Max Entropy Theorems

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Theorem 1 (Epstein) *Let $(X\times Y,\mu\times\nu)$ be a product computable measure space. Let $A:\mathbb{N}\to X$, $B:\mathbb{N}\to Y$ be injective functions with $\mathbf{I}(\langle A,B\rangle:\mathcal{H})<\infty$. For $s\in\mathbb{N}$, $m<s$, there exists 2^{s-m} indices $t<2^s$ with* $\max\{\mathbf{G}_\mu(A(t)),\mathbf{G}_\nu(B(t))\}<-m+O(\log s)$.

Theorem 2 (Epstein) *Let L be the Lebesgue measure over $\mathbb{R}$, $(\mathcal{X},\mu)$, $(\mathcal{Y},\nu)$ be non-atomic computable measure spaces. Let $A:[0,1]\to\mathcal{X}$ and $B:[0,1]\to\mathcal{Y}$ be continuous. Let* $\mathbf{I}(\langle A,B\rangle:\mathcal{H})<\infty$. *There is a constant c with* $L\{t\in[0,1]:\max\{\mathbf{G}_\mu(A(t)),\mathbf{G}_\nu(B(t))\}<\log\mu(X)-n\}>2^{-n-\mathbf{K}(n)-c}$.

Theorem 3 (Epstein) *Let $(\mathcal{X},\mu)$, $(\mathcal{Y},\nu)$, and $(\mathcal{Z},\rho)$ be non-atomic computable measure spaces. Let $A:\mathcal{Z}\to\mathcal{X}$ and $B:\mathcal{Z}\to\mathcal{Y}$ be continuous. Let $\mathbf{I}(\langle A,B\rangle:\mathcal{H})<\infty$. There is a constant c with* $\rho\{\alpha:\max\{\mathbf{G}_\mu(A(\alpha)),\mathbf{G}_\nu(B(\alpha))\}<\log\mu(X)-n\}>2^{-n-\mathbf{K}(n)-c}$.