AIT Blog

On Creating Pairs of Derandomization Theorems

Samuel Epstein*

November 14, 2022

I've uploaded a new paper to my site, with the intention of eventually uploading it to arXiv and submission for publication. The main contribution is a resource bounded EL Theorem and a general formula for resource bounded randomization, in the sense of [Eps22]. In this blog, I show an example of taking non-constructive probabilistic proofs and producing a pair of derandomization proofs, one that is resource free and one that is resource bounded.

A hypergraph is a pair J=(V,E) of vertices V and edges $E\subseteq \mathcal{P}(V)$. Thus each edge can connect ≥ 2 vertices. A hypergraph is k-uniform of the size |e|=k for all edges $e\in E$. A 2-uniform hypergraph is just a simple graph. A valid C-coloring of a hypergraph (V,E) is a mapping $f:V\to\{1,\ldots,C\}$ where every edge $e\in E$ is not monochromatic $|\{f(v):v\in e\}|>1$. The following classic result uses Lovasz Local Lemma.

Theorem. [Probabilistic Method] Let G = (V, E) be a k-regular hypergraph. If for each edge f, there are at most $2^{k-1}/e-1$ edges $h \in E$ such that $h \cap f \neq \emptyset$, then there exists a valid 2-coloring of G.

We can now use derandomization, in the sense of [Eps22], to produce bounds on the simpliest such 2-coloring of G.

Theorem. [Derandomization] Let G = (V, E) be a k-regular hypergraph. If, for each edge f, there are at most $2^{k-1}/e - 1$ edges $h \in E$ such that $h \cap f \neq \emptyset$, then there exists a valid 2-coloring x of G with

$$\mathbf{K}(x) <^{\log} \mathbf{K}(n,k) + ne/2^{k-1} + \mathbf{I}(G;\mathcal{H}).$$

The function **K** is the prefix free Kolmogorov complexity. $\mathbf{I}(G;\mathcal{H}) = \mathbf{K}(G) - \mathbf{K}(G|\mathcal{H})$ is the amount of asymmetric information the halting sequence $\mathcal{H} \in \{0,1\}^{\infty}$ has about the graph G. We can now use resource derandomization, introduced in http://www.jptheorygroup.org/doc/Resource.pdf, to achieve bounds for time-bounded Kolmogorov complexity $\mathbf{K}^t(x) = \min\{p : U(p) = x \text{ in } t(\|x\|) \text{ steps}\}.$

Theorem. [Resource Bounded Derandomization] Let $G_n = (V, E)$ be a k-regular hypergraph where ||V|| = n, uniformly polynomial time computable. Furthermore, for each edge f in G_n there are at most $2^{k-1}/e - 1$ edges $h \in E$ such that $h \cap f \neq \emptyset$. Then there is a polynomial p, and a valid 2-coloring x of G_n with

$$\mathbf{K}^p(x) <^{\log n} e/2^{k-1} + O(\log n).$$

^{*}JP Theory Group. samepst@jptheorygroup.org

References

[Eps22] S. Epstein. 22 examples of solution compression via derandomization. CoRR, abs/2208.11562, 2022.