

# Max Entropy Theorems

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**Theorem 1 (Epstein)** *Let  $(X \times Y, \mu \times \nu)$  be a product computable measure space. Let  $A : \mathbb{N} \rightarrow X$ ,  $B : \mathbb{N} \rightarrow Y$  be injective functions with  $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$ . For  $s \in \mathbb{N}$ ,  $m < s$ , there exists  $2^{s-m}$  indices  $t < 2^s$  with  $\max\{\mathbf{G}_\mu(A(t)), \mathbf{G}_\nu(B(t))\} < -m + O(\log s)$ .*

**Theorem 2 (Epstein)** *Let  $L$  be the Lebesgue measure over  $\mathbb{R}$ ,  $(\mathcal{X}, \mu)$ ,  $(\mathcal{Y}, \nu)$  be non-atomic computable measure spaces. Let  $A : [0, 1] \rightarrow \mathcal{X}$  and  $B : [0, 1] \rightarrow \mathcal{Y}$  be continuous. Let  $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$ . There is a constant  $c$  with  $L\{t \in [0, 1] : \max\{\mathbf{G}_\mu(A(t)), \mathbf{G}_\nu(B(t))\} < \log \mu(X) - n\} > 2^{-n-\mathbf{K}(n)-c}$ .*

**Theorem 3**  *$(\mathcal{X}, \mu)$ ,  $(\mathcal{Y}, \nu)$ , and  $(\mathcal{Z}, \rho)$  be non-atomic computable measure spaces. Let  $A : \mathcal{Z} \rightarrow \mathcal{X}$  and  $B : \mathcal{Z} \rightarrow \mathcal{Y}$  be continuous. Let  $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$ . There is a constant  $c$  with  $\rho\{\alpha : \max\{\mathbf{G}_\mu(A(\alpha)), \mathbf{G}_\nu(B(\alpha))\} < \log \mu(X) - n\} > 2^{-n-\mathbf{K}(n)-c}$ .*