

Principle of Nonlocality and the Halting Sequence

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Theorem 1 *Let (\mathcal{X}, μ) , (\mathcal{Y}, ν) be non-atomic computable measure spaces with $U = \log \mu(\mathcal{X}) = \log \nu(\mathcal{Y})$. Let $\{\Pi_i\}$ and $\{\Gamma_i\}$ be infinite partitions over \mathcal{X} and \mathcal{Y} respectively. Let w be a balanced computable probability over \mathbb{N} . There is a constant c with $w\{m : \max\{\mathbf{G}_\mu(\Pi_m), \mathbf{G}_\nu(\Gamma_m)\} < U - n\} > 2^{-n-\mathbf{K}(n)-c}$.*

Theorem 2 *Let (\mathcal{X}, μ) and (\mathcal{Y}, ν) be non-atomic computable measure spaces with $U = \log \mu(\mathcal{X}) = \log \nu(\mathcal{Y})$. Let (\mathcal{Z}, ρ) be a non-atomic computable probability space. Let $A : \mathcal{Z} \rightarrow \mathcal{X}$ and $B : \mathcal{Z} \rightarrow \mathcal{Y}$ be continuous. Let $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$. There is a constant c with $\rho\{\alpha : \max\{\mathbf{G}_\mu(A(\alpha)), \mathbf{G}_\nu(B(\alpha))\} < U - n\} > 2^{-n-\mathbf{K}(n)-c}$.*

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If one has access to the halting sequence, then information can pass between spacelike events.

Example

Let (\mathcal{X}, μ) and (\mathcal{Y}, ν) be computable measure spaces and (\mathcal{Z}, ρ) be a computable probability space. Let $A : \mathcal{Z} \rightarrow \mathcal{X}$ and $B : \mathcal{Z} \rightarrow \mathcal{Y}$ be computable functions. Let $\{X_n, Y_n\}_{n=1}^\infty$ be random subsets of \mathcal{X} and \mathcal{Y} of size n that created from independently sampling \mathcal{Z} with ρ and then applying A and B respectively. Let $X_n^m = \{\alpha \in X_n : \mathbf{G}_\mu(\alpha) < -m\}$ and $Y_n^m = \{\alpha \in Y_n : \mathbf{G}_\nu(\alpha) < -m\}$. Using Theorem 2, there exists a c where

$$\lim_{n \rightarrow \infty} |\{t : X_n(t) \in X_n^m \cap Y_n(t) \in Y_n^m\}| / n > 2^{-m-\mathbf{K}(m)-c}.$$

Assume \mathbf{G} is computable, let $m \in \mathbb{N}$, and let $n \rightarrow \infty$. For each n , one can compute X_n^m and using Theorem 2, one can infer that $|\{t : X_n(t) \in X_n^m \cap Y_n(t) \in Y_n^m\}| / n > 2^{-m-\mathbf{K}(m)-c}$. Thus with access to the halting sequence, one can learn information across spacelike events.

One gets the following example with coarse grained entropy. Given is a set S of systems each with partitions. Given is a source of bits represented by a balanced probability w . The bits b are sent to each system in S . Each system goes to a state in the partition cell indexed by b . There is a function f whose input is w and S and outputs a number such that every system that makes $f(S, w)$ coarse grained entropy measurements M can compute $A(M) \in \{0, 1\}^*$ information about the coarse grained entropy of every other system, where A is an algorithm. These systems can be in distant galaxies.

Using a slight modification of the algorithmic entropy max entropy theorem, one gets another interesting example. Given a source of energy which propagates at the speed of light to a set of distant systems that all have spacelike separations. Each pulse is sent according to a distribution. Each system changes according to a function of a pulse and the previous state. There is an algorithm A that each system can use such that given enough algorithmic entropy measurements M can output information $A(M) \in \{0, 1\}^*$ about the algorithmic entropy measurements of all the other systems. Again, these systems can be in distant galaxies.