Principle of Nonlocality and the Halting Sequence

Samuel Epstein

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Theorem 1 *Let $(\mathcal{X},\mu)$ and $(\mathcal{Y},\nu)$ be computable measure spaces. Let $A:\mathbb{N}\to X$, $B:\mathbb{N}\to Y$ be injective functions with $\mathbf{I}(\langle A,B\rangle:\mathcal{H})<\infty$. For $s\in\mathbb{N}$, $m<s$, there exists 2^{s-m} indices $t<2^s$ with* $\max\{\mathbf{G}_\mu(A(t)),\mathbf{G}_\nu(B(t))\}<-m+O(\log s)$.

Theorem 2 *Let L be the Lebesgue measure over $\mathbb{R}$, $(\mathcal{X},\mu)$, $(\mathcal{Y},\nu)$ be non-atomic computable measure spaces with $U=\log\mu(\mathcal{X})=\log\nu(\mathcal{Y})$. Let $A:[0,1]\to\mathcal{X}$ and $B:[0,1]\to\mathcal{Y}$ be continuous. Let $\mathbf{I}(\langle A,B\rangle:\mathcal{H})<\infty$. There is a constant c with $L\{t\in[0,1]:\max\{\mathbf{G}_\mu(A(t)),\mathbf{G}_\nu(B(t))\}<U-n\}>2^{-n-\mathbf{K}(n)-c}$.*

Theorem 3 *Let $(\mathcal{X},\mu)$ and $(\mathcal{Y},\nu)$ be non-atomic computable measure spaces with $U=\log\mu(\mathcal{X})=\log\nu(\mathcal{Y})$. Let $(\mathcal{Z},\rho)$ be a non-atomic computable probability space. Let $A:\mathcal{Z}\to\mathcal{X}$ and $B:\mathcal{Z}\to\mathcal{Y}$ be continuous. Let $\mathbf{I}(\langle A,B\rangle:\mathcal{H})<\infty$. There is a constant c with $\rho\{\alpha:\max\{\mathbf{G}_\mu(A(\alpha)),\mathbf{G}_\nu(B(\alpha))\}<U-n\}>2^{-n-\mathbf{K}(n)-c}$.*

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If one has access to the halting sequence, then information can pass between spacelike events.

Discrete Example

Given is two computable measure spaces, each being the Cantor space paired with the uniform measure λ. The two sampling methods, $A:\mathbb{N}\to\{0,1\}^\infty$ and $B:\mathbb{N}\to\{0,1\}^\infty$ are defined using a single random infinite sequence α with $\mathbf{I}(\alpha:\mathcal{H})<\infty$. The even bits of α are used to create an infinite list $\{\beta_i\}_{i=1}^\infty$ in the standard way. Furthermore, $A(i)=\beta_i$. In an identical fashion, the odd bits of α are used to define B. Thus $\mathbf{I}(\langle A,B\rangle:\mathcal{H})<\infty$.

Let $\mathbf{G}_\lambda(\beta)$ be the algorithmic entropy of a sequence β in the Cantor space with the uniform measure λ. By properties of universal tests, $\lambda\{\beta:\mathbf{G}_\lambda(\beta)<-n\}<2^{-n}$. Let b be a small positive constant. For all $c\in(0,1)$, as $s\to\infty$,

$$|\{t\in[1,2^s]:\mathbf{G}_\lambda(A(t))<-cs+b\log s\}|<2^{(1-c)s+b\log s}$$
$$|\{t\in[1,2^s]:\mathbf{G}_\lambda(B(t))<-cs+b\log s\}|<2^{(1-c)s+b\log s}.$$

Furthermore, from Theorem 1,

$$|\{t\in[1,2^s]:\max\{\mathbf{G}_\lambda(A(t)),\mathbf{G}_\lambda(B(t))\}<-cs+b\log s\}|>2^{(1-c)s}.$$

Assume $\mathbf{G}_\lambda$ is computable, fix a rational $c\in(0,1)$, and let $s\to\infty$. Suppose one computes $\mathbf{G}_\lambda(A(t))$ for $t\in[1,2^s]$. One can compute at most $s^b2^{(1-c)s}$ indices t such that $\mathbf{G}_\lambda(A(t))<-cs+b\log s$. From Theorem 1, one know that there is a subset T of those indices, where $|T|>2^{(1-c)s}$ and for each $t\in T$, $\mathbf{G}_\lambda(B(t))<-cs+b\log s$. Thus by knowing the $\mathbf{G}$ values of sequences in the range of A, one knows information about the $\mathbf{G}_\lambda$ values in the range of B.

Continuous Example

Let $(\mathcal{X},\mu)$ and $(\mathcal{Y},\nu)$ be computable measure spaces and $(\mathcal{Z},\rho)$ be a computable probability space. Let $A:\mathcal{Z}\to\mathcal{X}$ and $\mathcal{Z}\to\mathcal{Y}$ be computable functions. Let $\{X_n,Y_n\}_{n=1}^\infty$ be random subsets of $\mathcal{X}$ and $\mathcal{Y}$ of size n that created from independently sampling $\mathcal{Z}$ with ρ and then applying A and B respectively. Let $X_n^m=\{\alpha\in X_n:\mathbf{G}_\mu(\alpha)<-m\}$ and $Y_n^m=\{\alpha\in Y_n:\mathbf{G}_\nu(\alpha)<-m\}$. Using Theorem 3, there exists a c where

$$\lim_{n\to\infty}\left|\{t:X_n(t)\in X_n^m\cap Y_n(t)\in Y_n^m\}\right|/n>2^{-m-2\log m-c}.$$

Assume $\mathbf{G}$ is computable, let $m\in\mathbb{N}$, and let $n\to\infty$. For each n, one can compute X_n^m and using Theorem 3, one can infer that $\left|\{t:X_n(t)\in X_n^m\cap Y_n(t)\in Y_n^m\}\right|/n>2^{-m-2\log m-c}$. Thus with access to the halting sequence, one can learn information across spacelike events.

Conclusion

Using the discrete max entropy theorem, one can get the following example. Given two spacelike systems, if one were to sample states of each system according to two independent computable probabilities, then knowing the algorithmic entropies of one system states will reveal information about the algorithmic entropies of the other system.

Using a slight modification of the continuous max entropy theorem, one gets another interesting example. Given a source of energy which propagates at the speed of light to two distant systems that have a spacelike separation. Each pulse is sent according to a distribution. Each system changes according to a function of a pulse and the previous state. If the algorithmic entropy of one system is computable then this system can infer information about the algorithmic entropies of the other system. These two systems can even be in distant galaxies.