Algorithms and Data Structures Coursework

5. For each of the following recurrences, give an expression for the runtime $T(n)$ if the recurrence can be solved with the Master Theorem. Otherwise, state why the Master theorem cannot be applied. You should justify your answers.

   (a) $T(n) = 9T(n/3) + n^2$ [3]

   My Solution:

   $$T(n) = 9T\left(\frac{n}{3}\right) + n^2$$

   $a = 9, b = 3, f(n) = n^2, \log_b a = 2$

   $$f(n) = \Theta(n^2)$$
   $$T(n) = \Theta(n^2 \log n)$$

   (b) $T(n) = 4T(n/2) + 100n$ [3]

   My Solution:

   $$T(n) = 4T\left(\frac{n}{2}\right) + 100n$$

   $a = 4, b = 2, f(n) = 100n, \log_b a = 2$

   $$f(n) = O(n^{2-1})$$
   $$T(n) = \Theta(n^2)$$

   (c) $T(n) = 2^n T(n/2) + n^3$ [3]

   My Solution:
   As a is not a number this cannot be solved using master theorem

   (d) $T(n) = 3T(n/3) + c \cdot n$ [3]

   My Solution:
   Under the assumption that c is a constant, otherwise this cannot be solved using master theorem

   $$T(n) = 3T\left(\frac{n}{3}\right) + c \cdot n$$

   $a = 3, b = 3, f(n) = c \cdot n, \log_b a = 1$

   $$f(n) = \Theta(n^1)$$
   $$T(n) = \Theta(n \log n)$$

   (e) $T(n) = 0.99T(n/7) + 1/(n^2)$ [3]

   My Solution:

   $$T(n) = 0.99T\left(\frac{n}{7}\right) + \frac{1}{n^2}$$

   $a < 1$ so Master theorem cannot be performed.