# Drag Motion of Falling Spherical Body

## 1 Objective

Study graphically the motion of a falling spherical body under various effects of medium (viscous drag, buoyancy, and air drag)

## 2 Introduction

A moving object in a viscous fluid is equivalent to a stationary object in a flowing fluid stream. The flow of the stationary fluid around a moving object may be laminar, turbulent, or a combination of the two.

## 3 Parameters Required for the C++ Code

To simulate the motion of a falling spherical body under various effects of medium using C++, we need to list the parameters involved in its dynamics:

1. A body with **mass** and **radius (r)**(only spherical body is considered)
2. To observe the body's motion in a medium, the medium's **density** and **coefficient of viscosity** ($\mu$) is required.
3. In this problem, the body is falling from a certain height, and hence **height** is an important parameter here.
4. Another important parameter is the **drag coefficient (drag_coeff)** of the body. The drag coefficient mainly depends on the shape of the body, but the coefficient also contains the complex dependencies of object inclination, the effects of air viscosity, and compressibility. In this simulation, we have considered a simpler case where the body only has a spherical shape.

| **Parameters Input by User** | **Symbol** |
|---|---|
| Mass | mass |
| Radius | r |
| Density of the Medium | density |
| Acceleration due to Gravity of the Medium | g |
| Coefficient of Viscosity of the Medium | $\mu$ |
| Drag Coefficient | drag_coeff |
| Initial Height | x |
| Initial value of velocity | v |

**Table 1**

The only predefined variable is the step size (h) here.

## 4 Important Definitions

Let us define a few important quantities and parameters.

- **Viscosity**: It is the property of a fluid that opposes the relative motion between two surfaces of the fluid in a fluid that is moving with different velocities.

- **Coefficient of viscosity**: It is defined as the tangential force per unit area of the layer of a fluid required to maintain a unit velocity gradient newline The S.I. unit of the coefficient of viscosity is Pa-s. newline The C.G.S. unit is poise (P), 1 Pa-s = 10 P
- **Stokes' law** states that viscous force (a type of drag force) acting on the spherical body is given by the equation,

$$\begin{aligned} F_d &= 6\pi\eta rv, \\ \text{where } \eta &= \text{Coefficient of viscosity,} \\ r &= \text{Radius of spherical body,} \\ v &= \text{Relative velocity of body} \end{aligned}$$

- **Drag Force**: Drag, or fluid resistance, is an opposing force acting on an object moving relative to a surrounding fluid. Drag force depends on velocity. It exists only when there is relative motion between an object and a fluid.
- **Drag Coefficient**: The drag coefficient describes the resistance of an object in a fluid. It is a dimensionless quantity and depends upon the surface area of the object. The drag coefficient of a rough sphere is about 0.46.
- **Drag Equation-** It quantifies the dependence of fluid resistance on surface area and relative velocity.

$$\begin{aligned} F_d &= \frac{1}{2} * \rho v^2 C_d A, \\ \text{where } \rho &= \text{Density of fluid,} \\ v &= \text{speed of the object relative to the fluid,} \\ C_d &= \text{Drag coefficient, and} \\ A &= \text{Cross-sectional area} \end{aligned}$$

- **Buoyant Force-** Buoyancy or upthrust describes the tendency of an object to float or to rise in a fluid when submerged. The buoyant force is defined as the upward force exerted by a fluid on an object immersed in it.

$$\begin{aligned} F_b &= \rho g V, \\ \text{where } \rho &= \text{Density of fluid,} \\ g &= \text{Acceleration due to gravity, and} \\ V &= \text{Volume of Fluid displaced} \end{aligned}$$

# 5 Motion of body under the effects of viscous drag and buoyancy

Consider a spherical body of radius r, density $\rho$ is falling through a fluid of density $\sigma$. The forces acting on the body are

1. $$W = mg = \frac{4}{3}\pi r^3 \rho g, \tag{1}$$

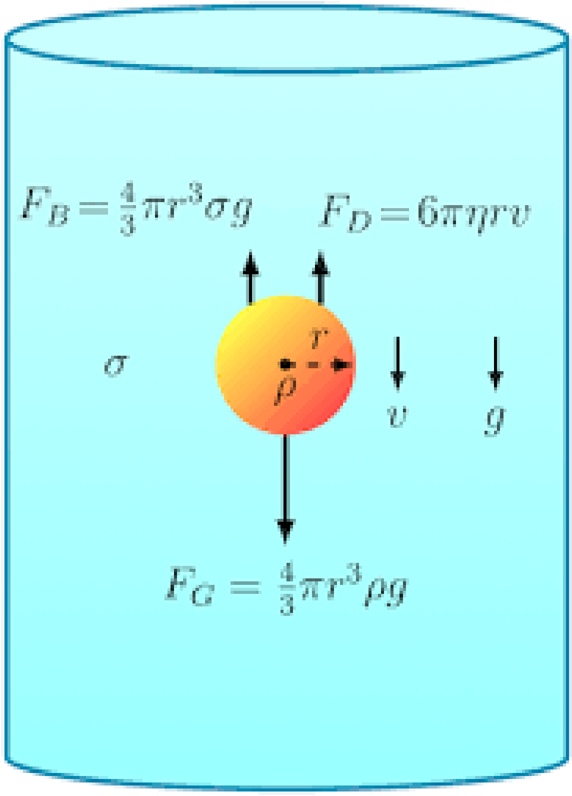
   acting in the downward direction

2. $$F_d = 6\pi\eta rv, \tag{2}$$

   acting in the upward direction (notice that the dependence of drag force is on v only)

3. $$F_b = \frac{4}{3}\pi r^3 \sigma g, \tag{3}$$

   acting in the upward direction




Hence, the net or resultant force acting on the body is

$$F_r = W - F_d - F_b \tag{4}$$

The equation for retardation is given as:

$$a = g - \frac{6\pi\eta r v}{m} - \frac{4}{3m}\pi r^3 \sigma g \tag{5}$$

If the body attains a constant velocity called terminal velocity, then the net force acting on the body is zero.

$$W - (F_d + F_b) = 0 \implies F_d = W - F_b \implies 6\pi\eta r v_t = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g \tag{6}$$

$$v_t = \frac{2g}{9\eta r}(r^2\rho - r^2\sigma) \tag{7}$$

This is the expression for the terminal velocity of a body falling through a viscous medium. As buoyant force is constant throughout the motion so it does not affect the path of the body during motion.
Finally, to study the motion of the body, we consider the equation for resultant force as,

$$F_r = W - F_d \implies ma = mg - 6\pi\eta r v \implies a = g - \frac{6\pi\eta r v}{m} \tag{8}$$

## 6 Motion under Air drag

Considering the gravitational force acting on an object. But in reality, the gravitational force is not the only force that acts on objects under free fall.
Our atmosphere has circulation, and air exerts a buoyant force on the falling objects. Sometimes the wind forces may be heavy and in such cases, the objects fall in a direction that is resultant to the vertical gravitational force and the horizontal wind force. The resistance due to air is a complicated function to evaluate. It depends on so many factors like temperature, physical dimensions of the objects, density of air, etc. In most cases, the quantum of air-resistive force is so small that it can be neglected in most cases. Till now, we have been discussing a free fall motion ignoring the air resistance. It is safe in most

cases. However in certain cases, where the area of the object is large the buoyant force of air offers a considerable resistance. Thus the net force of fall gets reduced and given by,

$$F = mg - F_a - F_b \tag{9}$$

where 'm' is the mass of the object and 'k' is a constant, called as 'air resistance coefficient' for the particular type of motion and F_b is the buoyant force due to air (although the value of buoyant force exerted by air is considerably less as compared to that of water or other fluids, but still the force exists). Now in above equation, the component F_a must be equal to the force due to the air drag (notice that the dependence of drag force, in this case, is on $v^2$ and the expression for this force is given as

$$F_a = \frac{1}{2}\sigma\pi c_d r^2 v^2 \tag{10}$$

where,

$$\begin{aligned} \sigma &= \text{Density of fluid,} \\ c_d &= \text{Drag coefficient,} \\ v &= \text{Velocity of the body, and} \\ r &= \text{Radius of body} \end{aligned}$$

Hence,

$$ma = mg - \frac{1}{2}\sigma\pi c_d r^2 v^2 \implies a = g - \frac{1}{2}\frac{\sigma c_d v^2 a}{m} - \frac{4}{3m}\pi r^3 \sigma g \tag{11}$$

where a= area = $\pi\ r^2$

# 7 Algorithm

1. **Include all the required header files**.
2. **Define a class 'falling_body'** with all the above-listed parameters in Table 1 and pre-defined parameters declared as private members.
3. **Declare public member functions** in the class that can be accessed from outside the class using an object or instance of the class defined in the main function (the compiler starts the execution of any C++ program from the main() function)
4. **Definition of Member function 1:** falling_body(){//}
   The constructor initializes parameters such as radius, mass, density, gravity, drag coefficient, viscosity, initial height, and velocity.
   - Return data-type: none
     A constructor is a special member function having the same name as the class and it gets invoked automatically as the instance/object of the class is declared.
   - The function doesn't accept any parameters
5. **Definition of Member function 2:** float retard(float) {//}
   The function implements the formula for total acceleration due to gravity, air drag, Stokes' drag, and buoyancy drag.
   - Return data-type: float
     This function returns the computed value of retardation depending upon the value of velocity passed to it by the calc() function from where the function call has been made.
   - The function accepts 1 parameter
     The accepted parameter by the function is velocity v at a particular time step.
   - The function calculates the retardation value in all of the cases that are under consideration depending on the parameters input by the user. Following are the two cases:

- Case 1: Body falling in the presence of air drag i.e., the coefficient of viscosity is zero (for air, $\mu = 1.8$*e-5 Pa·s at room temp.) -
  For this case, the governing equation is eq.(10).
- Case 2: Body falling in a fluid exerting Stokes' drag i.e., the drag coefficient is zero (this is to ensure that the equation with only v dependence is used and not the one with quadratic dependence on v) -
  For this case, the governing equation is eq.(5).

6. **Definition of Member function 3:** void calc() {//}
   This function calculates the values of position, velocity, and, body's deceleration for different time steps via Euler Method.
   - Return data-type: void This function doesn't return any value.
   - The function doesn't accept any parameter
   - The function performs important computations required for the final result:
     - An 'if-condition' is used in the function to check whether the density of the body is less than or equal to the density of the medium. If the density of the body is less than that of the medium, then the body will float. In that case, the user has to enter the values again.
     - A for-loop is used to implement the Euler method for the calculation of displacement and velocity. Also, the function call to retard (float) is made by this function. The loop gets terminated when the height reaches zero, i.e., the object hits the ground.
     - The function finally creates a .txt data file and enters the data computed to this file, which includes time step, displacement, velocity, and acceleration.
7. **Define the main function:** `int main()`
   - Create an instance of the 'falling_body' class, which automatically calls the constructor.
   - The instance created here makes function calls to all necessary functions.
   - Return 0 to indicate successful execution.
8. The final step in the algorithm involves a crucial step i.e., plotting the data saved in the file by the calc() function using **Gnuplot software**.
   Gnuplot is a graphing utility for Linux, OS/2, MS Windows, OSX, VMS, and many other platforms. Its source code is freely distributed and is extensively used for data visualization.
   Follow the following steps to plot data on Gnuplot:
   (a) Download Gnuplot.
   (b) Go to File → Change directory.
   Select the folder where the 'drag.txt' data file is stored.
   (c) Write the following command in Gnuplot:

```
plot "drag.txt" u 1:2 w
\* This command asks the Gnuplot to plot Column 2 with respect to Column 1
    with a line-type graph. There are many other variations to this command,
    you can check them out at the official website of Gnuplot or watch some
    free tutorials on YouTube.*/
```

# References

[1] @misc author = "Damilare Stephen Adepehin,L C Anu,Ayodeji Babinisi,Akintayo Ikusika", title = "African Scientific Reports Effect of viscosity on reservoir deliverability of green field, Niger-Delta, Nigeria", year = "2024", howpublished = "`https://www.researchgate.net/figure/Litho-stratigraphic-sequence-of-the-Niger-Delta_fig1_381830610`", note = "[Online; accessed 13-August-2024]"