 79669    PRETRIAL  Risk and Prescreen            Intake

  legal_status  … marital_status screening_date rec_supervision_level  \
0     Pretrial  …         Single     2013-08-14                     1
1     Pretrial  …         Single     2013-08-14                     1
2     Pretrial  …         Single     2013-08-14                     1
3     Pretrial  …         Single     2014-12-31                     1
4     Pretrial  …         Single     2014-12-31                     1

   rec_supervision_level_text score_text scale_id         type_of_assessment  \
0                        Low        Low        7            Risk of Violence
1                        Low        Low        8          Risk of Recidivism
2                        Low        Low       18  Risk of Failure to Appear
3                        Low        Low        7            Risk of Violence
4                        Low     Medium        8          Risk of Recidivism

  raw_score  decile_score  person_id
0        -4             1          1
1        -2             1          1
2        13             1          1
3        -2             2          2
4         0             5          2

[5 rows x 21 columns]
```

[13]:
```
compas.columns
```

[13]:
```
Index(['id', 'first', 'last', 'compas_person_id', 'compas_case_id',
       'compas_assessment_id', 'agency_text', 'scale_set', 'assessment_reason',
       'legal_status', 'custody_status', 'marital_status', 'screening_date',
       'rec_supervision_level', 'rec_supervision_level_text', 'score_text',
       'scale_id', 'type_of_assessment', 'raw_score', 'decile_score',
       'person_id'],
      dtype='object')
```

[14]:
```
# Inner join the tables. Compare empirical data (times reoffended) with␣
↪predicted (compas risk)

df = pandas.read_sql('''SELECT race,is_violent_recid,agency_text,compas.
↪score_text
```

```
FROM people JOIN compas ON
person_id = people.id''', engine)
df.head()
```

[14]:

	race	is_violent_recid	agency_text	score_text
0	Other	0	PRETRIAL	Low
1	Other	0	PRETRIAL	Low
2	Other	0	PRETRIAL	Low
3	Caucasian	0	PRETRIAL	Low
4	Caucasian	0	PRETRIAL	Medium

[15]:
```
relative_counts = pandas.DataFrame(
    {i: d.score_text.value_counts() / d.score_text.count()
     for i, d in df.groupby(['race', 'is_violent_recid'])})
relative_counts
```

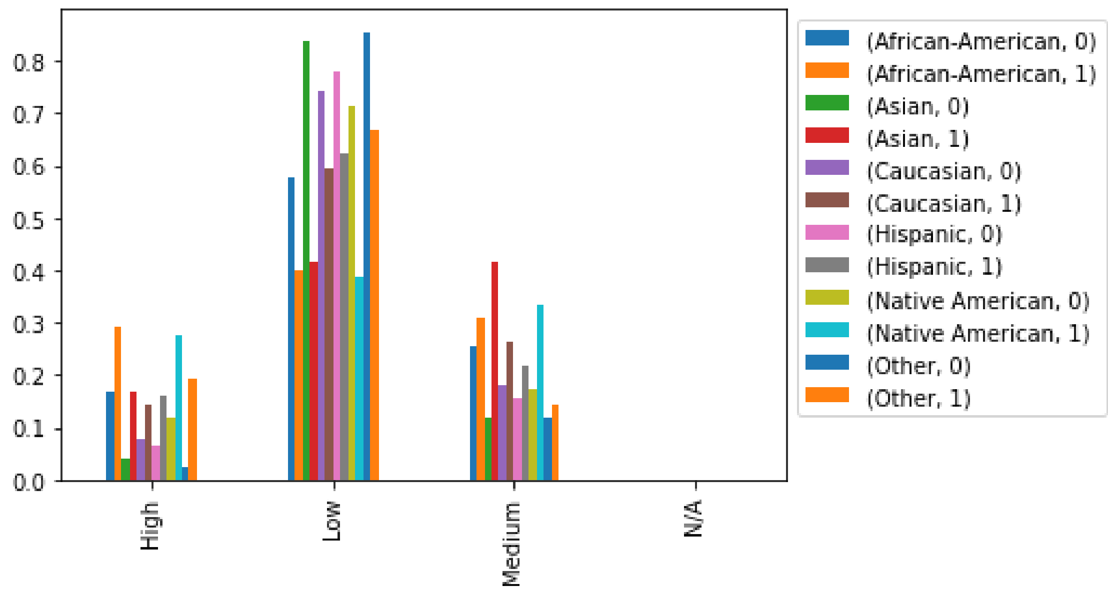
[15]:

	African-American		Asian		Caucasian		\
	0	1	0	1	0	1	
High	0.168173	0.290386	0.041667	0.166667	0.076904	0.142686	
Low	0.576348	0.399707	0.839286	0.416667	0.742984	0.594724	
Medium	0.255006	0.309907	0.119048	0.416667	0.179191	0.262590	
N/A	0.000473	NaN	NaN	NaN	0.000922	NaN	

	Hispanic		Native American		Other	
	0	1	0	1	0	1
High	0.065251	0.159204	0.117117	0.277778	0.025402	0.191489
Low	0.780240	0.621891	0.711712	0.388889	0.855365	0.666667
Medium	0.154509	0.218905	0.171171	0.333333	0.118196	0.141844
N/A	NaN	NaN	NaN	NaN	0.001037	NaN

[16]:
```
%matplotlib inline
relative_counts.plot.bar().legend(bbox_to_anchor = (1,1))
```

[16]: <matplotlib.legend.Legend at 0x7ffe74567350>



[17]:
```
# this will be your prediction
df['scored_high'] = 0 + (df.score_text == 'High')
```

[18]:
```
from statsmodels.formula.api import logit
#this will be the model you used to generate scores
# C() for categorical
model = logit('scored_high ~ C(race) + is_violent_recid + agency_text', df).
 ↪fit()
model.summary()
```

```
Optimization terminated successfully.
         Current function value: 0.365167
         Iterations 7
```

[18]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                           Logit Regression Results
==============================================================================
Dep. Variable:            scored_high   No. Observations:                37578
Model:                          Logit   Df Residuals:                    37568
Method:                           MLE   Df Model:                            9
Date:                Mon, 14 Dec 2020   Pseudo R-squ.:                 0.04752
Time:                        13:19:34   Log-Likelihood:                -13722.
converged:                       True   LL-Null:                       -14407.
Covariance Type:            nonrobust   LLR p-value:                 3.681e-289
================================================================================
==============
```

```
                                 coef    std err          z      P>|z|
[0.025      0.975]
--------------------------------------------------------------------------------
--------------
Intercept                     -0.9779      0.230     -4.248      0.000
-1.429      -0.527
C(race)[T.Asian]              -1.4142      0.343     -4.122      0.000
-2.087      -0.742
C(race)[T.Caucasian]          -0.8806      0.038    -23.374      0.000
-0.954      -0.807
C(race)[T.Hispanic]           -1.0405      0.069    -15.008      0.000
-1.176      -0.905
C(race)[T.Native American]    -0.3605      0.256     -1.406      0.160
-0.863       0.142
C(race)[T.Other]              -1.7172      0.119    -14.478      0.000
-1.950      -1.485
agency_text[T.DRRD]            0.2336      0.294      0.795      0.427
-0.342       0.809
agency_text[T.PRETRIAL]       -0.6537      0.231     -2.836      0.005
-1.105      -0.202
agency_text[T.Probation]      -0.2184      0.241     -0.905      0.366
-0.692       0.255
is_violent_recid               0.7479      0.045     16.515      0.000
0.659       0.837
================================================================================
==============
"""
```

[19]:
```
score = model.predict(df)
```

[20]:
```
bw_score = score[(df["race"] == "African-American") | (df["race"] ==␣
↪"Caucasian") ]
```

[21]:
```
max_score = max(bw_score)
max_score
```

[21]: 0.500899933158618

[22]:
```
min_score = min(bw_score)
min_score
```

[22]: 0.07501360138832201

2.1 (a) Independence

[23]:
```
df_BW = df[(df["race"] == "Caucasian") | (df["race"] == "African-American")]
df_BW = df_BW.assign(R=(df_BW["scored_high"] >= np.
↪mean(df_BW["scored_high"]))*1)
```

[24]:
```
P_W = sum((df_BW["race"] == "African-American") & (df_BW["R"] == 1))/
↪sum((df_BW["race"] == "African-American"))
P_B = sum((df_BW["race"] == "Caucasian") & (df_BW["R"] == 1))/
↪sum((df_BW["race"] == "Caucasian"))
P_W/P_B
```

[24]:
```
2.2336060238455064
```

[25]:
```
np.mean(df_BW["scored_high"])
```

[25]:
```
0.1410753094585656
```

Using the empirical threshold of 0.141, the ratio evaluates to $2.23 > 0.8$. Therefore, the model violates disparate impact law

2.2 (b) Separation

[26]:
```
def calc_TFPR(true, pred, n_thresh) :
    x = np.linspace(min_score,max_score,n_thresh+1)
    TPRs = []
    FPRs = []
    for thresh in x[0:n_thresh]:
        converted_pred = (pred >= thresh)*1
        TP = sum((true == 1) & (converted_pred == 1))
        act_P = sum(true == 1)
        TPR = TP/act_P
        TPRs.append(TPR)
        FP = sum((true == 0) & (converted_pred == 1))
        act_N = sum(true == 0)
        FPR = FP/act_N
        FPRs.append(FPR)
    df = pandas.DataFrame(zip(x[0:n_thresh], TPRs, FPRs),
                columns =['Lower Threshold', 'TPR', 'FPR'])
    return(df)
```

[27]:
```
W_filter = df["race"] == "Caucasian"
W = df[W_filter]
W_score = score[W_filter]
TFPR_W = calc_TFPR(W["is_violent_recid"], W_score, 100)
TFPR_W
```

[27]:

```
    Lower Threshold  TPR       FPR
0          0.075014  1.0  1.000000
1          0.079272  1.0  0.036441
2          0.083531  1.0  0.036441
3          0.087790  1.0  0.036441
4          0.092049  1.0  0.036441
..              …   …        …
95         0.479606  0.0  0.000000
96         0.483864  0.0  0.000000
97         0.488123  0.0  0.000000
98         0.492382  0.0  0.000000
99         0.496641  0.0  0.000000

[100 rows x 3 columns]
```

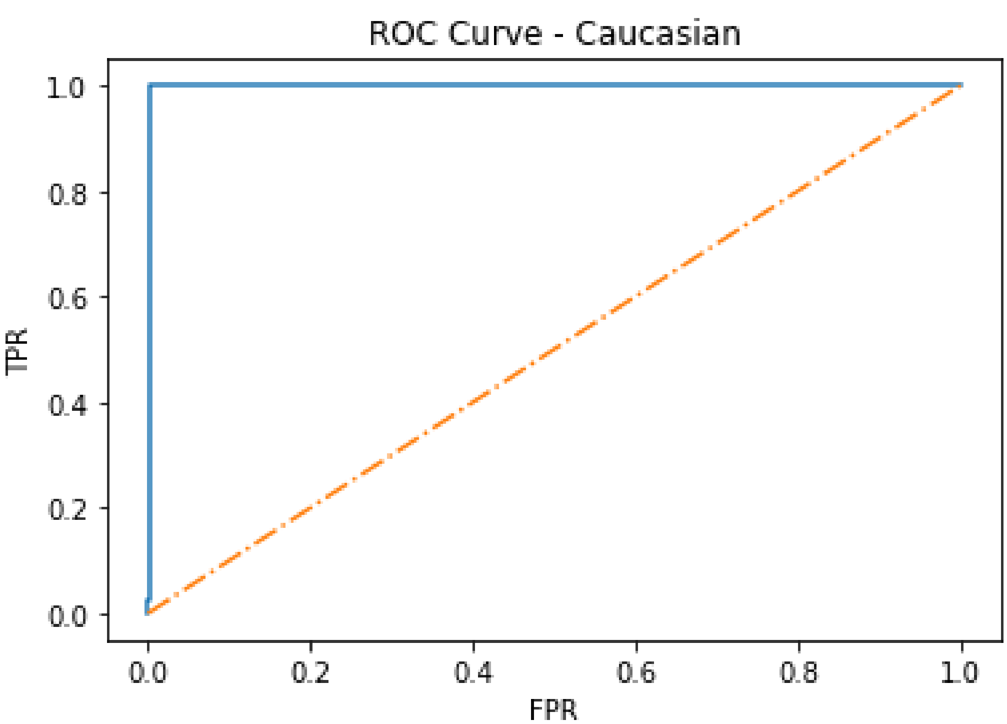
[28]:

```
import matplotlib.pyplot as plt

plt.plot(TFPR_W["FPR"], TFPR_W["TPR"])
plt.plot([0,1],[0,1],linestyle='-.')
plt.xlabel("FPR")
plt.ylabel("TPR")
plt.title("ROC Curve - Caucasian")
```

[28]:

```
Text(0.5, 1.0, 'ROC Curve - Caucasian')
```



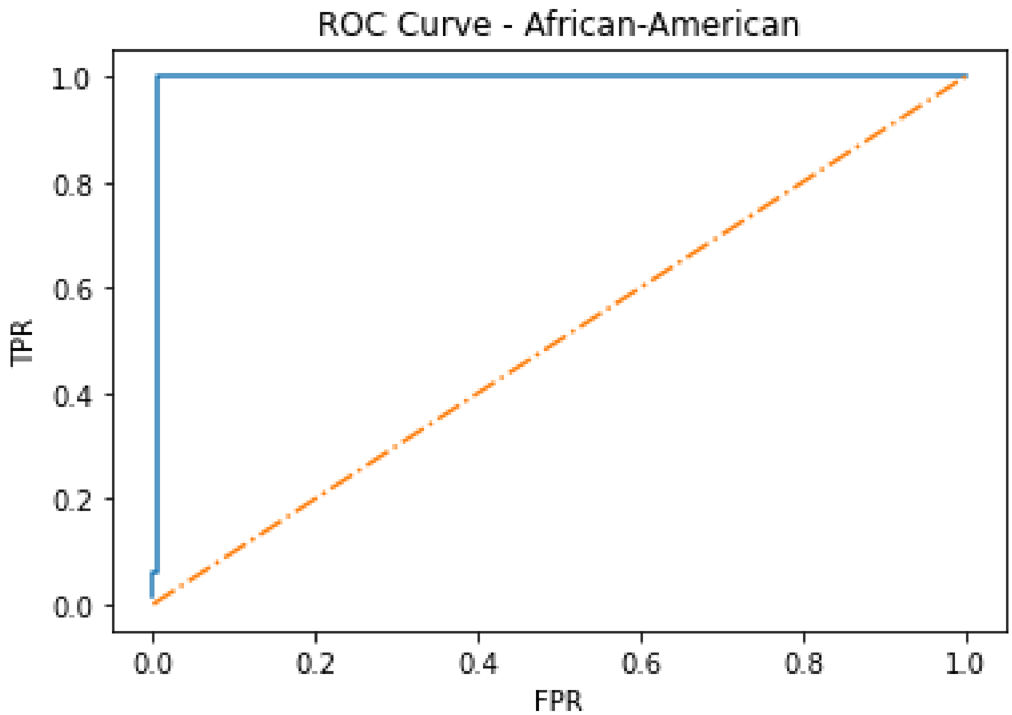
[29]:
```
B_filter = df["race"] == "African-American"
B = df[B_filter]
B_score = score[B_filter]
TFPR_B = calc_TFPR(B["is_violent_recid"], B_score, 100)
TFPR_B
```

[29]:
```
    Lower Threshold       TPR  FPR
0          0.075014  1.000000  1.0
1          0.079272  1.000000  1.0
2          0.083531  1.000000  1.0
3          0.087790  1.000000  1.0
4          0.092049  1.000000  1.0
..              …         …    …
95         0.479606  0.013177  0.0
96         0.483864  0.013177  0.0
97         0.488123  0.013177  0.0
98         0.492382  0.013177  0.0
99         0.496641  0.013177  0.0

[100 rows x 3 columns]
```

[30]:
```
plt.plot(TFPR_B["FPR"], TFPR_B["TPR"])
plt.plot([0,1],[0,1],linestyle='-.')
plt.xlabel("FPR")
plt.ylabel("TPR")
plt.title("ROC Curve - African-American")
```

[30]:
```
Text(0.5, 1.0, 'ROC Curve - African-American')
```



2.3 (c) Achieve Separation by Post-Processing

The ideal TPR is 1 and the ideal FPR is 0. This would mean we should look for separation errors of 0. However, if the TPR for both groups is 0 and the FPR is 1, this also gives 0, and likewise if both the TPR and the FPR are 0. In terms of the ROC curve, we want the TPR and FPR corresponding to the upper left hand corner, with TPR $= 1$, FPR ≈ 0.

[31]:
```
# oz stands for one zero
oz_TFPR_W = TFPR_W[(TFPR_W["TPR"] > 0) & (TFPR_W["FPR"] < 1)]
oz_TFPR_B = TFPR_B[(TFPR_B["TPR"] > 0) & (TFPR_B["FPR"] < 1)]
```

[32]:
```
oz_TFPR_W.head()
```

[32]:

	Lower Threshold	TPR	FPR
1	0.079272	1.0	0.036441
2	0.083531	1.0	0.036441
3	0.087790	1.0	0.036441
4	0.092049	1.0	0.036441
5	0.096308	1.0	0.036441

[33]:
```
oz_TFPR_B.head()
```

[33]:

	Lower Threshold	TPR	FPR
21	0.164450	1.0	0.041645
22	0.168709	1.0	0.041645
23	0.172967	1.0	0.041645
24	0.177226	1.0	0.041645
25	0.181485	1.0	0.041645

[34]:
```
TPR_term = np.abs(np.subtract.outer(list(oz_TFPR_W["TPR"]),
 ↪list(oz_TFPR_B["TPR"])))
FPR_term = np.abs(np.subtract.outer(list(oz_TFPR_W["FPR"]),
 ↪list(oz_TFPR_B["FPR"])))
sep_error = TPR_term + FPR_term
```

[35]:
```
x = oz_TFPR_W["Lower Threshold"]
y = oz_TFPR_B["Lower Threshold"]
coords = np.argwhere(sep_error == np.min(sep_error))
pairs = [[x[w],y[b]] for w, b in zip(coords[:,0], coords[:,1])]
```

[36]:
```
pairs = pandas.DataFrame(pairs, columns=["Caucasian", "African-American"])
pairs
```

[36]:

	Caucasian	African-American
0	0.109085	0.185744
1	0.109085	0.190003
2	0.109085	0.194262
3	0.109085	0.198521
4	0.109085	0.202780
5	0.113343	0.185744
6	0.113343	0.190003
7	0.113343	0.194262
8	0.113343	0.198521
9	0.113343	0.202780
10	0.117602	0.185744
11	0.117602	0.190003
12	0.117602	0.194262
13	0.117602	0.198521
14	0.117602	0.202780
15	0.121861	0.185744
16	0.121861	0.190003
17	0.121861	0.194262
18	0.121861	0.198521
19	0.121861	0.202780
20	0.126120	0.185744
21	0.126120	0.190003
22	0.126120	0.194262
23	0.126120	0.198521
24	0.126120	0.202780

25	0.130379	0.185744
26	0.130379	0.190003
27	0.130379	0.194262
28	0.130379	0.198521
29	0.130379	0.202780

The table above are pairs of thresholds that would give the lowest separation error while maximizing TPR and minimizing FPR. The first column is the threshold for Caucasians and the second is for African-American. We can see from the table that the threshold is lower for Caucasians than African-Americans. We can also see this in the mean and median below. Recall that the score, the output from logistic regression, is the predicted probability of scoring high. Our goal is to look for bias in the COMPAS scoring system. We can see that this model implies that African-Americans more likely to get a high COMPAS score. Therefore, it suggests that there is bias in the COMPAS score, as race makes a big difference in the model.

[37]:
```
pairs.describe()
```

[37]:

	Caucasian	African-American
count	30.000000	30.000000
mean	0.119732	0.194262
std	0.007398	0.006126
min	0.109085	0.185744
25%	0.113343	0.190003
50%	0.119732	0.194262
75%	0.126120	0.198521
max	0.130379	0.202780