Project #4: Measuring the Charge-to-mass Ratio of the Electron

In this project you will analyze data taken by students in a Modern Physics lab using the Thomson apparatus in order to measure the charge-to-mass ratio of the electron. In the experiment a beam of electrons moves in a circle in a magnetic field created by Helmholtz coils. The radius of the circle r is measured as a function of accelerating voltage V and Helmholtz coil current I. The charge-to-mass ratio e/m for the electrons can be shown to be

$$e/m = \frac{125V}{32}\left(\frac{a}{rI\mu_0 N}\right)^2 \tag{1}$$

where $a = 0.15$m is the radius of the Helmholtz coils, $N = 130$ is the number of turns in the coils, and $\mu_0 = 1.26\times10^{-6}$ Tm/A is the permeability constant. There are two sets of measurements on Canvas in the files em1.dat and em2.dat. The columns in both files are V in volts, I in amps, and r in cm (remember to convert r to meters to make the units work). First, calculate a column of e/m values for each file. To make sure that everything worked, compare your values to the accepted value of e/m for the electron. Calculate the mean and standard deviation for each set of values and make a histogram showing their distributions. Next, we would like to understand where the variation in the e/m values is coming from. The obvious hypothesis is that it comes from measurement uncertainties in V, I, and r. For V and I, we can assume that the measurement uncertainties are about 1 in the last digit (for example,1.52 could be assumed to have an uncertainty of 0.01). For r, assume that the measurement uncertainty is about 0.5mm. Propagate the uncertainties and obtain an estimate of the standard deviation σ that we would expect for each value of e/m in each of the two datasets. Note that since V and I were changed for each measurement, we can't average V and I measurements and so can't do the error propagation once for each data set.

Next, calculate the standard deviation of the set of mean e/m values for each data set. This gives us a direct measure of how much our values of e/m vary. Now, if our hypothesis is correct, then the σ values we got from error propagation should be similar (the same order of magnitude) as the directly calculated standard deviation. If the directly calculated standard deviation was larger, it would say that either there is some something other than measurement errors making our values vary and we would have to try to understand what this is, or that we have underestimated our measurement errors.

The value we will report for e/m is the mean $\pm\sigma_{mean}$, where mean is the mean value of e/m from the dataset and σ_{mean} is the standard deviation of the mean, also known as the standard error. You should use the directly calculated standard deviation to calculate the standard error, since this is our most direct measurement of the variation in our values. When you report your value for e/m, make sure that you truncate the mean to remove digits that are beyond the standard deviation, since these digits are pure noise. For example, it would not make sense to report 2.16 ± 0.1, since the 6 is below the uncertainty. This should instead be reported as 2.2 ± 0.1. Further, you should factor the same power of 10 out of both your mean and standard error. Thus rather than reporting a number as 1.23×10^6 N $\pm 2 \times 10^4$ N, it is much clearer to write $(1.23 \pm 0.02) \times 10^6$ N.

Next, calculate the probability for each result of getting a value of e/m that deviates as much or more from the accepted value (which you can calculate from the accepted values of e and $m_{electron}$). See, for example, exercise 4.7 from Worksheet 4. Calculate the probability that the two datasets are consistent with each other, e.g. that they were taken from the same system in the same way. See Exercise 4.9 in Worksheet 4. Finally, one concern of this experiment is that the resulting value of e/m might depend on what voltage V is being used in the experiment. To investigate this possibility, plot e/m vs. V and calculate the Pearson's correlation coefficient for V and e/m.

Discussion: Begin your discussion section by repeating the results of the two datasets for the value of e/m and its uncertainty. Are the values of e/m distributed in a roughly Gaussian way? Discuss whether the error propagation calculation supports the idea that the variation in e/m values is mostly coming from measurement uncertainties. If not, what might be going on? Discuss whether the values for e/m obtained for each dataset are consistent with the accepted value and whether they are consistent with each other. Discuss the possible role that a systematic error might play in these results. A systematic error differs from random errors in that they push the results of a measurement in a specific direction. For example, in this experiment a systematic error would arise if one of the experimenters systematically read the beam radius from the middle rather than the outer edge of the beam. This would cause all the r measurements to be systematically small, which would cause all the e/m values to be systematically large. Finally, discuss the result of your correlation analysis and whether it suggests that e/m is correlated with V. If so, how could you improve the experiment?