# Principle of Uniform Nonlocality and the Halting Sequence

Samuel Epstein

February 25, 2025

**Theorem 1** *Let $(\mathcal{X},\mu)$, $(\mathcal{Y},\nu)$ be non-atomic computable measure spaces with $U=\log\mu(\mathcal{X})=\log\nu(\mathcal{Y})$. Let $\{\Pi_i\}$ and $\{\Gamma_i\}$ be infinite partitions over $\mathcal{X}$ and $\mathcal{Y}$ respectively. Let $w$ be a balanced computable probability over $\mathbb{N}$. There is a constant $c$ with $w\{m:\max\{\mathbf{G}_\mu(\Pi_m),\mathbf{G}_\nu(\Gamma_m)\}<U-n\}>2^{-n-\mathbf{K}(n)-c}$.*

**Theorem 2** *Let $(\mathcal{X},\mu)$ and $(\mathcal{Y},\nu)$ be non-atomic computable measure spaces with $U=\log\mu(\mathcal{X})=\log\nu(\mathcal{Y})$. Let $(\mathcal{Z},\rho)$ be a non-atomic computable probability space. Let $A:\mathcal{Z}\to\mathcal{X}$ and $B:\mathcal{Z}\to\mathcal{Y}$ be continuous. Let $\mathbf{I}(\langle A,B\rangle:\mathcal{H})<\infty$. There is a constant $c$ with $\rho\{\alpha:\max\{\mathbf{G}_\mu(A(\alpha)),\mathbf{G}_\nu(B(\alpha))\}<U-n\}>2^{-n-\mathbf{K}(n)-c}$.*

## Principle of Uniform Nonlocality and the Halting Sequence

*If one has access to the halting sequence, then information can pass between spacelike events.*

## Example

Let $(\mathcal{X},\mu)$ and $(\mathcal{Y},\nu)$ be computable measure spaces and $(\mathcal{Z},\rho)$ be a computable probability space. Let $A:\mathcal{Z}\to\mathcal{X}$ and $\mathcal{Z}\to\mathcal{Y}$ be computable functions. Let $\{X_n,Y_n\}_{n=1}^\infty$ be random subsets of $\mathcal{X}$ and $\mathcal{Y}$ of size $n$ that created from independently sampling $\mathcal{Z}$ with $\rho$ and then applying $A$ and $B$ respectively. Let $X_n^m=\{\alpha\in X_n:\mathbf{G}_\mu(\alpha)<-m\}$ and $Y_n^m=\{\alpha\in Y_n:\mathbf{G}_\nu(\alpha)<-m\}$. Using Theorem 2, there exists a $c$ where

$$\lim_{n\to\infty}\left|\{t:X_n(t)\in X_n^m\cap Y_n(t)\in Y_n^m\}\right|/n>2^{-m-\mathbf{K}(m)-c}.$$

Assume $\mathbf{G}$ is computable, let $m\in\mathbb{N}$, and let $n\to\infty$. For each $n$, one can compute $X_n^m$ and using Theorem 2, one can infer that $\left|\{t:X_n(t)\in X_n^m\cap Y_n(t)\in Y_n^m\}\right|/n>2^{-m-\mathbf{K}(m)-c}$. Thus with access to the halting sequence, one can learn information across spacelike events.

One gets the following example with coarse grained entropy. Given is a set $S$ of systems each with partitions. Given is a source of bits represented by a balanced probability $w$. The bits $b$ are sent to each system in $S$. Each system goes to a state in the partition cell indexed by $b$. Epvery system that makes enough (dependent on $w$ and $S$) coarse grained entropy measurements $M$ can compute $A(M)\in\{0,1\}^*$ information about the coarse grained entropy of every other system, where $A$ is an algorithm. These systems can be in distant galaxies.

Using a slight modification of the algorithmic entropy max entropy theorem, one gets another interesting example. Given a source of energy which propagates at the speed of light to a set of distant systems that all have spacelike separations. Each pulse is sent according to a distribution. Each system changes according to a function of a pulse and the previous state. There is an algorithm $A$ that each system can use such that given enough algorithmc entropy measurements $M$ can output information $A(M)\in\{0,1\}^*$ about the algorithmic entropy measurements of all the other systems. Again, these systems can be in distant galaxies.