Quantum Decoherence Mostly Results in White Noise

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April 3, 2023

Abstract

An overwhelming majority of quantum (pure and mixed) states, when undertaking decoherence, will result in a classical probability with no algorithmic information. Thus most quantum states decohere into white noise. This can be seen as a consequence of the vastness of Hilbert spaces.

Information non-growth laws say information about a target source cannot be increased with randomized processing. In classical information theory, we have [CT91]

$$I(X : g(Y)) \leq I(X : Y).$$

where g is a randomized function, X and Y are random variables, and I is the mutual information function. Thus processing a channel at its output will not increase its capacity. Information conservation carries over into the algorithmic domain, with the inequalities [Lev84, Eps22]

$$\mathbf{I}(f(x) : y) <^+ \mathbf{I}(x : y), \qquad \mathbf{I}(f(a); \mathcal{H}) <^+ \mathbf{I}(a; \mathcal{H}).$$

These inequalities ensure target information cannot be obtained by processing. If for example the second inequality was not true, then one can potentially obtain information about the halting sequence $\mathcal{H}$ with simple functions. Obtaining information about $\mathcal{H}$ violates the Independence Postulate, (see [Lev13]). Information non growth laws can be extended to signals [Eps23] which can be modeled as probabilities over $\mathbb{N}$ or Euclidean space[1]. The "signal strength" of a probability p over $\mathbb{N}$ is measured by its self information.

$$\mathbf{I}_{\mathrm{Prob}}(p : p) = \log \sum_{i,j} 2^{\mathbf{I}(i:j)} p(i)p(j).$$

A signal, when undergoing randomized processing f, will lose its cohesion[2]. Thus any signal going through a classical channel will become less coherent [Eps23].

$$\mathbf{I}_{\mathrm{Prob}}(f(p) : f(p)) <^+ \mathbf{I}_{\mathrm{Prob}}(p : p).$$

In Euclidean space, probabilities that undergo convolutions with probability kernels will lose self information. For example a signal spike at a random position will spread out when convoluted with the Gaussian function, and lose self information. The above inequalities deal with classical

[1] In [Eps23] probabilities over $\{0,1\}^\infty$ and T_0 second countable topologies were also studied.

[2] A probability p, when processed by a channel $f : \{0,1\}^* \times \{0,1\}^* \to \mathbb{R}_{\geq 0}$ is a new probability $fp(x) = \sum_z f(x|z)p(z)$.

transformations. One can ask, is whether, quantum information processing can add new surprises to how information signals occur and evolve.

One can start with the measure-and-prepare channel, also known as a Holevo-form channel. Alice starts with a random variable X that can take values $\{1,\dots,n\}$ with corresponding probabilities $\{p_1,\dots,p_n\}$. Alice prepares a quantum state, corresponding to density matrix ρ_X, chosen from $\{\rho_1,\dots,\rho_n\}$ according to X. Bob performs a measurement on the state ρ_X, getting a classical outcome, denoted by Y. Though it uses quantum mechanics, this is a classical channel $X \to Y$. So using the above inequality, cohesion will deteriorate regardless of $X's$ probability, with

$$\mathbf{I}_{\mathrm{Prob}}(Y:Y) <^+ \mathbf{I}_{\mathrm{Prob}}(X:X).$$

There remains a second option, constructing a signal directly from a mixed state. This involves constructing a mixed state, i.e. density matrix σ, and then performing a measurement E on the state. The probability is $q(k) = \mathrm{Tr}\sigma E_k$. However from [Eps23], for elementary (even enumerable) probabilities q,

$$\mathbf{I}_{\mathrm{Prob}}(q:q) <^+ \mathbf{K}(q).$$

Thus for simply defined density matrices and measurements, no signal will appear. So experiments that are simple will result in simple measurements, or white noise. However it could be that a larger number of uncomputable pure or mixed states produce coherent signals. However, theorems in [Eps] say otherwise, in that given a POVM measurement E, a vast majority of pure and mixed states will have negligible self-information. Thus for uniform distributions Λ and μ over pure and mixed states[3],

$$\int 2^{\mathbf{I}_{\mathrm{Prob}}(E|\psi\rangle:E|\psi\rangle)}d\Lambda = O(1), \qquad \int 2^{\mathbf{I}_{\mathrm{Prob}}(E\sigma:E\sigma)}d\mu(\sigma) = O(1).$$

This can be seen as a consequence of the vastness of Hilbert spaces as opposed to the limited discriminatory power of quantum measurements. In addition, there could be non-uniform distributions of pure or mixed states that could be of research interest. In quantum decoherence, a quantum state becomes entangled with the environment, losing decoherence. The off diagonal elements of the mixed state become dampened, as the state becomes more like a classical mixture of states. Let p_σ be the idealized classical probability that σ decoheres to, with $p_\sigma(i) = \sigma_{ii}$. The following theorem from [Eps] states that for an overwhelming majority of pure or mixed states σ, p_σ is noise, that is, has negligible self-information. Let Λ be the uniform distribution on the unit sphere of an n qubit space.

$$\int 2^{\mathbf{I}_{\mathrm{Prob}}\left(p_{|\psi\rangle}:p_{|\psi\rangle}\right)}d\Lambda = O(1) \qquad \int 2^{\mathbf{I}_{\mathrm{Prob}}(p_\sigma:p_\sigma)}d\mu(\sigma) = O(1).$$

To recap, information in the classical world obeys strong conservation laws, both probabilistically and algorithmically. This can be extended to probabilities over $\mathbb{N}$ and $\mathbb{R}^n$. Holevo-form channels cannot sidestep conservation laws. Simply prepared and measured states produce no strong signals, and an overwhelming majority of quantum states produce white noise with respect to a measurement. Quantum decoherence causes most states to devolve into white noise.

These results mark a continuing investigation of algorithmic information and physics. The results of this survey show that the boundary between the quantum world and the classical world is permeated by white noise, so how is information transmitted from the former to the latter?

[3]The mixed state integral is $\int f(\sigma)d\mu(\sigma) = \int_{\Delta_M}\int_{\Lambda_1}\cdots\int_{\Lambda_M} f\left(\sum_{i=1}^M p_i \left|\psi_i\right\rangle\left\langle\psi_i\right|\right) d\Lambda_1\dots d\Lambda_M d\eta(p_1,\dots,p_M)$, where η is any distribution over the M-simplex Δ_M.

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