

# Max Entropy Theorems

Samuel Epstein

February 18, 2025

**Theorem 1** *Let  $(\mathcal{X}, \mu)$  and  $(\mathcal{Y}, \nu)$  be computable measure spaces. Let  $A : \mathbb{N} \rightarrow X$ ,  $B : \mathbb{N} \rightarrow Y$  be injective functions with  $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$ . For  $s \in \mathbb{N}$ ,  $m < s$ , there exists  $2^{s-m}$  indices  $t < 2^s$  with  $\max\{\mathbf{G}_\mu(A(t)), \mathbf{G}_\nu(B(t))\} < -m + O(\log s)$ .*

**Theorem 2** *Let  $L$  be the Lebesgue measure over  $\mathbb{R}$ ,  $(\mathcal{X}, \mu)$ ,  $(\mathcal{Y}, \nu)$  be non-atomic computable measure spaces with  $U = \log \mu(\mathcal{X}) = \log \nu(\mathcal{Y})$ . Let  $A : [0, 1] \rightarrow \mathcal{X}$  and  $B : [0, 1] \rightarrow \mathcal{Y}$  be continuous. Let  $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$ . There is a constant  $c$  with  $L\{t \in [0, 1] : \max\{\mathbf{G}_\mu(A(t)), \mathbf{G}_\nu(B(t))\} < U - n\} > 2^{-n - \mathbf{K}(n) - c}$ .*

**Theorem 3** *Let  $(\mathcal{X}, \mu)$ ,  $(\mathcal{Y}, \nu)$ , and  $\rho$  be non-atomic computable measure spaces with  $U = \log \mu(\mathcal{X}) = \log \nu(\mathcal{Y})$ . Let  $(\mathcal{Z}, \rho)$  be a non-atomic computable probability space. Let  $A : \mathcal{Z} \rightarrow \mathcal{X}$  and  $B : \mathcal{Z} \rightarrow \mathcal{Y}$  be continuous. Let  $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$ . There is a constant  $c$  with  $\rho\{\alpha : \max\{\mathbf{G}_\mu(A(\alpha)), \mathbf{G}_\nu(B(\alpha))\} < \log U - n\} > 2^{-n - \mathbf{K}(n) - c}$ .*