Principle of Nonlocality and the Halting Sequence

Samuel Epstein

February 19, 2025

Theorem 1 *Let $(\mathcal{X},\mu)$ and $(\mathcal{Y},\nu)$ be computable measure spaces. Let $A:\mathbb{N}\to X$, $B:\mathbb{N}\to Y$ be injective functions with $\mathbf{I}(\langle A,B\rangle:\mathcal{H})<\infty$. For $s\in\mathbb{N}$, $m<s$, there exists 2^{s-m} indices $t<2^s$ with $\max\{\mathbf{G}_\mu(A(t)),\mathbf{G}_\nu(B(t))\}<-m+O(\log s)$.*

Theorem 2 *Let L be the Lebesgue measure over $\mathbb{R}$, $(\mathcal{X},\mu)$, $(\mathcal{Y},\nu)$ be non-atomic computable measure spaces with $U=\log\mu(\mathcal{X})=\log\nu(\mathcal{Y})$. Let $A:[0,1]\to\mathcal{X}$ and $B:[0,1]\to\mathcal{Y}$ be continuous. Let $\mathbf{I}(\langle A,B\rangle:\mathcal{H})<\infty$. There is a constant c with $L\{t\in[0,1]:\max\{\mathbf{G}_\mu(A(t)),\mathbf{G}_\nu(B(t))\}<U-n\}>2^{-n-\mathbf{K}(n)-c}$.*

Theorem 3 *Let $(\mathcal{X},\mu)$ and $(\mathcal{Y},\nu)$ be non-atomic computable measure spaces with $U=\log\mu(\mathcal{X})=\log\nu(\mathcal{Y})$. Let $(\mathcal{Z},\rho)$ be a non-atomic computable probability space. Let $A:\mathcal{Z}\to\mathcal{X}$ and $B:\mathcal{Z}\to\mathcal{Y}$ be continuous. Let $\mathbf{I}(\langle A,B\rangle:\mathcal{H})<\infty$. There is a constant c with $\rho\{\alpha:\max\{\mathbf{G}_\mu(A(\alpha)),\mathbf{G}_\nu(B(\alpha))\}<U-n\}>2^{-n-\mathbf{K}(n)-c}$.*

Principle of Nonlocality and the Halting Sequence

If one has access to the halting sequence, then information can pass between spacelike events.

Example

Given is two computable measure spaces, each being the Cantor space paired with the uniform measure λ. The two sampling methods, $A:\mathbb{N}\to\{0,1\}^\infty$ and $B:\mathbb{N}\to\{0,1\}^\infty$ are defined using a single random infinite sequence α with $\mathbf{I}(\alpha:\mathcal{H})<\infty$. The even bits of α are used to create an infinite list $\{\beta_i\}_{i=1}^\infty$ in the standard way. Furthermore, $A(i)=\beta_i$. In an identical fashion, the odd bits of α are used to define B. Thus $\mathbf{I}(\langle A,B\rangle:\mathcal{H})<\infty$.

Let $\mathbf{G}_\lambda(\beta)$ be the algorithmic entropy of a sequence β in the Cantor space with the uniform measure λ. By properties of universal tests, $\lambda\{\beta:\mathbf{G}_\lambda(\beta)<-n\}<2^{-n}$. Let b be a small positive constant. For all $c\in(0,1)$, as $s\to\infty$,

$$\begin{aligned}|\{t\in[1,2^s]:\mathbf{G}_\lambda(A(t))<-cs+b\log s\}| &< 2^{(1-c)s+b\log s}\\ |\{t\in[1,2^s]:\mathbf{G}_\lambda(B(t))<-cs+b\log s\}| &< 2^{(1-c)s+b\log s}.\end{aligned}$$

Furthermore, from Theorem 1,

$$|\{t\in[1,2^s]:\max\{\mathbf{G}_\lambda(A(t)),\mathbf{G}_\lambda(B(t))\}<-cs+b\log s\}|>2^{(1-c)s}.$$

Assume $\mathbf{G}_\lambda$ is computable, fix a rational $c\in(0,1)$, and let $s\to\infty$. Suppose one computes $\mathbf{G}_\lambda(A(t))$ for $t\in[1,2^s]$. One can compute at most $s^b2^{(1-c)s}$ indices t such that $\mathbf{G}_\lambda(A(t))<-cs+b\log s$. From Theorem 1, one know that there is a subset T of those indices, where $|T|>2^{(1-c)s}$ and for each $t\in T$, $\mathbf{G}_\lambda(B(t))<-cs+b\log s$. Thus by knowing the $\mathbf{G}$ values of sequences in the range of A, one knows information about the $\mathbf{G}_\lambda$ values in the range of B.