Principle of Uniform Nonlocality and the Halting Sequence

Samuel Epstein

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Theorem 1 *Let $(\mathcal{X},\mu)$ and $(\mathcal{Y},\nu)$ be non-atomic computable measure spaces with $U=\log\mu(\mathcal{X})=\log\nu(\mathcal{Y})$. Let $(\mathcal{Z},\rho)$ be a non-atomic computable probability space. Let $A:\mathcal{Z}\to\mathcal{X}$ and $B:\mathcal{Z}\to\mathcal{Y}$ be continuous. Let $\mathbf{I}(\langle A,B\rangle:\mathcal{H})<\infty$. There is a constant c with $\rho\{\alpha:\max\{\mathbf{G}_\mu(A(\alpha)),\mathbf{G}_\nu(B(\alpha))\}<U-n\}>2^{-n-\mathbf{K}(n)-c}$.*

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If one has access to the halting sequence, then non-trivial information can be inferred between spacelike events.

Example

Let $(\mathcal{X},\mu)$ and $(\mathcal{Y},\nu)$ be computable measure spaces and $(\mathcal{Z},\rho)$ be a computable probability space. Let $A:\mathcal{Z}\to\mathcal{X}$ and $\mathcal{Z}\to\mathcal{Y}$ be computable functions. Let $\{X_n,Y_n\}_{n=1}^\infty$ be random subsets of $\mathcal{X}$ and $\mathcal{Y}$ of size n that created from independently sampling $\mathcal{Z}$ with ρ and then applying A and B respectively. Let $X_n^m=\{\alpha\in X_n:\mathbf{G}_\mu(\alpha)<-m\}$ and $Y_n^m=\{\alpha\in Y_n:\mathbf{G}_\nu(\alpha)<-m\}$. Using Theorem 1, there exists a c where

$$\lim_{n\to\infty}\left|\{t:X_n(t)\in X_n^m\cap Y_n(t)\in Y_n^m\}\right|/n>2^{-m-\mathbf{K}(m)-c}.$$

Assume $\mathbf{G}$ is computable, let $m\in\mathbb{N}$, and let $n\to\infty$. For each n, one can compute X_n^m and using Theorem 1, one can infer that $\left|\{t:X_n(t)\in X_n^m\cap Y_n(t)\in Y_n^m\}\right|/n>2^{-m-\mathbf{K}(m)-c}$. Thus with access to the halting sequence, one can learn information across spacelike events.

Using a slight modification of the max entropy theorem, one gets another interesting example. Given a source s of energy which propagates at the speed of light to a set S of distant systems that all have spacelike separations. Each pulse is sent according to a distribution. Each system changes according to a function of a pulse and the previous state. There is an algorithm that on input of the halting sequence and a large enough amount of pulses can output, with high probability, information about the algorithmic entropies of all other systems. As the number of pulses approaches infinity, the probability of learning information approaches unity. The systems can be in distant galaxies.