Quantum Decoherence Mostly Results in White Noise

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Abstract

An overwhelming majority of quantum (pure and mixed) states, when undertaking decoherence, will result in a classical probability with no algorithmic information. Thus most quantum states decohere into white noise. This can be seen as a consequence of the vastness of Hilbert spaces.

Information non-growth laws say information about a target source cannot be increased with randomized processing. In classical information theory, we have [CT91]

$$I(g(X):Y) \leq I(X:Y).$$

where g is a randomized function, X and Y are random variables, and I is the mutual information function. Thus processing a channel at its output will not increase its capacity. Information conservation carries over into the algorithmic domain, with the inequalities [Lev84, Eps22]

$$\mathbf{I}(f(x):y) <^+ \mathbf{I}(x:y); \qquad \mathbf{I}(f(a);\mathcal{H}) <^+ \mathbf{I}(a;\mathcal{H}).$$

The information function is $\mathbf{I}(x:y) = \mathbf{K}(x)+\mathbf{K}(y)-\mathbf{K}(x,y)$, where $\mathbf{K}$ is Kolmogorov complexity. The other term is $\mathbf{I}(a;\mathcal{H}) = \mathbf{K}(a)-\mathbf{K}(a|\mathcal{H})$, where $\mathcal{H} \in \{0,1\}^\infty$ is the halting sequence. These inequalities ensure target information cannot be obtained by processing. If for example the second inequality was not true, then one can potentially obtain information about $\mathcal{H}$ with simple functions. Obtaining information about $\mathcal{H}$ violates the Independence Postulate, (see [Lev13]). Information non growth laws can be extended to signals [Eps23b] which can be modeled as probabilities over $\mathbb{N}$ or Euclidean space[1]. The "signal strength" of a probability p over $\mathbb{N}$ is measured by its self information.

$$\mathbf{I}_{\mathrm{Prob}}(p:p) = \log \sum_{i,j} 2^{\mathbf{I}(i:j)} p(i)p(j).$$

A signal, when undergoing randomized processing f, will lose its cohesion[2]. Thus any signal going through a classical channel will become less coherent [Eps23b].

$$\mathbf{I}_{\mathrm{Prob}}(f(p):f(p)) <^+ \mathbf{I}_{\mathrm{Prob}}(p:p).$$

In Euclidean space, probabilities that undergo convolutions with probability kernels will lose self information. For example a signal spike at a random position will spread out when convoluted with the Gaussian function, and lose self information. The above inequalities deal with classical

[1] In [Eps23b] probabilities over $\{0,1\}^\infty$ and T_0 second countable topologies were also studied.

[2] A probability p, when processed by a channel $f : \{0,1\}^* \times \{0,1\}^* \to \mathbb{R}_{\geq 0}$ is a new probability $fp(x) = \sum_z f(x|z)p(z)$.

transformations. One can ask, is whether, quantum information processing can add new surprises to how information signals occur and evolve.

One can start with the prepare-and-measure channel, also known as a Holevo-form channel. Alice starts with a random variable X that can take values $\{1,\dots,n\}$ with corresponding probabilities $\{p_1,\dots,p_n\}$. Alice prepares a quantum state, corresponding to density matrix ρ_X, chosen from $\{\rho_1,\dots,\rho_n\}$ according to X. Bob performs a measurement on the state ρ_X, getting a classical outcome, denoted by Y. Though it uses quantum mechanics, this is a classical channel $X \to Y$. So using the above inequality, cohesion will deteriorate regardless of $X's$ probability, with Th

$$\mathbf{I}_{\mathrm{Prob}}(Y:Y) <^+ \mathbf{I}_{\mathrm{Prob}}(X:X).$$

There remains a second option, constructing a signal directly from a mixed state. This involves constructing a mixed state, i.e. density matrix σ, and then performing a POVM measurement[3] E on the state, inducing the probability $E\sigma(\cdot)$. However from [Eps23b], for elementary (even enumerable) probabilities $E\sigma$,

$$\mathbf{I}_{\mathrm{Prob}}(E\sigma : E\sigma) <^+ \mathbf{K}(\sigma, E).$$

Thus for simply defined density matrices and measurements, no signal can appear. So experiments that are simple will result in simple measurements, or white noise. However it could be that a larger number of uncomputable pure or mixed states produce coherent signals. However, Theorems in [Eps23a] say otherwise, in that given a POVM measurement E, a vast majority of pure and mixed states will have negligible self-information. Thus for uniform distributions Λ and μ over pure and mixed states[45],

$$\int 2^{\mathbf{I}_{\mathrm{Prob}}(E|\psi\rangle:E|\psi\rangle)} d\Lambda = O(1); \qquad \int 2^{\mathbf{I}_{\mathrm{Prob}}(E\sigma:E\sigma)} d\mu(\sigma) = O(1).$$

This can be seen as a consequence of the vastness of Hilbert spaces as opposed to the limited discriminatory power of quantum measurements. In addition, there could be non-uniform distributions of pure or mixed states that could be of research interest. In quantum decoherence, a quantum state becomes entangled with the environment, losing decoherence. The off diagonal elements of the mixed state become dampened, as the state becomes more like a classical mixture of states. Let p_σ be the idealized classical probability that σ decoheres to, with $p_\sigma(i) = \sigma_{ii}$. The following theorem from [Eps23a] states that for an overwhelming majority of pure or mixed states σ, p_σ is noise, that is, has negligible self-information.

$$\int 2^{\mathbf{I}_{\mathrm{Prob}}\left(p_{|\psi\rangle}:p_{|\psi\rangle}\right)} d\Lambda = O(1); \qquad \int 2^{\mathbf{I}_{\mathrm{Prob}}(p_\sigma:p_\sigma)} d\mu(\sigma) = O(1).$$

However the measurement procress has a surprising consquence, in that it causes an uptake in self information. Let F be a PVM of size 2^{n-c}, of an n qubit space and let Λ_F be the distribution of pure states when F is measured over the uniform distribution Λ over n qubit spaces. Thus Λ_F represents the F-collapsed states from Λ. A theorem from [Eps23a] states

$$2^{n-2c} \overset{*}{<} \int 2^{\mathbf{I}(F|\psi\rangle:F|\psi\rangle)} d\Lambda_F.$$

[3]A POVM measurement E is a collection of positive-semi definite Hermitian matrices $\{E_k\}$ such that $\sum_k E_k = 1$. Given a state σ, E induces a probability over the measurements of the form $E\sigma(k) = \mathrm{Tr}E_k\sigma$.

[4]The mixed state integral is $\int f(\sigma)d\mu(\sigma) = \int_{\Delta_M}\int_{\Lambda_1}\cdots\int_{\Lambda_M} f\left(\sum_{i=1}^M p_i |\psi_i\rangle\langle\psi_i|\right) d\Lambda_1 \dots d\Lambda_M d\eta(p_1,\dots,p_M)$, where η is any distribution over the M-simplex Δ_M.

[5]The proof to these inequalities is in the running for the strangest in AIT, relying on a lower computable combination of *upper* computable tests.

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