

# Quantum Decoherence Mostly Produces White Noise

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## Abstract

An overwhelming majority of quantum (pure and mixed) states, when undertaking decoherence, will result in a classical probability with no algorithmic information. Thus most quantum states decohere into white noise. This can be seen as a consequence of the vastness of Hilbert spaces.

Information non-growth laws say information about a target source cannot be increased with randomized processing. In classical information theory, we have [CT91]

$$I(g(X) : Y) \leq I(X : Y).$$

where  $g$  is a randomized function,  $X$  and  $Y$  are random variables, and  $I$  is the mutual information function. Thus processing a channel at its output will not increase its capacity. Information conservation carries over into the algorithmic domain, with the inequalities [Lev84, Eps22]

$$\mathbf{I}(f(x) : y) <^+ \mathbf{I}(x : y); \quad \mathbf{I}(f(a); \mathcal{H}) <^+ \mathbf{I}(a; \mathcal{H}).$$

The information function is  $\mathbf{I}(x : y) = \mathbf{K}(x) + \mathbf{K}(y) - \mathbf{K}(x, y)$ , where  $\mathbf{K}$  is Kolmogorov complexity. The other term is  $\mathbf{I}(a; \mathcal{H}) = \mathbf{K}(a) - \mathbf{K}(a|\mathcal{H})$ , where  $\mathcal{H} \in \{0, 1\}^\infty$  is the halting sequence. These inequalities ensure target information cannot be obtained by processing. If for example the second inequality was not true, then one can potentially obtain information about  $\mathcal{H}$  with simple functions. Obtaining information about  $\mathcal{H}$  violates the Independence Postulate, (see [Lev13]). Information non growth laws can be extended to signals [Eps23b] which can be modeled as probabilities over  $\mathbb{N}$  or Euclidean space<sup>1</sup>. The “signal strength” of a probability  $p$  over  $\mathbb{N}$  is measured by its self information.

$$\mathbf{I}_{\text{Prob}}(p : p) = \log \sum_{i,j} 2^{\mathbf{I}(i:j)} p(i)p(j).$$

A signal, when undergoing randomized processing  $f$ , will lose its cohesion<sup>2</sup>. Thus any signal going through a classical channel will become less coherent [Eps23b].

$$\mathbf{I}_{\text{Prob}}(f(p) : f(p)) <^+ \mathbf{I}_{\text{Prob}}(p : p).$$

In Euclidean space, probabilities that undergo convolutions with probability kernels will lose self information. For example a signal spike at a random position will spread out when convoluted with the Gaussian function, and lose self information. The above inequalities deal with classical

<sup>1</sup>In [Eps23b] probabilities over  $\{0, 1\}^\infty$  and  $T_0$  second countable topologies were also studied.

<sup>2</sup>A probability  $p$ , when processed by a channel  $f : \{0, 1\}^* \times \{0, 1\}^* \rightarrow \mathbb{R}_{\geq 0}$  is a new probability  $fp(x) = \sum_z f(x|z)p(z)$ .

transformations. One can ask, is whether, quantum information processing can add new surprises to how information signals occur and evolve.

One can start with the prepare-and-measure channel, also known as a Holevo-form channel. Alice starts with a random variable  $X$  that can take values  $\{1, \dots, n\}$  with corresponding probabilities  $\{p_1, \dots, p_n\}$ . Alice prepares a quantum state, corresponding to density matrix  $\rho_X$ , chosen from  $\{\rho_1, \dots, \rho_n\}$  according to  $X$ . Bob performs a measurement on the state  $\rho_X$ , getting a classical outcome, denoted by  $Y$ . Though it uses quantum mechanics, this is a classical channel  $X \rightarrow Y$ . So using the above inequality, cohesion will deteriorate regardless of  $X$ 's probability, with Th

$$\mathbf{I}_{\text{Prob}}(Y : Y) <^+ \mathbf{I}_{\text{Prob}}(X : X).$$

There remains a second option, constructing a signal directly from a mixed state. This involves constructing a mixed state, i.e. density matrix  $\sigma$ , and then performing a POVM measurement<sup>3</sup>  $E$  on the state, inducing the probability  $E\sigma(\cdot)$ . However from [Eps23b], for elementary (even enumerable) probabilities  $E\sigma$ ,

$$\mathbf{I}_{\text{Prob}}(E\sigma : E\sigma) <^+ \mathbf{K}(\sigma, E).$$

Thus for simply defined density matrices and measurements, no signal can appear. So experiments that are simple will result in simple measurements, or white noise. However it could be that a larger number of uncomputable pure or mixed states produce coherent signals. However, Theorems in [Eps23a] say otherwise, in that given a POVM measurement  $E$ , a vast majority of pure and mixed states will have negligible self-information. Thus for uniform distributions  $\Lambda$  and  $\mu$  over pure and mixed states<sup>45</sup>,

$$\int 2^{\mathbf{I}_{\text{Prob}}(E|\psi\rangle : E|\psi\rangle)} d\Lambda = O(1); \quad \int 2^{\mathbf{I}_{\text{Prob}}(E\sigma : E\sigma)} d\mu(\sigma) = O(1).$$

This can be seen as a consequence of the vastness of Hilbert spaces as opposed to the limited discriminatory power of quantum measurements. In addition, there could be non-uniform distributions of pure or mixed states that could be of research interest. In quantum decoherence, a quantum state becomes entangled with the environment, losing decoherence. The off diagonal elements of the mixed state become dampened, as the state becomes more like a classical mixture of states. Let  $p_\sigma$  be the idealized classical probability that  $\sigma$  decoheres to, with  $p_\sigma(i) = \sigma_{ii}$ . The following theorem from [Eps23a] states that for an overwhelming majority of pure or mixed states  $\sigma$ ,  $p_\sigma$  is noise, that is, has negligible self-information.

$$\int 2^{\mathbf{I}_{\text{Prob}}(p_{|\psi\rangle} : p_{|\psi\rangle})} d\Lambda = O(1); \quad \int 2^{\mathbf{I}_{\text{Prob}}(p_\sigma : p_\sigma)} d\mu(\sigma) = O(1).$$

However the measurement process has a surprising consequence, in that the wave function collapse causes a massive uptake in algorithmic signal strength. Let  $F$  be a PVM<sup>6</sup>, of size  $2^{n-c}$ , of an  $n$  qubit space and let  $\Lambda_F$  be the distribution of pure states when  $F$  is measured over the uniform distribution  $\Lambda$ . Thus  $\Lambda_F$  represents the  $F$ -collapsed states from  $\Lambda$ . A theorem from [Eps23a] states

$$n - 2c <^{\log \log} \int 2^{\mathbf{I}_{\text{Prob}}(F|\psi\rangle : F|\psi\rangle)} d\Lambda_F.$$

<sup>3</sup>A POVM measurement  $E$  is a collection of positive-semi definite Hermitian matrices  $\{E_k\}$  such that  $\sum_k E_k = 1$ . Given a state  $\sigma$ ,  $E$  induces a probability over the measurements of the form  $E\sigma(k) = \text{Tr} E_k \sigma$ .

<sup>4</sup>The distribution  $\eta$  over mixed states is  $\mu(\sum_{i=1}^M p_i |\psi_i\rangle \langle \psi_i|) = \eta(p_1, \dots, p_M) \prod_{i=1}^M \Lambda(|\psi_i\rangle)$ , where  $\eta$  is any probability over the  $M$ -simplex.

<sup>5</sup>The proof to these inequalities is in the running for the strangest in AIT, relying on a lower computable combination of *upper* computable tests.

<sup>6</sup>A PVM measurement is a POVM measurement where the measurement operators are projectors.

## References

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