Max Entropy Theorems

Samuel Epstein

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Theorem 1 *Let* $(\mathcal{X}, \mu)$ *and* $(\mathcal{Y}, \nu)$ *be computable measure spaces. Let* $A : \mathbb{N} \to X$, $B : \mathbb{N} \to Y$ *be injective functions with* $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$. *For* $s \in \mathbb{N}$, $m < s$, *there exists* 2^{s-m} *indices* $t < 2^s$ *with* $\max\{\mathbf{G}_\mu(A(t)), \mathbf{G}_\nu(B(t))\} < -m + O(\log s)$.

Theorem 2 *Let* L *be the Lebesgue measure over* $\mathbb{R}$, $(\mathcal{X}, \mu)$, $(\mathcal{Y}, \nu)$ *be non-atomic computable measure spaces with* $U = \log \mu(\mathcal{X}) = \log \nu(\mathcal{Y})$. *Let* $A : [0, 1] \to \mathcal{X}$ *and* $B : [0, 1] \to \mathcal{Y}$ *be continuous. Let* $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$. *There is a constant* c *with* $L\{t \in [0, 1] : \max\{\mathbf{G}_\mu(A(t)), \mathbf{G}_\nu(B(t))\} < U - n\} > 2^{-n-\mathbf{K}(n)-c}$.

Theorem 3 *Let* $(\mathcal{X}, \mu)$ *and* $(\mathcal{Y}, \nu)$ *be non-atomic computable measure spaces with* $U = \log \mu(\mathcal{X}) = \log \nu(\mathcal{Y})$. *Let* $(\mathcal{Z}, \rho)$ *be a non-atomic computable probability space. Let* $A : \mathcal{Z} \to \mathcal{X}$ *and* $B : \mathcal{Z} \to \mathcal{Y}$ *be continuous. Let* $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$. *There is a constant* c *with* $\rho\{\alpha : \max\{\mathbf{G}_\mu(A(\alpha)), \mathbf{G}_\nu(B(\alpha))\} < U - n\} > 2^{-n-\mathbf{K}(n)-c}$.