Max Entropy Theorems

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Theorem 1 *Let $(\mathcal{X}, \mu)$ and $(\mathcal{Y}, \nu)$ be computable measure spaces. Let $A : \mathbb{N} \to X$, $B : \mathbb{N} \to Y$ be injective functions with $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$. For $s \in \mathbb{N}$, $m < s$, there exists 2^{s-m} indices $t < 2^s$ with $\max\{\mathbf{G}_\mu(A(t)), \mathbf{G}_\nu(B(t))\} < -m + O(\log s)$.*

Theorem 2 *Let L be the Lebesgue measure over $\mathbb{R}$, $(\mathcal{X}, \mu)$, $(\mathcal{Y}, \nu)$ be non-atomic computable measure spaces with $U = \log \mu(\mathcal{X}) = \log \nu(\mathcal{Y})$. Let $A : [0, 1] \to \mathcal{X}$ and $B : [0, 1] \to \mathcal{Y}$ be continuous. Let $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$. There is a constant c with $L\{t \in [0, 1] : \max\{\mathbf{G}_\mu(A(t)), \mathbf{G}_\nu(B(t))\} < U - n\} > 2^{-n-\mathbf{K}(n)-c}$.*

Theorem 3 *Let $(\mathcal{X}, \mu)$, $(\mathcal{Y}, \nu)$, and be non-atomic computable measure spaces with $U = \log \mu(\mathcal{X}) = \log \nu(\mathcal{Y})$. Let $(\mathcal{Z}, \rho)$ be a non-atomic computable probability space. Let $A : \mathcal{Z} \to \mathcal{X}$ and $B : \mathcal{Z} \to \mathcal{Y}$ be continuous. Let $\mathbf{I}(\langle A, B \rangle : \mathcal{H}) < \infty$. There is a constant c with $\rho\{\alpha : \max\{\mathbf{G}_\mu(A(\alpha)), \mathbf{G}_\nu(B(\alpha))\} < U - n\} > 2^{-n-\mathbf{K}(n)-c}$.*