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Homework 8  
Computer Performance Modeling

1. Suppose an interactive system is supporting 100 users with 15 second think times and a system throughput of 5 interactions/second. a. What is the response time of the system? b. Suppose that the service demands of the workload evolve over time so that system throughput drops to 50% of its former value (i.e., to 2.5 interactions/second). Assuming that there still are 100 users with 15 second think times, what would their response time be? c. How do you account for the fact that response time in (b) is more than twice as large as that in (a)?

A )  $(100/5) - 15 = 5$  seconds.

B)  $(100/2.5) - 15 = 25$  seconds

2) You're going to develop a bounded model for the proxy and web server system. You should follow Algorithm 5.1 in the book "Quantitative System Performance" ([PDF on the Moodle](#)).

1. Write down the visit count for the Proxy, Router, Web server A and B; these will be as a function of the probability of a hit in the proxy. Write this down in your PDF writeup. You need to properly account for the fact that you either use either web server A or B, but not both, averaged.

$$V_p = 1 + (1 * .3) = 1.3$$

$$V_r = .6 = .3 * 2$$

$$V_a = .15 = .3 * .5$$

$$V_b = .15 = .3 * .5$$

2. Write down the Demand ( $D_i$ ) for each service center - this is based on the visits  $V_i$  and the service time  $S_i$ . The "delay center" for the transmission time should be included in the total demand  $D$ , but is not eligible for the maximum delay center  $D_{max}$  because there is no queuing for the transmission time, so it's not the bottleneck.

```
print "proxy " + str(Vp * sProxy)
```

```
print "router " + str(Vr * sRouter)
```

```
print "A " + str(Va * sA)
```

```
print "B " + str(Vb * sB)
```

**proxy 13.0**

**router 3.0**

**A 22.5**

**B 15.0**

3. Examine case study 5.3.1 for the process of calculating the job bounds, and calculate the upper bounds on throughput ( $X$ ) and the lower bounds on response time ( $R$ ). Write the bounds down and also graph those bounds (two lines for each).

```
def AsymptoticlowboundThroughput(N,Dmax,D):
```

```
    low = N/(N*D+Z)
```

```
    return low
```

```
def AsymptoticlowboundResponse(N,Dmax,D):
```

```
    return max(N*Dmax-Z,D)
```

```
def AsymptoticupboundThroughput(N,Dmax,D):
```

```
    #1/Dmax
```

```
    up = N/(D+Z)
```

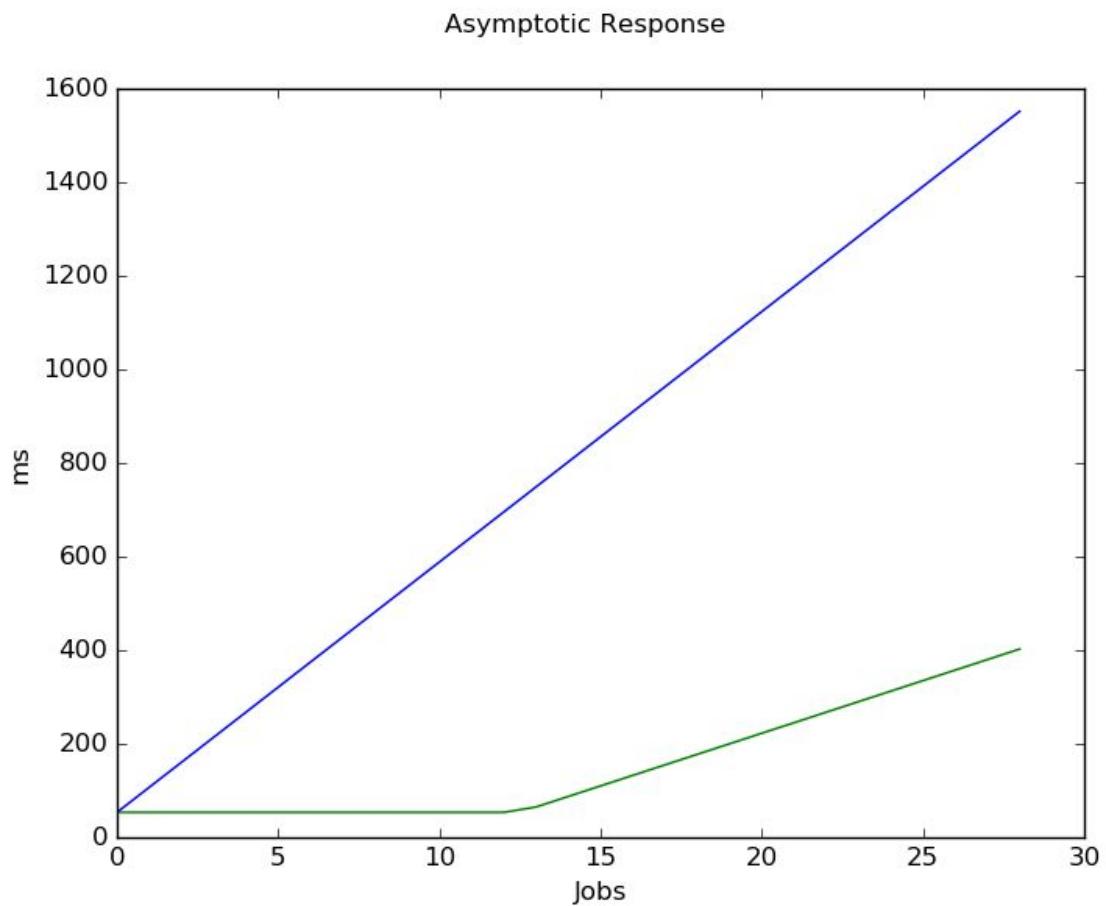
```
    return min(1/Dmax,up)
```

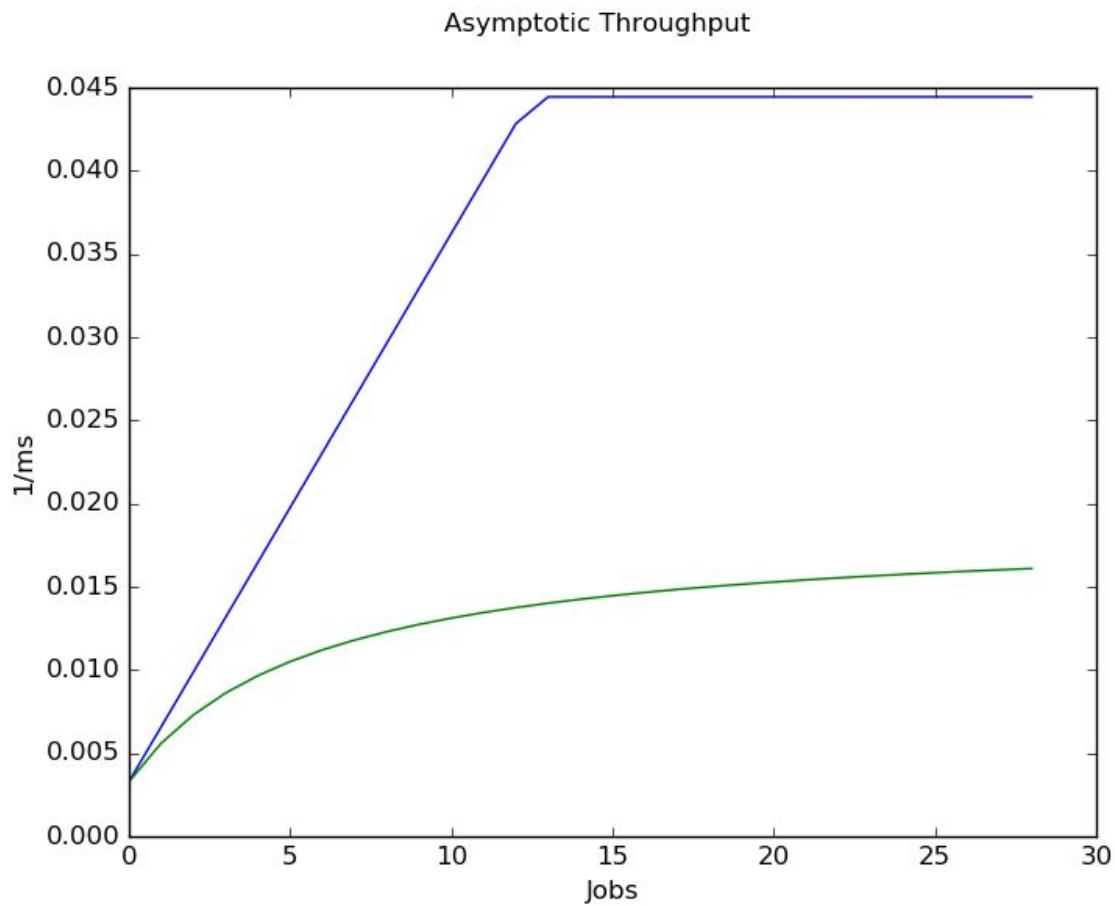
```
def AsymptoticupboundResponse(N,Dmax,D):
```

```
    return N*D
```

**0.031 ms lower bound throughput**

**400 ms lower bound response time**

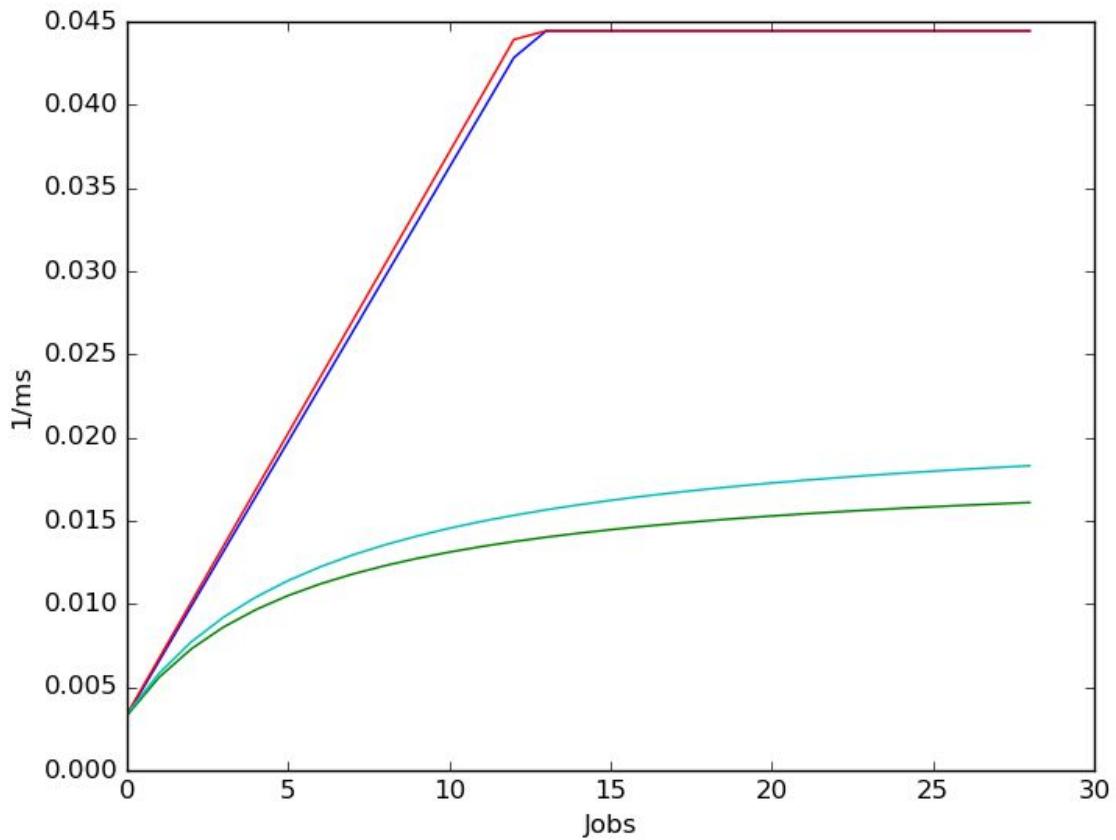


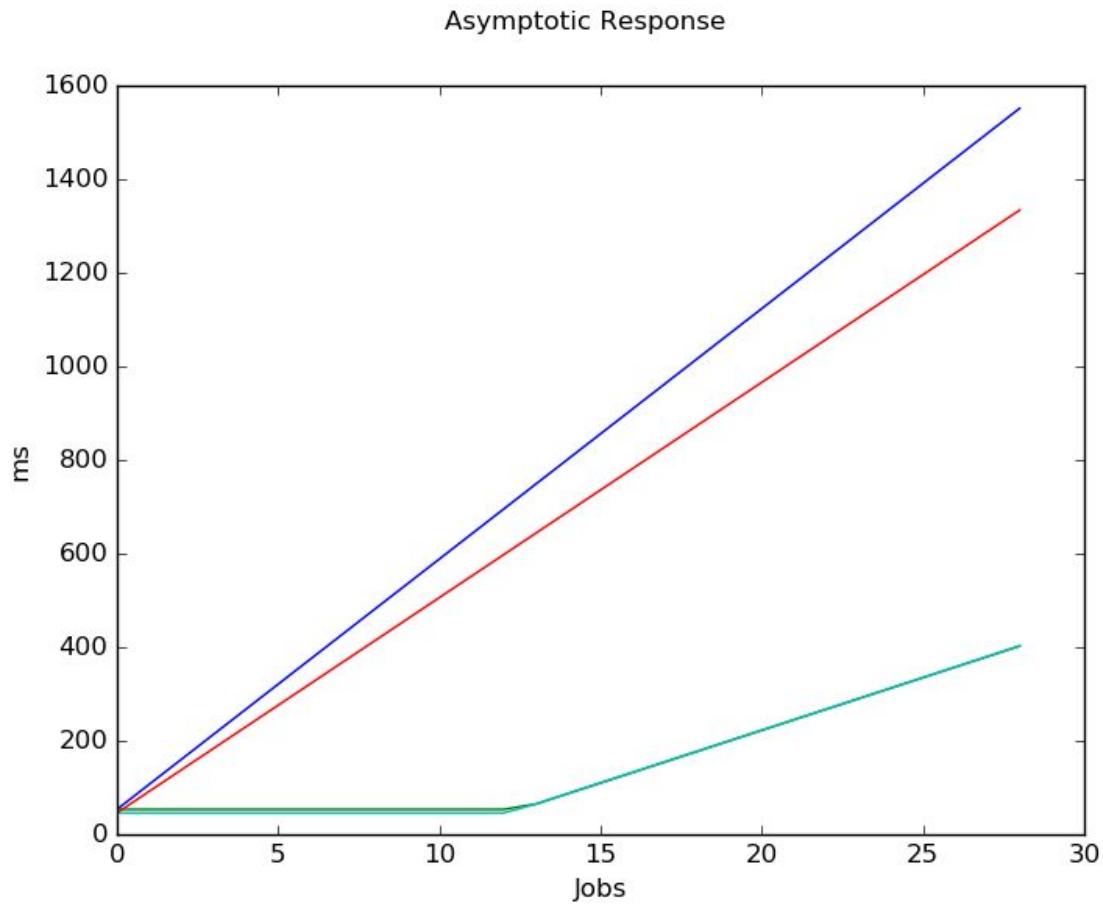


4. *Modify the model in a method similar to 5.3.3 of the QSP text. In your case, assume that the two origin servers are of equal performance. Write down the bounds equations and graph them again, overlaying them with the original bounds from part 3.*

I used the same functions as in the prior problem but changed the service time on A to be the same b.

### Asymptotic Throughput





5. Now, determine the lower bounds on throughput ( $X$ ) and the upper bounds on response time ( $R$ ) for the original system using the balanced job bounds equations summarized in Table 5.2. Produce a single graph with the upper and lower bounds of  $X$  and  $R$ .

```
def balancedlowboundThroughput(N,Dmax,D):
```

$$\text{low} = N/(D+Z+((N-1)*Dmax)/(1+Z/(N*D)))$$

```
return low
```

```
def balancedlowboundResponse(N,Dmax,D):
```

$$\text{low} = D + ((N-1)*Dave)/(1+Z/(D))$$

```
return max(N*Dmax-Z,low)
```

```
def balancedupboundThroughput(N,Dmax,D):
```

```
    #1/Dmax
```

```
    up = N/(D+Z+((N-1)*Dave)/(1+Z/(D)))
```

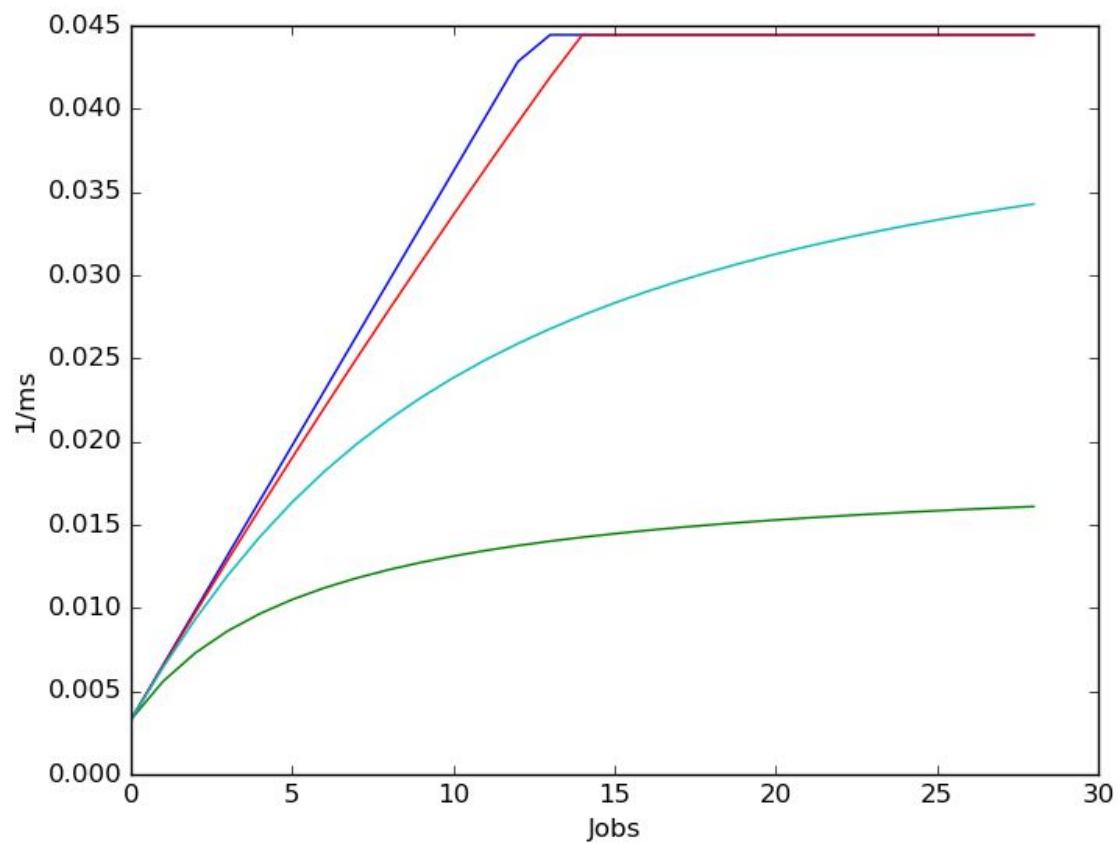
```
    return min(1/Dmax,up)
```

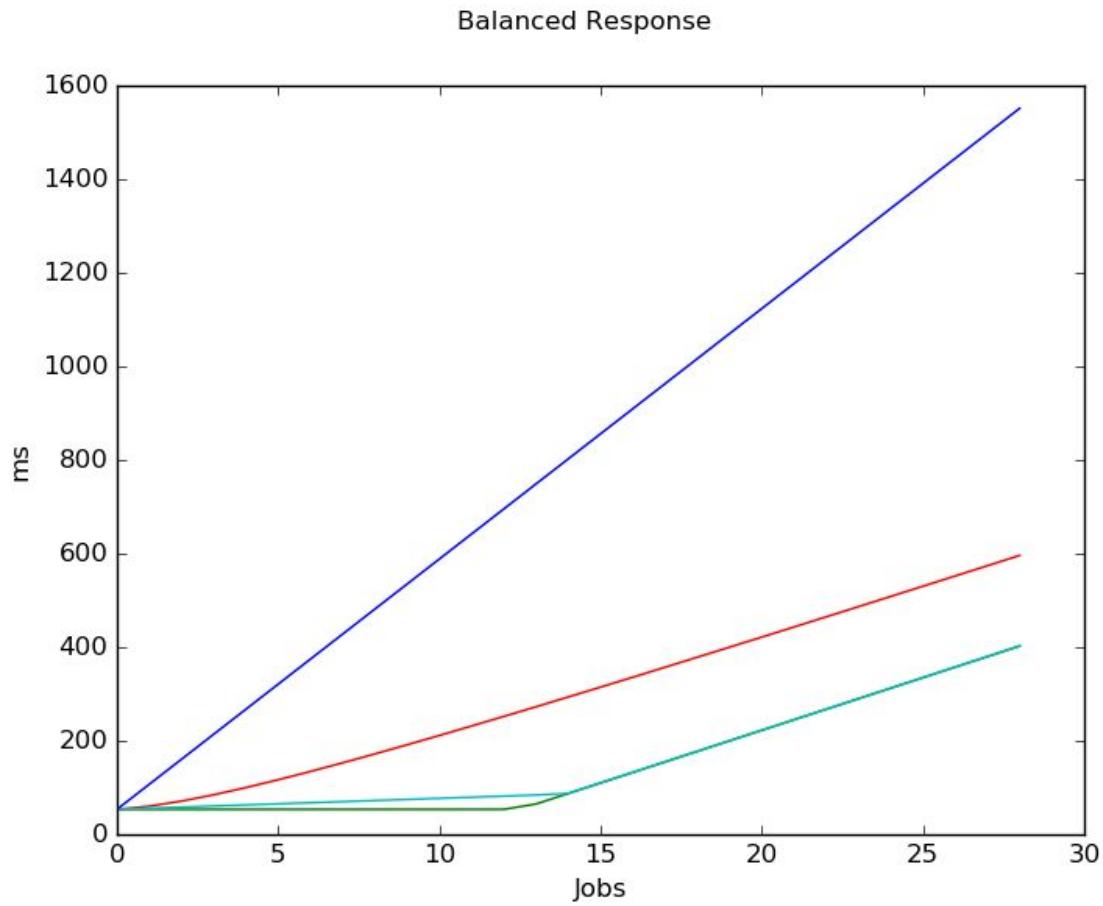
```
def balancedupboundResponse(N,Dmax,D):
```

```
    low = D+ ((N-1)*Dmax)/(1+Z/(N*D))
```

```
    return low
```

Balanced Throughput





3.

1. You will use the  $V_i$  and  $D_i$  from the prior problem.
2. Write down the equations for the Response for the individual components for  $N$  customers in the system, using the appropriate service times (again, in the PDF document) and the Queue length for  $N-1$  customers in the system

$$R_{proxy} = D_{proxy} * (1 + Q_{proxy}(N-1))$$

$$Q_{proxy}(N) = X(N) * R_{proxy}(N)$$

$$R_{router} = D_{router} * (1 + Q_{router}(N-1))$$

$$Q_{router}(N) = X(N) * R_{router}(N)$$

$$Ra = Da * (1 + Qa(N-1))$$

$$Qa(N) = X(N) * Ra(N)$$

$$Rb = Db * (1 + Qb(N-1))$$

$$Qb(N) = X(N) * Rb(N)$$

$$Rwan = Dwan * (1 + Qwan(N-1))$$

$$Qwan(N) = X(N) * Rwan(N)$$

4. Write down the equation for the System Response, which combines the visit ratios and the individual response times

$$\text{systemresponsetime} = N/X - Z$$

$$\text{averagenumberinsystem} = N - XZ$$

5. Write down the equation for the Throughput.

$$X(N) = \frac{N}{Z + \sum_{k=1}^K R_k(N)}$$

6. Write down the equations for the Q (number in queue) for N customers in the system

$$Q_k(N) = X(N) R_k(N)$$

7. Now, wrap all that in into a lovely Excel spreadsheet and drag out the computation until at least N=30; cut and paste or do whatever it is that gets it into your single PDF document that is easy to grade. The example in Table 6.2 of the QSP book shows the structure to follow (although that example is "sideways", or use the PPT example from class.

	Respon se					Queue					
Jobs	Proxy	Router	A	B	Wan	Proxy	Router	A	B	Wan	X(N)
0						0	0	0	0	0	
1	13	3	22.5	15	21.6	0.0399 88	0.0092 28	0.0692 09	0.0461 4	0.0664 41	0.0030 76
2	13.519 84	3.0276 84	24.057 21	15.692 09	21.6	0.0824 64	0.0184 67	0.1467 36	0.0957 14	0.1317 49	0.0060 99
3	14.072 03	3.0554 02	25.801 57	16.435 7	21.6	0.1275 55	0.0276 95	0.2338 76	0.1489 8	0.1957 91	0.0090 64
4	14.658 21	3.0830 86	27.762 21	17.234 7	21.6	0.1753 7	0.0368 86	0.3321 45	0.2061 95	0.2584 21	0.0119 64
5	15.279 81	3.1106 58	29.973 27	18.092 92	21.6	0.2259 95	0.0460 08	0.4433 17	0.2676 02	0.3194 73	0.0147 9
6	15.937 93	3.1380 24	32.474 64	19.014 03	21.6	0.2794 78	0.0550 27	0.5694 56	0.3334 19	0.3787 65	0.0175 35
7	16.633 22	3.1650 8	35.312 77	20.001 28	21.6	0.3358 19	0.0639 02	0.7129 52	0.4038 19	0.4360 96	0.0201 9
8	17.365 64	3.1917 06	38.541 43	21.057 28	21.6	0.3949 47	0.0725 89	0.8765 49	0.4789 07	0.4912 5	0.0227 43
9	18.134 32	3.2177 67	42.222 35	22.183 6	21.6	0.4567 1	0.0810 39	1.0633 63	0.5586 9	0.5439 92	0.0251 85
10	18.937 23	3.2431 17	46.425 66	23.380 35	21.6	0.5208 45	0.0891 98	1.2768 81	0.6430 48	0.5940 82	0.0275 04
11	19.770 99	3.2675 94	51.229 83	24.645 72	21.6	0.5869 71	0.0970 1	1.5209 36	0.7316 94	0.6412 71	0.0296 88
12	20.630 62	3.2910 3	56.721 05	25.975 41	21.6	0.6545 63	0.1044 17	1.7996 3	0.8241 41	0.6853 19	0.0317 28
13	21.509 31	3.3132 51	62.991 67	27.362 11	21.6	0.7229 53	0.1113 62	2.1172 23	0.9196 72	0.7260 01	0.0336 11
14	22.398 39	3.3340 87	70.137 52	28.795 08	21.6	0.7913 32	0.1177 93	2.4779 5	1.0173 27	0.7631 26	0.0353 3

15	23.287 32	3.3533 79	78.253 89	30.259 9	21.6	0.8587 73	0.1236 63	2.8857 91	1.1159 03	0.7965 49	0.0368 77
16	24.164 05	3.3709 9	87.430 29	31.738 55	21.6	0.9242 68	0.1289 39	3.3441 83	1.2139 9	0.8261 94	0.0382 5
17	25.015 48	3.3868 18	97.744 11	33.209 85	21.6	0.9867 9	0.1336	3.8557 27	1.3100 34	0.8520 59	0.0394 47
18	25.828 27	3.4008 01	109.25 39	34.650 51	21.6	1.0453 65	0.1376 43	4.4219 06	1.4024 34	0.8742 32	0.0404 74
19	26.589 75	3.4129 29	121.99 29	36.036 51	21.6	1.0991 51	0.1410 82	5.0428 7	1.4896 56	0.8928 88	0.0413 37
20	27.288 97	3.4232 45	135.96 46	37.344 84	21.6	1.1475 07	0.1439 48	5.7173 42	1.5703 59	0.9082 85	0.0420 5
21	27.917 6	3.4318 45	151.14 02	38.555 39	21.6	1.1900 45	0.1462 89	6.4426 6	1.6435 02	0.9207 44	0.0426 27
22	28.470 58	3.4388 68	167.45 98	39.652 53	21.6	1.2266 47	0.1481 63	7.2149 61	1.7084 18	0.9306 3	0.0430 85
23	28.946 41	3.4444 88	184.83 66	40.626 28	21.6	1.2574 61	0.1496 32	8.0294 87	1.7648 46	0.9383 26	0.0434 41
24	29.346 99	3.4488 96	203.16 35	41.472 69	21.6	1.2828 54	0.1507 63	8.8809 44	1.8129 08	0.9442 07	0.0437 13
25	29.677 1	3.4522 88	222.32 12	42.193 62	21.6	1.3033 55	0.1516 17	9.7638 78	1.8530 54	0.9486 26	0.0439 18
26	29.943 62	3.4548 51	242.18 72	42.795 81	21.6	1.3195 91	0.1522 52	10.672 99	1.8859 76	0.9518 94	0.0440 69
27	30.154 68	3.4567 57	262.64 23	43.289 64	21.6	1.3322 18	0.1527 18	11.603 4	1.9125 14	0.9542 77	0.0441 79
28	30.318 83	3.4581 53	283.57 66	43.687 71	21.6	1.3418 78	0.1530 54	12.550 78	1.9335 7	0.9559 92	0.0442 59
29	30.444 41	3.4591 62	304.89 26	44.003 54	21.6	1.3491 57	0.1532 94	13.511 45	1.9500 36	0.9572 13	0.0443 15
30	30.539 04	3.4598 83	326.50 75	44.250 53	21.6	1.3545 68	0.1534 64	14.482 33	1.9627 45	0.9580 74	0.0443 55

8. Now, repeat the analysis for a system where the origin servers have balanced service times. If you structure your spreadsheet correctly, this should be just changing a single cell that defines the service time of the web servers.

	Response					Queue					
Jobs	Proxy	Router	A	B	Wan	Proxy	Router	A	B	Wan	X(N)
0						0	0	0	0	0	
1	13	3	22.5	15	21.6	0.0399 88	0.0092 28	0.0692 09	0.0461 4	0.0664 41	0.0030 76
2	13.519 84	3.0276 84	24.057 21	15.692 09	21.6	0.0824 64	0.0184 67	0.1467 36	0.0957 14	0.1317 49	0.0060 99
3	14.072 03	3.0554 02	25.801 57	16.435 7	21.6	0.1275 55	0.0276 95	0.2338 76	0.1489 8	0.1957 91	0.0090 64
4	14.658 21	3.0830 86	27.762 21	17.234 7	21.6	0.1753 7	0.0368 86	0.3321 45	0.2061 95	0.2584 21	0.0119 64
5	15.279 81	3.1106 58	29.973 27	18.092 92	21.6	0.2259 95	0.0460 08	0.4433 17	0.2676 02	0.3194 73	0.0147 9
6	15.937 93	3.1380 24	32.474 64	19.014 03	21.6	0.2794 78	0.0550 27	0.5694 56	0.3334 19	0.3787 65	0.0175 35
7	16.633 22	3.1650 8	35.312 77	20.001 28	21.6	0.3358 19	0.0639 02	0.7129 52	0.4038 19	0.4360 96	0.0201 9
8	17.365 64	3.1917 06	38.541 43	21.057 28	21.6	0.3949 47	0.0725 89	0.8765 49	0.4789 07	0.4912 5	0.0227 43
9	18.134 32	3.2177 67	42.222 35	22.183 6	21.6	0.4567 1	0.0810 39	1.0633 63	0.5586 9	0.5439 92	0.0251 85
10	18.937 23	3.2431 17	46.425 66	23.380 35	21.6	0.5208 45	0.0891 98	1.2768 81	0.6430 48	0.5940 82	0.0275 04
11	19.770 99	3.2675 94	51.229 83	24.645 72	21.6	0.5869 71	0.0970 1	1.5209 36	0.7316 94	0.6412 71	0.0296 88
12	20.630 62	3.2910 3	56.721 05	25.975 41	21.6	0.6545 63	0.1044 17	1.7996 3	0.8241 41	0.6853 19	0.0317 28
13	21.509 31	3.3132 51	62.991 67	27.362 11	21.6	0.7229 53	0.1113 62	2.1172 23	0.9196 72	0.7260 01	0.0336 11
14	22.398 39	3.3340 87	70.137 52	28.795 08	21.6	0.7913 32	0.1177 93	2.4779 5	1.0173 27	0.7631 26	0.0353 3
15	23.287 32	3.3533 79	78.253 89	30.259 9	21.6	0.8587 73	0.1236 63	2.8857 91	1.1159 03	0.7965 49	0.0368 77
16	24.164	3.3709	87.430	31.738	21.6	0.9242	0.1289	3.3441	1.2139	0.8261	0.0382

	05	9	29	55		68	39	83	9	94	5
17	25.015 48	3.3868 18	97.744 11	33.209 85	21.6	0.9867 9	0.1336	3.8557 27	1.3100 34	0.8520 59	0.0394 47
18	25.828 27	3.4008 01	109.25 39	34.650 51	21.6	1.0453 65	0.1376 43	4.4219 06	1.4024 34	0.8742 32	0.0404 74
19	26.589 75	3.4129 29	121.99 29	36.036 51	21.6	1.0991 51	0.1410 82	5.0428 7	1.4896 56	0.8928 88	0.0413 37
20	27.288 97	3.4232 45	135.96 46	37.344 84	21.6	1.1475 07	0.1439 48	5.7173 42	1.5703 59	0.9082 85	0.0420 5
21	27.917 6	3.4318 45	151.14 02	38.555 39	21.6	1.1900 45	0.1462 89	6.4426 6	1.6435 02	0.9207 44	0.0426 27
22	28.470 58	3.4388 68	167.45 98	39.652 53	21.6	1.2266 47	0.1481 63	7.2149 61	1.7084 18	0.9306 3	0.0430 85
23	28.946 41	3.4444 88	184.83 66	40.626 28	21.6	1.2574 61	0.1496 32	8.0294 87	1.7648 46	0.9383 26	0.0434 41
24	29.346 99	3.4488 96	203.16 35	41.472 69	21.6	1.2828 54	0.1507 63	8.8809 44	1.8129 08	0.9442 07	0.0437 13
25	29.677 1	3.4522 88	222.32 12	42.193 62	21.6	1.3033 55	0.1516 17	9.7638 78	1.8530 54	0.9486 26	0.0439 18
26	29.943 62	3.4548 51	242.18 72	42.795 81	21.6	1.3195 91	0.1522 52	10.672 99	1.8859 76	0.9518 94	0.0440 69
27	30.154 68	3.4567 57	262.64 23	43.289 64	21.6	1.3322 18	0.1527 18	11.603 4	1.9125 14	0.9542 77	0.0441 79
28	30.318 83	3.4581 53	283.57 66	43.687 71	21.6	1.3418 78	0.1530 54	12.550 78	1.9335 7	0.9559 92	0.0442 59
29	30.444 41	3.4591 62	304.89 26	44.003 54	21.6	1.3491 57	0.1532 94	13.511 45	1.9500 36	0.9572 13	0.0443 15
30	30.539 04	3.4598 83	326.50 75	44.250 53	21.6	1.3545 68	0.1534 64	14.482 33	1.9627 45	0.9580 74	0.0443 55

9. Compare the results from the simulation model, MVA model and the balanced job bounds. Do the bounds correctly bound the MVA model? How different are the MVA results than the simulation?

The bounds look to have properly bound the MVA model. The MVA results are different than the simulation but not excessively so.