Principle of Nonlocality and the Halting Sequence

Samuel Epstein

February 24, 2025

Theorem 1 *Let $(\mathcal{X},\mu)$ and $(\mathcal{Y},\nu)$ be non-atomic computable measure spaces with $U = \log\mu(\mathcal{X}) = \log\nu(\mathcal{Y})$. Let $(\mathcal{Z},\rho)$ be a non-atomic computable probability space. Let $A:\mathcal{Z}\to\mathcal{X}$ and $B:\mathcal{Z}\to\mathcal{Y}$ be continuous. Let $\mathbf{I}(\langle A,B\rangle:\mathcal{H})<\infty$. There is a constant c with $\rho\{\alpha:\max\{\mathbf{G}_\mu(A(\alpha)),\mathbf{G}_\nu(B(\alpha))\}<U-n\}>2^{-n-\mathbf{K}(n)-c}$.*

Principle of Nonlocality and the Halting Sequence

If one has access to the halting sequence, then information can pass between spacelike events.

Example

Let $(\mathcal{X},\mu)$ and $(\mathcal{Y},\nu)$ be computable measure spaces and $(\mathcal{Z},\rho)$ be a computable probability space. Let $A:\mathcal{Z}\to\mathcal{X}$ and $\mathcal{Z}\to\mathcal{Y}$ be computable functions. Let $\{X_n,Y_n\}_{n=1}^\infty$ be random subsets of $\mathcal{X}$ and $\mathcal{Y}$ of size n that created from independently sampling $\mathcal{Z}$ with ρ and then applying A and B respectively. Let $X_n^m=\{\alpha\in X_n:\mathbf{G}_\mu(\alpha)<-m\}$ and $Y_n^m=\{\alpha\in Y_n:\mathbf{G}_\nu(\alpha)<-m\}$. Using Theorem 1, there exists a c where

$$\lim_{n\to\infty}\left|\{t:X_n(t)\in X_n^m\cap Y_n(t)\in Y_n^m\}\right|/n>2^{-m-\mathbf{K}(m)-c}.$$

Assume $\mathbf{G}$ is computable, let $m\in\mathbb{N}$, and let $n\to\infty$. For each n, one can compute X_n^m and using Theorem 1, one can infer that $\left|\{t:X_n(t)\in X_n^m\cap Y_n(t)\in Y_n^m\}\right|/n>2^{-m-\mathbf{K}(m)-c}$. Thus with access to the halting sequence, one can learn information across spacelike events.

Using a slight modification of the max entropy theorem, one gets another interesting example. Given a source of energy which propagates at the speed of light to two distant systems that have a spacelike separation. Each pulse is sent according to a distribution. Each system changes according to a function of a pulse and the previous state. If the algorithmic entropy of one system is computable then this system can infer information about the algorithmic entropies of the other system. These two systems can even be in distant galaxies.