Algorithmic Game Theory Summative Assignment – Individual Component

1. Consider the following instance of the load balancing game where the number of tasks is equal to the number of machines, and in particular we have:

   - m identical machines $M_1, M_2, \ldots, M_m$ (all of speed 1),
   - m identical tasks $w_1 = w_2 = \cdots = w_m = 1$.

   Consider also the mixed strategy profile A where each of the tasks is assigned to all machines equiprobably (i.e. with probability $1/m$).

   (a) Calculate the ratio $cost(A)/cost(OPT)$ in the special case where $m = 2$. [3]

   Answer:

   - There are $2^2 = 4$ possible assignments of 2 tasks to 2 machines
   - In two of these both tasks are assigned to 1 machine (time 2)
   - In the other two one task is assigned to each machine (time 1)
   - The Cost of A is therefore $1/4(2 + 2 + 1) = 1.5$
   - However the optimal cost is where one is assigned to each machine 1
   - The ratio is therefore 1.5

   (b) Calculate the ratio $cost(A)/cost(OPT)$ in the special case where $m = 3$. [3]

   Answer:

   (c) Discuss what this ratio is for arbitrary m. What does this imply about the Price of Anarchy on identical machines for mixed Nash equilibria? [5]

   Answer:

Total for Question 1: 11