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Introduction

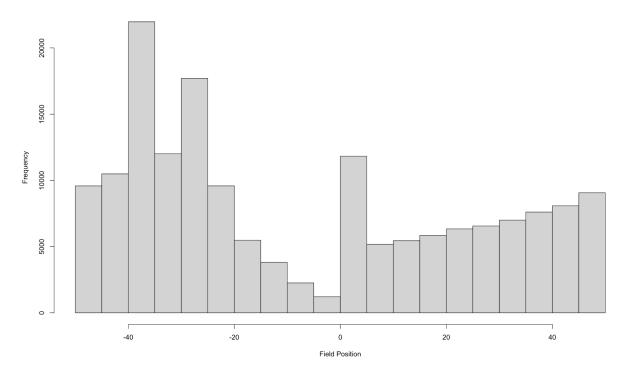
- A. Question of Interest
 - a. In the 1st and 3rd quarters of a college football game, at what point on the field can coaches confidently make the decision to go for it on a 4th and 1 situation?
- B. Why is the question significant?
 - a. In 2021, the Virginia team averaged 78.5 plays per game. Logically, it would seem that teams that run more plays per game are also the teams that score more points since they theoretically have more scoring opportunities per game. Thus, by going for it on 4th down plays and converting, teams naturally increase their plays per game. At the same time, however, going for it on 4th down can be a risky decision as it can give the opposing team great field position and consequently a higher likelihood of scoring on their drive. With our project, we wish to give college coaches, like Virginia's Tony Elliot, a sort of cost-benefit analysis on going for it on 4th and 1 plays so they can make informed decisions on whether the benefits of extending a drive outweighs the cost of turning the ball over on downs.

Methods

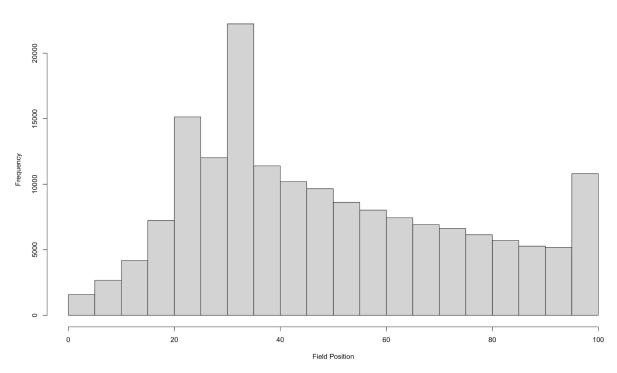
- A. Modifications to our simulation
 - a. Initial Modifications
 - i. Modifications to Field Position Variable
 - 1. To effectively model 4th and 1 situations in a college football game, we first needed to modify how the information of a team's field position was recorded in the data set. Prior to modification, the field position variable was recorded from the perspective of the team that had possession first where the support for the variable ranged from -50 (representing the team that had possession first's own goalline) to the 50 (representing the opposing team's goal line). As a result we seeked to modify the field position variable so that the field position of either team was based on the same number line. Thus we transformed this variable, so 0 represented situations when either team was at their own goal line and 100 represented situations when either team was at their opponent's goal line.

From this point on, field position is discussed from this perspective, where a value X on a 0-100 number line indicates the distance a team has already traveled in order to score a touchdown

Distribution of Original Field Position Variable



Distribution of Transformed Field Position Variable



ii. Modifications to Yards Gained

- 1. We first modeled yards gained on offense using an extremely conservative approach, where the vards gained on 1st, 2nd, and 3rd down were normally distributed with a mean of 3 and a standard deviation of 2 at all field position points on the field. After carrying out some much needed data exploration, however, we very soon modified our approach to modeling yards gained on offense by incorporating mixture modeling theory. This theory suggests that one can model the probabilistic outcome of a certain variable by modeling the probabilistic outcomes of that variable within certain subpopulations that make up an overall population. In our case, that variable is yards gained on offense and the different subpopulations are the various groups of plays that make up all possible play types for a team on offense. Yards gained on offense is a continuous variable; however it has a different probability distribution depending upon the specific play that was carried out. In our model, these plays are either a running play, a passing play, or a sack. Thus we use a discrete distribution to model which play will occur on each down on offense and then we model yards gained on that down based on the specific probability distributions of each of these 3 plays.
- b. Since our model seeks to answer a question relating to 4th down situations, the main modifications to our model dealt with incorporating the decision to punt vs kick a field goal on 4th down and the outcomes of each decision. Our model assumes that the decision to punt vs kick a field goal is binary, where teams will always punt on 4th down when they are at or before 60 yard line and teams will always attempt to kick a field goal when they are past the 60 yard line.
 - i. Modifications to Punting
 - 1. The decision to punt a football on 4th down in our model has three distinct outcomes: the opposing team blocks the punt, the punt goes into the endzone and there is a touchback, or the opposing team returns the punt. Since the distribution of these outcomes is not uniformly distributed across the given field range, our model first conditions on a team's field position. Said team's field position is grouped into four bins: 1-15 yard line, 16-30 yard line, 31-45 yard line, 46--60 yard line. At each distinct field bin, there is a unique discrete distribution of # of blocked punts, # of touchbacks, and # of returns.

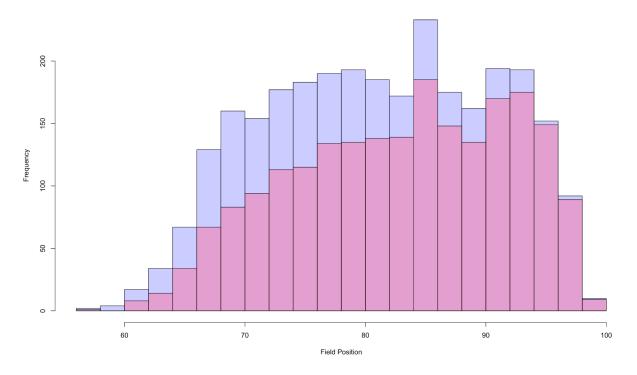
- a. If a punt is blocked, then there is a turnover on downs and the opposing team takes possession at the yard line where the team that had possession previously punted.
- b. If there is a touchback, then the opposing team takes possession at their own 25 yard line.
- c. If there is a return, then the yardline at which the opposing team takes possession is modeled based on a continuous distribution conditioned on the punting team's field position. These distributions are ...
 - i. Normal if punt is kicked from the 0-15 yard line
 - ii. Normal if punt is kicked from the 16-30 yard line
 - iii. Normal if punt is kicked from the 31-45 yard line
 - iv. Weibull if punt is kicked from the 46-60 yard line

ii. Modifications to Field Goals

1. The decision to kick a field goal in our model has a distinct binary outcome: either the team with possession converts the field goal and they earn 3 points, or the team with possession misses the field goal and they earn 0 points. Because this outcome is binary, we chose to model whether a team converts or misses a field goal using logistic regression, with a team's field position as our unique regressor variable. As expected, our model suggests that as a team's field position moves closer and closer into the opponent's territory, the probability of converting a field goal approaches 1.

B. Why are these modifications correct?

a. These modifications are correct because they follow a basic assumption in football that a team's field position is the variable that most explains a team's decision-making and the ensuing outcomes of these decisions. With punting, for example, a team's likelihood to punt on 4th down generally increases as they move closer to their own end zone. On the other hand, with field goals, a team's likelihood to kick a field goal on 4th down generally increases as they move closer to their opponent's end zone. That being said, the decision to restrict punting and kicking a field goal to binary sections of the field does lead to our model being less flexible for situations when team's are between the 50 yard line and the 60 yard line. At the same time, according to the distribution of field goals attempted from the 2019 college football season, very few kicks are attempted when teams have not yet reached the 60 yard line. Moreover, when attempting a field goal on 4th down, the probability of missing said field goal begins to decrease around the 70 yard line. Below is a histogram that displays this information.



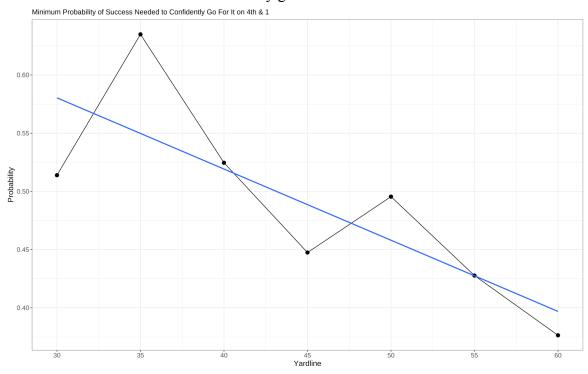
Results

A. Report Findings

a. Summary

i. After simulating the three different outcomes of 4th and 1 plays in our model 10,000 times for each 5 yard increment between the 30 and 60 yard lines (for a total of 21 simulations with 10,000 iterations each) we found that the minimum probability of success needed to confidently go for it on 4th and 1 substantially decreases overall between the 30 and 60 yard lines. There is a slight increase in the minimum probability needed at the 35 yard line which could be due to some sort of externality or inconsistency in the data; however, using the information we have on hand, we are confident in our results as we have run through a sufficient amount of simulations. Our findings show that as the team with possession travels into their opponent's territory or past the 50 yard line, the minimum probability of success suggested in order to confidently go for it on 4th and 1 decreases. Thus, teams should feel more comfortable making the decision to go for it on a 4th and 1 play as they get closer to their opponent's goal line or the 100 yard line. This makes sense, as the closer a team is to their opponent's goal line the higher likelihood they have to score and the less they would risk by giving it up on a turnover on downs. Below is a visualization that plots this change in the minimum probability

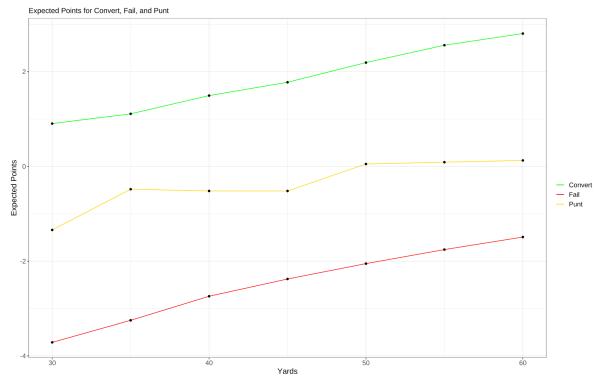
for success suggested in order to go for it on 4th and 1. The scatter plot also includes a trendline, displaying a decreasing linear relationship between a teams field position and the minimum probability suggested in order to confidently go for it on a 4th and 1.



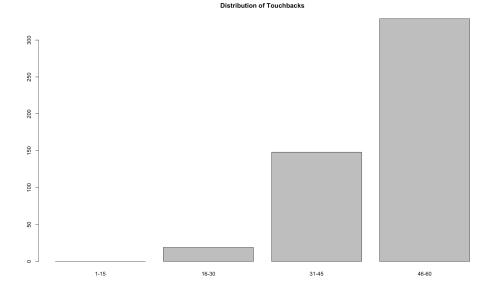
- b. Investigating the spike in probability at the 35 yard line
 - i. Below is a visualization that details our model's expected points for a team with a 4th and 1 situation based on whether they go for it and convert a 1st down, punt the ball, or go for it and fail to convert a first down between the 30 and 60 yard lines. As seen, the expected points for a team when they convert on 4th and 1 increases almost linearly as field position increases. In the same way, the expected points for a team when they fail to convert on 4th and 1 also increases almost linearly as field position increases. Yet while these two relationships are almost parallel with a similar slope, the expected points for a team when they punt on 4th down displays an overall increasing trend as field position increases but this relationship is not consistent throughout the support. Instead, we notice an abrupt spike in the expected points of a team punting on 4th and 1 at the 35 and 50 yard lines. This phenomenon is consistent with the abrupt spikes seen in the above plot.

**We model expected points from the perspective of the team that has possession first.

Consequently, an increase in expected points for the team that has possession translates to a decrease in expected points for the team that has possession second and vice versa**



ii. Thus, we then pose the question of why abrupt spikes are seen in the expected points of a team punting the football on 4th and 1 at the 35 and 50 yard lines. To investigate this secondary question, we turn to the distribution of touchbacks in our model, or the probability that a punt return is downed in/kicked out of the endzone and the ball is brought to the 25 yard line for the start of the opposing team's drive. As mentioned in the modifications section, to model the outcome of punting, we chose to group the field position variable into four bins: 1-15 yard line, 16-30 yard line, 31-45 yard line, 46-60 yard line. Then with each unique bin we modeled a discrete distribution of the likelihood that a punt is returned, blocked, or touchbacked. As seen in the visualization below of the distribution of touchbacks at these bins, there are large increases in the number of touchbacks between the 16-30 yard line bin and the 31-45 yard line bin as well as between the 31-45 yard line bin and the 46-60 yard line bin. Since our final question observed expected points at 5 yard increments from the 30-60 yard line, we see that the expected points from a punt on a 4th and 1 at the 30 yard line draws from a different distribution of touchbacks than the expected points of a punt on 4th and 1 at the 35 yard line. The same can be said about the expected points from 4th and 1 situations at the 50 and 55 yard lines. In both cases, the likelihood of a touchback occurring in the model differs dramatically depending on whether a team has 5 more yards or 5 less yards.



iii. In conclusion, we posit that the spikes in expected points at the 35 and 50 yard lines result from our method of binning the field position variable for punting situations. More specifically, the spikes may be attributed to the variance in the number of touchbacks between these 4 bins. Logically, a touchback becomes less and less valuable to the team punting the football as they move into their opponent's territory past the 50 yard line because at that point a punter is generally capable of landing his kick between his opponent's goal line and 25 yard line. In other words, the value of a touchback to the team punting the football depends upon what field position they are punting the football from. At the 30 and 35 yard line, for example, a punt for a touchback would be rather valuable to the team punting the ball because they would flip the field position by 40-45 yards. That being said, according to the distribution of touchbacks in our model, there is a significant difference between the likelihood of kicking a touchback depending on whether the punting team is at the 30 or 35 yard line. In reality this is not the case, but because our model implements this binning strategy, with just 5 more yards of field position, teams are sometimes expected to gain a significant amount of value.

Discussion

- A. Recap Question of Interest
 - a. In the 1st and 3rd quarters of a college football game, at what point on the field can coaches confidently make the decision to go for it on a 4th and 1 situation?
- B. Why are our findings interesting?
 - a. Our findings are interesting because they leave a substantial amount of room for interpretation up to the head coach. To make use of our findings, a head coach

would have to judge for him or herself whether a certain probability of converting on 4th and 1 at X field position is a high threshold or a low threshold. If they deem this probability to be a high threshold, then they may not go for it on 4th and 1, but if they deem this probability to be a low threshold, then they may. These thresholds, however, are completely subjective because each head coach will have a different evaluation of his team's ability to convert a 1st down on a 4th and 1 play. If Tony Elliot, for example, knew that his offense was exceptional at QB sneaks then perhaps his threshold for going for it on 4th and 1 would be fairly high. In the context of our model, Coach Elliot may deem that if the minimum probability suggested for going for it on a 1st down is almost 0.5 at the 50 yard line, then his offense should go for it in these situations because he evaluates them as having a higher than 50% chance of converting a 4th and 1. At the same time, another coach in the ACC may not highly rate his offense's ability to execute the QB sneak and therefore this 0.5 threshold for him may be too high because he believes his offense has a less than 50% chance of converting a 4th and 1. In summary, although our findings are quantitative, the ways in which coaches will make use of our findings will be based on qualitative observations.

C. Limitations of study

a. The main limitation to this study is its practicality in the sphere of college football. In a perfect world, for a coach to get the most use out of our findings, they would want to have an exact idea of his or her team's probability to convert a 4th and 1 play. With this probability, a coach would be able to decide at the 30, 35, 40, 45, 50, 55, and 60 yard lines of whether or not his team's actual probability of converting meets this minimum probability suggested. That being said, arriving at this exact idea of a team's probability to convert a 4th and 1 play is easier said than done. First, a coach would have to decide if he or she is arriving at this exact probability from outcomes in practice or outcomes in games. Each of these, however, comes with its own inherent flaws. If a coach were to use practice outcomes to simulate his or her team's relative success on 4th and 1 plays, then they would be drawing conclusions from a likely biased sample that included no in-game pressure and potentially a sub-standard defense. If a coach were to use game outcomes to simulate his or her team's relative success on 4th and 1 plays, then they would likely not generate a large enough sample to responsibly represent the team's ability in that specific year.