

AIT Blog

A Complication for the Many Worlds Theory

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The Many Worlds Theory (**MWT**) was formulated by Hugh Everett [Eve57] as a solution to the measurement problem of Quantum Mechanics. Branching (a.k.a splitting of worlds) occurs during any process that magnifies microscopic superpositions to the macro-scale. This occurs in events including human measurements such as the double slit experiments, or natural processes such as radiation resulting in cell mutations.

One question is if **MWT** causes issues with the foundations of computer science. The physical Church Turing Thesis (**PCTT**) states that any functions computed by a physical system can be simulated by a Turing machine. A straw man argument for showing **MWT** and **PCTT** are in conflict is an experiment that measures the spin of an unending number of electrons, with each measurement bifurcating the current branch into two sub-branches. This results in a single branch in which the halting sequence is outputted. However this branch has Born probability converging to 0, and can be seen as a deviant, atypical branch.

In fact, conflicts do emerge between **MWT** and Algorithmic Information Theory. In particular, the Independence Postulate (**IP**) is a finitary Church-Turing thesis, postulating that certain infinite and *finite* sequences cannot be found in nature, a.k.a. have high “addresses”. If a forbidden sequence is found in nature, an information leak will occur. However **MWT** represents a theory in which such information leaks can occur. This blog entry covers the main arguments of this conflict.

Many Worlds Theory

Some researchers believe there is an inherent problem in quantum mechanics. On one hand, the dynamics of quantum states is prescribed by unitary evolution. This evolution is deterministic and linear. On the other hand, measurements result in the collapse of the wavefunction. This evolution is non-linear and nondeterministic. This conflict is called the measurement problem of quantum mechanics.

The time of the collapse is undefined and the criteria for the kind of collapse are strange. The Born rule assigns probabilities to macroscopic outcomes. The projection postulate assigns new microscopic states to the system measured, depending on the the macroscopic outcome. One could argue that the apparatus itself should be modeled in quantum mechanics. However it’s dynamics is deterministic. Probabilities only enter the conventional theory with the measurement postulates.

MWT was proposed by Everett as a way to remove the measurement postulate from quantum mechanics. The theory consists of unitary evolutions of quantum states without measurement collapses. For **MWT**, the collapse of the wave function is the change in dynamical influence of one

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part of the wavefunction over another, the decoherence of one part from the other. The result is a branching structure of the wavefunction and a collapse only in the phenomenological sense.

Branching Worlds

An example of a branching of universes can be seen in an idealized Stern-Gerlach experiment with a single electron with spin $|\phi_\uparrow\rangle$ and $|\phi_\downarrow\rangle$. This description can be found in [SBKW10]. There is a measuring apparatus \mathcal{A} , which is in an initial state of $|\psi_{\text{ready}}^{\mathcal{A}}\rangle$. After \mathcal{A} reads spin-up or spin-down then it is in state $|\psi_{\text{reads spin } \uparrow}^{\mathcal{A}}\rangle$ or $|\psi_{\text{reads spin } \downarrow}^{\mathcal{A}}\rangle$, respectively. The evolution for when the electron is solely spin-up or spin-down is

$$\begin{aligned} |\phi_\uparrow\rangle \otimes |\psi_{\text{ready}}^{\mathcal{A}}\rangle &\xrightarrow{\text{unitary}} |\phi_{\text{absorbed}}\rangle \otimes |\psi_{\text{reads spin } \uparrow}^{\mathcal{A}}\rangle \\ |\phi_\downarrow\rangle \otimes |\psi_{\text{ready}}^{\mathcal{A}}\rangle &\xrightarrow{\text{unitary}} |\phi_{\text{absorbed}}\rangle \otimes |\psi_{\text{reads spin } \downarrow}^{\mathcal{A}}\rangle. \end{aligned}$$

Furthermore, one can model the entire quantum state of an observer \mathcal{O} of the apparatus, with

$$\begin{aligned} &|\phi_\uparrow\rangle \otimes |\psi_{\text{ready}}^{\mathcal{A}}\rangle \otimes |\xi_{\text{ready}}^{\mathcal{O}}\rangle \\ &\xrightarrow{\text{unitary}} |\phi_{\text{absorbed}}\rangle \otimes |\psi_{\text{reads spin } \uparrow}^{\mathcal{A}}\rangle \otimes |\xi_{\text{ready}}^{\mathcal{O}}\rangle \\ &\xrightarrow{\text{unitary}} |\phi_{\text{absorbed}}\rangle \otimes |\psi_{\text{reads spin } \uparrow}^{\mathcal{A}}\rangle \otimes |\xi_{\text{reads spin } \uparrow}^{\mathcal{O}}\rangle \\ &|\phi_\downarrow\rangle \otimes |\psi_{\text{ready}}^{\mathcal{A}}\rangle \otimes |\xi_{\text{ready}}^{\mathcal{O}}\rangle \\ &\xrightarrow{\text{unitary}} |\phi_{\text{absorbed}}\rangle \otimes |\psi_{\text{reads spin } \downarrow}^{\mathcal{A}}\rangle \otimes |\xi_{\text{ready}}^{\mathcal{O}}\rangle \\ &\xrightarrow{\text{unitary}} |\phi_{\text{absorbed}}\rangle \otimes |\psi_{\text{reads spin } \downarrow}^{\mathcal{A}}\rangle \otimes |\xi_{\text{reads spin } \downarrow}^{\mathcal{O}}\rangle. \end{aligned}$$

For the general case, the electron is in a state $|\phi\rangle = a|\phi_\uparrow\rangle + b|\phi_\downarrow\rangle$, where $|a|^2 + |b|^2 = 1$. In this case, the final superposition would be of the form:

$$\begin{aligned} &a|\phi_{\text{absorbed}}\rangle \otimes |\psi_{\text{reads spin } \uparrow}^{\mathcal{A}}\rangle \otimes |\xi_{\text{reads spin } \uparrow}^{\mathcal{O}}\rangle \\ &+ b|\phi_{\text{absorbed}}\rangle \otimes |\psi_{\text{reads spin } \downarrow}^{\mathcal{A}}\rangle \otimes |\xi_{\text{reads spin } \downarrow}^{\mathcal{O}}\rangle. \end{aligned}$$

This is a superposition of two branches, each of which describes a perfectly reasonable physical story. This bifurcation is one method on how the quantum state of universe bifurcates into two branches.

Deriving the Born Rule

In my opinion, one of the glaring problem of **MWT** is its reconciliation of the Born rule, for which no proposed solution has universal consensus. In standard quantum mechanics, measurements are probabilistic operations. Measurements on a state vector $|\psi\rangle$, which is a unit vector over Hilbert space \mathcal{H} , are self-adjoint operators \mathcal{O} on \mathcal{H} . Observables are real numbers that are the spectrum $\text{Sp}(\mathcal{O})$ of \mathcal{O} . A measurement outcome is a subset $E \subseteq \text{Sp}(\mathcal{O})$ with associated projector P_E on \mathcal{H} . Outcome E is observed on measurement of \mathcal{O} on $|\psi\rangle$ with probability $P(E) = \langle\psi|P_E|\psi\rangle$. This is known as the Born rule. After this measurement, the new state becomes $P_E|\psi\rangle/\sqrt{\langle\psi|P_E|\psi\rangle}$. This is known as the projection postulate.

However, the Born rule and the projection postulate are not assumed by **MWT**. The dynamics are totally deterministic. Each branch is equally real to the observers in it. To address these issues, Everett first derived a typicality-measure that weights each branch of a state's superposition. Assuming a set of desirable constraints, Everett derived the typicality-measure to be equal to the norm-squared of the coefficients of each branch, i.e. the Born probability of each branch. Everett then drew a distinction between typical branches that have high typicality-measure and exotic atypical branches of decreasing typicality-measure. For the repeated measurements of the spin of an electron $|\phi\rangle = a|\phi_\uparrow\rangle + b|\phi_\downarrow\rangle$, the relative frequencies of up and down spin measurements in a typical branch converge to $|a|^2$ and $|b|^2$, respectively. The notion of typicality can be extended to measurements with many observables.

In a more recent resolution to the relation between **MWT** and probability, Deutsch introduced a decision theoretic interpretation [Deu99] that obtains the Born rule from the non-probabilistic axioms of quantum theory and non-probabilistic axioms of decision theory. Deutsch proved that rational actors are compelled to adopt the Born rule as the probability measure associated with their available actions. This approach is highly controversial, as some critics say the idea has circular logic.

Another attempt uses subjective probability [Vai98]. The experimenter puts on a blindfold before he finishes performing the experiment. After he finishes the experiment, he has uncertainty about what world he is in. This uncertainty is the foundation of a probability measure over the measurements. However, the actual form of the probability measure needs to be postulated:

Probability Postulate. *An observer should set his subjective probability of the outcome of a quantum experiment in proportion to the total measure of existence of all worlds with that outcome.*

Whichever explanation of the Born rule one adopts, the following section shows there is an issue with **MWT** and **IP**. There exist branches of substantial Born probability where information leaks occurs.

Violating the Independence Postulate

In [Lev84, Lev13], the Independence Postulate, **IP**, was introduced:

Let $\alpha \in \{0,1\}^{\infty}$ be a sequence defined with an n -bit mathematical statement (e.g., in Peano Arithmetic or Set Theory), and a sequence $\beta \in \{0,1\}^{*\infty}$ can be located in the physical world with a k -bit instruction set (e.g., ip-address). Then $\mathbf{I}(\alpha : \beta) < k + n + c_{\text{IP}}$, for some small absolute constant c_{IP} .*

The **I** term is an information measure in Algorithmic Information Theory. For this blog, the information term we use is $\mathbf{I}(x : y) = \mathbf{K}(x) + \mathbf{K}(y) - \mathbf{K}(x, y)$, where **K** is the prefix-free Kolmogorov complexity. We can use this definition of **I** because we only deal with finite sequences.

IP can be violated in the following idealized Stern-Gerlach experiment measuring the spin $|\phi_\uparrow\rangle$ and $|\phi_\downarrow\rangle$ of N isolated electrons. We denote $|\phi_0\rangle$ for $|\phi_\uparrow\rangle$ and $|\phi_1\rangle$ for $|\phi_\downarrow\rangle$. The “address” (in the sense of **IP**) of this experiment is $< O(\log n)$. There is a measuring apparatus \mathcal{A} with initial state of $|\psi^{\mathcal{A}}\rangle$, and after reading N spins of N electrons, it is in the state $|\psi^{\mathcal{A}}[x]\rangle$, where $x \in \{0,1\}^N$, whose i th bit is 1 iff the i th measurement returns $|\phi_1\rangle$. The experiment evolves according to the following unitary transformation:

$$\bigotimes_{i=1}^N |\phi\rangle \otimes |\psi^A\rangle \otimes \xrightarrow{\text{unitary}} \sum_{a_1, \dots, a_N \in \{0,1\}^N} 2^{-N/2} \bigotimes_{i=1}^N |\phi_{a_i}\rangle \otimes |\psi^A[a_1 a_2 \dots a_n]\rangle.$$

If the bits returned are the first N bits of Chaitin's Omega, then a memory leak of size $n - O(\log n)$ has occurred. Thus

$$\text{Born-Probability}(\text{a memory leak of size } n - O(\log n) \text{ occurred}) \geq 2^{-n}.$$

Conclusion

There are multiple variations of **MWT** when it comes to consistency across universes. In one formulation, all universes conform to the same physical laws. In another model, each universe has its own laws, for example different values of gravity, etc. However, the experiment in the previous section shows that mathematics itself is different between universes, regardless of which model is used. In some universes, **IP** holds and there is no way to create information leaks. In other universes information leaks occur, and there are tasks where randomized algorithms fail but non-algorithmic physical methods succeeds. One such task is finding new axioms of mathematics. This was envisioned as a possibility by Gödel [Gö61], but there is a universal consensus of the impossibility of this task. Not any more! In addition, because information leaks are finite events, the Born probability of worlds containing them is not insignificant. In such worlds, **IP** cannot be formulated, and the foundations of Algorithmic Information Theory itself become detached from reality.

Formulated another way, let us suppose the Born probability is derived from the probability postulate. We have a “blindfolded mathematician” who performs the experiment described above. Before the mathematician takes off her blindfold, she states the Independence Postulate. By the probability postulate, with measure 2^{-n} over all worlds, there is a memory leak of size $n - O(\log n)$ and **IP** statement by the mathematician is in error.

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