Algorithmic Game Theory Summative Assignment – Individual Component

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Exercise 1. Consider the following instance of the load balancing game where the number of tasks is equal to the number of machines, and in particular we have:

- m identical machines $M_1, M_2, \ldots, M_m$ (all of speed 1),
- m identical tasks $w_1 = w_2 = \cdots = w_m = 1$.

Consider also the mixed strategy profile A where each of the tasks is assigned to all machines equiprobably (i.e. with probability $1/m$).

(a) Calculate the ratio $cost(A)/cost(OPT)$ in the special case where $m = 2$. **[3 marks]**

- There are $2^2 = 4$ possible assignments of 2 tasks to 2 machines
- In two of these both tasks are assigned to 1 machine (time 2)
- In the other two one task is assigned to each machine (time 1)
- The Cost of A is therefore $1/4(2 + 2 + 1) = 1.5$
- However the optimal cost is where one is assigned to each machine 1
- The ratio is therefore 1.5

(b) Calculate the ratio $cost(A)/cost(OPT)$ in the special case where $m = 3$. **[3 marks]**

(c) Discuss what this ratio is for arbitrary m. What does this imply about the Price of Anarchy on identical machines for mixed Nash equilibria? **[5 marks]**

Exercise 2. We consider a second-price sealed-bid auction where there are n bidders who bid as follows:

- Bidders 1 up to $n - 1$ bid either 1 dollar or $r > 1$ dollars equiprobably and independently of the rest.
- Bidder n bids h dollars, where $h > r$.

The seller's expected revenue R is the expectation of the second highest value.

(a) What is the value that R is approaching when n is very large? **[1 marks]**

(b) Justify your answer by taking the limit. **[9 marks]**

Exercise 3. Mary and Alice are buying items for Sunday lunch. Mary buys either chicken (C) or beef (B) for the main course and Alice buys either juice (J) or wine (W). Both people prefer wine with beef and juice with chicken. The opposite alternatives are equally displeasing. However, Mary prefers beef over chicken, while Alice prefers chicken over beef.

We assume that Mary buys first and then tells Alice what she bought, so when Alice makes her decision, she knows if the main course is beef or chicken.

(a) Express the above preferences as payoffs by using numbers (e.g. $u_M(B, W) = 2$, $u_A(B, W) = \ldots$ etc.) **[2 marks]**

(b) Write down a bimatrix game with Mary as the row player and Alice as the column player, using your chosen payoffs. **[4 marks]**

(c) Write down a game tree representing this game as an extended game. **[4 marks]**

(d) Find a solution for the extended game using backward induction. Describe your steps. **[5 marks]**

Exercise 4. We consider a (matching) market of k sellers and k buyers, where k is an integer, $k > 0$. Each seller sells an item and the prices of the items are initially all zero. Buyer i has valuation $k - i + 1$ for the first item and valuation 0 for every other item, as shown in the following diagram.

Buyers	Valuations (for items 1 to k)			
x_1	k,	0,	$\ldots$,	0
x_2	$k-1$,	0,	$\ldots$,	0
$\vdots$			$\vdots$	
x_k	1,	0,	$\ldots$,	0

The sellers find the market-clearing prices using the procedure discussed in the lectures.

(a) What are the prices of the sellers' items (1^{st} item, 2^{nd} item, $\ldots$, k^{th} item) when the market clears? Which buyer gets the 1^{st} item and at what price? **[3 marks]**

(b) Justify your answers to (a). **[6 marks]**

(c) Which kind of auction does the construction of market-clearing prices procedure implement in this case? **[3 marks]**