

Max Entropy Theorems

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Theorem 1 (Epstein) *Let (\mathcal{X}, μ) and (\mathcal{Y}, ν) be computable measure spaces. Let $A : \mathbb{N} \rightarrow X$, $B : \mathbb{N} \rightarrow Y$ be injective functions with $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$. For $s \in \mathbb{N}$, $m < s$, there exists 2^{s-m} indices $t < 2^s$ with $\max\{\mathbf{G}_\mu(A(t)), \mathbf{G}_\nu(B(t))\} < -m + O(\log s)$.*

Theorem 2 (Epstein) *Let L be the Lebesgue measure over \mathbb{R} , (\mathcal{X}, μ) , (\mathcal{Y}, ν) be non-atomic computable measure spaces. Let $A : [0, 1] \rightarrow \mathcal{X}$ and $B : [0, 1] \rightarrow \mathcal{Y}$ be continuous. Let $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$. There is a constant c with $L\{t \in [0, 1] : \max\{\mathbf{G}_\mu(A(t)), \mathbf{G}_\nu(B(t))\} < \log \mu(X) - n\} > 2^{-n-\mathbf{K}(n)-c}$.*

Theorem 3 (Epstein) *Let (\mathcal{X}, μ) , (\mathcal{Y}, ν) , and (\mathcal{Z}, ρ) be non-atomic computable measure spaces. Let $A : \mathcal{Z} \rightarrow \mathcal{X}$ and $B : \mathcal{Z} \rightarrow \mathcal{Y}$ be continuous. Let $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$. There is a constant c with $\rho\{\alpha : \max\{\mathbf{G}_\mu(A(\alpha)), \mathbf{G}_\nu(B(\alpha))\} < \log \mu(X) - n\} > 2^{-n-\mathbf{K}(n)-c}$.*