Theory Problems

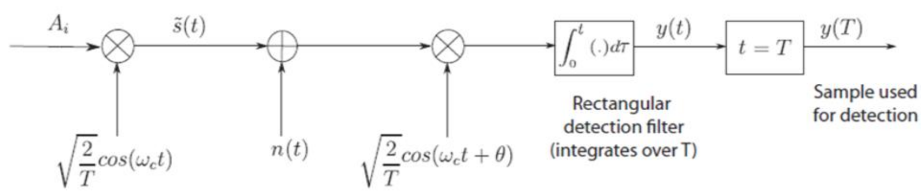


Figure 1: Problem 1: BPSK w/ a phase offset error.

$$\underline{y}(T) = \frac{2}{T} \int_0^T A_i |p(t)|^2 cos\left(\omega_c t\right) cos\left(\omega_c t + \theta\right) dt + \underline{N}(T)$$

$$\underline{N}(T) = \sqrt{\frac{2}{T}} \int_0^T n(t) cos(\omega_c t + \theta) dt$$

$$p(t) = \begin{cases} 1 & 0 \leq t \leq T \\ 0 & else \end{cases}$$

$$E_b = \frac{1}{T} \int_0^T |p(t)|^2 dt$$

and $n(t)$ is passband noise given by:

$$\underline{n}(t) = 2\left[\underline{n_I}(t) cos(\omega_c t) - \underline{n_Q}(t) sin(\omega_c t)\right]$$

$$\underline{n_I}(t), \underline{n_Q}(t) \sim N\left(0, \sigma_n^2\right)$$

$$\sigma_n^2 = \frac{N_0}{2}$$

where $\underline{n_I}(t)$ and $\underline{n_Q}(t)$ are uncorrelated. This leads to the following expression for $\underline{N}(T)$

$$\underline{N}(T) = \sqrt{\frac{2}{T}} \int_0^T \underline{n}(t) p(t) cos\left(\omega_c t + \theta\right) dt$$

where θ is the phase error caused by phase noise. If the phase noise is slowly varying, it is can be approximated by a random variable $\underline{\theta}$ modeled as a zero-mean gaussian random variable with a variance σ_θ^2.

a) Derive expressions for the test statistics $y_1(T)$ when $A_i = 1$ and $y_0(T)$ when $A_i = -1$.

Mean of $\underline{y}(T)$ = $E\{\frac{2}{T}\int_0^T A_i|p(t)|^2 \cos^2(\omega_c t+\theta)\,dt + \underline{N}(T)\}$

$= \frac{2}{T}A_i\int_0^T |p(t)|^2(\frac{1}{2} + E\{\frac{1}{2}\cos 2(\omega_c t+\theta)\}\,dt + E\{\underline{N}(T)\}$, as $E\{\cos 2(\omega_c t+\theta)\}$ & $E\{\underline{N}(T)\}$=0

$= \frac{2}{T}A_i\int_0^T |p(t)|^2\left(\frac{1}{2}\right)dt = A_i$

Variance of y(T) = $E\{y(T)^2\} - E\{y(T)\}^2 = E\{\frac{4}{T^2}A_i^2\int_0^T |p(t)|^4(\frac{1}{2}+\frac{1}{2}\cos 2(\omega_c t+\theta))(\frac{1}{2}+\frac{1}{2}\cos 2(\omega_c t+\theta))\,dt - A_i^2$

$= \frac{4}{T^2}A_i^2\int_0^T |p(t)|^4(\frac{1}{2})(\frac{1}{2})\,dt - A_i^2$

$= \frac{1}{T^2}A_i^2\int_0^T |p(t)|^4 dt - A_i^2$

$= A_i^2(\frac{1-T}{T}) = \frac{1-T}{T}$

$test\ statistics\ of\ y_1(T) = test\ statistics\ of\ y_0(T) = \frac{T}{\sqrt{2\pi(1-T)}}\exp\left[-(\frac{T^2}{2(1-T)^2})(x-1)^2\right]$

b) Derive an expression for the conditioned probability of error bit error $Pr\{e|\underline{\theta}_e = \theta\}$ denoted $p_{e|\theta}(e|\theta)$.

$\sigma^2_{n_I} = \sigma^2_{n_Q} = \sigma^2_n = No/2$

$\sigma^2_{n_{I+Q}} = \frac{4\sigma^2_n}{2} = 2\sigma^2_n = No$

So, $p_{e|\theta}(e|\theta) = \frac{1}{2}erfc(\sqrt{E_b/2No}\cos\theta)$

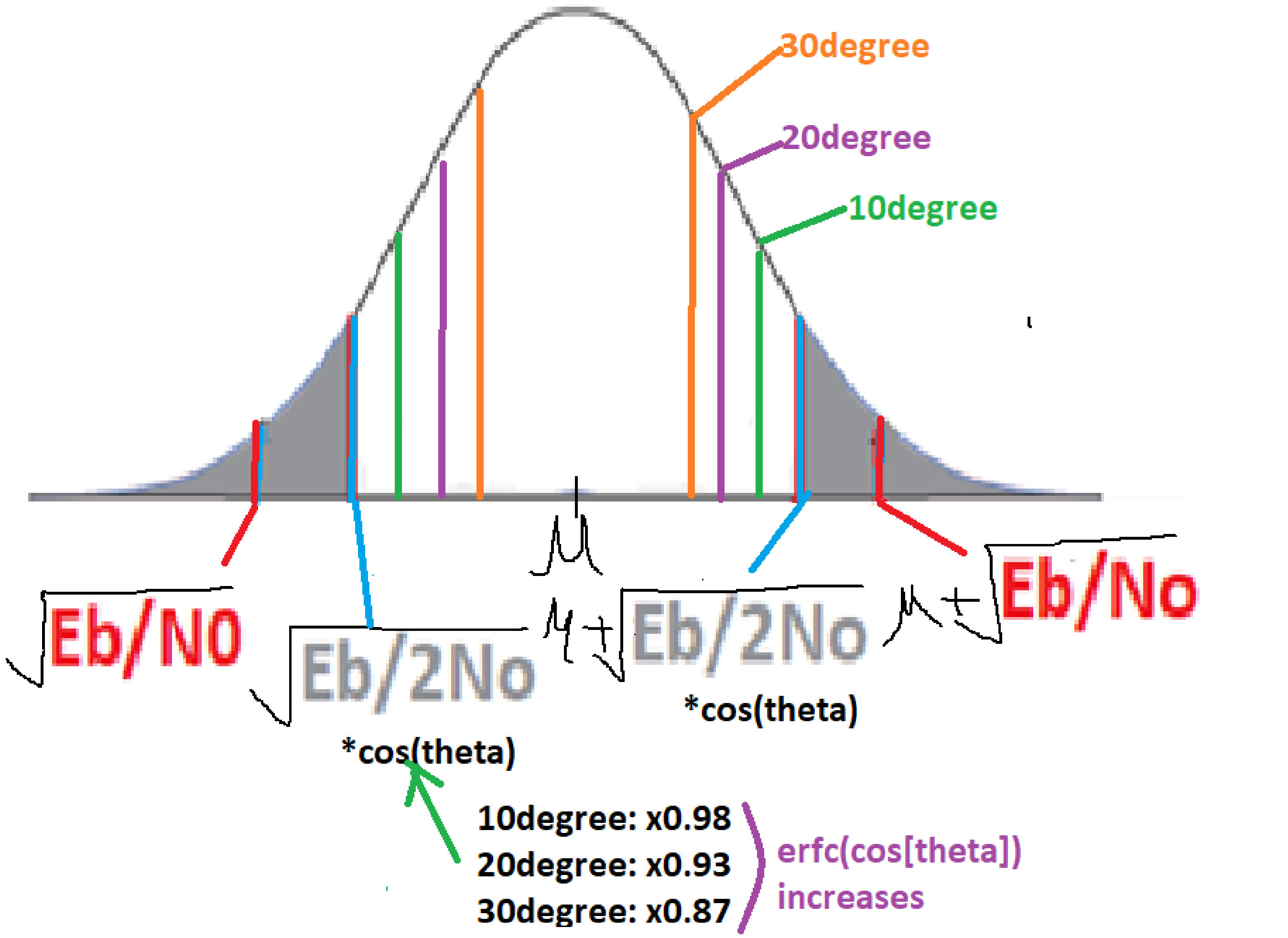
Where Eb = $\frac{1}{T}\int_0^T |p(t)|^2 dt$

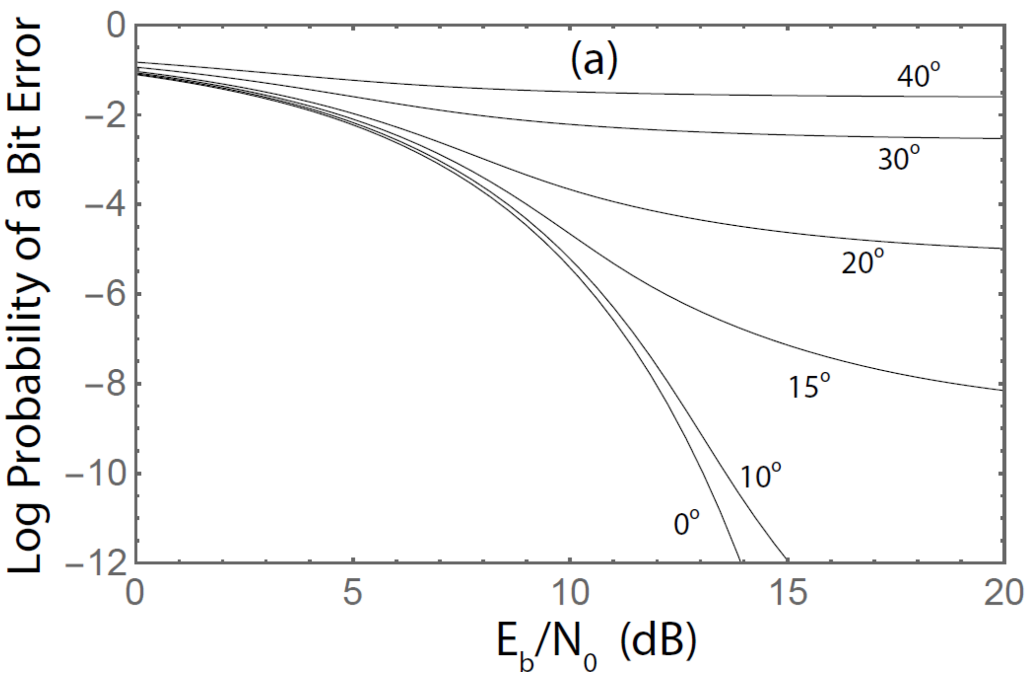
c) Using your result from part b.) determine the unconditioned probability of a bit error p_e when the phase noise is modeled as a zero-mean gaussian distribution with a root-mean-squared phase noise σ_θ expressed in degrees. Do this for $\sigma_\theta = 10^o, 20^o, 30^o$. Compare your results to Figure 4.11a in Papen/Blahut.

Note: The unconditioned probability of error is given by $p_e = \int_{\forall\theta} p_{e|\theta}(e|\theta)p_\theta(\theta)d\theta$, and $p_\theta(\theta)$ is the pdf of the phase noise.

$\sigma_\theta = \sigma_{\theta\_radian} * 180/\pi$

$Pe = \int_0^T p(e|\theta)f(\theta)\,d\theta = \frac{1}{2\sqrt{2\pi}\sigma_\theta}\int_{-\infty}^{\infty} e^{-\theta^2(t)/\sigma^2_\theta}erfc(\sqrt{E_b/2No}\cos\theta)d\theta$





new Eb/No will be *0.5*(cos[theta])^0.5

2. *Error in timing recovery*

Suppose that the output pulse is filtered by a matched filter and that the resulting filtered waveform is sampled nine times per symbol interval. Determine the fraction of the energy that is lost if the timing recovery is off by one sample from the optimal value for the following pulse waveforms:

a) The **input** to the matched filter is a rectangular pulse

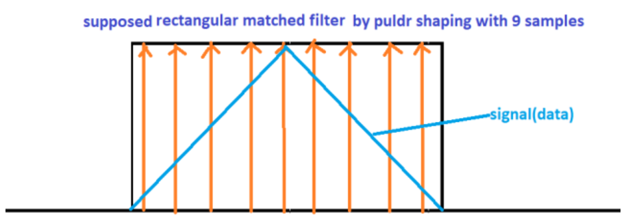
b) The **output** of the matched filter is a raised cosine pulse with $\beta = 0.5$
Note: You may assume that the output of the matched filter is in units of energy.

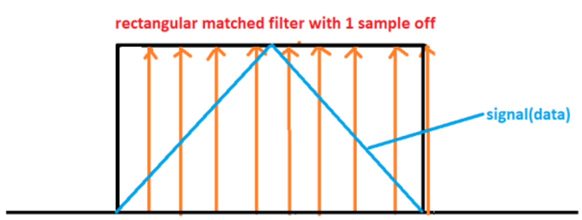
c) For BPSK modulation using Eq. (3.6.14a) in Papen/Blahut, the BER for matched filter detection is given by

$$p_e = \tfrac{1}{2}\mathrm{erfc}\left(\sqrt{d_{10}^2/4N_0}\right) = \tfrac{1}{2}\mathrm{erfc}\left(\sqrt{E_b/N_0}\right) = Q\left(\sqrt{2E_b/N_0}\right)$$

where $d_{\min}^2 = d_{10}^2 = (s_1 - s_0)^2 = 4E_b$ and $\sigma^2 = N_0/2$. Suppose that at the optimal sampling time, the BER is equal to 10^{-9}. Using the results from part (a) and (b), determine the increase in the BER when the sampling time is off by one sample compared to the optimal sampling time.

a)





$\underline{r_k} = \int_{-\infty}^{\infty} r(t)rect\left(\frac{kT-t}{T}\right)dt = \int_{\left(k-\frac{1}{2}\right)T}^{(k+1/2)T} r(t)dt$

as T → 8/9T, $\underline{r_k}$ amplitude will be 8/9 of correctly received signal.

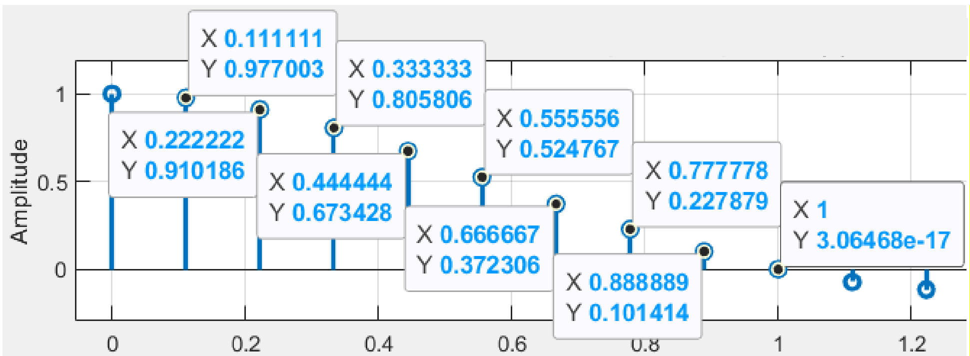
Then P = $\underline{r_k}^2$ = 64/81(79.01%) of original signal power(21% energy loss).

b) Raised cosine function

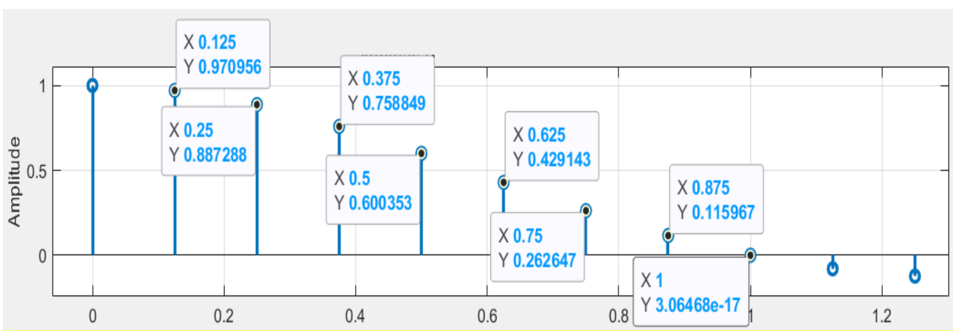
$$q(t) \quad = \quad \frac{\sin(\pi t)\cos(\beta\pi t)}{\pi t\left(1-(2\beta t)^2\right)},$$

With $\beta = 0.5$ q(t) = $\sin(\pi t)\cos(0.5\pi t)\,/(\pi t(1 - t^2))$

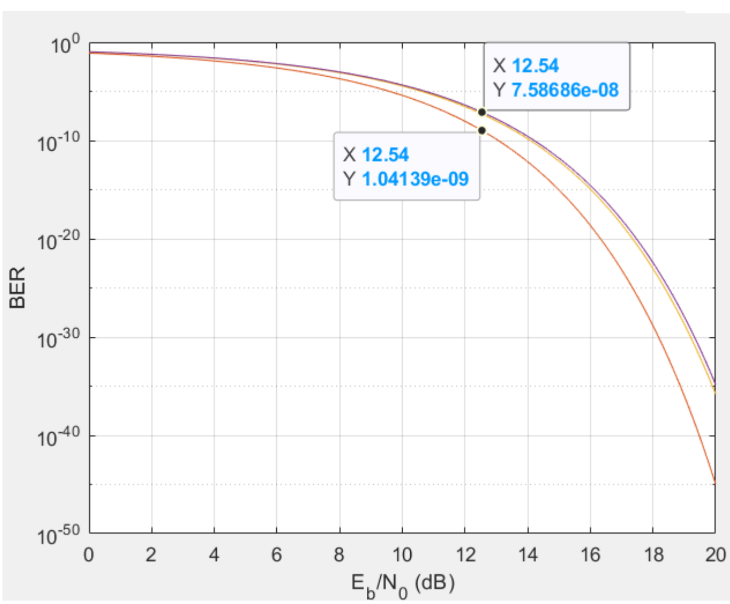
< 9 sample case > voltage sum = 4.592, P = v^2 = 21.093



< 8 sample case > voltage sum = 4.025, P = v^2 = 16.202(76.8%, 23.2% energy loss)



c) 1x10^-9 ➔ 7.59x10^-8



3. *Maximum likelihood carrier phase estimation*

Consider the following linear model in which we have a constant amplitude level A in the presence of complex circular-symmetric additive white gaussian noise:

$$x(k) = Ae^{j\theta} + n(k)$$

$$n(k) = n_I(k) + jn_Q(k)$$
$$n_{I,Q}(k) \sim N(0, \sigma^2)$$

and the observation interval is over a symbol that spans K samples.

$$0 \leq k \leq K-1$$

a) Write down the likelihood function $L(\theta; \boldsymbol{x})$.

$$L(\theta; \boldsymbol{x}) = f_{x_0,x_1,\ldots,x_{K-1}}(x_0, x_1, \ldots, x_{K-1}|\theta)$$

$X(k) = r_I(k) + jr_Q(k) = A\cos\theta + jA\sin\theta + n_I(k) + jn_Q(k)$

Assumes $n_{I,Q}(k) \sim N(0,\sigma^2)$ means n_I and n_Q combined noise distribution is $N(0,\sigma^2)$

$L(\theta; x) = (1/2\,\pi\sigma^2)^{k/2}\exp[-1/2\sigma^2 \sum_{k=0}^{k-1}(x_k - s(k;\theta))^2]$, $s(k;\theta) = A\cos\theta + jA\sin\theta$

$$\lim_{k\to\infty} -1/2\sigma^2 \sum_{k=0}^{k-1}(x_k - s(k;\theta))^2 T_s/T_s$$
$$= -1/No\int_{Ts}\left(x(t) - s(t;\theta)\right)^2 dt, \text{ as } \sigma^2/(fs/2) = No$$

$L(\theta; x(t)) = (1/2\,\pi\sigma^2)^{k/2} \exp[-1/No\int_{Ts}\left(x(t) - s(t;\theta)\right)^2 dt]$

b) Describe how you would solve the for the maximum likelihood estimator for θ.

i. What does the parameter θ signify?

1) Find the joint-pdf of observations
2) Find the likelihood function $L(\theta; x)$ or log-likelihood function could be found if it helps.
3) Optimize likelihood function or log-likelihood function.
   : set the derivative of it to zero and second order derivation to be negative to find the max point.

i. estimated θ is arctangent of I-channel integration/ Q-channel integration .
   So, the balance between I/Q should be taken care of for the wanted phase.

c) (*Extra Credit)* Calculate the maximum likelihood estimator for the parameter θ.

i. Compare your results to the Eq. (5.2.2) in Papen/Blahut. Explain the differences if any.

$\text{Log}(\,\text{L}(\theta;x)) = \text{C}_\text{L} - 1/\text{No}\int_{Ts}\left(\text{x}\,(\text{t}) - \text{s}(\text{t};\theta)\right)^2\text{dt}$

To maximize it it`s derivative should be zero as follows.

$$\frac{\partial \text{Log}(\,\text{L}(\theta;x))}{\partial\theta} = 0\ - 1/\text{No}\int_{Ts}\frac{\partial}{\partial\theta}\left(x(t) - \text{s}(\text{t};\theta)\right)^2\text{dt} =\ 0$$

from $\frac{\partial}{\partial\theta}\left(x(t) - \text{s}(\text{t};\theta)\right)^2$, $\quad$ *only* $x(t)s(t;\theta)$ *term matters*

Let set $\text{J}(\theta) = \int_{Ts}x(t)\text{s}(\text{t};\theta)\text{dt}$

$$\frac{\partial(\,\text{J}(\theta))}{\partial\theta} = \int_{Ts}x(t)\frac{\partial}{\partial\theta}\text{s}(\text{t};\theta)\text{dt} = \int_{Ts}\left(r_\text{I}\,(\text{t}) + \text{j}r_\text{Q}\,(\text{t}) + n_\text{I}(t)\ + jn_\text{Q}(t)\ \right)(\text{Asin}\theta - jAcos\theta)\text{dt} = 0$$

$$\text{Asin}\theta\int_{Ts}r_\text{I}\,(\text{t}) + \text{j}r_\text{Q}\,(\text{t}) + n_\text{I}(t)\ + jn_\text{Q}(t)\ \text{dt} = jAcos\theta\int_{Ts}r_\text{I}\,(\text{t}) + \text{j}r_\text{Q}\,(\text{t}) + n_\text{I}(t)\ + jn_\text{Q}(t)\ \text{dt}$$

$$\frac{\text{Asin}\theta}{Acos\theta} = tan\theta = j\int_{Ts}r_\text{I}\,(\text{t}) + \text{j}r_\text{Q}\,(\text{t}) + n_\text{I}(t)\ + jn_\text{Q}(t)\ \text{dt}\ /\int_{Ts}r_\text{I}\,(\text{t}) + \text{j}r_\text{Q}\,(\text{t}) + n_\text{I}(t)\ + jn_\text{Q}(t)\ \text{dt}$$

So, $\hat{\theta}(t) =\ tan^{-1}\left(\text{j}\,\frac{\int_{Ts}r_\text{I}\,(\text{t})+\text{j}r_\text{Q}\,(\text{t})+n_\text{I}(t)\ +jn_\text{Q}(t)\ dt}{\int_{Ts}r_\text{I}\,(\text{t})+\text{j}r_\text{Q}\,(\text{t})+n_\text{I}(t)\ +jn_\text{Q}(t)\ dt}\right)$

It`s different from the 5.5.2 below. If imaginary terms are taken with out 'j' it`s the same but don`t know why is that.

$$\hat{\theta}(t) =\ tan^{-1}\left(\frac{\int_{Ts}\left(r_\text{Q}\,(\text{t})\right)dt}{\int_{Ts}r_\text{I}\,(\text{t})dt}\right)\ (5.5.2)$$

4. *Gardner Algorithm*

Assume that a demodulated signal has a phase error $\Delta\theta$ such that the complex baseband signal is

$$w(t) = [y_I(t) + j y_Q(t)]\, e^{j\Delta\theta}$$

Show that substituting

$$x_I(t) = y_I(t)\cos\Delta\theta - y_Q(t)\sin\Delta\theta$$
$$x_Q(t) = y_I(t)\sin\Delta\theta + y_Q(t)\cos\Delta\theta$$

into

$$\begin{aligned} u_\ell(t) &= x_I(t-T/2)\,[x_I(t) - x_I(t-T)] + \\ &\quad x_Q(t-T/2)\,[x_Q(t) - x_Q(t-T)] \end{aligned}$$

produces

$$\begin{aligned} u_\ell(t) &= y_I(t-T/2)\,[y_I(t) - y_I(t-T)] + \\ &\quad y_Q(t-T/2)\,[y_Q(t) - y_Q(t-T)] \end{aligned}$$

and thus the error signal $u_\ell(t)$ of the Gardner symbol timing recovery algorithm is insensitive to a carrier phase error $\Delta\theta$ when both the in-phase and quadrature components are demodulated.

$$u_I(t) = x_I\left(t-\frac{T}{2}\right)[x_I(t) - x_I(t-T)] +$$
$$x_Q\left(t-\frac{T}{2}\right)[x_Q(t) - x_Q(t-T)]$$

$$= \left[y_I\left(t-\frac{T}{2}\right)cos\Delta\theta - y_Q\left(t-\frac{T}{2}\right)sin\Delta\theta\right]\left[y_I(t)cos\Delta\theta - y_Q(t)sin\Delta\theta - \left(y_I(t-T)cos\Delta\theta + y_Q(t-T)sin\Delta\theta\right)\right] +$$
$$\left[y_I\left(t-\frac{T}{2}\right)sin\Delta\theta + y_Q\left(t-\frac{T}{2}\right)cos\Delta\theta\right]\left[y_I(t)sin\Delta\theta + y_Q(t)cos\Delta\theta - \left(y_I(t-T)sin\Delta\theta - y_Q(t-T)cos\Delta\theta\right)\right]$$

$$= y_I\left(t-\frac{T}{2}\right)cos\Delta\theta\, y_I(t)cos\Delta\theta - y_I\left(t-\frac{T}{2}\right)cos\Delta\theta\, y_Q(t)sin\Delta\theta - y_I\left(t-\frac{T}{2}\right)cos\Delta\theta\, y_I(t-T)cos\Delta\theta +$$
$$y_I\left(t-\frac{T}{2}\right)cos\Delta\theta\, y_Q(t-T)sin\Delta\theta) - y_Q\left(t-\frac{T}{2}\right)sin\Delta\theta\, y_I(t)cos\Delta\theta + y_Q\left(t-\frac{T}{2}\right)sin\Delta\theta\, y_Q(t)sin\Delta\theta +$$
$$y_Q\left(t-\frac{T}{2}\right)sin\Delta\theta\, y_I(t-T)cos\Delta\theta - y_Q\left(t-\frac{T}{2}\right)sin\Delta\theta\, y_Q(t-T)sin\Delta\theta) +$$
$$y_I\left(t-\frac{T}{2}\right)sin\Delta\theta\, y_I(t)sin\Delta\theta + y_I\left(t-\frac{T}{2}\right)sin\Delta\theta\, y_Q(t)cos\Delta\theta - y_I\left(t-\frac{T}{2}\right)sin\Delta\theta\, y_I(t-T)sin\Delta\theta$$
$$- y_I\left(t-\frac{T}{2}\right)sin\Delta\theta\, y_Q(t-T)cos\Delta\theta)$$
$$+ y_Q\left(t-\frac{T}{2}\right)cos\Delta\theta\, y_I(t)sin\Delta\theta + y_Q\left(t-\frac{T}{2}\right)cos\Delta\theta\, y_Q(t)cos\Delta\theta - y_Q\left(t-\frac{T}{2}\right)cos\Delta\theta\, y_I(t-T)sin\Delta\theta -$$
$$y_Q\left(t-\frac{T}{2}\right)cos\Delta\theta\, y_Q(t-T)cos\Delta\theta)$$

$$= y_I\left(t-\frac{T}{2}\right)[\cos\Delta\theta\, y_I(t)\cos + \sin\Delta\theta\, y_I(t)\sin\Delta\theta - \cos\Delta\theta\, y_Q(t)\sin\Delta\theta - \cos\Delta\theta\, y_I(t-T)\cos\Delta\theta +$$

$$\cos\Delta\theta\, y_Q(t-T)\sin\Delta\theta) + \sin\Delta\theta\, y_Q(t)\cos\Delta\theta - \sin\Delta\theta\, y_I(t-T)\sin\Delta\theta - \sin\Delta\theta\, y_Q(t-T)\cos\Delta\theta)] +$$

$$y_Q\left(t-\frac{T}{2}\right)[\cos\Delta\theta\, y_I(t)\sin\Delta\theta + \cos\Delta\theta\, y_Q(t)\cos\Delta\theta - \cos\Delta\theta\, y_I(t-T)\sin\Delta\theta - \cos\Delta\theta\, y_Q(t-T)\cos\Delta\theta)$$

$$-\sin\Delta\theta\, y_I(t)\cos\Delta\theta + \sin\Delta\theta\, y_Q(t)\sin\Delta\theta + \sin\Delta\theta(y_I(t-T)\cos\Delta\theta - \sin\Delta\theta\, y_Q(t-T)\sin\Delta\theta)]$$

$$= y_I\left(t-\frac{T}{2}\right)[y_I(t) - y_I(t-T)] +$$

$$y_Q\left(t-\frac{T}{2}\right)[y_Q(t) - y_Q(t-T)]$$

Matlab problem 1

- PulseShape = **Rectangular**

– SamplesPerSymbol = **8**

– SyncSequence = **1000010010110011111000110111010**

– SyncSequenceFormat = *BPSK*

– SyncSeqLength = **31 bits**

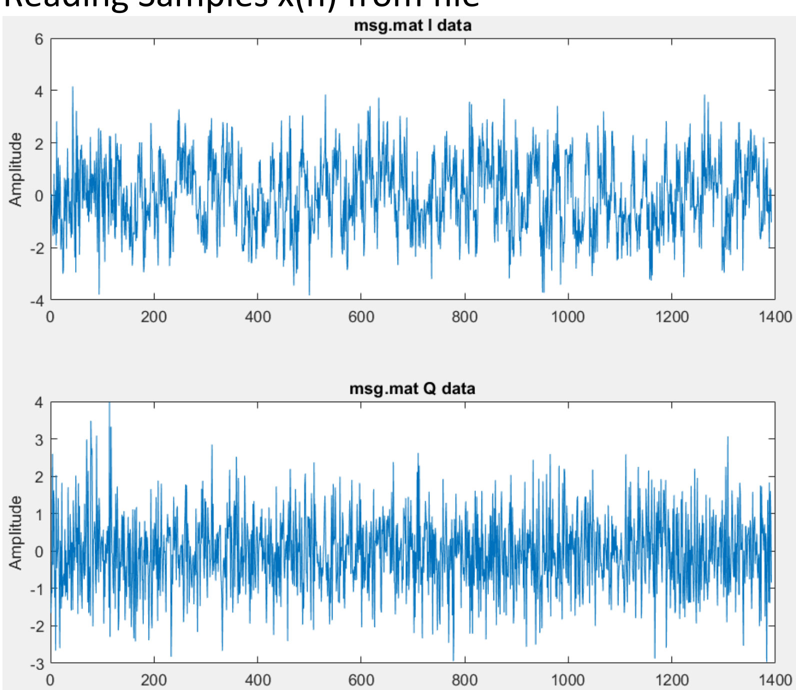
– NumDataBits = **96 bits**

– PacketFormat =[SYNC SEQ (31bits)][**DATA BITS (96 bits)**][SYNC SEQ (31bits)]

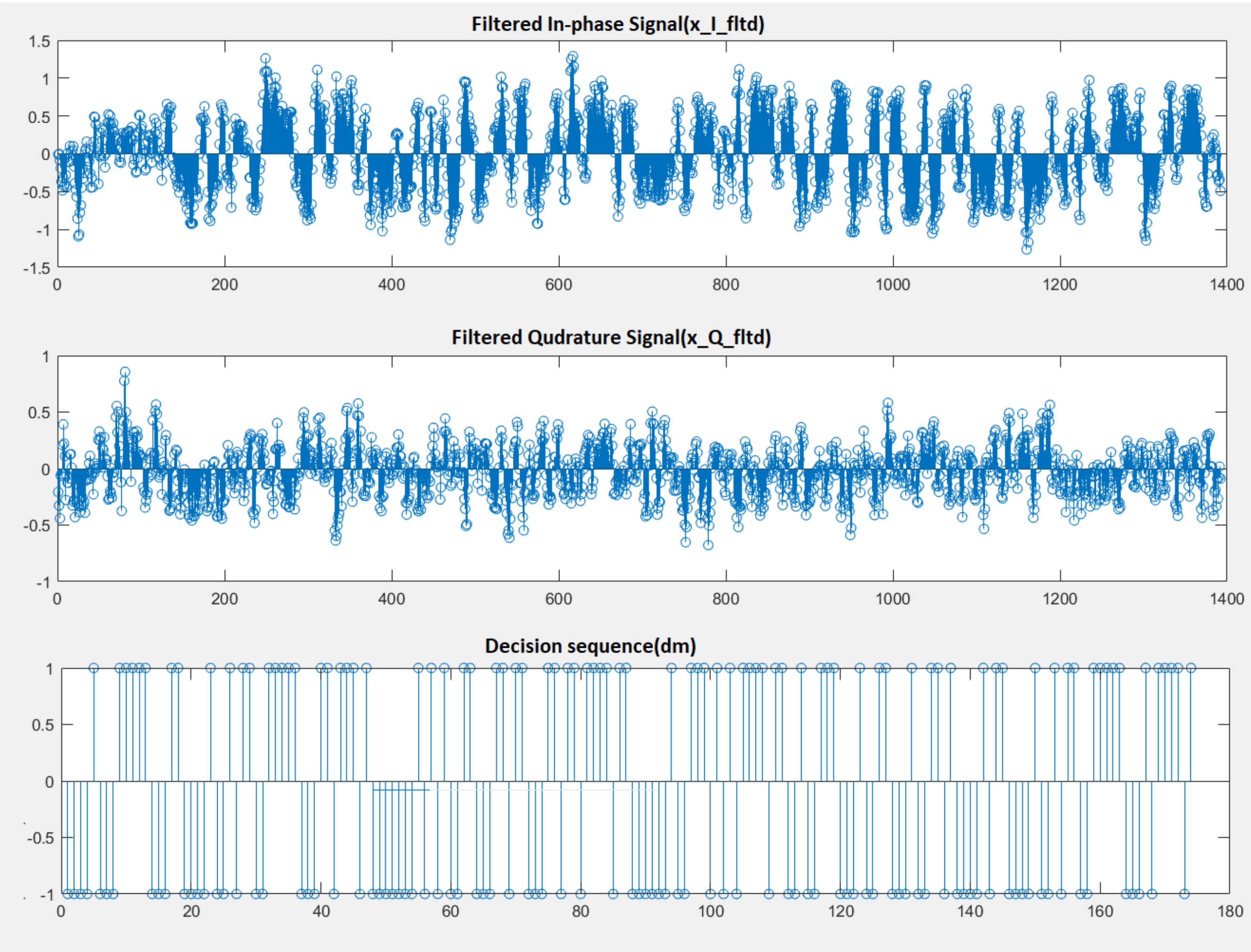
– DataFile = *msg.mat*

– DataFileFormat = Complex Baseband Samples

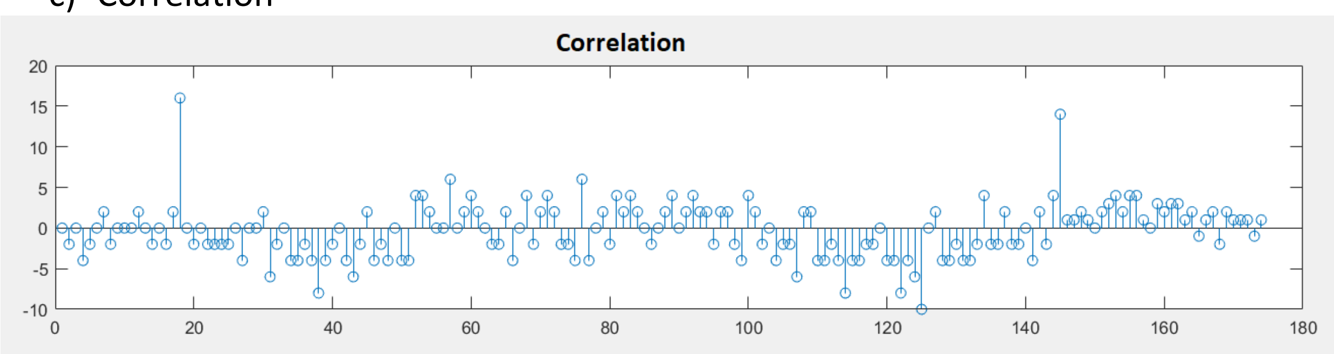
a) Reading Samples x(n) from file



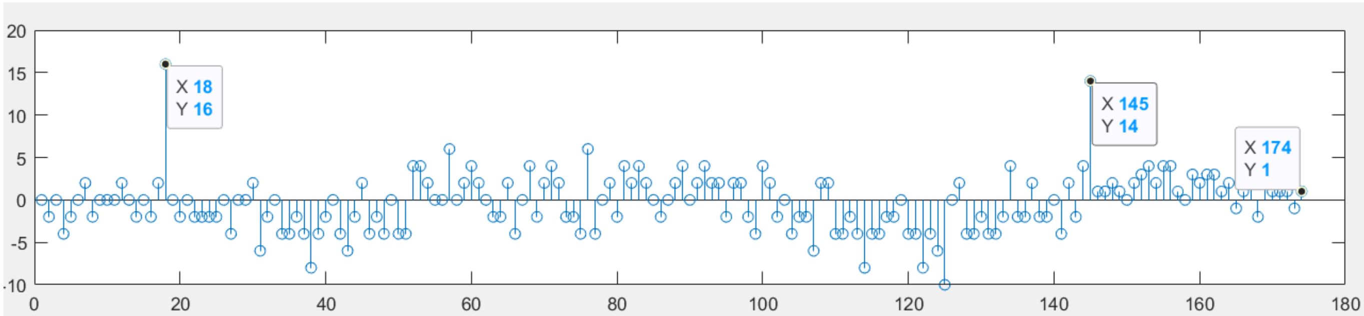
b) Matched Filter h(n) and Decision d(m)



c) Correlation

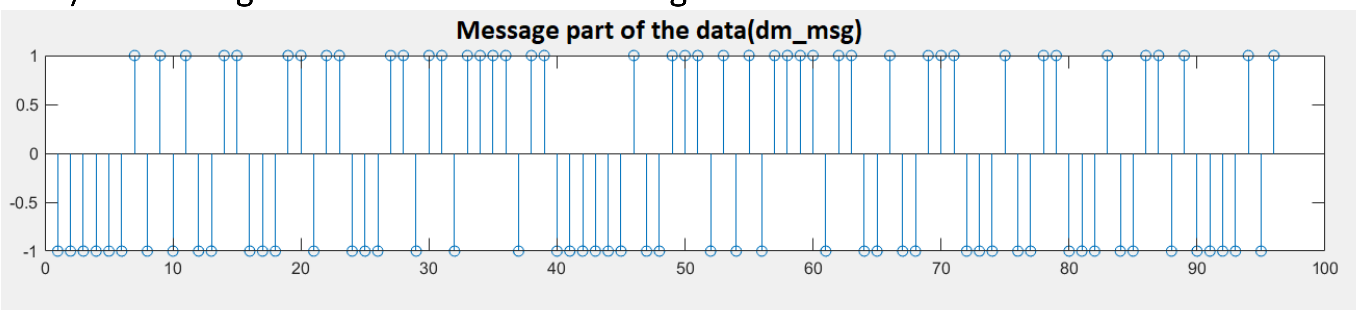


d) Examining the Correlation Peaks



m1 = 18, m2 = 145

e) Removing the Headers and Extracting the Data Bits



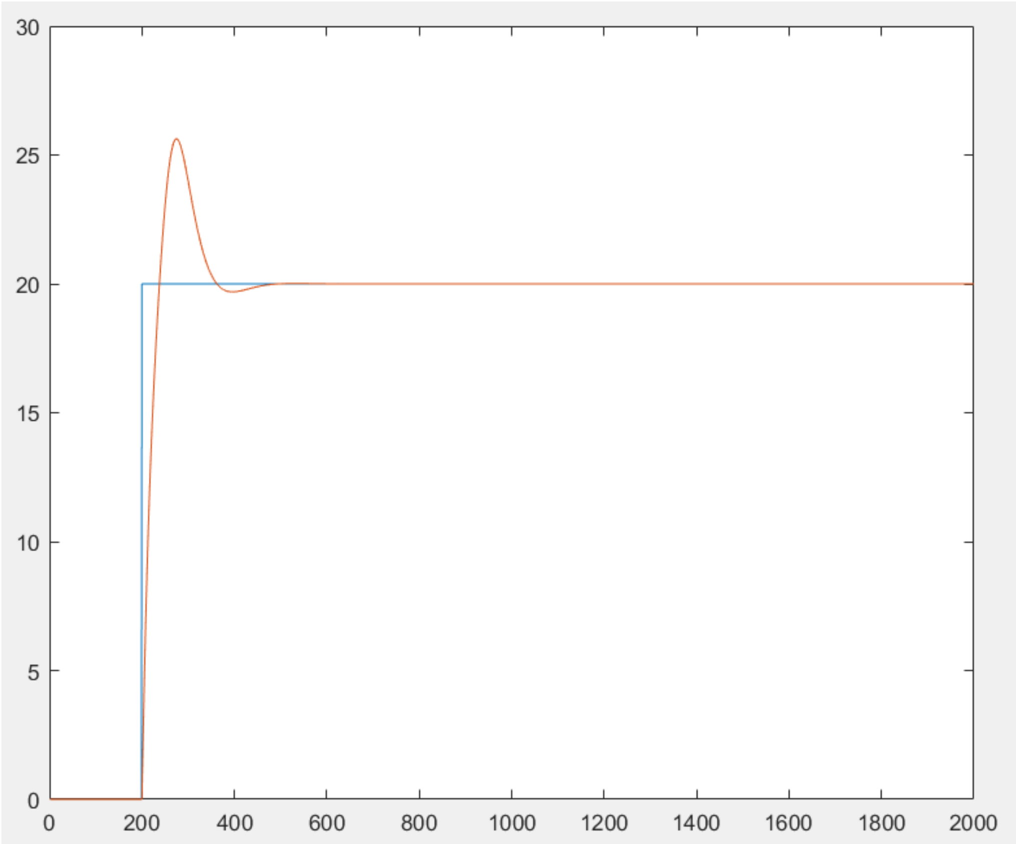
f) (By MSB first for each 8 bits)

00000010: @
10100110: e
00110110: l
00110110: l
11110110: o
00000100: space
11101010: W
11110110: o
01001110: r
00100110: d
00100110: d
10000101: ¡

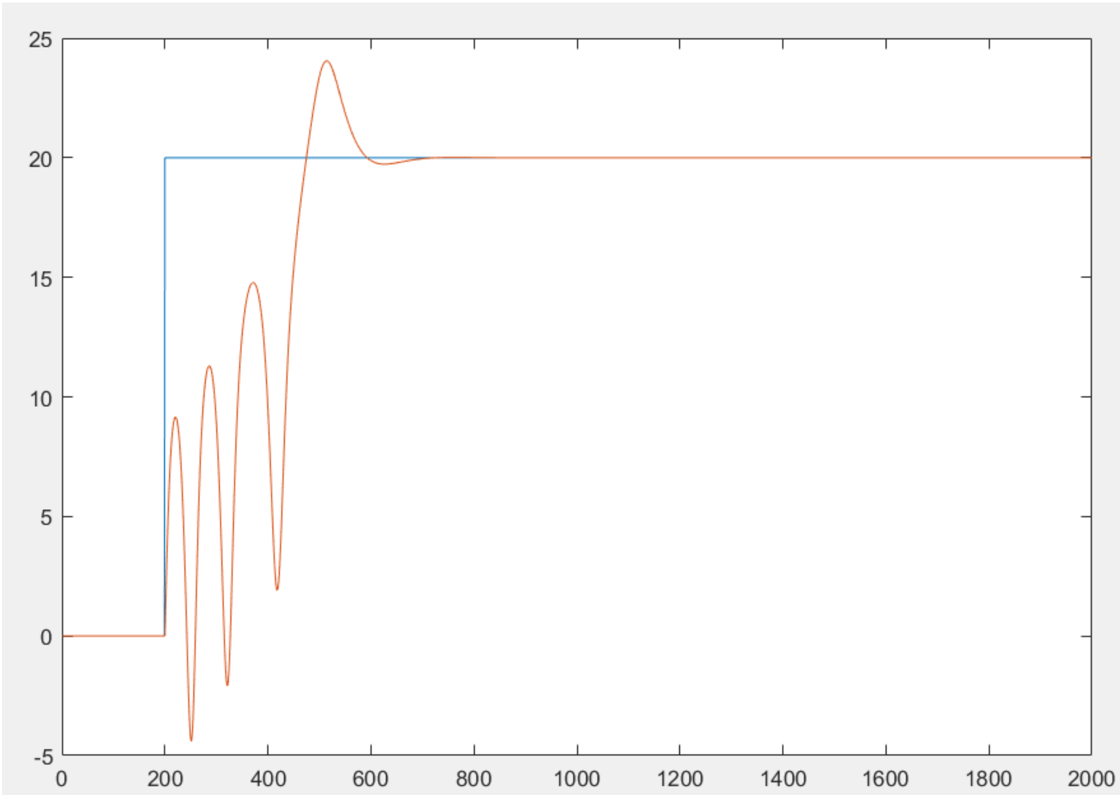
(I can tell it should e Hello World¡ but was not able to get it maybe due to 'dm' was not correct.)

2. Implementation of a PLL

< Standard PLL>



< Costas PLL >



<code for Matlab P1>

```
x=load("msg.mat");
x_I = real(x.x);
x_Q = imag(x.x);
figure(1)
subplot(211)
plot(x_I)
title(['msg.mat I data'])
ylabel('Amplitude')
subplot(212)
plot(x_Q)
title(['msg.mat Q data'])
ylabel('Amplitude')

figure(2)

fs=8;N=128/fs; B =0.999;NN=1392/2; fftlen = 1024;
ndx=(-N:1/fs:N);              % Impulse Response Sample Span
h=(sin(pi*ndx).*cos(B*pi*ndx))./((pi*ndx).*(1-(2*B*ndx).^2));
h(round(fs*N+1))=1.0;         % Insert value 1.0 at Midpoint
```

```
hc=zeros(1,NN*2);          % Array of zeros
hc(NN+1+(-N*fs:N*fs))=h;   % Center impulse response in array
hc=fftshift(hc)/fs;       % Shift center to Origin, now Non Causal, and scale

subplot(3,1,1)
stem(ndx,h,'linewidth',2);    % Plot non causal impulse response
grid on;
axis([-5 5 -0.3 1.2])
title(['Impulse Response, h(n)',num2str(20*N+1),' Tap Filter'])
text(-0.5,0,'Time Index');
ylabel('Amplitude')

subplot(3,1,2)
Hc = fftshift(real(fft(hc,fftlen)))/max(fftshift(real(fft(hc,fftlen))));%/3.3027;
plot((-0.5:1/fftlen:0.5-1/fftlen)*fs,Hc,'linewidth',2)  % Mag Freq Resp
hold on
plot([-0.5*fs -0.5 -0.5 0.5 0.5 0.5*fs],[0 0 1 1 0 0],'r','linewidth',2)
hold off;
grid on;
axis([-0.5*fs 0.5*fs -0.2 1.2])
set(gca,'XTick',[-5:0.5:5])
title(['Magnitude Frequency Response, ',num2str(20*N+1),' Tap Filter'])
ylabel('Magnitude')

subplot(3,1,3)
plot((-0.5:1/fftlen:0.5-1/fftlen)*fs,20*log10(Hc),'linewidth',2) % Log Mag Freq Resp
hold on
plot([-0.5*fs -0.5 -0.5 0.5 0.5 0.5*fs],[-20 -20 0 0 -20 -20],'r','linewidth',2)
hold off
grid on
axis([-0.5*fs 0.5*fs -20 10])
set(gca,'XTick',[-0.5*fs:0.5:-0.5*fs])
title(['Log Magnitude Frequency Response, ',num2str(20*N+1),' Tap Filter'])
xlabel('Frequency')
ylabel('Log Mag (dB)')

%%
x_I_ip=zeros(1,NN*2);            % Array of zeros
x_I_ip(1:1:1392)=x_I;   % Center impulse response in array
x_Q_ip=zeros(1,NN*2);            % Array of zeros
x_Q_ip(1:1:1392)=x_Q;   % Center impulse response in array
y_I_fltd = filter(hc,1,x_I_ip);
y_Q_fltd = filter(hc,1,x_Q_ip);

figure(3)
subplot(511)
stem(y_I_fltd)
subplot(512)
stem(y_Q_fltd)
subplot(513)
dm = zeros(1,NN*2);
dm = ceil(y_I_fltd(1:8:end))*2-1;
dm = dm./abs(dm);
stem(dm)
```

```
%% correlate
subplot(514)
x = [ 1 0 0 0 0 1 0 0 1 0 1 1 0 0 1 1 1 1 1 0 0 0 1 1 0 1 1 1 0 1 0];
z = correlate(x,dm);
stem(z)
subplot(515)
dm_msg = zeros(1,96);
dm_msg =dm(49:1:144)
stem(dm_msg)
dm_msg_out = zeros(1,96);
dm_msg_out = dm_msg(144:1:49)
```

<code for Matlab P2>

```
clear all; close all;
nn = 2000;
n = 1:1:nn;
%n_1 = 2:1:nn+1;
fs = 2000; % samples per second
Ts = 1/fs; % period
delF = 20; % Hz
fn = 10;   % Hz
zeta = 0.707 ;  %0.707 ;
n0 = 200;  % samples
n1 = 2000; % samples

%%Initilize PLL Loop
% (Setting up a Simulation to test the step response)
     %st = ones(1,nn);
for i = 1:nn
     if  i < n0
        st(i) = 0;
     elseif i> n0-1
        st(i) = delF;
     end
end
%% xc definition
xc0 = zeros(1,nn);
xc = zeros(2,nn);
for j = 1:nn

    if j< n0+1
        xc0(j) = 0;
    elseif j > n0-1
        xc0(j) = exp(1i*2*pi*delF*(j-n0)*Ts);       % n0 <= n < n
    end
      xI(j) = real(xc0(j)); xQ(j) = imag(xc0(j));

     xc(1,j) = xI(j);
     xc(2,j) = xQ(j);
end

e = zeros(1,length(xc));
fi = zeros(1,length(xc)+1);
```

```
fint = zeros(1,length(xc)+1)
fo = zeros(1,length(xc));
vi = zeros(1,length(xc));
vo = zeros(1,length(xc));
th = zeros(1,length(xc));

%% Kt
Kt = 4*pi*zeta*fn;
%% Ka
Ka = pi*fn/zeta;

%% PLL implementation
for l = 2:length(xc)
    %% phase rotator

    %ro(1) = [1  0; 0 1];
    ro= [ cos(th(l-1)) -sin(th(l-1)) ; sin(th(l-1)) cos(th(l-1))];
    %xc(l) = ro*xc(1,l)
     s(1,l) = ro(1,1)*xc(1,l)+ro(1,2)*xc(2,l);
     s(2,l) = ro(2,1)*xc(1,l)+ro(2,2)*xc(2,l);

    sI(l) = s(1,l) ; sQ(l) =s(2,l);
    sI(l) =s(1,l);
    sQ(l) =s(2,l);

    %% phase comparator
    %standard PLL
    e(l) = s(2,l);%atan(abs(s(1,l)/abs(s(2,l))))-atan(abs(xc(1,l)/abs(xc(2,l))));%
    %costas PLL
    %e(l) = s(2,l)*s(1,l);%sI(n)sQ(n;
    fi(l) = e(l)*Kt;
    fint(l) =fint(l-1) + Ka*Ts/2*(fi(l)+fi(l-1));
    fo(l) = fi(l) + fint(l);

    %% Numerically Controlled Oscillator

    vi(l) = fo(l);
    vo(l) = vo(l-1) + Ts/2.*(vi(l)+vi(l-1));
    th(l+1) = -vo(l);


    %% loop filter
    fint(l+1) =fint(l) + Ka*Ts/2*(fi(l+1)+fi(l));
end

figure(1)
% plot(1:length(xc), xc)
% hold on
plot(1:length(st), st)
hold on
plot(1:length(fo), fo/2/pi)
% hold on
% plot(1:length(th), th)
```