

# Principle of Nonlocality and the Halting Sequence

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**Theorem 1** *Let  $(\mathcal{X}, \mu)$ ,  $(\mathcal{Y}, \nu)$  be non-atomic computable measure spaces with  $U = \log \mu(\mathcal{X}) = \log \nu(\mathcal{Y})$ . Let  $\{\Pi_i\}$  and  $\{\Gamma_i\}$  be infinite partitions over  $\mathcal{X}$  and  $\mathcal{Y}$  respectively. Let  $w$  be a balanced computable probability over  $\mathbb{N}$ . There is a constant  $c$  with  $w\{m : \max\{\mathbf{G}_\mu(\Pi_m), \mathbf{G}_\nu(\Gamma_m)\} < U - n\} > 2^{-n-\mathbf{K}(n)-c}$ .*

**Theorem 2** *Let  $(\mathcal{X}, \mu)$  and  $(\mathcal{Y}, \nu)$  be non-atomic computable measure spaces with  $U = \log \mu(\mathcal{X}) = \log \nu(\mathcal{Y})$ . Let  $(\mathcal{Z}, \rho)$  be a non-atomic computable probability space. Let  $A : \mathcal{Z} \rightarrow \mathcal{X}$  and  $B : \mathcal{Z} \rightarrow \mathcal{Y}$  be continuous. Let  $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$ . There is a constant  $c$  with  $\rho\{\alpha : \max\{\mathbf{G}_\mu(A(\alpha)), \mathbf{G}_\nu(B(\alpha))\} < U - n\} > 2^{-n-\mathbf{K}(n)-c}$ .*

## Principle of Nonlocality and the Halting Sequence

*If one has access to the halting sequence, then information can pass between spacelike events.*

### Example

Let  $(\mathcal{X}, \mu)$  and  $(\mathcal{Y}, \nu)$  be computable measure spaces and  $(\mathcal{Z}, \rho)$  be a computable probability space. Let  $A : \mathcal{Z} \rightarrow \mathcal{X}$  and  $\mathcal{Z} \rightarrow \mathcal{Y}$  be computable functions. Let  $\{X_n, Y_n\}_{n=1}^\infty$  be random subsets of  $\mathcal{X}$  and  $\mathcal{Y}$  of size  $n$  that created from independently sampling  $\mathcal{Z}$  with  $\rho$  and then applying  $A$  and  $B$  respectively. Let  $X_n^m = \{\alpha \in X_n : \mathbf{G}_\mu(\alpha) < -m\}$  and  $Y_n^m = \{\alpha \in Y_n : \mathbf{G}_\nu(\alpha) < -m\}$ . Using Theorem 2, there exists a  $c$  where

$$\lim_{n \rightarrow \infty} |\{t : X_n(t) \in X_n^m \cap Y_n(t) \in Y_n^m\}| / n > 2^{-m-\mathbf{K}(m)-c}.$$

Assume  $\mathbf{G}$  is computable, let  $m \in \mathbb{N}$ , and let  $n \rightarrow \infty$ . For each  $n$ , one can compute  $X_n^m$  and using Theorem 2, one can infer that  $|\{t : X_n(t) \in X_n^m \cap Y_n(t) \in Y_n^m\}| / n > 2^{-m-\mathbf{K}(m)-c}$ . Thus with access to the halting sequence, one can learn information across spacelike events.

One gets the following example with coarse grained entropy. Given is a set  $S$  of systems each with partitions. Given is a source of bits represented by a balanced probability  $w$ . The bits  $b$  are sent to each system in  $S$ . Each system goes to a state in the partition cell indexed by  $b$ . Every system that makes enough (dependent on  $w$  and  $S$ ) coarse grained entropy measurements  $M$  can compute  $A(M) \in \{0, 1\}^*$  information about the coarse grained entropy of every other system, where  $A$  is an algorithm. These systems can be in distant galaxies.

Using a slight modification of the algorithmic entropy max entropy theorem, one gets another interesting example. Given a source of energy which propagates at the speed of light to a set of distant systems that all have spacelike separations. Each pulse is sent according to a distribution. Each system changes according to a function of a pulse and the previous state. There is an algorithm  $A$  that each system can use such that given enough algorithmic entropy measurements  $M$  can output information  $A(M) \in \{0, 1\}^*$  about the algorithmic entropy measurements of all the other systems. Again, these systems can be in distant galaxies.