Algorithmic Nonlocality

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Abstract

Locality inspired both Maxwell and Einstein in their formulations of electromagnetism and relativity. In this paper, we review our recent discovery of three nonlocality theorems in thermodynamics. Thermodynamics is dynamically nonlocal, in that two systems with space-like separations evolving over time cannot have synchronized algorithmic entropies. Thermodynamics is uniformly nonlocal, implying that algorithms with oracle access to the halting problem can infer non-trivial information about the algorithmic entropies of systems across space-like separations. Thermodynamics has ergodic nonlocality. This means that typical states of ergodic processes across spacelike separations are not synchronized.

1 Introduction

This paper starts with a survey of traditional notions of causality and locality. This paper introduces the first non-trivial synthesis of special relativity and computer science, detailing the work in [Eps] which contains three theorems proving that *classical physics itself is nonlocal*. Thus, the notions of local causation and locality detailed in Sections 2 and 3 can no longer be applied to classical physics.

With *dynamical nonlocality*, we will detail how algorithmic entropy, a semi-computable definition of thermodynamics, is nonlocal. All closed and isolated systems evolving over time throughout the universe have algorithmic entropies that are not synchronized. With *uniform nonlocality*, there exists an algorithm, when given access to the halting sequence, can infer the algorithmic entropy scores of systems with space-like separations. With *ergodic nonlocality*, all typical states of ergodic processes cannot be synchronized.

2 Causation and Locality

In this section, we review the historical view of causation in classical physics. Generally, causation has been assumed to be a local phenomenon, where an object at one location affects only its neighboring objects. By the end of this paper, we will show that three new nonlocal theorems implies that traditional notions of causation can no longer be applied to classical physics. We start with looking back to 1739, where Hume, in "Treatise of Human Nature" [Hum78], stated:

> *The idea then, of causation must be derived from some relation among objects; and that relation we must now endeavor to discover. I find in the first place, that whatever objects are considered as causes or effects, are contiguous and that nothing can operate in a time or place which is ever so little removed from those of its existence. Though distant objects may sometimes seem productive of each other, they are commonly found upon examination to be linked by a chain of causes, which are contiguous by themselves,*
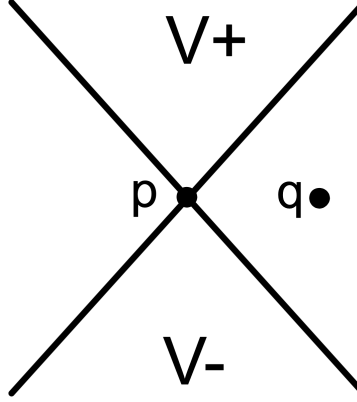
Figure 1: The forward cone V^+ represents the region in spacetime on which the event p can have a causal influence on. The backward cone V^- represents the region in spacetime in which events could influence p. The events p and q have a space-like separation and thus have no causal influence on one another.

and to the distant objects; and when in any particular instance we cannot discover this connection; we still presume it to exist. We may therefore consider the relation of contiguouty as essential to that of causation.

Thus Hume assigned causation a contiguous nature. This gives rise to the notion of *locality*, which is discussed in [Haa92] to have the following properties.

The German term "Nahwirkungsprinzip" is more impressive than the somewhat colourless word "locality". Certainly the idea behind these words, proposed by Faraday around 1830, initiated the most significant conceptual advance in physics after Newton's Principia. It guided Maxwell in his formulation of the law of electrodynamics, was sharpened by Einstein in the the theory of special relativity and again it was the strict adherence to this idea which led Einstein ultimately to his theory of gravitation, the general theory of relativity.

Towards this end, the notion of fields was introduced. Each point of a field represents a random variable, which is a function of its neighboring points. In electromagnetic theory, they are vectors representing electric and magnetic strength. In special relativity, Einstein introduced the general principle that propagation of effects has a limiting velocity of the speed of light.

In special relativity a point p in spacetime segments spacetime into three regions, as shown in Figure 1. The forward cone V^+ contains all events which can be causally influenced by p, a backward cone V^- from which all events from which an influence on p. The compliment of $V^+ \cup V^-$ is the set of all events q which have space-like distance from p. With respect to one frame of reference, p occurred before q. With respect to another frame of reference, q occurred before p. Thus no causal relationship can be assigned to the two points. In 1990, Bell gave the following precise definition of *local causality*, [Bel90], with the following axioms.

1. The direct causes (and effects) of events are near by, and even the indirect causes (and effects) are no further away than permitted by the velocity of light.

2. A theory will be said to be locally causal if the probabilities attached to values of local event $b1$ is unaltered by specification of values of space-like separated event $b2$, when what
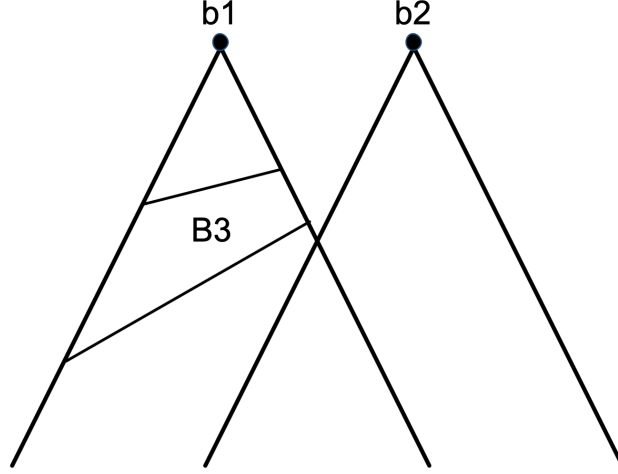
Figure 2: Local causality states that event $b1$ is independent of $b2$ conditioned on the events in region $B3$.

happens in the backward light cone of $b1$ is already sufficiently specified, for example by a full specification of local events in a spacetime region $B3$, with $P(b1|b2, B3) = P(b1|B3)$. This can be seen in Figure 2.

2.1 Reichenbach's Common Cause Principle

Let events A and B be positively correlated with

$$p(A \cap B) > p(A)p(B).$$

Also, let us assume that neither event is a cause of each other. In this case, we get the following statement:

Reichenbach's Common Cause Principle (RCCP)

Events A and B have a common cause that renders them conditionally independent.

Reichenbauch used this principle [Rei99] to formulate a new theory of causation and used this principle to formulate a macroscale theory of the irreversibility of time analogously to the second law of thermodynamics. Reichenbach incorporated his RCCP into a new probablistic theory of causation, and used it to describe a (purported) macrostatistical temporal asymmetry in analogy with the second law of thermodynamics. It has widely believed that RCCP applies to classical physics but not quantum physics. This paper details why this is not true; RCCP cannot be applied to classical physics.

3 Nonlocality of Quantum Mechanics

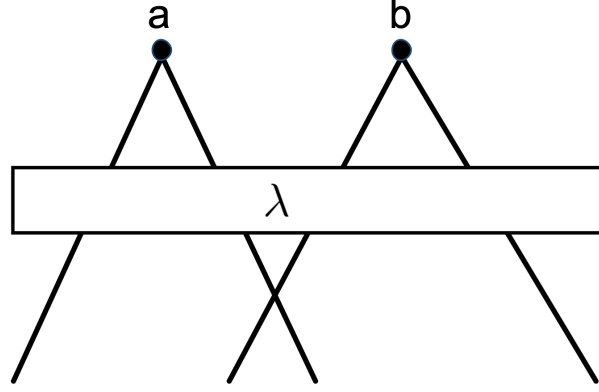In their famous paper [EPR35] Einstein, A., Podolsky, B., and Rosen gave criteria for *completeness* of a theory.

Figure 3: This figure shows how local causation implies that measurements can be put in a form congruent with Bell's hidden variable model.

> *Whatever the meaning assigned to the term complete, the following requirement for a complete theory seems to be a reasonable one: every element of the physical reality must have a counterpart in the physical theory. We shall call this the condition of completeness. Physical reality is defined as such: "If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity".*

However, due to the Heisenberg uncertainty principle, a particle cannot have a precisely defined position and momentum on an x axis. Thus, since position cannot have simultaneous defined values, Einstein et al. declared that quantum mechanics is incomplete. Put another way, either quantum theory is incomplete or there can be no simultaneously real values for incompatible quantities. Thus Einstein et al. conjectured the existence of so-,called "hidden variables" to supplement Scrödinger's wave function. With these variables, one could potentially create a complete theory of quantum mechanics. In particular, they were looking for a complete, locally causal theory of quantum mechanics.

However, these hopes were dashed by Bell's inequality [Bel66]. In this paper, a locally causal hidden variable model was proposed, called Bell locality. With this definition, every correlation between space-like events has a local explanation in that there is a common source of particles which has affected both sites. Let λ be the hidden parameter. A pair of measurements. (a_1, a_2) is Bell local if it is factorable in the following way

$$P(a_1, a_2) = \int P_1(a_1|\lambda)P_2(a_2|\lambda)dP(\lambda). \tag{1}$$

Bell's notion of local causation, defined in Section 2 implies all measurements can be put in the form of Equation 1. This can be seen in Figure 3. In [Bel66], it was shown that there are measurements in quantum mechanics that cannot be put the form of Equation 1. In addition, the 2022 Nobel Prize was awarded to Alain Aspect, John Clauser, and Anton Zeilinger, who experimentally confirmed a violation of these inequalities. Thus quantum mechanics is nonlocal and incomplete. Einstein never made his peace with this fact. In a letter to Born, dated March 3, 1947, he states

> *I cannot make a case for my attitude in physics which you would consider at all reasonable. I admit, of course, that there is considerable amount of validity in the statistical*

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theory which you were the first to recognize clearly as necessary, given the framework of the existing formalism. I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance.

Bell's inequalities have shown that RCCP (Section 2.1) captures assumptions about the behavior of classical systems that are violated in quantum mechanics. Unfortunately, this paper will add to Einstein's troubles. Not only is quantum information nonlocal, but classical physics itself is also not local. In particular, we show that a new thermodynamic definition, *algorithmic entropy* is nonlocal.

4 Thermodynamics

Thermodynamics deals with the amount of disorder, or *entropy* of systems. Each system is associated with a phase space Ω and a volume measure L. There is a measure preserving discrete time dynamics D^t. In the case of a finite rectangular container contains n particles, the state space is $6n$ dimensional. Each particle specifies six real numbers: three numbers for the position and three numbers for the velocity. Thermodynamic systems are defined by a finite number of macroscopic parameters. Some examples are volume, temperature, energy, and pressure. Systems are assumed to be in equilibrium.

One partitions the phase space Ω into cells by discretizing the macroscopic parameters. This induces irreversibility of dynamics when a state $s \in S_1$ in an astronomically small cell moves to a state $D^t(s) \in S_2$ with a large cell. The probability of the particle moving back from S_2 to S_1 is so infinitesimal that one would need to wait for the end of universe for this to occur. Boltzmann associates with every macro state the Boltzmann entropy:

$$S(M) = k \log L(M).$$

where k is Boltzmann's constant and $L(M)$ is the volume of the macro state.

When the system is at equilibrium, Boltzmann entropy agrees with the macroscopic thermodynamic entropy of Rudolf Clausius. We can expect to see unusual events such as gases unseparating themselves, only if one waits for inconceivable amount of times.

4.1 Algorithmic Entropy

Algorithmic entropy was introduced in [Gac94] and refined in [Eps] as a modern update to Boltzmann entropy. What is required of this definition is that there is a maximum saturation value where states with this score are maximally entropic. Lower scores represent some degree of non-randomness. To this end, it makes sense to define algorithmic entropy as the negative logarithm of an L-test t. Thus, this $t : \Omega \to \mathbb{R}_{\geq 0} \cup \{\infty\}$ is a lower-semicontinuous function such that $\int_\Omega t(\alpha)dL(\alpha) \leq 1$. In the selection of t, it makes sense to choose one that is universal among a class of tests $\mathcal{T}$, where $t \in \mathcal{T}$ and for each $t' \in \mathcal{T}$, there is a c where $ct > t'$.

Since tests are lower-semicontinuous, we will choose the set of all *lower-computable* tests. A function $f : \mathbb{N} \to \mathbb{N} \cup \{\infty\}$ is lower-computable if there is an algorithm $A : \mathbb{N} \times \mathbb{N} \to \mathbb{N} \cup \{\infty\}$ such that $A(x,1) = 1$, $A(x,y) \leq A(x, y+1)$, and $\lim_{y \to \infty} A(x,y) = f(x)$. There are functions that are lower-computable but not computable. One example is a universal lower-computable test s with respect to a probability p over natural numbers $\mathbb{N}$. The algorithm for s can enumerate all lower-computable p tests $\{s_i\}$ for p in the standard way in algorithmic information theory (see

[LO97]). Then one can use the following definition, with

$$s(a) = \sum_{i=1}^{\infty} \frac{1}{2^i} s_i(a).$$

This function s is not computable, but it is lower-computable. Similarly, one can define a universal lower-computable test $\mathbf{t}$ over Ω. A simple function f over Ω assigns a positive rational value to a point in a basic (ball) open set and zero to all points outside of this ball. A lower-computable L-test t has $\int_{\Omega} t(\alpha) dL(\alpha) \leq 1$ and is the supremum of a set of simple functions. Using advanced techniques detailed in [Gź1, HR09], one can prove the existence a universal lower-computable L test $\mathbf{t}$, such that for each lower-computable L test t, there is a $c \in \mathbb{R}_{\geq 0}$ where for all $\alpha \in \Omega$, $\mathbf{t}(\alpha) \geq ct(\alpha)$. From this test, we get the following definition for algorithmic entropy.

Definition (Algorithmic Entropy) $\mathbf{G}_L(\alpha) = \lfloor -\log \mathbf{t}(\alpha) \rfloor$.

Algorithmic entropy has a max positive value of $\log L(\Omega) + O(\log \log L(\Omega))$. Algorithmic entropy can take arbitrary negative values, even negative infinity. Algorithmic entropy has many interesting properties, including that it will oscillate in a very balance manner [Eps] under any combination of physical dynamics and systems. However, this paper will focus on the nonlocality of algorithmic entropy. Another important definition is entropy information.

Definition (Entropy Information) $\mathbf{I}_{L_1, L_2}(\alpha, \beta) = \mathbf{G}_{L_1}(\alpha) + \mathbf{G}_{L_2}(\beta) - \mathbf{G}_{L_1 \times L_2}(\alpha, \beta)$.

5 Computability of Terms

The phase spaces, measures, dynamics, and random functions between spaces are assumed to be *computable* in that there is an algorithm that can compute the measures and transformations. Computability for a continuous phase space and Borel measure is not a trivial definition, and its properties are derived in [HR09] and slightly refined in [Eps].

In fact, one can go one step further and only require that the some constructs in the theorem only have small *mutual information with the halting sequence*. The halting sequence $\mathcal{H}$ is infinite list of 1's and 0's where $\mathcal{H}[i] = 1$ if and only if the ith program input causes a universal algorithm to halt. There is no algorithm that can compute $\mathcal{H}$. When we say "mutual information", we mean an *algorithmic definition* which is the following function $\mathbf{I} : \{0,1\}^\infty \times \{0,1\}^\infty \to \mathbb{R}_{\geq 0}$, with [Lev74]

Definition (Information) $\mathbf{I}(\alpha : \beta) = \log \sum_{x,y \in \mathbb{N}} 2^{\mathbf{i}(x:y) - \mathbf{K}(x|\alpha) - \mathbf{K}(y|\beta)}$.

The term $\mathbf{K}(\cdot|\cdot)$ is the conditional Kolmogorov complexity and information is $\mathbf{i}(x : y) = \mathbf{K}(x) + \mathbf{K}(y) - \mathbf{K}(x, y)$. As infinite sequences become more alike, their $\mathbf{I}$ scores will increase. For example, for strings $x, y \in \{0,1\}^*$, $\mathbf{I}(\langle x \rangle 0^\infty : \langle y \rangle 0^\infty) = \mathbf{i}(x : y) + O(1)$. For a random sequence $\alpha \in \{0,1\}^\infty$, $\mathbf{I}(x\alpha : y\alpha) = \infty$. For more information about $\mathbf{I}$, we refer readers to [Eps].

For some terms, computability is not required, only small mutual information with the halting sequence is needed. The constructs D (such as dynamics) are not themselves infinite sequences, but one can easily define $\mathbf{I}(D : \mathcal{H})$ to be the infimum of the mutual information of $\mathcal{H}$ with every sequence in the set of all possible encodings of D.

6 The Fundamental Independence Axioms of Physics

This section details two independence postulates in physics, **IP1** and **IP2**. **IP1** is from [Lev84, Lev13] and **IP2** is a new contribution. Both axioms upper bound the independence of constructs by their addresses. Section 9 shows how the axioms, combined with theorems about **G** will lead to provable nonlocality properties. Section 10 shows that the postulates contradict notions of causation and locality discussed in Sections 2 and 3.

6.1 First Independence Postulate

The first independence postulate, **IP1**, is defined in the context of the celebrated Church Turing thesis (**CT**). **CT** relates mechanical methods to functions computed from Turing machines. A method, M, for achieving some desired result is "effective" or "mechanical" if it can be carried out by a human with a pencil and paper. The Church-Turing thesis states

CT: *A method is effective if and only if it can be computed by a Turing machine.*

One well known variant of **CT** is the physical Church-Turing thesis, which states *all physically computable functions are Turing-computable.* However there are several drawbacks associated with **CT**. The notion of an "effective method" is vague, admitting multiple different interpretations. On such early assessment of this fact can be found in [Kle52],

> *Since our original notion of effective calculability of a function . . . is a somewhat vague intuitive one, the thesis cannot be proved. . . . While we cannot prove Church's thesis, since its role is to delimit precisely an hitherto vaguely conceived totality, we require evidence.*

Turing himself had reservations about his thesis, [Tur36]

> *. . . fundamentally, appeals to intuition, and for this reason rather unsatisfactory mathematically.*

The first postulate, **IP1**, [Lev84, Lev13], is an unprovable inequality on the information measure of two sequences. Among other applications, **IP1** is a finitary Church Turing Thesis, postulating that certain infinite and *finite* sequences cannot be found in nature, a.k.a. have high "physical addresses". **IP1** provides a solution to the concerns of the somewhat vague formulation of **CT**. The statement of the **IP1** is as follows.

IP1*: With respect to inertial frame R, if α is a sequence defined with an n-bit mathematical statement, and a sequence β can be located in the physical world with a k-bit instruction set, then $\mathbf{I}(\alpha : \beta) < k + n + c_R$, with low expected c_R with respect to the uniform distribution over inertial frames.*

We take **I** to be the informations terms over finite or infinite sequences. The address system is physical in that the maximum velocity is the speed of light. One consequence of **IP1** is a finite version of **CT**. This advantage was mentioned by L. A. Levin in [Lev13],

> **IP1** *is simpler,* **CT** *more abstract. All sequences we ever see are computable just by being finite:* **CT** *is useless for them!* **IP1** *works equally well for finite and infinite sequences.*

IP1 says that the only finite sequences that can be found in nature (i.e. have short physical addresses) will have non-recursive descriptions that are equal in length to their recursive descriptions. This can be seen when **IP1** is applied to the case when $\alpha = \beta \in \{0,1\}^*$ is a finite sequence which has a non-recursive description of length $\mathbf{NR}(\alpha)$ that is much shorter than its recursive description $\mathbf{K}(\alpha)$, with $\mathbf{NR}(\alpha) \ll \mathbf{K}(\alpha)$. Let k be the shortest physical address of α. Then by **IP1**, with $\beta = \alpha$,

$$\mathbf{K}(\alpha) <^+ \mathbf{I}(\alpha : \alpha) <^+ k + \mathbf{NR}(\alpha) + c$$
$$\mathbf{K}(\alpha) - \mathbf{NR}(\alpha) - c <^+ k. \tag{2}$$

Thus k is large and α cannot be easily located in the physical world. The only sequences α with short physical addresses must have $\mathbf{NR}(\alpha) \approx \mathbf{K}(\alpha)$.

6.2 Second Independence Postulate

The second independence axiom pertains to algorithmic entropies. Imagine the following setup. Given is an empty box with no particles. It would be very easy to reproduce this state and have two such empty boxes. Now imagine a box full of particles at maximum algorithmic entropy. There exists no realistic way for someone to reproduce the exact contents of this box at high fidelity. This impossibility is generalized with the following assertion.

A system with two high entropy states that have joint low entropy is unphysical.

With this statement in mind, one gets the second independence postulate.

IP2: *With respect to inertial frame R, If system $(\Omega_1 \times \Omega_2, L_1 \times L_2)$ with state (α, β) can be reached with an address of size k, then $\mathbf{I}_{L_1,L_2}(\alpha : \beta) < k + c_R$, with low expected c_R with respect to the uniform distribution over inertial frames.*

6.3 Conservation Inequalities

IP1 and **IP2** are unprovable postulates, supported by provable arguments. In particular, conservation inequalities prevent deterministic or probabilistic processing to increase information between two targets. There is no way to increase information over sequences with respect to deterministic or random processing.

- For function f, $\mathbf{I}(f(x) : y) <^+ \mathbf{I}(x : y)$.

- Let g be a function that transforms a sequence β using a random seed ω. For uniform measure $\mathcal{U}$ over $\{0,1\}^\infty$, $\mathbf{E}_{\omega \sim \mathcal{U}}[\mathbf{I}(g(\beta, \omega) : \alpha)] <^+ \mathbf{I}(\beta : \alpha)$.

Similarly given two system states α and β, processing one state by a computable measure preserving function D cannot increase their information

$$\mathbf{I}_{L_1 \times L_2}(D\alpha : \beta) <^+ \mathbf{I}_{L_1 \times L_2}(\alpha : \beta).$$

7 Dynamical Nonlocality

In late 2023, the author discovered a new nonlocal property of physics, the first such achievement since [EPR35]. This discovery was incorporated into the *Algorithmic Physics* manuscript, [Eps].

Whats most striking is that this nonlocal property is a proven theorem statement about classical physics, namely over algorithmic entropy in the field of thermodynamics. It is difficult to imagine how Einstein would have received this work, as he spent decades refusing to accept quantum mechanics because of, in part, its nonlocal properties. For example, Einstein, in an article entitled "Physics and Reality" in 1936 in the Journal of the Franklin Institute, stated:

> There is no doubt that quantum mechanics has seized hold of a beautiful element of truth, and that it will be a test stone for any future theoretical basis...However, I do not believe that quantum mechanics will be the starting point in the search.

The nonlocality of classical physics can be described as *dynamical nonlocality*. Given is two closed and isolated systems located anywhere in the universe. One system can be in the Milky Way galaxy and another system can be in the Andromeda galaxy. Furthermore, one can assume that the systems have not received any common physical phenomena. The first of three nonlocal properties is as follows:

Dynamical Nonlocality: *For any such pair of systems undergoing dynamics, their algorithmic entropies are not synchronized.*

This statement is a proven fact. It is Theorem 110 in [Eps] The theorem for this is as follows.

Theorem 1 *For phase spaces and measures (Ω_1, L_1) and (Ω_2, L_2), discrete dynamics D_1^t and D_2^t, and physical states $\alpha_1 \in \Omega_1$, $\alpha_2 \in \Omega_2$, such that $\mathbf{G}_{L_1 \times L_2}(\alpha, \beta) > -\infty$ and $\mathbf{I}((\alpha, \beta) : \mathcal{H}) < \infty$, $\sup_t \left| \mathbf{G}_{L_1}(D_1^t(\alpha_1)) - \mathbf{G}_{L_1}(D_2^t(\alpha_2)) \right| = \infty$.*

States with infinite negative entropy cannot be reached in any manner with inertial frames R with finite c_R. Inertial frames with infinite c_R are pathological and have measure 0 with respect to the uniform distribution over all inertial frames. Thus there is no possible way to reach two states with dynamics that have synchronized entropies at any level

8   Uniform Nonlocality

For the second newly discovered nonlocality property of thermodynamics, *uniform nonlocality*, inference of information about algorithmic entropies of the states of systems in space-like separations is actually possible. However to do so, one needs access to the halting sequence, a physical impossibility. Uniform nonlocality implies a fascinating and deep connection between computability and special relativity.

Uniform Nonlocality: *If one has access to the halting sequence, then non-trivial information about the algorithmic entropy of systems with space-like separations can be inferred.*

Uniform nonlocality is a principle supported by the following result, which is Corollary 45 in [Eps].

Theorem 2 *For phase spaces and measures (Ω_1, L_1) and (Ω_2, L_2), probability space (Λ, λ), bicomputable injections $A : \Lambda \to \Omega_1$, $B : \Lambda \to \Omega_2$, there is a c where $\lambda\{\alpha : \max\{\mathbf{G}_{L_1}(A(\alpha)), \mathbf{G}_{L_2}(B(\alpha))\} < -n\} > 2^{-n-2\log n - c}$.*

The error term c is dependent on (Ω_1, L_1), (Ω_2, L_2), (A, B), and $\mathbf{I}((\Lambda, \lambda) : \mathcal{H})$. We describe an algorithm that, given the halting sequence, can infer the algorithmic entropies of states of systems

across space-like separations. Assume that one sample elements according to the probability space (Λ, λ) and sends the results at the speed of light to two systems $(\mathcal{X}, \mu)$ and $(\mathcal{Y}, \nu)$ in distant galaxies. Let $X_n^m = \{\alpha \in X_n : \mathbf{G}_\mu(\alpha) < -m\}$ and $Y_n^m = \{\alpha \in Y_n : \mathbf{G}_\nu(\alpha) < -m\}$. Using Theorem 2, there exists a c where

$$\lim_{n \to \infty} |\{t : X_n(t) \in X_n^m \cap Y_n(t) \in Y_n^m\}| / n > 2^{-m - 2\log m - c}.$$

Assume $\mathbf{G}$ is computable, let $m \in \mathbb{N}$, and let $n \to \infty$. For each n, one can compute X_n^m and using Theorem 2, one can infer that $|\{t : X_n(t) \in X_n^m \cap Y_n(t) \in Y_n^m\}| / n > 2^{-m - 2\log m - c}$. Thus, with access to the halting sequence, one can learn information across space-like events.

The properties of c implies that the probability space and the random functions can have conditional terms. With this in mind, one gets the following setup. Given a source of energy which propagates at the speed of light to a set of distant systems that all have spacelike separations. Each pulse is sent according to a distribution that is conditional to the previous pulse. Each system changes according to a function of this pulse. There is an algorithm that on input of the halting sequence and a large enough amount of pulses can output, with high probability, information about the algorithmic entropies of all other systems. As the number of pulses approaches infinity, the probability of learning information approaches unity. The rate of convergence not dependent on the conditional pulse distribution. The systems can be in distant galaxies.

9 Ergodic Nonlocality

In Section 7, it was shown that two systems undergoing measure-preserving dynamics cannot have entropies that are in sync. In this section, nonlocality with respect to ergodic processes is explored. Such processes are sufficiently mixing in nature, and used throughout physics. This section shows that it is very atypical for two ergodic systems to be in sync.

Theorem 3 *Given two phase spaces (Ω_1, L_1) and (Ω_2, L_2) and ergodic transforms T_1^t and T_2^t, for open sets $A \subset \mathcal{X}$ and $B \subset \mathcal{Y}$ and $(\alpha, \beta) \in (\Omega_1, \Omega_2)$, if for all $t \in \mathbb{N}$, $T_1^t(\alpha) \in A$ iff $T_2^t(\beta) \in B$, then $\mathbf{G}_{L_1 \times L_2}(\alpha, \beta) < \log L_1(A) + \mathbf{K}(A, B) + \mathbf{I}((A, \alpha, L_1(A)) : \mathcal{H})$.*

An example can be seen with black holes. Black holes are highly nonlocal because their small number implies small addresses. The phase space of black holes can be modeled with $SU(n)$, the space of special unitary transformations. Black holes evolve according to ergodic transformations. Thus, using Theorem 3, in the context of **IP1** and **IP2**, one can come to the following conclusion.

Two typical black holes cannot have synchronized states.

10 Provable Nonlocality

Assuming the postulates **IP1** and **IP2**, one can get a provable contradiction with the following notions of causality.

1. Hume's notion of contiguous causality.

2. Bell's notion of local causality.

3. Bell's notion of local hidden variable model.

4. Reichenbach's Common Cause Principle.

The reasoning for the contradiction with Reichenbach's Common Cause Principle is as follows. Take two Schwarzschild eternal black holes that no common causing factors. Take a particular set of inertial frames $\{R_t\}_{t=1}^{\infty}$, which share the same spatial point and are increasing in time in fixed intervals (say one second). As t approaches infinity, take the average of the states of the black holes. Due to Theorem 3, the limit probabilities of the states of the two black holes are correlated. However, due to the assumptions of the setup, there is no causing agent for the two black holes. This violates RCCP. Contradictions for the other definitions of causality follow from similar reasoning. The consequence of this is that classical physics does have causality. Note that this contradiction is valid if **IP1** and **IP2** is generalized to any finite expected c_R.

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11   Conclusion

This paper represents the first successful synthesis of special relativity and computer science. Whats remarkable about the dynamical nonlocality is that this is a requirement for all systems in the universe, no common information or physical phenomena is required. So, the question is what is the mechanism connecting to these systems that is enforcing this constraint? By examining Theorem 110 in [Eps], one realizes that systems that have synchronized algorithmic entropies either have negative infinite entropies or infinite mutual information with the halting sequence, where both properties are considered unphysical. Thus only in degenerate "non-algorithmic" universes can two systems be synchronized. What other properties can be deduced about such non-degenerate universes?

Another astonishing fact about Algorithmic Physics is that the halting sequence enables someone to glean information about the algorithmic entropies of distant systems. This is possible because upper-computable terms are heavily correlated when embedded in constructs that are either computable or have low mutual information with the halting sequence.

We conjecture that there exists a vast research field pertaining to the application of upper-computable entropy terms to physics. In fact, in [Eps], dynamical nonlocality is proved with coarse grained entropy and an additional two other nonequivalent upper-computable measures of entropy of a system state. It appears that upper-computable terms have profound properties that imply deep non-causal correlations throughout the universe. Another area of study could be into general relativity. Are there properties of tensor fields that can be combined with upper-computable terms?

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